

Chapter 20 – Student Solutions Manual

5. (a) Since the gas is ideal, its pressure p is given in terms of the number of moles n , the volume V , and the temperature T by $p = nRT/V$. The work done by the gas during the isothermal expansion is

$$W = \int_{V_1}^{V_2} p \, dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \frac{V_2}{V_1}.$$

We substitute $V_2 = 2.00V_1$ to obtain

$$W = nRT \ln 2.00 = (4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(400 \text{ K}) \ln 2.00 = 9.22 \times 10^3 \text{ J}.$$

(b) Since the expansion is isothermal, the change in entropy is given by

$$\Delta S = \int (1/T) \, dQ = Q/T,$$

where Q is the heat absorbed. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22 \times 10^3 \text{ J}}{400 \text{ K}} = 23.1 \text{ J/K}.$$

(c) $\Delta S = 0$ for all reversible adiabatic processes.

7. (a) The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a (T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w (T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a (T_{ai} - T_f) = m_w c_w (T_f - T_{wi}) \Rightarrow T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The specific heat of aluminum is $900 \text{ J/kg} \cdot \text{K}$ and the specific heat of water is $4190 \text{ J/kg} \cdot \text{K}$. Thus,

$$T_f = \frac{(0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K})(100^\circ\text{C}) + (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(20^\circ\text{C})}{(0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) + (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})} = 57.0^\circ\text{C} = 330 \text{ K}.$$

(b) Now temperatures must be given in Kelvins: $T_{ai} = 393 \text{ K}$, $T_{wi} = 293 \text{ K}$, and $T_f = 330 \text{ K}$. For the aluminum, $dQ = m_a c_a dT$ and the change in entropy is

$$\Delta S_a = \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} = (0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{330 \text{ K}}{373 \text{ K}} \right) = -22.1 \text{ J/K}.$$

(c) The entropy change for the water is

$$\Delta S_w = \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} = (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{330 \text{ K}}{293 \text{ K}} \right) = +24.9 \text{ J/K}.$$

(d) The change in the total entropy of the aluminum-water system is

$$\Delta S = \Delta S_a + \Delta S_w = -22.1 \text{ J/K} + 24.9 \text{ J/K} = +2.8 \text{ J/K}.$$

25. (a) The efficiency is

$$\varepsilon = \frac{T_H - T_L}{T_H} = \frac{(235 - 115) \text{ K}}{(235 + 273) \text{ K}} = 0.236 = 23.6\%.$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_H|$, the work done is given by

$$|W| = \varepsilon |Q_H| = 0.236(6.30 \times 10^4 \text{ J}) = 1.49 \times 10^4 \text{ J}.$$

29. (a) Energy is added as heat during the portion of the process from a to b . This portion occurs at constant volume (V_b), so $Q_{in} = nC_V \Delta T$. The gas is a monatomic ideal gas, so $C_V = 3R/2$ and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a) V_b.$$

Thus, $Q_{in} = \frac{3}{2}(p_b - p_a)V_b$. V_b and p_b are given. We need to find p_a . Now p_a is the same as p_c and points c and b are connected by an adiabatic process. Thus, $p_c V_c^\gamma = p_b V_b^\gamma$ and

$$p_a = p_c = \left(\frac{V_b}{V_c} \right)^\gamma p_b = \left(\frac{1}{8.00} \right)^{5/3} (1.013 \times 10^6 \text{ Pa}) = 3.167 \times 10^4 \text{ Pa}.$$

The energy added as heat is

$$Q_{\text{in}} = \frac{3}{2} (1.013 \times 10^6 \text{ Pa} - 3.167 \times 10^4 \text{ Pa}) (1.00 \times 10^{-3} \text{ m}^3) = 1.47 \times 10^3 \text{ J}.$$

(b) Energy leaves the gas as heat during the portion of the process from c to a . This is a constant pressure process, so

$$\begin{aligned} Q_{\text{out}} &= nC_p \Delta T = \frac{5}{2} (p_a V_a - p_c V_c) = \frac{5}{2} p_a (V_a - V_c) \\ &= \frac{5}{2} (3.167 \times 10^4 \text{ Pa}) (-7.00) (1.00 \times 10^{-3} \text{ m}^3) = -5.54 \times 10^2 \text{ J}, \end{aligned}$$

or $|Q_{\text{out}}| = 5.54 \times 10^2 \text{ J}$. The substitutions $V_a - V_c = V_a - 8.00 V_a = -7.00 V_a$ and $C_p = \frac{5}{2} R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_{\text{in}} = (9.18 \times 10^2 \text{ J}) / (1.47 \times 10^3 \text{ J}) = 0.624 = 62.4\%.$$

37. A Carnot refrigerator working between a hot reservoir at temperature T_H and a cold reservoir at temperature T_L has a coefficient of performance K that is given by

$$K = \frac{T_L}{T_H - T_L}.$$

For the refrigerator of this problem, $T_H = 96^\circ \text{ F} = 309 \text{ K}$ and $T_L = 70^\circ \text{ F} = 294 \text{ K}$, so $K = (294 \text{ K}) / (309 \text{ K} - 294 \text{ K}) = 19.6$. The coefficient of performance is the energy Q_L drawn from the cold reservoir as heat divided by the work done: $K = |Q_L|/|W|$. Thus,

$$|Q_L| = K|W| = (19.6)(1.0 \text{ J}) = 20 \text{ J}.$$

39. The coefficient of performance for a refrigerator is given by $K = |Q_L|/|W|$, where Q_L is the energy absorbed from the cold reservoir as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields $Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W , so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|W|$ is $|W| = |Q_H|/(K + 1)$. In one hour,

$$|W| = \frac{7.54 \text{ MJ}}{3.8 + 1} = 1.57 \text{ MJ}.$$

The rate at which work is done is $(1.57 \times 10^6 \text{ J})/(3600 \text{ s}) = 440 \text{ W}$.

47. (a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. There are $N!$ arrangements of the N molecules, but $n_L!$ are simply rearrangements of the n_L molecules in the left third, $n_C!$ are rearrangements of the n_C molecules in the center third, and $n_R!$ are rearrangements of the n_R molecules in the right third. These rearrangements do not produce a new configuration. Thus, the multiplicity is

$$W = \frac{N!}{n_L! n_C! n_R!}.$$

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!(N/2)!}.$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!(N/3)!(N/3)!}.$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!(N/2)!}{(N/3)!(N/3)!(N/3)!}.$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!50!}{33!33!34!} = 4.2 \times 10^{16}.$$

49. Using Eq. 19-34 and Eq. 19-35, we arrive at

$$\Delta v = (\sqrt{3} - \sqrt{2})\sqrt{RT/M}$$

(a) We find, with $M = 28 \text{ g/mol} = 0.028 \text{ kg/mol}$ (see Table 19-1), $\Delta v_i = 87 \text{ m/s}$ at 250 K,

(b) and $\Delta v_f = 122 \approx 1.2 \times 10^2 \text{ m/s}$ at 500 K.

(c) The expression above for Δv implies

$$T = \frac{M}{R(\sqrt{3} - \sqrt{2})^2} (\Delta v)^2$$

which we can plug into Eq. 20-4 to yield

$$\Delta S = nR \ln(V_f/V_i) + nC_V \ln(T_f/T_i) = 0 + nC_V \ln[(\Delta v_f)^2/(\Delta v_i)^2] = 2nC_V \ln(\Delta v_f/\Delta v_i).$$

Using Table 19-3 to get $C_V = 5R/2$ (see also Table 19-2) we then find, for $n = 1.5$ mol, $\Delta S = 22$ J/K.

55. Except for the phase change (which just uses Eq. 20-2), this has some similarities with Sample Problem 20-2. Using constants available in the Chapter 19 tables, we compute

$$\Delta S = m[c_{\text{ice}} \ln(273/253) + \frac{L_f}{273} + c_{\text{water}} \ln(313/273)] = 1.18 \times 10^3 \text{ J/K}.$$

63. (a) Eq. 20-15 can be written as $|Q_H| = |Q_L|(1 + 1/K_C) = (35)(1 + \frac{1}{4.6}) = 42.6$ kJ.

(b) Similarly, Eq. 20-14 leads to $|W| = |Q_L|/K = 35/4.6 = 7.61$ kJ.

67. We adapt the discussion of §20-7 to 3 and 5 particles (as opposed to the 6 particle situation treated in that section).

(a) The least multiplicity configuration is when all the particles are in the same half of the box. In this case, using Eq. 20-20, we have

$$W = \frac{3!}{3!0!} = 1.$$

(b) Similarly for box B, $W = 5!/(5!0!) = 1$ in the “least” case.

(c) The most likely configuration in the 3 particle case is to have 2 on one side and 1 on the other. Thus,

$$W = \frac{3!}{2!1!} = 3.$$

(d) The most likely configuration in the 5 particle case is to have 3 on one side and 2 on the other. Thus,

$$W = \frac{5!}{3!2!} = 10.$$

(e) We use Eq. 20-21 with our result in part (c) to obtain

$$S = k \ln W = (1.38 \times 10^{-23}) \ln 3 = 1.5 \times 10^{-23} \text{ J/K}.$$

(f) Similarly for the 5 particle case (using the result from part (d)), we find

$$S = k \ln 10 = 3.2 \times 10^{-23} \text{ J/K}.$$