

Chapter 15 – Student Solutions Manual

3. (a) The amplitude is half the range of the displacement, or $x_m = 1.0$ mm.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi (120 \text{ Hz}) (1.0 \times 10^{-3} \text{ m}) = 0.75 \text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi (120 \text{ Hz}))^2 (1.0 \times 10^{-3} \text{ m}) = 5.7 \times 10^2 \text{ m/s}^2.$$

7. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500 \text{ s}) = 2.00$ Hz.

(c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00 \text{ Hz}) = 12.6$ rad/s.

(d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k and obtain

$$k = m\omega^2 = (0.500 \text{ kg})(12.6 \text{ rad/s})^2 = 79.0 \text{ N/m}.$$

(e) Let x_m be the amplitude. The maximum speed is

$$v_m = \omega x_m = (12.6 \text{ rad/s})(0.350 \text{ m}) = 4.40 \text{ m/s}.$$

(f) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0 \text{ N/m})(0.350 \text{ m}) = 27.6 \text{ N}$.

9. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude.

(a) The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g/x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{x_m}} = \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2}{1.0 \times 10^{-6} \text{ m}}} = 498 \text{ Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds g for some part of the motion.

17. The maximum force that can be exerted by the surface must be less than $\mu_s F_N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by

$$F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m,$$

where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8 \text{ m/s}^2)}{(2\pi \times 2.0 \text{ Hz})^2} = 0.031 \text{ m}.$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

19. (a) Let

$$x_1 = \frac{A}{2} \cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2} \cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A , the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50$ s,

$$x_1 = \frac{A}{2} \cos\left(\frac{2\pi \times 0.50 \text{ s}}{1.5 \text{ s}}\right) = -0.25A$$

and

$$x_2 = \frac{A}{2} \cos\left(\frac{2\pi \times 0.50 \text{ s}}{1.5 \text{ s}} + \frac{\pi}{6}\right) = -0.43A.$$

Their separation at that time is $x_1 - x_2 = -0.25A + 0.43A = 0.18A$.

(b) The velocities of the particles are given by

$$v_1 = \frac{dx_1}{dt} = \frac{\pi A}{T} \sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{dx_2}{dt} = \frac{\pi A}{T} \sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right).$$

We evaluate these expressions for $t = 0.50$ s and find they are both negative-valued, indicating that the particles are moving in the same direction.

27. When the block is at the end of its path and is momentarily stopped, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E = \frac{1}{2} kx_m^2 = \frac{1}{2} (1.3 \times 10^2 \text{ N/m})(0.024 \text{ m})^2 = 3.7 \times 10^{-2} \text{ J}.$$

29. The total energy is given by $E = \frac{1}{2} kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first.

(a) The fraction of the energy that is kinetic is

$$\frac{K}{E} = \frac{E - U}{E} = 1 - \frac{U}{E} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

where the result from part (b) has been used.

(b) When $x = \frac{1}{2} x_m$ the potential energy is $U = \frac{1}{2} kx^2 = \frac{1}{8} kx_m^2$. The ratio is

$$\frac{U}{E} = \frac{kx_m^2/8}{kx_m^2/2} = \frac{1}{4} = 0.25.$$

(c) Since $E = \frac{1}{2} kx_m^2$ and $U = \frac{1}{2} kx^2$, $U/E = x^2/x_m^2$. We solve $x^2/x_m^2 = 1/2$ for x . We should get $x = x_m / \sqrt{2}$.

39. (a) We take the angular displacement of the wheel to be $\theta = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_m \sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi \text{ rad})}{0.500 \text{ s}} = 39.5 \text{ rad/s}.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1 - \cos^2(2\pi t/T)} = \sqrt{1 - (1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500 \text{ s}}\right)(\pi \text{ rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2 \text{ rad/s}.$$

During another portion of the cycle its angular speed is $+34.2 \text{ rad/s}$ when its angular displacement is $\pi/2 \text{ rad}$.

(c) The angular acceleration is

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta.$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500 \text{ s}}\right)^2 \left(\frac{\pi}{4}\right) = -124 \text{ rad/s}^2,$$

or $|\alpha| = 124 \text{ rad/s}^2$.

43. (a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}Mr^2$, where M is its mass and r is its radius. The disk of this problem rotates about a point that is displaced from its center by $r+L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}Mr^2 + \frac{1}{2}M(L+r)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$\begin{aligned} I &= \frac{1}{2}Mr^2 + M(L+r)^2 + \frac{1}{3}mL^2 \\ &= \frac{1}{2}(0.500\text{kg})(0.100\text{m})^2 + (0.500\text{kg})(0.500\text{m} + 0.100\text{m})^2 + \frac{1}{3}(0.270\text{kg})(0.500\text{m})^2 \\ &= 0.205\text{kg}\cdot\text{m}^2. \end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + r = 0.500 \text{ m} + 0.100 \text{ m} = 0.600 \text{ m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500 \text{ m})/2 = 0.250 \text{ m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M + m} = \frac{(0.500 \text{ kg})(0.600 \text{ m}) + (0.270 \text{ kg})(0.250 \text{ m})}{0.500 \text{ kg} + 0.270 \text{ kg}} = 0.477 \text{ m}.$$

(c) The period of oscillation is

$$T = 2\pi \sqrt{\frac{I}{(M + m)gd}} = 2\pi \sqrt{\frac{0.205 \text{ kg} \cdot \text{m}^2}{(0.500 \text{ kg} + 0.270 \text{ kg})(9.80 \text{ m/s}^2)(0.477 \text{ m})}} = 1.50 \text{ s}.$$

51. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is

$$T = 2\pi / \omega = 2\pi \sqrt{I/C},$$

where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ , with the left end moving away from the wall. This end is now $(L/2) \sin \theta$ further from the wall and has moved a distance $(L/2)(1 - \cos \theta)$ to the right. The length of the spring is now

$$\ell = \sqrt{(L/2)^2(1 - \cos \theta)^2 + [\ell_0 + (L/2)\sin \theta]^2}.$$

If the angle θ is small we may approximate $\cos \theta$ with 1 and $\sin \theta$ with θ in radians. Then the length of the spring is given by $\ell \approx \ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. See Table 10-2. Thus the period of oscillation is

$$T = 2\pi \sqrt{\frac{I}{C}} = 2\pi \sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi \sqrt{\frac{m}{3k}}.$$

With $m = 0.600 \text{ kg}$ and $k = 1850 \text{ N/m}$, we obtain $T = 0.0653 \text{ s}$.

57. (a) We want to solve $e^{-bt/2m} = 1/3$ for t . We take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Therefore, $t = -(2m/b) \ln(1/3) = (2m/b) \ln 3$. Thus,

$$t = \frac{2(1.50 \text{ kg})}{0.230 \text{ kg/s}} \ln 3 = 14.3 \text{ s}.$$

(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00 \text{ N/m}}{1.50 \text{ kg}} - \frac{(0.230 \text{ kg/s})^2}{4(1.50 \text{ kg})^2}} = 2.31 \text{ rad/s}.$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31 \text{ rad/s}) = 2.72 \text{ s}$ and the number of oscillations is

$$t/T = (14.3 \text{ s})/(2.72 \text{ s}) = 5.27.$$

75. (a) The frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. This gives

$$f = (1/2\pi)\sqrt{(9.80 \text{ m/s}^2)/(2.0 \text{ m})} = 0.35 \text{ Hz}.$$

(b) The forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where m is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g} - \vec{a}_e) + \vec{T} = m\vec{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g} - \vec{a}_e$. Since $\vec{g}$ and $\vec{a}_e$ are along the same line and in opposite directions we can find the frequency for small amplitude oscillations by replacing g with $g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus

$$f = \frac{1}{2\pi} \sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2 + 2.0 \text{ m/s}^2}{2.0 \text{ m}}} = 0.39 \text{ Hz}.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g} - \vec{a}_e = 0$. To find the frequency for small amplitude oscillations, replace g with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

83. We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is $180/(60 \text{ s}) = 3.0 \text{ Hz}$ and the amplitude is half the stroke, or $x_m = 0.38 \text{ m}$. Thus,

$$v_m = 2\pi(3.0 \text{ Hz})(0.38 \text{ m}) = 7.2 \text{ m/s}.$$

89. (a) The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky = mg$, where $y = 0.096 \text{ m}$ is the elongation of the spring at equilibrium, k is the spring constant, and $m = 1.3 \text{ kg}$ is the mass of the block. Thus

$$k = mg/y = (1.3 \text{ kg})(9.8 \text{ m/s}^2)/(0.096 \text{ m}) = 1.33 \times 10^2 \text{ N/m}.$$

(b) The period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.3 \text{ kg}}{133 \text{ N/m}}} = 0.62 \text{ s}.$$

(c) The frequency is $f = 1/T = 1/0.62 \text{ s} = 1.6 \text{ Hz}$.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) The block has maximum speed as it passes the equilibrium point. At the initial position, the block is not moving but it has potential energy

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3 \text{ kg})(9.8 \text{ m/s}^2)(0.146 \text{ m}) + \frac{1}{2}(133 \text{ N/m})(0.146 \text{ m})^2 = -0.44 \text{ J}.$$

When the block is at the equilibrium point, the elongation of the spring is $y = 9.6 \text{ cm}$ and the potential energy is

$$U_f = -mgy + \frac{1}{2}ky^2 = -(1.3 \text{ kg})(9.8 \text{ m/s}^2)(0.096 \text{ m}) + \frac{1}{2}(133 \text{ N/m})(0.096 \text{ m})^2 = -0.61 \text{ J}.$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v :

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44 \text{ J} + 0.61 \text{ J})}{1.3 \text{ kg}}} = 0.51 \text{ m/s}.$$

91. We note that for a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given $v = 5.2 \text{ m/s}$ at $x = 0$ is the maximum value v_m (which equals ωx_m where $\omega = \sqrt{k/m} = 20 \text{ rad/s}$).

(a) Since $\omega = 2\pi f$, we find $f = 3.2$ Hz.

(b) We have $v_m = 5.2$ m/s $= (20$ rad/s) x_m , which leads to $x_m = 0.26$ m.

(c) With meters, seconds and radians understood,

$$\begin{aligned}x &= (0.26 \text{ m}) \cos(20t + \phi) \\v &= -(5.2 \text{ m/s}) \sin(20t + \phi).\end{aligned}$$

The requirement that $x = 0$ at $t = 0$ implies (from the first equation above) that either $\phi = +\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that $v > 0$ when $t = 0$; that choice is $\phi = -\pi/2$. Therefore,

$$x = 0.26 \cos\left(20t - \frac{\pi}{2}\right) = 0.26 \sin(20t).$$