

Chapter 10 – Student Solutions Manual

13. We take $t = 0$ at the start of the interval and take the sense of rotation as positive. Then at the end of the $t = 4.0$ s interval, the angular displacement is $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$. We solve for the angular velocity at the start of the interval:

$$\omega_0 = \frac{\theta - \frac{1}{2} \alpha t^2}{t} = \frac{120 \text{ rad} - \frac{1}{2} (3.0 \text{ rad/s}^2) (4.0 \text{ s})^2}{4.0 \text{ s}} = 24 \text{ rad/s}.$$

We now use $\omega = \omega_0 + \alpha t$ (Eq. 10-12) to find the time when the wheel is at rest:

$$t = -\frac{\omega_0}{\alpha} = -\frac{24 \text{ rad/s}}{3.0 \text{ rad/s}^2} = -8.0 \text{ s}.$$

That is, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

21. (a) We obtain

$$\omega = \frac{(200 \text{ rev/min})(2\pi \text{ rad/rev})}{60 \text{ s/min}} = 20.9 \text{ rad/s}.$$

(b) With $r = 1.20/2 = 0.60$ m, Eq. 10-18 leads to $v = r\omega = (0.60)(20.9) = 12.5$ m/s.

(c) With $t = 1$ min, $\omega = 1000$ rev/min and $\omega_0 = 200$ rev/min, Eq. 10-12 gives

$$\alpha = \frac{\omega - \omega_0}{t} = 800 \text{ rev/min}^2.$$

(d) With the same values used in part (c), Eq. 10-15 becomes

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(200 + 1000)(1) = 600 \text{ rev}.$$

29. (a) Earth makes one rotation per day and 1 d is $(24 \text{ h})(3600 \text{ s/h}) = 8.64 \times 10^4$ s, so the angular speed of Earth is

$$\omega = \frac{2\pi \text{ rad}}{8.64 \times 10^4 \text{ s}} = 7.3 \times 10^{-5} \text{ rad/s}.$$

(b) We use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R \cos 40^\circ$, where R is the radius of Earth (6.4×10^6 m). Therefore, its speed is

$$v = \omega(R \cos 40^\circ) = (7.3 \times 10^{-5} \text{ rad/s})(6.4 \times 10^6 \text{ m}) \cos 40^\circ = 3.5 \times 10^2 \text{ m/s}.$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.3×10^{-5} rad/s).

(d) The latitude is 0° and the speed is

$$v = \omega R = (7.3 \times 10^{-5} \text{ rad/s})(6.4 \times 10^6 \text{ m}) = 4.6 \times 10^2 \text{ m/s}.$$

33. The kinetic energy (in J) is given by $K = \frac{1}{2} I \omega^2$, where I is the rotational inertia (in $\text{kg} \cdot \text{m}^2$) and ω is the angular velocity (in rad/s). We have

$$\omega = \frac{(602 \text{ rev / min})(2\pi \text{ rad / rev})}{60 \text{ s / min}} = 63.0 \text{ rad / s}.$$

Consequently, the rotational inertia is

$$I = \frac{2K}{\omega^2} = \frac{2(24400 \text{ J})}{(63.0 \text{ rad / s})^2} = 12.3 \text{ kg} \cdot \text{m}^2.$$

35. We use the parallel axis theorem: $I = I_{\text{com}} + Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 10-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies $h = 0.50 \text{ m} - 0.20 \text{ m} = 0.30 \text{ m}$. We find

$$I_{\text{com}} = \frac{1}{12} ML^2 = \frac{1}{12} (0.56 \text{ kg})(1.0 \text{ m})^2 = 4.67 \times 10^{-2} \text{ kg} \cdot \text{m}^2.$$

Consequently, the parallel axis theorem yields

$$I = 4.67 \times 10^{-2} \text{ kg} \cdot \text{m}^2 + (0.56 \text{ kg})(0.30 \text{ m})^2 = 9.7 \times 10^{-2} \text{ kg} \cdot \text{m}^2.$$

37. Since the rotational inertia of a cylinder is $I = \frac{1}{2} MR^2$ (Table 10-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2} I \omega^2 = \frac{1}{4} MR^2 \omega^2.$$

(a) For the smaller cylinder, we have $K = \frac{1}{4} (1.25)(0.25)^2 (235)^2 = 1.1 \times 10^3 \text{ J}$.

(b) For the larger cylinder, we obtain $K = \frac{1}{4} (1.25)(0.75)^2 (235)^2 = 9.7 \times 10^3 \text{ J}$.

41. We use the parallel-axis theorem. According to Table 10-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{\text{com}} = \frac{M}{12}(a^2 + b^2).$$

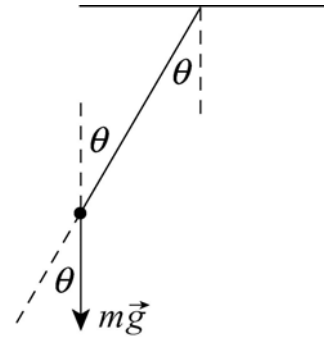
A parallel axis through the corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center. Therefore,

$$\begin{aligned} I &= I_{\text{com}} + Mh^2 = \frac{M}{12}(a^2 + b^2) + \frac{M}{4}(a^2 + b^2) = \frac{M}{3}(a^2 + b^2) \\ &= \frac{0.172 \text{ kg}}{3}[(0.035 \text{ m})^2 + (0.084 \text{ m})^2] = 4.7 \times 10^{-4} \text{ kg} \cdot \text{m}^2. \end{aligned}$$

45. Two forces act on the ball, the force of the rod and the force of gravity. No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is $mg \sin \theta$. If ℓ is the length of the rod, then the torque associated with this force has magnitude

$$\tau = mg\ell \sin \theta = (0.75)(9.8)(1.25) \sin 30^\circ = 4.6 \text{ N} \cdot \text{m}.$$

For the position shown, the torque is counter-clockwise.



47. We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1 F_1 \sin \theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2 F_2 \sin \theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1 F_1 \sin \theta_1 - r_2 F_2 \sin \theta_2.$$

Substituting the given values, we obtain

$$\tau = (1.30 \text{ m})(4.20 \text{ N}) \sin 75^\circ - (2.15 \text{ m})(4.90 \text{ N}) \sin 60^\circ = -3.85 \text{ N} \cdot \text{m}.$$

49. (a) We use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity, α is the angular acceleration, and t is the time. This gives

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20 \text{ rad/s}}{220 \times 10^{-3} \text{ s}} = 28.2 \text{ rad/s}^2.$$

(b) If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is

$$\tau = I\alpha = (12.0 \text{ kg} \cdot \text{m}^2)(28.2 \text{ rad/s}^2) = 3.38 \times 10^2 \text{ N} \cdot \text{m}.$$

63. We use ℓ to denote the length of the stick. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $\frac{1}{2}mg\ell$, where m is its mass. Its initial kinetic energy is zero. Its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity just before it hits the floor. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{mg\ell}{I}}.$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 10-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}.$$

Using Table 10-2 and the parallel-axis theorem, the rotational inertia is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3(9.8 \text{ m/s}^2)(1.00 \text{ m})} = 5.42 \text{ m/s}.$$

69. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1 , m_2 and (in the form of Eq. 10-45) to M , respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2).

$$\begin{aligned} m_1g - T_1 &= m_1a_1 \\ T_2 - f_2 &= m_2a_2 \\ T_1R - T_2R &= I\alpha \end{aligned}$$

(a) From Eq. 10-13 (with $\omega_0 = 0$) we find

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow \alpha = \frac{2\theta}{t^2} = \frac{2(1.30 \text{ rad})}{(0.0910 \text{ s})^2} = 314 \text{ rad/s}^2.$$

(b) From the fact that $a = R\alpha$ (noted above), we obtain

$$a = \frac{2R\theta}{t^2} = \frac{2(0.024 \text{ m})(1.30 \text{ rad})}{(0.0910 \text{ s})^2} = 7.54 \text{ m/s}^2.$$

(c) From the first of the above equations, we find

$$T_1 = m_1(g - a_1) = M \left(g - \frac{2R\theta}{t^2} \right) = (6.20 \text{ kg}) \left(9.80 \text{ m/s}^2 - \frac{2(0.024 \text{ m})(1.30 \text{ rad})}{(0.0910 \text{ s})^2} \right) = 14.0 \text{ N}.$$

(d) From the last of the above equations, we obtain the second tension:

$$T_2 = T_1 - \frac{I\alpha}{R} = M \left(g - \frac{2R\theta}{t^2} \right) - \frac{2I\theta}{Rt^2} = 14.0 \text{ N} - \frac{(7.40 \times 10^{-4} \text{ kg} \cdot \text{m}^2)(314 \text{ rad/s}^2)}{0.024 \text{ m}} = 4.36 \text{ N}.$$

87. With rightward positive for the block and clockwise negative for the wheel (as is conventional), then we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P - T = ma$, where $m = 2.0 \text{ kg}$. Applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$, where $I = 0.050 \text{ kg} \cdot \text{m}^2$.

Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \Rightarrow T = a \frac{I}{R^2}.$$

Adding this to the above equation (for the block) leads to $P = (m + I/R^2)a$.

Thus, $a = 0.92 \text{ m/s}^2$ and therefore $\alpha = -4.6 \text{ rad/s}^2$ (or $|\alpha| = 4.6 \text{ rad/s}^2$), where the negative sign in α should not be mistaken for a deceleration (it simply indicates the clockwise sense to the motion).

89. We assume the sense of initial rotation is positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued.

(a) The angular acceleration is constant, so we can apply Eq. 10-12 ($\omega = \omega_0 + \alpha t$). To obtain the requested units, we have $t = 30/60 = 0.50 \text{ min}$. Thus,

$$\alpha = -\frac{33.33 \text{ rev/min}}{0.50 \text{ min}} = -66.7 \text{ rev/min}^2 \approx -67 \text{ rev/min}^2.$$

(b) We use Eq. 10-13: $\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = (33.33)(0.50) + \frac{1}{2}(-66.7)(0.50)^2 = 8.3 \text{ rev.}$

91. (a) According to Table 10-2, the rotational inertia formulas for the cylinder (radius R) and the hoop (radius r) are given by

$$I_C = \frac{1}{2} MR^2 \quad \text{and} \quad I_H = Mr^2.$$

Since the two bodies have the same mass, then they will have the same rotational inertia if

$$R^2 / 2 = R_H^2 \rightarrow R_H = R / \sqrt{2}.$$

(b) We require the rotational inertia to be written as $I = Mk^2$, where M is the mass of the given body and k is the radius of the “equivalent hoop.” It follows directly that $k = \sqrt{I / M}$.

115. We employ energy methods in this solution; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, so that

$$K_{\text{box}} = \frac{1}{2} m_{\text{box}} v^2 \Rightarrow v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = 1.41 \text{ m/s}$$

implies that the angular speed is $\omega = 1.41/0.20 = 0.71 \text{ rad/s}$. Thus, the kinetic energy of rotation is $\frac{1}{2} I \omega^2 = 10.0 \text{ J}$.

(b) Since it was released from rest at what we will consider to be the reference position for gravitational potential, then (with SI units understood) energy conservation requires

$$K_0 + U_0 = K + U \Rightarrow 0 + 0 = (6.0 + 10.0) + m_{\text{box}} g(-h).$$

Therefore, $h = 16.0/58.8 = 0.27 \text{ m}$.