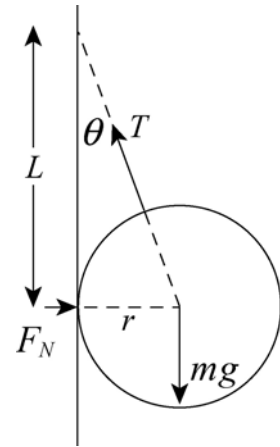


Chapter 12 – Student Solutions Manual

5. Three forces act on the sphere: the tension force $\vec{T}$ of the rope (acting along the rope), the force of the wall $\vec{F}_N$ (acting horizontally away from the wall), and the force of gravity $m\vec{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then Newton's second law gives

$$\text{vertical component : } T \cos \theta - mg = 0$$

$$\text{horizontal component: } F_N - T \sin \theta = 0.$$



(a) We solve the first equation for the tension: $T = mg / \cos \theta$. We substitute $\cos \theta = L / \sqrt{L^2 + r^2}$ to obtain

$$T = \frac{mg\sqrt{L^2 + r^2}}{L} = \frac{(0.85 \text{ kg})(9.8 \text{ m/s}^2)\sqrt{(0.080 \text{ m})^2 + (0.042 \text{ m})^2}}{0.080 \text{ m}} = 9.4 \text{ N}.$$

(b) We solve the second equation for the normal force: $F_N = T \sin \theta$.

Using $\sin \theta = r / \sqrt{L^2 + r^2}$, we obtain

$$F_N = \frac{Tr}{\sqrt{L^2 + r^2}} = \frac{mg\sqrt{L^2 + r^2}}{L} \frac{r}{\sqrt{L^2 + r^2}} = \frac{mgr}{L} = \frac{(0.85 \text{ kg})(9.8 \text{ m/s}^2)(0.042 \text{ m})}{(0.080 \text{ m})} = 4.4 \text{ N}.$$

7. We take the force of the left pedestal to be F_1 at $x = 0$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = d$. W is the weight of the diver, located at $x = L$. The following two equations result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

$$F_1 + F_2 - W = 0$$

$$F_1 d + W(L - d) = 0$$

(a) The second equation gives

$$F_1 = -\frac{L-d}{d}W = -\left(\frac{3.0 \text{ m}}{1.5 \text{ m}}\right)(580 \text{ N}) = -1160 \text{ N}$$

which should be rounded off to $F_1 = -1.2 \times 10^3 \text{ N}$. Thus, $|F_1| = 1.2 \times 10^3 \text{ N}$.

(b) Since F_1 is negative, indicating that this force is downward.

(c) The first equation gives $F_2 = W - F_1 = 580 \text{ N} + 1160 \text{ N} = 1740 \text{ N}$

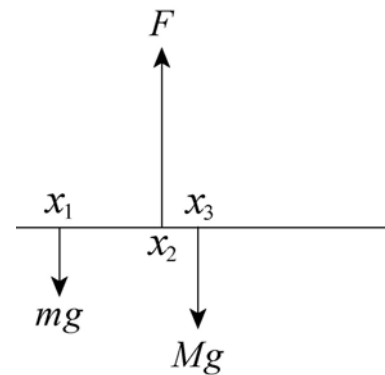
which should be rounded off to $F_2 = 1.7 \times 10^3 \text{ N}$. Thus, $|F_2| = 1.7 \times 10^3 \text{ N}$.

(d) The result is positive, indicating that this force is upward.

(e) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched.

(f) The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

11. The x axis is along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown on the diagram below. The nickels are at $x = x_1 = 0.120 \text{ m}$, and m is their total mass. The knife edge is at $x = x_2 = 0.455 \text{ m}$ and exerts force $\vec{F}$. The mass of the meter stick is M , and the force of gravity acts at the center of the stick, $x = x_3 = 0.500 \text{ m}$. Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish:

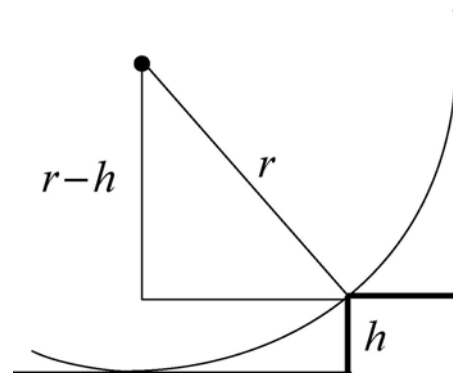
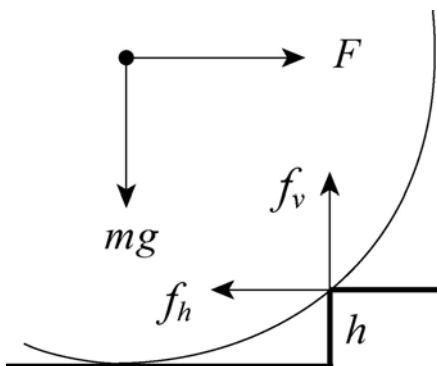


$$Mg(x_3 - x_2) - mg(x_2 - x_1) = 0.$$

Thus,

$$M = \frac{x_2 - x_1}{x_3 - x_2} m = \left(\frac{0.455 \text{ m} - 0.120 \text{ m}}{0.500 \text{ m} - 0.455 \text{ m}} \right) (10.0 \text{ g}) = 74.4 \text{ g}.$$

21. We consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v . If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.



We calculate the torque around the step corner. The second diagram indicates that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step.

The distance from the line of mg to the corner is $\sqrt{r^2 + (r - h)^2} = \sqrt{2rh - h^2}$. Thus,

$$F(r - h) - mg\sqrt{2rh - h^2} = 0.$$

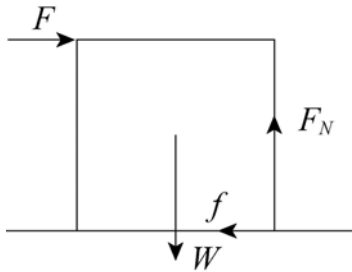
The solution for F is

$$F = \frac{\sqrt{2rh - h^2}}{r - h} mg = \frac{\sqrt{2(6.00 \times 10^{-2} \text{ m})(3.00 \times 10^{-2} \text{ m}) - (3.00 \times 10^{-2} \text{ m})^2}}{(6.00 \times 10^{-2} \text{ m}) - (3.00 \times 10^{-2} \text{ m})} (0.800 \text{ kg})(9.80 \text{ m/s}^2) = 13.6 \text{ N}.$$

33. We examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled F_N on the diagram below. The force of friction is denoted by f , the applied force by F , and the force of gravity by W . Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes:

$$FL - WL/2 = 0.$$

Here L is the length of a side of the box and the origin was chosen to be at the lower right edge.



(a) From the torque equation, we find

$$F = \frac{W}{2} = \frac{890 \text{ N}}{2} = 445 \text{ N}.$$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the equations of equilibrium $F_N = W = 890 \text{ N}$ and $f = F = 445 \text{ N}$, so

$$\mu_s = \frac{445 \text{ N}}{890 \text{ N}} = 0.50.$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes

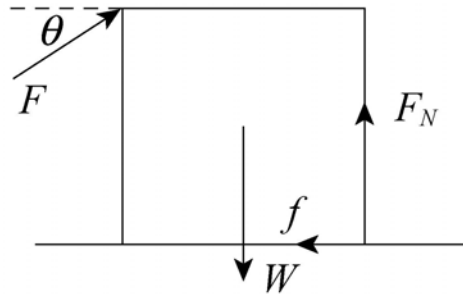
$$FL \cos \theta + FL \sin \theta - WL/2 = 0,$$

with the solution

$$F = \frac{W}{2(\cos \theta + \sin \theta)}.$$

We want $\cos \theta + \sin \theta$ to have the largest possible value. This occurs if $\theta = 45^\circ$, a result we can prove by setting the derivative of $\cos \theta + \sin \theta$ equal to zero and solving for θ . The minimum force needed is

$$F = \frac{W}{4 \cos 45^\circ} = \frac{890 \text{ N}}{4 \cos 45^\circ} = 315 \text{ N}.$$



43. (a) The shear stress is given by F/A , where F is the magnitude of the force applied parallel to one face of the aluminum rod and A is the cross-sectional area of the rod. In this case F is the weight of the object hung on the end: $F = mg$, where m is the mass of the object. If r is the radius of the rod then $A = \pi r^2$. Thus, the shear stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1200 \text{ kg})(9.8 \text{ m/s}^2)}{\pi (0.024 \text{ m})^2} = 6.5 \times 10^6 \text{ N/m}^2.$$

(b) The shear modulus G is given by

$$G = \frac{F / A}{\Delta x / L}$$

where L is the protrusion of the rod and Δx is its vertical deflection at its end. Thus,

$$\Delta x = \frac{(F/A)L}{G} = \frac{(6.5 \times 10^6 \text{ N/m}^2)(0.053 \text{ m})}{3.0 \times 10^{10} \text{ N/m}^2} = 1.1 \times 10^{-5} \text{ m}.$$

55. (a) The forces acting on bucket are the force of gravity, down, and the tension force of cable A, up. Since the bucket is in equilibrium and its weight is

$$W_B = m_B g = (817 \text{ kg})(9.80 \text{ m/s}^2) = 8.01 \times 10^3 \text{ N},$$

the tension force of cable A is $T_A = 8.01 \times 10^3 \text{ N}$.

(b) We use the coordinates axes defined in the diagram. Cable A makes an angle of $\theta_2 = 66.0^\circ$ with the negative y axis, cable B makes an angle of 27.0° with the positive y axis, and cable C is along the x axis. The y components of the forces must sum to zero since the knot is in equilibrium. This means $T_B \cos 27.0^\circ - T_A \cos 66.0^\circ = 0$ and

$$T_B = \frac{\cos 66.0^\circ}{\cos 27.0^\circ} T_A = \left(\frac{\cos 66.0^\circ}{\cos 27.0^\circ} \right) (8.01 \times 10^3 \text{ N}) = 3.65 \times 10^3 \text{ N}.$$

(c) The x components must also sum to zero. This means $T_C + T_B \sin 27.0^\circ - T_A \sin 66.0^\circ = 0$ and

$$\begin{aligned} T_C &= T_A \sin 66.0^\circ - T_B \sin 27.0^\circ = (8.01 \times 10^3 \text{ N}) \sin 66.0^\circ - (3.65 \times 10^3 \text{ N}) \sin 27.0^\circ \\ &= 5.66 \times 10^3 \text{ N}. \end{aligned}$$

61. We denote the mass of the slab as m , its density as ρ , and volume as $V = LTW$. The angle of inclination is $\theta = 26^\circ$.

(a) The component of the weight of the slab along the incline is

$$\begin{aligned} F_1 &= mg \sin \theta = \rho V g \sin \theta \\ &= (3.2 \times 10^3 \text{ kg/m}^3)(43 \text{ m})(2.5 \text{ m})(12 \text{ m})(9.8 \text{ m/s}^2) \sin 26^\circ \approx 1.8 \times 10^7 \text{ N}. \end{aligned}$$

(b) The static force of friction is

$$\begin{aligned} f_s &= \mu_s F_N = \mu_s mg \cos \theta = \mu_s \rho V g \cos \theta \\ &= (0.39)(3.2 \times 10^3 \text{ kg/m}^3)(43 \text{ m})(2.5 \text{ m})(12 \text{ m})(9.8 \text{ m/s}^2) \cos 26^\circ \approx 1.4 \times 10^7 \text{ N}. \end{aligned}$$

(c) The minimum force needed from the bolts to stabilize the slab is

$$F_2 = F_1 - f_s = 1.77 \times 10^7 \text{ N} - 1.42 \times 10^7 \text{ N} = 3.5 \times 10^6 \text{ N}.$$

If the minimum number of bolts needed is n , then $F_2 / nA \leq 3.6 \times 10^8 \text{ N/m}^2$, or

$$n \geq \frac{3.5 \times 10^6 \text{ N}}{(3.6 \times 10^8 \text{ N/m}^2)(6.4 \times 10^{-4} \text{ m}^2)} = 15.2$$

Thus 16 bolts are needed.

63. Analyzing forces at the knot (particularly helpful is a graphical view of the vector right-triangle with horizontal “side” equal to the static friction force f_s and vertical “side” equal to the weight $m_B g$ of block B), we find $f_s = m_B g \tan \theta$ where $\theta = 30^\circ$. For f_s to be at its maximum value, then it must equal $\mu_s m_A g$ where the weight of block A is $m_A g = (10 \text{ kg})(9.8 \text{ m/s}^2)$. Therefore,

$$\mu_s m_A g = m_B g \tan \theta \Rightarrow \mu_s = \frac{5.0}{10} \tan 30^\circ = 0.29.$$

65. With the pivot at the hinge, Eq. 12-9 leads to

$$-mg \sin \theta_1 \frac{L}{2} + TL \sin(180^\circ - \theta_1 - \theta_2) = 0.$$

where $\theta_1 = 60^\circ$ and $T = mg/2$. This yields $\theta_2 = 60^\circ$.

81. When it is about to move, we are still able to apply the equilibrium conditions, but (to obtain the critical condition) we set static friction equal to its maximum value and picture the normal force $\vec{F}_N$ as a concentrated force (upward) at the bottom corner of the cube, directly below the point O where P is being applied. Thus, the line of action of $\vec{F}_N$ passes through point O and exerts no torque about O (of course, a similar observation applied to the pull P). Since $F_N = mg$ in this problem, we have $f_{s\max} = \mu mg$ applied a distance h away from O . And the line of action of force of gravity (of magnitude mg), which is best pictured as a concentrated force at the center of the cube, is a distance $L/2$ away from O . Therefore, equilibrium of torques about O produces

$$\mu mgh = mg \left(\frac{L}{2} \right) \Rightarrow \mu = \frac{L}{2h} = \frac{(8.0 \text{ cm})}{2(7.0 \text{ cm})} = 0.57$$

for the critical condition we have been considering. We now interpret this in terms of a range of values for μ .

(a) For it to slide but not tip, a value of μ *less* than that derived above is needed, since then — static friction will be exceeded for a smaller value of P , before the pull is strong enough to cause it to tip. Thus, $\mu < L/2h = 0.57$ is required.

(b) And for it to tip but not slide, we need μ *greater* than that derived above is needed, since now — static friction will not be exceeded even for the value of P which makes the

cube rotate about its front lower corner. That is, we need to have $\mu > L/2h = 0.57$ in this case.

85. We choose an axis through the top (where the ladder comes into contact with the wall), perpendicular to the plane of the figure and take torques that would cause counterclockwise rotation as positive. Note that the line of action of the applied force $\vec{F}$ intersects the wall at a height of $(8.0 \text{ m})/5 = 1.6 \text{ m}$; in other words, the *moment arm* for the applied force (in terms of where we have chosen the axis) is $r_{\perp} = (4/5)(8.0 \text{ m}) = 6.4 \text{ m}$. The moment arm for the weight is half the horizontal distance from the wall to the base of the ladder; this works out to be $\sqrt{(10 \text{ m})^2 - (8 \text{ m})^2} / 2 = 3.0 \text{ m}$. Similarly, the moment arms for the x and y components of the force at the ground ($\vec{F}_g$) are 8.0 m and 6.0 m , respectively. Thus, with lengths in meters, we have

$$\sum \tau_z = F(6.4 \text{ m}) + W(3.0 \text{ m}) + F_{gx}(8.0 \text{ m}) - F_{gy}(6.0 \text{ m}) = 0.$$

In addition, from balancing the vertical forces we find that $W = F_{gy}$ (keeping in mind that the wall has no friction). Therefore, the above equation can be written as

$$\sum \tau_z = F(6.4 \text{ m}) + W(3.0 \text{ m}) + F_{gx}(8.0 \text{ m}) - W(6.0 \text{ m}) = 0.$$

(a) With $F = 50 \text{ N}$ and $W = 200 \text{ N}$, the above equation yields $F_{gx} = 35 \text{ N}$. Thus, in unit vector notation we obtain

$$\vec{F}_g = (35 \text{ N})\hat{i} + (200 \text{ N})\hat{j}.$$

(b) With $F = 150 \text{ N}$ and $W = 200 \text{ N}$, the above equation yields $F_{gx} = -45 \text{ N}$. Therefore, in unit vector notation we obtain

$$\vec{F}_g = (-45 \text{ N})\hat{i} + (200 \text{ N})\hat{j}.$$

(c) Note that the phrase “start to move towards the wall” implies that the friction force is pointed away from the wall (in the $-\hat{i}$ direction). Now, if $f = -F_{gx}$ and $F_N = F_{gy} = 200 \text{ N}$ are related by the (maximum) static friction relation ($f = f_{s,\max} = \mu_s F_N$) with $\mu_s = 0.38$, then we find $F_{gx} = -76 \text{ N}$. Returning this to the above equation, we obtain

$$F = \frac{(200 \text{ N})(3.0 \text{ m}) + (76 \text{ N})(8.0 \text{ m})}{6.4 \text{ m}} = 1.9 \times 10^2 \text{ N}.$$