

Chapter 11

- 11-1** For the deep-groove 02-series ball bearing with $R = 0.90$, the design life x_D , in multiples of rating life, is

$$x_D = \frac{30\,000(300)(60)}{10^6} = 540 \quad \text{Ans.}$$

The design radial load F_D is

$$F_D = 1.2(1.898) = 2.278 \text{ kN}$$

From Eq. (11-6),

$$C_{10} = 2.278 \left\{ \frac{540}{0.02 + 4.439[\ln(1/0.9)]^{1/1.483}} \right\}^{1/3}$$

$$= 18.59 \text{ kN} \quad \text{Ans.}$$

Table 11-2: Choose a 02-30 mm with $C_{10} = 19.5 \text{ kN}$. *Ans.*

Eq. (11-18):

$$R = \exp \left\{ - \left[\frac{540(2.278/19.5)^3 - 0.02}{4.439} \right]^{1.483} \right\}$$

$$= 0.919 \quad \text{Ans.}$$

- 11-2** For the Angular-contact 02-series ball bearing as described, the rating life multiple is

$$x_D = \frac{50\,000(480)(60)}{10^6} = 1440$$

The design load is radial and equal to

$$F_D = 1.4(610) = 854 \text{ lbf} = 3.80 \text{ kN}$$

Eq. (11-6):

$$C_{10} = 854 \left\{ \frac{1440}{0.02 + 4.439[\ln(1/0.9)]^{1/1.483}} \right\}^{1/3}$$

$$= 9665 \text{ lbf} = 43.0 \text{ kN}$$

Table 11-2: Select a 02-55 mm with $C_{10} = 46.2 \text{ kN}$. *Ans.*

Using Eq. (11-18),

$$R = \exp \left\{ - \left[\frac{1440(3.8/46.2)^3 - 0.02}{4.439} \right]^{1.483} \right\}$$

$$= 0.927 \quad \text{Ans.}$$

- 11-3** For the straight-Roller 03-series bearing selection, $x_D = 1440$ rating lives from Prob. 11-2 solution.

$$F_D = 1.4(1650) = 2310 \text{ lbf} = 10.279 \text{ kN}$$

$$C_{10} = 10.279 \left(\frac{1440}{1} \right)^{3/10} = 91.1 \text{ kN}$$

Table 11-3: Select a 03-55 mm with $C_{10} = 102 \text{ kN}$. *Ans.*

Using Eq. (11-18),

$$R = \exp \left\{ - \left[\frac{1440(10.28/102)^{10/3} - 0.02}{4.439} \right]^{1.483} \right\} = 0.942 \quad \text{Ans.}$$

- 11-4** We can choose a reliability goal of $\sqrt{0.90} = 0.95$ for each bearing. We make the selections, find the existing reliabilities, multiply them together, and observe that the reliability goal is exceeded due to the roundup of capacity upon table entry.

Another possibility is to use the reliability of one bearing, say R_1 . Then set the reliability goal of the second as

$$R_2 = \frac{0.90}{R_1}$$

or vice versa. This gives three pairs of selections to compare in terms of cost, geometry implications, etc.

- 11-5** Establish a reliability goal of $\sqrt{0.90} = 0.95$ for each bearing. For a 02-series angular contact ball bearing,

$$C_{10} = 854 \left\{ \frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}} \right\}^{1/3}$$

$$= 11\,315 \text{ lbf} = 50.4 \text{ kN}$$

Select a 02-60 mm angular-contact bearing with $C_{10} = 55.9 \text{ kN}$.

$$R_A = \exp \left\{ - \left[\frac{1440(3.8/55.9)^3 - 0.02}{4.439} \right]^{1.483} \right\} = 0.969$$

For a 03-series straight-roller bearing,

$$C_{10} = 10.279 \left\{ \frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}} \right\}^{3/10} = 105.2 \text{ kN}$$

Select a 03-60 mm straight-roller bearing with $C_{10} = 123 \text{ kN}$.

$$R_B = \exp \left\{ - \left[\frac{1440(10.28/123)^{10/3} - 0.02}{4.439} \right]^{1.483} \right\} = 0.977$$

The overall reliability is $R = 0.969(0.977) = 0.947$, which exceeds the goal. Note, using R_A from this problem, and R_B from Prob. 11-3, $R = 0.969(0.942) = 0.913$, which still exceeds the goal. Likewise, using R_B from this problem, and R_A from Prob. 11-2, $R = 0.927(0.977) = 0.906$.

The point is that the designer has choices. Discover them before making the selection decision. Did the answer to Prob. 11-4 uncover the possibilities?

- 11-6** Choose a 02-series ball bearing from manufacturer #2, having a service factor of 1. For $F_r = 8$ kN and $F_a = 4$ kN

$$x_D = \frac{5000(900)(60)}{10^6} = 270$$

Eq. (11-5):

$$C_{10} = 8 \left\{ \frac{270}{0.02 + 4.439[\ln(1/0.90)]^{1/1.483}} \right\}^{1/3} = 51.8 \text{ kN}$$

Trial #1: From Table (11-2) make a tentative selection of a deep-groove 02-70 mm with $C_0 = 37.5$ kN.

$$\frac{F_a}{C_0} = \frac{4}{37.5} = 0.107$$

Table 11-1:

$$F_a/(V F_r) = 0.5 > e$$

$$X_2 = 0.56, \quad Y_2 = 1.46$$

Eq. (11-9):

$$F_e = 0.56(1)(8) + 1.46(4) = 10.32 \text{ kN}$$

Eq. (11-6): For $R = 0.90$,

$$C_{10} = 10.32 \left(\frac{270}{1} \right)^{1/3} = 66.7 \text{ kN} > 61.8 \text{ kN}$$

Trial #2: From Table 11-2 choose a 02-80 mm having $C_{10} = 70.2$ and $C_0 = 45.0$.

Check:

$$\frac{F_a}{C_0} = \frac{4}{45} = 0.089$$

Table 11-1: $X_2 = 0.56$, $Y_2 = 1.53$

$$F_e = 0.56(8) + 1.53(4) = 10.60 \text{ kN}$$

Eq. (11-6):

$$C_{10} = 10.60 \left(\frac{270}{1} \right)^{1/3} = 68.51 \text{ kN} < 70.2 \text{ kN}$$

$\therefore$ Selection stands.

Decision: Specify a 02-80 mm deep-groove ball bearing. *Ans.*

11-7 From Prob. 11-6, $x_D = 270$ and the final value of F_e is 10.60 kN.

$$C_{10} = 10.6 \left\{ \frac{270}{0.02 + 4.439[\ln(1/0.96)]^{1/1.483}} \right\}^{1/3} = 84.47 \text{ kN}$$

Table 11-2: Choose a deep-groove ball bearing, based upon C_{10} load ratings.

Trial #1:

Tentatively select a 02-90 mm.

$$C_{10} = 95.6, \quad C_0 = 62 \text{ kN}$$

$$\frac{F_a}{C_0} = \frac{4}{62} = 0.0645$$

From Table 11-1, interpolate for Y_2 .

F_a/C_0	Y_2
0.056	1.71
0.0645	Y_2
0.070	1.63

$$\frac{Y_2 - 1.71}{1.63 - 1.71} = \frac{0.0645 - 0.056}{0.070 - 0.056} = 0.607$$

$$Y_2 = 1.71 + 0.607(1.63 - 1.71) = 1.661$$

$$F_e = 0.56(8) + 1.661(4) = 11.12 \text{ kN}$$

$$C_{10} = 11.12 \left\{ \frac{270}{0.02 + 4.439[\ln(1/0.96)]^{1/1.483}} \right\}^{1/3} \\ = 88.61 \text{ kN} < 95.6 \text{ kN}$$

Bearing is OK.

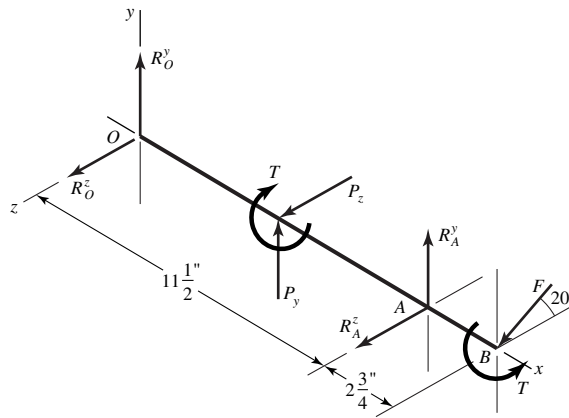
Decision: Specify a deep-groove 02-90 mm ball bearing. *Ans.*

11-8 For the straight cylindrical roller bearing specified with a service factor of 1, $R = 0.90$ and $F_r = 12 \text{ kN}$

$$x_D = \frac{4000(750)(60)}{10^6} = 180$$

$$C_{10} = 12 \left(\frac{180}{1} \right)^{3/10} = 57.0 \text{ kN} \quad \text{Ans.}$$

11-9



Assume concentrated forces as shown.

$$P_z = 8(24) = 192 \text{ lbf}$$

$$P_y = 8(30) = 240 \text{ lbf}$$

$$T = 192(2) = 384 \text{ lbf} \cdot \text{in}$$

$$\sum T^x = -384 + 1.5F \cos 20^\circ = 0$$

$$F = \frac{384}{1.5(0.940)} = 272 \text{ lbf}$$

$$\sum M_O^z = 5.75P_y + 11.5R_A^y - 14.25F \sin 20^\circ = 0;$$

thus

$$5.75(240) + 11.5R_A^y - 14.25(272)(0.342) = 0$$

$$R_A^y = -4.73 \text{ lbf}$$

$$\sum M_O^y = -5.75P_z - 11.5R_A^z - 14.25F \cos 20^\circ = 0;$$

thus

$$-5.75(192) - 11.5R_A^z - 14.25(272)(0.940) = 0$$

$$R_A^z = -413 \text{ lbf}; \quad R_A = [(-413)^2 + (-4.73)^2]^{1/2} = 413 \text{ lbf}$$

$$\sum F^z = R_O^z + P_z + R_A^z + F \cos 20^\circ = 0$$

$$R_O^z + 192 - 413 + 272(0.940) = 0$$

$$R_O^z = -34.7 \text{ lbf}$$

$$\sum F^y = R_O^y + P_y + R_A^y - F \sin 20^\circ = 0$$

$$R_O^y + 240 - 4.73 - 272(0.342) = 0$$

$$R_O^y = -142 \text{ lbf}$$

$$R_O = [(-34.6)^2 + (-142)^2]^{1/2} = 146 \text{ lbf}$$

So the reaction at A governs.

Reliability Goal: $\sqrt{0.92} = 0.96$

$$F_D = 1.2(413) = 496 \text{ lbf}$$

$$x_D = 30\,000(300)(60/10^6) = 540$$

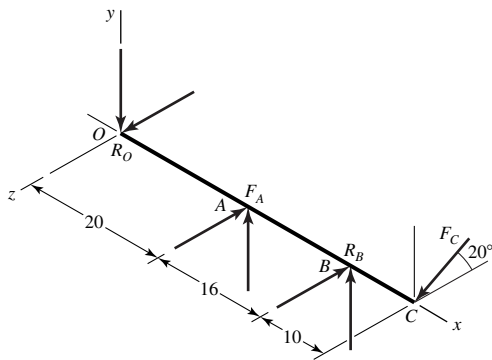
$$C_{10} = 496 \left\{ \frac{540}{0.02 + 4.439[\ln(1/0.96)]^{1/1.483}} \right\}^{1/3}$$

$$= 4980 \text{ lbf} = 22.16 \text{ kN}$$

A 02-35 bearing will do.

Decision: Specify an angular-contact 02-35 mm ball bearing for the locations at A and O. Check combined reliability. *Ans.*

11-10 For a combined reliability goal of 0.90, use $\sqrt{0.90} = 0.95$ for the individual bearings.



$$x_0 = \frac{50\,000(480)(60)}{10^6} = 1440$$

The resultant of the given forces are $R_O = [(-387)^2 + 467^2]^{1/2} = 607 \text{ lbf}$ and $R_B = [316^2 + (-1615)^2]^{1/2} = 1646 \text{ lbf}$.

At O: $F_e = 1.4(607) = 850 \text{ lbf}$

Ball:

$$C_{10} = 850 \left\{ \frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}} \right\}^{1/3}$$

$$= 11\,262 \text{ lbf} \quad \text{or} \quad 50.1 \text{ kN}$$

Select a 02-60 mm angular-contact ball bearing with a basic load rating of 55.9 kN. *Ans.*

At B: $F_e = 1.4(1646) = 2304 \text{ lbf}$

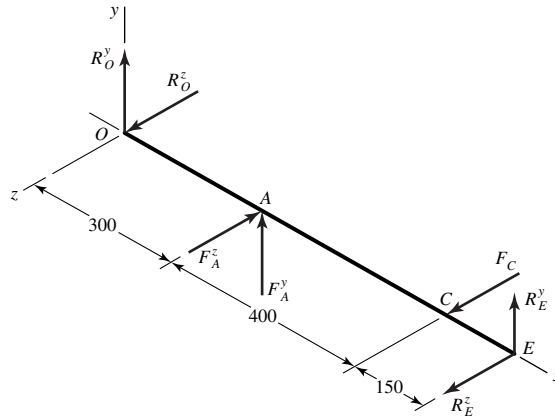
Roller:

$$C_{10} = 2304 \left\{ \frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}} \right\}^{3/10}$$

$$= 23\,576 \text{ lbf} \quad \text{or} \quad 104.9 \text{ kN}$$

Select a 02-80 mm cylindrical roller or a 03-60 mm cylindrical roller. The 03-series roller has the same bore as the 02-series ball. *Ans.*

11-11 The reliability of the individual bearings is $R = \sqrt{0.999} = 0.9995$



From statics,

$$R_O^y = -163.4 \text{ N}, \quad R_O^z = 107 \text{ N}, \quad R_O = 195 \text{ N}$$

$$R_E^y = -89.2 \text{ N}, \quad R_E^z = -174.4 \text{ N}, \quad R_E = 196 \text{ N}$$

$$x_D = \frac{60\,000(1200)(60)}{10^6} = 4320$$

$$C_{10} = 0.196 \left\{ \frac{4340}{0.02 + 4.439[\ln(1/0.9995)]^{1/1.483}} \right\}^{1/3}$$

$$= 8.9 \text{ kN}$$

A 02-25 mm deep-groove ball bearing has a basic load rating of 14.0 kN which is ample. An extra-light bearing could also be investigated.

11-12 Given:

$$F_{rA} = 560 \text{ lbf} \quad \text{or} \quad 2.492 \text{ kN}$$

$$F_{rB} = 1095 \text{ lbf} \quad \text{or} \quad 4.873 \text{ kN}$$

Trial #1: Use $K_A = K_B = 1.5$ and from Table 11-6 choose an indirect mounting.

$$\frac{0.47F_{rA}}{K_A} < ? > \frac{0.47F_{rB}}{K_B} - (-1)(0)$$

$$\frac{0.47(2.492)}{1.5} < ? > \frac{0.47(4.873)}{1.5}$$

$0.781 < 1.527$ Therefore use the upper line of Table 11-6.

$$F_{aA} = F_{aB} = \frac{0.47F_{rB}}{K_B} = 1.527 \text{ kN}$$

$$P_A = 0.4F_{rA} + K_A F_{aA} = 0.4(2.492) + 1.5(1.527) = 3.29 \text{ kN}$$

$$P_B = F_{rB} = 4.873 \text{ kN}$$

Fig. 11-16:

$$f_T = 0.8$$

Fig. 11-17:

$$f_V = 1.07$$

Thus,

$$a_{3l} = f_T f_V = 0.8(1.07) = 0.856$$

Individual reliability: $R_i = \sqrt{0.9} = 0.95$

Eq. (11-17):

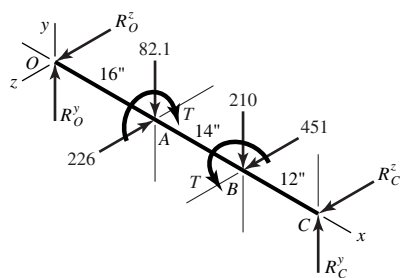
$$(C_{10})_A = 1.4(3.29) \left[\frac{40\,000(400)(60)}{4.48(0.856)(1 - 0.95)^{2/3}(90)(10^6)} \right]^{0.3}$$

$$= 11.40 \text{ kN}$$

$$(C_{10})_B = 1.4(4.873) \left[\frac{40\,000(400)(60)}{4.48(0.856)(1 - 0.95)^{2/3}(90)(10^6)} \right]^{0.3}$$

$$= 16.88 \text{ kN}$$

From Fig. 11-15, choose cone 32 305 and cup 32 305 which provide $F_r = 17.4 \text{ kN}$ and $K = 1.95$. With $K = 1.95$ for both bearings, a second trial validates the choice of cone 32 305 and cup 32 305. *Ans.*

11-13

$$R = \sqrt{0.95} = 0.975$$

$$T = 240(12)(\cos 20^\circ) = 2706 \text{ lbf} \cdot \text{in}$$

$$F = \frac{2706}{6 \cos 25^\circ} = 498 \text{ lbf}$$

In xy -plane:

$$\sum M_O = -82.1(16) - 210(30) + 42R_C^y = 0$$

$$R_C^y = 181 \text{ lbf}$$

$$R_O^y = 82 + 210 - 181 = 111 \text{ lbf}$$

In xz -plane:

$$\sum M_O = 226(16) - 452(30) - 42R_C^z = 0$$

$$R_C^z = -237 \text{ lbf}$$

$$R_O^z = 226 - 451 + 237 = 12 \text{ lbf}$$

$$R_O = (111^2 + 12^2)^{1/2} = 112 \text{ lbf} \quad \text{Ans.}$$

$$R_C = (181^2 + 237^2)^{1/2} = 298 \text{ lbf} \quad \text{Ans.}$$

$$F_{eO} = 1.2(112) = 134.4 \text{ lbf}$$

$$F_{eC} = 1.2(298) = 357.6 \text{ lbf}$$

$$x_D = \frac{40\,000(200)(60)}{10^6} = 480$$

$$\begin{aligned}
 (C_{10})_O &= 134.4 \left\{ \frac{480}{0.02 + 4.439[\ln(1/0.975)]^{1/1.483}} \right\}^{1/3} \\
 &= 1438 \text{ lbf} \quad \text{or} \quad 6.398 \text{ kN} \\
 (C_{10})_C &= 357.6 \left\{ \frac{480}{0.02 + 4.439[\ln(1/0.975)]^{1/1.483}} \right\}^{1/3} \\
 &= 3825 \text{ lbf} \quad \text{or} \quad 17.02 \text{ kN}
 \end{aligned}$$

Bearing at *O*: Choose a deep-groove 02-12 mm. *Ans.*

Bearing at *C*: Choose a deep-groove 02-30 mm. *Ans.*

There may be an advantage to the identical 02-30 mm bearings in a gear-reduction unit.

- 11-14** Shafts subjected to thrust can be constrained by bearings, one of which supports the thrust. The shaft floats within the endplay of the second (Roller) bearing. Since the thrust force here is larger than any radial load, the bearing absorbing the thrust is heavily loaded compared to the other bearing. The second bearing is thus oversized and does not contribute measurably to the chance of failure. This is predictable. The reliability goal is not $\sqrt{0.99}$, but 0.99 for the ball bearing. The reliability of the roller is 1. Beginning here saves effort.

Bearing at A (Ball)

$$F_r = (36^2 + 212^2)^{1/2} = 215 \text{ lbf} = 0.957 \text{ kN}$$

$$F_a = 555 \text{ lbf} = 2.47 \text{ kN}$$

Trial #1:

Tentatively select a 02-85 mm angular-contact with $C_{10} = 90.4 \text{ kN}$ and $C_0 = 63.0 \text{ kN}$.

$$\frac{F_a}{C_0} = \frac{2.47}{63.0} = 0.0392$$

$$x_D = \frac{25\,000(600)(60)}{10^6} = 900$$

Table 11-1: $X_2 = 0.56$, $Y_2 = 1.88$

$$F_e = 0.56(0.957) + 1.88(2.47) = 5.18 \text{ kN}$$

$$F_D = f_A F_e = 1.3(5.18) = 6.73 \text{ kN}$$

$$\begin{aligned}
 C_{10} &= 6.73 \left\{ \frac{900}{0.02 + 4.439[\ln(1/0.99)]^{1/1.483}} \right\}^{1/3} \\
 &= 107.7 \text{ kN} > 90.4 \text{ kN}
 \end{aligned}$$

Trial #2:

Tentatively select a 02-95 mm angular-contact ball with $C_{10} = 121 \text{ kN}$ and $C_0 = 85 \text{ kN}$.

$$\frac{F_a}{C_0} = \frac{2.47}{85} = 0.029$$

Table 11-1: $Y_2 = 1.98$

$$F_e = 0.56(0.957) + 1.98(2.47) = 5.43 \text{ kN}$$

$$F_D = 1.3(5.43) = 7.05 \text{ kN}$$

$$C_{10} = 7.05 \left\{ \frac{900}{0.02 + 4.439[\ln(1/0.99)]^{1/1.483}} \right\}^{1/3}$$

$$= 113 \text{ kN} < 121 \text{ kN} \quad O.K.$$

Select a 02-95 mm angular-contact ball bearing. *Ans.*

Bearing at B (Roller): Any bearing will do since $R = 1$. Let's prove it. From Eq. (11-18) when

$$\left(\frac{a_f F_D}{C_{10}} \right)^3 x_D < x_0 \quad R = 1$$

The smallest 02-series roller has a $C_{10} = 16.8 \text{ kN}$ for a basic load rating.

$$\left(\frac{0.427}{16.8} \right)^3 (900) < ? > 0.02$$

$$0.0148 < 0.02 \quad \therefore R = 1$$

Spotting this early avoided rework from $\sqrt{0.99} = 0.995$.

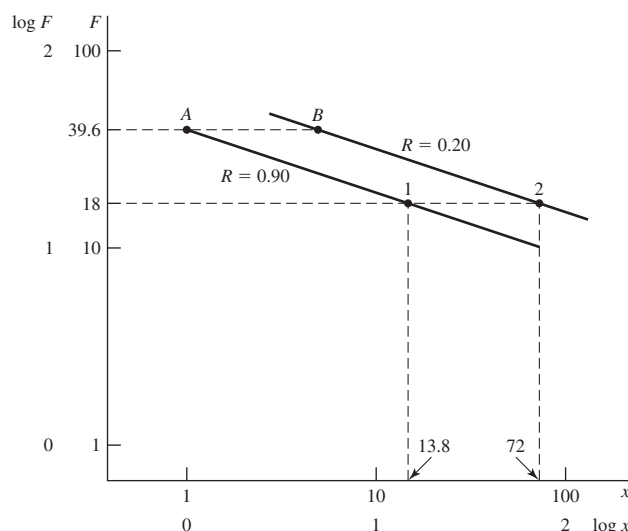
Any 02-series roller bearing will do. Same bore or outside diameter is a common choice. (Why?) *Ans.*

11-15 Hoover Ball-bearing Division uses the same 2-parameter Weibull model as Timken: $b = 1.5, \theta = 4.48$. We have some data. Let's estimate parameters b and θ from it. In Fig. 11-5, we will use line AB. In this case, B is to the right of A.

For $F = 18 \text{ kN}$,

$$(x)_1 = \frac{115(2000)(16)}{10^6} = 13.8$$

This establishes point 1 on the $R = 0.90$ line.



The $R = 0.20$ locus is above and parallel to the $R = 0.90$ locus. For the two-parameter Weibull distribution, $x_0 = 0$ and points A and B are related by [see Eq. (20-25)]:

$$x_A = \theta [\ln(1/0.90)]^{1/b} \quad (1)$$

$$x_B = \theta [\ln(1/0.20)]^{1/b}$$

and x_B/x_A is in the same ratio as 600/115. Eliminating θ

$$b = \frac{\ln[\ln(1/0.20)/\ln(1/0.90)]}{\ln(600/115)} = 1.65 \quad \text{Ans.}$$

Solving for θ in Eq. (1)

$$\theta = \frac{x_A}{[\ln(1/R_A)]^{1/1.65}} = \frac{1}{[\ln(1/0.90)]^{1/1.65}} = 3.91 \quad \text{Ans.}$$

Therefore, for the data at hand,

$$R = \exp \left[- \left(\frac{x}{3.91} \right)^{1.65} \right]$$

Check R at point B : $x_B = (600/115) = 5.217$

$$R = \exp \left[- \left(\frac{5.217}{3.91} \right)^{1.65} \right] = 0.20$$

Note also, for point 2 on the $R = 0.20$ line.

$$\begin{aligned} \log(5.217) - \log(1) &= \log(x_m)_2 - \log(13.8) \\ (x_m)_2 &= 72 \end{aligned}$$

11-16 This problem is rich in useful variations. Here is one.

Decision: Make straight roller bearings identical on a given shaft. Use a reliability goal of $(0.99)^{1/6} = 0.9983$.

Shaft a

$$F_A^r = (239^2 + 111^2)^{1/2} = 264 \text{ lbf} \quad \text{or} \quad 1.175 \text{ kN}$$

$$F_B^r = (502^2 + 1075^2)^{1/2} = 1186 \text{ lbf} \quad \text{or} \quad 5.28 \text{ kN}$$

Thus the bearing at B controls

$$x_D = \frac{10\,000(1200)(60)}{10^6} = 720$$

$$0.02 + 4.439[\ln(1/0.9983)]^{1/1.483} = 0.080\,26$$

$$C_{10} = 1.2(5.2) \left(\frac{720}{0.080\,26} \right)^{0.3} = 97.2 \text{ kN}$$

Select either a 02-80 mm with $C_{10} = 106 \text{ kN}$ or a 03-55 mm with $C_{10} = 102 \text{ kN}$. *Ans.*

Shaft *b*

$$F_C^r = (874^2 + 2274^2)^{1/2} = 2436 \text{ lbf} \quad \text{or} \quad 10.84 \text{ kN}$$

$$F_D^r = (393^2 + 657^2)^{1/2} = 766 \text{ lbf} \quad \text{or} \quad 3.41 \text{ kN}$$

The bearing at *C* controls

$$x_D = \frac{10\,000(240)(60)}{10^6} = 144$$

$$C_{10} = 1.2(10.84) \left(\frac{144}{0.0826} \right)^{0.3} = 122 \text{ kN}$$

Select either a 02-90 mm with $C_{10} = 142 \text{ kN}$ or a 03-60 mm with $C_{10} = 123 \text{ kN}$. *Ans.*

Shaft *c*

$$F_E^r = (1113^2 + 2385^2)^{1/2} = 2632 \text{ lbf} \quad \text{or} \quad 11.71 \text{ kN}$$

$$F_F^r = (417^2 + 895^2)^{1/2} = 987 \text{ lbf} \quad \text{or} \quad 4.39 \text{ kN}$$

The bearing at *E* controls

$$x_D = 10\,000(80)(60/10^6) = 48$$

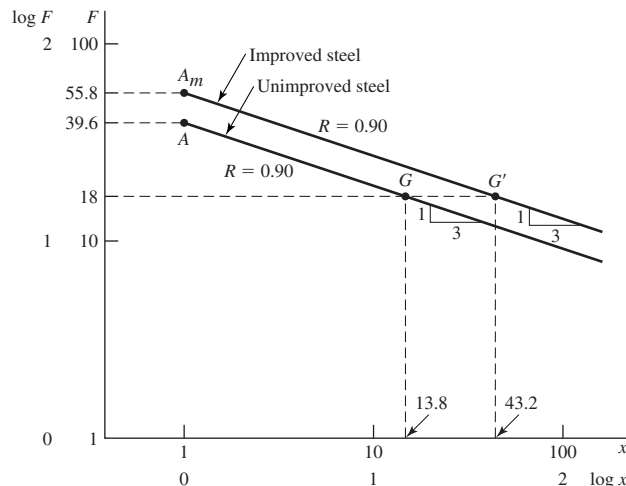
$$C_{10} = 1.2(11.71) \left(\frac{48}{0.0826} \right)^{0.3} = 94.8 \text{ kN}$$

Select a 02-80 mm with $C_{10} = 106 \text{ kN}$ or a 03-60 mm with $C_{10} = 123 \text{ kN}$. *Ans.*

- 11-17** The horizontal separation of the $R = 0.90$ loci in a $\log F$ - $\log x$ plot such as Fig. 11-5 will be demonstrated. We refer to the solution of Prob. 11-15 to plot point *G* ($F = 18 \text{ kN}$, $x_G = 13.8$). We know that $(C_{10})_1 = 39.6 \text{ kN}$, $x_1 = 1$. This establishes the unimproved steel $R = 0.90$ locus, line *AG*. For the improved steel

$$(x_m)_1 = \frac{360(2000)(60)}{10^6} = 43.2$$

We plot point G' ($F = 18 \text{ kN}$, $x_{G'} = 43.2$), and draw the $R = 0.90$ locus $A_m G'$ parallel to *AG*



We can calculate $(C_{10})_m$ by similar triangles.

$$\frac{\log(C_{10})_m - \log 18}{\log 43.2 - \log 1} = \frac{\log 39.6 - \log 18}{\log 13.8 - \log 1}$$

$$\log(C_{10})_m = \frac{\log 43.2}{\log 13.8} \log \left(\frac{39.6}{18} \right) + \log 18$$

$$(C_{10})_m = 55.8 \text{ kN}$$

The usefulness of this plot is evident. The improvement is $43.2/13.8 = 3.13$ fold in life. This result is also available by $(L_{10})_m/(L_{10})_1$ as $360/115$ or 3.13 fold, but the plot shows the improvement is for all loading. Thus, the manufacturer's assertion that there is at least a 3-fold increase in life has been demonstrated by the sample data given. *Ans.*

11-18 Express Eq. (11-1) as

$$F_1^a L_1 = C_{10}^a L_{10} = K$$

For a ball bearing, $a = 3$ and for a 02-30 mm angular contact bearing, $C_{10} = 20.3 \text{ kN}$.

$$K = (20.3)^3 (10^6) = 8.365(10^9)$$

At a load of 18 kN, life L_1 is given by:

$$L_1 = \frac{K}{F_1^a} = \frac{8.365(10^9)}{18^3} = 1.434(10^6) \text{ rev}$$

For a load of 30 kN, life L_2 is:

$$L_2 = \frac{8.365(10^9)}{30^3} = 0.310(10^6) \text{ rev}$$

In this case, Eq. (7-57) – the Palmgren-Miner cycle ratio summation rule – can be expressed as

$$\frac{l_1}{L_1} + \frac{l_2}{L_2} = 1$$

Substituting,

$$\frac{200\,000}{1.434(10^6)} + \frac{l_2}{0.310(10^6)} = 1$$

$$l_2 = 0.267(10^6) \text{ rev} \quad \text{Ans.}$$

11-19 *Total life in revolutions*

Let:

l = total turns

f_1 = fraction of turns at F_1

f_2 = fraction of turns at F_2

From the solution of Prob. 11-18, $L_1 = 1.434(10^6)$ rev and $L_2 = 0.310(10^6)$ rev.

Palmgren-Miner rule:

$$\frac{l_1}{L_1} + \frac{l_2}{L_2} = \frac{f_1 l}{L_1} + \frac{f_2 l}{L_2} = 1$$

from which

$$l = \frac{1}{f_1/L_1 + f_2/L_2}$$

$$l = \frac{1}{\{0.40/[1.434(10^6)]\} + \{0.60/[0.310(10^6)]\}}$$

$$= 451\,585 \text{ rev} \quad \text{Ans.}$$

Total life in loading cycles

$$4 \text{ min at } 2000 \text{ rev/min} = 8000 \text{ rev}$$

$$\frac{6 \text{ min}}{10 \text{ min/cycle}} \text{ at } 2000 \text{ rev/min} = \frac{12\,000 \text{ rev}}{20\,000 \text{ rev/cycle}}$$

$$\frac{451\,585 \text{ rev}}{20\,000 \text{ rev/cycle}} = 22.58 \text{ cycles} \quad \text{Ans.}$$

Total life in hours

$$\left(10 \frac{\text{min}}{\text{cycle}}\right) \left(\frac{22.58 \text{ cycles}}{60 \text{ min/h}}\right) = 3.76 \text{ h} \quad \text{Ans.}$$

11-20 While we made some use of the log F -log x plot in Probs. 11-15 and 11-17, the principal use of Fig. 11-5 is to understand equations (11-6) and (11-7) in the discovery of the catalog basic load rating for a case at hand.

Point D

$$F_D = 495.6 \text{ lbf}$$

$$\log F_D = \log 495.6 = 2.70$$

$$x_D = \frac{30\,000(300)(60)}{10^6} = 540$$

$$\log x_D = \log 540 = 2.73$$

$$K_D = F_D^3 x_D = (495.6)^3 (540)$$

$$= 65.7(10^9) \text{ lbf}^3 \cdot \text{turns}$$

$$\log K_D = \log[65.7(10^9)] = 10.82$$

F_D has the following uses: F_{design} , F_{desired} , F_e when a thrust load is present. It can include application factor a_f , or not. It depends on context.

Point B

$$x_B = 0.02 + 4.439[\ln(1/0.99)]^{1/1.483}$$

$$= 0.220 \text{ turns}$$

$$\log x_B = \log 0.220 = -0.658$$

$$F_B = F_D \left(\frac{x_D}{x_B} \right)^{1/3} = 495.6 \left(\frac{540}{0.220} \right)^{1/3} = 6685 \text{ lbf}$$

Note: Example 11-3 used Eq. (11-7). Whereas, here we basically used Eq. (11-6).

$$\log F_B = \log(6685) = 3.825$$

$$K_D = 6685^3(0.220) = 65.7(10^9) \text{ lbf}^3 \cdot \text{turns} \quad (\text{as it should})$$

Point A

$$F_A = F_B = C_{10} = 6685 \text{ lbf}$$

$$\log C_{10} = \log(6685) = 3.825$$

$$x_A = 1$$

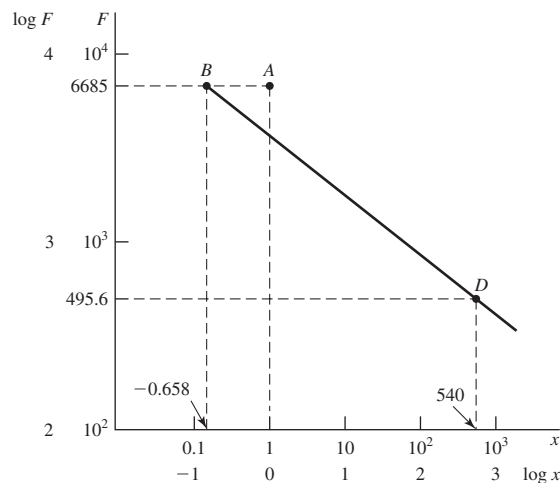
$$\log x_A = \log(1) = 0$$

$$K_{10} = F_A^3 x_A = C_{10}^3(1) = 6685^3 = 299(10^9) \text{ lbf}^3 \cdot \text{turns}$$

Note that $K_D/K_{10} = 65.7(10^9)/[299(10^9)] = 0.220$, which is x_B . This is worth knowing since

$$K_{10} = \frac{K_D}{x_B}$$

$$\log K_{10} = \log[299(10^9)] = 11.48$$



Now $C_{10} = 6685 \text{ lbf} = 29.748 \text{ kN}$, which is required for a reliability goal of 0.99. If we select an angular contact 02-40 mm ball bearing, then $C_{10} = 31.9 \text{ kN} = 7169 \text{ lbf}$.