

Chapter 16

16-1

(a) $\theta_1 = 0^\circ$, $\theta_2 = 120^\circ$, $\theta_a = 90^\circ$, $\sin \theta_a = 1$, $a = 5$ in

$$\text{Eq. (16-2): } M_f = \frac{0.28 p_a (1.5)(6)}{1} \int_{0^\circ}^{120^\circ} \sin \theta (6 - 5 \cos \theta) d\theta$$

$$= 17.96 p_a \text{ lbf} \cdot \text{in}$$

$$\text{Eq. (16-3): } M_N = \frac{p_a (1.5)(6)(5)}{1} \int_{0^\circ}^{120^\circ} \sin^2 \theta d\theta = 56.87 p_a \text{ lbf} \cdot \text{in}$$

$$c = 2(5 \cos 30^\circ) = 8.66 \text{ in}$$

$$\text{Eq. (16-4): } F = \frac{56.87 p_a - 17.96 p_a}{8.66} = 4.49 p_a$$

$$p_a = F/4.49 = 500/4.49 = 111.4 \text{ psi for cw rotation}$$

$$\text{Eq. (16-7): } 500 = \frac{56.87 p_a + 17.96 p_a}{8.66}$$

$$p_a = 57.9 \text{ psi for ccw rotation}$$

A maximum pressure of 111.4 psi occurs on the RH shoe for cw rotation. *Ans.*

(b) *RH shoe:*

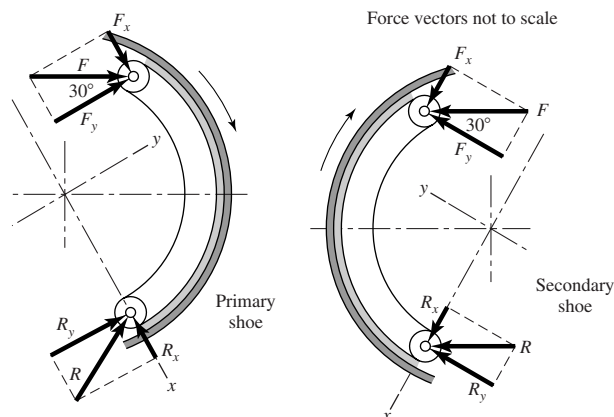
$$\text{Eq. (16-6): } T_R = \frac{0.28(111.4)(1.5)(6)^2(\cos 0^\circ - \cos 120^\circ)}{1} = 2530 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

LH shoe:

$$\text{Eq. (16-6): } T_L = \frac{0.28(57.9)(1.5)(6)^2(\cos 0^\circ - \cos 120^\circ)}{1} = 1310 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

$$T_{\text{total}} = 2530 + 1310 = 3840 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

(c)



RH shoe: $F_x = 500 \sin 30^\circ = 250 \text{ lbf}, \quad F_y = 500 \cos 30^\circ = 433 \text{ lbf}$

Eqs. (16-8): $A = \left(\frac{1}{2} \sin^2 \theta \right)_{0^\circ}^{120^\circ} = 0.375, \quad B = \left(\frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right)_0^{2\pi/3 \text{ rad}} = 1.264$

Eqs. (16-9): $R_x = \frac{111.4(1.5)(6)}{1} [0.375 - 0.28(1.264)] - 250 = -229 \text{ lbf}$

$$R_y = \frac{111.4(1.5)(6)}{1} [1.264 + 0.28(0.375)] - 433 = 940 \text{ lbf}$$

$$R = [(-229)^2 + (940)^2]^{1/2} = 967 \text{ lbf} \quad \text{Ans.}$$

LH shoe: $F_x = 250 \text{ lbf}, \quad F_y = 433 \text{ lbf}$

Eqs. (16-10): $R_x = \frac{57.9(1.5)(6)}{1} [0.375 + 0.28(1.264)] - 250 = 130 \text{ lbf}$

$$R_y = \frac{57.9(1.5)(6)}{1} [1.264 - 0.28(0.375)] - 433 = 171 \text{ lbf}$$

$$R = [(130)^2 + (171)^2]^{1/2} = 215 \text{ lbf} \quad \text{Ans.}$$

16-2 $\theta_1 = 15^\circ, \quad \theta_2 = 105^\circ, \quad \theta_a = 90^\circ, \quad \sin \theta_a = 1, \quad a = 5 \text{ in}$

Eq. (16-2): $M_f = \frac{0.28 p_a (1.5)(6)}{1} \int_{15^\circ}^{105^\circ} \sin \theta (6 - 5 \cos \theta) d\theta = 13.06 p_a$

Eq. (16-3): $M_N = \frac{p_a (1.5)(6)(5)}{1} \int_{15^\circ}^{105^\circ} \sin^2 \theta d\theta = 46.59 p_a$

$$c = 2(5 \cos 30^\circ) = 8.66 \text{ in}$$

Eq. (16-4): $F = \frac{46.59 p_a - 13.06 p_a}{8.66} = 3.872 p_a$

RH shoe:

$$p_a = 500/3.872 = 129.1 \text{ psi} \quad \text{on RH shoe for cw rotation} \quad \text{Ans.}$$

Eq. (16-6): $T_R = \frac{0.28(129.1)(1.5)(6^2)(\cos 15^\circ - \cos 105^\circ)}{1} = 2391 \text{ lbf} \cdot \text{in}$

LH shoe:

$$500 = \frac{46.59 p_a + 13.06 p_a}{8.66} \Rightarrow p_a = 72.59 \text{ psi} \quad \text{on LH shoe for ccw rotation} \quad \text{Ans.}$$

$$T_L = \frac{0.28(72.59)(1.5)(6^2)(\cos 15^\circ - \cos 105^\circ)}{1} = 1344 \text{ lbf} \cdot \text{in}$$

$$T_{\text{total}} = 2391 + 1344 = 3735 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

Comparing this result with that of Prob. 16-1, a 2.7% reduction in torque is achieved by using 25% less braking material.

16-3 Given: $\theta_1 = 0^\circ$, $\theta_2 = 120^\circ$, $\theta_a = 90^\circ$, $\sin \theta_a = 1$, $a = R = 90$ mm, $f = 0.30$, $F = 1000$ N = 1 kN, $r = 280/2 = 140$ mm, counter-clockwise rotation.

LH shoe:

$$\begin{aligned} M_f &= \frac{fpabr}{\sin \theta_a} \left[r(1 - \cos \theta_2) - \frac{a}{2} \sin^2 \theta_2 \right] \\ &= \frac{0.30p_a(0.030)(0.140)}{1} \left[0.140(1 - \cos 120^\circ) - \frac{0.090}{2} \sin^2 120^\circ \right] \\ &= 0.000\,222p_a \text{ N} \cdot \text{m} \end{aligned}$$

$$\begin{aligned} M_N &= \frac{pabra}{\sin \theta_a} \left[\frac{\theta_2}{2} - \frac{1}{4} \sin 2\theta_2 \right] \\ &= \frac{p_a(0.030)(0.140)(0.090)}{1} \left[\frac{120^\circ}{2} \left(\frac{\pi}{180} \right) - \frac{1}{4} \sin 2(120^\circ) \right] \\ &= 4.777(10^{-4})p_a \text{ N} \cdot \text{m} \end{aligned}$$

$$c = 2r \cos \left(\frac{180^\circ - \theta_2}{2} \right) = 2(0.090) \cos 30^\circ = 0.155\,88 \text{ m}$$

$$F = 1 = p_a \left[\frac{4.777(10^{-4}) - 2.22(10^{-4})}{0.155\,88} \right] = 1.64(10^{-3})p_a$$

$$p_a = 1/1.64(10^{-3}) = 610 \text{ kPa}$$

$$\begin{aligned} T_L &= \frac{fpabr^2(\cos \theta_1 - \cos \theta_2)}{\sin \theta_a} \\ &= \frac{0.30(610)(10^3)(0.030)(0.140^2)}{1} [1 - (-0.5)] \\ &= 161.4 \text{ N} \cdot \text{m} \quad \text{Ans.} \end{aligned}$$

RH shoe:

$$M_f = 2.22(10^{-4})p_a \text{ N} \cdot \text{m}$$

$$M_N = 4.77(10^{-4})p_a \text{ N} \cdot \text{m}$$

$$c = 0.155\,88 \text{ m}$$

$$F = 1 = p_a \left[\frac{4.77(10^{-4}) + 2.22(10^{-4})}{0.155\,88} \right] = 4.49(10^{-3})p_a$$

$$p_a = \frac{1}{4.49(10^{-3})} = 222.8 \text{ kPa} \quad \text{Ans.}$$

$$T_R = (222.8/610)(161.4) = 59.0 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

16-4

- (a) Given: $\theta_1 = 10^\circ$, $\theta_2 = 75^\circ$, $\theta_a = 75^\circ$, $p_a = 10^6$ Pa, $f = 0.24$,
 $b = 0.075$ m (shoe width), $a = 0.150$ m, $r = 0.200$ m, $d = 0.050$ m, $c = 0.165$ m.
 Some of the terms needed are evaluated as:

$$A = \left[r \int_{\theta_1}^{\theta_2} \sin \theta d\theta - a \int_{\theta_1}^{\theta_2} \sin \theta \cos \theta d\theta \right] = r [-\cos \theta]_{\theta_1}^{\theta_2} - a \left[\frac{1}{2} \sin^2 \theta \right]_{\theta_1}^{\theta_2}$$

$$= 200 [-\cos \theta]_{10^\circ}^{75^\circ} - 150 \left[\frac{1}{2} \sin^2 \theta \right]_{10^\circ}^{75^\circ} = 77.5 \text{ mm}$$

$$B = \int_{\theta_1}^{\theta_2} \sin^2 \theta d\theta = \left[\frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right]_{10\pi/180 \text{ rad}}^{75\pi/180 \text{ rad}} = 0.528$$

$$C = \int_{\theta_1}^{\theta_2} \sin \theta \cos \theta d\theta = 0.4514$$

Now converting to pascals and meters, we have from Eq. (16-2),

$$M_f = \frac{f p_a b r}{\sin \theta_a} A = \frac{0.24[(10)^6](0.075)(0.200)}{\sin 75^\circ} (0.0775) = 289 \text{ N} \cdot \text{m}$$

From Eq. (16-3),

$$M_N = \frac{p_a b r a}{\sin \theta_a} B = \frac{[(10)^6](0.075)(0.200)(0.150)}{\sin 75^\circ} (0.528) = 1230 \text{ N} \cdot \text{m}$$

Finally, using Eq. (16-4), we have

$$F = \frac{M_N - M_f}{c} = \frac{1230 - 289}{0.165} = 5.70 \text{ kN} \quad \text{Ans.}$$

- (b) Use Eq. (16-6) for the primary shoe.

$$T = \frac{f p_a b r^2 (\cos \theta_1 - \cos \theta_2)}{\sin \theta_a}$$

$$= \frac{0.24[(10)^6](0.075)(0.200)^2 (\cos 10^\circ - \cos 75^\circ)}{\sin 75^\circ} = 541 \text{ N} \cdot \text{m}$$

For the secondary shoe, we must first find p_a .

Substituting

$$M_N = \frac{1230}{10^6} p_a \quad \text{and} \quad M_f = \frac{289}{10^6} p_a \quad \text{into Eq. (16-7),}$$

$$5.70 = \frac{(1230/10^6) p_a + (289/10^6) p_a}{0.165}, \quad \text{solving gives} \quad p_a = 619(10)^3 \text{ Pa}$$

Then

$$T = \frac{0.24[0.619(10)^6](0.075)(0.200)^2 (\cos 10^\circ - \cos 75^\circ)}{\sin 75^\circ} = 335 \text{ N} \cdot \text{m}$$

so the braking capacity is $T_{\text{total}} = 2(541) + 2(335) = 1750 \text{ N} \cdot \text{m} \quad \text{Ans.}$

(c) *Primary shoes:*

$$R_x = \frac{p_a b r}{\sin \theta_a} (C - f B) - F_x$$

$$= \frac{(10^6)(0.075)(0.200)}{\sin 75^\circ} [0.4514 - 0.24(0.528)](10)^{-3} - 5.70 = -0.658 \text{ kN}$$

$$R_y = \frac{p_a b r}{\sin \theta_a} (B + f C) - F_y$$

$$= \frac{(10^6)(0.075)(0.200)}{\sin 75^\circ} [0.528 + 0.24(0.4514)](10)^{-3} - 0 = 9.88 \text{ kN}$$

Secondary shoes:

$$R_x = \frac{p_a b r}{\sin \theta_a} (C + f B) - F_x$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75^\circ} [0.4514 + 0.24(0.528)](10)^{-3} - 5.70$$

$$= -0.143 \text{ kN}$$

$$R_y = \frac{p_a b r}{\sin \theta_a} (B - f C) - F_y$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75^\circ} [0.528 - 0.24(0.4514)](10)^{-3} - 0$$

$$= 4.03 \text{ kN}$$

Note from figure that +y for secondary shoe is opposite to +y for primary shoe.

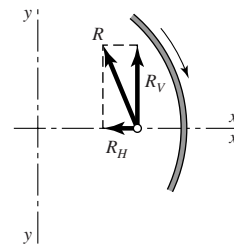
Combining horizontal and vertical components,

$$R_H = -0.658 - 0.143 = -0.801 \text{ kN}$$

$$R_V = 9.88 - 4.03 = 5.85 \text{ kN}$$

$$R = \sqrt{(0.801)^2 + (5.85)^2}$$

$$= 5.90 \text{ kN} \quad \text{Ans.}$$



16-5 Preliminaries: $\theta_1 = 45^\circ - \tan^{-1}(150/200) = 8.13^\circ$, $\theta_2 = 98.13^\circ$

$$\theta_a = 90^\circ, \quad a = [(150)^2 + (200)^2]^{1/2} = 250 \text{ mm}$$

Eq. (16-8): $A = \frac{1}{2} (\sin^2 \theta)_{8.13^\circ}^{98.13^\circ} = 0.480$

Let $C = \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta = -(\cos \theta)_{8.13^\circ}^{98.13^\circ} = 1.1314$

Eq. (16-2):

$$M_f = \frac{fpabr}{\sin \theta_a}(rC - aA) = \frac{0.25p_a(0.030)(0.150)}{\sin 90^\circ}[0.15(1.1314) - 0.25(0.48)]$$

$$= 5.59(10^{-5})p_a \text{ N} \cdot \text{m}$$

Eq. (16-8): $B = \left(\frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right)_{8.13\pi/180 \text{ rad}}^{98.13\pi/180 \text{ rad}} = 0.925$

Eq. (16-3): $M_N = \frac{pabra}{\sin \theta_a}B = \frac{p_a(0.030)(0.150)(0.250)}{1}(0.925)$

$$= 1.0406(10^{-3})p_a \text{ N} \cdot \text{m}$$

Using $F = (M_N - M_f)/c$, we obtain

$$400 = \frac{104.06 - 5.59}{0.5(10^5)}p_a \quad \text{or} \quad p_a = 203 \text{ kPa} \quad \text{Ans.}$$

$$T = \frac{fpabr^2C}{\sin \theta_a} = \frac{0.25(203)(10^3)(0.030)(0.150)^2}{1}(1.1314)$$

$$= 38.76 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

16-6 For $+3\hat{\sigma}_f$:

$$f = \bar{f} + 3\hat{\sigma}_f = 0.25 + 3(0.025) = 0.325$$

$$M_f = 5.59(10^{-5})p_a \left(\frac{0.325}{0.25} \right) = 7.267(10^{-5})p_a$$

Eq. (16-4):

$$400 = \frac{104.06 - 7.267}{10^5(0.500)}p_a$$

$$p_a = 207 \text{ kPa}$$

$$T = 38.75 \left(\frac{207}{203} \right) \left(\frac{0.325}{0.25} \right) = 51.4 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

Similarly, for $-3\hat{\sigma}_f$:

$$f = \bar{f} - 3\hat{\sigma}_f = 0.25 - 3(0.025) = 0.175$$

$$M_f = 3.913(10^{-5})p_a$$

$$p_a = 200 \text{ kPa}$$

$$T = 26.7 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

16-7 Preliminaries: $\theta_2 = 180^\circ - 30^\circ - \tan^{-1}(3/12) = 136^\circ$, $\theta_1 = 20^\circ - \tan^{-1}(3/12) = 6^\circ$,
 $\theta_a = 90^\circ$, $a = [(3)^2 + (12)^2]^{1/2} = 12.37 \text{ in}$, $r = 10 \text{ in}$, $f = 0.30$, $b = 2 \text{ in}$.

Eq. (16-2): $M_f = \frac{0.30(150)(2)(10)}{\sin 90^\circ} \int_{6^\circ}^{136^\circ} \sin \theta (10 - 12.37 \cos \theta) d\theta$

$$= 12\,800 \text{ lbf} \cdot \text{in}$$

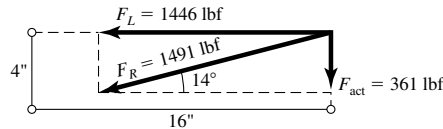
$$\text{Eq. (16-3): } M_N = \frac{150(2)(10)(12.37)}{\sin 90^\circ} \int_{6^\circ}^{136^\circ} \sin^2 \theta d\theta = 53\,300 \text{ lbf} \cdot \text{in}$$

LH shoe:

$$c_L = 12 + 12 + 4 = 28 \text{ in}$$

Now note that M_f is cw and M_N is ccw. Thus,

$$F_L = \frac{53\,300 - 12\,800}{28} = 1446 \text{ lbf}$$



$$\text{Eq. (16-6): } T_L = \frac{0.30(150)(2)(10)^2(\cos 6^\circ - \cos 136^\circ)}{\sin 90^\circ} = 15\,420 \text{ lbf} \cdot \text{in}$$

RH shoe:

$$M_N = 53\,300 \left(\frac{p_a}{150} \right) = 355.3 p_a, \quad M_f = 12\,800 \left(\frac{p_a}{150} \right) = 85.3 p_a$$

On this shoe, both M_N and M_f are ccw.

$$\text{Also } c_R = (24 - 2 \tan 14^\circ) \cos 14^\circ = 22.8 \text{ in}$$

$$F_{act} = F_L \sin 14^\circ = 361 \text{ lbf} \quad \text{Ans.}$$

$$F_R = F_L / \cos 14^\circ = 1491 \text{ lbf}$$

$$\text{Thus } 1491 = \frac{355.3 + 85.3}{22.8} p_a \Rightarrow p_a = 77.2 \text{ psi}$$

$$\text{Then } T_R = \frac{0.30(77.2)(2)(10)^2(\cos 6^\circ - \cos 136^\circ)}{\sin 90^\circ} = 7940 \text{ lbf} \cdot \text{in}$$

$$T_{\text{total}} = 15\,420 + 7940 = 23\,400 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

16-8

$$M_f = 2 \int_0^{\theta_2} (f dN)(a' \cos \theta - r) \quad \text{where } dN = pbr d\theta$$

$$= 2fpbr \int_0^{\theta_2} (a' \cos \theta - r) d\theta = 0$$

From which

$$a' \int_0^{\theta_2} \cos \theta d\theta = r \int_0^{\theta_2} d\theta$$

$$a' = \frac{r\theta_2}{\sin \theta_2} = \frac{r(60^\circ)(\pi/180)}{\sin 60^\circ} = 1.209r$$

Eq. (16-15)

$$a = \frac{4r \sin 60^\circ}{2(60)(\pi/180) + \sin[2(60)]} = 1.170r$$

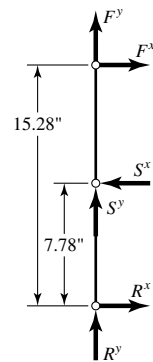
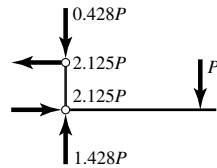
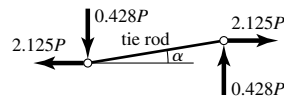
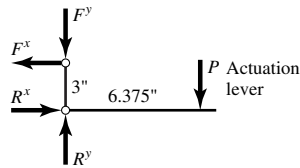
16-9

(a) Counter-clockwise rotation, $\theta_2 = \pi/4$ rad, $r = 13.5/2 = 6.75$ in

$$a = \frac{4r \sin \theta_2}{2\theta_2 + \sin 2\theta_2} = \frac{4(6.75) \sin(\pi/4)}{2\pi/4 + \sin(2\pi/4)} = 7.426 \text{ in}$$

$$e = 2(7.426) = 14.85 \text{ in} \quad \text{Ans.}$$

(b)



$$\alpha = \tan^{-1}(3/14.85) = 11.4^\circ$$

$$\sum M_R = 0 = 3F^x - 6.375P$$

$$F^x = 2.125P$$

$$\sum F_x = 0 = -F^x + R^x$$

$$R^x = F^x = 2.125P$$

$$F^y = F^x \tan 11.4^\circ = 0.428P$$

$$\sum F_y = -P - F^y + R^y$$

$$R^y = P + 0.428P = 1.428P$$

Left shoe lever.

$$\sum M_R = 0 = 7.78S^x - 15.28F^x$$

$$S^x = \frac{15.28}{7.78}(2.125P) = 4.174P$$

$$S^y = f S^x = 0.30(4.174P)$$

$$= 1.252P$$

$$\sum F_y = 0 = R^y + S^y + F^y$$

$$R^y = -F^y - S^y$$

$$= -0.428P - 1.252P$$

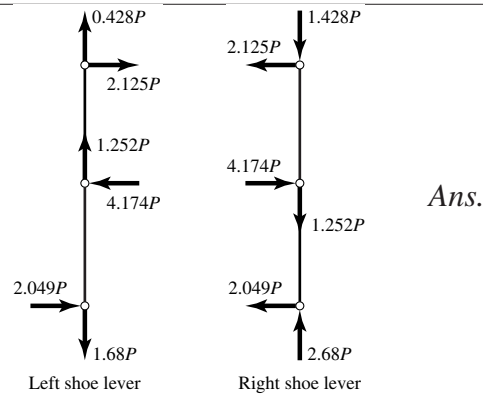
$$= -1.68P$$

$$\sum F_x = 0 = R^x - S^x + F^x$$

$$R^x = S^x - F^x$$

$$= 4.174P - 2.125P$$

$$= 2.049P$$



Ans.

- (c) The direction of brake pulley rotation affects the sense of S^y , which has no effect on the brake shoe lever moment and hence, no effect on S^x or the brake torque.

The brake shoe levers carry identical bending moments but the left lever carries a tension while the right carries compression (column loading). The right lever is designed and used as a left lever, producing interchangeable levers (identical levers). But do not infer from these identical loadings.

16-10 $r = 13.5/2 = 6.75$ in, $b = 7.5$ in, $\theta_2 = 45^\circ$

From Table 16-3 for a rigid, molded nonasbestos use a conservative estimate of $p_a = 100$ psi, $f = 0.31$.

In Eq. (16-16):

$$2\theta_2 + \sin 2\theta_2 = 2(\pi/4) + \sin 2(45^\circ) = 2.571$$

From Prob. 16-9 solution,

$$N = S^x = 4.174P = \frac{p_a b r}{2}(2.571) = 1.285 p_a b r$$

$$P = \frac{1.285}{4.174}(100)(7.5)(6.75) = 1560 \text{ lbf} \quad \text{Ans.}$$

Applying Eq. (16-18) for two shoes,

$$\begin{aligned} T &= 2afN = 2(7.426)(0.31)(4.174)(1560) \\ &= 29\,980 \text{ lbf} \cdot \text{in} \quad \text{Ans.} \end{aligned}$$

16-11 From Eq. (16-22),

$$P_1 = \frac{p_a b D}{2} = \frac{90(4)(14)}{2} = 2520 \text{ lbf} \quad \text{Ans.}$$

$$f\phi = 0.25(\pi)(270^\circ/180^\circ) = 1.178$$

Eq. (16-19): $P_2 = P_1 \exp(-f\phi) = 2520 \exp(-1.178) = 776 \text{ lbf} \quad \text{Ans.}$

$$T = \frac{(P_1 - P_2)D}{2} = \frac{(2520 - 776)14}{2} = 12\,200 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

16-12 Given: $D = 300 \text{ mm}$, $f = 0.28$, $b = 80 \text{ mm}$, $\phi = 270^\circ$, $P_1 = 7600 \text{ N}$.

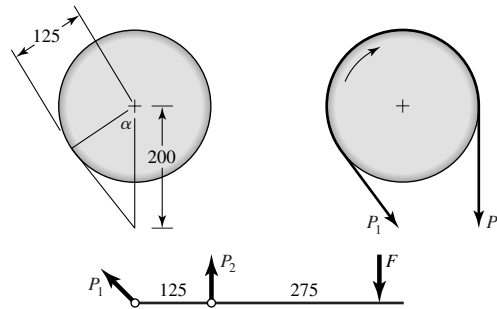
$$f\phi = 0.28(\pi)(270^\circ/180^\circ) = 1.319$$

$$P_2 = P_1 \exp(-f\phi) = 7600 \exp(-1.319) = 2032 \text{ N}$$

$$p_a = \frac{2P_1}{bD} = \frac{2(7600)}{80(300)} = 0.6333 \text{ N/mm}^2 \quad \text{or} \quad 633 \text{ kPa} \quad \text{Ans.}$$

$$T = (P_1 - P_2) \frac{D}{2} = (7600 - 2032) \frac{300}{2} \\ = 835\,200 \text{ N} \cdot \text{mm} \quad \text{or} \quad 835.2 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

16-13



$$\alpha = \cos^{-1} \left(\frac{125}{200} \right) = 51.32^\circ$$

$$\phi = 270^\circ - 51.32^\circ = 218.7^\circ$$

$$f\phi = 0.30(218.7) \frac{\pi}{180^\circ} = 1.145$$

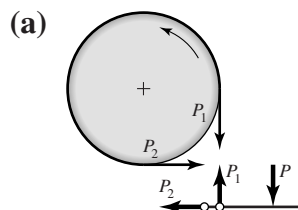
$$P_2 = \frac{(125 + 275)F}{125} = \frac{(125 + 275)400}{125} = 1280 \text{ N} \quad \text{Ans.}$$

$$P_1 = P_2 \exp(f\phi) = 1280 \exp(1.145) = 4022 \text{ N}$$

$$T = (P_1 - P_2) \frac{D}{2} = (4022 - 1280) \frac{250}{2}$$

$$= 342\,750 \text{ N} \cdot \text{mm} \quad \text{or} \quad 343 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

16-14



$$D = 16", \quad b = 3"$$

$$n = 200 \text{ rev/min}$$

$$f = 0.20, \quad p_a = 70 \text{ psi}$$

Eq. (16-22):

$$P_1 = \frac{p_a b D}{2} = \frac{70(3)(16)}{2} = 1680 \text{ lbf}$$

$$f\phi = 0.20(3\pi/2) = 0.942$$

Eq. (16-14): $P_2 = P_1 \exp(-f\phi) = 1680 \exp(-0.942) = 655 \text{ lbf}$

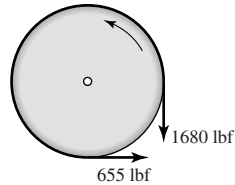
$$T = (P_1 - P_2) \frac{D}{2} = (1680 - 655) \frac{16}{2}$$

$$= 8200 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

$$H = \frac{Tn}{63\,025} = \frac{8200(200)}{63\,025} = 26.0 \text{ hp} \quad \text{Ans.}$$

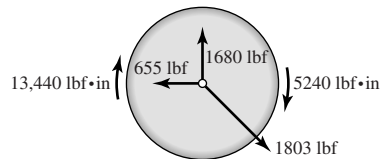
$$P = \frac{3P_1}{10} = \frac{3(1680)}{10} = 504 \text{ lbf} \quad \text{Ans.}$$

(b)



Force of belt on the drum:

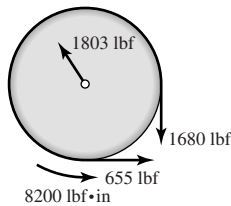
$$R = (1680^2 + 655^2)^{1/2} = 1803 \text{ lbf}$$



Force of shaft on the drum: 1680 and 655 lbf

$$T_{P_1} = 1680(8) = 13\,440 \text{ lbf} \cdot \text{in}$$

$$T_{P_2} = 655(8) = 5240 \text{ lbf} \cdot \text{in}$$



Net torque on drum due to brake band:

$$\begin{aligned} T &= T_{P_1} - T_{P_2} \\ &= 13\,440 - 5240 \\ &= 8200 \text{ lbf} \cdot \text{in} \end{aligned}$$

The radial load on the bearing pair is 1803 lbf. If the bearing is straddle mounted with the drum at center span, the bearing radial load is $1803/2 = 901 \text{ lbf}$.

(c) Eq. (16-22):

$$p = \frac{2P}{bD}$$

$$p|_{\theta=0^\circ} = \frac{2P_1}{3(16)} = \frac{2(1680)}{3(16)} = 70 \text{ psi} \quad \text{Ans.}$$

As it should be

$$p|_{\theta=270^\circ} = \frac{2P_2}{3(16)} = \frac{2(655)}{3(16)} = 27.3 \text{ psi} \quad \text{Ans.}$$

16-15 Given: $\phi = 270^\circ$, $b = 2.125 \text{ in}$, $f = 0.20$, $T = 150 \text{ lbf} \cdot \text{ft}$, $D = 8.25 \text{ in}$, $c_2 = 2.25 \text{ in}$
Notice that the pivoting rocker is not located on the vertical centerline of the drum.

(a) To have the band tighten for ccw rotation, it is necessary to have $c_1 < c_2$. When friction is fully developed,

$$\frac{P_1}{P_2} = \exp(f\phi) = \exp[0.2(3\pi/2)] = 2.566$$

If friction is not fully developed

$$P_1/P_2 \leq \exp(f\phi)$$

To help visualize what is going on let's add a force W parallel to P_1 , at a lever arm of c_3 . Now sum moments about the rocker pivot.

$$\sum M = 0 = c_3 W + c_1 P_1 - c_2 P_2$$

From which

$$W = \frac{c_2 P_2 - c_1 P_1}{c_3}$$

The device is self locking for ccw rotation if W is no longer needed, that is, $W \leq 0$. It follows from the equation above

$$\frac{P_1}{P_2} \geq \frac{c_2}{c_1}$$

When friction is fully developed

$$\begin{aligned} 2.566 &= 2.25/c_1 \\ c_1 &= \frac{2.25}{2.566} = 0.877 \text{ in} \end{aligned}$$

When P_1/P_2 is less than 2.566, friction is not fully developed. Suppose $P_1/P_2 = 2.25$, then

$$c_1 = \frac{2.25}{2.25} = 1 \text{ in}$$

We don't want to be at the point of slip, and we need the band to tighten.

$$\frac{c_2}{P_1/P_2} \leq c_1 \leq c_2$$

When the developed friction is very small, $P_1/P_2 \rightarrow 1$ and $c_1 \rightarrow c_2$ Ans.

(b) Rocker has $c_1 = 1$ in

$$\begin{aligned} \frac{P_1}{P_2} &= \frac{c_2}{c_1} = \frac{2.25}{1} = 2.25 \\ f &= \frac{\ln(P_1/P_2)}{\phi} = \frac{\ln 2.25}{3\pi/2} = 0.172 \end{aligned}$$

Friction is not fully developed, no slip.

$$T = (P_1 - P_2) \frac{D}{2} = P_2 \left(\frac{P_1}{P_2} - 1 \right) \frac{D}{2}$$

Solve for P_2

$$P_2 = \frac{2T}{[(P_1/P_2) - 1]D} = \frac{2(150)(12)}{(2.25 - 1)(8.25)} = 349 \text{ lbf}$$

$$P_1 = 2.25 P_2 = 2.25(349) = 785 \text{ lbf}$$

$$p = \frac{2P_1}{bD} = \frac{2(785)}{2.125(8.25)} = 89.6 \text{ psi} \quad \text{Ans.}$$

(c) The torque ratio is $150(12)/100$ or 18-fold.

$$P_2 = \frac{349}{18} = 19.4 \text{ lbf}$$

$$P_1 = 2.25 P_2 = 2.25(19.4) = 43.6 \text{ lbf}$$

$$p = \frac{89.6}{18} = 4.98 \text{ psi} \quad \text{Ans.}$$

Comment:

As the torque opposed by the locked brake increases, P_2 and P_1 increase (although ratio is still 2.25), then p follows. The brake can self-destruct. Protection could be provided by a shear key.

16-16

(a) From Eq. (16-23), since

$$F = \frac{\pi p_a d}{2}(D - d)$$

then

$$p_a = \frac{2F}{\pi d(D - d)}$$

and it follows that

$$\begin{aligned} p_a &= \frac{2(5000)}{\pi(225)(300 - 225)} \\ &= 0.189 \text{ N/mm}^2 \quad \text{or} \quad 189\,000 \text{ N/m}^2 \quad \text{or} \quad 189 \text{ kPa} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} T &= \frac{Ff}{4}(D + d) = \frac{5000(0.25)}{4}(300 + 225) \\ &= 164\,043 \text{ N} \cdot \text{mm} \quad \text{or} \quad 164 \text{ N} \cdot \text{m} \quad \text{Ans.} \end{aligned}$$

(b) From Eq. (16-26),

$$\begin{aligned} F &= \frac{\pi p_a}{4}(D^2 - d^2) \\ p_a &= \frac{4F}{\pi(D^2 - d^2)} = \frac{4(5000)}{\pi(300^2 - 225^2)} \\ &= 0.162 \text{ N/mm}^2 = 162 \text{ kPa} \quad \text{Ans.} \end{aligned}$$

From Eq. (16-27),

$$\begin{aligned} T &= \frac{\pi}{12} f p_a (D^3 - d^3) = \frac{\pi}{12} (0.25)(162)(10^3)(300^3 - 225^3)(10^{-3})^3 \\ &= 166 \text{ N} \cdot \text{m} \quad \text{Ans.} \end{aligned}$$

16-17

(a) Eq. (16-23):

$$F = \frac{\pi p_a d}{2}(D - d) = \frac{\pi(120)(4)}{2}(6.5 - 4) = 1885 \text{ lbf} \quad \text{Ans.}$$

Eq. (16-24):

$$T = \frac{\pi f p_a d}{8} (D^2 - d^2) N = \frac{\pi (0.24)(120)(4)}{8} (6.5^2 - d^2)(6)$$

$$= 7125 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

(b)
$$T = \frac{\pi (0.24)(120d)}{8} (6.5^2 - d^2)(6)$$

$d, \text{ in}$	$T, \text{ lbf} \cdot \text{in}$	
2	5191	
3	6769	
4	7125	Ans.
5	5853	
6	2545	

- (c) The torque-diameter curve exhibits a stationary point maximum in the range of diameter d . The clutch has nearly optimal proportions.

16-18

(a)
$$T = \frac{\pi f p_a d (D^2 - d^2) N}{8} = C D^2 d - C d^3$$

Differentiating with respect to d and equating to zero gives

$$\frac{dT}{dd} = C D^2 - 3C d^2 = 0$$

$$d^* = \frac{D}{\sqrt{3}} \quad \text{Ans.}$$

$$\frac{d^2 T}{dd^2} = -6 C d$$

which is negative for all positive d . We have a stationary point *maximum*.

(b)
$$d^* = \frac{6.5}{\sqrt{3}} = 3.75 \text{ in} \quad \text{Ans.}$$

$$T^* = \frac{\pi (0.24)(120)}{8} \frac{(6.5/\sqrt{3})}{(6.5/\sqrt{3})} [6.5^2 - (6.5^2/3)](6) = 7173 \text{ lbf} \cdot \text{in}$$

- (c) The table indicates a maximum within the range:

$$3 \leq d \leq 5 \text{ in}$$

(d) Consider:
$$0.45 \leq \frac{d}{D} \leq 0.80$$

Multiply through by D

$$0.45D \leq d \leq 0.80D$$

$$0.45(6.5) \leq d \leq 0.80(6.5)$$

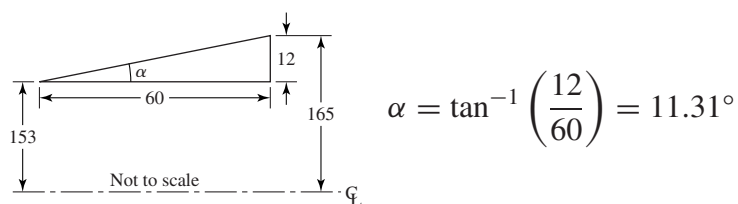
$$2.925 \leq d \leq 5.2 \text{ in}$$

$$\left(\frac{d}{D}\right)^* = d^*/D = \frac{1}{\sqrt{3}} = 0.577$$

which lies within the common range of clutches.

Yes. *Ans.*

16-19 Given: $d = 0.306 \text{ m}$, $l = 0.060 \text{ m}$, $T = 0.200 \text{ kN} \cdot \text{m}$, $D = 0.330 \text{ m}$, $f = 0.26$.



Uniform wear

Eq. (16-45):

$$0.200 = \frac{\pi(0.26)(0.306)p_a}{8 \sin 11.31^\circ} (0.330^2 - 0.306^2) = 0.002432 p_a$$

$$p_a = \frac{0.200}{0.002432} = 82.2 \text{ kPa} \quad \text{Ans.}$$

Eq. (16-44):

$$F = \frac{\pi p_a d}{2} (D - d) = \frac{\pi(82.2)(0.306)}{2} (0.330 - 0.306) = 0.949 \text{ kN} \quad \text{Ans.}$$

Uniform pressure

Eq. (16-48):

$$0.200 = \frac{\pi(0.26)p_a}{12 \sin 11.31^\circ} (0.330^3 - 0.306^3) = 0.00253 p_a$$

$$p_a = \frac{0.200}{0.00253} = 79.1 \text{ kPa} \quad \text{Ans.}$$

Eq. (16-47):

$$F = \frac{\pi p_a}{4} (D^2 - d^2) = \frac{\pi(79.1)}{4} (0.330^2 - 0.306^2) = 0.948 \text{ kN} \quad \text{Ans.}$$

16-20 *Uniform wear*

Eq. (16-34):

$$T = \frac{1}{2}(\theta_2 - \theta_1) f p_a r_i (r_o^2 - r_i^2)$$

Eq. (16-33): $F = (\theta_2 - \theta_1) p_a r_i (r_o - r_i)$

Thus,
$$\frac{T}{fFD} = \frac{(1/2)(\theta_2 - \theta_1) f p_a r_i (r_o^2 - r_i^2)}{f(\theta_2 - \theta_1) p_a r_i (r_o - r_i)(D)}$$

$$= \frac{r_o + r_i}{2D} = \frac{D/2 + d/2}{2D} = \frac{1}{4} \left(1 + \frac{d}{D} \right) \quad O.K. \quad Ans.$$

Uniform pressure

Eq. (16-38): $T = \frac{1}{3}(\theta_2 - \theta_1) f p_a (r_o^3 - r_i^3)$

Eq. (16-37):

$$F = \frac{1}{2}(\theta_2 - \theta_1) p_a (r_o^2 - r_i^2)$$

$$\frac{T}{fFD} = \frac{(1/3)(\theta_2 - \theta_1) f p_a (r_o^3 - r_i^3)}{(1/2)f(\theta_2 - \theta_1) p_a (r_o^2 - r_i^2)D} = \frac{2}{3} \left\{ \frac{(D/2)^3 - (d/2)^3}{[(D/2)^2 - (d/2)^2]D} \right\}$$

$$= \frac{2(D/2)^3(1 - (d/D)^3)}{3(D/2)^2[1 - (d/D)^2]D} = \frac{1}{3} \left[\frac{1 - (d/D)^3}{1 - (d/D)^2} \right] \quad O.K. \quad Ans.$$

16-21

$$\omega = 2\pi n/60 = 2\pi \cdot 500/60 = 52.4 \text{ rad/s}$$

$$T = \frac{H}{\omega} = \frac{2(10^3)}{52.4} = 38.2 \text{ N} \cdot \text{m}$$

Key:

$$F = \frac{T}{r} = \frac{38.2}{12} = 3.18 \text{ kN}$$

Average shear stress in key is

$$\tau = \frac{3.18(10^3)}{6(40)} = 13.2 \text{ MPa} \quad Ans.$$

Average bearing stress is

$$\sigma_b = -\frac{F}{A_b} = -\frac{3.18(10^3)}{3(40)} = -26.5 \text{ MPa} \quad Ans.$$

Let one jaw carry the entire load.

$$r_{av} = \frac{1}{2} \left(\frac{26}{2} + \frac{45}{2} \right) = 17.75 \text{ mm}$$

$$F = \frac{T}{r_{av}} = \frac{38.2}{17.75} = 2.15 \text{ kN}$$

The bearing and shear stress estimates are

$$\sigma_b = \frac{-2.15(10^3)}{10(22.5 - 13)} = -22.6 \text{ MPa} \quad \text{Ans.}$$

$$\tau = \frac{2.15(10^3)}{10[0.25\pi(17.75)^2]} = 0.869 \text{ MPa} \quad \text{Ans.}$$

16-22

$$\omega_1 = 2\pi n/60 = 2\pi(1800)/60 = 188.5 \text{ rad/s}$$

$$\omega_2 = 0$$

From Eq. (16-51),

$$\frac{I_1 I_2}{I_1 + I_2} = \frac{T t_1}{\omega_1 - \omega_2} = \frac{320(8.3)}{188.5 - 0} = 14.09 \text{ N} \cdot \text{m} \cdot \text{s}^2$$

Eq. (16-52):

$$E = 14.09 \left(\frac{188.5^2}{2} \right) (10^{-3}) = 250 \text{ kJ}$$

Eq. (16-55):

$$\Delta T = \frac{E}{C_p m} = \frac{250(10^3)}{500(18)} = 27.8^\circ\text{C} \quad \text{Ans.}$$

16-23

$$n = \frac{n_1 + n_2}{2} = \frac{260 + 240}{2} = 250 \text{ rev/min}$$

$$C_s = \frac{260 - 240}{250} = 0.08 \quad \text{Ans.}$$

$$\omega = 2\pi(250)/60 = 26.18 \text{ rad/s}$$

$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{5000(12)}{0.08(26.18)^2} = 1094 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

$$I_x = \frac{m}{8} (d_o^2 + d_i^2) = \frac{W}{8g} (d_o^2 + d_i^2)$$

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(1094)}{60^2 + 56^2} = 502 \text{ lbf}$$

$$w = 0.260 \text{ lbf/in}^3 \quad \text{for cast iron}$$

$$V = \frac{W}{w} = \frac{502}{0.260} = 1931 \text{ in}^3$$

Also,
$$V = \frac{\pi t}{4} (d_o^2 - d_i^2) = \frac{\pi t}{4} (60^2 - 56^2) = 364t \text{ in}^3$$

Equating the expressions for volume and solving for t ,

$$t = \frac{1931}{364} = 5.3 \text{ in} \quad \text{Ans.}$$

16-24 (a) The useful work performed in one revolution of the crank shaft is

$$U = 35(2000)(8)(0.15) = 84(10^3) \text{ in} \cdot \text{lbf}$$

Accounting for friction, the total work done in one revolution is

$$U = 84(10^3)/(1 - 0.16) = 100(10^3) \text{ in} \cdot \text{lbf}$$

Since 15% of the crank shaft stroke is 7.5% of a crank shaft revolution, the energy fluctuation is

$$E_2 - E_1 = 84(10^3) - 100(10^3)(0.075) = 76.5(10^3) \text{ in} \cdot \text{lbf} \quad \text{Ans.}$$

(b) For the flywheel

$$n = 6(90) = 540 \text{ rev/min}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(540)}{60} = 56.5 \text{ rad/s}$$

Since

$$C_s = 0.10$$

$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{76.5(10^3)}{0.10(56.5)^2} = 239.6 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

Assuming all the mass is concentrated at the effective diameter, d ,

$$I = \frac{md^2}{4}$$

$$W = \frac{4gI}{d^2} = \frac{4(386)(239.6)}{48^2} = 161 \text{ lbf} \quad \text{Ans.}$$

16-25 Use Ex. 16-6 and Table 16-6 data for one cylinder of a 3-cylinder engine.

$$C_s = 0.30$$

$$n = 2400 \text{ rev/min or } 251 \text{ rad/s}$$

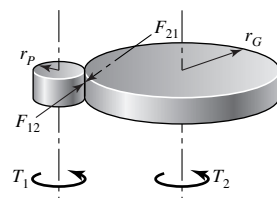
$$T_m = \frac{3(3368)}{4\pi} = 804 \text{ in} \cdot \text{lbf} \quad \text{Ans.}$$

$$E_2 - E_1 = 3(3531) = 10\,590 \text{ in} \cdot \text{lbf}$$

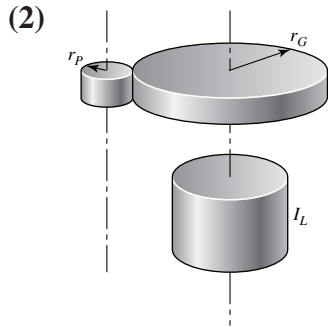
$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{10\,590}{0.30(251^2)} = 0.560 \text{ in} \cdot \text{lbf} \cdot \text{s}^2 \quad \text{Ans.}$$

16-26 (a)

(1)



$$(T_2)_1 = -F_{21}r_P = -\frac{T_2}{r_G}r_P = \frac{T_2}{-n} \quad \text{Ans.}$$



Equivalent energy

$$(1/2)I_2\omega_2^2 = (1/2)(I_2)_1(w_1^2)$$

$$(I_2)_1 = \frac{\omega_2^2}{\omega_1^2} I_2 = \frac{I_2}{n^2} \quad \text{Ans.}$$

$$(3) \quad \frac{I_G}{I_P} = \left(\frac{r_G}{r_P}\right)^2 \left(\frac{m_G}{m_P}\right) = \left(\frac{r_G}{r_P}\right)^2 \left(\frac{r_G}{r_P}\right)^2 = n^4$$

From (2) $(I_2)_1 = \frac{I_G}{n^2} = \frac{n^4 I_P}{n^2} = n^2 I_P \quad \text{Ans.}$

$$(b) \quad I_e = I_M + I_P + n^2 I_P + \frac{I_L}{n^2} \quad \text{Ans.}$$

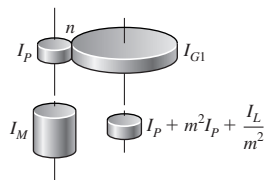
$$(c) \quad I_e = 10 + 1 + 10^2(1) + \frac{100}{10^2}$$

$$= 10 + 1 + 100 + 1 = 112$$

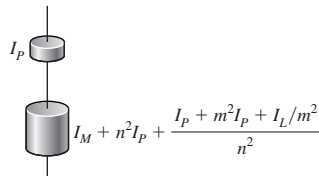
$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 armature inertia pinion inertia reflected gear inertia reflected load inertia

Ans.

16-27 (a) Reflect I_L, I_{G2} to the center shaft



Reflect the center shaft to the motor shaft



$$I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{m^2}{n^2} I_P + \frac{I_L}{m^2 n^2} \quad \text{Ans.}$$

(b) For $R = \text{constant} = nm$, $I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{R^2 I_P}{n^4} + \frac{I_L}{R^2}$ Ans.

(c) For $R = 10$, $\frac{\partial I_e}{\partial n} = 0 + 0 + 2n(1) - \frac{2(1)}{n^3} - \frac{4(10^2)(1)}{n^5} + 0 = 0$

$$n^6 - n^2 - 200 = 0$$

From which

$$n^* = 2.430 \quad \text{Ans.}$$

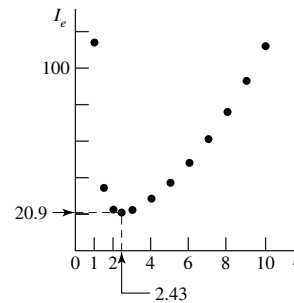
$$m^* = \frac{10}{2.430} = 4.115 \quad \text{Ans.}$$

Notice that n^* and m^* are independent of I_L .

16-28 From Prob. 16-27,

$$\begin{aligned} I_e &= I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{R^2 I_P}{n^4} + \frac{I_L}{R^2} \\ &= 10 + 1 + n^2(1) + \frac{1}{n^2} + \frac{100(1)}{n^4} + \frac{100}{10^2} \\ &= 10 + 1 + n^2 + \frac{1}{n^2} + \frac{100}{n^4} + 1 \end{aligned}$$

n	I_e
1.00	114.00
1.50	34.40
2.00	22.50
2.43	20.90
3.00	22.30
4.00	28.50
5.00	37.20
6.00	48.10
7.00	61.10
8.00	76.00
9.00	93.00
10.00	112.02



Optimizing the partitioning of a double reduction lowered the gear-train inertia to $20.9/112 = 0.187$, or to 19% of that of a single reduction. This includes the two additional gears.

16-29 Figure 16-29 applies,

$$t_2 = 10 \text{ s}, \quad t_1 = 0.5 \text{ s}$$

$$\frac{t_2 - t_1}{t_1} = \frac{10 - 0.5}{0.5} = 19$$

The load torque, as seen by the motor shaft (Rule 1, Prob. 16-26), is

$$T_L = \left| \frac{1300(12)}{10} \right| = 1560 \text{ lbf} \cdot \text{in}$$

The rated motor torque T_r is

$$T_r = \frac{63\,025(3)}{1125} = 168.07 \text{ lbf} \cdot \text{in}$$

For Eqs. (16-65):

$$\omega_r = \frac{2\pi}{60}(1125) = 117.81 \text{ rad/s}$$

$$\omega_s = \frac{2\pi}{60}(1200) = 125.66 \text{ rad/s}$$

$$a = \frac{-T_r}{\omega_s - \omega_r} = -\frac{168.07}{125.66 - 117.81} = -21.41$$

$$b = \frac{T_r \omega_s}{\omega_s - \omega_r} = \frac{168.07(125.66)}{125.66 - 117.81} \\ = 2690.4 \text{ lbf} \cdot \text{in}$$

The linear portion of the squirrel-cage motor characteristic can now be expressed as

$$T_M = -21.41\omega + 2690.4 \text{ lbf} \cdot \text{in}$$

Eq. (16-68):

$$T_2 = 168.07 \left(\frac{1560 - 168.07}{1560 - T_2} \right)^{19}$$

One root is 168.07 which is for infinite time. The root for 10 s is wanted. Use a successive substitution method

T_2	New T_2
0.00	19.30
19.30	24.40
24.40	26.00
26.00	26.50
26.50	26.67

Continue until convergence.

$$T_2 = 26.771$$

Eq. (16-69):

$$I = \frac{-21.41(10 - 0.5)}{\ln(26.771/168.07)} = 110.72 \text{ in} \cdot \text{lbf} \cdot \text{s/rad}$$

$$\omega = \frac{T - b}{a}$$

$$\omega_{\max} = \frac{T_2 - b}{a} = \frac{26.771 - 2690.4}{-21.41} = 124.41 \text{ rad/s} \quad \text{Ans.}$$

$$\omega_{\min} = 117.81 \text{ rad/s} \quad \text{Ans.}$$

$$\bar{\omega} = \frac{124.41 + 117.81}{2} = 121.11 \text{ rad/s}$$

$$C_s = \frac{\omega_{\max} - \omega_{\min}}{(\omega_{\max} + \omega_{\min})/2} = \frac{124.41 - 117.81}{(124.41 + 117.81)/2} = 0.0545 \quad \text{Ans.}$$

$$E_1 = \frac{1}{2} I \omega_r^2 = \frac{1}{2} (110.72) (117.81)^2 = 768\,352 \text{ in} \cdot \text{lb}$$

$$E_2 = \frac{1}{2} I \omega_2^2 = \frac{1}{2} (110.72) (124.41)^2 = 856\,854 \text{ in} \cdot \text{lb}$$

$$\Delta E = E_1 - E_2 = 768\,352 - 856\,854 = -88\,502 \text{ in} \cdot \text{lb}$$

Eq. (16-64):

$$\begin{aligned} \Delta E &= C_s I \bar{\omega}^2 = 0.0545 (110.72) (121.11)^2 \\ &= 88\,508 \text{ in} \cdot \text{lb}, \quad \text{close enough} \quad \text{Ans.} \end{aligned}$$

During the punch

$$T = \frac{63\,025 H}{n}$$

$$H = \frac{T_L \bar{\omega} (60/2\pi)}{63\,025} = \frac{1560 (121.11) (60/2\pi)}{63\,025} = 28.6 \text{ hp}$$

The gear train has to be sized for 28.6 hp under shock conditions since the flywheel is on the motor shaft. From Table A-18,

$$I = \frac{m}{8} (d_o^2 + d_i^2) = \frac{W}{8g} (d_o^2 + d_i^2)$$

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(110.72)}{d_o^2 + d_i^2}$$

If a mean diameter of the flywheel rim of 30 in is acceptable, try a rim thickness of 4 in

$$d_i = 30 - (4/2) = 28 \text{ in}$$

$$d_o = 30 + (4/2) = 32 \text{ in}$$

$$W = \frac{8(386)(110.72)}{32^2 + 28^2} = 189.1 \text{ lbf}$$

Rim volume V is given by

$$V = \frac{\pi l}{4} (d_o^2 - d_i^2) = \frac{\pi l}{4} (32^2 - 28^2) = 188.5l$$

where l is the rim width as shown in Table A-18. The specific weight of cast iron is $\gamma = 0.260 \text{ lbf} \cdot \text{in}^3$, therefore the volume of cast iron is

$$V = \frac{W}{\gamma} = \frac{189.1}{0.260} = 727.3 \text{ in}^3$$

Thus

$$188.5l = 727.3$$

$$l = \frac{727.3}{188.5} = 3.86 \text{ in wide}$$

Proportions can be varied.

16-30 Prob. 16-29 solution has I for the motor shaft flywheel as

$$I = 110.72 \text{ in} \cdot \text{lbf} \cdot \text{s}^2/\text{rad}$$

A flywheel located on the crank shaft needs an inertia of $10^2 I$ (Prob. 16-26, rule 2)

$$I = 10^2(110.72) = 11\,072 \text{ in} \cdot \text{lbf} \cdot \text{s}^2/\text{rad}$$

A 100-fold inertia increase. On the other hand, the gear train has to transmit 3 hp under shock conditions.

Stating the problem is most of the solution. Satisfy yourself that on the crankshaft:

$$T_L = 1300(12) = 15\,600 \text{ lbf} \cdot \text{in}$$

$$T_r = 10(168.07) = 1680.7 \text{ lbf} \cdot \text{in}$$

$$\omega_r = 117.81/10 = 11.781 \text{ rad/s}$$

$$\omega_s = 125.66/10 = 12.566 \text{ rad/s}$$

$$a = -21.41(100) = -2141$$

$$b = 2690.35(10) = 26903.5$$

$$T_M = -2141\omega_c + 26\,903.5 \text{ lbf} \cdot \text{in}$$

$$T_2 = 1680.6 \left(\frac{15\,600 - 1680.5}{15\,600 - T_2} \right)^{19}$$

The root is $10(26.67) = 266.7 \text{ lbf} \cdot \text{in}$

$$\bar{\omega} = 121.11/10 = 12.111 \text{ rad/s}$$

$$C_s = 0.0549 \quad (\text{same})$$

$$\omega_{\max} = 121.11/10 = 12.111 \text{ rad/s} \quad \text{Ans.}$$

$$\omega_{\min} = 117.81/10 = 11.781 \text{ rad/s} \quad \text{Ans.}$$

E_1 , E_2 , ΔE and peak power are the same.

From Table A-18

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(11\,072)}{d_o^2 + d_i^2}$$

Scaling will affect d_o and d_i , but the gear ratio changed I . Scale up the flywheel in the Prob. 16-29 solution by a factor of 2.5. Thickness becomes $4(2.5) = 10$ in.

$$\bar{d} = 30(2.5) = 75 \text{ in}$$

$$d_o = 75 + (10/2) = 80 \text{ in}$$

$$d_i = 75 - (10/2) = 70 \text{ in}$$

$$W = \frac{8(386)(11\,072)}{80^2 + 70^2} = 3026 \text{ lbf}$$

$$v = \frac{3026}{0.26} = 11\,638 \text{ in}^3$$

$$V = \frac{\pi}{4}l(80^2 - 70^2) = 1178l$$

$$l = \frac{11\,638}{1178} = 9.88 \text{ in}$$

Proportions can be varied. The weight has increased $3026/189.1$ or about 16-fold while the moment of inertia I increased 100-fold. The gear train transmits a steady 3 hp. But the motor armature has its inertia magnified 100-fold, and during the punch there are deceleration stresses in the train. With no motor armature information, we cannot comment.

16-31 This can be the basis for a class discussion.