

Chapter 16 – Student Solutions Manual

15. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope:

$$\mu = m/L = (0.0600 \text{ kg})/(2.00 \text{ m}) = 0.0300 \text{ kg/m}.$$

Thus,

$$v = \sqrt{\frac{500 \text{ N}}{0.0300 \text{ kg/m}}} = 129 \text{ m/s}.$$

17. (a) The amplitude of the wave is $y_m = 0.120 \text{ mm}$.

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f \sqrt{\frac{\mu}{\tau}} = 2\pi(100 \text{ Hz}) \sqrt{\frac{0.50 \text{ kg/m}}{10 \text{ N}}} = 141 \text{ m}^{-1}.$$

(c) The frequency is $f = 100 \text{ Hz}$, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100 \text{ Hz}) = 628 \text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction. In summary, the wave can be expressed as

$$y = (0.120 \text{ mm}) \sin \left[(141 \text{ m}^{-1})x + (628 \text{ s}^{-1})t \right].$$

21. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15 \text{ cm}$ and again with the same slope at about $x = 55 \text{ cm}$, so

$$\lambda = (55 \text{ cm} - 15 \text{ cm}) = 40 \text{ cm} = 0.40 \text{ m}.$$

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6 \text{ N}}{25 \times 10^{-3} \text{ kg/m}}} = 12 \text{ m/s.}$$

(d) The frequency is $f = v/\lambda = (12 \text{ m/s})/(0.40 \text{ m}) = 30 \text{ Hz}$ and the period is

$$T = 1/f = 1/(30 \text{ Hz}) = 0.033 \text{ s.}$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30 \text{ Hz})(5.0 \text{ cm}) = 940 \text{ cm/s} = 9.4 \text{ m/s.}$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40 \text{ m}) = 16 \text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30 \text{ Hz}) = 1.9 \times 10^2 \text{ rad/s}$

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is $4.0 \times 10^{-2} \text{ m}$. The formula for the displacement gives $y(0, 0) = y_m \sin \phi$. We wish to select ϕ so that $5.0 \times 10^{-2} \sin \phi = 4.0 \times 10^{-2}$. The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93 \text{ rad}$.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. Using the results obtained above, the expression for the displacement is

$$y(x, t) = (5.0 \times 10^{-2} \text{ m}) \sin[(16 \text{ m}^{-1})x + (190 \text{ s}^{-1})t + 0.93].$$

31. The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) = 2y_m \cos\left(\frac{1}{2}\phi\right) \sin\left(kx - \omega t + \frac{1}{2}\phi\right),$$

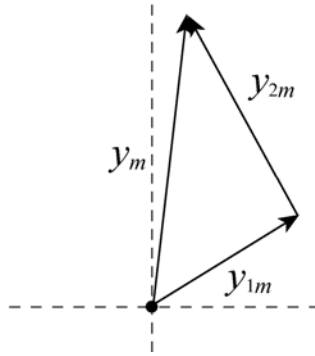
where $\phi = \pi/2$. The amplitude is

$$A = 2y_m \cos\left(\frac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41y_m.$$

35. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0 \text{ cm})^2 + (4.0 \text{ cm})^2 = (5.0 \text{ cm})^2.$$

Thus $y_m = 5.0 \text{ cm}$.



41. Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f_n = \frac{n}{2L} \sqrt{\frac{\tau L}{M}} = \frac{n}{2} \sqrt{\frac{\tau}{LM}} = \frac{n}{2} \sqrt{\frac{250 \text{ N}}{(10.0 \text{ m})(0.100 \text{ kg})}} = n (7.91 \text{ Hz}).$$

(a) The lowest frequency is $f_1 = 7.91 \text{ Hz}$.

(b) The second lowest frequency is $f_2 = 2(7.91 \text{ Hz}) = 15.8 \text{ Hz}$.

(c) The third lowest frequency is $f_3 = 3(7.91 \text{ Hz}) = 23.7 \text{ Hz}$.

43. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0 \text{ N})(8.40 \text{ m})}{0.120 \text{ kg}}} = 82.0 \text{ m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L = 2(8.40 \text{ m}) = 16.8 \text{ m}$.

(c) The frequency is $f = v/\lambda = (82.0 \text{ m/s})/(16.8 \text{ m}) = 4.88 \text{ Hz}$.

47. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n . Then, since there are no resonant frequencies between, the higher frequency is associated

with $n + 1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n + 1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n}.$$

The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315 \text{ Hz}}{420 \text{ Hz} - 315 \text{ Hz}} = 3.$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315 \text{ Hz})/3 = 105 \text{ Hz}$.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then

$$v = \lambda f = 2Lf = 2(0.75 \text{ m})(105 \text{ Hz}) = 158 \text{ m/s}.$$

53. (a) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The amplitude y_m is half the maximum displacement of the standing wave, or $5.0 \times 10^{-3} \text{ m}$.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With $L = 3.0 \text{ m}$, $\lambda = 2.0 \text{ m}$. The angular wave number is $k = 2\pi/\lambda = 2\pi/(2.0 \text{ m}) = 3.1 \text{ m}^{-1}$.

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100 \text{ m/s})}{2(3.0 \text{ m})} = 50 \text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or $\omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ rad/s}$.

(d) The two waves are

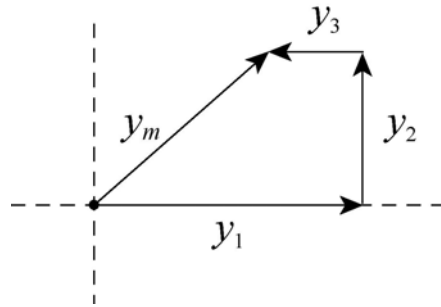
$$y_1 = (5.0 \times 10^{-3} \text{ m}) \sin \left[(3.14 \text{ m}^{-1})x - (314 \text{ s}^{-1})t \right]$$

and

$$y_2 = (5.0 \times 10^{-3} \text{ m}) \sin \left[(3.14 \text{ m}^{-1})x + (314 \text{ s}^{-1})t \right].$$

Thus, if one of the waves has the form $y(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y'(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.

61. (a) The phasor diagram is shown here: y_1 , y_2 , and y_3 represent the original waves and y_m represents the resultant wave.



The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1.$$

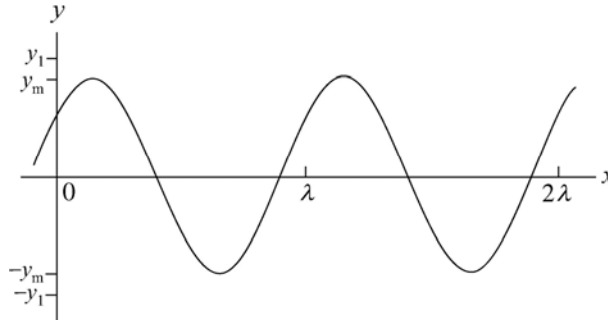
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\left(\frac{y_{mv}}{y_{mh}}\right) = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 0.644 \text{ rad} = 37^\circ.$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



69. (a) We take the form of the displacement to be $y(x, t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is $u(x, t) = \partial y / \partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda / T = \omega / k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

77. (a) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{120 \text{ N}}{8.70 \times 10^{-3} \text{ kg} / 1.50 \text{ m}}} = 144 \text{ m/s}.$$

(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50 \text{ m}) = 3.00 \text{ m}$.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50 \text{ m}$.

(d) The frequency for the one-loop wave is $f_1 = v / \lambda_1 = (144 \text{ m/s}) / (3.00 \text{ m}) = 48.0 \text{ Hz}$.

(e) The frequency for the two-loop wave is $f_2 = v / \lambda_2 = (144 \text{ m/s}) / (1.50 \text{ m}) = 96.0 \text{ Hz}$.

78. We use $P = \frac{1}{2} \mu v \omega^2 y_m^2 \propto v f^2 \propto \sqrt{\tau} f^2$.

(a) If the tension is quadrupled, then $P_2 = P_1 \sqrt{\frac{\tau_2}{\tau_1}} = P_1 \sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1$.

(b) If the frequency is halved, then $P_2 = P_1 \left(\frac{f_2}{f_1} \right)^2 = P_1 \left(\frac{f_1/2}{f_1} \right)^2 = \frac{1}{4} P_1$.

87. (a) From the frequency information, we find $\omega = 2\pi f = 10\pi$ rad/s. A point on the rope undergoing simple harmonic motion (discussed in Chapter 15) has maximum speed as it passes through its "middle" point, which is equal to $y_m\omega$. Thus,

$$5.0 \text{ m/s} = y_m\omega \Rightarrow y_m = 0.16 \text{ m}.$$

(b) Because of the oscillation being in the *fundamental* mode (as illustrated in Fig. 16-23(a) in the textbook), we have $\lambda = 2L = 4.0$ m. Therefore, the speed of waves along the rope is $v = f\lambda = 20$ m/s. Then, with $\mu = m/L = 0.60$ kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = \mu v^2 = 240 \text{ N} \approx 2.4 \times 10^2 \text{ N}.$$

(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$, and we observe that the anti-node having zero displacement at $t = 0$ suggests the use of sine instead of cosine for the simple harmonic motion factor. Now, if the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16 \text{ m}) \sin\left(\frac{\pi x}{2}\right) \sin(10\pi t) = (0.16 \text{ m}) \sin[(1.57 \text{ m}^{-1})x] \sin[(31.4 \text{ rad/s})t]$$

89. (a) The wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta\ell}{m/(\ell + \Delta\ell)}} = \sqrt{\frac{k\Delta\ell(\ell + \Delta\ell)}{m}}.$$

(b) The time required is

$$t = \frac{2\pi(\ell + \Delta\ell)}{v} = \frac{2\pi(\ell + \Delta\ell)}{\sqrt{k\Delta\ell(\ell + \Delta\ell)/m}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{1 + \frac{\ell}{\Delta\ell}}.$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$; and if $\ell/\Delta\ell \ll 1$, then $t \simeq 2\pi\sqrt{m/k} = \text{const.}$