

Chapter 18 – Student Solutions Manual

8. The change in length for the aluminum pole is

$$\Delta \ell = \ell_0 \alpha_{Al} \Delta T = (33\text{m})(23 \times 10^{-6} / \text{C}^\circ)(15^\circ \text{C}) = 0.011\text{m}.$$

15. If V_c is the original volume of the cup, α_a is the coefficient of linear expansion of aluminum, and ΔT is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a V_c \Delta T$. See Eq. 18-11. If β is the coefficient of volume expansion for glycerin then the change in the volume of glycerin is $\Delta V_g = \beta V_c \Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\begin{aligned} \Delta V_g - \Delta V_c &= (\beta - 3\alpha_a) V_c \Delta T = \left[(5.1 \times 10^{-4} / \text{C}^\circ) - 3(23 \times 10^{-6} / \text{C}^\circ) \right] (100\text{cm}^3)(6.0^\circ \text{C}) \\ &= 0.26\text{cm}^3. \end{aligned}$$

21. Consider half the bar. Its original length is $\ell_0 = L_0 / 2$ and its length after the temperature increase is $\ell = \ell_0 + \alpha \ell_0 \Delta T$. The old position of the half-bar, its new position, and the distance x that one end is displaced form a right triangle, with a hypotenuse of length ℓ , one side of length ℓ_0 , and the other side of length x . The Pythagorean theorem yields $x^2 = \ell^2 - \ell_0^2 = \ell_0^2 (1 + \alpha \Delta T)^2 - \ell_0^2$. Since the change in length is small we may approximate $(1 + \alpha \Delta T)^2$ by $1 + 2\alpha \Delta T$, where the small term $(\alpha \Delta T)^2$ was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2 \alpha \Delta T - \ell_0^2 = 2\ell_0^2 \alpha \Delta T$$

and

$$x = \ell_0 \sqrt{2\alpha \Delta T} = \frac{3.77\text{m}}{2} \sqrt{2(25 \times 10^{-6} / \text{C}^\circ)(32^\circ \text{C})} = 7.5 \times 10^{-2}\text{m}.$$

25. The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0°C ($= 288\text{K}$) to 1235 K. This requires heat

$$Q = cm(T_f - T_i) = (236\text{J/kg} \cdot \text{K})(0.130\text{kg})(1235^\circ \text{C} - 288^\circ \text{C}) = 2.91 \times 10^4\text{J}.$$

Now the silver at its melting point must be melted. If L_F is the heat of fusion for silver this requires

$$Q = mL_F = (0.130\text{kg})(105 \times 10^3\text{J/kg}) = 1.36 \times 10^4\text{J}.$$

The total heat required is $(2.91 \times 10^4\text{J} + 1.36 \times 10^4\text{J}) = 4.27 \times 10^4\text{J}$.

27. The mass $m = 0.100 \text{ kg}$ of water, with specific heat $c = 4190 \text{ J/kg}\cdot\text{K}$, is raised from an initial temperature $T_i = 23^\circ\text{C}$ to its boiling point $T_f = 100^\circ\text{C}$. The heat input is given by $Q = cm(T_f - T_i)$. This must be the power output of the heater P multiplied by the time t ; $Q = Pt$. Thus,

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190 \text{ J/kg}\cdot\text{K})(0.100 \text{ kg})(100^\circ\text{C} - 23^\circ\text{C})}{200 \text{ J/s}} = 160 \text{ s}.$$

41. (a) We work in Celsius temperature, which poses no difficulty for the $\text{J/kg}\cdot\text{K}$ values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. There are three possibilities:

- None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.
- The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.
- All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First, suppose that no ice melts. The temperature of the water decreases from $T_{wi} = 25^\circ\text{C}$ to some final temperature T_f and the temperature of the ice increases from $T_{li} = -15^\circ\text{C}$ to T_f . If m_w is the mass of the water and c_w is its specific heat then the water rejects heat

$$|Q| = c_w m_w (T_{wi} - T_f).$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{li}).$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_w m_w (T_{wi} - T_f) = c_I m_I (T_f - T_{li}).$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_w m_w T_{wi} + c_I m_I T_{li}}{c_w m_w + c_I m_I} \\ &= \frac{(4190 \text{ J/kg}\cdot\text{K})(0.200 \text{ kg})(25^\circ\text{C}) + (2220 \text{ J/kg}\cdot\text{K})(0.100 \text{ kg})(-15^\circ\text{C})}{(4190 \text{ J/kg}\cdot\text{K})(0.200 \text{ kg}) + (2220 \text{ J/kg}\cdot\text{K})(0.100 \text{ kg})} \\ &= 16.6^\circ\text{C}. \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at $T_f = 0^\circ\text{C}$, with mass m ($< m_I$) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi},$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + mL_F,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + mL_F.$$

This equation can be solved for the mass m of ice melted:

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190 \text{ J/kg} \cdot \text{K})(0.200 \text{ kg})(25^\circ\text{C}) + (2220 \text{ J/kg} \cdot \text{K})(0.100 \text{ kg})(-15^\circ\text{C})}{333 \times 10^3 \text{ J/kg}} \\ &= 5.3 \times 10^{-2} \text{ kg} = 53 \text{ g}. \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C . This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F.$$

The first term is the energy required to raise the temperature of the ice to 0°C , the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f , and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F.$$

The solution for T_f is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)}.$$

Inserting the given values, we obtain $T_f = 2.5^\circ\text{C}$.

43. Over a cycle, the internal energy is the same at the beginning and end, so the heat Q absorbed equals the work done: $Q = W$. Over the portion of the cycle from A to B the pressure p is a linear function of the volume V and we may write

$$p = \frac{10}{3} \text{ Pa} + \left(\frac{20}{3} \text{ Pa/m}^3 \right) V,$$

where the coefficients were chosen so that $p = 10 \text{ Pa}$ when $V = 1.0 \text{ m}^3$ and $p = 30 \text{ Pa}$ when $V = 4.0 \text{ m}^3$. The work done by the gas during this portion of the cycle is

$$\begin{aligned} W_{AB} &= \int_1^4 p dV = \int_1^4 \left(\frac{10}{3} + \frac{20}{3} V \right) dV = \left(\frac{10}{3} V + \frac{10}{3} V^2 \right) \Big|_1^4 \\ &= \left(\frac{40}{3} + \frac{160}{3} - \frac{10}{3} - \frac{10}{3} \right) \text{ J} = 60 \text{ J}. \end{aligned}$$

The BC portion of the cycle is at constant pressure and the work done by the gas is

$$W_{BC} = p \Delta V = (30 \text{ Pa})(1.0 \text{ m}^3 - 4.0 \text{ m}^3) = -90 \text{ J}.$$

The CA portion of the cycle is at constant volume, so no work is done. The total work done by the gas is $W = W_{AB} + W_{BC} + W_{CA} = 60 \text{ J} - 90 \text{ J} + 0 = -30 \text{ J}$ and the total heat absorbed is $Q = W = -30 \text{ J}$. This means the gas loses 30 J of energy in the form of heat.

49. (a) The change in internal energy ΔE_{int} is the same for path iaf and path ibf . According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along iaf

$$\Delta E_{\text{int}} = Q - W = 50 \text{ cal} - 20 \text{ cal} = 30 \text{ cal}.$$

Along ibf ,

$$W = Q - \Delta E_{\text{int}} = 36 \text{ cal} - 30 \text{ cal} = 6.0 \text{ cal}.$$

(b) Since the curved path is traversed from f to i the change in internal energy is -30 cal and $Q = \Delta E_{\text{int}} + W = -30 \text{ cal} - 13 \text{ cal} = -43 \text{ cal}$.

(c) Let $\Delta E_{\text{int}} = E_{\text{int}, f} - E_{\text{int}, i}$. Then, $E_{\text{int}, f} = \Delta E_{\text{int}} + E_{\text{int}, i} = 30 \text{ cal} + 10 \text{ cal} = 40 \text{ cal}$.

(d) The work W_{bf} for the path bf is zero, so $Q_{bf} = E_{\text{int}, f} - E_{\text{int}, b} = 40 \text{ cal} - 22 \text{ cal} = 18 \text{ cal}$.

(e) For the path ibf , $Q = 36 \text{ cal}$ so $Q_{ib} = Q - Q_{bf} = 36 \text{ cal} - 18 \text{ cal} = 18 \text{ cal}$.

51. The rate of heat flow is given by

$$P_{\text{cond}} = kA \frac{T_H - T_C}{L},$$

where k is the thermal conductivity of copper ($401 \text{ W/m}\cdot\text{K}$), A is the cross-sectional area (in a plane perpendicular to the flow), L is the distance along the direction of flow between the points where the temperature is T_H and T_C . Thus,

$$P_{\text{cond}} = \frac{(401 \text{ W/m}\cdot\text{K})(90.0 \times 10^{-4} \text{ m}^2)(125^\circ\text{C} - 10.0^\circ\text{C})}{0.250 \text{ m}} = 1.66 \times 10^3 \text{ J/s}.$$

The thermal conductivity is found in Table 18-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

65. Let h be the thickness of the slab and A be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{h}$$

where k is the thermal conductivity of ice, T_H is the temperature of the water (0°C), and T_C is the temperature of the air above the ice (-10°C). The heat leaving the water freezes it, the heat required to freeze mass m of water being $Q = L_F m$, where L_F is the heat of fusion for water. Differentiate with respect to time and recognize that $dQ/dt = P_{\text{cond}}$ to obtain

$$P_{\text{cond}} = L_F \frac{dm}{dt}.$$

Now, the mass of the ice is given by $m = \rho Ah$, where ρ is the density of ice and h is the thickness of the ice slab, so $dm/dt = \rho A(dh/dt)$ and

$$P_{\text{cond}} = L_F \rho A \frac{dh}{dt}.$$

We equate the two expressions for P_{cond} and solve for dh/dt :

$$\frac{dh}{dt} = \frac{k(T_H - T_C)}{L_F \rho h}.$$

Since 1 cal = 4.186 J and 1 cm = 1×10^{-2} m, the thermal conductivity of ice has the SI value

$$k = (0.0040 \text{ cal/s}\cdot\text{cm}\cdot\text{K}) (4.186 \text{ J/cal}) / (1 \times 10^{-2} \text{ m/cm}) = 1.674 \text{ W/m}\cdot\text{K}.$$

The density of ice is $\rho = 0.92 \text{ g/cm}^3 = 0.92 \times 10^3 \text{ kg/m}^3$. Thus,

$$\frac{dh}{dt} = \frac{(1.674 \text{ W/m}\cdot\text{K})(0^\circ\text{C} + 10^\circ\text{C})}{(333 \times 10^3 \text{ J/kg})(0.92 \times 10^3 \text{ kg/m}^3)(0.050 \text{ m})} = 1.1 \times 10^{-6} \text{ m/s} = 0.40 \text{ cm/h}.$$

73. The work (the “area under the curve”) for process 1 is $4p_i V_i$, so that

$$U_b - U_a = Q_1 - W_1 = 6p_i V_i$$

by the First Law of Thermodynamics.

(a) Path 2 involves more work than path 1 (note the triangle in the figure of area $\frac{1}{2}(4V_i)(p_i/2) = p_i V_i$). With $W_2 = 4p_i V_i + p_i V_i = 5p_i V_i$, we obtain

$$Q_2 = W_2 + U_b - U_a = 5p_i V_i + 6p_i V_i = 11p_i V_i.$$

(b) Path 3 starts at a and ends at b so that $\Delta U = U_b - U_a = 6p_i V_i$.

75. The volume of the disk (thought of as a short cylinder) is $\pi r^2 L$ where $L = 0.50 \text{ cm}$ is its thickness and $r = 8.0 \text{ cm}$ is its radius. Eq. 18-10, Eq. 18-11 and Table 18-2 (which gives $\alpha = 3.2 \times 10^{-6}/^\circ\text{C}$) lead to

$$\Delta V = (\pi r^2 L)(3\alpha)(60^\circ\text{C} - 10^\circ\text{C}) = 4.83 \times 10^{-2} \text{ cm}^3.$$

77. We have $W = \int p dV$ (Eq. 18-24). Therefore,

$$W = a \int V^2 dV = \frac{a}{3}(V_f^3 - V_i^3) = 23 \text{ J}.$$

81. Following the method of Sample Problem 18-4 (particularly its third [Key Idea](#)), we have

$$(900 \frac{\text{J}}{\text{kg}\cdot^\circ\text{C}})(2.50 \text{ kg})(T_f - 92.0^\circ\text{C}) + (4190 \frac{\text{J}}{\text{kg}\cdot^\circ\text{C}})(8.00 \text{ kg})(T_f - 5.0^\circ\text{C}) = 0$$

where Table 18-3 has been used. Thus we find $T_f = 10.5^\circ\text{C}$.

82. We use $Q = -\lambda_F m_{ice} = W + \Delta E_{int}$. In this case $\Delta E_{int} = 0$. Since $\Delta T = 0$ for the ideal gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333 \text{ J/g})(100 \text{ g}) = 33.3 \text{ kJ}.$$

83. This is similar to Sample Problem 18-3. An important difference with part (b) of that sample problem is that, in this case, the final state of the H_2O is *all liquid* at $T_f > 0$. As discussed in part (a) of that sample problem, there are three steps to the total process:

$$Q = m (c_{ice} (0 \text{ C}^\circ - (-150 \text{ C}^\circ)) + L_F + c_{liquid} (T_f - 0 \text{ C}^\circ))$$

Thus,

$$T_f = \frac{Q/m - (c_{ice}(150^\circ) + L_F)}{c_{liquid}} = 79.5^\circ\text{C}.$$