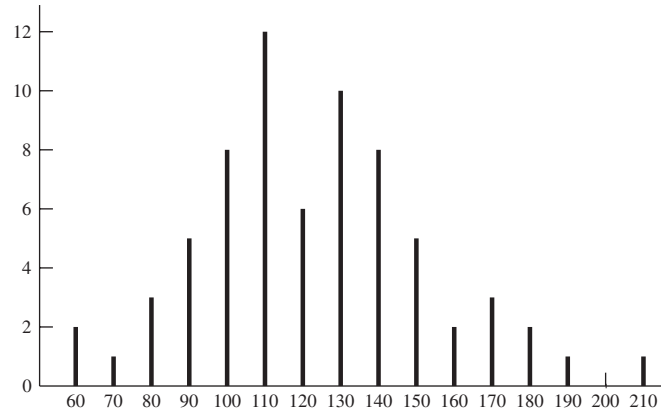


Chapter 20

20-1

(a)



(b) $f/(N\Delta x) = f/(69 \cdot 10) = f/690$

x	f	fx	fx^2	$f/(N\Delta x)$
60	2	120	7200	0.0029
70	1	70	4900	0.0015
80	3	240	19 200	0.0043
90	5	450	40 500	0.0072
100	8	800	80 000	0.0116
110	12	1320	145 200	0.0174
120	6	720	86 400	0.0087
130	10	1300	169 000	0.0145
140	8	1120	156 800	0.0116
150	5	750	112 500	0.0174
160	2	320	51 200	0.0029
170	3	510	86 700	0.0043
180	2	360	64 800	0.0029
190	1	190	36 100	0.0015
200	0	0	0	0
210	1	210	44 100	0.0015
	<u>69</u>	<u>8480</u>	<u>1 104 600</u>	

Eq. (20-9) $\bar{x} = \frac{8480}{69} = 122.9 \text{ kcycles}$

Eq. (20-10) $s_x = \left[\frac{1\,104\,600 - 8480^2/69}{69 - 1} \right]^{1/2}$
 $= 30.3 \text{ kcycles} \quad \text{Ans.}$

20-2 Data represents a 7-class histogram with $N = 197$.

x	f	fx	fx^2
174	6	1044	181 656
182	9	1638	298 116
190	44	8360	1 588 400
198	67	13 266	2 626 688
206	53	10 918	2 249 108
214	12	2568	549 552
220	6	1320	290 400
	<u>197</u>	<u>39 114</u>	<u>7 789 900</u>

$$\bar{x} = \frac{39\,114}{197} = 198.55 \text{ kpsi} \quad \text{Ans.}$$

$$s_x = \left[\frac{7\,783\,900 - 39\,114^2/197}{197 - 1} \right]^{1/2}$$

$$= 9.55 \text{ kpsi} \quad \text{Ans.}$$

20-3

Form a table:

x	f	fx	fx^2
64	2	128	8192
68	6	408	27 744
72	6	432	31 104
76	9	684	51 984
80	19	1520	121 600
84	10	840	70 560
88	4	352	30 976
92	2	184	16 928
	<u>58</u>	<u>4548</u>	<u>359 088</u>

$$\bar{x} = \frac{4548}{58} = 78.4 \text{ kpsi}$$

$$s_x = \left[\frac{359\,088 - 4548^2/58}{58 - 1} \right]^{1/2} = 6.57 \text{ kpsi}$$

From Eq. (20-14)

$$f(x) = \frac{1}{6.57\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - 78.4}{6.57} \right)^2 \right]$$

20-4 (a)

y	f	fy	fy^2	y	$f/(Nw)$	$f(y)$	$g(y)$
5.625	1	5.625	31.64063	5.625	0.072727	0.001262	0.000295
5.875	0	0	0	5.875	0	0.008586	0.004088
6.125	0	0	0	6.125	0	0.042038	0.031194
6.375	3	19.125	121.9219	6.375	0.218182	0.148106	0.140262
6.625	3	19.875	131.6719	6.625	0.218182	0.375493	0.393667
6.875	6	41.25	283.5938	6.875	0.436364	0.685057	0.725002
7.125	14	99.75	710.7188	7.125	1.018182	0.899389	0.915128
7.375	15	110.625	815.8594	7.375	1.090909	0.849697	0.822462
7.625	10	76.25	581.4063	7.625	0.727273	0.577665	0.544251
7.875	2	15.75	124.0313	7.875	0.145455	0.282608	0.273138
8.125	1	8.125	66.01563	8.125	0.072727	0.099492	0.10672
	<u>55</u>	<u>396.375</u>	<u>2866.859</u>				

For a normal distribution,

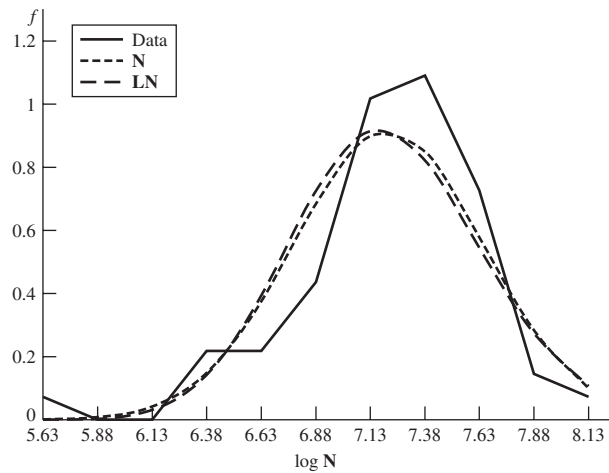
$$\bar{y} = 396.375/55 = 7.207, \quad s_y = \left(\frac{2866.859 - (396.375^2/55)}{55 - 1} \right)^{1/2} = 0.4358$$

$$f(y) = \frac{1}{0.4358\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - 7.207}{0.4358} \right)^2 \right]$$

For a lognormal distribution,

$$\bar{x} = \ln 7.206818 - \ln \sqrt{1 + 0.060474^2} = 1.9732, \quad s_x = \ln \sqrt{1 + 0.060474^2} = 0.0604$$

$$g(y) = \frac{1}{x(0.0604)(\sqrt{2\pi})} \exp \left[-\frac{1}{2} \left(\frac{\ln x - 1.9732}{0.0604} \right)^2 \right]$$

(b) Histogram

20-5 Distribution is uniform in interval 0.5000 to 0.5008 in, range numbers are $a = 0.5000$, $b = 0.5008$ in.

(a) Eq. (20-22)
$$\mu_x = \frac{a + b}{2} = \frac{0.5000 + 0.5008}{2} = 0.5004$$

Eq. (20-23)
$$\sigma_x = \frac{b - a}{2\sqrt{3}} = \frac{0.5008 - 0.5000}{2\sqrt{3}} = 0.000231$$

(b) PDF from Eq. (20-20)

$$f(x) = \begin{cases} 1250 & 0.5000 \leq x \leq 0.5008 \text{ in} \\ 0 & \text{otherwise} \end{cases}$$

(c) CDF from Eq. (20-21)

$$F(x) = \begin{cases} 0 & x < 0.5000 \\ (x - 0.5)/0.0008 & 0.5000 \leq x \leq 0.5008 \\ 1 & x > 0.5008 \end{cases}$$

If all smaller diameters are removed by inspection, $a = 0.5002$, $b = 0.5008$

$$\mu_x = \frac{0.5002 + 0.5008}{2} = 0.5005 \text{ in}$$

$$\hat{\sigma}_x = \frac{0.5008 - 0.5002}{2\sqrt{3}} = 0.000173 \text{ in}$$

$$f(x) = \begin{cases} 1666.7 & 0.5002 \leq x \leq 0.5008 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0.5002 \\ 1666.7(x - 0.5002) & 0.5002 \leq x \leq 0.5008 \\ 1 & x > 0.5008 \end{cases}$$

20-6 Dimensions produced are due to tool dulling and wear. When parts are mixed, the distribution is uniform. From Eqs. (20-22) and (20-23),

$$a = \mu_x - \sqrt{3}s = 0.6241 - \sqrt{3}(0.000581) = 0.6231 \text{ in}$$

$$b = \mu_x + \sqrt{3}s = 0.6241 + \sqrt{3}(0.000581) = 0.6251 \text{ in}$$

We suspect the dimension was $\frac{0.623}{0.625}$ in *Ans.*

20-7 $F(x) = 0.555x - 33 \text{ mm}$

(a) Since $F(x)$ is linear, the distribution is uniform at $x = a$

$$F(a) = 0 = 0.555(a) - 33$$

$\therefore a = 59.46 \text{ mm}$. Therefore, at $x = b$

$$F(b) = 1 = 0.555b - 33$$

$\therefore b = 61.26 \text{ mm}$. Therefore,

$$F(x) = \begin{cases} 0 & x < 59.46 \text{ mm} \\ 0.555x - 33 & 59.46 \leq x \leq 61.26 \text{ mm} \\ 1 & x > 61.26 \text{ mm} \end{cases}$$

The PDF is dF/dx , thus the range numbers are:

$$f(x) = \begin{cases} 0.555 & 59.46 \leq x \leq 61.26 \text{ mm} \\ 0 & \text{otherwise} \end{cases} \quad \text{Ans.}$$

From the range numbers,

$$\mu_x = \frac{59.46 + 61.26}{2} = 60.36 \text{ mm} \quad \text{Ans.}$$

$$\hat{\sigma}_x = \frac{61.26 - 59.46}{2\sqrt{3}} = 0.520 \text{ mm} \quad \text{Ans.}$$

(b) σ is an uncorrelated quotient $\bar{F} = 3600 \text{ lbf}$, $\bar{A} = 0.112 \text{ in}^2$

$$C_F = 300/3600 = 0.08333, \quad C_A = 0.001/0.112 = 0.008929$$

From Table 20-6, for σ

$$\bar{\sigma} = \frac{\mu_F}{\mu_A} = \frac{3600}{0.112} = 32143 \text{ psi} \quad \text{Ans.}$$

$$\hat{\sigma}_\sigma = 32143 \left[\frac{(0.08333^2 + 0.008929^2)}{(1 + 0.008929^2)} \right]^{1/2} = 2694 \text{ psi} \quad \text{Ans.}$$

$$C_\sigma = 2694/32143 = 0.0838 \quad \text{Ans.}$$

Since **F** and **A** are lognormal, division is closed and σ is lognormal too.

$$\sigma = \text{LN}(32143, 2694) \text{ psi} \quad \text{Ans.}$$

20-8 Cramer's rule

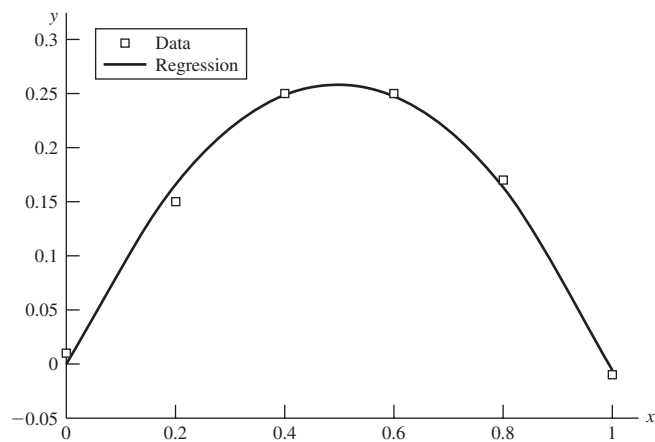
$$a_1 = \frac{\begin{vmatrix} \Sigma y & \Sigma x^2 \\ \Sigma xy & \Sigma x^3 \end{vmatrix}}{\begin{vmatrix} \Sigma x & \Sigma x^2 \\ \Sigma x^2 & \Sigma x^3 \end{vmatrix}} = \frac{\Sigma y \Sigma x^3 - \Sigma xy \Sigma x^2}{\Sigma x \Sigma x^3 - (\Sigma x^2)^2} \quad \text{Ans.}$$

$$a_2 = \frac{\begin{vmatrix} \Sigma x & \Sigma y \\ \Sigma x^2 & \Sigma xy \end{vmatrix}}{\begin{vmatrix} \Sigma x & \Sigma x^2 \\ \Sigma x^2 & \Sigma x^3 \end{vmatrix}} = \frac{\Sigma x \Sigma xy - \Sigma y \Sigma x^2}{\Sigma x \Sigma x^3 - (\Sigma x^2)^2} \quad \text{Ans.}$$

x	y	x^2	x^3	xy
0	0.01	0	0	0
0.2	0.15	0.04	0.008	0.030
0.4	0.25	0.16	0.064	0.100
0.6	0.25	0.36	0.216	0.150
0.8	0.17	0.64	0.512	0.136
1.0	-0.01	1.00	1.000	-0.010
3.0	0.82	2.20	1.800	0.406

$$a_1 = 1.040\,714 \quad a_2 = -1.046\,43 \quad \text{Ans.}$$

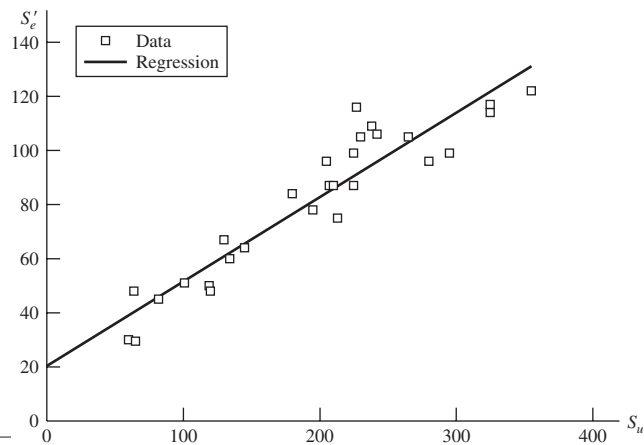
x	Data	Regression
	y	y
0	0.01	0
0.2	0.15	0.166\,286
0.4	0.25	0.248\,857
0.6	0.25	0.247\,714
0.8	0.17	0.162\,857
1.0	-0.01	-0.005\,71



20-9

S_u	Data	Regression	S_u^2	$S_u S'_e$
	S'_e	S'_e		
0		20.35675		
60	30	39.08078	3 600	1 800
64	48	40.32905	4 096	3 072
65	29.5	40.64112	4 225	1 917.5
82	45	45.94626	6 724	3 690
101	51	51.87554	10 201	5 151
119	50	57.49275	14 161	5 950
120	48	57.80481	14 400	5 760
130	67	60.92548	16 900	8 710
134	60	62.17375	17 956	8 040
145	64	65.60649	21 025	9 280
180	84	76.52884	32 400	15 120
195	78	81.20985	38 025	15 210
205	96	84.33052	42 025	19 680
207	87	84.95466	42 849	18 009
210	87	85.89086	44 100	18 270
213	75	86.82706	45 369	15 975
225	99	90.57187	50 625	22 275
225	87	90.57187	50 625	19 575
227	116	91.196	51 529	26 332
230	105	92.1322	52 900	24 150
238	109	94.62874	56 644	25 942
242	106	95.87701	58 564	25 652
265	105	103.0546	70 225	27 825
280	96	107.7356	78 400	26 880
295	99	112.4166	87 025	29 205
325	114	121.7786	105 625	37 050
325	117	121.7786	105 625	38 025
355	122	131.1406	126 025	43 310
5462	2274.5		1 251 868	501 855.5

$$m = 0.312067 \quad b = 20.35675 \quad \text{Ans.}$$



20-10

$$\mathcal{E} = \sum (y - a_0 - a_2 x^2)^2$$

$$\frac{\partial \mathcal{E}}{\partial a_0} = -2 \sum (y - a_0 - a_2 x^2) = 0$$

$$\sum y - na_0 - a_2 \sum x^2 = 0 \Rightarrow \sum y = na_0 + a_2 \sum x^2$$

$$\frac{\partial \mathcal{E}}{\partial a_2} = 2 \sum (y - a_0 - a_2 x^2) (2x) = 0 \Rightarrow \sum xy = a_0 \sum x + a_2 \sum x^3$$

Ans.

Cramer's rule

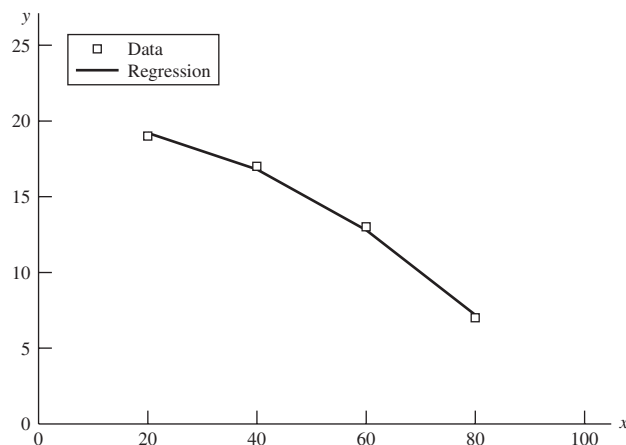
$$a_0 = \frac{\begin{vmatrix} \sum y & \sum x^2 \\ \sum xy & \sum x^3 \end{vmatrix}}{\begin{vmatrix} n & \sum x^2 \\ \sum x & \sum x^3 \end{vmatrix}} = \frac{\sum x^3 \sum y - \sum x^2 \sum xy}{n \sum x^3 - \sum x \sum x^2}$$

$$a_2 = \frac{\begin{vmatrix} n & \sum y \\ \sum x & \sum xy \end{vmatrix}}{\begin{vmatrix} n & \sum x^2 \\ \sum x & \sum x^3 \end{vmatrix}} = \frac{n \sum xy - \sum x \sum y}{n \sum x^3 - \sum x \sum x^2}$$

Data		Regression			
x	y	y	x^2	x^3	xy
20	19	19.2	400	8 000	380
40	17	16.8	1600	64 000	680
60	13	12.8	3600	216 000	780
80	7	7.2	6400	512 000	560
200	56		12 000	800 000	2400

$$a_0 = \frac{800\,000(56) - 12\,000(2400)}{4(800\,000) - 200(12\,000)} = 20$$

$$a_2 = \frac{4(2400) - 200(56)}{4(800\,000) - 200(12\,000)} = -0.002$$

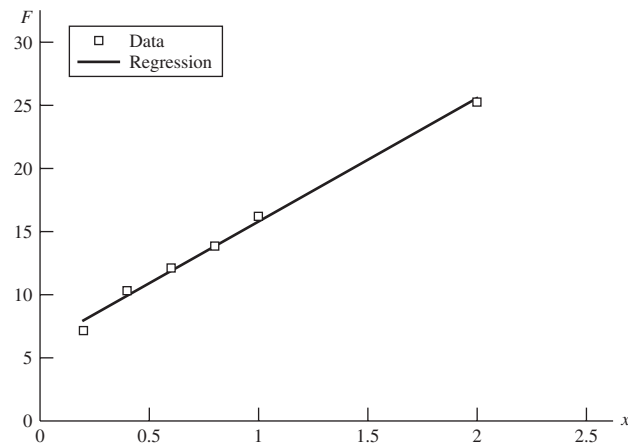


20-11

x	Data y	Regression y	x^2	y^2	xy	$x - \bar{x}$	$(x - \bar{x})^2$
0.2	7.1	7.931 803	0.04	50.41	1.42	-0.633 333	0.401 111 111
0.4	10.3	9.884 918	0.16	106.09	4.12	-0.433 333	0.187 777 778
0.6	12.1	11.838 032	0.36	146.41	7.26	-0.233 333	0.054 444 444
0.8	13.8	13.791 147	0.64	190.44	11.04	-0.033 333	0.001 111 111
1	16.2	15.744 262	1.00	262.44	16.20	0.166 666	0.027 777 778
2	25.2	25.509 836	4.00	635.04	50.40	1.166 666	1.361 111 111
5	84.7		6.2	1390.83	90.44	0	2.033 333 333

$$\hat{m} = \bar{k} = \frac{6(90.44) - 5(84.7)}{6(6.2) - (5)^2} = 9.7656$$

$$\hat{b} = \bar{F}_i = \frac{84.7 - 9.7656(5)}{6} = 5.9787$$



(a) $\bar{x} = \frac{5}{6}; \quad \bar{y} = \frac{84.7}{6} = 14.117$

Eq. (20-37)

$$s_{yx} = \sqrt{\frac{1390.83 - 5.9787(84.7) - 9.7656(90.44)}{6 - 2}}$$

$$= 0.556$$

Eq. (20-36)

$$s_{\hat{b}} = 0.556 \sqrt{\frac{1}{6} + \frac{(5/6)^2}{2.0333}} = 0.3964 \text{ lbf}$$

$$F_i = (5.9787, 0.3964) \text{ lbf} \quad \text{Ans.}$$

(b) Eq. (20-35)

$$s_{\hat{m}} = \frac{0.556}{\sqrt{2.0333}} = 0.3899 \text{ lbf/in}$$

$$k = (9.7656, 0.3899) \text{ lbf/in} \quad \text{Ans.}$$

20-12 The expression $\epsilon = \delta/l$ is of the form x/y . Now $\delta = (0.0015, 0.000\,092)$ in, unspecified distribution; $l = (2.000, 0.0081)$ in, unspecified distribution;

$$C_x = 0.000\,092/0.0015 = 0.0613$$

$$C_y = 0.0081/2.000 = 0.000\,75$$

From Table 20-6, $\bar{\epsilon} = 0.0015/2.000 = 0.000\,75$

$$\begin{aligned}\hat{\sigma}_{\epsilon} &= 0.000\,75 \left[\frac{0.0613^2 + 0.004\,05^2}{1 + 0.004\,05^2} \right]^{1/2} \\ &= 4.607(10^{-5}) = 0.000\,046\end{aligned}$$

We can predict $\bar{\epsilon}$ and $\hat{\sigma}_{\epsilon}$ but not the distribution of ϵ .

20-13 $\sigma = \epsilon E$

$\epsilon = (0.0005, 0.000\,034)$ distribution unspecified; $E = (29.5, 0.885)$ Mpsi, distribution unspecified;

$$C_x = 0.000\,034/0.0005 = 0.068,$$

$$C_y = 0.0885/29.5 = 0.030$$

σ is of the form x, y

Table 20-6

$$\begin{aligned}\bar{\sigma} &= \bar{\epsilon} \bar{E} = 0.0005(29.5)10^6 = 14\,750 \text{ psi} \\ \hat{\sigma}_{\sigma} &= 14\,750(0.068^2 + 0.030^2 + 0.068^2 + 0.030^2)^{1/2} \\ &= 1096.7 \text{ psi} \\ C_{\sigma} &= 1096.7/14\,750 = 0.074\,35\end{aligned}$$

20-14

$$\delta = \frac{Fl}{AE}$$

$F = (14.7, 1.3)$ kip, $A = (0.226, 0.003)$ in², $l = (1.5, 0.004)$ in, $E = (29.5, 0.885)$ Mpsi distributions unspecified.

$$C_F = 1.3/14.7 = 0.0884; \quad C_A = 0.003/0.226 = 0.0133; \quad C_l = 0.004/1.5 = 0.00267;$$

$$C_E = 0.885/29.5 = 0.03$$

Mean of δ :

$$\delta = \frac{Fl}{AE} = Fl \left(\frac{1}{A} \right) \left(\frac{1}{E} \right)$$

From Table 20-6,

$$\begin{aligned}\bar{\delta} &= \bar{F}\bar{l}(1/\bar{A})(1/\bar{E}) \\ \bar{\delta} &= 14\,700(1.5)\frac{1}{0.226}\frac{1}{29.5(10^6)} \\ &= 0.003\,31 \text{ in } \textit{Ans.}\end{aligned}$$

For the standard deviation, using the first-order terms in Table 20-6,

$$\begin{aligned}\hat{\sigma}_{\delta} &\doteq \frac{\bar{F}\bar{l}}{\bar{A}\bar{E}} (C_F^2 + C_l^2 + C_A^2 + C_E^2)^{1/2} = \bar{\delta} (C_F^2 + C_l^2 + C_A^2 + C_E^2)^{1/2} \\ \hat{\sigma}_{\delta} &= 0.003\,31(0.0884^2 + 0.00267^2 + 0.0133^2 + 0.03^2)^{1/2} \\ &= 0.000\,313 \text{ in } \textit{Ans.}\end{aligned}$$

COV

$$C_{\delta} = 0.000\,313/0.003\,31 = 0.0945 \textit{ Ans.}$$

Force COV dominates. There is no distributional information on δ .

20-15 $\mathbf{M} = (15\,000, 1350) \text{ lbf} \cdot \text{in}$, distribution unspecified; $\mathbf{d} = (2.00, 0.005) \text{ in}$ distribution unspecified.

$$\sigma = \frac{32\mathbf{M}}{\pi\mathbf{d}^3}, \quad C_M = \frac{1350}{15\,000} = 0.09, \quad C_d = \frac{0.005}{2.00} = 0.0025$$

σ is of the form $\mathbf{x}/\mathbf{y}$, Table 20-6.

Mean:

$$\begin{aligned}\bar{\sigma} &= \frac{32\bar{M}}{\pi\bar{d}^3} \doteq \frac{32\bar{M}}{\pi\bar{d}^3} = \frac{32(15\,000)}{\pi(2^3)} \\ &= 19\,099 \text{ psi } \textit{Ans.}\end{aligned}$$

Standard Deviation:

$$\hat{\sigma}_{\sigma} = \bar{\sigma} [(C_M^2 + C_{d^3}^2)/(1 + C_{d^3}^2)]^{1/2}$$

From Table 20-6, $C_{d^3} \doteq 3C_d = 3(0.0025) = 0.0075$

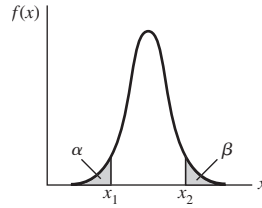
$$\begin{aligned}\hat{\sigma}_{\sigma} &= \bar{\sigma} [(C_M^2 + (3C_d)^2)/(1 + (3C_d)^2)]^{1/2} \\ &= 19\,099[(0.09^2 + 0.0075^2)/(1 + 0.0075^2)]^{1/2} \\ &= 1725 \text{ psi } \textit{Ans.}\end{aligned}$$

COV:

$$C_{\sigma} = \frac{1725}{19\,099} = 0.0903 \textit{ Ans.}$$

Stress COV dominates. No information of distribution of σ .

20-16



Fraction discarded is $\alpha + \beta$. The area under the PDF was unity. Having discarded $\alpha + \beta$ fraction, the ordinates to the truncated PDF are multiplied by a .

$$a = \frac{1}{1 - (\alpha + \beta)}$$

New PDF, $g(x)$, is given by

$$g(x) = \begin{cases} f(x)/[1 - (\alpha + \beta)] & x_1 \leq x \leq x_2 \\ 0 & \text{otherwise} \end{cases}$$

More formal proof: $g(x)$ has the property

$$1 = \int_{x_1}^{x_2} g(x) dx = a \int_{x_1}^{x_2} f(x) dx$$

$$1 = a \left[\int_{-\infty}^{\infty} f(x) dx - \int_0^{x_1} f(x) dx - \int_{x_2}^{\infty} f(x) dx \right]$$

$$1 = a \{1 - F(x_1) - [1 - F(x_2)]\}$$

$$a = \frac{1}{F(x_2) - F(x_1)} = \frac{1}{(1 - \beta) - \alpha} = \frac{1}{1 - (\alpha + \beta)}$$

20-17

(a) $d = U[0.748, 0.751]$

$$\mu_d = \frac{0.751 + 0.748}{2} = 0.7495 \text{ in}$$

$$\hat{\sigma}_d = \frac{0.751 - 0.748}{2\sqrt{3}} = 0.000866 \text{ in}$$

$$f(x) = \frac{1}{b - a} = \frac{1}{0.751 - 0.748} = 333.3 \text{ in}^{-1}$$

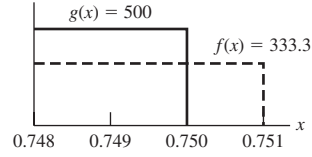
$$F(x) = \frac{x - 0.748}{0.751 - 0.748} = 333.3(x - 0.748)$$

(b)

$$F(x_1) = F(0.748) = 0$$

$$F(x_2) = (0.750 - 0.748)333.3 = 0.6667$$

If $g(x)$ is truncated, PDF becomes



$$g(x) = \frac{f(x)}{F(x_2) - F(x_1)} = \frac{333.3}{0.6667 - 0} = 500 \text{ in}^{-1}$$

$$\mu_x = \frac{a' + b'}{2} = \frac{0.748 + 0.750}{2} = 0.749 \text{ in}$$

$$\hat{\sigma}_x = \frac{b' - a'}{2\sqrt{3}} = \frac{0.750 - 0.748}{2\sqrt{3}} = 0.000577 \text{ in}$$

20-18 From Table A-10, 8.1% corresponds to $z_1 = -1.4$ and 5.5% corresponds to $z_2 = +1.6$.

$$k_1 = \mu + z_1 \hat{\sigma}$$

$$k_2 = \mu + z_2 \hat{\sigma}$$

From which

$$\begin{aligned} \mu &= \frac{z_2 k_1 - z_1 k_2}{z_2 - z_1} = \frac{1.6(9) - (-1.4)11}{1.6 - (-1.4)} \\ &= 9.933 \end{aligned}$$

$$\begin{aligned} \hat{\sigma} &= \frac{k_2 - k_1}{z_2 - z_1} \\ &= \frac{11 - 9}{1.6 - (-1.4)} = 0.6667 \end{aligned}$$

The original density function is

$$f(k) = \frac{1}{0.6667\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{k - 9.933}{0.6667} \right)^2 \right] \quad \text{Ans.}$$

20-19 From Prob. 20-1, $\mu = 122.9$ kcycles and $\hat{\sigma} = 30.3$ kcycles.

$$z_{10} = \frac{x_{10} - \mu}{\hat{\sigma}} = \frac{x_{10} - 122.9}{30.3}$$

$$x_{10} = 122.9 + 30.3 z_{10}$$

From Table A-10, for 10 percent failure, $z_{10} = -1.282$

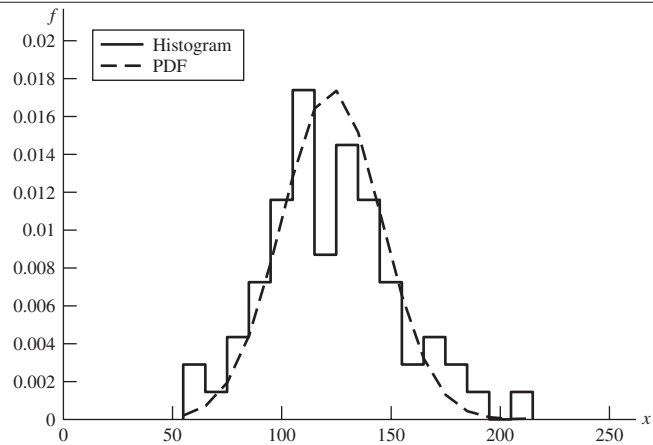
$$\begin{aligned} x_{10} &= 122.9 + 30.3(-1.282) \\ &= 84.1 \text{ kcycles} \quad \text{Ans.} \end{aligned}$$

20-20

x	f	fx	fx^2	x	$f/(Nw)$	$f(x)$
60	2	120	7200	60	0.002 899	0.000 399
70	1	70	4900	70	0.001 449	0.001 206
80	3	240	19 200	80	0.004 348	0.003 009
90	5	450	40 500	90	0.007 246	0.006 204
100	8	800	80 000	100	0.011 594	0.010 567
110	12	1320	145 200	110	0.017 391	0.014 871
120	6	720	86 400	120	0.008 696	0.017 292
130	10	1300	169 000	130	0.014 493	0.016 612
140	8	1120	156 800	140	0.011 594	0.013 185
150	5	750	112 500	150	0.007 246	0.008 647
160	2	320	51 200	160	0.002 899	0.004 685
170	3	510	86 700	170	0.004 348	0.002 097
180	2	360	64 800	180	0.002 899	0.000 776
190	1	190	36 100	190	0.001 449	0.000 237
200	0	0	0	200	0	5.98E-05
210	1	210	44 100	210	0.001 449	1.25E-05
	69	8480				

$$\bar{x} = 122.8986 \quad s_x = 22.887 19$$

x	$f/(Nw)$	$f(x)$	x	$f/(Nw)$	$f(x)$
55	0	0.000 214	145	0.011 594	0.010 935
55	0.002 899	0.000 214	145	0.007 246	0.010 935
65	0.002 899	0.000 711	155	0.007 246	0.006 518
65	0.001 449	0.000 711	155	0.002 899	0.006 518
75	0.001 449	0.001 951	165	0.002 899	0.003 21
75	0.004 348	0.001 951	165	0.004 348	0.003 21
85	0.004 348	0.004 425	175	0.004 348	0.001 306
85	0.007 246	0.004 425	175	0.002 899	0.001 306
95	0.007 246	0.008 292	185	0.002 899	0.000 439
95	0.011 594	0.008 292	185	0.001 449	0.000 439
105	0.011 594	0.012 839	195	0.001 449	0.000 122
105	0.017 391	0.012 839	195	0	0.000 122
115	0.017 391	0.016 423	205	0	2.8E-05
115	0.008 696	0.016 423	205	0.001 499	2.8E-05
125	0.008 696	0.017 357	215	0.001 499	5.31E-06
125	0.014 493	0.017 357	215	0	5.31E-06
135	0.014 493	0.015 157			
135	0.011 594	0.015 157			

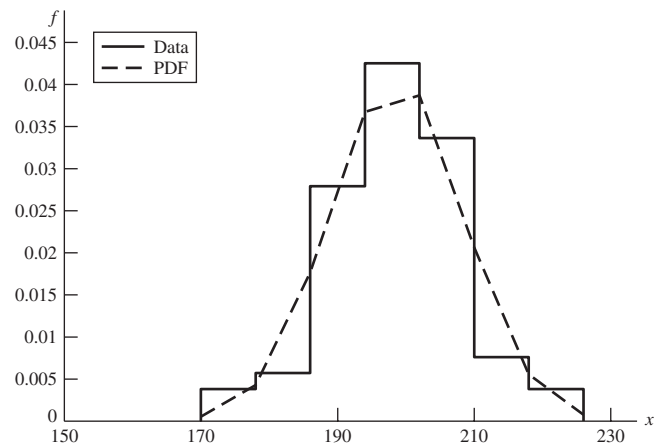


20-21

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
174	6	1044	181 656	0.003 807	0.001 642
182	9	1638	298 116	0.005 711	0.009 485
190	44	8360	1 588 400	0.027 919	0.027 742
198	67	13 266	2 626 668	0.042 513	0.041 068
206	53	10 918	2 249 108	0.033 629	0.030 773
214	12	2 568	549 552	0.007 614	0.011 671
222	6	1 332	295 704	0.003 807	0.002 241
1386	197	39 126	7 789 204		

$$\bar{x} = 198.6091 \quad s_x = 9.695 071$$

x	$f/(Nw)$	$f(x)$
170	0	0.000 529
170	0.003 807	0.000 529
178	0.003 807	0.004 297
178	0.005 711	0.004 297
186	0.005 711	0.017 663
186	0.027 919	0.017 663
194	0.027 919	0.036 752
194	0.042 513	0.036 752
202	0.042 513	0.038 708
202	0.033 629	0.038 708
210	0.033 629	0.020 635
210	0.007 614	0.020 635
218	0.007 614	0.005 568
218	0.003 807	0.005 568
226	0.003 807	0.000 76
226	0	0.000 76

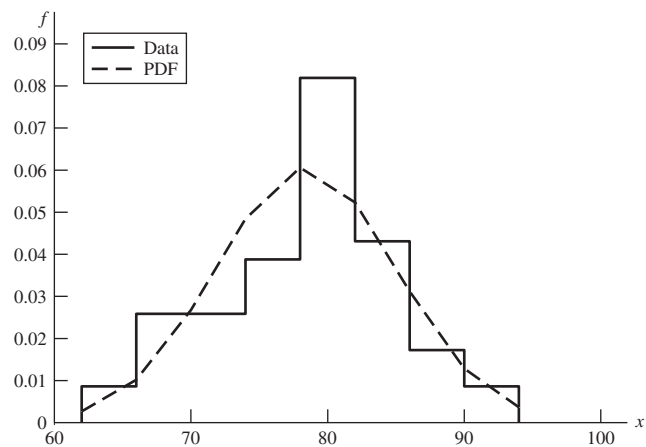


20-22

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
64	2	128	8192	0.008 621	0.005 48
68	6	408	27 744	0.025 862	0.017 299
72	6	432	31 104	0.025 862	0.037 705
76	9	684	51 984	0.038 793	0.056 742
80	19	1520	121 600	0.081 897	0.058 959
84	10	840	70 560	0.043 103	0.042 298
88	4	352	30 976	0.017 241	0.020 952
92	2	184	16 928	0.008 621	0.007 165
624	58	4548	359 088		

$$\bar{x} = 78.41379 \quad s_x = 6.572229$$

x	$f/(Nw)$	$f(x)$	x	$f/(Nw)$	$f(x)$
62	0	0.002 684	82	0.081 897	0.052 305
62	0.008 621	0.002 684	82	0.043 103	0.052 305
66	0.008 621	0.010 197	86	0.043 103	0.031 18
66	0.025 862	0.010 197	86	0.017 241	0.031 18
70	0.025 862	0.026 749	90	0.017 241	0.012 833
70	0.025 862	0.026 749	90	0.008 621	0.012 833
74	0.025 862	0.048 446	94	0.008 621	0.003 647
74	0.038 793	0.048 446	94	0	0.003 647
78	0.038 793	0.060 581			
78	0.081 897	0.060 581			



20-23

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(40)}{\pi(1^2)} = 50.93 \text{ kpsi}$$

$$\hat{\sigma}_{\sigma} = \frac{4\hat{\sigma}_P}{\pi d^2} = \frac{4(8.5)}{\pi(1^2)} = 10.82 \text{ kpsi}$$

$$\hat{\sigma}_{s_y} = 5.9 \text{ kpsi}$$

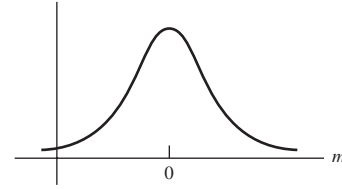
For no yield, $m = S_y - \sigma \geq 0$

$$z = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - \mu_m}{\hat{\sigma}_m} = -\frac{\mu_m}{\hat{\sigma}_m}$$

$$\mu_m = \bar{S}_y - \bar{\sigma} = 27.47 \text{ kpsi,}$$

$$\hat{\sigma}_m = \left(\hat{\sigma}_\sigma^2 + \hat{\sigma}_{S_y}^2 \right)^{1/2} = 12.32 \text{ kpsi}$$

$$z = \frac{-27.47}{12.32} = -2.230$$



From Table A-10, $p_f = 0.0129$

$$R = 1 - p_f = 1 - 0.0129 = 0.987 \quad \text{Ans.}$$

20-24 For a lognormal distribution,

Eq. (20-18) $\mu_y = \ln \mu_x - \ln \sqrt{1 + C_x^2}$

Eq. (20-19) $\hat{\sigma}_y = \sqrt{\ln(1 + C_x^2)}$

From Prob. (20-23)

$$\mu_m = \bar{S}_y - \bar{\sigma} = \mu_x$$

$$\mu_y = \left(\ln \bar{S}_y - \ln \sqrt{1 + C_{S_y}^2} \right) - \left(\ln \bar{\sigma} - \ln \sqrt{1 + C_\sigma^2} \right)$$

$$= \ln \left[\frac{\bar{S}_y}{\bar{\sigma}} \sqrt{\frac{1 + C_\sigma^2}{1 + C_{S_y}^2}} \right]$$

$$\hat{\sigma}_y = \left[\ln \left(1 + C_{S_y}^2 \right) + \ln \left(1 + C_\sigma^2 \right) \right]^{1/2}$$

$$= \sqrt{\ln \left[\left(1 + C_{S_y}^2 \right) \left(1 + C_\sigma^2 \right) \right]}$$

$$z = -\frac{\mu}{\hat{\sigma}} = -\frac{\ln \left(\frac{\bar{S}_y}{\bar{\sigma}} \sqrt{\frac{1 + C_\sigma^2}{1 + C_{S_y}^2}} \right)}{\sqrt{\ln \left[\left(1 + C_{S_y}^2 \right) \left(1 + C_\sigma^2 \right) \right]}}$$

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(30)}{\pi(1^2)} = 38.197 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{4\hat{\sigma}_P}{\pi d^2} = \frac{4(5.1)}{\pi(1^2)} = 6.494 \text{ kpsi}$$

$$C_\sigma = \frac{6.494}{38.197} = 0.1700$$

$$C_{S_y} = \frac{3.81}{49.6} = 0.07681$$

$$z = -\frac{\ln \left[\frac{49.6}{38.197} \sqrt{\frac{1 + 0.170^2}{1 + 0.07681^2}} \right]}{\sqrt{\ln[(1 + 0.07681^2)(1 + 0.170^2)]}} = -1.470$$

From Table A-10

$$p_f = 0.0708$$

$$R = 1 - p_f = 0.929 \quad \text{Ans.}$$

20-25

x	n	nx	nx^2
93	19	1767	164 311
95	25	2375	225 625
97	38	3685	357 542
99	17	1683	166 617
101	12	1212	122 412
103	10	1030	106 090
105	5	525	55 125
107	4	428	45 796
109	4	436	47 524
111	2	222	24 624
	<u>136</u>	<u>13 364</u>	<u>1315 704</u>

$$\bar{x} = 13\,364/136 = 98.26 \text{ kpsi}$$

$$s_x = \left(\frac{1\,315\,704 - 13\,364^2/136}{135} \right)^{1/2} = 4.30 \text{ kpsi}$$

Under normal hypothesis,

$$z_{0.01} = (x_{0.01} - 98.26)/4.30$$

$$x_{0.01} = 98.26 + 4.30z_{0.01}$$

$$= 98.26 + 4.30(-2.3267)$$

$$= 88.26 \doteq 88.3 \text{ kpsi} \quad \text{Ans.}$$

20-26 From Prob. 20-25, $\mu_x = 98.26$ kpsi, and $\hat{\sigma}_x = 4.30$ kpsi.

$$C_x = \hat{\sigma}_x/\mu_x = 4.30/98.26 = 0.043\,76$$

From Eqs. (20-18) and (20-19),

$$\mu_y = \ln(98.26) - 0.043\,76^2/2 = 4.587$$

$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.043\,76^2)} = 0.043\,74$$

For a yield strength exceeded by 99% of the population,

$$z_{0.01} = (\ln x_{0.01} - \mu_y)/\hat{\sigma}_y \quad \Rightarrow \quad \ln x_{0.01} = \mu_y + \hat{\sigma}_y z_{0.01}$$

From Table A-10, for 1% failure, $z_{0.01} = -2.326$. Thus,

$$\ln x_{0.01} = 4.587 + 0.04374(-2.326) = 4.485$$

$$x_{0.01} = 88.7 \text{ kpsi} \quad \text{Ans.}$$

The normal PDF is given by Eq. (20-14) as

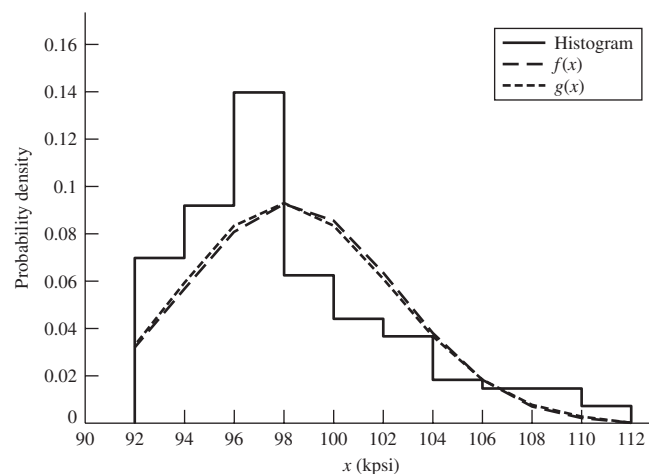
$$f(x) = \frac{1}{4.30\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - 98.26}{4.30} \right)^2 \right]$$

For the lognormal distribution, from Eq. (20-17), defining $g(x)$,

$$g(x) = \frac{1}{x(0.04374)\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\ln x - 4.587}{0.04374} \right)^2 \right]$$

x (kpsi)	$f/(Nw)$	$f(x)$	$g(x)$	x (kpsi)	$f/(Nw)$	$f(x)$	$g(x)$
92	0.00000	0.03215	0.03263	102	0.03676	0.06356	0.06134
92	0.06985	0.03215	0.03263	104	0.03676	0.03806	0.03708
94	0.06985	0.05680	0.05890	104	0.01838	0.03806	0.03708
94	0.09191	0.05680	0.05890	106	0.01838	0.01836	0.01869
96	0.09191	0.08081	0.08308	106	0.01471	0.01836	0.01869
96	0.13971	0.08081	0.08308	108	0.01471	0.00713	0.00793
98	0.13971	0.09261	0.09297	108	0.01471	0.00713	0.00793
98	0.06250	0.09261	0.09297	110	0.01471	0.00223	0.00286
100	0.06250	0.08548	0.08367	110	0.00735	0.00223	0.00286
100	0.04412	0.08548	0.08367	112	0.00735	0.00056	0.00089
102	0.04412	0.06356	0.06134	112	0.00000	0.00056	0.00089

Note: rows are repeated to draw histogram



The normal and lognormal are almost the same. However the data is quite skewed and perhaps a Weibull distribution should be explored. For a method of establishing the

Weibull parameters see Shigley, J. E., and C. R. Mischke, *Mechanical Engineering Design*, McGraw-Hill, 5th ed., 1989, Sec. 4-12.

20-27 Let $\mathbf{x} = (\mathbf{S}'_{fe})_{10^4}$

$$x_0 = 79 \text{ kpsi}, \quad \theta = 86.2 \text{ kpsi}, \quad b = 2.6$$

Eq. (20-28)

$$\bar{x} = x_0 + (\theta - x_0)\Gamma(1 + 1/b)$$

$$\begin{aligned}\bar{x} &= 79 + (86.2 - 79)\Gamma(1 + 1/2.6) \\ &= 79 + 7.2\Gamma(1.38)\end{aligned}$$

From Table A-34, $\Gamma(1.38) = 0.88854$

$$\bar{x} = 79 + 7.2(0.88854) = 85.4 \text{ kpsi} \quad \text{Ans.}$$

Eq. (20-29)

$$\begin{aligned}\hat{\sigma}_x &= (\theta - x_0)[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2} \\ &= (86.2 - 79)[\Gamma(1 + 2/2.6) - \Gamma^2(1 + 1/2.6)]^{1/2} \\ &= 7.2[0.92376 - 0.88854^2]^{1/2} \\ &= 2.64 \text{ kpsi} \quad \text{Ans.}\end{aligned}$$

$$C_x = \frac{\hat{\sigma}_x}{\bar{x}} = \frac{2.64}{85.4} = 0.031 \quad \text{Ans.}$$

20-28

$$\mathbf{x} = \mathbf{S}_{ut}$$

$$x_0 = 27.7, \quad \theta = 46.2, \quad b = 4.38$$

$$\begin{aligned}\mu_x &= 27.7 + (46.2 - 27.7)\Gamma(1 + 1/4.38) \\ &= 27.7 + 18.5\Gamma(1.23) \\ &= 27.7 + 18.5(0.91075) \\ &= 44.55 \text{ kpsi} \quad \text{Ans.}\end{aligned}$$

$$\begin{aligned}\hat{\sigma}_x &= (46.2 - 27.7)[\Gamma(1 + 2/4.38) - \Gamma^2(1 + 1/4.38)]^{1/2} \\ &= 18.5[\Gamma(1.46) - \Gamma^2(1.23)]^{1/2} \\ &= 18.5[0.8856 - 0.91075^2]^{1/2} \\ &= 4.38 \text{ kpsi} \quad \text{Ans.}\end{aligned}$$

$$C_x = \frac{4.38}{44.55} = 0.098 \quad \text{Ans.}$$

From the Weibull survival equation

$$R = \exp \left[- \left(\frac{x - x_0}{\theta - x_0} \right)^b \right] = 1 - p$$

$$R_{40} = \exp \left[- \left(\frac{x_{40} - x_0}{\theta - x_0} \right)^b \right] = 1 - p_{40}$$

$$= \exp \left[- \left(\frac{40 - 27.7}{46.2 - 27.7} \right)^{4.38} \right] = 0.846$$

$$p_{40} = 1 - R_{40} = 1 - 0.846 = 0.154 = 15.4\% \quad \text{Ans.}$$

20-29

$$\mathbf{x} = \mathbf{S}_{ut}$$

$$x_0 = 151.9, \theta = 193.6, b = 8$$

$$\mu_x = 151.9 + (193.6 - 151.9)\Gamma(1 + 1/8)$$

$$= 151.9 + 41.7\Gamma(1.125)$$

$$= 151.9 + 41.7(0.94176)$$

$$= 191.2 \text{ kpsi} \quad \text{Ans.}$$

$$\hat{\sigma}_x = (193.6 - 151.9)[\Gamma(1 + 2/8) - \Gamma^2(1 + 1/8)]^{1/2}$$

$$= 41.7[\Gamma(1.25) - \Gamma^2(1.125)]^{1/2}$$

$$= 41.7[0.90640 - 0.94176^2]^{1/2}$$

$$= 5.82 \text{ kpsi} \quad \text{Ans.}$$

$$C_x = \frac{5.82}{191.2} = 0.030$$

20-30

$$\mathbf{x} = \mathbf{S}_{ut}$$

$$x_0 = 47.6, \theta = 125.6, b = 11.84$$

$$\bar{x} = 47.6 + (125.6 - 47.6)\Gamma(1 + 1/11.84)$$

$$\bar{x} = 47.6 + 78\Gamma(1.08)$$

$$= 47.6 + 78(0.95973) = 122.5 \text{ kpsi}$$

$$\hat{\sigma}_x = (125.6 - 47.6)[\Gamma(1 + 2/11.84) - \Gamma^2(1 + 1/11.84)]^{1/2}$$

$$= 78[\Gamma(1.08) - \Gamma^2(1.17)]^{1/2}$$

$$= 78(0.95973 - 0.93670^2)^{1/2}$$

$$= 22.4 \text{ kpsi}$$

From Prob. 20-28

$$p = 1 - \exp \left[- \left(\frac{x - x_0}{\theta - x_0} \right)^b \right]$$

$$= 1 - \exp \left[- \left(\frac{100 - 47.6}{125.6 - 47.6} \right)^{11.84} \right]$$

$$= 0.0090 \quad \text{Ans.}$$

$$y = S_y$$

$$y_0 = 64.1, \theta = 81.0, b = 3.77$$

$$\begin{aligned}\bar{y} &= 64.1 + (81.0 - 64.1)\Gamma(1 + 1/3.77) \\ &= 64.1 + 16.9\Gamma(1.27) \\ &= 64.1 + 16.9(0.90250) \\ &= 79.35 \text{ kpsi}\end{aligned}$$

$$\sigma_y = (81 - 64.1)[\Gamma(1 + 2/3.77) - \Gamma(1 + 1/3.77)]^{1/2}$$

$$\begin{aligned}\sigma_y &= 16.9[(0.88757) - 0.90250^2]^{1/2} \\ &= 4.57 \text{ kpsi}\end{aligned}$$

$$p = 1 - \exp \left[- \left(\frac{y - y_0}{\theta - y_0} \right)^{3.77} \right]$$

$$p = 1 - \exp \left[- \left(\frac{70 - 64.1}{81 - 64.1} \right)^{3.77} \right] = 0.019 \quad \text{Ans.}$$

20-31 $x = S_{ut} = \mathbf{W}[122.3, 134.6, 3.64]$ kpsi, $p(x > 120) = 1 = 100\%$ since $x_0 > 120$ kpsi

$$\begin{aligned}p(x > 133) &= \exp \left[- \left(\frac{133 - 122.3}{134.6 - 122.3} \right)^{3.64} \right] \\ &= 0.548 = 54.8\% \quad \text{Ans.}\end{aligned}$$

20-32 Using Eqs. (20-28) and (20-29) and Table A-34,

$$\mu_n = n_0 + (\theta - n_0)\Gamma(1 + 1/b) = 36.9 + (133.6 - 36.9)\Gamma(1 + 1/2.66) = 122.85 \text{ kcycles}$$

$$\hat{\sigma}_n = (\theta - n_0)[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)] = 34.79 \text{ kcycles}$$

For the Weibull density function, Eq. (2-27),

$$f_W(n) = \frac{2.66}{133.6 - 36.9} \left(\frac{n - 36.9}{133.6 - 36.9} \right)^{2.66-1} \exp \left[- \left(\frac{n - 36.9}{133.6 - 36.9} \right)^{2.66} \right]$$

For the lognormal distribution, Eqs. (20-18) and (20-19) give,

$$\mu_y = \ln(122.85) - (34.79/122.85)^2/2 = 4.771$$

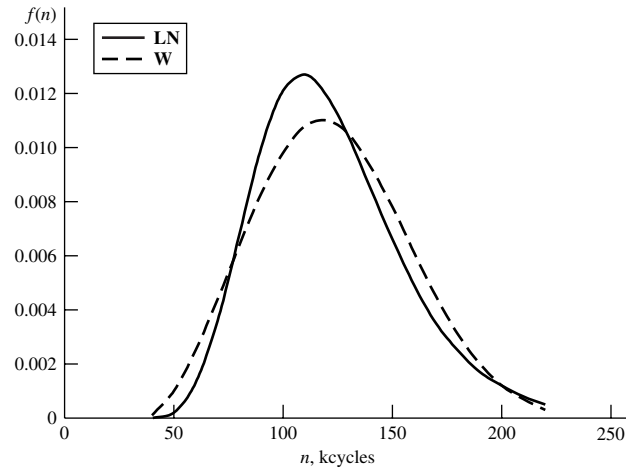
$$\hat{\sigma}_y = \sqrt{[1 + (34.79/122.85)^2]} = 0.2778$$

From Eq. (20-17), the lognormal PDF is

$$f_{LN}(n) = \frac{1}{0.2778 \, n \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\ln n - 4.771}{0.2778} \right)^2 \right]$$

We form a table of densities $f_W(n)$ and $f_{LN}(n)$ and plot.

n (kcycles)	$f_W(n)$	$f_{LN}(n)$
40	9.1E-05	1.82E-05
50	0.000991	0.000241
60	0.002498	0.001233
70	0.004380	0.003501
80	0.006401	0.006739
90	0.008301	0.009913
100	0.009822	0.012022
110	0.010750	0.012644
120	0.010965	0.011947
130	0.010459	0.010399
140	0.009346	0.008492
150	0.007827	0.006597
160	0.006139	0.004926
170	0.004507	0.003564
180	0.003092	0.002515
190	0.001979	0.001739
200	0.001180	0.001184
210	0.000654	0.000795
220	0.000336	0.000529



The Weibull L10 life comes from Eq. (20-26) with a reliability of $R = 0.90$. Thus,

$$n_{0.10} = 36.9 + (133 - 36.9)[\ln(1/0.90)]^{1/2.66} = 78.1 \text{ kcycles} \quad \text{Ans.}$$

The lognormal L10 life comes from the definition of the z variable. That is,

$$\ln n_0 = \mu_y + \hat{\sigma}_y z \quad \text{or} \quad n_0 = \exp(\mu_y + \hat{\sigma}_y z)$$

From Table A-10, for $R = 0.90$, $z = -1.282$. Thus,

$$n_0 = \exp[4.771 + 0.2778(-1.282)] = 82.7 \text{ kcycles} \quad \text{Ans.}$$

20-33 Form a table

i	x $L(10^{-5})$	f_i	$f_i x(10^{-5})$	$f_i x^2(10^{-10})$	$g(x)$ (10^5)
1	3.05	3	9.15	27.9075	0.0557
2	3.55	7	24.85	88.2175	0.1474
3	4.05	11	44.55	180.4275	0.2514
4	4.55	16	72.80	331.24	0.3168
5	5.05	21	106.05	535.5525	0.3216
6	5.55	13	72.15	400.4325	0.2789
7	6.05	13	78.65	475.8325	0.2151
8	6.55	6	39.30	257.415	0.1517
9	7.05	2	14.10	99.405	0.1000
10	7.55	0	0	0	0.0625
11	8.05	4	32.20	259.21	0.0375
12	8.55	3	25.65	219.3075	0.0218
13	9.05	0	0	0	0.0124
14	9.55	0	0	0	0.0069
15	10.05	1	10.05	101.0025	0.0038
		<u>100</u>	<u>529.50</u>	<u>2975.95</u>	

$$\bar{x} = 529.5(10^5)/100 = 5.295(10^5) \text{ cycles} \quad \text{Ans.}$$

$$s_x = \left[\frac{2975.95(10^{10}) - [529.5(10^5)]^2/100}{100 - 1} \right]^{1/2}$$

$$= 1.319(10^5) \text{ cycles} \quad \text{Ans.}$$

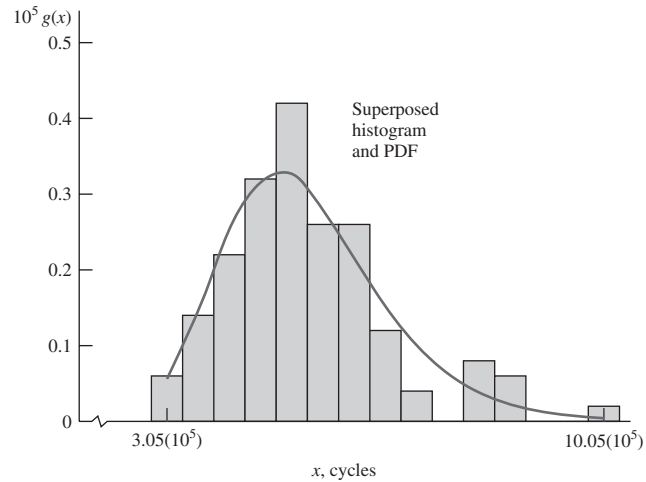
$$C_x = s/\bar{x} = 1.319/5.295 = 0.249$$

$$\mu_y = \ln 5.295(10^5) - 0.249^2/2 = 13.149$$

$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.249^2)} = 0.245$$

$$g(x) = \frac{1}{x\hat{\sigma}_y\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\ln x - \mu_y}{\hat{\sigma}_y} \right)^2 \right]$$

$$g(x) = \frac{1.628}{x} \exp \left[-\frac{1}{2} \left(\frac{\ln x - 13.149}{0.245} \right)^2 \right]$$

**20-34**

$$\mathbf{x} = \mathbf{S}_u = \mathbf{W}[70.3, 84.4, 2.01]$$

$$\begin{aligned} \text{Eq. (20-28)} \quad \mu_x &= 70.3 + (84.4 - 70.3)\Gamma(1 + 1/2.01) \\ &= 70.3 + (84.4 - 70.3)\Gamma(1.498) \\ &= 70.3 + (84.4 - 70.3)0.88617 \\ &= 82.8 \text{ kpsi} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} \text{Eq. (20-29)} \quad \hat{\sigma}_x &= (84.4 - 70.3)[\Gamma(1 + 2/2.01) - \Gamma^2(1 + 1/2.01)]^{1/2} \\ \hat{\sigma}_x &= 14.1[0.99791 - 0.88617^2]^{1/2} \\ &= 6.502 \text{ kpsi} \end{aligned}$$

$$C_x = \frac{6.502}{82.8} = 0.079 \quad \text{Ans.}$$

20-35 Take the Weibull equation for the standard deviation

$$\hat{\sigma}_x = (\theta - x_0)[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2}$$

and the mean equation solved for $\bar{x} - x_0$

$$\bar{x} - x_0 = (\theta - x_0)\Gamma(1 + 1/b)$$

Dividing the first by the second,

$$\frac{\hat{\sigma}_x}{\bar{x} - x_0} = \frac{[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2}}{\Gamma(1 + 1/b)}$$

$$\frac{4.2}{49 - 33.8} = \sqrt{\frac{\Gamma(1 + 2/b)}{\Gamma^2(1 + 1/b)} - 1} = \sqrt{R} = 0.2763$$

Make a table and solve for b iteratively

b	$1 + 2/b$	$1 + 1/b$	$\Gamma(1 + 2/b)$	$\Gamma(1 + 1/b)$	
3	1.67	1.33	0.903 30	0.893 38	0.363
4	1.5	1.25	0.886 23	0.906 40	0.280
4.1	1.49	1.24	0.885 95	0.908 52	0.271

$$b \doteq 4.068 \quad \text{Using MathCad} \quad \text{Ans.}$$

$$\begin{aligned} \theta &= x_0 + \frac{\bar{x} - x_0}{\Gamma(1 + 1/b)} = 33.8 + \frac{49 - 33.8}{\Gamma(1 + 1/4.068)} \\ &= 49.8 \text{ kpsi} \quad \text{Ans.} \end{aligned}$$

20-36

$$\mathbf{x} = \mathbf{S}_y = \mathbf{W}[34.7, 39, 2.93] \text{ kpsi}$$

$$\bar{x} = 34.7 + (39 - 34.7)\Gamma(1 + 1/2.93)$$

$$= 34.7 + 4.3\Gamma(1.34)$$

$$= 34.7 + 4.3(0.892\ 22) = 38.5 \text{ kpsi}$$

$$\hat{\sigma}_x = (39 - 34.7)[\Gamma(1 + 2/2.93) - \Gamma^2(1 + 1/2.93)]^{1/2}$$

$$= 4.3[\Gamma(1.68) - \Gamma^2(1.34)]^{1/2}$$

$$= 4.3[0.905\ 00 - 0.892\ 22^2]^{1/2}$$

$$= 1.42 \text{ kpsi} \quad \text{Ans.}$$

$$C_x = 1.42/38.5 = 0.037 \quad \text{Ans.}$$

20-37

x (Mrev)	f	fx	fx^2
1	11	11	11
2	22	44	88
3	38	114	342
4	57	228	912
5	31	155	775
6	19	114	684
7	15	105	735
8	12	96	768
9	11	99	891
10	9	90	900
11	7	77	847
12	5	60	720
Sum	78	237	7673

$$\mu_x = 1193(10^6)/237 = 5.034(10^6) \text{ cycles}$$

$$\hat{\sigma}_x = \sqrt{\frac{7673(10^{12}) - [1193(10^6)]^2/237}{237 - 1}} = 2.658(10^6) \text{ cycles}$$

$$C_x = 2.658/5.034 = 0.528$$

From Eqs. (20-18) and (20-19),

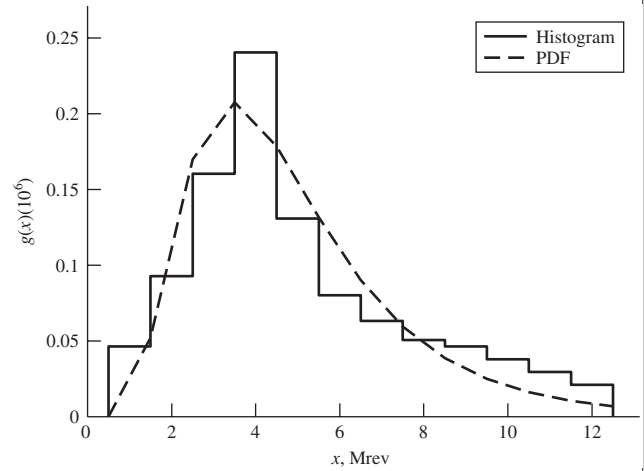
$$\mu_y = \ln[5.034(10^6)] - 0.528^2/2 = 15.292$$

$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.528^2)} = 0.496$$

From Eq. (20-17), defining $g(x)$,

$$g(x) = \frac{1}{x(0.496)\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\ln x - 15.292}{0.496} \right)^2 \right]$$

x (Mrev)	$f/(Nw)$	$g(x) \cdot (10^6)$
0.5	0.00000	0.00011
0.5	0.04641	0.00011
1.5	0.04641	0.05204
1.5	0.09283	0.05204
2.5	0.09283	0.16992
2.5	0.16034	0.16992
3.5	0.16034	0.20754
3.5	0.24051	0.20754
4.5	0.24051	0.17848
4.5	0.13080	0.17848
5.5	0.13080	0.13158
5.5	0.08017	0.13158
6.5	0.08017	0.09011
6.5	0.06329	0.09011
7.5	0.06329	0.05953
7.5	0.05063	0.05953
8.5	0.05063	0.03869
8.5	0.04641	0.03869
9.5	0.04641	0.02501
9.5	0.03797	0.02501
10.5	0.03797	0.01618
10.5	0.02954	0.01618
11.5	0.02954	0.01051
11.5	0.02110	0.01051
12.5	0.02110	0.00687
12.5	0.00000	0.00687



$$z = \frac{\ln x - \mu_y}{\hat{\sigma}_y} \Rightarrow \ln x = \mu_y + \hat{\sigma}_y z = 15.292 + 0.496z$$

L_{10} life, where 10% of bearings fail, from Table A-10, $z = -1.282$. Thus,

$$\ln x = 15.292 + 0.496(-1.282) = 14.66$$

$$\therefore x = 2.32 \times 10^6 \text{ rev} \quad \text{Ans.}$$