

Chapter 9 – Student Solutions Manual

15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0y} - gt$ and this is zero at time $t = v_{0y}/g = (v_0/g) \sin \theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0t \cos \theta_0 = \frac{v_0^2}{g} \sin \theta_0 \cos \theta_0 = \frac{(20 \text{ m/s})^2}{9.8 \text{ m/s}^2} \sin 60^\circ \cos 60^\circ = 17.7 \text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2 \theta_0 = \frac{1}{2}\frac{(20 \text{ m/s})^2}{9.8 \text{ m/s}^2}\sin^2 60^\circ = 15.3 \text{ m}.$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0 \cos \theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then $Mv_0 \cos \theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0 \cos \theta_0 = 2(20 \text{ m/s}) \cos 60^\circ = 20 \text{ m/s}.$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7 \text{ m}$, $y_0 = 15.3 \text{ m}$. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7 \text{ m} + (20 \text{ m/s})\sqrt{\frac{2(15.3 \text{ m})}{9.8 \text{ m/s}^2}} = 53 \text{ m}.$$

23. The initial direction of motion is in the $+x$ direction. The magnitude of the average force F_{avg} is given by

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{32.4 \text{ N}\cdot\text{s}}{2.70 \times 10^{-2} \text{ s}} = 1.20 \times 10^3 \text{ N}$$

The force is in the negative direction. Using the linear momentum-impulse theorem stated in Eq. 9-31, we have

$$-F_{\text{avg}}\Delta t = mv_f - mv_i.$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. Thus,

$$v_f = \frac{mv_i - F_{\text{avg}} \Delta t}{m} = \frac{(0.40 \text{ kg})(14 \text{ m/s}) - (1200 \text{ N})(27 \times 10^{-3} \text{ s})}{0.40 \text{ kg}} = -67 \text{ m/s}.$$

(a) The final speed of the ball is $|v_f| = 67 \text{ m/s}$.

(b) The negative sign indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{\text{avg}} = 1.20 \times 10^3 \text{ N}$.

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

35. (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$\begin{aligned} J &= \int_0^{3.0 \times 10^{-3}} F dt = \int_0^{3.0 \times 10^{-3}} [(6.0 \times 10^6)t - (2.0 \times 10^9)t^2] dt \\ &= \left[\frac{1}{2} (6.0 \times 10^6)t^2 - \frac{1}{3} (2.0 \times 10^9)t^3 \right] \bigg|_0^{3.0 \times 10^{-3}} \\ &= 9.0 \text{ N} \cdot \text{s}. \end{aligned}$$

(b) Since $J = F_{\text{avg}} \Delta t$, we find

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.0 \text{ N} \cdot \text{s}}{3.0 \times 10^{-3} \text{ s}} = 3.0 \times 10^3 \text{ N}.$$

(c) To find the time at which the maximum force occurs, we set the derivative of F with respect to time equal to zero – and solve for t . The result is $t = 1.5 \times 10^{-3} \text{ s}$. At that time the force is

$$F_{\text{max}} = (6.0 \times 10^6)(1.5 \times 10^{-3}) - (2.0 \times 10^9)(1.5 \times 10^{-3})^2 = 4.5 \times 10^3 \text{ N}.$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let m be the mass of the ball and v its speed as it leaves the foot. Then,

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0 \text{ N} \cdot \text{s}}{0.45 \text{ kg}} = 20 \text{ m/s}.$$

39. No external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, so the total momentum of the system is conserved. Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked; let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then

$$m_s v_s + m_m v_m = 0 \rightarrow v_m = -m_s v_s / m_m.$$

We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{(0.068 \text{ kg})(4.0 \text{ m/s})}{91 \text{ kg}} = -3.0 \times 10^{-3} \text{ m/s},$$

or $|v_m| = 3.0 \times 10^{-3} \text{ m/s}$. The negative sign indicates that the man moves in the direction opposite to the direction of motion of the stone.

47. Our notation is as follows: the mass of the original body is $M = 20.0 \text{ kg}$; its initial velocity is $\vec{v}_0 = 200\hat{i}$ in SI units (m/s); the mass of one fragment is $m_1 = 10.0 \text{ kg}$; its velocity is $\vec{v}_1 = -100\hat{j}$ in SI units; the mass of the second fragment is $m_2 = 4.0 \text{ kg}$; its velocity is $\vec{v}_2 = -500\hat{i}$ in SI units; and, the mass of the third fragment is $m_3 = 6.00 \text{ kg}$.

(a) Conservation of linear momentum requires $M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$, which (using the above information) leads to

$$\vec{v}_3 = (1.00 \times 10^3 \hat{i} - 0.167 \times 10^3 \hat{j}) \text{ m/s}$$

in SI units. The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{1000^2 + (-167)^2} = 1.01 \times 10^3 \text{ m/s}$. It points at $\tan^{-1}(-167/1000) = -9.48^\circ$ (that is, at 9.5° measured clockwise from the $+x$ axis).

(b) We are asked to calculate ΔK or

$$\left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 \right) - \frac{1}{2} M v_0^2 = 3.23 \times 10^6 \text{ J}.$$

61. (a) Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}.$$

Using SI units (so $m_1 = 0.34 \text{ kg}$), we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{1.2 \text{ m/s} - 0.66 \text{ m/s}}{1.2 \text{ m/s} + 0.66 \text{ m/s}} \right) (0.34 \text{ kg}) = 0.099 \text{ kg}.$$

(b) The velocity of the second cart is given by Eq. 9-68:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(0.34 \text{ kg})}{0.34 \text{ kg} + 0.099 \text{ kg}} \right) (1.2 \text{ m/s}) = 1.9 \text{ m/s}.$$

(c) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34)(1.2) + 0}{0.34 + 0.099} = 0.93 \text{ m/s}.$$

Values for the initial velocities were used but the same result is obtained if values for the final velocities are used.

63. (a) Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}.$$

We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i}} m_1.$$

We combine this with $v_{1f} = v_{1i} / 4$ to obtain $m_2 = 3m_1/5 = 3(2.0)/5 = 1.2 \text{ kg}$.

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(2.0)(4.0)}{2.0 + 1.2} = 2.5 \text{ m/s}.$$

77. (a) The thrust of the rocket is given by $T = Rv_{\text{rel}}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. For this problem $R = 480 \text{ kg/s}$ and $v_{\text{rel}} = 3.27 \times 10^3 \text{ m/s}$, so

$$T = (480 \text{ kg/s})(3.27 \times 10^3 \text{ m/s}) = 1.57 \times 10^6 \text{ N}.$$

(b) The mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$, where Δt is the time interval of the burn. Thus, $M_{\text{fuel}} = (480 \text{ kg/s})(250 \text{ s}) = 1.20 \times 10^5 \text{ kg}$. The mass of the rocket after the burn is

$$M_f = M_i - M_{\text{fuel}} = (2.55 \times 10^5 \text{ kg}) - (1.20 \times 10^5 \text{ kg}) = 1.35 \times 10^5 \text{ kg}.$$

(c) Since the initial speed is zero, the final speed is given by

$$v_f = v_{\text{rel}} \ln \frac{M_i}{M_f} = (3.27 \times 10^3) \ln \left(\frac{2.55 \times 10^5}{1.35 \times 10^5} \right) = 2.08 \times 10^3 \text{ m/s}.$$

79. (a) We consider what must happen to the coal that lands on the faster barge during one minute ($\Delta t = 60 \text{ s}$). In that time, a total of $m = 1000 \text{ kg}$ of coal must experience a change of velocity $\Delta v = 20 \text{ km/h} - 10 \text{ km/h} = 10 \text{ km/h} = 2.8 \text{ m/s}$, where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{m \Delta \vec{v}}{\Delta t} = \frac{(1000 \text{ kg})(2.8 \text{ m/s})}{60 \text{ s}} = 46 \text{ N}$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$.

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

91. (a) If m is the mass of a pellet and v is its velocity as it hits the wall, then its momentum is $p = mv = (2.0 \times 10^{-3} \text{ kg})(500 \text{ m/s}) = 1.0 \text{ kg} \cdot \text{m/s}$, toward the wall.

(b) The kinetic energy of a pellet is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.0 \times 10^{-3} \text{ kg})(500 \text{ m/s})^2 = 2.5 \times 10^2 \text{ J}.$$

(c) The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers $p = 1.0 \text{ kg} \cdot \text{m/s}$. If ΔN pellets hit in time Δt , then the average rate at which momentum is transferred is

$$F_{\text{avg}} = \frac{p \Delta N}{\Delta t} = (1.0 \text{ kg} \cdot \text{m/s})(10 \text{ s}^{-1}) = 10 \text{ N}.$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If Δt is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F_{\text{avg}} = \frac{p}{\Delta t} = \frac{1.0 \text{ kg} \cdot \text{m/s}}{0.6 \times 10^{-3} \text{ s}} = 1.7 \times 10^3 \text{ N}.$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

93. (a) The initial momentum of the car is

$$\vec{p}_i = m\vec{v}_i = (1400 \text{ kg})(5.3 \text{ m/s})\hat{j} = (7400 \text{ kg} \cdot \text{m/s})\hat{j}$$

and the final momentum is $\vec{p}_f = (7400 \text{ kg} \cdot \text{m/s})\hat{i}$. The impulse on it equals the change in its momentum: $\vec{J} = \vec{p}_f - \vec{p}_i = (7400 \text{ N} \cdot \text{s})(\hat{i} - \hat{j})$.

(b) The initial momentum of the car is $\vec{p}_i = (7400 \text{ kg} \cdot \text{m/s})\hat{i}$ and the final momentum is $\vec{p}_f = 0$. The impulse acting on it is $\vec{J} = \vec{p}_f - \vec{p}_i = (-7.4 \times 10^3 \text{ N} \cdot \text{s})\hat{i}$.

(c) The average force on the car is

$$\vec{F}_{\text{avg}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400 \text{ kg} \cdot \text{m/s})(\hat{i} - \hat{j})}{4.6 \text{ s}} = (1600 \text{ N})(\hat{i} - \hat{j})$$

and its magnitude is $F_{\text{avg}} = (1600 \text{ N})\sqrt{2} = 2.3 \times 10^3 \text{ N}$.

(d) The average force is

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{(-7400 \text{ kg} \cdot \text{m/s})\hat{i}}{350 \times 10^{-3} \text{ s}} = (-2.1 \times 10^4 \text{ N})\hat{i}$$

and its magnitude is $F_{\text{avg}} = 2.1 \times 10^4 \text{ N}$.

(e) The average force is given above in unit vector notation. Its x and y components have equal magnitudes. The x component is positive and the y component is negative, so the force is 45° below the positive x axis.

97. Let m_F be the mass of the freight car and v_F be its initial velocity. Let m_C be the mass of the caboose and v be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to $m_F v_F = (m_F + m_C)v$, so $v = v_F m_F / (m_F + m_C)$. The initial kinetic energy of the system is

$$K_i = \frac{1}{2} m_F v_F^2$$

and the final kinetic energy is

$$K_f = \frac{1}{2} (m_F + m_C) v^2 = \frac{1}{2} (m_F + m_C) \frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2} \frac{m_F^2 v_F^2}{(m_F + m_C)}.$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73 K_i$. Thus,

$$\frac{1}{2} \frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73) \left(\frac{1}{2} m_F v_F^2 \right).$$

Simplifying, we obtain $m_F / (m_F + m_C) = 0.73$, which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73} m_F = 0.37 m_F = (0.37)(3.18 \times 10^4 \text{ kg}) = 1.18 \times 10^4 \text{ kg}.$$

101. The mass of each ball is m , and the initial speed of one of the balls is $v_{1i} = 2.2 \text{ m/s}$. We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m v_{1i} &= m v_{1f} \cos \theta_1 + m v_{2f} \cos \theta_2 \\ 0 &= m v_{1f} \sin \theta_1 - m v_{2f} \sin \theta_2 \end{aligned}$$

The mass m cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

(a) The y -momentum equation can be rewritten as, using $\theta_2 = 60^\circ$ and $v_{2f} = 1.1 \text{ m/s}$,

$$v_{1f} \sin \theta_1 = (1.1 \text{ m/s}) \sin 60^\circ = 0.95 \text{ m/s}.$$

and the x -momentum equation yields

$$v_{1f} \cos \theta_1 = (2.2 \text{ m/s}) - (1.1 \text{ m/s}) \cos 60^\circ = 1.65 \text{ m/s}.$$

Dividing these two equations, we find $\tan \theta_1 = 0.576$ which yields $\theta_1 = 30^\circ$. We plug the value into either equation and find $v_{1f} \approx 1.9 \text{ m/s}$.

(b) From the above, we have $\theta_1 = 30^\circ$.

(c) One can check to see if this an elastic collision by computing

$$\frac{2K_i}{m} = v_{1i}^2 \quad \text{and} \quad \frac{2K_f}{m} = v_{1f}^2 + v_{2f}^2$$

and seeing if they are equal (they are), but one must be careful not to use rounded-off values. Thus, it is useful to note that the answer in part (a) can be expressed “exactly” as $v_{1f} = \frac{1}{2} v_{1i} \sqrt{3}$ (and of course $v_{2f} = \frac{1}{2} v_{1i}$ “exactly” — which makes it clear that these two kinetic energy expressions are indeed equal).

107. (a) Noting that the initial velocity of the system is zero, we use Eq. 9-19 and Eq. 2-15 (adapted to two dimensions) to obtain

$$\vec{d} = \frac{1}{2} \left(\frac{\vec{F}_1 + \vec{F}_2}{m_1 + m_2} \right) t^2 = \frac{1}{2} \left(\frac{-2\hat{i} + \hat{j}}{0.006} \right) (0.002)^2$$

which has a magnitude of 0.745 mm.

(b) The angle of $\vec{d}$ is 153° counterclockwise from +x-axis.

(c) A similar calculation using Eq. 2-11 (adapted to two dimensions) leads to a center of mass velocity of $\vec{v} = 0.7453 \text{ m/s}$ at 153° . Thus, the center of mass kinetic energy is

$$K_{\text{com}} = \frac{1}{2} (m_1 + m_2) v^2 = 0.00167 \text{ J}.$$

113. By conservation of momentum, the final speed v of the sled satisfies

$$(2900 \text{ kg})(250 \text{ m/s}) = (2900 \text{ kg} + 920 \text{ kg})v$$

which gives $v = 190 \text{ m/s}$.

115. (a) We locate the coordinate origin at the center of Earth. Then the distance r_{com} of the center of mass of the Earth-Moon system is given by

$$r_{\text{com}} = \frac{m_M r_M}{m_M + m_E}$$

where m_M is the mass of the Moon, m_E is the mass of Earth, and r_M is their separation. These values are given in Appendix C. The numerical result is

$$r_{\text{com}} = \frac{(7.36 \times 10^{22} \text{ kg})(3.82 \times 10^8 \text{ m})}{7.36 \times 10^{22} \text{ kg} + 5.98 \times 10^{24} \text{ kg}} = 4.64 \times 10^6 \text{ m} \approx 4.6 \times 10^3 \text{ km}.$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6 \text{ m}$, so $r_{\text{com}} / R_E = 0.73 = 73\%$.

117. (a) The thrust is Rv_{rel} where $v_{\text{rel}} = 1200 \text{ m/s}$. For this to equal the weight Mg where $M = 6100 \text{ kg}$, we must have $R = (6100)(9.8)/1200 \approx 50 \text{ kg/s}$.

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{\text{rel}} - Mg = Ma$$

so that requiring $a = 21 \text{ m/s}^2$ leads to $R = (6100)(9.8 + 21)/1200 = 1.6 \times 10^2 \text{ kg/s}$.

129. Using Eq. 9-68 with $m_1 = 3.0 \text{ kg}$, $v_{1i} = 8.0 \text{ m/s}$ and $v_{2f} = 6.0 \text{ m/s}$, then

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \Rightarrow m_2 = m_1 \left(\frac{2v_{1i}}{v_{2f}} - 1 \right)$$

leads to $m_2 = M = 5.0 \text{ kg}$.