Appendix A

Units and Definitions of Material Properties

A.1 UNIT SYSTEMS

The traditional unit system in the United States has been the Imperial system, often referred to as the British system, although in United Kingdom the Imperial system was replaced by the SI International System (Système Internationale, French). In the United States, the engineering societies are in favor of adopting SI, and most engineering publications and textbooks currently use SI units. Many engineering companies are in transition from Imperial to SI units, so engineers must be familiar with the two systems. For this reason, this text uses both systems, although most of the example problems are presented in SI units.

The SI is based on three units: mass, length, and time. The unit of mass is the kilogram (kg), that of length is the meter (m), and that for time is the second (s). The unit of force is the Newton (N), which is defined by Newton's second law as the force required to accelerate 1 kg of mass at the rate of 1 m^2/s.

Gravitational acceleration is $g = 9.81\ m^2/s$, so the weight (force exerted by gravity at the earth's surface) of 1 kg mass is

$$F = mg = 1 \times 9.81 = 9.81\ N \qquad \text{(A-1)}$$

The unit of energy (or work) is the Joule (J), which is equivalent to N-m. The unit of power, which is energy per unit of time, is the watt (W). The watt is equivalent to J/s, or, in basic SI units, N-m/s.

Pressure or stress is force per unit area. The SI unit is the pascal (Pa), which is equivalent to N/m^2. This is a small unit, and prefixes such as kPa (10^3 Pa) and MPa (10^6 Pa) are often used.

In SI units, very large or very small numbers are often needed in practical problems, and the following prefixes serve to indicate multiplication of units by various powers of 10:

μ (micro-) $= 10^{-6}$	k (kilo-) $= 10^3$
m (milli-) $= 10^{-3}$	M (mega-) $= 10^6$
c (centi-) $= 10^{-2}$	G (giga-) $= 10^9$

For example, the well-known Imperial unit of pressure is psi ($lb_f/in.^2$).

$$1 \text{ psi is} = 6895 \text{ N/m}^2 \text{ (Pa)} = 6.895 \text{ kPa.}$$

A second example is the modulus of elasticity of the steel:

$$E = 2.05 \times 10^{11} \text{ Pa (N/m}^2) = 2.05 \times 10^5 \text{ MPa,} = 2.05 \times 10^2 \text{ GPa.}$$

A.2 DEFINITIONS OF MATERIAL PROPERTIES

A.2.1 Density, ρ

Material density ρ is mass per unit volume. The SI unit of density is kg/m^3. In Imperial units, the density is lb_m/ft^3, or lb_m/in^3. For example, the density of water at 4°C is 1000 kg/m^3, and in imperial units it is 62.43 $lb_m/in.^3$.

The conversion is

$$1 \text{ kg/m}^3 = 0.06243 \text{ lb}_m/\text{ft}^3.$$

A.2.2 Specific Weight, γ

Specific weight, γ, is the gravity force (weight) per unit volume of the material

$$\gamma = \rho g \tag{A-2}$$

The SI unit of density is N/m^3. For example, the specific weight γ of water at 4°C is 9810 N/m^3, obtained by the equation

$$\gamma_{water} = \rho g = 1000 \times 9.81 = 9810 \text{ N/m}^3.$$

The Imperial unit of specific density is lb_f/ft^3, or lb_f/in^3. For example, the specific weight γ of water at 4°C is 62.4 lb_f/ft^3. The conversion is

$$1 \text{ lb}_f/\text{ft}^3 = 157.1 \text{ N/m}^3$$
$$1 \text{ N/m}^3 = 0.00636 \text{ lb}_f/\text{ft}^3$$

A.2.3 Specific Gravity, S

Specific gravity, S, of a material is the ratio of its specific weight to the specific weight of water at 4°C. It is also the ratio of its density to the density of water at 4°C. For example, if the density of a steel is 7800 kg/m^3, its specific density is $7800/1000 = 7.8$ (specific gravity is a dimensionless ratio).

A.2.4 Specific Heat, c

Specific heat, c, is the amount of heat that must be transferred to a unit of mass of a material to raise its temperature by one degree. For gas, the specific heat depends if the unit of mass has a constant pressure, c_p, or if the unit of mass has a constant volume c_v. The specific heat of a material is a function of its temperature. The SI unit of specific heat is J/Kg-°C (a widely used unit is KJ/Kg-°C), and the Imperial unit is BTU/lb_m°F.

The conversion ratio is

$$1\ \mathrm{BTU/lb_m{}^\circ F} = 2326\ \mathrm{J/Kg\text{-}^\circ C}$$
$$1\ \mathrm{BTU/lb_m{}^\circ F} = 2.326\ \mathrm{KJ/Kg\text{-}^\circ C}$$

A.2.5 Thermal Conductivity, k

The thermal conductivity is a measure of the rate of heat transfer through a material. It is the coefficient k in the Fourier Law of heat conduction

$$q = -kA\frac{\partial T}{\partial x} \qquad \text{(A-3)}$$

where q is the rate of heat transfer, A is the area normal to the temperature gradient $\partial T/\partial x$.

The SI unit of thermal conductivity is Watt per meter per Celsius degree, W/m-C. The Imperial unit of thermal conductivity is BTU/h-ft-°F. The conversion ratio is

$$1\ \mathrm{W/m\text{-}C} = 0.57782\ \mathrm{BTU/h\text{-}ft\text{-}^\circ F}.$$

A.2.6 Absolute Viscosity, μ

The absolute viscosity, μ, is a measure of the fluid resistance to flow. The viscosity and its units are presented in Chap. 2. The SI unit of absolute viscosity is $N\text{-}s/m^2$ (or Pa-s). An additional widely used unit is the poise, (P) (after Poiseuille), which is $dyne\text{-}s/cm^2$, and a smaller traditional unit is centipoise (cP).

$$1\ \text{centipoise, (cP)} = 10^{-2} \times \text{poise}$$

An Imperial unit for the viscosity is the reyn (after Osborne Reynolds), which is lbf-s/in.^{2}.

Conversions

1. 1 centipoise is equal to 1.45×10^{-7} reyn
2. 1 centipoise is equal to 0.001 N-s/m^2
3. 1 centipoise is equal to 0.01 poise
4. 1 reyn is equal to 6.895×10^3 N-s/m^2
5. 1 reyn is equal to 6.895×10^6 centipoise
6. 1 N-s/m^2 is equal to 10^3 centipoise
7. 1 N-s/m^2 is equal to 1.45×10^{-4} reyn

A.2.7 Kinematic Viscosity, ν

The kinematic viscosity, ν, is the ratio of the absolute viscosity and density

$$\nu = \frac{\mu}{\rho} \tag{A-4}$$

The SI unit of kinematic viscosity is m^2/s. Additional widely used traditional unit is the stokes (St) (after Stokes), which is cm^2/s, and a smaller unit is the centistokes (cSt), which is mm^2/s.

The common Imperial unit is in.^{2}/s.

Conversions

1 centistokes, cSt = 10^6 m^2/s
1 stokes, St = 10^4 m^2/s
1 m^2/s = 6.452×10^{-4} in.^{2}/s
1 m^2/s = 10^{-4} stokes
1 in.^{2}/s = 0.00155 cSt

Appendix B

Numerical Integration

The pressure wave along the bearing is solved by integration of Eq. 4-13. Although some integrals can be solved analytically, complex functions can be solved by numerical integration. This appendix is a survey of the various methods for numerical integration, and examples are presented. A simple numerical integration is demonstrated by means of a spreadsheet computer program, which is favored by engineers and students for its simplicity, and because the spreadsheet program can be used for graphic presentation of the pressure wave.

The methods of approximate numerical integration are based on a summation of small areas of width Δx below the curve, which are approximated by various methods that include the midpoint rule, rectangle rule, trapezoidal rule, and Simpson rule.

B.1 MIDPOINT RULE

Integration by midpoint rule is an approximation. The area below the curve is approximated by the sum of the rectangular areas, as shown in Fig. B-1.

The integral is approximated by the following equation:

$$\int_a^b f(x)dx \approx \frac{b-a}{n} \sum_{i=1}^{n} f(x_j)$$

$$x_j = \frac{x_{i-1} + x_i}{2}$$

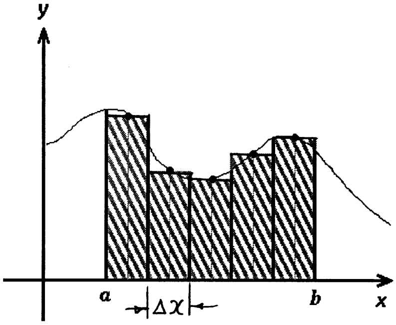

FIG. B-1 Integration by midpoint rule.

where a is the lower limit and b is upper limit of the integration, and n is the number of columns.

B.2 RECTANGLE RULE (FIG. B-2)

The integral is approximated by the following equation:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

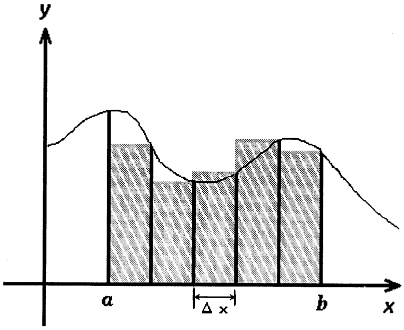

FIG. B-2 Rectangle rule.

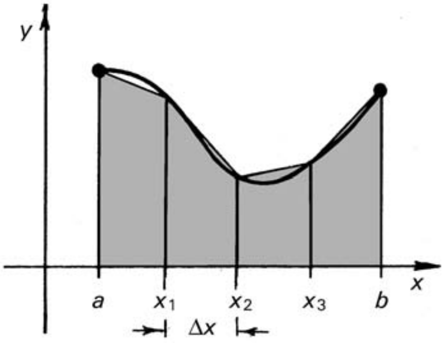

FIG. B-3 Integration by the trapezoidal rule.

B-3 TRAPEZOIDAL RULE (FIG. B-3)

The integral is approximated by the following equation:

$$\int_a^b f(x)dx \approx T$$

$$T = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots 2f(x_{i-1}) + f(x_i)]$$

$$\Delta x = \frac{b-a}{n}$$

The endpoints, at points *a* and *b*, are counted only once, while all the other points have the coefficient 2.

B-4 SIMPSON RULE (FIG. B-4)

The Simpson rule is based on approximating the graph by parabolas rather than straight lines. The parabola is determined each time by the three consecutive points through which it passes.

$$f(x_{i-1}), f(x_i) \textit{ and } f(x_{i+1})$$

The area from (x_{i-1}) to (x_{i+1}) is

$$A_i = \frac{\Delta x}{3}(f(x_{i-1}) + 4f(x_i) + f(x_{i+1}))$$

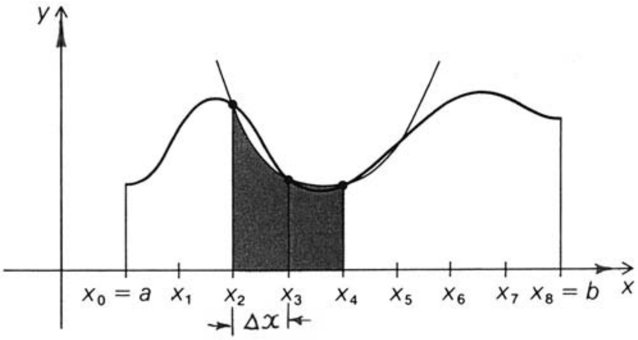

FIG. B-4 Integration by the Simpson rule.

If this procedure is repeated for every three adjacent points, the following Simpson rule for approximate integration is obtained:

$$\int_a^b f(x)dx \approx S$$

$$S = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) \ldots 2f(x_{i-2})$$
$$+ 4f(x_{i-1}) + f(x_i)]$$

$$\Delta x = \frac{b-a}{n}$$

$$x_{i-1} = a + (n-1)\Delta x$$

$$x_i = b$$

The endpoints, at points a and b, are counted only once, while all the others are counted according to the coefficients 4 and 2 in the Simpson rule. For the Simpson rule, the n must be an even number. The Simpson rule yields the best approximation.

Example Problem B-1

Numerical Integration Using a Spreadsheet Program

Integrate the function $f(x) = 3x^2$ in the boundaries $0 \le x \le 2$. Use the approximate rectangle rule, and solve the summation with the aid of a spreadsheet.

Compare with the trapezoidal and Simpson rules.

Solution

The concept of numerical integration is to section the area under the curve into a large number of rectangles, calculating the area of each individual rectangle, and

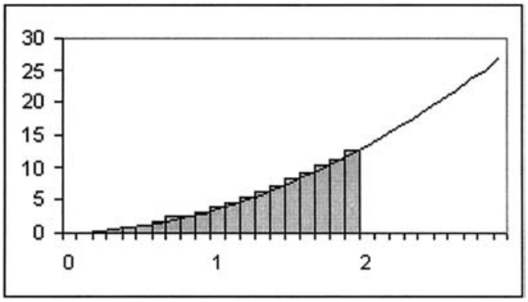

FIG. B-5 Approximate integration by summation of rectangles.

finally summing the rectangular areas to obtain the total area under the curve (see Fig. B-5).

The numerical integration is according to the equation

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(x_i)\Delta x_i$$

$$\Delta x_i = \frac{b-a}{n}$$

In this problem, the function is

$$f(x) = 3x^2$$

$$0 \le x \le 2$$

$$\int_0^2 3x^2 dx = \sum_{i=1}^{n} f(x_i)\Delta x_i$$

$$x_i = x_{i-1} + \Delta x$$

The summation is performed with the aid of a spreadsheet program (Table B-1). The first and second rows are added for explanation. The number of rectangles is selected ($n = 200$), resulting in uniform $\Delta x_i = 0.01$. The third column shows the values x_i, and the fourth column shows the respective values of the function $f(x_i)$. The fifth column lists the areas of the rectangles obtained by the product $f(x_i)\Delta x_i$. The sixth (last) column lists the sum of the rectangles to the last x_i. The solution of this numerical integration is at the bottom of this column. The exact solution of this integration is 8, and the errors of the various methods are compared in Table B-2. The best precision is obtained using the Simpson method.

TABLE B-1 Numerical integration with a spreadsheet program

Column

Row

Rectangular Method

Number	Rectangle Width	Distance from coordinate origin	Height of rectangle	Area of rectangle	Summation of rectangular Areas
Number	**Δx**	**x_i**	**$f(x) = 3x_i^2$**	**$f(x_i)(\Delta x)$**	**$\sum f(x_i)\,\Delta x$**
=A2 + 1	=B2	=C2 + B2	=3 * (C2^2)	=D2 * B2	=SUM( )
1	0.01	0.01	0.0003	0.000003	0.000003
2	0.01	0.02	0.0012	0.000012	0.000015
3	0.01	0.03	0.0027	0.000027	0.000042
4	0.01	0.04	0.0048	0.000048	0.00009
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---------------	---------------	---------------	---------------	---------------	---------------
199	0.01	1.99	11.8803	0.118803	7.9401
200	**0.01**	**2**	**12**	**0.12**	**8.0601**

Trapezoidal Method

Number	Rectangle Width	Distance from coordinate origin	Height of rectangle	Trapezoidal Coefficient C		Summation of rectangular areas
i	**Δx**	**X_i**	**$f(x) = 3X_i^2$**	**C**	**$C\,f(X_i)$**	**$(\Delta x / 2) \sum f(X_i)\,C$**
1	0.01	0.01	0.0003	1	0.0003	0.0000015
2	0.01	0.02	0.0012	2	0.0024	0.0000135
3	0.01	0.03	0.0027	2	0.0054	0.0000405
4	0.01	0.04	0.0048	2	0.0096	0.0000885
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------	------	------	------	------	------	------
199	0.01	1.99	11.8803	2	23.7606	7.9400985
200	**0.01**	**2**	**12**	**1**	**12**	**8.0000985**

Simpson Method

Number	Rectangle Width	Distance from coordinate origin	Height of rectangle	Simpson Coefficient C		Summation of rectangular areas
i	**Δx**	**X_i**	**$f(x) = 3X_i^2$**	**C**	**$C\,f(X_i)$**	**$(\Delta x / 3) \sum f(X_i)\,C$**
1	0.01	0.01	0.0003	1	0.0003	0.000001
2	0.01	0.02	0.0012	2	0.0024	0.000009
3	0.01	0.03	0.0027	4	0.0108	0.000045
4	0.01	0.04	0.0048	2	0.0096	0.000077
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199	0.01	1.99	11.8803	4	47.5212	7.959997
200	**0.01**	**2**	**12**	**1**	**12**	**7.999997**

TABLE B-2 Errors of Various Numerical Integration Methods

Method	Solution	Percent Error
Actual solution	8.000	0%
Rectangular method	**8.06010**	0.751%
Trapezoidal method	**8.0000985**	0.00123%
Simpson method	**7.999997**	0.0000375%