Gas Dynamics

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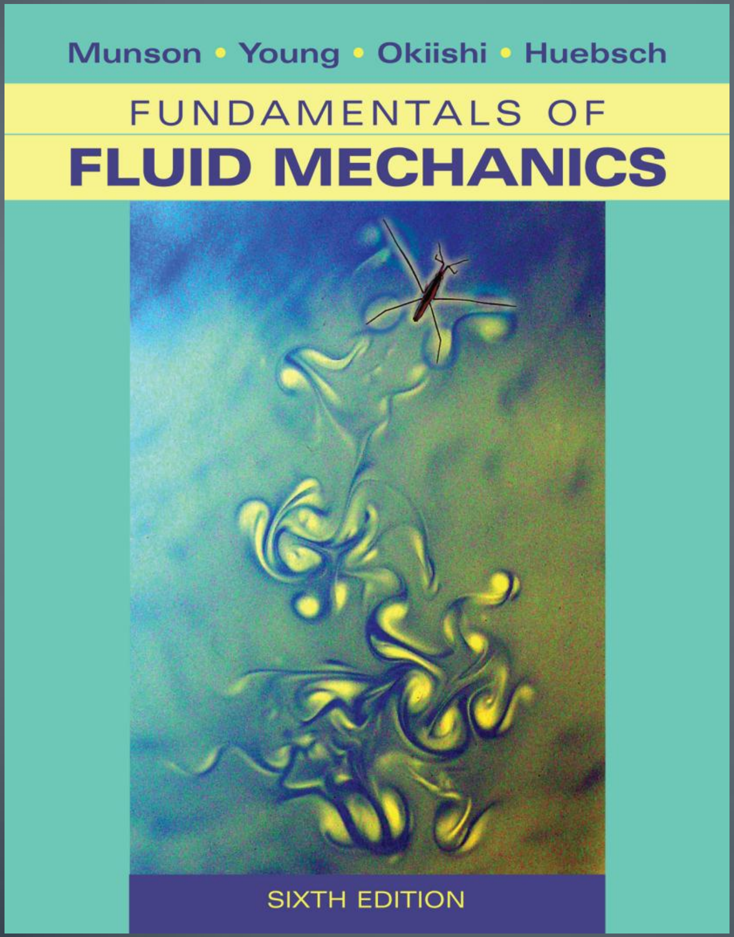
Textbooks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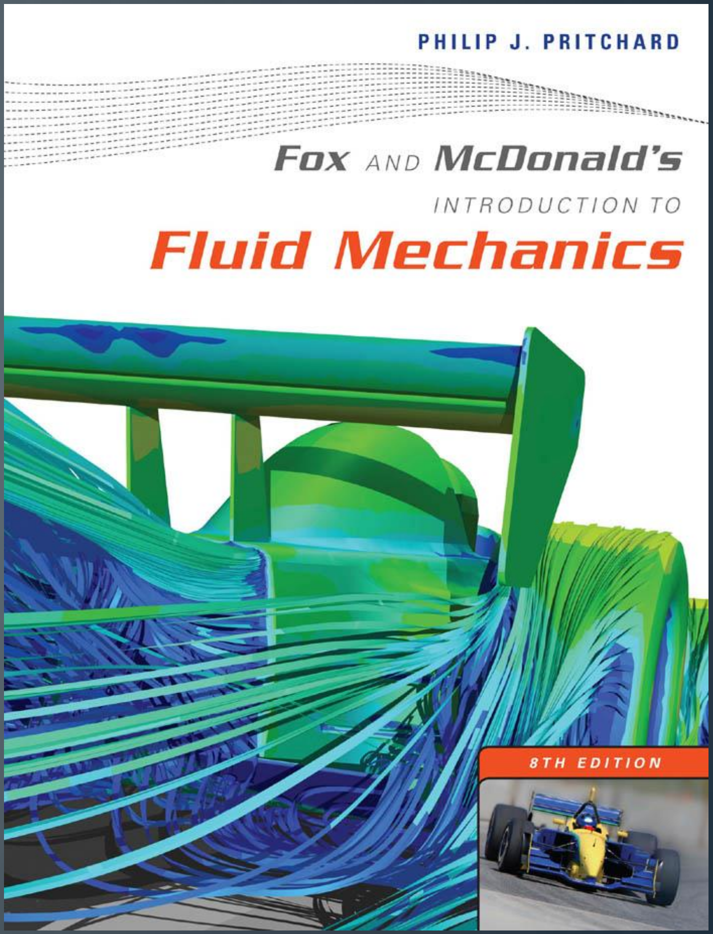
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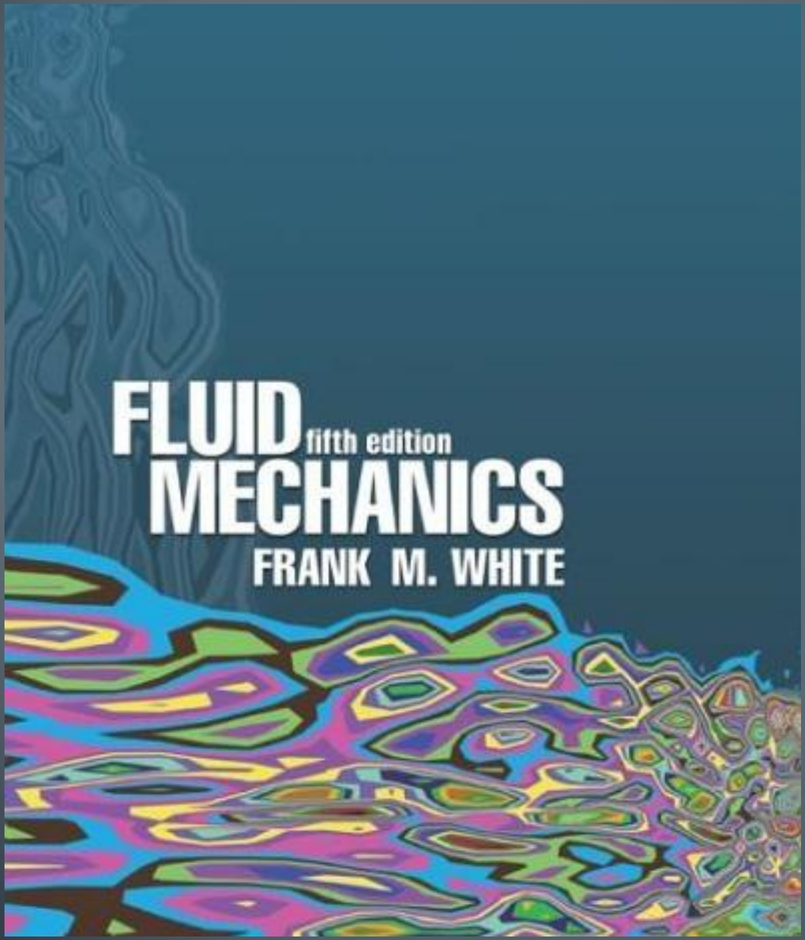
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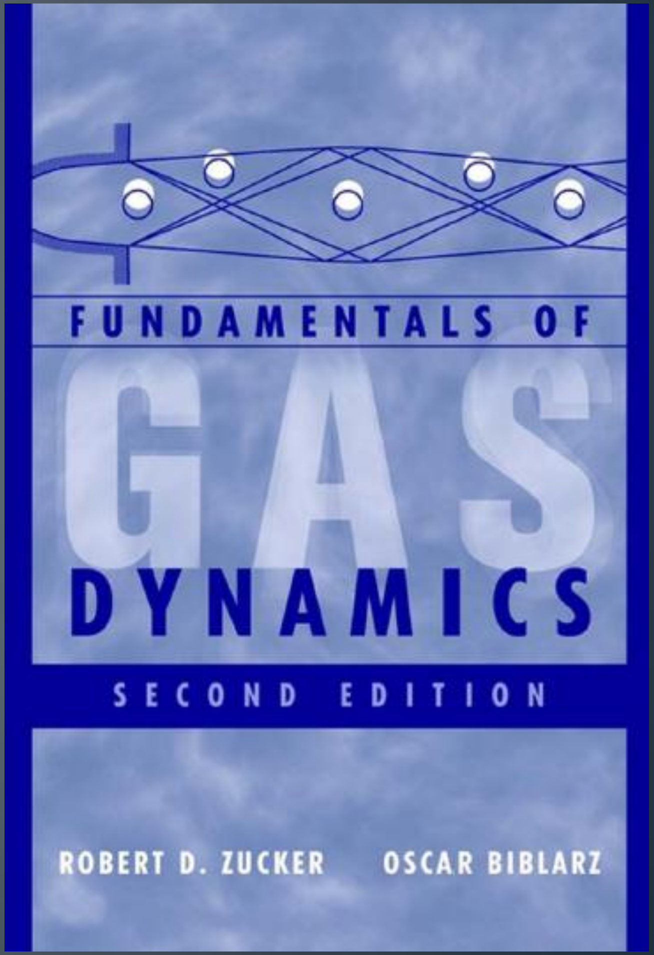
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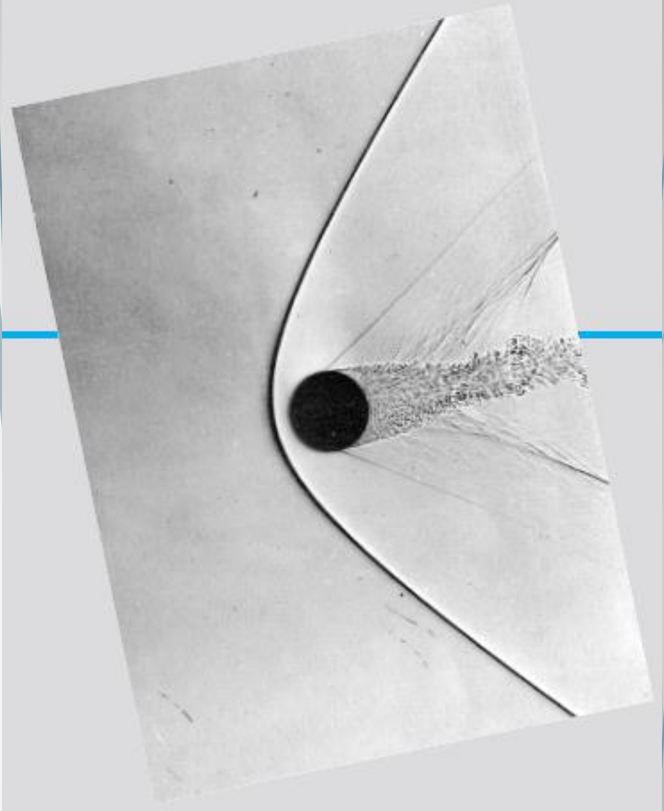


Compressible Flow

Ideal Gas Relationships

$$p = \rho R T$$

$$\check{u}_2 - \check{u}_1 = c_v(T_2 - T_1)$$

$$\check{h}_2 - \check{h}_1 = c_p(T_2 - T_1)$$

$$c_p - c_v = R$$

$$c_p = \frac{Rk}{k - 1}$$

$$c_v = \frac{R}{k - 1}$$

Ideal Gas Relationships

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{\rho_1}{\rho_2}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

- For *isentropic flow*

$$\frac{p}{\rho^k} = \text{constant}$$

$$\left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{\rho_2}{\rho_1}\right)^k = \left(\frac{p_2}{p_1}\right)$$

Mach Number and Speed of Sound

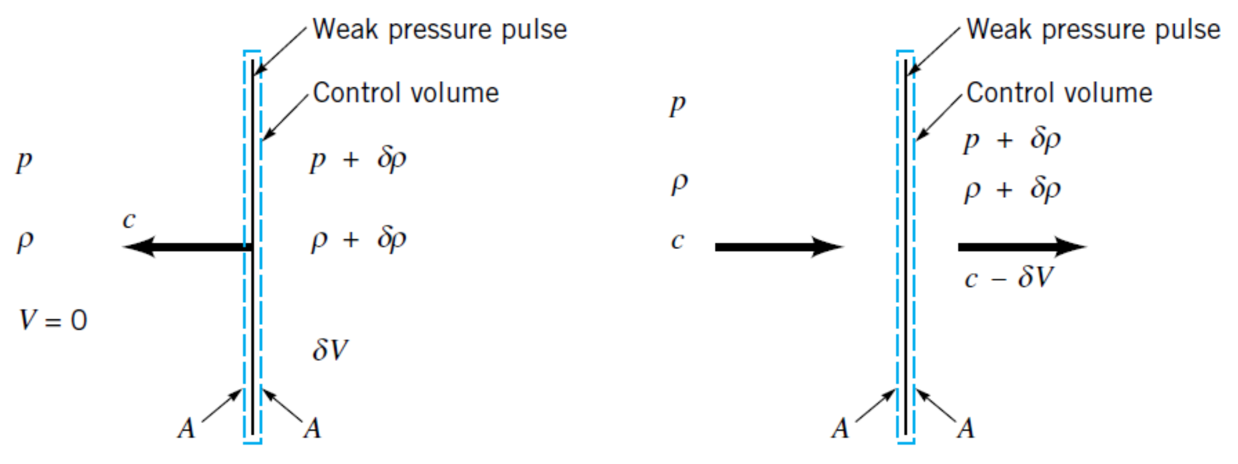
- The Mach number is defined as the ratio of the value of the local flow velocity, V, to the local speed of sound, c. In other words,

$$\mathrm{Ma} = \frac{V}{c}$$

- What we perceive as sound generally consists of weak pressure pulses that move through air. When our ear drums respond to a succession of moving pressure pulses, we hear sounds.

- To better understand the notion of speed of sound, we analyze the one-dimensional fluid mechanics of an infinitesimally thin, weak pressure pulse moving at the speed of sound through a fluid at rest.

Mach Number and Speed of Sound



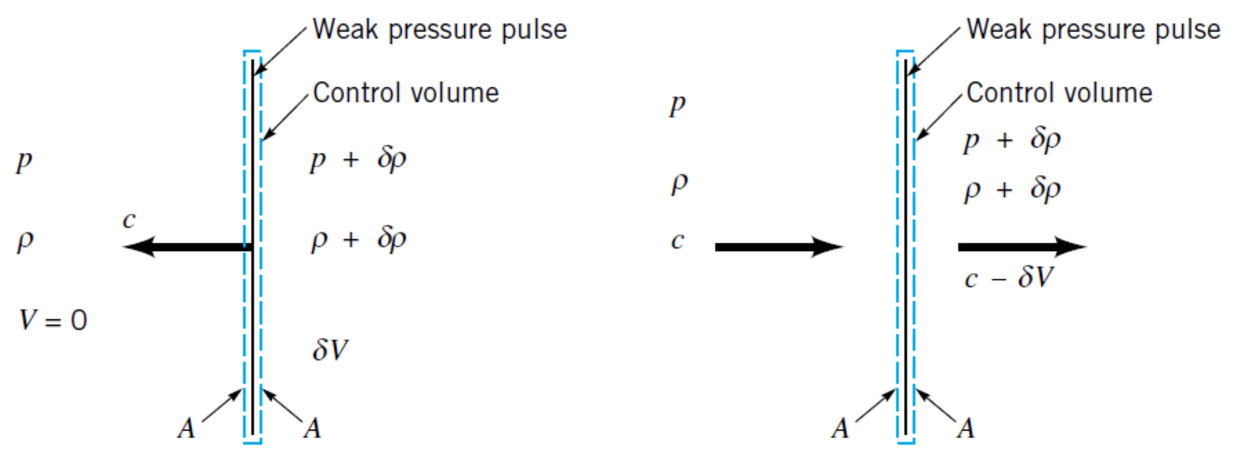
- When the continuity equation is applied to the flow through this control volume, the result is

$$\rho A c = (\rho + \delta\rho) A (c - \delta V)$$

$$\rho c = \rho c - \rho\, \delta V + c\, \delta\rho - (\delta\rho)(\delta V)$$

$$\rho\, \delta V = c\, \delta\rho$$

Mach Number and Speed of Sound



- The linear momentum equation can also be applied to the flow through the control volume. The result is

$$-c\rho cA + (c - \delta V)(\rho + \delta\rho)(c - \delta V)A = pA - (p + \delta p)A$$

$$-c\rho cA + (c - \delta V)\rho Ac = -\delta pA$$

$$\rho\delta V = \frac{\delta p}{c}$$

Mach Number and Speed of Sound

- From continuity and linear momentum we obtain

$$c^2 = \frac{\delta p}{\delta \rho}$$

$$c = \sqrt{\frac{\delta p}{\delta \rho}}$$

- If the energy equation is used for the flow through this control volume, the result is

$$\frac{\delta p}{\rho} + \delta\left(\frac{V^2}{2}\right) + g\,\delta z = \delta(\text{loss})$$

$$\frac{\delta p}{\rho} + \frac{(c - \delta V)^2}{2} - \frac{c^2}{2} = 0$$

Mach Number and Speed of Sound

$$\rho \, \delta V = \frac{\delta p}{c}$$

- By combining continuity and energy we again find that

$$c = \sqrt{\frac{\delta p}{\delta \rho}}$$

- Thus, the conservation of linear momentum and the conservation of energy principles lead to the same result. If we further assume that the frictionless flow through the control volume is adiabatic (no heat transfer), then the flow is isentropic.

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$$

Mach Number and Speed of Sound

- For the isentropic flow of an ideal gas, we have

$$p = (\text{constant})(\rho^k)$$

$$\left(\frac{\partial p}{\partial \rho}\right)_s = (\text{constant})\, k\rho^{k-1} = \frac{p}{\rho^k}\, k\rho^{k-1} = \frac{p}{\rho}\, k = RTk$$

$$c = \sqrt{RTk}$$

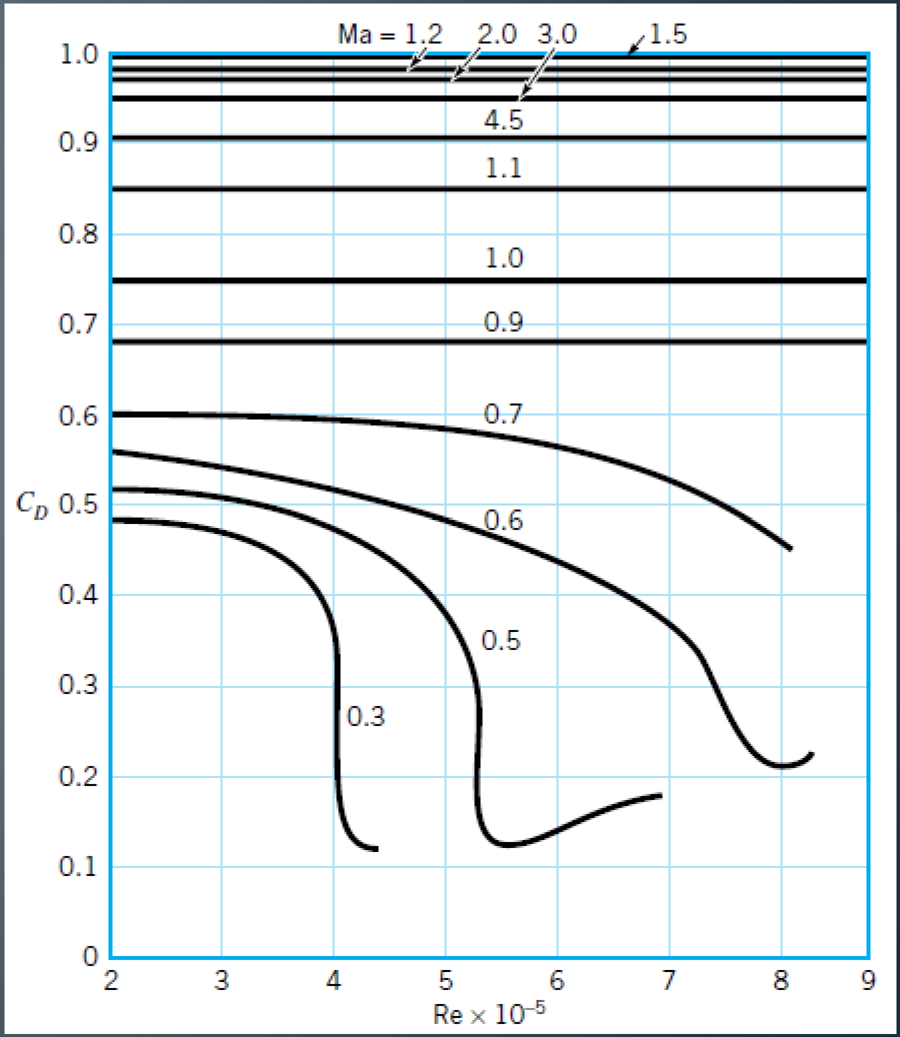
- More generally, the bulk modulus of elasticity, of any fluid including liquids is defined as

$$E_v = \frac{dp}{d\rho/\rho} = \rho\left(\frac{\partial p}{\partial \rho}\right)_s$$

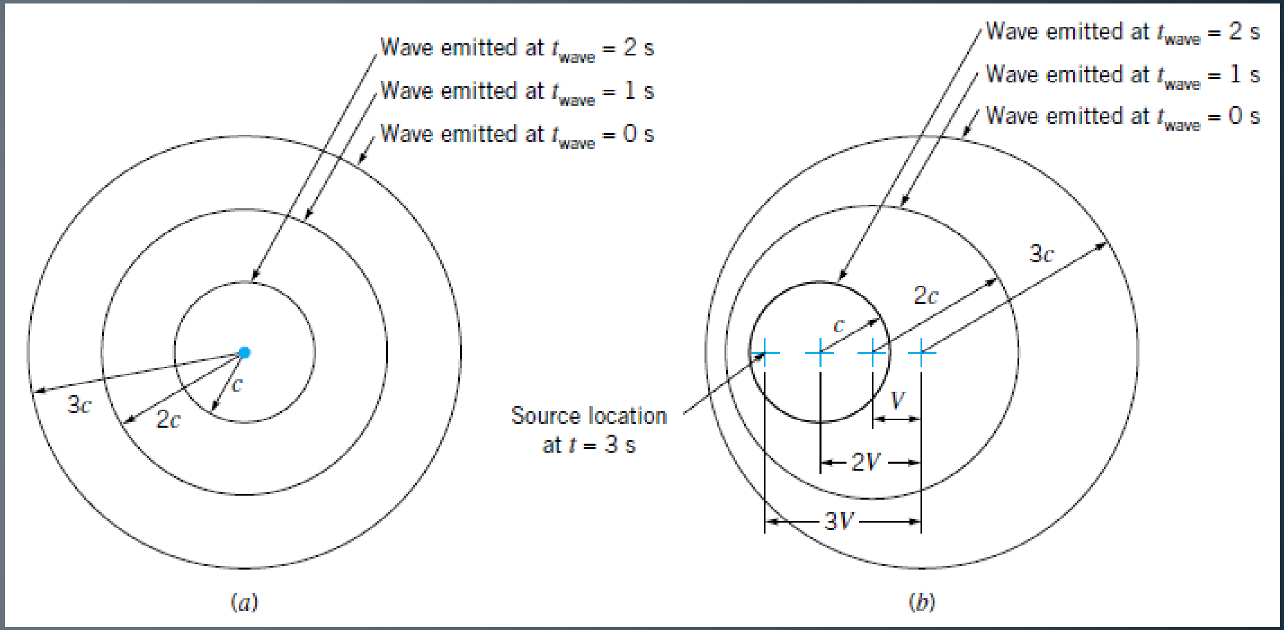
$$c = \sqrt{\frac{E_v}{\rho}}$$

Categories of Compressible Flow

- The variation of drag coefficient with Reynolds number and Mach number is shown for air flow over a sphere. Compressibility effects can be of considerable importance.

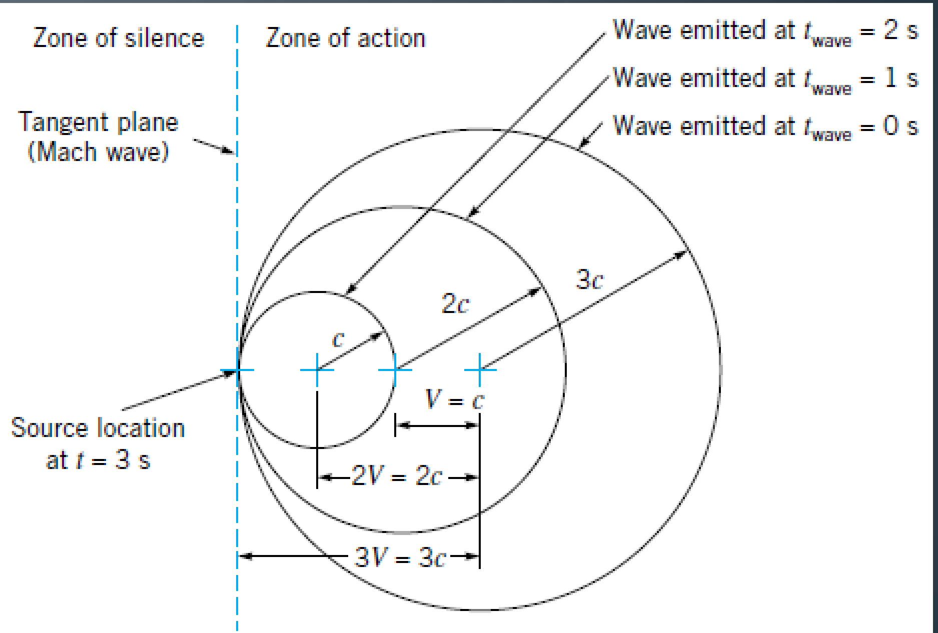


Categories of Compressible Flow



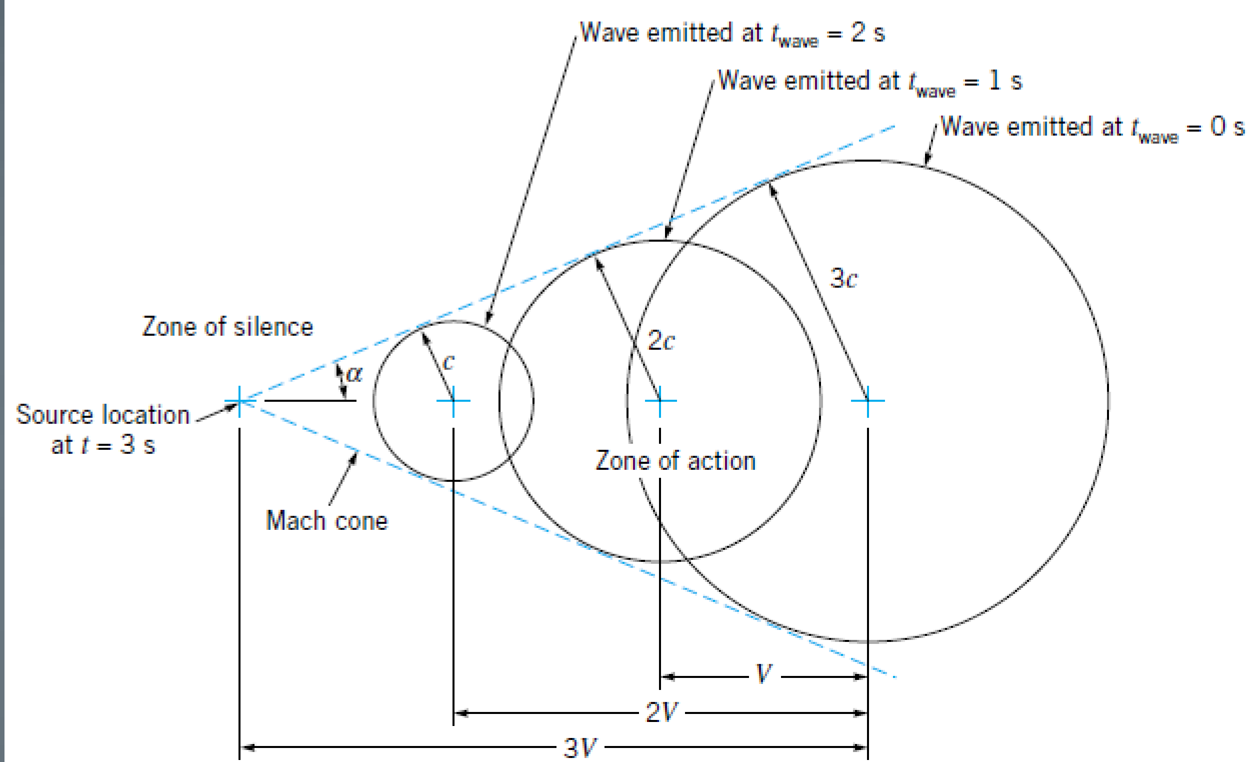
$$r = (t - t_{\text{wave}})c$$

Categories of Compressible Flow



(c)

Categories of Compressible Flow



(d)

$$\sin \alpha = \frac{c}{V} = \frac{1}{\mathrm{Ma}}$$

Categories of Compressible Flow

- This discussion about pressure wave patterns suggests the following categories of fluid flow:

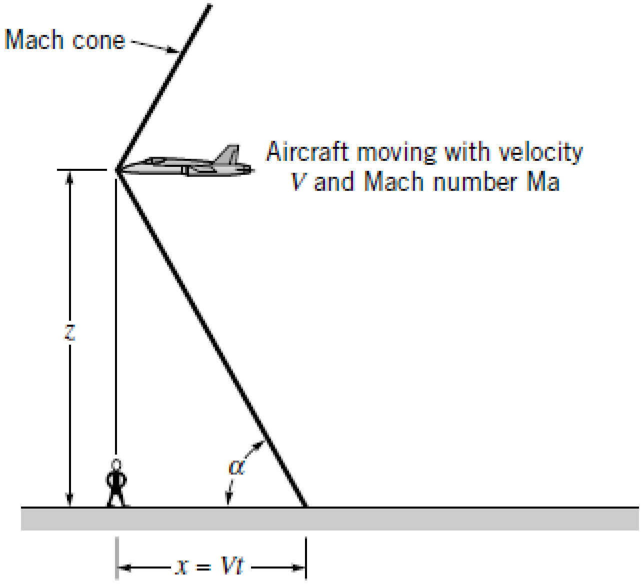
- 1. Incompressible flow: $Ma \leq 0.3$. Unrestricted, nearly symmetrical and instantaneous pressure communication.
- 2. Compressible subsonic flow: $0.3 < Ma < 1$. Unrestricted but noticeably asymmetrical pressure communication.
- 3. Compressible supersonic flow: $Ma \geq 1$. Formation of Mach wave; pressure communication restricted to zone of action.
- In addition to the above-mentioned categories of flows, two other regimes are commonly referred to: namely, transonic flows ($0.9 \leq Ma \leq 1.2$) and hypersonic flows ($Ma \geq 5$).

EXAMPLE 11.4

An aircraft cruising at 1000-m elevation, z, above you moves past in a flyby. How many seconds after the plane passes overhead do you expect to wait before you hear the aircraft if it is moving with a Mach number equal to 1.5 and the ambient temperature is 20 °C?

SOLUTION

Since the aircraft is moving supersonically (Ma > 1), we can imagine a Mach cone originating from the forward tip of the craft as is illustrated in Fig. E11.4. When the surface of



■ FIGURE E11.4

the cone reaches the observer, the "sound" of the aircraft is perceived. The angle α in Fig. E11.4 is related to the elevation of the plane, z, and the ground distance, x, by

$$\alpha = \tan^{-1}\frac{z}{x} = \tan^{-1}\frac{1000}{Vt} \quad (1)$$

Also, assuming negligible change of Mach number with elevation, we can use Eq. 11.39 to relate Mach number to the angle α. Thus,

$$\mathrm{Ma} = \frac{1}{\sin \alpha} \tag{2}$$

Combining Eqs. 1 and 2 we obtain

$$\mathrm{Ma} = \frac{1}{\sin\left[\tan^{-1}(1000/Vt)\right]} \tag{3}$$

The speed of the aircraft can be related to the Mach number with

$$V = (\mathrm{Ma})c \tag{4}$$

where c is the speed of sound. From Table B.4, $c = 343.3$ m/s. Using Ma = 1.5, we get from Eqs. 3 and 4

$$1.5 = \frac{1}{\sin\left\{\tan^{-1}\left[\dfrac{1000 \text{ m}}{(1.5)(343.3 \text{ m/s})t}\right]\right\}}$$

or

$$t = 2.17 \text{ s} \qquad \textbf{(Ans)}$$

Isentropic Flow of an Ideal Gas

- Effect of Variations in Flow Cross-Sectional Area

$$\dot{m} = \rho A V = \text{constant}$$

- The equation of motion in the streamwise direction for the steady, onedimensional, and isentropic (adiabatic and frictionless) flow of an ideal gas is

$$dp + \tfrac{1}{2}\rho\, d(V^2) + \gamma\, dz = 0$$

$$\frac{dp}{\rho V^2} = -\frac{dV}{V}$$

- If we form the logarithm of both sides of the continuity equation, the result is

$$\ln \rho + \ln A + \ln V = \text{constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

Isentropic Flow of an Ideal Gas

- Effect of Variations in Flow Cross-Sectional Area

$$-\frac{dV}{V} = \frac{d\rho}{\rho} + \frac{dA}{A}$$

$$\frac{dp}{\rho V^2}\left(1 - \frac{V^2}{dp/d\rho}\right) = \frac{dA}{A}$$

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$$

$$\mathrm{Ma} = \frac{V}{c}$$

$$\frac{dp}{\rho V^2}(1 - \mathrm{Ma}^2) = \frac{dA}{A}$$

Isentropic Flow of an Ideal Gas

- Effect of Variations in Flow Cross-Sectional Area

$$\frac{dV}{V} = -\frac{dA}{A}\frac{1}{(1 - \mathrm{Ma}^2)}$$

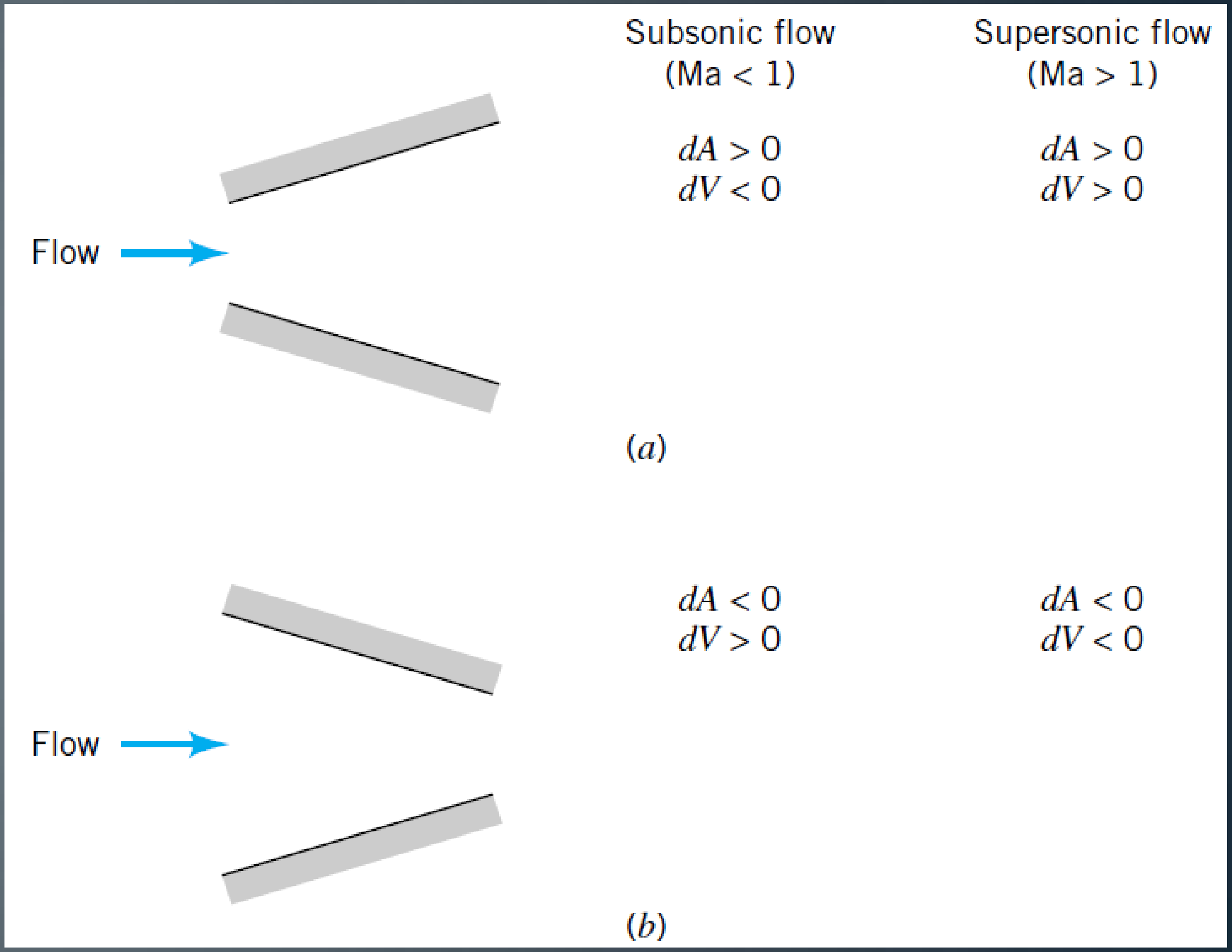
- We can use the above to conclude that when the flow is subsonic velocity and section area changes are in opposite directions, and when the flow is supersonic velocity and area changes are in the same direction.

$$\frac{d\rho}{\rho} = \frac{dA}{A}\frac{\mathrm{Ma}^2}{(1 - \mathrm{Ma}^2)}$$

- We can conclude that for subsonic flows density and area changes are in the same direction, whereas for supersonic flows density and area changes are in opposite directions.

• Effect of Variations in Flow Cross-Sectional Area

Isentropic Flow of an Ideal Gas



Isentropic Flow of an Ideal Gas

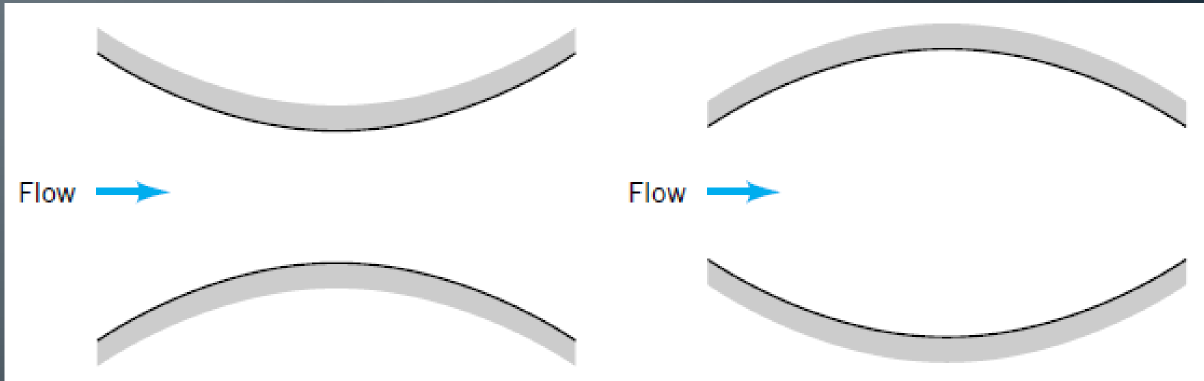
- Effect of Variations in Flow Cross-Sectional Area

$$\frac{dV}{V} = -\frac{dA}{A}\frac{1}{(1 - \text{Ma}^2)}$$

- By rearranging the above equation, we can obtain

$$\frac{dA}{dV} = -\frac{A}{V}(1 - \text{Ma}^2)$$

- This result suggests that the area associated with Ma = 1 is either a minimum or a maximum amount.



(a) (b)

Converging-Diverging Duct Flow

- It is convenient to use the stagnation state of the fluid as a reference state for compressible flow calculations.
- The stagnation state is associated with zero flow velocity and an entropy value that corresponds to the entropy of the flowing fluid.
- The stagnation state can also be achieved by isentropically decelerating a flow to zero velocity.

$$\frac{p}{\rho^k} = \text{constant} = \frac{p_0}{\rho_0^k}$$

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) = 0$$

$$\frac{p_0^{1/k}}{\rho_0}\frac{dp}{(p)^{1/k}} + d\left(\frac{V^2}{2}\right) = 0$$

Converging-Diverging Duct Flow

$$\frac{k}{k-1}\left(\frac{p_0}{\rho_0}-\frac{p}{\rho}\right)-\frac{V^2}{2}=0$$

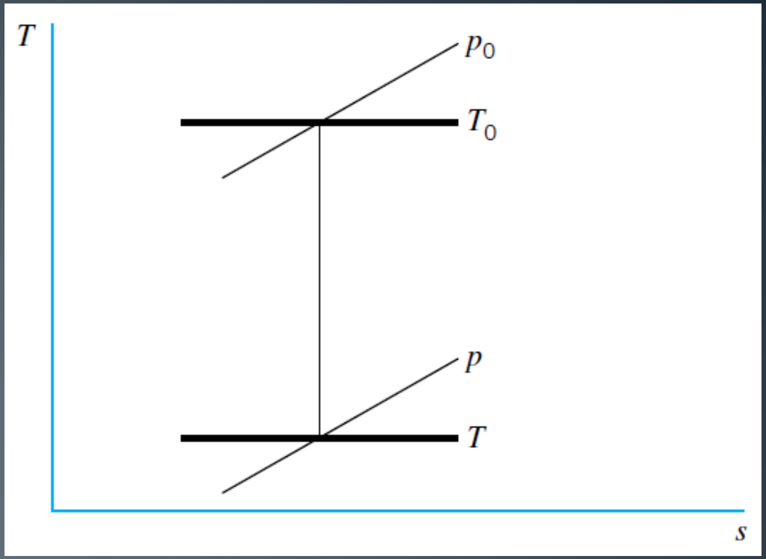
$$\frac{kR}{k-1}(T_0-T)-\frac{V^2}{2}=0$$

$$c_p\,(T_0-T)-\frac{V^2}{2}=0$$

$$\check{h}_0-\left(\check{h}+\frac{V^2}{2}\right)=0$$

- The definition of Mach number and the speed of sound relationship for ideal gases can be combined with the above equation yield

Converging-Diverging Duct Flow



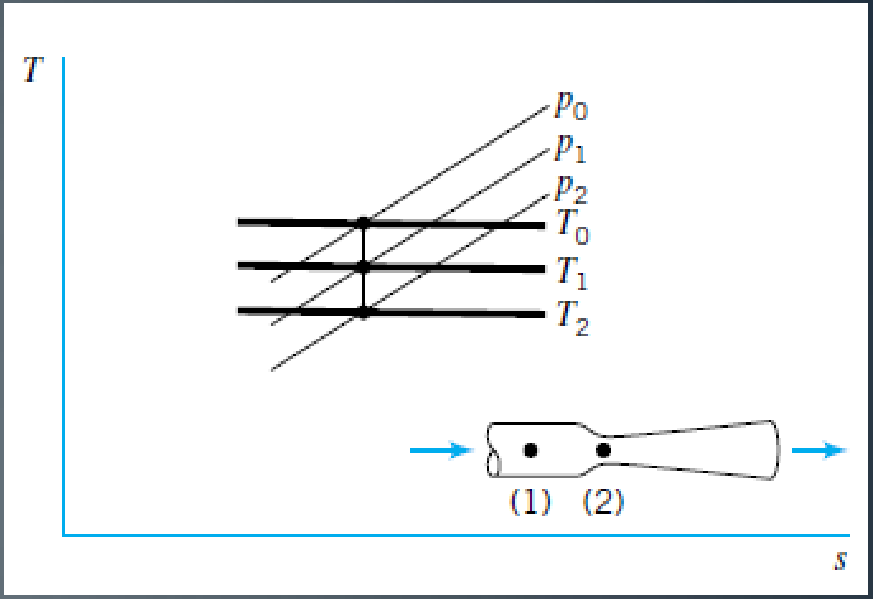
$$\frac{T}{T_0} = \frac{1}{1 + [(k-1)/2]\mathrm{Ma}^2}$$

$$\frac{p}{p_0} = \left\{\frac{1}{1 + [(k-1)/2]\mathrm{Ma}^2}\right\}^{k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left\{\frac{1}{1 + [(k-1)/2]\mathrm{Ma}^2}\right\}^{1/(k-1)}$$

Converging-Diverging Duct Flow

- A very useful means of keeping track of the states of an isentropic flow of an ideal gas involves a temperature-entropy diagram, as is shown below



- When the duct back pressure is lowered sufficiently, the Mach number at the throat of the duct will be 1.
- Any further decrease of the back pressure will not affect the flow in the converging portion of the duct because information about pressure cannot move upstream when Ma=1.
- When Ma=1 at the throat of the converging-diverging duct, we have a condition called choked flow.

Converging-Diverging Duct Flow

- We have already used the *stagnation state* for which Ma=0 as a reference condition.
- It will prove helpful to us to use the state associated with Ma=1 and the same entropy level as the flowing fluid as another reference condition we shall call the *critical state*.
- The ratio of pressure at the converging-diverging duct throat for choked flow, to stagnation pressure, is referred to as the *critical pressure ratio*.

$$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

- For k=1.4 the nominal value of k for air,

$$\left(\frac{p^*}{p_0}\right)_{k=1.4} = 0.528$$

Converging-Diverging Duct Flow

- Because the stagnation pressure for our converging-diverging duct example is the atmospheric pressure, the throat pressure for choked air flow is,

$$p^*_{k=1.4} = 0.528 p_{atm}$$

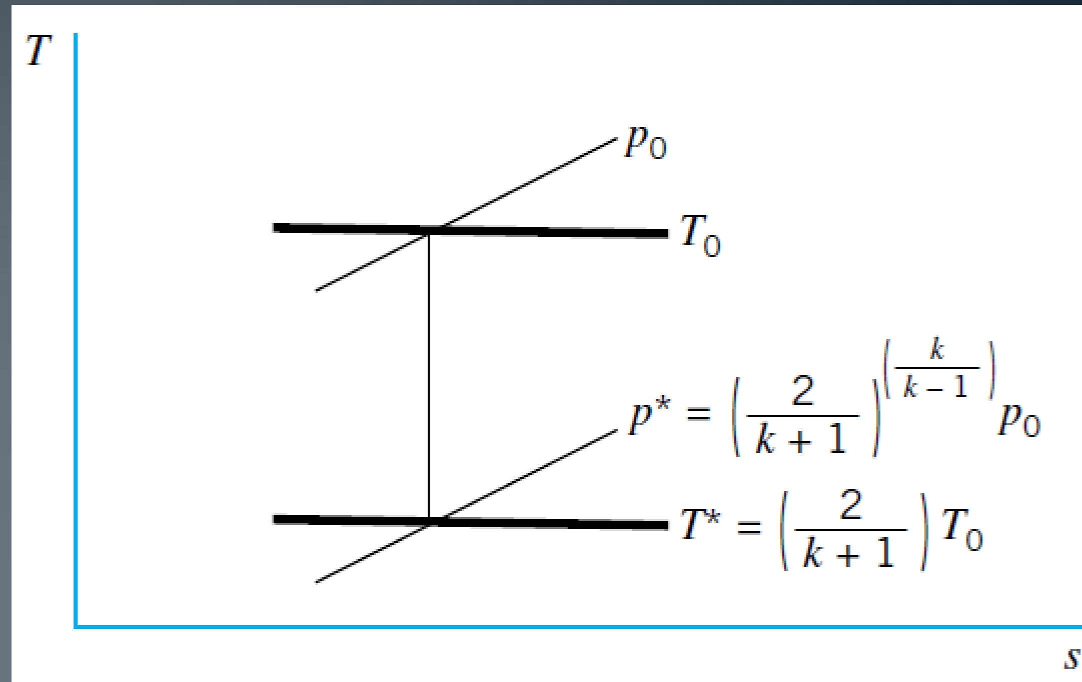
- We can get a relationship for the critical temperature ratio by the same way,

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

$$\left(\frac{T^*}{T_0}\right)_{k=1.4} = 0.833$$

$$T^*_{k=1.4} = 0.833 T_{atm}$$

Converging-Diverging Duct Flow



$$\frac{\rho^*}{\rho_0} = \left(\frac{p^*}{T^*}\right)\left(\frac{T_0}{p_0}\right) = \left(\frac{2}{k+1}\right)^{k/(k-1)}\left(\frac{k+1}{2}\right) = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

$$\left(\frac{\rho^*}{\rho_0}\right)_{k=1.4} = 0.634$$

Converging-Diverging Duct Flow

- For choked flow through the converging-diverging duct, the conservation of mass equation yields

$$\rho A V = \rho^* A^* V^*$$

$$\frac{A}{A^*} = \left(\frac{\rho^*}{\rho}\right)\left(\frac{V^*}{V}\right)$$

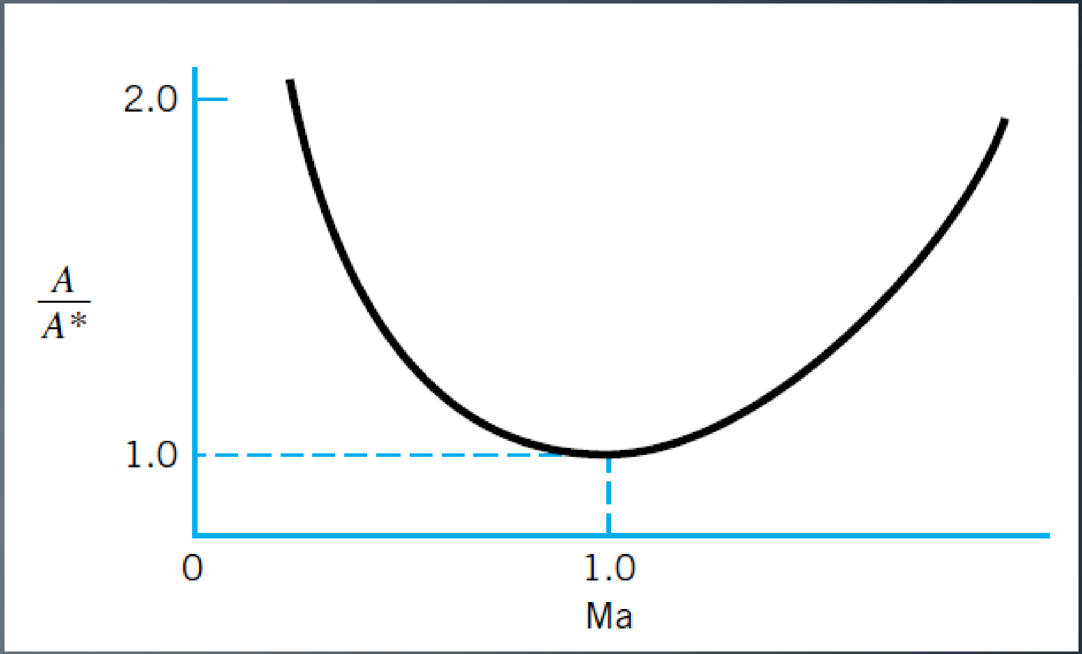
$$V^* = \sqrt{RT^*k}$$

$$V = \mathrm{Ma}\sqrt{RTk}$$

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left(\frac{\rho^*}{\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)\sqrt{\frac{(T^*/T_0)}{(T/T_0)}}$$

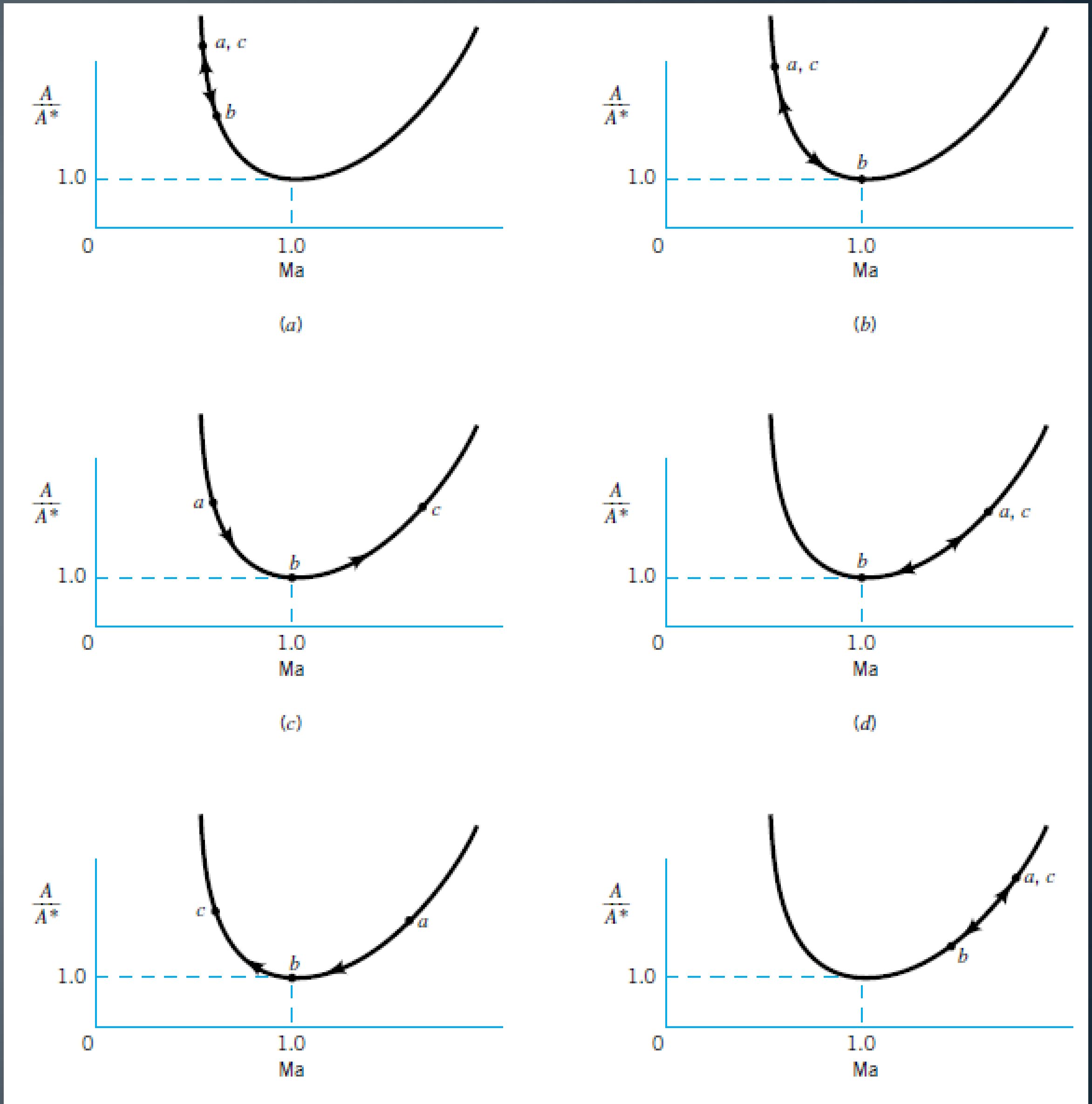
$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left\{\frac{1 + [(k-1)/2]\mathrm{Ma}^2}{1 + [(k-1)/2]}\right\}^{(k+1)/[2(k-1)]}$$

Converging-Diverging Duct Flow

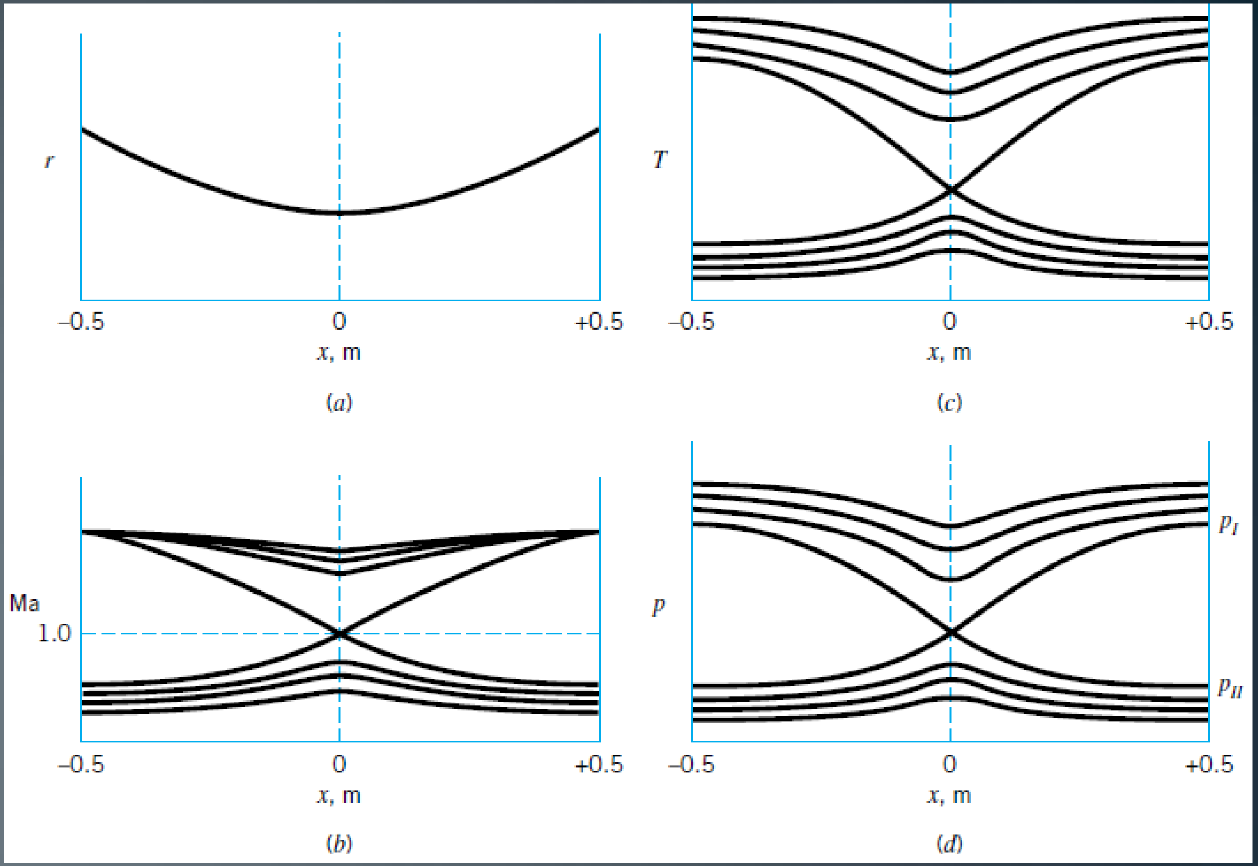


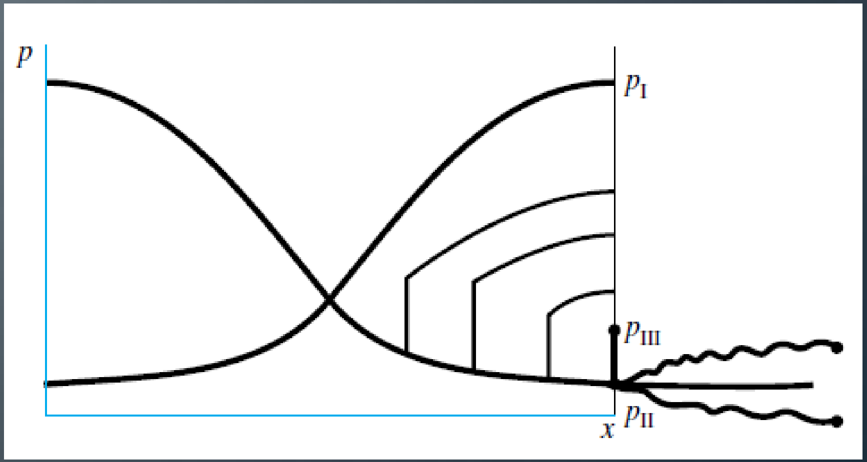
$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}} \left\{ \frac{1 + [(k-1)/2]\mathrm{Ma}^2}{1 + [(k-1)/2]} \right\}^{(k+1)/[2(k-1)]}$$

Converging-Diverging Duct Flow



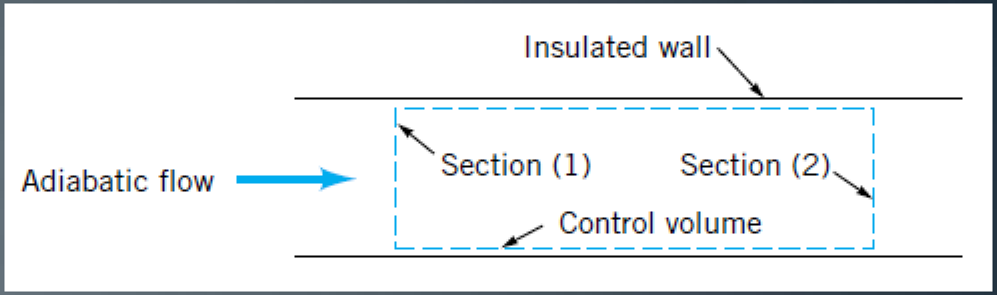
Converging-Diverging Duct Flow





Nonisentropic Flow of an Ideal Gas

- Adiabatic Constant-Area Duct Flow with Friction (Fanno Flow)



$$\dot{m}\left[\check{h}_2 - \check{h}_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] = \dot{Q}_{\substack{\text{net}\\ \text{in.}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}$$

$g(z_2 - z_1)$: 0 (negligibly small for gas flow)

$\dot{Q}_{\text{net in.}}$: 0 (flow is adiabatic)

$\dot{W}_{\text{shaft net in}}$: 0 (flow is steady throughout)

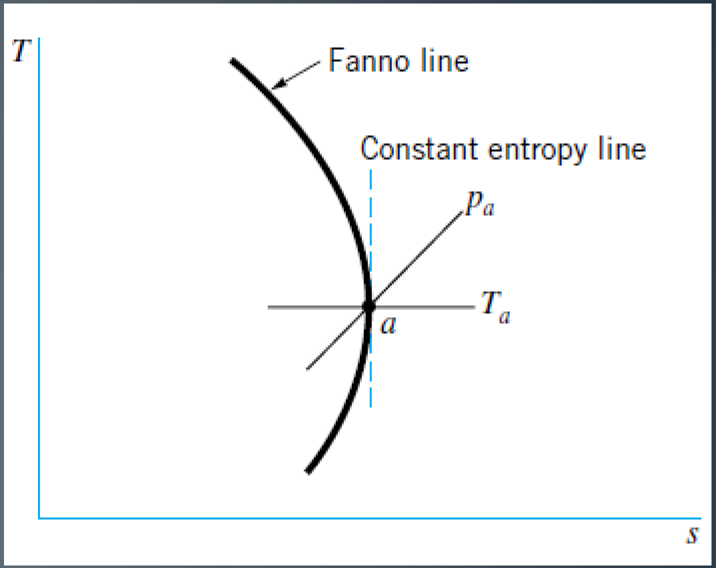
$$\check{h} + \frac{V^2}{2} = \check{h}_0 = \text{constant}$$

$$\check{h} - \check{h}_0 = c_p(T - T_0)$$

$$T + \frac{V^2}{2c_p} = T_0 = \text{constant}$$

Nonisentropic Flow of an Ideal Gas

- Adiabatic Constant-Area Duct Flow with Friction (Fanno Flow)

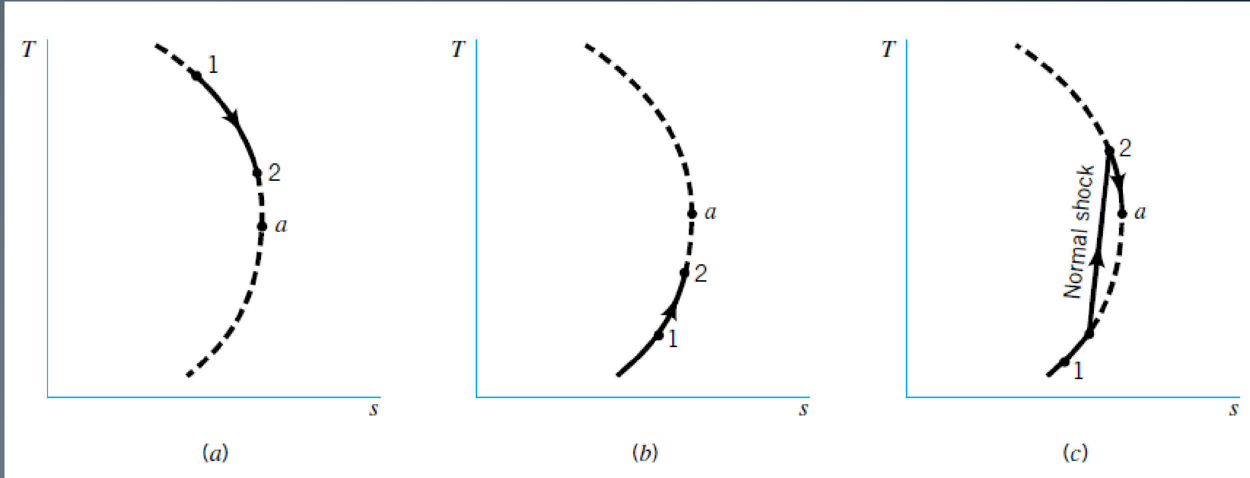


$$T + \frac{(\rho V)^2 T^2}{2c_p(p^2/R^2)} = T_0 = \text{constant}$$

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p}{p_1}$$

Nonisentropic Flow of an Ideal Gas

- Adiabatic Constant-Area Duct Flow with Friction (Fanno Flow)

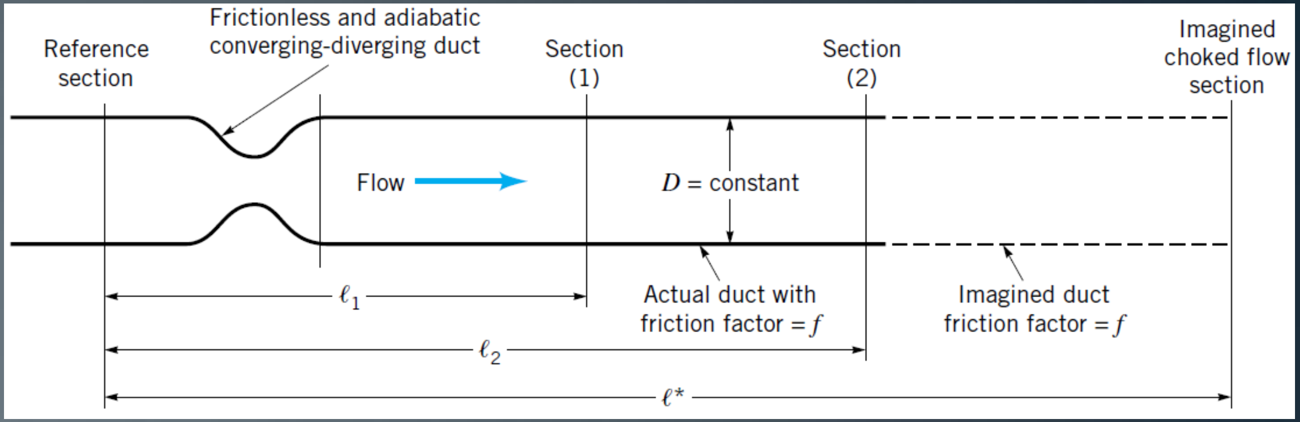


Summary of Fanno Flow Behavior

Parameter	Flow: Subsonic Flow	Flow: Supersonic Flow
Stagnation temperature	Constant	Constant
Ma	Increases (maximum is 1)	Decreases (minimum is 1)
Friction	Accelerates flow	Decelerates flow
Pressure	Decreases	Increases
Temperature	Decreases	Increases

Nonisentropic Flow of an Ideal Gas

- Adiabatic Constant-Area Duct Flow with Friction (Fanno Flow)

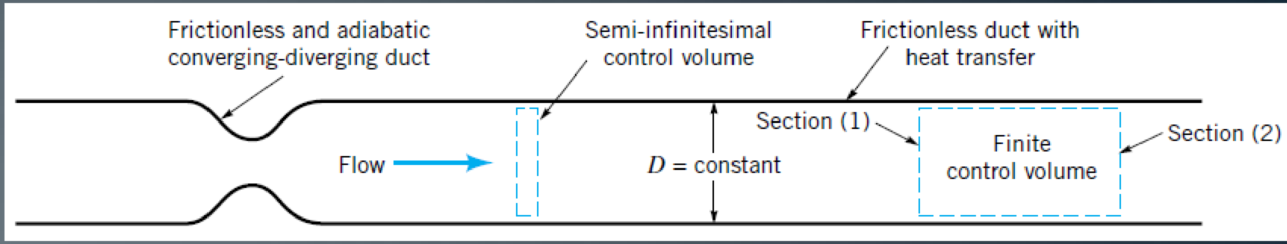


$$\frac{T}{T^*} = \frac{(k+1)/2}{1 + [(k-1)/2]\mathrm{Ma}^2}$$

$$\frac{p}{p^*} = \frac{1}{\mathrm{Ma}} \left\{ \frac{(k+1)/2}{1 + [(k-1)/2]\mathrm{Ma}^2} \right\}^{1/2}$$

Nonisentropic Flow of an Ideal Gas

- Frictionless Constant-Area Duct Flow with Heat Transfer (Rayleigh Flow)



$$p_1A_1 + \dot{m}V_1 = p_2A_2 + \dot{m}V_2 + \cancel{R_x} \quad \text{0(frictionless flow)}$$

$$p + \frac{(\rho V)^2}{\rho} = \text{constant}$$

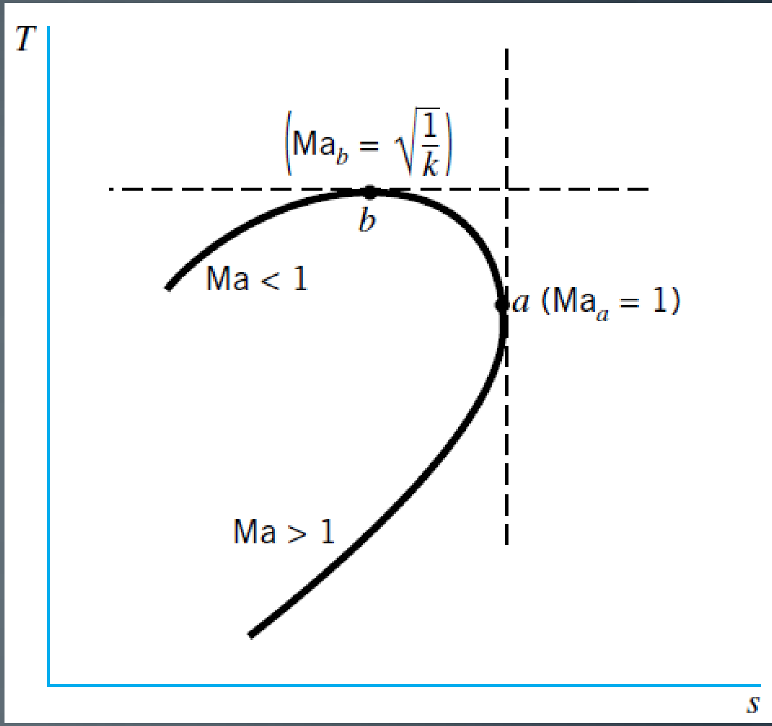
$$p + \frac{(\rho V)^2\, RT}{p} = \text{constant}$$

- Since the flow cross-sectional area remains constant for Rayleigh flow,from the continuity equation we conclude that

$$\rho V = \text{constant}$$

Nonisentropic Flow of an Ideal Gas

- Frictionless Constant-Area Duct Flow with Heat Transfer (Rayleigh Flow)

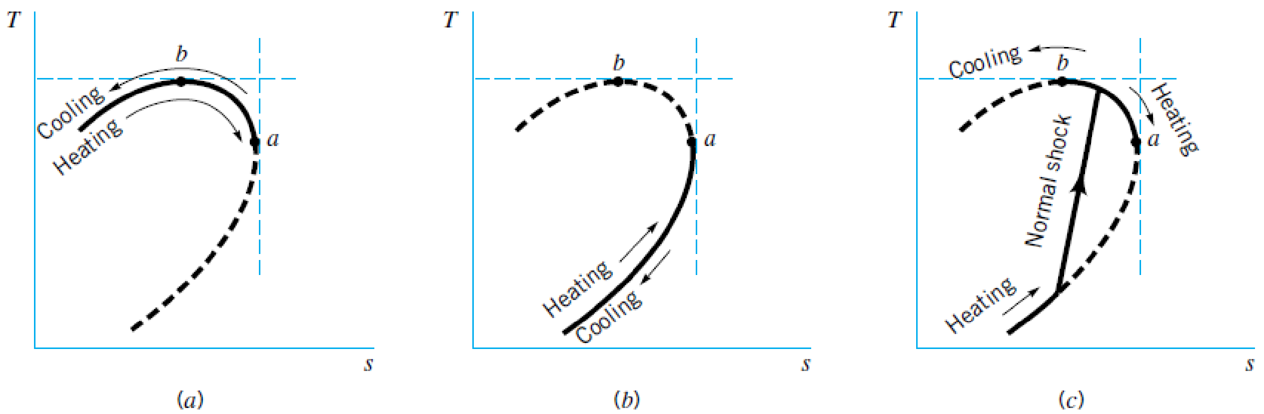


$$p + \frac{(\rho V)^2 RT}{p} = \text{constant}$$

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p}{p_1}$$

Nonisentropic Flow of an Ideal Gas

- Frictionless Constant-Area Duct Flow with Heat Transfer (Rayleigh Flow)



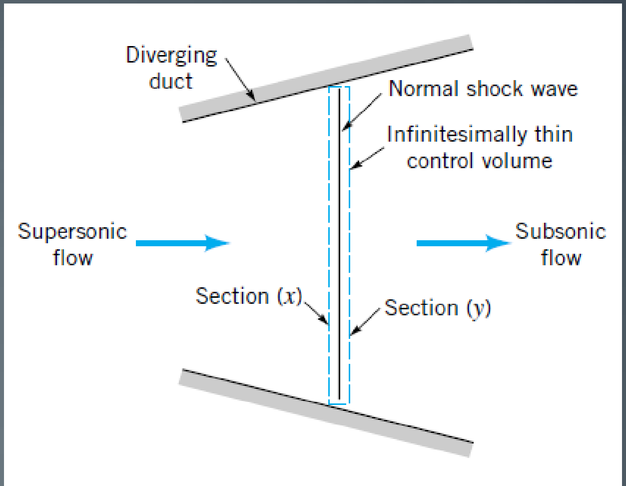
	Heating		Cooling	
	Subsonic	Supersonic	Subsonic	Supersonic
V	Increase	Decrease	Decrease	Increase
Ma	Increase	Decrease	Decrease	Increase
T	Increase for $0 \le \text{Ma} \le \sqrt{1/k} = 0.845$; Decrease for $\sqrt{1/k} \le \text{Ma} \le 1$	Increase	Decrease for $0 \le \text{Ma} \le \sqrt{1/k} = 0.845$; Increase for $\sqrt{1/k} \le \text{Ma} \le 1$	Decrease
T_0	Increase	Increase	Decrease	Decrease
p	Decrease	Increase	Increase	Decrease
p_0	Decrease	Decrease	Increase	Increase

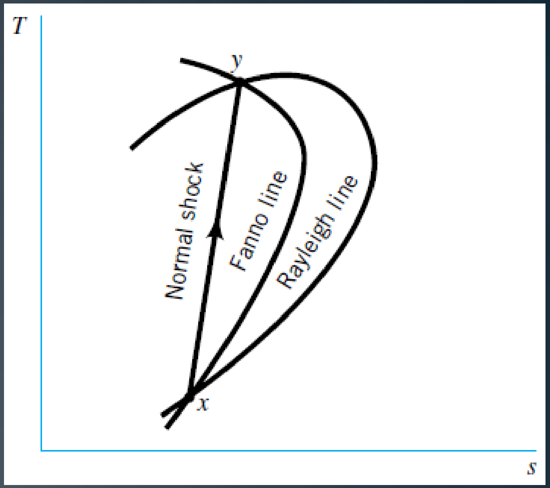
$$\frac{p}{p_a} = \frac{1 + k}{1 + k\text{Ma}^2}$$

$$\frac{T}{T_a} = \left[\frac{(1 + k)\text{Ma}}{1 + k\text{Ma}^2}\right]^2$$

- Normal Shock Waves

Nonisentropic Flow of an Ideal Gas





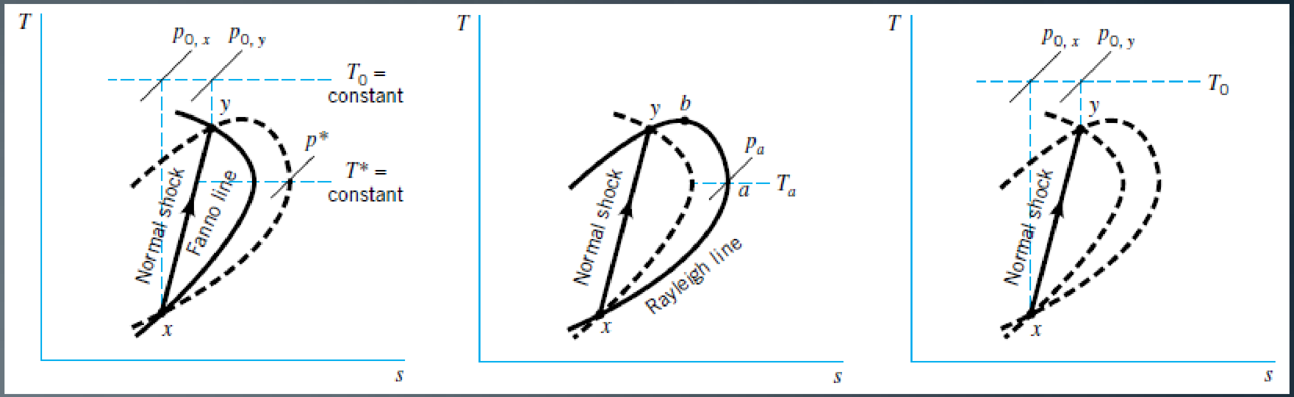
$$\rho V = \text{constant}$$

$$p + \frac{(\rho V)^2 RT}{p} = \text{constant}$$

$$T + \frac{(\rho V)^2 T^2}{2c_p(p^2/R^2)} = T_0 = \text{constant}$$

Normal Shock Waves

Nonisentropic Flow of an Ideal Gas



Summary of Normal Shock Wave Characteristics

Variable	Change Across Normal Shock Wave
Mach number	Decrease
Static pressure	Increase
Stagnation pressure	Decrease
Static temperature	Increase
Stagnation temperature	Constant
Density	Increase
Velocity	Decrease

$$\frac{p_y}{p_x} = \frac{1 + k\mathrm{Ma}_x^2}{1 + k\mathrm{Ma}_y^2}$$

$$\frac{T_y}{T_x} = \frac{1 + [(k-1)/2]\mathrm{Ma}_x^2}{1 + [(k-1)/2]\mathrm{Ma}_y^2}$$