

Chapter 13 – Student Solutions Manual

1. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r :

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.2 \text{ kg})(2.4 \text{ kg})}{2.3 \times 10^{-12} \text{ N}}} = 19 \text{ m}$$

7. At the point where the forces balance $GM_em/r_1^2 = GM_sm/r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{(d - r_1)^2}.$$

Taking the positive square root of both sides, we solve for r_1 . A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s} + \sqrt{M_e}} = \frac{(150 \times 10^9 \text{ m})\sqrt{5.98 \times 10^{24} \text{ kg}}}{\sqrt{1.99 \times 10^{30} \text{ kg}} + \sqrt{5.98 \times 10^{24} \text{ kg}}} = 2.60 \times 10^8 \text{ m}.$$

Values for M_e , M_s , and d can be found in Appendix C.

17. The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain $a_g = GM/(R + h)^2$. We solve for h and obtain $h = \sqrt{GM/a_g} - R$. According to Appendix C, $R = 6.37 \times 10^6 \text{ m}$ and $M = 5.98 \times 10^{24} \text{ kg}$, so

$$h = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3 / \text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{(4.9 \text{ m/s}^2)}} - 6.37 \times 10^6 \text{ m} = 2.6 \times 10^6 \text{ m}.$$

29. (a) The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. The ratio of the density of Mars to the density of Earth is

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E} \frac{R_E^3}{R_M^3} = 0.11 \left(\frac{0.65 \times 10^4 \text{ km}}{3.45 \times 10^3 \text{ km}} \right)^3 = 0.74.$$

(b) The value of a_g at the surface of a planet is given by $a_g = GM/R^2$, so the value for Mars is

$$a_g M = \frac{M_M}{M_E} \frac{R_E^2}{R_M^2} a_{gE} = 0.11 \left(\frac{0.65 \times 10^4 \text{ km}}{3.45 \times 10^3 \text{ km}} \right)^2 (9.8 \text{ m/s}^2) = 3.8 \text{ m/s}^2.$$

(c) If v is the escape speed, then, for a particle of mass m

$$\frac{1}{2}mv^2 = G \frac{mM}{R} \Rightarrow v = \sqrt{\frac{2GM}{R}}.$$

For Mars, the escape speed is

$$v = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(0.11)(5.98 \times 10^{24} \text{ kg})}{3.45 \times 10^6 \text{ m}}} = 5.0 \times 10^3 \text{ m/s}.$$

37. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields $-GMm/R + \frac{1}{2}mv^2 = 0$. We replace GM/R with $a_g R$, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_g R + \frac{1}{2}v^2 = 0$. We solve for v :

$$v = \sqrt{2a_g R} = \sqrt{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m})} = 1.7 \times 10^3 \text{ m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R + h}.$$

We replace GM with $a_g R^2$ and cancel m in the energy equation to obtain

$$-a_g R + \frac{1}{2}v^2 = -\frac{a_g R^2}{(R + h)}.$$

The solution for h is

$$h = \frac{2a_g R^2}{2a_g R - v^2} - R = \frac{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m})^2}{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m}) - (1000 \text{ m/s})^2} - (500 \times 10^3 \text{ m})$$

$$= 2.5 \times 10^5 \text{ m}.$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R + h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

We substitute $a_g R^2$ for GM and cancel m , obtaining

$$-\frac{a_g R^2}{R + h} = -a_g R + \frac{1}{2}v^2.$$

The solution for v is

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R + h}} = \sqrt{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m}) - \frac{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m})^2}{(500 \times 10^3 \text{ m}) + (1000 \times 10^3 \text{ m})}}$$

$$= 1.4 \times 10^3 \text{ m/s}.$$

39. (a) The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -2GM^2/r_i$ since the final separation is $r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2.$$

The solution for v is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(10^{30} \text{ kg})}{10^{10} \text{ m}}} = 8.2 \times 10^4 \text{ m/s}.$$

(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5 \text{ m}$, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes $-GM^2/r_i = -GM^2/r_f + Mv^2$. The solution for v is

$$v = \sqrt{GM \left(\frac{1}{r_f} - \frac{1}{r_i} \right)} = \sqrt{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(10^{30} \text{ kg}) \left(\frac{1}{2 \times 10^5 \text{ m}} - \frac{1}{10^{10} \text{ m}} \right)}$$

$$= 1.8 \times 10^7 \text{ m/s.}$$

45. Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is $N M$ and the magnitude of the gravitational force acting on the Sun is $F = GNM^2/r^2$. The force points toward the galactic center. The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields $GNM^2/R^2 = 4\pi^2 MR/T^2$. The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M}.$$

The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2 (2.2 \times 10^{20} \text{ m})^3}{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(7.88 \times 10^{15} \text{ s})^2 (2.0 \times 10^{30} \text{ kg})} = 5.1 \times 10^{10}.$$

47. (a) The greatest distance between the satellite and Earth's center (the apogee distance) is $R_a = (6.37 \times 10^6 \text{ m} + 360 \times 10^3 \text{ m}) = 6.73 \times 10^6 \text{ m}$. The least distance (perigee distance) is $R_p = (6.37 \times 10^6 \text{ m} + 180 \times 10^3 \text{ m}) = 6.55 \times 10^6 \text{ m}$. Here $6.37 \times 10^6 \text{ m}$ is the radius of Earth. From Fig. 13-13, we see that the semi-major axis is

$$a = \frac{R_a + R_p}{2} = \frac{6.73 \times 10^6 \text{ m} + 6.55 \times 10^6 \text{ m}}{2} = 6.64 \times 10^6 \text{ m}.$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73 \times 10^6 \text{ m} - 6.55 \times 10^6 \text{ m}}{6.73 \times 10^6 \text{ m} + 6.55 \times 10^6 \text{ m}} = 0.0136.$$

61. (a) We use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the Sun ($1.99 \times 10^{30} \text{ kg}$) and r is the radius of the orbit. The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_e = 2(150 \times 10^9 \text{ m}) = 300 \times 10^9 \text{ m}$. Thus,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (300 \times 10^9 \text{ m})^3}{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(1.99 \times 10^{30} \text{ kg})}} = 8.96 \times 10^7 \text{ s}.$$

Dividing by (365 d/y) (24 h/d) (60 min/h) (60 s/min), we obtain $T = 2.8$ y.

(b) The kinetic energy of any asteroid or planet in a circular orbit of radius r is given by $K = GMm/2r$, where m is the mass of the asteroid or planet. We note that it is proportional to m and inversely proportional to r . The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is $K/K_e = (m/m_e) (r_e/r)$. We substitute $m = 2.0 \times 10^{-4}m_e$ and $r = 2r_e$ to obtain $K/K_e = 1.0 \times 10^{-4}$.

75. (a) Using Kepler's law of periods, we obtain

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right) r^3} = 2.15 \times 10^4 \text{ s}.$$

(b) The speed is constant (before she fires the thrusters), so $v_0 = 2\pi r/T = 1.23 \times 10^4$ m/s.

(c) A two percent reduction in the previous value gives $v = 0.98v_0 = 1.20 \times 10^4$ m/s.

(d) The kinetic energy is $K = \frac{1}{2}mv^2 = 2.17 \times 10^{11}$ J.

(e) The potential energy is $U = -GmM/r = -4.53 \times 10^{11}$ J.

(f) Adding these two results gives $E = K + U = -2.35 \times 10^{11}$ J.

(g) Using Eq. 13-42, we find the semi-major axis to be

$$a = \frac{-GMm}{2E} = 4.04 \times 10^7 \text{ m}.$$

(h) Using Kepler's law of periods for elliptical orbits (using a instead of r) we find the new period is

$$T' = \sqrt{\left(\frac{4\pi^2}{GM}\right) a^3} = 2.03 \times 10^4 \text{ s}.$$

This is smaller than our result for part (a) by $T - T' = 1.22 \times 10^3$ s.

(i) Elliptical orbit has a smaller period.

79. We use $F = Gm_s m_m / r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d = 15 \text{ m} + 3 \text{ m} = 18 \text{ m}$. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor. Thus,

$$F = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(20 \text{ kg})(7.0 \text{ kg})}{(18 \text{ m})^2} = 2.9 \times 10^{-11} \text{ N}.$$

83. (a) We write the centripetal acceleration (which is the same for each, since they have identical mass) as $r\omega^2$ where ω is the unknown angular speed. Thus,

$$\frac{G(M)(M)}{(2r)^2} = \frac{GM^2}{4r^2} = Mr\omega^2$$

which gives $\omega = \frac{1}{2}\sqrt{MG/r^3} = 2.2 \times 10^{-7} \text{ rad/s}$.

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 13-28). If m is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \Rightarrow v = \sqrt{\frac{4GM}{r}} = 8.9 \times 10^4 \text{ m/s}.$$

87. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed v_f . The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F dr = \int (-Kr) dr$ (using the notation of that Sample Problem referred to in the problem statement). Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F dr = \int_R^0 (-Kr) dr = \frac{1}{2}KR^2$$

where R is the radius of Earth. Solving for the final speed, we obtain $v_f = R \sqrt{K/m}$. We note that the acceleration of gravity $a_g = g = 9.8 \text{ m/s}^2$ on the surface of Earth is given by $a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2$, where ρ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$v_f = R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})} = 7.9 \times 10^3 \text{ m/s}.$$

93. The magnitude of the net gravitational force on one of the smaller stars (of mass m) is

$$\frac{GMm}{r^2} + \frac{Gmm}{(2r)^2} = \frac{Gm}{r^2} \left(M + \frac{m}{4} \right).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2} \left(M + \frac{m}{4} \right) = m \frac{v^2}{r} \quad \text{where } v = \frac{2\pi r}{T}.$$

Plugging in for speed v , we arrive at an equation for period T :

$$T = \frac{2\pi r^{3/2}}{\sqrt{G(M + m/4)}}.$$