

## Chapter 13

**13-1**

$$d_P = 17/8 = 2.125 \text{ in}$$

$$d_G = \frac{N_2}{N_3} d_P = \frac{1120}{544} (2.125) = 4.375 \text{ in}$$

$$N_G = P d_G = 8(4.375) = 35 \text{ teeth} \quad \text{Ans.}$$

$$C = (2.125 + 4.375)/2 = 3.25 \text{ in} \quad \text{Ans.}$$

**13-2**

$$n_G = 1600(15/60) = 400 \text{ rev/min} \quad \text{Ans.}$$

$$p = \pi m = 3\pi \text{ mm} \quad \text{Ans.}$$

$$C = [3(15 + 60)]/2 = 112.5 \text{ mm} \quad \text{Ans.}$$

**13-3**

$$N_G = 20(2.80) = 56 \text{ teeth} \quad \text{Ans.}$$

$$d_G = N_G m = 56(4) = 224 \text{ mm} \quad \text{Ans.}$$

$$d_P = N_P m = 20(4) = 80 \text{ mm} \quad \text{Ans.}$$

$$C = (224 + 80)/2 = 152 \text{ mm} \quad \text{Ans.}$$

**13-4** Mesh:

$$a = 1/P = 1/3 = 0.3333 \text{ in} \quad \text{Ans.}$$

$$b = 1.25/P = 1.25/3 = 0.4167 \text{ in} \quad \text{Ans.}$$

$$c = b - a = 0.0834 \text{ in} \quad \text{Ans.}$$

$$p = \pi/P = \pi/3 = 1.047 \text{ in} \quad \text{Ans.}$$

$$t = p/2 = 1.047/2 = 0.523 \text{ in} \quad \text{Ans.}$$

*Pinion Base-Circle:*

$$d_1 = N_1/P = 21/3 = 7 \text{ in}$$

$$d_{1b} = 7 \cos 20^\circ = 6.578 \text{ in} \quad \text{Ans.}$$

*Gear Base-Circle:*

$$d_2 = N_2/P = 28/3 = 9.333 \text{ in}$$

$$d_{2b} = 9.333 \cos 20^\circ = 8.770 \text{ in} \quad \text{Ans.}$$

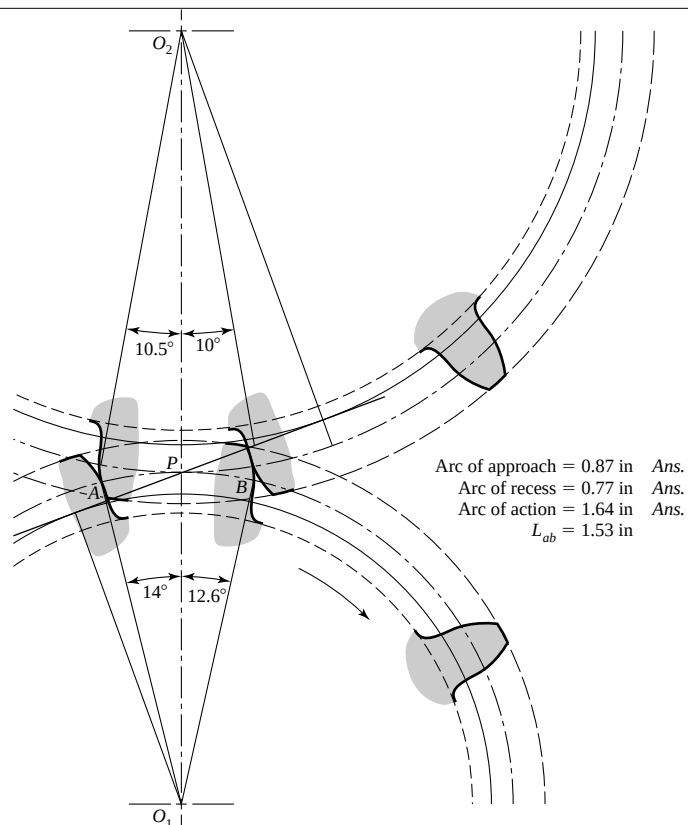
*Base pitch:*

$$p_b = p_c \cos \phi = (\pi/3) \cos 20^\circ = 0.984 \text{ in} \quad \text{Ans.}$$

*Contact Ratio:*

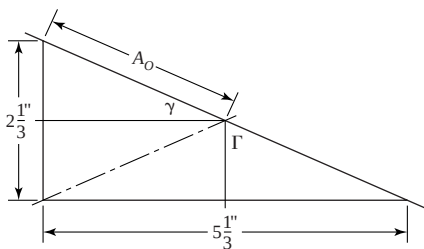
$$m_c = L_{ab}/p_b = 1.53/0.984 = 1.55 \quad \text{Ans.}$$

See the next page for a drawing of the gears and the arc lengths.



## 13-5

$$(a) A_O = \left[ \left( \frac{14/6}{2} \right)^2 + \left( \frac{32/6}{2} \right)^2 \right]^{1/2} = 2.910 \text{ in} \quad \text{Ans.}$$



$$(b) \gamma = \tan^{-1}(14/32) = 23.63^\circ \quad \text{Ans.}$$

$$\square = \tan^{-1}(32/14) = 66.37^\circ \quad \text{Ans.}$$

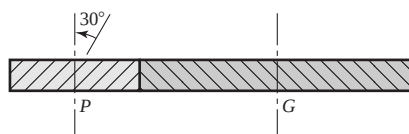
$$(c) d_P = 14/6 = 2.333 \text{ in,}$$

$$d_G = 32/6 = 5.333 \text{ in} \quad \text{Ans.}$$

$$(d) \text{ From Table 13-3, } 0.3A_O = 0.873 \text{ in and } 10/P = 10/6 = 1.67$$

$$0.873 < 1.67 \quad \therefore F = 0.873 \text{ in} \quad \text{Ans.}$$

## 13-6



$$(a) p_n = \pi/5 = 0.6283 \text{ in}$$

$$p_t = p_n / \cos \psi = 0.6283 / \cos 30^\circ = 0.7255 \text{ in}$$

$$p_x = p_t / \tan \psi = 0.7255 / \tan 30^\circ = 1.25 \text{ in}$$

(b) Eq. (13-7):  $p_{nb} = p_n \cos \phi_n = 0.6283 \cos 20^\circ = 0.590$  in *Ans.*

(c)  $P_t = P_n \cos \psi = 5 \cos 30^\circ = 4.33$  teeth/in

$$\phi_t = \tan^{-1}(\tan \phi_n / \cos \psi) = \tan^{-1}(\tan 20^\circ / \cos 30^\circ) = 22.8^\circ \quad \text{Ans.}$$

(d) Table 13-4:

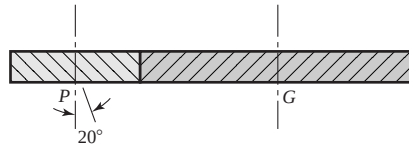
$$a = 1/5 = 0.200 \text{ in} \quad \text{Ans.}$$

$$b = 1.25/5 = 0.250 \text{ in} \quad \text{Ans.}$$

$$d_P = \frac{17}{5 \cos 30^\circ} = 3.926 \text{ in} \quad \text{Ans.}$$

$$d_G = \frac{34}{5 \cos 30^\circ} = 7.852 \text{ in} \quad \text{Ans.}$$

**13-7**



$N_P = 19$  teeth,  $N_G = 57$  teeth,  $\phi_n = 14.5^\circ$ ,  $P_n = 10$  teeth/in

(a)  $p_n = \pi/10 = 0.3142$  in *Ans.*

$$p_t = \frac{p_n}{\cos \psi} = \frac{0.3142}{\cos 20^\circ} = 0.3343 \text{ in} \quad \text{Ans.}$$

$$p_x = \frac{p_t}{\tan \psi} = \frac{0.3343}{\tan 20^\circ} = 0.9185 \text{ in} \quad \text{Ans.}$$

(b)  $P_t = P_n \cos \psi = 10 \cos 20^\circ = 9.397$  teeth/in *Ans.*

$$\phi_t = \tan^{-1} \left( \frac{\tan 14.5^\circ}{\cos 20^\circ} \right) = 15.39^\circ \quad \text{Ans.}$$

(c)  $a = 1/10 = 0.100$  in *Ans.*

$$b = 1.25/10 = 0.125 \text{ in} \quad \text{Ans.}$$

$$d_P = \frac{19}{10 \cos 20^\circ} = 2.022 \text{ in} \quad \text{Ans.}$$

$$d_G = \frac{57}{10 \cos 20^\circ} = 6.066 \text{ in} \quad \text{Ans.}$$

**13-8 (a)** The smallest pinion tooth count that will run with itself is found from Eq. (13-10)

$$\begin{aligned} N_P &\geq \frac{2k}{3 \sin^2 \phi} \left( 1 + \sqrt{1 + 3 \sin^2 \phi} \right) \\ &\geq \frac{2(1)}{3 \sin^2 20^\circ} \left( 1 + \sqrt{1 + 3 \sin^2 20^\circ} \right) \\ &\geq 12.32 \rightarrow 13 \text{ teeth } \textit{Ans.} \end{aligned}$$

**(b)** The smallest pinion that will mesh with a gear ratio of  $m_G = 2.5$ , from Eq. (13-11) is

$$\begin{aligned} N_P &\geq \frac{2(1)}{[1 + 2(2.5)] \sin^2 20^\circ} \left\{ 2.5 + \sqrt{2.5^2 + [1 + 2(2.5)] \sin^2 20^\circ} \right\} \\ &\geq 14.64 \rightarrow 15 \text{ pinion teeth } \textit{Ans.} \end{aligned}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-12) is

$$\begin{aligned} N_G &\leq \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi} \\ &\leq \frac{15^2 \sin^2 20^\circ - 4(1)^2}{4(1) - 2(15) \sin^2 20^\circ} \\ &\leq 45.49 \rightarrow 45 \text{ teeth } \textit{Ans.} \end{aligned}$$

**(c)** The smallest pinion that will mesh with a rack, from Eq. (13-13)

$$\begin{aligned} N_P &\geq \frac{2k}{\sin^2 \phi} = \frac{2(1)}{\sin^2 20^\circ} \\ &\geq 17.097 \rightarrow 18 \text{ teeth } \textit{Ans.} \end{aligned}$$

**13-9**  $\phi_n = 20^\circ$ ,  $\psi = 30^\circ$ ,  $\phi_t = \tan^{-1}(\tan 20^\circ / \cos 30^\circ) = 22.80^\circ$

**(a)** The smallest pinion tooth count that will run itself is found from Eq. (13-21)

$$\begin{aligned} N_P &\geq \frac{2k \cos \psi}{3 \sin^2 \phi_t} \left( 1 + \sqrt{1 + 3 \sin^2 \phi_t} \right) \\ &\geq \frac{2(1) \cos 30^\circ}{3 \sin^2 22.80^\circ} \left( 1 + \sqrt{1 + 3 \sin^2 22.80^\circ} \right) \\ &\geq 8.48 \rightarrow 9 \text{ teeth } \textit{Ans.} \end{aligned}$$

**(b)** The smallest pinion that will mesh with a gear ratio of  $m = 2.5$ , from Eq. (13-22) is

$$\begin{aligned} N_P &\geq \frac{2(1) \cos 30^\circ}{[1 + 2(2.5)] \sin^2 22.80^\circ} \left\{ 2.5 + \sqrt{2.5^2 + [1 + 2(2.5)] \sin^2 22.80^\circ} \right\} \\ &\geq 9.95 \rightarrow 10 \text{ teeth } \textit{Ans.} \end{aligned}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-23) is

$$\begin{aligned} N_G &\leq \frac{10^2 \sin^2 22.80^\circ - 4(1) \cos^2 30^\circ}{4(1) \cos^2 30^\circ - 2(20) \sin^2 22.80^\circ} \\ &\leq 26.08 \rightarrow 26 \text{ teeth } \textit{Ans.} \end{aligned}$$

(c) The smallest pinion that will mesh with a rack, from Eq. (13-24) is

$$N_P \geq \frac{2(1) \cos 30^\circ}{\sin^2 22.80^\circ} \\ \geq 11.53 \rightarrow 12 \text{ teeth } \textit{Ans.}$$

**13-10** Pressure Angle:  $\varphi_t = \tan^{-1} \left( \frac{\tan 20^\circ}{\cos 30^\circ} \right) = 22.796^\circ$

Program Eq. (13-24) on a computer using a spreadsheet or code and increment  $N_P$ . The first value of  $N_P$  that can be doubled is  $N_P = 10$  teeth, where  $N_G \leq 26.01$  teeth. So  $N_G = 20$  teeth will work. Higher tooth counts will work also, for example 11:22, 12:24, etc.

Use 10:20 *Ans.*

**13-11** Refer to Prob. 13-10 solution. The first value of  $N_P$  that can be multiplied by 6 is  $N_P = 11$  teeth where  $N_G \leq 93.6$  teeth. So  $N_G = 66$  teeth.

Use 11:66 *Ans.*

**13-12** Begin with the more general relation, Eq. (13-24), for full depth teeth.

$$N_G = \frac{N_P^2 \sin^2 \varphi_t - 4 \cos^2 \psi}{4 \cos \psi - 2 N_P \sin^2 \varphi_t}$$

For a rack, set the denominator to zero

$$4 \cos \psi - 2 N_P \sin^2 \varphi_t = 0$$

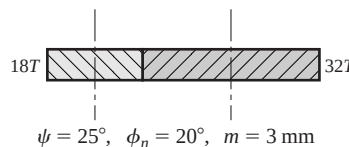
From which

$$\sin \varphi_t = \sqrt{\frac{2 \cos \psi}{N_P}} \\ \varphi_t = \sin^{-1} \sqrt{\frac{2 \cos \psi}{N_P}}$$

For  $N_P = 9$  teeth and  $\psi = 0$  for spur gears,

$$\varphi_t = \sin^{-1} \sqrt{\frac{2(1)}{9}} = 28.126^\circ \textit{ Ans.}$$

**13-13**



(a)  $p_n = \pi m_n = 3\pi \text{ mm } \textit{Ans.}$

$p_t = 3\pi / \cos 25^\circ = 10.4 \text{ mm } \textit{Ans.}$

$p_x = 10.4 / \tan 25^\circ = 22.3 \text{ mm } \textit{Ans.}$

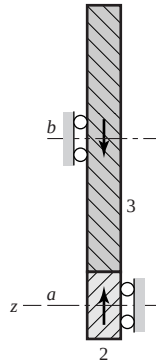
(b)  $m_t = 10.4/\pi = 3.310 \text{ mm}$  *Ans.*

$$\phi_t = \tan^{-1} \frac{\tan 20^\circ}{\cos 25^\circ} = 21.88^\circ \text{ } Ans.$$

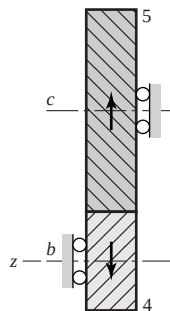
(c)  $d_P = 3.310(18) = 59.58 \text{ mm}$  *Ans.*

$$d_G = 3.310(32) = 105.92 \text{ mm} \text{ } Ans.$$

- 13-14** (a) The axial force of 2 on shaft *a* is in the negative direction. The axial force of 3 on shaft *b* is in the positive direction of *z*. *Ans.*



The axial force of gear 4 on shaft *b* is in the positive *z*-direction. The axial force of gear 5 on shaft *c* is in the negative *z*-direction. *Ans.*



(b)  $n_c = n_5 = \frac{14}{54} \left( \frac{16}{36} \right) (900) = +103.7 \text{ rev/min ccw}$  *Ans.*

(c)  $d_{P2} = 14/(10 \cos 30^\circ) = 1.6166 \text{ in}$

$$d_{G3} = 54/(10 \cos 30^\circ) = 6.2354 \text{ in}$$

$$C_{ab} = \frac{1.6166 + 6.2354}{2} = 3.926 \text{ in} \text{ } Ans.$$

$$d_{P4} = 16/(6 \cos 25^\circ) = 2.9423 \text{ in}$$

$$d_{G5} = 36/(6 \cos 25^\circ) = 6.6203 \text{ in}$$

$$C_{bc} = 4.781 \text{ in} \text{ } Ans.$$

**13-15**

$$e = \frac{20}{40} \left( \frac{8}{17} \right) \left( \frac{20}{60} \right) = \frac{4}{51}$$

$$n_d = \frac{4}{51} (600) = 47.06 \text{ rev/min cw} \text{ } Ans.$$

**13-16**

$$e = \frac{6}{10} \left( \frac{18}{38} \right) \left( \frac{20}{48} \right) \left( \frac{3}{36} \right) = \frac{3}{304}$$

$$n_a = \frac{3}{304}(1200) = 11.84 \text{ rev/min cw } \textit{Ans.}$$

**13-17**

$$(a) \ n_c = \frac{12}{40} \cdot \frac{1}{1}(540) = 162 \text{ rev/min cw about } x. \textit{ Ans.}$$

$$(b) \ d_P = 12/(8 \cos 23^\circ) = 1.630 \text{ in}$$

$$d_G = 40/(8 \cos 23^\circ) = 5.432 \text{ in}$$

$$\frac{d_P + d_G}{2} = 3.531 \text{ in } \textit{Ans.}$$

$$(c) \ d = \frac{32}{4} = 8 \text{ in at the large end of the teeth. } \textit{Ans.}$$

**13-18** (a) The planet gears act as keys and the wheel speeds are the same as that of the ring gear. Thus

$$n_A = n_3 = 1200(17/54) = 377.8 \text{ rev/min } \textit{Ans.}$$

$$(b) \quad n_F = n_5 = 0, \quad n_L = n_6, \quad e = -1$$

$$-1 = \frac{n_6 - 377.8}{0 - 377.8}$$

$$377.8 = n_6 - 377.8$$

$$n_6 = 755.6 \text{ rev/min } \textit{Ans.}$$

Alternatively, the velocity of the center of gear 4 is  $v_{4c} \square N_6 n_3$ . The velocity of the left edge of gear 4 is zero since the left wheel is resting on the ground. Thus, the velocity of the right edge of gear 4 is  $2v_{4c} \square 2N_6 n_3$ . This velocity, divided by the radius of gear 6  $\square N_6$ , is angular velocity of gear 6—the speed of wheel 6.

$$\therefore n_6 = \frac{2N_6 n_3}{N_6} = 2n_3 = 2(377.8) = 755.6 \text{ rev/min } \textit{Ans.}$$

(c) The wheel spins freely on icy surfaces, leaving no traction for the other wheel. The car is stalled. *Ans.*

**13-19** (a) The motive power is divided equally among four wheels instead of two.

(b) Locking the center differential causes 50 percent of the power to be applied to the rear wheels and 50 percent to the front wheels. If one of the rear wheels, rests on a slippery surface such as ice, the other rear wheel has no traction. But the front wheels still provide traction, and so you have two-wheel drive. However, if the rear differential is locked, you have 3-wheel drive because the rear-wheel power is now distributed 50-50.

**13-20** Let gear 2 be first, then  $n_F = n_2 = 0$ . Let gear 6 be last, then  $n_L = n_6 = -12$  rev/min.

$$e = \frac{20}{30} \left( \frac{16}{34} \right) = \frac{16}{51}, \quad e = \frac{n_L - n_A}{n_F - n_A}$$

$$(0 - n_A) \frac{16}{51} = -12 - n_A$$

$$n_A = \frac{-12}{35/51} = -17.49 \text{ rev/min (negative indicates cw) } \text{Ans.}$$

**13-21** Let gear 2 be first, then  $n_F = n_2 = 180$  rev/min. Let gear 6 be last, then  $n_L = n_6 = 0$ .

$$e = \frac{20}{30} \left( \frac{16}{34} \right) = \frac{16}{51}, \quad e = \frac{n_L - n_A}{n_F - n_A}$$

$$(180 - n_A) \frac{16}{51} = (0 - n_A)$$

$$n_A = \left( -\frac{16}{35} \right) 180 = -82.29 \text{ rev/min}$$

The negative sign indicates opposite  $n_2 \therefore n_A = 82.29$  rev/min cw *Ans.*

**13-22**  $N_5 = 12 + 2(16) + 2(12) = 68$  teeth *Ans.*

Let gear 2 be first,  $n_F = n_2 = 320$  rev/min. Let gear 5 be last,  $n_L = n_5 = 0$

$$e = \frac{12}{16} \left( \frac{16}{12} \right) \left( \frac{12}{68} \right) = \frac{3}{17}, \quad e = \frac{n_L - n_A}{n_F - n_A}$$

$$320 - n_A = \frac{17}{3}(0 - n_A)$$

$$n_A = -\frac{3}{14}(320) = -68.57 \text{ rev/min}$$

The negative sign indicates opposite of  $n_2 \therefore n_A = 68.57$  rev/min cw *Ans.*

**13-23** Let  $n_F = n_2$  then  $n_L = n_7 = 0$ .

$$e = -\frac{24}{18} \left( \frac{18}{30} \right) \left( \frac{36}{54} \right) = -\frac{8}{15}$$

$$e = \frac{n_L - n_5}{n_F - n_5} = -\frac{8}{15}$$

$$\frac{0 - 5}{n_2 - 5} = -\frac{8}{15} \quad \square \quad n_2 = 5 + \frac{15}{8}(5) = 14.375 \text{ turns in same direction}$$

**13-24 (a)**  $\omega = 2\pi n/60$

$$H = T\omega = 2\pi Tn/60 \quad (T \text{ in N} \cdot \text{m}, H \text{ in W})$$

So 
$$T = \frac{60H(10^3)}{2\pi n}$$

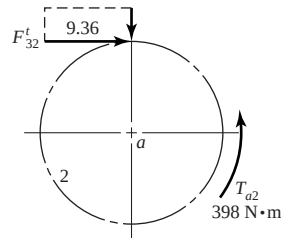
$$= 9550H/n \quad (H \text{ in kW}, n \text{ in rev/min})$$

$$T_a = \frac{9550(75)}{1800} = 398 \text{ N} \cdot \text{m}$$

$$r_2 = \frac{mN_2}{2} = \frac{5(17)}{2} = 42.5 \text{ mm}$$

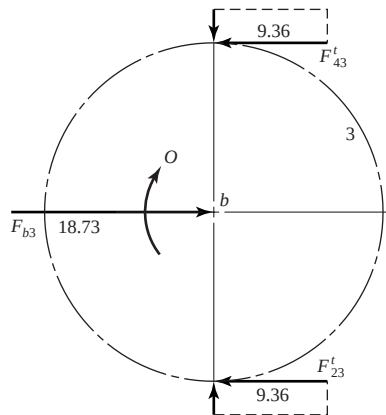
So

$$F_{32}^t = \frac{T_a}{r_2} = \frac{398}{42.5} = 9.36 \text{ kN}$$



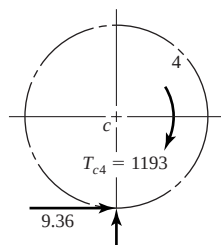
$F_{3b} = -F_{b3} = 2(9.36) = 18.73 \text{ kN}$  in the positive  $x$ -direction. *Ans.*  
See the figure in part (b).

(b) 
$$r_4 = \frac{mN_4}{2} = \frac{5(51)}{2} = 127.5 \text{ mm}$$



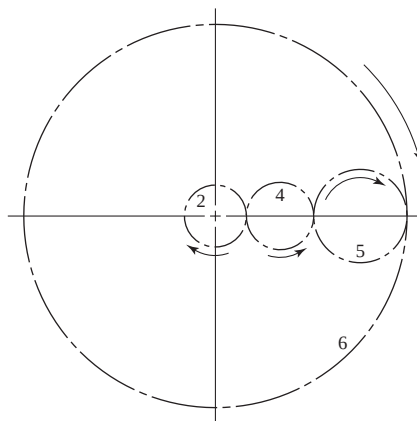
$$T_{c4} = 9.36(127.5) = 1193 \text{ N} \cdot \text{m ccw}$$

$\therefore T_{4c} = 1193 \text{ N} \cdot \text{m cw}$  *Ans.*



Note: The solution is independent of the pressure angle.

13-25



$$d = \frac{N}{6}$$

$$d_2 = 4 \text{ in}, \quad d_4 = 4 \text{ in}, \quad d_5 = 6 \text{ in}, \quad d_6 = 24 \text{ in}$$

$$e = \frac{24}{24} \left( \frac{24}{36} \right) \left( \frac{36}{144} \right) = 1/6, \quad n_P = n_2 = 1000 \text{ rev/min}$$

$$n_L = n_6 = 0$$

$$e = \frac{n_L - n_A}{n_F - n_A} = \frac{0 - n_A}{1000 - n_A}$$

$$n_A = -200 \text{ rev/min}$$

Input torque:

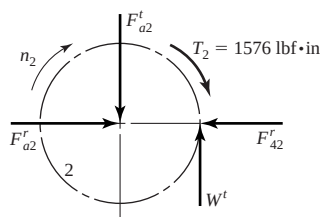
$$T_2 = \frac{63\,025H}{n}$$

$$T_2 = \frac{63\,025(25)}{1000} = 1576 \text{ lbf} \cdot \text{in}$$

For 100 percent gear efficiency

$$T_{\text{arm}} = \frac{63\,025(25)}{200} = 7878 \text{ lbf} \cdot \text{in}$$

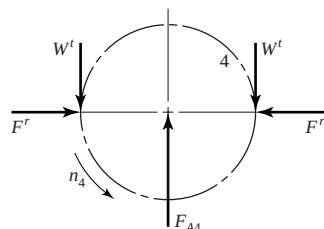
Gear 2



$$W^t = \frac{1576}{2} = 788 \text{ lbf}$$

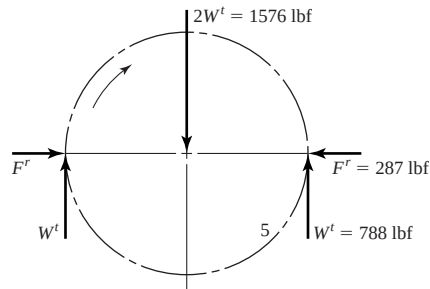
$$F_{32}^r = 788 \tan 20^\circ = 287 \text{ lbf}$$

Gear 4

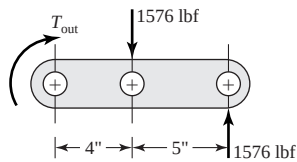


$$F_{A4} = 2W^t = 2(788) = 1576 \text{ lbf}$$

## Gear 5



## Arm

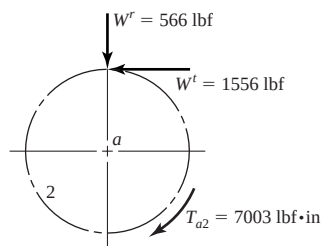


$$T_{\text{out}} = 1576(9) - 1576(4) = 7880 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

**13-26** Given:  $P = 2$  teeth/in,  $n_P = 1800$  rev/min cw,  $N_2 = 18T$ ,  $N_3 = 32T$ ,  $N_4 = 18T$ ,  $N_5 = 48T$ .

Pitch Diameters:  $d_2 = 18/2 = 9$  in;  $d_3 = 32/2 = 16$  in;  $d_4 = 18/2 = 9$  in;  $d_5 = 48/2 = 24$  in.

## Gear 2

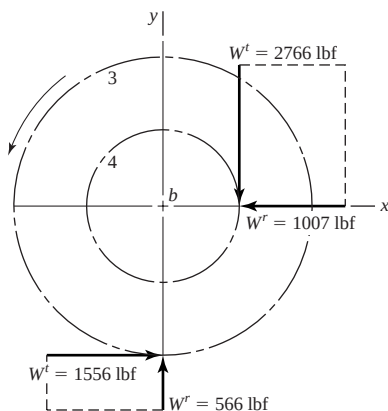


$$T_{a2} = 63\,025(200)/1800 = 7003 \text{ lbf} \cdot \text{in}$$

$$W^t = 7003/4.5 = 1556 \text{ lbf}$$

$$W^r = 1556 \tan 20^\circ = 566 \text{ lbf}$$

## Gears 3 and 4



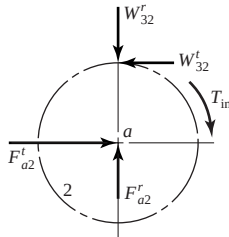
$$W^t(4.5) = 1556(8), \quad W^t = 2766 \text{ lbf}$$

$$W^r = 2766 \tan 20^\circ = 1007 \text{ lbf}$$

Ans.

**13-27** Given:  $P = 5$  teeth/in,  $N_2 = 18T$ ,  $N_3 = 45T$ ,  $\phi_n = 20^\circ$ ,  $H = 32$  hp,  $n_2 = 1800$  rev/min.

*Gear 2*



$$T_{in} = \frac{63\,025(32)}{1800} = 1120 \text{ lbf} \cdot \text{in}$$

$$d_P = \frac{18}{5} = 3.600 \text{ in}$$

$$d_G = \frac{45}{5} = 9.000 \text{ in}$$

$$W_{32}^t = \frac{1120}{3.6/2} = 622 \text{ lbf}$$

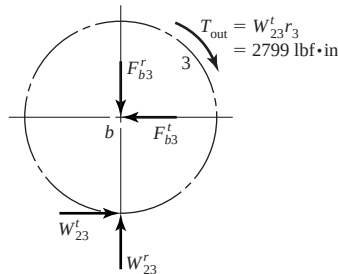
$$W_{32}^r = 622 \tan 20^\circ = 226 \text{ lbf}$$

$$F_{a2}^t = W_{32}^t = 622 \text{ lbf}, \quad F_{a2}^r = W_{32}^r = 226 \text{ lbf}$$

$$F_{a2} = (622^2 + 226^2)^{1/2} = 662 \text{ lbf}$$

Each bearing on shaft  $a$  has the same radial load of  $R_A = R_B = 662/2 = 331$  lbf.

*Gear 3*



$$W_{23}^t = W_{32}^t = 622 \text{ lbf}$$

$$W_{23}^r = W_{32}^r = 226 \text{ lbf}$$

$$F_{b3} = F_{b2} = 662 \text{ lbf}$$

$$R_C = R_D = 662/2 = 331 \text{ lbf}$$

Each bearing on shaft  $b$  has the same radial load which is equal to the radial load of bearings,  $A$  and  $B$ . Thus, all four bearings have the same radial load of 331 lbf. *Ans.*

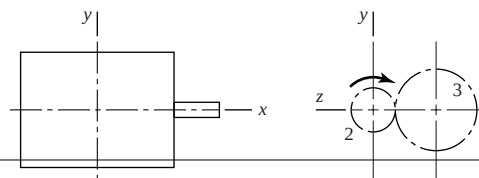
**13-28** Given:  $P = 4$  teeth/in,  $\phi_n = 20^\circ$ ,  $N_P = 20T$ ,  $n_2 = 900$  rev/min.

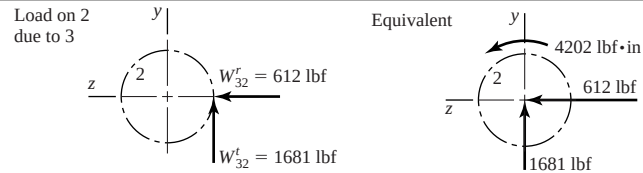
$$d_2 = \frac{N_P}{P} = \frac{20}{4} = 5.000 \text{ in}$$

$$T_{in} = \frac{63\,025(30)(2)}{900} = 4202 \text{ lbf} \cdot \text{in}$$

$$W_{32}^t = T_{in}/(d_2/2) = 4202/(5/2) = 1681 \text{ lbf}$$

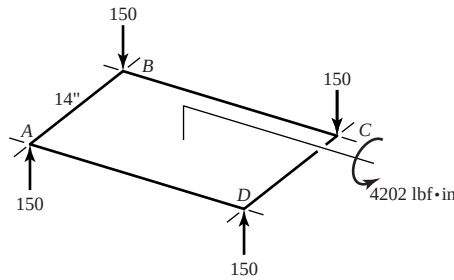
$$W_{32}^r = 1681 \tan 20^\circ = 612 \text{ lbf}$$





The motor mount resists the equivalent forces and torque. The radial force due to torque

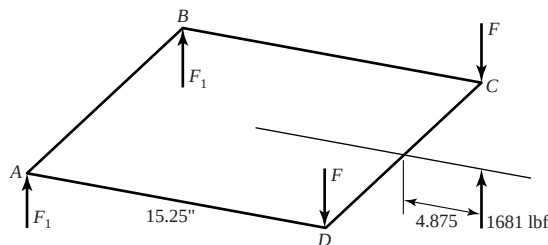
$$F^r = \frac{4202}{14(2)} = 150 \text{ lbf}$$



Forces reverse with rotational sense as torque reverses.

The compressive loads at A and D are absorbed by the base plate, not the bolts. For  $W_{32}^t$ , the tensions in C and D are

$$\sum M_{AB} = 0 \quad 1681(4.875 + 15.25) - 2F(15.25) = 0 \quad F = 1109 \text{ lbf}$$



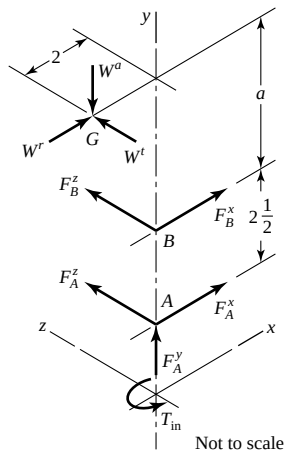
If  $W_{32}^t$  reverses, 15.25 in changes to 13.25 in, 4.815 in changes to 2.875 in, and the forces change direction. For A and B,

$$1681(2.875) - 2F_1(13.25) = 0 \quad \square \quad F_1 = 182.4 \text{ lbf}$$

For  $W_{32}^r$



**13-29**  $T_{in} = 63\,025H/n = 63\,025(2.5)/240 = 656.5 \text{ lbf} \cdot \text{in}$



$$W^t = T/r = 656.5/2 = 328.3 \text{ lbf}$$

$$\gamma = \tan^{-1}(2/4) = 26.565^\circ$$

$$\phi = \tan^{-1}(4/2) = 63.435^\circ$$

$$a = 2 + (1.5 \cos 26.565^\circ)/2 = 2.67 \text{ in}$$

$$W^r = 328.3 \tan 20^\circ \cos 26.565^\circ = 106.9 \text{ lbf}$$

$$W^a = 328.3 \tan 20^\circ \sin 26.565^\circ = 53.4 \text{ lbf}$$

$$\mathbf{W} = 106.9\mathbf{i} - 53.4\mathbf{j} + 328.3\mathbf{k} \text{ lbf}$$

$$\mathbf{R}_{AG} = -2\mathbf{i} + 5.17\mathbf{j}, \quad \mathbf{R}_{AB} = 2.5\mathbf{j}$$

$$\sum \mathbf{M}_4 = \mathbf{R}_{AG} \times \mathbf{W} + \mathbf{R}_{AB} \times \mathbf{F}_B + \mathbf{T} = \mathbf{0}$$

Solving gives

$$\mathbf{R}_{AB} \times \mathbf{F}_B = 2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k}$$

$$\mathbf{R}_{AG} \times \mathbf{W} = 1697\mathbf{i} + 656.6\mathbf{j} - 445.9\mathbf{k}$$

So

$$(1697\mathbf{i} + 656.6\mathbf{j} - 445.9\mathbf{k}) + (2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k} + T\mathbf{j}) = \mathbf{0}$$

$$F_B^z = -1697/2.5 = -678.8 \text{ lbf}$$

$$T = -656.6 \text{ lbf} \cdot \text{in}$$

$$F_B^x = -445.9/2.5 = -178.4 \text{ lbf}$$

So

$$F_B = [(-678.8)^2 + (-178.4)^2]^{1/2} = 702 \text{ lbf} \quad \text{Ans.}$$

$$\mathbf{F}_A = -(\mathbf{F}_B + \mathbf{W})$$

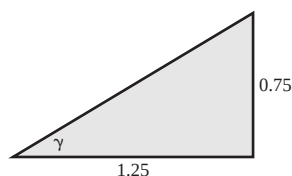
$$= -(-178.4\mathbf{i} - 678.8\mathbf{k} + 106.9\mathbf{i} - 53.4\mathbf{j} + 328.3\mathbf{k})$$

$$= 71.5\mathbf{i} + 53.4\mathbf{j} + 350.5\mathbf{k}$$

$$F_A (\text{radial}) = (71.5^2 + 350.5^2)^{1/2} = 358 \text{ lbf} \quad \text{Ans.}$$

$$F_A (\text{thrust}) = 53.4 \text{ lbf} \quad \text{Ans.}$$

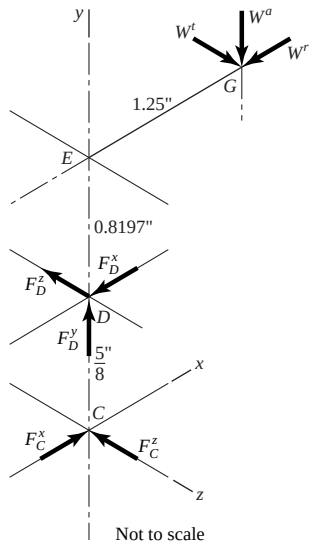
**13-30**



$$d_2 = 15/10 = 1.5 \text{ in}, \quad W^t = 30 \text{ lbf}, \quad d_3 = \frac{25}{10} = 2.5 \text{ in}$$

$$\gamma = \tan^{-1} \frac{0.75}{1.25} = 30.96^\circ, \quad \phi = 59.04^\circ$$

$$DE = \frac{9}{16} + 0.5 \cos 59.04^\circ = 0.8197 \text{ in}$$



$$W^r = 30 \tan 20^\circ \cos 59.04^\circ = 5.617 \text{ lbf}$$

$$W^a = 30 \tan 20^\circ \sin 59.04^\circ = 9.363 \text{ lbf}$$

$$\mathbf{W} = -5.617\mathbf{i} - 9.363\mathbf{j} + 30\mathbf{k}$$

$$\mathbf{R}_{DG} = 0.8197\mathbf{j} + 1.25\mathbf{i}$$

$$\mathbf{R}_{DC} = -0.625\mathbf{j}$$

$$\sum \mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{DG} \times \mathbf{W} = 24.591\mathbf{i} - 37.5\mathbf{j} - 7.099\mathbf{k}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = -0.625F_C^z\mathbf{i} + 0.625F_C^x\mathbf{k}$$

$$T = 37.5 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

$$\mathbf{F}_C = 11.4\mathbf{i} + 39.3\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

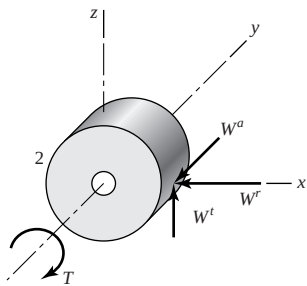
$$F_C = (11.4^2 + 39.3^2)^{1/2} = 40.9 \text{ lbf} \quad \text{Ans.}$$

$$\sum \mathbf{F} = \mathbf{0} \quad \mathbf{F}_D = -5.78\mathbf{i} + 9.363\mathbf{j} - 69.3\mathbf{k} \text{ lbf}$$

$$F_D (\text{radial}) = [(-5.78)^2 + (-69.3)^2]^{1/2} = 69.5 \text{ lbf} \quad \text{Ans.}$$

$$F_D (\text{thrust}) = W^a = 9.363 \text{ lbf} \quad \text{Ans.}$$

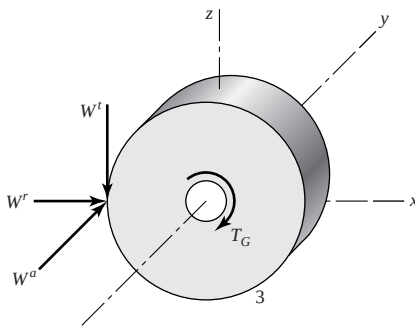
### 13-31 Sketch gear 2 pictorially.



Sketch gear 3 pictorially,

$$P_t = P_n \cos \psi = 4 \cos 30^\circ = 3.464 \text{ teeth/in}$$

$$\phi_t = \tan^{-1} \frac{\tan \phi_n}{\cos \psi} = \tan^{-1} \frac{\tan 20^\circ}{\cos 30^\circ} = 22.80^\circ$$



$$d_P = \frac{18}{3.464} = 5.196 \text{ in}$$

*Pinion (Gear 2)*

$$W^r = W^t \tan \phi_t = 800 \tan 22.80^\circ = 336 \text{ lbf}$$

$$W^a = W^t \tan \psi = 800 \tan 30^\circ = 462 \text{ lbf}$$

$$\mathbf{W} = -336\mathbf{i} - 462\mathbf{j} + 800\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

$$W = [(-336)^2 + (-462)^2 + 800^2]^{1/2} = 983 \text{ lbf} \quad \text{Ans.}$$

*Gear 3*

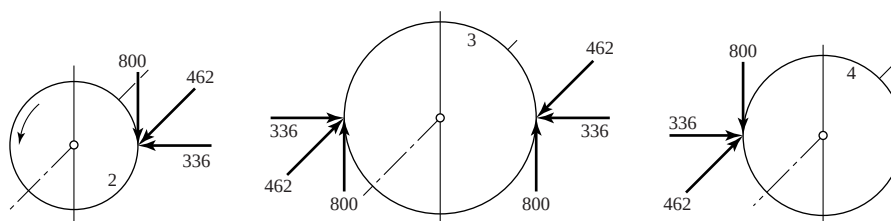
$$\mathbf{W} = 336\mathbf{i} + 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

$$W = 983 \text{ lbf} \quad \text{Ans.}$$

$$d_G = \frac{32}{3.464} = 9.238 \text{ in}$$

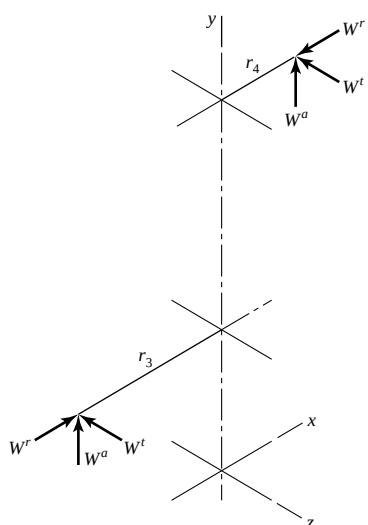
$$T_G = W^t r = 800(9.238) = 7390 \text{ lbf} \cdot \text{in}$$

**13-32** From Prob. 13-31 solution,



Notice that the idler shaft reaction contains a couple tending to turn the shaft end-over-end. Also the idler teeth are bent both ways. Idlers are more severely loaded than other gears, belying their name. Thus be cautious.

**13-33**



*Gear 3:*

$$P_t = P_n \cos \psi = 7 \cos 30^\circ = 6.062 \text{ teeth/in}$$

$$\tan \phi_t = \frac{\tan 20^\circ}{\cos 30^\circ} = 0.4203, \quad \phi_t = 22.8^\circ$$

$$d_3 = \frac{54}{6.062} = 8.908 \text{ in}$$

$$W^t = 500 \text{ lbf}$$

$$W^a = 500 \tan 30^\circ = 288.7 \text{ lbf}$$

$$W^r = 500 \tan 22.8^\circ = 210.2 \text{ lbf}$$

$$\mathbf{W}_3 = 210.2\mathbf{i} + 288.7\mathbf{j} - 500\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

*Gear 4:*

$$d_4 = \frac{14}{6.062} = 2.309 \text{ in}$$

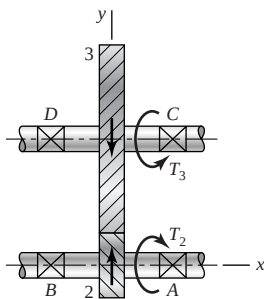
$$W^t = 500 \frac{8.908}{2.309} = 1929 \text{ lbf}$$

$$W^a = 1929 \tan 30^\circ = 1114 \text{ lbf}$$

$$W^r = 1929 \tan 22.8^\circ = 811 \text{ lbf}$$

$$\mathbf{W}_4 = -811\mathbf{i} + 1114\mathbf{j} - 1929\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

13-34



$$P_t = 6 \cos 30^\circ = 5.196 \text{ teeth/in}$$

$$d_3 = \frac{42}{5.196} = 8.083 \text{ in}$$

$$\phi_t = 22.8^\circ$$

$$d_2 = \frac{16}{5.196} = 3.079 \text{ in}$$

$$T_2 = \frac{63\,025(25)}{1720} = 916 \text{ lbf} \cdot \text{in}$$

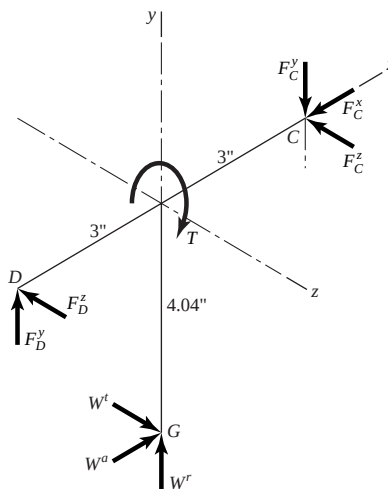
$$W^t = \frac{T}{r} = \frac{916}{3.079/2} = 595 \text{ lbf}$$

$$W^a = 595 \tan 30^\circ = 344 \text{ lbf}$$

$$W^r = 595 \tan 22.8^\circ = 250 \text{ lbf}$$

$$\mathbf{W} = 344\mathbf{i} + 250\mathbf{j} + 595\mathbf{k} \text{ lbf}$$

$$\mathbf{R}_{DC} = 6\mathbf{i}, \quad \mathbf{R}_{DG} = 3\mathbf{i} - 4.04\mathbf{j}$$



$$\sum \mathbf{M}_D = \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{T} = \mathbf{0} \quad (1)$$

$$\mathbf{R}_{DG} \times \mathbf{W} = -2404\mathbf{i} - 1785\mathbf{j} + 2140\mathbf{k}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = -6F_C^z\mathbf{j} + 6F_C^y\mathbf{k}$$

Substituting and solving Eq. (1) gives

$$\mathbf{T} = 2404\mathbf{i} \text{ lbf} \cdot \text{in}$$

$$F_C^z = -297.5 \text{ lbf}$$

$$F_C^y = -356.7 \text{ lbf}$$

$$\sum \mathbf{F} = \mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0}$$

Substituting and solving gives

$$F_C^x = -344 \text{ lbf}$$

$$F_D^y = 106.7 \text{ lbf}$$

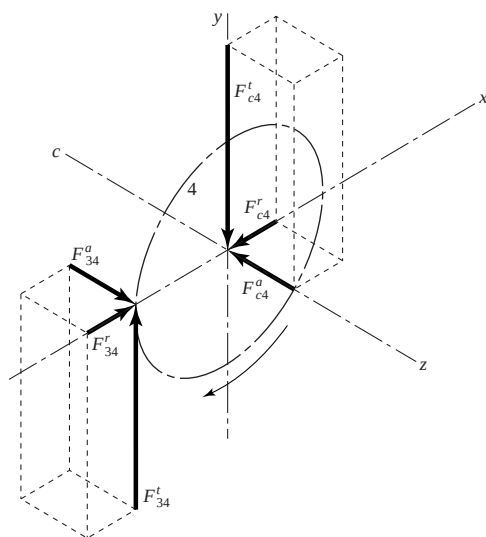
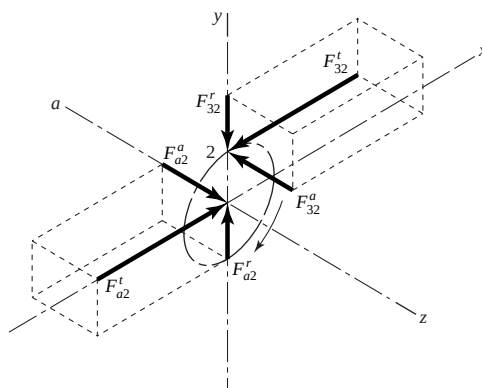
$$F_D^z = -297.5 \text{ lbf}$$

So

$$\mathbf{F}_C = -344\mathbf{i} - 356.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

$$\mathbf{F}_D = 106.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

**13-35**  $P_t = 8 \cos 15^\circ = 7.727 \text{ teeth/in}$



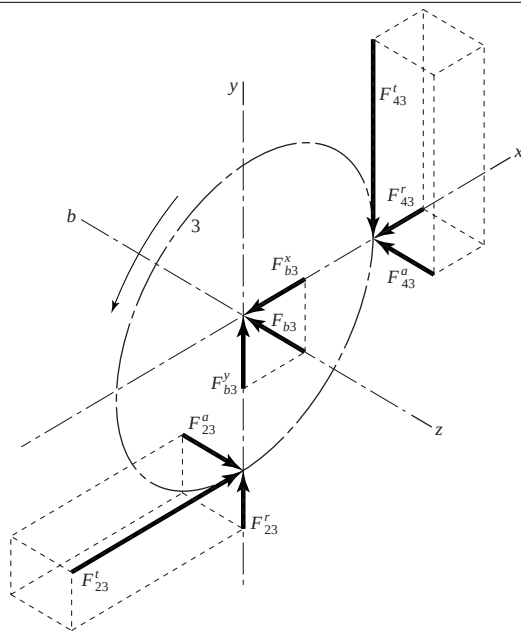
$$d_2 = 16/7.727 = 2.07 \text{ in}$$

$$d_3 = 36/7.727 = 4.66 \text{ in}$$

$$d_4 = 28/7.727 = 3.62 \text{ in}$$

$$T_2 = \frac{63\,025(7.5)}{1720} = 274.8 \text{ lbf} \cdot \text{in}$$

$$W^t = \frac{274.8}{2.07/2} = 266 \text{ lbf}$$



$$W^r = 266 \tan 20^\circ = 96.8 \text{ lbf}$$

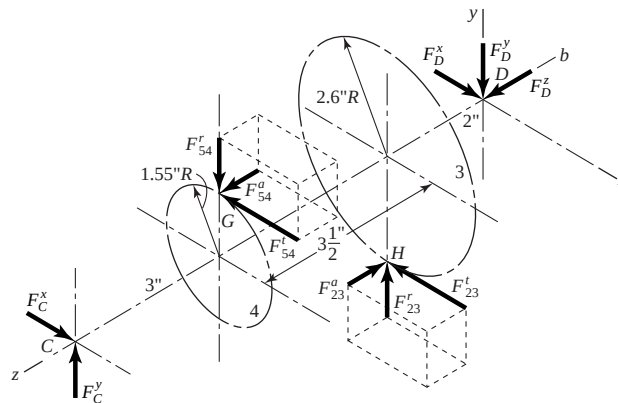
$$W^a = 266 \tan 15^\circ = 71.3 \text{ lbf}$$

$$\mathbf{F}_{2a} = -266\mathbf{i} - 96.8\mathbf{j} - 71.3\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

$$\begin{aligned} \mathbf{F}_{3b} &= (266 - 96.8)\mathbf{i} - (266 - 96.8)\mathbf{j} \\ &= 169\mathbf{i} - 169\mathbf{j} \text{ lbf} \quad \text{Ans.} \end{aligned}$$

$$\mathbf{F}_{4c} = 96.8\mathbf{i} + 266\mathbf{j} + 71.3\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

## 13-36



$$d_2 = \frac{N}{P_n \cos \psi} = \frac{14}{8 \cos 30^\circ} = 2.021 \text{ in}, \quad d_3 = \frac{36}{8 \cos 30^\circ} = 5.196 \text{ in}$$

$$d_4 = \frac{15}{5 \cos 15^\circ} = 3.106 \text{ in}, \quad d_5 = \frac{45}{5 \cos 15^\circ} = 9.317 \text{ in}$$

For gears 2 and 3:  $\phi_t = \tan^{-1}(\tan \phi_n / \cos \psi) = \tan^{-1}(\tan 20^\circ / \cos 30^\circ) = 22.8^\circ$ ,

For gears 4 and 5:  $\phi_t = \tan^{-1}(\tan 20^\circ / \cos 15^\circ) = 20.6^\circ$ ,

$$F_{23}^t = T_2 / r = 1200 / (2.021/2) = 1188 \text{ lbf}$$

$$F_{54}^t = 1188 \frac{5.196}{3.106} = 1987 \text{ lbf}$$

$$F_{23}^r = F_{23}^t \tan \phi_t = 1188 \tan 22.8^\circ = 499 \text{ lbf}$$

$$F_{54}^r = 1986 \tan 20.6^\circ = 746 \text{ lbf}$$

$$F_{23}^a = F_{23}^t \tan \psi = 1188 \tan 30^\circ = 686 \text{ lbf}$$

$$F_{54}^a = 1986 \tan 15^\circ = 532 \text{ lbf}$$

Next, designate the points of action on gears 4 and 3, respectively, as points  $G$  and  $H$ , as shown. Position vectors are

$$\mathbf{R}_{CG} = 1.553\mathbf{j} - 3\mathbf{k}$$

$$\mathbf{R}_{CH} = -2.598\mathbf{j} - 6.5\mathbf{k}$$

$$\mathbf{R}_{CD} = -8.5\mathbf{k}$$

Force vectors are

$$\mathbf{F}_{54} = -1986\mathbf{i} - 748\mathbf{j} + 532\mathbf{k}$$

$$\mathbf{F}_{23} = -1188\mathbf{i} + 500\mathbf{j} - 686\mathbf{k}$$

$$\mathbf{F}_C = F_C^x\mathbf{i} + F_C^y\mathbf{j}$$

$$\mathbf{F}_D = F_D^x\mathbf{i} + F_D^y\mathbf{j} + F_D^z\mathbf{k}$$

Now, a summation of moments about bearing  $C$  gives

$$\sum \mathbf{M}_C = \mathbf{R}_{CG} \times \mathbf{F}_{54} + \mathbf{R}_{CH} \times \mathbf{F}_{23} + \mathbf{R}_{CD} \times \mathbf{F}_D = \mathbf{0}$$

The terms for this equation are found to be

$$\mathbf{R}_{CG} \times \mathbf{F}_{54} = -1412\mathbf{i} + 5961\mathbf{j} + 3086\mathbf{k}$$

$$\mathbf{R}_{CH} \times \mathbf{F}_{23} = 5026\mathbf{i} + 7722\mathbf{j} - 3086\mathbf{k}$$

$$\mathbf{R}_{CD} \times \mathbf{F}_D = 8.5F_D^y\mathbf{i} - 8.5F_D^x\mathbf{j}$$

When these terms are placed back into the moment equation, the  $\mathbf{k}$  terms, representing the shaft torque, cancel. The  $\mathbf{i}$  and  $\mathbf{j}$  terms give

$$F_D^y = -\frac{3614}{8.5} = -425 \text{ lbf} \quad \text{Ans.}$$

$$F_D^x = \frac{(13\,683)}{8.5} = 1610 \text{ lbf} \quad \text{Ans.}$$

Next, we sum the forces to zero.

$$\sum \mathbf{F} = \mathbf{F}_C + \mathbf{F}_{54} + \mathbf{F}_{23} + \mathbf{F}_D = \mathbf{0}$$

Substituting, gives

$$(F_C^x\mathbf{i} + F_C^y\mathbf{j}) + (-1987\mathbf{i} - 746\mathbf{j} + 532\mathbf{k}) + (-1188\mathbf{i} + 499\mathbf{j} - 686\mathbf{k}) + (1610\mathbf{i} - 425\mathbf{j} + F_D^z\mathbf{k}) = \mathbf{0}$$

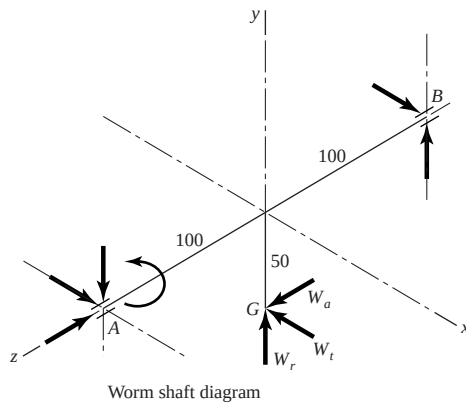
Solving gives

$$F_C^x = 1987 + 1188 - 1610 = 1565 \text{ lbf}$$

$$F_C^y = 746 - 499 + 425 = 672 \text{ lbf}$$

$$F_D^z = -532 + 686 = 154 \text{ lbf} \quad \text{Ans.}$$

13-37



$$V_W = \frac{\pi d_W n_W}{60} = \frac{\pi(0.100)(600)}{60} = \pi \text{ m/s}$$

$$W_{Wt} = \frac{H}{V_W} = \frac{2000}{\pi} = 637 \text{ N}$$

$$L = p_x N_W = 25(1) = 25 \text{ mm}$$

$$\lambda = \tan^{-1} \frac{L}{\pi d_W}$$

$$= \tan^{-1} \frac{25}{\pi(100)} = 4.550^\circ \quad \text{lead angle}$$

$$W = \frac{W_{Wt}}{\cos \phi_n \sin \lambda + f \cos \lambda}$$

$$V_S = \frac{V_W}{\cos \lambda} = \frac{\pi}{\cos 4.550^\circ} = 3.152 \text{ m/s}$$

In ft/min:  $V_S = 3.28(3.152) = 10.33 \text{ ft/s} = 620 \text{ ft/min}$

Use  $f = 0.043$  from curve A of Fig. 13-42. Then from the first of Eq. (13-43)

$$W = \frac{637}{\cos 14.5^\circ (\sin 4.55^\circ) + 0.043 \cos 4.55^\circ} = 5323 \text{ N}$$

$$W^y = W \sin \phi_n = 5323 \sin 14.5^\circ = 1333 \text{ N}$$

$$W^z = 5323 [\cos 14.5^\circ (\cos 4.55^\circ) - 0.043 \sin 4.55^\circ] = 5119 \text{ N}$$

The force acting against the worm is

$$\mathbf{W} = -637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} \text{ N}$$

Thus A is the thrust bearing. *Ans.*

$$\mathbf{R}_{AG} = -0.05\mathbf{j} - 0.10\mathbf{k}, \quad \mathbf{R}_{AB} = -0.20\mathbf{k}$$

$$\sum \mathbf{M}_A = \mathbf{R}_{AG} \times \mathbf{W} + \mathbf{R}_{AB} \times \mathbf{F}_B + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{AG} \times \mathbf{W} = -122.6\mathbf{i} + 63.7\mathbf{j} - 31.85\mathbf{k}$$

$$\mathbf{R}_{AB} \times \mathbf{F}_B = 0.2F_B^y\mathbf{i} - 0.2F_B^x\mathbf{j}$$

Substituting and solving gives

$$T = 31.85 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

$$F_B^x = 318.5 \text{ N}, \quad F_B^y = 613 \text{ N}$$

So  $\mathbf{F}_B = 318.5\mathbf{i} + 613\mathbf{j} \text{ N} \quad \text{Ans.}$

Or  $F_B = [(613)^2 + (318.5)^2]^{1/2} = 691 \text{ N radial}$

$$\sum \mathbf{F} = \mathbf{F}_A + \mathbf{W} + \mathbf{R}_B = \mathbf{0}$$

$$\begin{aligned} \mathbf{F}_A &= -(\mathbf{W} + \mathbf{F}_B) = -(-637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} + 318.5\mathbf{i} + 613\mathbf{j}) \\ &= 318.5\mathbf{i} - 1946\mathbf{j} - 5119\mathbf{k} \quad \text{Ans.} \end{aligned}$$

Radial  $\mathbf{F}_A^r = 318.5\mathbf{i} - 1946\mathbf{j}$  N,  
 $F_A^r = [(318.5)^2 + (-1946)^2]^{1/2} = 1972$  N  
 Thrust  $F_A^a = -5119$  N

**13-38** From Prob. 13-37

$$\mathbf{W}_G = 637\mathbf{i} - 1333\mathbf{j} - 5119\mathbf{k}$$

$$p_t = p_x$$

So 
$$d_G = \frac{N_G p_x}{\pi} = \frac{48(25)}{\pi} = 382 \text{ mm}$$

Bearing  $D$  to take thrust load

$$\sum \mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W}_G + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{DG} = -0.0725\mathbf{i} + 0.191\mathbf{j}$$

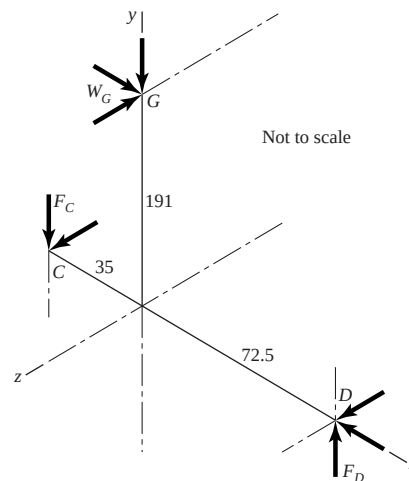
$$\mathbf{R}_{DC} = -0.1075\mathbf{i}$$

The position vectors are in meters.

$$\mathbf{R}_{DG} \times \mathbf{W}_G = -977.7\mathbf{i} - 371.1\mathbf{j} - 25.02\mathbf{k}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = 0.1075 F_C^z \mathbf{j} - 0.1075 F_C^y \mathbf{k}$$

Putting it together and solving



Gives

$$T = 977.7 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

$$\mathbf{F}_C = -233\mathbf{j} + 3450\mathbf{k} \text{ N}, \quad F_C = 3460 \text{ N} \quad \text{Ans.}$$

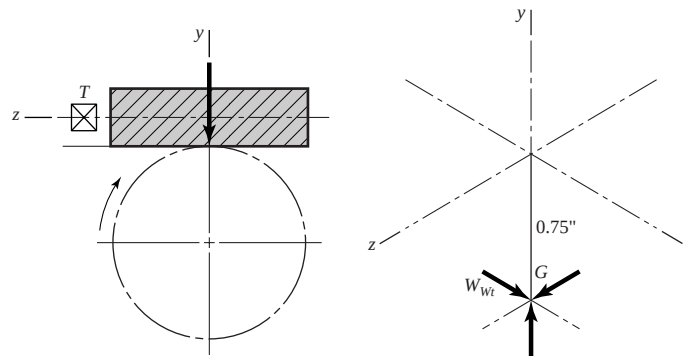
$$\sum \mathbf{F} = \mathbf{F}_C + \mathbf{W}_G + \mathbf{F}_D = \mathbf{0}$$

$$\mathbf{F}_D = -(\mathbf{F}_C + \mathbf{W}_G) = -637\mathbf{i} + 1566\mathbf{j} + 1669\mathbf{k} \text{ N} \quad \text{Ans.}$$

Radial  $\mathbf{F}_D^r = 1566\mathbf{j} + 1669\mathbf{k}$  N

Or  $F_D^r = 2289 \text{ N}$  (total radial)

$$\mathbf{F}_D^t = -637\mathbf{i} \text{ N} \quad \text{(thrust)}$$

**13-39**


$$V_W = \frac{\pi(1.5)(900)}{12} = 353.4 \text{ ft/min}$$

$$W^x = W_{Wt} = \frac{33\,000(0.5)}{353.4} = 46.69 \text{ lbf}$$

$$p_t = p_x = \frac{\pi}{10} = 0.314\,16 \text{ in}$$

$$L = 0.314\,16(2) = 0.628 \text{ in}$$

$$\lambda = \tan^{-1} \frac{0.628}{\pi(1.5)} = 7.59^\circ$$

$$W = \frac{46.7}{\cos 14.5^\circ \sin 7.59^\circ + 0.05 \cos 7.59^\circ} = 263 \text{ lbf}$$

$$W^y = 263 \sin 14.5^\circ = 65.8 \text{ lbf}$$

$$W^z = 263[\cos 14.5^\circ (\cos 7.59^\circ) - 0.05 \sin 7.59^\circ] = 251 \text{ lbf}$$

So

$$\mathbf{W} = 46.7\mathbf{i} + 65.8\mathbf{j} + 251\mathbf{k} \text{ lbf} \quad \text{Ans.}$$

$$T = 46.7(0.75) = 35 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

**13-40** Computer programs will vary.