

Chapter 11 – Student Solutions Manual

5. By Eq. 10-52, the work required to stop the hoop is the negative of the initial kinetic energy of the hoop. The initial kinetic energy is $K = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$ (Eq. 11-5), where $I = m R^2$ is its rotational inertia about the center of mass, $m = 140$ kg, and $v = 0.150$ m/s is the speed of its center of mass. Eq. 11-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K = \frac{1}{2} m R^2 \left(\frac{v^2}{R^2} \right) + \frac{1}{2} m v^2 = m v^2 = (140 \text{ kg})(0.150 \text{ m/s})^2$$

which implies that the work required is -3.15 J.

23. If we write $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}.$$

(a) Plugging in, we find $\vec{\tau} = [(3.0\text{ m})(6.0\text{ N}) - (4.0\text{ m})(-8.0\text{ N})]\hat{k} = (50 \text{ N} \cdot \text{m})\hat{k}$.

(b) We use Eq. 3-27, $|\vec{r} \times \vec{F}| = rF \sin \phi$, where ϕ is the angle between $\vec{r}$ and $\vec{F}$. Now $r = \sqrt{x^2 + y^2} = 5.0$ m and $F = \sqrt{F_x^2 + F_y^2} = 10$ N. Thus,

$$rF = (5.0 \text{ m})(10 \text{ N}) = 50 \text{ N} \cdot \text{m},$$

the same as the magnitude of the vector product calculated in part (a). This implies $\sin \phi = 1$ and $\phi = 90^\circ$.

29. (a) We use $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the object, $\vec{v}$ is its velocity vector, and m is its mass. Only the x and z components of the position and velocity vectors are nonzero, so Eq. 3-30 leads to $\vec{r} \times \vec{v} = (-xv_z + zv_x)\hat{j}$. Therefore,

$$\vec{\ell} = m(-xv_z + zv_x)\hat{j} = (0.25 \text{ kg})(-(2.0 \text{ m})(5.0 \text{ m/s}) + (-2.0 \text{ m})(-5.0 \text{ m/s}))\hat{j} = 0.$$

(b) If we write $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}.$$

With $x = 2.0$, $z = -2.0$, $F_y = 4.0$ and all other components zero (and SI units understood) the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = (8.0\hat{i} + 8.0\hat{k}) \text{ N} \cdot \text{m}.$$

33. If we write (for the general case) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{v}$ is equal to

$$(yv_z - zv_y)\hat{i} + (zv_x - xv_z)\hat{j} + (xv_y - yv_x)\hat{k}.$$

(a) The angular momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the particle, $\vec{v}$ is its velocity, and $m = 3.0 \text{ kg}$ is its mass. Substituting (with SI units understood) $x = 3, y = 8, z = 0, v_x = 5, v_y = -6$ and $v_z = 0$ into the above expression, we obtain

$$\vec{\ell} = (3.0)[(3.0)(-6.0) - (8.0)(5.0)]\hat{k} = (-1.7 \times 10^2 \text{ kg} \cdot \text{m}^2/\text{s})\hat{k}.$$

(b) The torque is given by Eq. 11-14, $\vec{\tau} = \vec{r} \times \vec{F}$. We write $\vec{r} = x\hat{i} + y\hat{j}$ and $\vec{F} = F_x\hat{i}$ and obtain

$$\vec{\tau} = (x\hat{i} + y\hat{j}) \times (F_x\hat{i}) = -yF_x\hat{k}$$

since $\hat{i} \times \hat{i} = 0$ and $\hat{j} \times \hat{i} = -\hat{k}$. Thus, we find

$$\vec{\tau} = -(8.0\text{m})(-7.0\text{N})\hat{k} = (56 \text{ N} \cdot \text{m})\hat{k}.$$

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is $56 \text{ kg} \cdot \text{m}^2/\text{s}^2$, in the positive z direction.

37. (a) Since $\tau = dL/dt$, the average torque acting during any interval Δt is given by $\tau_{\text{avg}} = (L_f - L_i)/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. Thus,

$$\tau_{\text{avg}} = \frac{0.800 \text{ kg} \cdot \text{m}^2/\text{s} - 3.00 \text{ kg} \cdot \text{m}^2/\text{s}}{1.50\text{s}} = -1.47 \text{ N} \cdot \text{m},$$

or $|\tau_{\text{avg}}| = 1.47 \text{ N} \cdot \text{m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) The angle turned is $\theta = \omega_0 t + \alpha t^2 / 2$. If the angular acceleration α is uniform, then so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\theta = \frac{L_i t + \tau t^2 / 2}{I} = \frac{(3.00 \text{ kg} \cdot \text{m}^2 / \text{s})(1.50 \text{ s}) + (-1.467 \text{ N} \cdot \text{m})(1.50 \text{ s})^2 / 2}{0.140 \text{ kg} \cdot \text{m}^2} = 20.4 \text{ rad}.$$

(c) The work done on the wheel is

$$W = \tau \theta = (-1.47 \text{ N} \cdot \text{m})(20.4 \text{ rad}) = -29.9 \text{ J}$$

where more precise values are used in the calculation than what is shown here. An equally good method for finding W is Eq. 10-52, which, if desired, can be rewritten as

$$W = (L_f^2 - L_i^2) / 2I.$$

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

43. (a) No external torques act on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved. Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i \omega_i = I_f \omega_f$ and

$$\omega_f = \left(\frac{I_i}{I_f} \right) \omega_i = \left(\frac{6.0 \text{ kg} \cdot \text{m}^2}{2.0 \text{ kg} \cdot \text{m}^2} \right) (1.2 \text{ rev/s}) = 3.6 \text{ rev/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2} I_i \omega_i^2$, the final kinetic energy is $K_f = \frac{1}{2} I_f \omega_f^2$, and their ratio is

$$\frac{K_f}{K_i} = \frac{I_f \omega_f^2 / 2}{I_i \omega_i^2 / 2} = \frac{(2.0 \text{ kg} \cdot \text{m}^2)(3.6 \text{ rev/s})^2 / 2}{(6.0 \text{ kg} \cdot \text{m}^2)(1.2 \text{ rev/s})^2 / 2} = 3.0.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.

45. (a) No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved. Let I_1 be the rotational inertia of the wheel that is originally spinning (at ω_i) and I_2 be the rotational inertia of the wheel that is initially at rest. Then $I_1 \omega_i = (I_1 + I_2) \omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2} \omega_i$$

where ω_f is the common final angular velocity of the wheels. Substituting $I_2 = 2I_1$ and $\omega_i = 800 \text{ rev/min}$, we obtain $\omega_f = 267 \text{ rev/min}$.

(b) The initial kinetic energy is $K_i = \frac{1}{2} I_1 \omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2} (I_1 + I_2) \omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2} (I_1 + 2I_1) \left(\frac{I_1 \omega_i}{I_1 + 2I_1} \right)^2 = \frac{1}{6} I \omega_i^2.$$

Therefore, the fraction lost, $(K_i - K_f)/K_i$ is

$$1 - \frac{K_f}{K_i} = 1 - \frac{I \omega_i^2 / 6}{I \omega_i^2 / 2} = \frac{2}{3} = 0.667.$$

49. No external torques act on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero. Let $I = MR^2$ be the rotational inertia of the wheel. Its final angular momentum is

$$\vec{L}_f = I \omega \hat{k} = -MR^2 |\omega| \hat{k},$$

where $\hat{k}$ is up in Fig. 11-47 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω . The linear speed of a point on the track is ωR and the speed of the train (going counterclockwise in Fig. 11-47 with speed v' relative to an outside observer) is therefore $v' = v - |\omega| R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $m(v - |\omega| R) R \hat{k}$. Conservation of angular momentum yields

$$0 = -MR^2 |\omega| \hat{k} + m(v - |\omega| R) R \hat{k}.$$

When this equation is solved for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{v}{(M/m+1)R} = \frac{(0.15 \text{ m/s})}{(1.1+1)(0.43 \text{ m})} = 0.17 \text{ rad/s}.$$

67. (a) If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad. The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where $d = 0.500 \text{ m}$ is the length of the rod. The angular momentum of the wad is $mv d/2$ where $m = 0.0500 \text{ kg}$ and $v = 3.00 \text{ m/s}$ are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls and the wad at its end. Conservation of angular momentum yields $mv d/2 = I\omega$ where

$$I = (2M + m)(d/2)^2$$

and $M = 2.00$ kg is the mass of each of the balls. We solve

$$mvd/2 = (2M + m)(d/2)^2 \omega$$

for the angular speed:

$$\omega = \frac{2mv}{(2M + m)d} = \frac{2(0.0500 \text{ kg})(3.00 \text{ m/s})}{(2(2.00 \text{ kg}) + 0.0500 \text{ kg})(0.500 \text{ m})} = 0.148 \text{ rad/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is $K_f/K_i = I\omega^2/mv^2$. When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, this becomes

$$\frac{K_f}{K_i} = \frac{m}{2M + m} = \frac{0.0500 \text{ kg}}{2(2.00 \text{ kg}) + 0.0500 \text{ kg}} = 0.0123.$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h , the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ , losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mgd/2$. If it swings up to the angular position θ , as measured from its lowest point, then its final height is $(d/2)(1 - \cos \theta)$ above the lowest point and its final potential energy is

$$U_f = mg(d/2)(1 - \cos \theta).$$

The initial kinetic energy is the sum of that of the balls and wad:

$$K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M + m)(d/2)^2 \omega^2.$$

At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$mg\frac{d}{2} + \frac{1}{2}(2M + m)\left(\frac{d}{2}\right)^2 \omega^2 = mg\frac{d}{2}(1 - \cos \theta).$$

When this equation is solved for $\cos \theta$, the result is

$$\cos \theta = -\frac{1}{2} \left(\frac{2M + m}{mg} \right) \left(\frac{d}{2} \right) \omega^2 = -\frac{1}{2} \left(\frac{2(2.00 \text{ kg}) + 0.0500 \text{ kg}}{(0.0500 \text{ kg})(9.8 \text{ m/s}^2)} \right) \left(\frac{0.500 \text{ m}}{2} \right) (0.148 \text{ rad/s})^2$$

$$= -0.0226.$$

Consequently, the result for θ is 91.3° . The total angle through which it has swung is $90^\circ + 91.3^\circ = 181^\circ$.

73. (a) The diagram below shows the particles and their lines of motion. The origin is marked O and may be anywhere. The angular momentum of particle 1 has magnitude

$$\ell_1 = mvr_1 \sin \theta_1 = mv(d + h)$$

and it is into the page. The angular momentum of particle 2 has magnitude

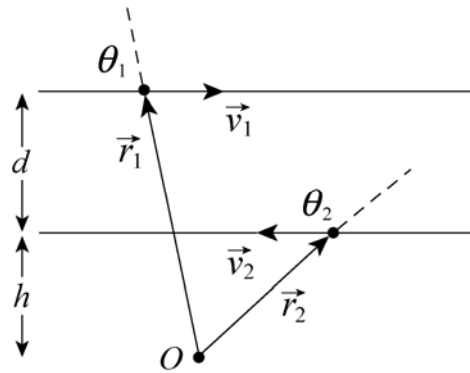
$$\ell_2 = mvr_2 \sin \theta_2 = mvh$$

and it is out of the page. The net angular momentum has magnitude

$$L = mv(d + h) - mvh = mvd$$

$$= (2.90 \times 10^{-4} \text{ kg})(5.46 \text{ m/s})(0.042 \text{ m})$$

$$= 6.65 \times 10^{-5} \text{ kg} \cdot \text{m}^2/\text{s}.$$



and is into the page. This result is independent of the location of the origin.

(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then

$$L = mv(d + h) + mvh = mv(d + 2h).$$

This result depends on h , the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $L = 0$.

(d) As we have seen in part (c), the result depends on the choice of origin.

77. As the wheel-axle system rolls down the inclined plane by a distance d , the decrease in potential energy is $\Delta U = mgd \sin \theta$. This must be equal to the total kinetic energy gained:

$$mgd \sin \theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Since the axel rolls without slipping, the angular speed is given by $\omega = v/r$, where r is the radius of the axel. The above equation then becomes

$$mgd \sin \theta = \frac{1}{2} I \omega^2 \left(\frac{mr^2}{I} + 1 \right) = K_{\text{rot}} \left(\frac{mr^2}{I} + 1 \right)$$

(a) With $m=10.0$ kg, $d = 2.00$ m, $r = 0.200$ m, and $I = 0.600$ kg $\cdot$ m², $mr^2/I = 2/3$, the rotational kinetic energy may be obtained as $98 \text{ J} = K_{\text{rot}} (5/3)$, or $K_{\text{rot}} = 58.8 \text{ J}$.

(b) The translational kinetic energy is $K_{\text{trans}} = (98 - 58.8) \text{ J} = 39.2 \text{ J}$.

85. (a) In terms of the radius of gyration k , the rotational inertia of the merry-go-round is $I = Mk^2$. We obtain

$$I = (180 \text{ kg}) (0.910 \text{ m})^2 = 149 \text{ kg} \cdot \text{m}^2.$$

(b) An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 11-21, mvR , where R is the radius of the merry-go-round. Therefore,

$$|\vec{L}_{\text{child}}| = (44.0 \text{ kg})(3.00 \text{ m/s})(1.20 \text{ m}) = 158 \text{ kg} \cdot \text{m}^2 / \text{s}.$$

(c) No external torques act on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved. The initial angular momentum is given by mvR ; the final angular momentum is given by $(I + mR^2) \omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = (I + mR^2) \omega$ and

$$\omega = \frac{mvR}{I + mR^2} = \frac{158 \text{ kg} \cdot \text{m}^2 / \text{s}}{149 \text{ kg} \cdot \text{m}^2 + (44.0 \text{ kg})(1.20 \text{ m})^2} = 0.744 \text{ rad/s}.$$

87. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r}' = x'\hat{i} + y'\hat{j}$, then (using Eq. 3-30) we find

$$\vec{r}' \times \vec{v} = (x'v_y - y'v_x)\hat{k}.$$

(a) Here, $\vec{r}'$ points in either the $+\hat{i}$ or the $-\hat{i}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{i} \times \hat{i} = 0$) that $\vec{\ell} = m(\vec{r}' \times \vec{v}) = 0$ in this case.

(b) The net force is in the $-\hat{i}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\vec{\tau} = \vec{r}' \times \vec{F} = 0$.

(c) Now, $\vec{r}' = \vec{r} - \vec{r}_0$ where $\vec{r}_0 = 2.0\hat{i} + 5.0\hat{j}$ (with SI units understood) and points from (2.0, 5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m(\vec{r}' \times \vec{v}) = -m(\vec{r}_0 \times \vec{v}) = -(3.0)\left((2.0)(0) - (5.0)(-2.0t^3)\right)\hat{k}$$

which yields $\vec{\ell} = -30t^3\hat{k}$ in SI units ($\text{kg} \cdot \text{m}^2/\text{s}$).

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\hat{i}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m(\vec{r}' \times \vec{a}) = -m(\vec{r}_0 \times \vec{a}) = -(3.0)\left((2.0)(0) - (5.0)(-6.0t^2)\right)\hat{k}$$

which yields $\vec{\tau} = -90t^2\hat{k}$ in SI units ($\text{N} \cdot \text{m}$).

(e) In this situation, $\vec{r}' = \vec{r} - \vec{r}_0$ where $\vec{r}_0 = 2.0\hat{i} - 5.0\hat{j}$ (with SI units understood) and points from (2.0, -5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m(\vec{r}' \times \vec{v}) = -m(\vec{r}_0 \times \vec{v}) = -(3.0)\left((2.0)(0) - (-5.0)(-2.0t^3)\right)\hat{k}$$

which yields $\vec{\ell} = 30t^3\hat{k}$ in SI units ($\text{kg} \cdot \text{m}^2/\text{s}$).

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\hat{i}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m(\vec{r}' \times \vec{a}) = -m(\vec{r}_0 \times \vec{a}) = -(3.0)\left((2.0)(0) - (-5.0)(-6.0t^2)\right)\hat{k}$$

which yields $\vec{\tau} = 90t^2\hat{k}$ in SI units ($\text{N} \cdot \text{m}$).

95. We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration is positive (as is the linear acceleration of the center of mass, since we take rightwards as positive).

(a) We approach this in the manner of Eq. 11-3 (*pure rotation* about point P) but use torques instead of energy. The torque (relative to point P) is $\tau = I_P \alpha$, where

$$I_P = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

with the use of the parallel-axis theorem and Table 10-2(c). The torque is due to the $F_{\text{app}} = 12 \text{ N}$ force and can be written as $\tau = F_{\text{app}}(2R)$. In this way, we find

$$\tau = I_P \alpha = \left(\frac{3}{2}MR^2 \right) \alpha = 2RF_{\text{app}}$$

which leads to

$$\alpha = \frac{2RF_{\text{app}}}{3MR^2/2} = \frac{4F_{\text{app}}}{3MR} = \frac{4(12 \text{ N})}{3(10 \text{ kg})(0.10 \text{ m})} = 16 \text{ rad/s}^2.$$

Hence, $a_{\text{com}} = R\alpha = 1.6 \text{ m/s}^2$.

(b) As shown above, $\alpha = 16 \text{ rad/s}^2$.

(c) Applying Newton's second law in its linear form yields $(12 \text{ N}) - f = Ma_{\text{com}}$.

Therefore, $f = -4.0 \text{ N}$. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* with magnitude 4.0 N , i.e., $\vec{f} = (4.0 \text{ N})\hat{i}$.