

Chapter 3 – Student Solutions Manual

1. A vector $\vec{a}$ can be represented in the *magnitude-angle* notation (a, θ) , where

$$a = \sqrt{a_x^2 + a_y^2}$$

is the magnitude and

$$\theta = \tan^{-1} \left(\frac{a_y}{a_x} \right)$$

is the angle $\vec{a}$ makes with the positive x axis.

(a) Given $A_x = -25.0$ m and $A_y = 40.0$ m, $A = \sqrt{(-25.0 \text{ m})^2 + (40.0 \text{ m})^2} = 47.2$ m

(b) Recalling that $\tan \theta = \tan (\theta + 180^\circ)$, $\tan^{-1} [40 / (-25)] = -58^\circ$ or 122° . Noting that the vector is in the third quadrant (by the signs of its x and y components) we see that 122° is the correct answer. The graphical calculator “shortcuts” mentioned above are designed to correctly choose the right possibility.

3. The x and the y components of a vector $\vec{a}$ lying on the xy plane are given by

$$a_x = a \cos \theta, \quad a_y = a \sin \theta$$

where $a = |\vec{a}|$ is the magnitude and θ is the angle between $\vec{a}$ and the positive x axis.

(a) The x component of $\vec{a}$ is given by $a_x = 7.3 \cos 250^\circ = -2.5$ m.

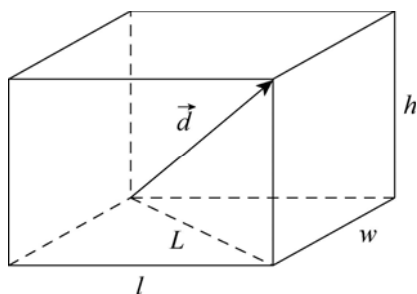
(b) and the y component is given by $a_y = 7.3 \sin 250^\circ = -6.9$ m.

In considering the variety of ways to compute these, we note that the vector is 70° below the $-x$ axis, so the components could also have been found from $a_x = -7.3 \cos 70^\circ$ and $a_y = -7.3 \sin 70^\circ$. In a similar vein, we note that the vector is 20° to the left from the $-y$ axis, so one could use $a_x = -7.3 \sin 20^\circ$ and $a_y = -7.3 \cos 20^\circ$ to achieve the same results.

7. The length unit meter is understood throughout the calculation.

(a) We compute the distance from one corner to the diametrically opposite corner:

$$d = \sqrt{3.00^2 + 3.70^2 + 4.30^2} = 6.42 .$$

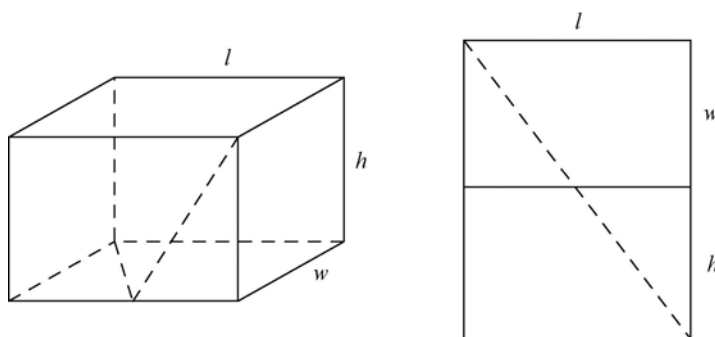


(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement.

(c) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h = 11.0$ m.

(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward. Then the x component of the displacement is $w = 3.70$, the y component of the displacement is 4.30 , and the z component is 3.00 . Thus $\vec{d} = 3.70\hat{i} + 4.30\hat{j} + 3.00\hat{k}$. An equally correct answer is gotten by interchanging the length, width, and height.



(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$L_{\min} = \sqrt{(w + h)^2 + \ell^2} = \sqrt{(3.70 + 3.00)^2 + 4.30^2} = 7.96 \text{ m}.$$

9. We write $\vec{r} = \vec{a} + \vec{b}$. When not explicitly displayed, the units here are assumed to be meters.

(a) The x and the y components of $\vec{r}$ are $r_x = a_x + b_x = (4.0 \text{ m}) - (13 \text{ m}) = -9.0 \text{ m}$ and $r_y = a_y + b_y = (3.0 \text{ m}) + (7.0 \text{ m}) = 10 \text{ m}$, respectively. Thus $\vec{r} = (-9.0 \text{ m})\hat{i} + (10 \text{ m})\hat{j}$.

(b) The magnitude of $\vec{r}$ is

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0 \text{ m})^2 + (10 \text{ m})^2} = 13 \text{ m}.$$

(c) The angle between the resultant and the $+x$ axis is given by

$$\theta = \tan^{-1}(r_y/r_x) = \tan^{-1}[(10 \text{ m})/(-9.0 \text{ m})] = -48^\circ \text{ or } 132^\circ.$$

Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

17. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the $+x$ axis is $30^\circ + 105^\circ = 135^\circ$ and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The x component of $\vec{r}$ is $r_x = (10.0 \text{ m}) \cos 30^\circ + (10.0 \text{ m}) \cos 135^\circ = 1.59 \text{ m}$.

(b) The y component of $\vec{r}$ is $r_y = (10.0 \text{ m}) \sin 30^\circ + (10.0 \text{ m}) \sin 135^\circ = 12.1 \text{ m}$.

(c) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{(1.59 \text{ m})^2 + (12.1 \text{ m})^2} = 12.2 \text{ m}$.

(d) The angle between $\vec{r}$ and the $+x$ direction is $\tan^{-1}[(12.1 \text{ m})/(1.59 \text{ m})] = 82.5^\circ$.

39. Since $ab \cos \phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos \phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab}.$$

The magnitudes of the vectors given in the problem are

$$a = |\vec{a}| = \sqrt{(3.00)^2 + (3.00)^2 + (3.00)^2} = 5.20$$

$$b = |\vec{b}| = \sqrt{(2.00)^2 + (1.00)^2 + (3.00)^2} = 3.74.$$

The angle between them is found from

$$\cos \phi = \frac{(3.00)(2.00) + (3.00)(1.00) + (3.00)(3.00)}{(5.20)(3.74)} = 0.926.$$

The angle is $\phi = 22^\circ$.

43. From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is 120° . Direct application of Eq. 3-5 yields the answers for this and the next few parts.

(a) $a_x = a \cos 0^\circ = a = 3.00 \text{ m}.$

(b) $a_y = a \sin 0^\circ = 0.$

(c) $b_x = b \cos 30^\circ = (4.00 \text{ m}) \cos 30^\circ = 3.46 \text{ m}.$

(d) $b_y = b \sin 30^\circ = (4.00 \text{ m}) \sin 30^\circ = 2.00 \text{ m}.$

(e) $c_x = c \cos 120^\circ = (10.0 \text{ m}) \cos 120^\circ = -5.00 \text{ m}.$

(f) $c_y = c \sin 120^\circ = (10.0 \text{ m}) \sin 120^\circ = 8.66 \text{ m}.$

(g) In terms of components (first x and then y), we must have

$$\begin{aligned} -5.00 \text{ m} &= p (3.00 \text{ m}) + q (3.46 \text{ m}) \\ 8.66 \text{ m} &= p (0) + q (2.00 \text{ m}). \end{aligned}$$

Solving these equations, we find $p = -6.67$.

(h) And $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

47. We apply Eq. 3-20 and Eq. 3-27.

(a) The scalar (dot) product of the two vectors is

$$\vec{a} \cdot \vec{b} = ab \cos \phi = (10) (6.0) \cos 60^\circ = 30.$$

(b) The magnitude of the vector (cross) product of the two vectors is

$$|\vec{a} \times \vec{b}| = ab \sin \phi = (10) (6.0) \sin 60^\circ = 52.$$

51. Let $\vec{A}$ represent the first part of his actual voyage (50.0 km east) and $\vec{C}$ represent the intended voyage (90.0 km north). We are looking for a vector $\vec{B}$ such that $\vec{A} + \vec{B} = \vec{C}$.

(a) The Pythagorean theorem yields $B = \sqrt{(50.0)^2 + (90.0)^2} = 103 \text{ km}$.

(b) The direction is $\tan^{-1}(50.0/90.0) = 29.1^\circ$ west of north (which is equivalent to 60.9° north of due west).

71. Given: $\vec{A} + \vec{B} = 6.0 \hat{i} + 1.0 \hat{j}$ and $\vec{A} - \vec{B} = -4.0 \hat{i} + 7.0 \hat{j}$. Solving these simultaneously leads to $\vec{A} = 1.0 \hat{i} + 4.0 \hat{j}$. The Pythagorean theorem then leads to $A = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$.