

Vibration Simulation Using

**MATLAB**  
and **ANSYS**

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M I C H A E L   R .   H A T C H

CHAPMAN & HALL/CRC

Boca Raton London New York Washington, D.C.

## Library of Congress Cataloging-in-Publication Data

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Hatch, Michael R.

Vibration simulation using MATLAB and ANSYS / Michael R. Hatch.

p. cm.

Includes bibliographical references and index.

ISBN 1-58488-205-0 (alk. paper)

1. Vibration--Computer simulation. 2. MATLAB. 3. ANSYS (Computer system) I.  
Title.

TJ177 .H38 2000

620.3'01'13--dc21

00-055517

CIP

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International Standard Book Number 1-58488-205-0

Library of Congress Card Number 00-055517

Printed in the United States of America 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

# **PREFACE**

## **Background**

This book resulted from using, documenting and teaching various analysis techniques during a 30-year mechanical engineering career in the disk drive industry. Disk drives use high performance servo systems to control actuator position. Both experimental and analytical techniques are used to understand the dynamic characteristics of the systems being controlled. Constant in-depth communications between mechanical and control engineers are required to bring high performance electro-mechanical systems to market. Having mechanical engineers who can discuss dynamic characteristics of mechanical systems with servo engineers is very valuable in bringing these high-performance systems into production. This book should be useful to both the mechanical and control communities in enhancing their communication.

## **Purpose of the Book**

The book has three main purposes. The first purpose is to collect in one document various methods of constructing and representing dynamic mechanical models. For someone learning dynamics for the first time or for an experienced engineer who uses the tools infrequently, the options available for modeling can be daunting: transfer function form, zpk form, state space form, modal form, state space modal form, etc. Seeing all the methods in one book, with background theory, an example problem and accompanying MATLAB ® (MathWorks, Inc., Natick, MA) code listing for each method, will help put them in perspective and make them readily available for quick reference. (Also, having equation listings with their accompanying MATLAB code is a good way to develop or reinforce MATLAB programming skills.)

The second purpose is to help the reader develop a strong understanding of modal analysis, where the total response of a system can be constructed by combinations of the individual modes of vibration.

The third purpose is to show how to take the results of large dynamic finite element models and build small MATLAB state space dynamic mechanical models for use in mechanical or servo/mechanical system models.

## **Audience / Prerequisites**

This book is meant to be used as a reference book in senior and early graduate-level vibration and servo courses as well as for practicing servo and mechanical engineers. It should be especially useful for engineers who have limited experience with state space. It assumes the reader has a background in basic vibration theory and elementary Laplace transforms.

For those with a strong linear systems background, the first 12 chapters will provide little new information. Chapters 13 and 14, the finite element chapters, may prove interesting for those with little familiarity with finite elements. Chapters 15 to 19 cover methods for creating state space MATLAB models from ANSYS finite element results, then reducing the models.

## Programs Used

It is assumed that the reader has access to MATLAB and the Control System Toolbox and is familiar with their basic use. The MATLAB block diagram graphical modeling tool Simulink is used for several examples through the book but is not required. Several excellent texts covering the basics of MATLAB usage can be found on the MathWorks Web page, [www.mathworks.com](http://www.mathworks.com). All the programs were developed using MATLAB Version 5.3.1.

Lumped mass and cantilever examples using the ANSYS® (ANSYS, Inc., Canonsburg, PA) finite element program are used throughout the text. Where ANSYS results are required for input into MATLAB models, they are available by download without having to run the ANSYS code. For those with access to ANSYS, input code is available by download. The last three chapters contain complete ANSYS/MATLAB dynamic analyses of SISO (Single Input Single Output) and MIMO (Multiple Input Multiple Output) disk drive actuator/suspension systems. Revisions 5.5 and 5.6 of ANSYS were used for the examples.

## Organization

The unifying theme throughout most of the book is a **three degree of freedom (tdof)** system, simple enough to be solved for all of its dynamic characteristics in closed form, but complex enough to be able to visualize mode shapes and to have interesting dynamics.

Chapters 1 to 16 contain background theoretical material, closed form solutions to the example problem and MATLAB and/or ANSYS code for solving the problems. All closed form solutions are shown in their entirety.

Chapters 17 to 19 analyze complete disk drive actuator/suspension systems using ANSYS and MATLAB. All chapters list and discuss the related MATLAB code, and all but the last three chapters list the related ANSYS code. All the MATLAB and ANSYS input codes, as well as selected output results, are available for downloading from both the MathWorks FTP site and the author's FTP site, both listed at the end of the preface. Reviewers have provided different inputs on the amount and location of MATLAB and ANSYS code in the book. Engineers for whom the material is new have

requested that the code be broken up, interspersed with the text and explained, section by section. Others for whom MATLAB code is second nature have suggested either removing the code listings altogether or providing them at the end of the chapters or in an appendix. My apologies to the latter, but I have chosen to intersperse code in the associated text for the new user.

A problem set accompanies the early chapters. A two degree of freedom system, very amenable to hand calculations, is used in the problem sets to allow one to follow through the derivations and codes with less work than the three degree of freedom (tdof) system used in the text. Some of the problems involve modifying the supplied tdof MATLAB code to simulate the two degree of freedom problem, allowing one to become familiar with MATLAB coding techniques and usage.

Following an introductory chapter, Chapter 2 starts with transfer function analysis. A systematic method for creating mass and stiffness matrices is introduced. Laplace transforms and the transfer function matrix are then discussed. The characteristic equation, poles and zeros are defined.

Chapter 3 develops an intuitive method of sketching frequency responses by hand, and the significance of the magnitudes and phases of various frequency ranges are discussed. Following a development of the imaginary plane and plotting of poles and zeros for the various transfer functions, the relationship between the transfer function and poles and zeros is discussed. Finally, mode shapes are defined, calculated and plotted.

Chapter 4 discusses the origin and interpretation of zeros in Single Input and Single Output (SISO) mechanical systems. Various transfer functions are taken for a lumped parameter system to show the origin of the zeros and how they vary depending on where the force is applied and where the output is taken. An ANSYS finite element model of a tip-loaded cantilever is analyzed and the results are converted into a MATLAB modal state space model to show an overlay of the poles of the “constrained” system and their relationship with the zeros of the original model.

Chapter 5, the state space chapter, takes the basic tdof model and uses it to develop the concept of state space representation of equations of motion. A detailed discussion of complex modes of vibration is then presented, including the use of Argand diagrams and individual mode transient responses.

Chapter 6 uses the state space formulation of Chapter 5 to solve for frequency responses and time domain responses. The matrix exponential is introduced both as an inverse Laplace transform and as a power series solution for a single degree of freedom (sdof) mass system. The tdof transient problem is

solved using both the MATLAB function `ode45` and a MATLAB Simulink model.

Chapter 7, the modal analysis chapter, begins with a definition of principal modes of vibration, then develops the eigenvalue problem. The relationship between the determinant of the coefficient matrix and the characteristic equation is shown. Eigenvectors are calculated and interpreted, and the modal matrix is defined. Next, the relationship between physical and principal coordinate systems is developed and the concept of diagonalizing or uncoupling the equations of motion is shown. Several methods of normalization are developed and compared. The transformation of initial conditions and forces from physical to principal coordinates is developed. Once the solution in principal coordinates is available, the back transformation to physical coordinates is shown. The chapter then goes on to develop various types of damping typically used in simulation and discusses damping requirements for the existence of principal modes. A two degree of freedom model is used to illustrate the form of the damping matrix when proportional damping is assumed, showing that the answer is not intuitive.

In Chapters 8 and 9 the *tdof* model is solved for both frequency responses and transient responses in closed form and using MATLAB. A description of how individual modes combine to create the overall frequency response is provided, one of several discussions throughout the book which will help to develop a strong mental image of the basics of the modal analysis method.

Chapter 10, the state space modal analysis chapter, shows how to solve the normal mode eigenvalue problem in state space form, discussing the interpretation of the resulting eigenvectors. Equations of motion are developed in the principal coordinates system and again, individual mode contributions to the overall frequency response are discussed. Real modes are discussed in the same context as for complex modes, using Argand diagrams and individual mode transient responses to illustrate.

Chapter 11 continues the modal state space form by solving for the frequency response. Chapter 12 covers time domain response in modal state space form using the MATLAB “`ode45`” command and “function” files.

Chapters 13 and 14 discuss the basics of static and dynamic analysis using finite elements, the generation of global stiffness and mass matrices from element matrices, mass matrix forms, static condensation and Guyan Reduction. The purpose of the finite element chapters is to familiarize the reader with basic analysis methods used in finite elements. This familiarity should allow a better understanding of how to interpret the results of the models without necessarily becoming a finite element practitioner. A cantilever beam is used as an example in both chapters. In Chapter 14 a

complete eigenvalue analysis with Guyan Reduction is carried out by hand for a two-element beam. Then, MATLAB and ANSYS are used to solve the eigenvalue problem with arbitrary cantilever models.

Chapters 15 and 16 use eigenvalue results from ANSYS beam models to develop state space MATLAB models for frequency and time domain analyses. Both chapters discuss simple methods for reducing the size of ANSYS finite element results to generate small, efficient MATLAB state space models which can be used to describe the dynamic mechanical portion of a servo-mechanical model.

Chapter 17 uses an ANSYS model of a single stage SISO disk drive actuator/suspension system to illustrate using dc or peak gains of individual modes to rank modes for elimination when creating a low order state space MATLAB model.

Chapter 18 introduces balanced reduction, another method of ranking modes for elimination, and uses it to produce a reduced model of the SISO disk drive actuator/suspension model from Chapter 17.

In Chapter 19 a complete ANSYS/MATLAB analysis of a two stage MIMO actuator/suspension system is carried out, with balanced reduction used to create a low order model.

Appendix 1 lists the names of all the MATLAB and ANSYS codes used in the book, separated by chapter. It also contains instruction for downloading the MATLAB and ANSYS files from the MathWorks FTP site as well as the author's Web site, [www.hatchcon.com](http://www.hatchcon.com).

Appendix 2 contains a short introduction to Laplace transforms.

For MATLAB product information, contact:

The MathWorks, Inc.  
3 Apple Hill Drive  
Natick, MA, 01760-2098 U.S.A.

Tel: 508-647-7000

Fax: 508-647-7101

E-mail: [info@mathworks.com](mailto:info@mathworks.com)

Web: [www.mathworks.com](http://www.mathworks.com)

For ANSYS product information, contact:

ANSYS, Inc.  
Southpointe  
275 Technology Drive  
Canonsburg, PA 15317

Tel: 724-746-3304

Fax: 724-514-9494

Web: [www.ansys.com](http://www.ansys.com)

## **Acknowledgments**

There are many people whom I would like to thank for their assistance in the creation of this book, some of whom contributed directly and some of whom contributed indirectly.

First, I would like to acknowledge the influence of the late William Weaver, Jr., Professor Emeritus, Civil Engineering Department, Stanford University. I first learned finite elements and modal analysis when taking Professor Weaver's courses in the early 1970s and his teachings have stood me in good stead for the last 30 years.

Dr. Haithum Hindi kindly allowed the use of a portion of his unpublished notes for the Laplace transform presentation in Appendix 2 and provided valuable feedback on the nuances of "modred" and balanced reduction.

I would like to thank my reviewers for their thorough and time-consuming reviews of the document: Stephen Birn, Marianne Crowder, Dr. Y.C. Fu, Dr. Haithum Hindi, Dr. Michael Lu, Dr. Babu Rahman, Kathryn Tao and Yimin Niu. Mark Rodamaker, an ANSYS distributor, kindly reviewed the book from an ANSYS perspective. My daughter-in-law, Stephanie Hatch, provided valuable editing input throughout the book.

I would also like to thank Dr. Wodek Gawronski for his words of encouragement and his helpful suggestions to a new author. Dr. Gawronski's two advanced texts on the subject are highly recommended for those wishing additional information (see References).

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