

Chapter 14 – Student Solutions Manual

1. The pressure increase is the applied force divided by the area: $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston. Thus

$$\Delta p = (42 \text{ N})/\pi(0.011 \text{ m})^2 = 1.1 \times 10^5 \text{ Pa}.$$

This is equivalent to 1.1 atm.

3. The air inside pushes outward with a force given by $p_i A$, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by $p_o A$, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$. Since $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$,

$$F = (1.0 \text{ atm} - 0.96 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(3.4 \text{ m})(2.1 \text{ m}) = 2.9 \times 10^4 \text{ N}.$$

19. When the levels are the same the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2 . The final situation can then be achieved by taking liquid with volume $A(h_1 - h)$ and mass $\rho A(h_1 - h)$, in the first vessel, and lowering it a distance $h - h_2$. The work done by the force of gravity is

$$W = \rho A(h_1 - h)g(h - h_2).$$

We substitute $h = (h_1 + h_2)/2$ to obtain

$$\begin{aligned} W &= \frac{1}{4} \rho g A (h_1 - h_2)^2 = \frac{1}{4} (1.30 \times 10^3 \text{ kg/m}^3) (9.80 \text{ m/s}^2) (4.00 \times 10^{-4} \text{ m}^2) (1.56 \text{ m} - 0.854 \text{ m})^2 \\ &= 0.635 \text{ J} \end{aligned}$$

27. (a) We use the expression for the variation of pressure with height in an incompressible fluid: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5 \text{ Pa}$, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. For this calculation, we take the density to be uniformly 1.3 kg/m^3 . Then,

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01 \times 10^5 \text{ Pa}}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 7.9 \times 10^3 \text{ m} = 7.9 \text{ km}.$$

(b) Let h be the height of the atmosphere. Now, since the density varies with altitude, we integrate

$$p_2 = p_1 - \int_0^h \rho g \, dy.$$

Assuming $\rho = \rho_0 (1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8 \text{ m/s}^2$ for $0 \leq y \leq h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2} \rho_0 g h.$$

Since $p_2 = 0$, this implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01 \times 10^5 \text{ Pa})}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 16 \times 10^3 \text{ m} = 16 \text{ km}.$$

31. (a) The anchor is completely submerged in water of density ρ_w . Its effective weight is $W_{\text{eff}} = W - \rho_w gV$, where W is its actual weight (mg). Thus,

$$V = \frac{W - W_{\text{eff}}}{\rho_w g} = \frac{200 \text{ N}}{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 2.04 \times 10^{-2} \text{ m}^3.$$

(b) The mass of the anchor is $m = \rho V$, where ρ is the density of iron (found in Table 14-1). Its weight in air is

$$W = mg = \rho Vg = (7870 \text{ kg/m}^3)(2.04 \times 10^{-2} \text{ m}^3)(9.80 \text{ m/s}^2) = 1.57 \times 10^3 \text{ N}.$$

35. (a) Let V be the volume of the block. Then, the submerged volume is $V_s = 2V/3$. Since the block is floating, the weight of the displaced water is equal to the weight of the block, so $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block. We substitute $V_s = 2V/3$ to obtain

$$\rho_b = 2\rho_w/3 = 2(1000 \text{ kg/m}^3)/3 \approx 6.7 \times 10^2 \text{ kg/m}^3.$$

(b) If ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s = \rho_b V$. We substitute $V_s = 0.90V$ to obtain $\rho_o = \rho_b/0.90 = 7.4 \times 10^2 \text{ kg/m}^3$.

37. (a) The downward force of gravity mg is balanced by the upward buoyant force of the liquid: $mg = \rho g V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where r_o is the outer radius. Therefore,

$$m = \frac{2\pi}{3} \rho r_o^3 = \left(\frac{2\pi}{3}\right)(800 \text{ kg/m}^3)(0.090 \text{ m})^3 = 1.22 \text{ kg}.$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3} (r_o^3 - r_i^3) = \frac{4\pi}{3} ((0.090 \text{ m})^3 - (0.080 \text{ m})^3) = 9.09 \times 10^{-4} \text{ m}^3.$$

The density is

$$\rho_m = \frac{1.22 \text{ kg}}{9.09 \times 10^{-4} \text{ m}^3} = 1.3 \times 10^3 \text{ kg/m}^3.$$

49. We use the equation of continuity. Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. $A_1 = \pi R^2$ is the cross-sectional area of the hose. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2 (N A_2) \Rightarrow v_2 = \frac{A_1}{N A_2} v_1 = \frac{R^2}{N r^2} v_1$$

where R is the radius of the hose and r is the radius of a hole. Noting that $R/r = D/d$ (the ratio of diameters) we find

$$v_2 = \frac{D^2}{N d^2} v_1 = \frac{(1.9 \text{ cm})^2}{24 (0.13 \text{ cm})^2} (0.91 \text{ m/s}) = 8.1 \text{ m/s}.$$

53. Suppose that a mass Δm of water is pumped in time Δt . The pump increases the potential energy of the water by Δmgh , where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2} \Delta m v^2$, where v is its final speed. The work it does is $\Delta W = \Delta mgh + \frac{1}{2} \Delta m v^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t} \left(gh + \frac{1}{2} v^2 \right).$$

Now the rate of mass flow is $\Delta m / \Delta t = \rho_w A v$, where ρ_w is the density of water and A is the area of the hose. The area of the hose is $A = \pi r^2 = \pi (0.010 \text{ m})^2 = 3.14 \times 10^{-4} \text{ m}^2$ and

$$\rho_w A v = (1000 \text{ kg/m}^3) (3.14 \times 10^{-4} \text{ m}^2) (5.00 \text{ m/s}) = 1.57 \text{ kg/s}.$$

Thus,

$$P = \rho A v \left(gh + \frac{1}{2} v^2 \right) = (1.57 \text{ kg/s}) \left((9.8 \text{ m/s}^2) (3.0 \text{ m}) + \frac{(5.0 \text{ m/s})^2}{2} \right) = 66 \text{ W}.$$

55. (a) We use the equation of continuity: $A_1 v_1 = A_2 v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. Thus

$$v_2 = (A_1/A_2) v_1 = [(4.0 \text{ cm}^2)/(8.0 \text{ cm}^2)] (5.0 \text{ m/s}) = 2.5 \text{ m/s}.$$

(b) We use the Bernoulli equation:

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 ,$$

where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude. Thus

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (h_1 - h_2) \\ &= 1.5 \times 10^5 \text{ Pa} + \frac{1}{2} (1000 \text{ kg/m}^3) [(5.0 \text{ m/s})^2 - (2.5 \text{ m/s})^2] + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(10 \text{ m}) \\ &= 2.6 \times 10^5 \text{ Pa.} \end{aligned}$$

59. (a) We use the Bernoulli equation: $p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$, where h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. ρ is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes $\rho g h_1 = \frac{1}{2} \rho v_2^2 + \rho g h_2$ and

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.8 \text{ m/s}^2)(0.30 \text{ m})} = 2.42 \text{ m/s.}$$

The flow rate is $A_2 v_2 = (6.5 \times 10^{-4} \text{ m}^2)(2.42 \text{ m/s}) = 1.6 \times 10^{-3} \text{ m}^3/\text{s}$.

(b) We use the equation of continuity: $A_2 v_2 = A_3 v_3$, where $A_3 = \frac{1}{2} A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole. Thus

$$v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84 \text{ m/s.}$$

The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2} \rho v_2^2 + \rho g h_2 = \frac{1}{2} \rho v_3^2 + \rho g h_3$. Thus

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84 \text{ m/s})^2 - (2.42 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.90 \text{ m.}$$

67. (a) The continuity equation yields $Av = aV$, and Bernoulli's equation yields $\Delta p + \frac{1}{2} \rho v^2 = \frac{1}{2} \rho V^2$, where $\Delta p = p_1 - p_2$. The first equation gives $V = (A/a)v$. We use this to substitute for V in the second equation, and obtain $\Delta p + \frac{1}{2} \rho v^2 = \frac{1}{2} \rho (A/a)^2 v^2$. We solve for v . The result is

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{2a^2\Delta p}{\rho(A^2 - a^2)}}.$$

(b) We substitute values to obtain

$$v = \sqrt{\frac{2(32 \times 10^{-4} \text{ m}^2)^2(55 \times 10^3 \text{ Pa} - 41 \times 10^3 \text{ Pa})}{(1000 \text{ kg/m}^3)((64 \times 10^{-4} \text{ m}^2)^2 - (32 \times 10^{-4} \text{ m}^2)^2)}} = 3.06 \text{ m/s}.$$

Consequently, the flow rate is

$$Av = (64 \times 10^{-4} \text{ m}^2)(3.06 \text{ m/s}) = 2.0 \times 10^{-2} \text{ m}^3/\text{s}.$$

75. If we examine both sides of the U-tube at the level where the low-density liquid (with $\rho = 0.800 \text{ g/cm}^3 = 800 \text{ kg/m}^3$) meets the water (with $\rho_w = 0.998 \text{ g/cm}^3 = 998 \text{ kg/m}^3$), then the pressures there on either side of the tube must agree:

$$\rho gh = \rho_w gh_w$$

where $h = 8.00 \text{ cm} = 0.0800 \text{ m}$, and Eq. 14-9 has been used. Thus, the height of the water column (as measured from that level) is $h_w = (800/998)(8.00 \text{ cm}) = 6.41 \text{ cm}$. The volume of water in that column is therefore $\pi r^2 h_w = \pi(1.50 \text{ cm})^2(6.41 \text{ cm}) = 45.3 \text{ cm}^3$.