10

Hydrostatic Bearings

10.1 INTRODUCTION

The design concept for hydrostatic bearings is the generation of a high-pressure fluid film by using an external pump. The hydrostatic system of a journal bearing is shown in Fig. 1-4. The fluid is fed from a pump into several recesses around the bore of the bearing. From the recesses, the fluid flows out through a thin clearance, h_0, between the journal and bearing surfaces, at the *lands* outside the recesses. Previous literature on hydrostatic bearings includes Opitz (1967), Rowe (1989), Bassani and Picicigallo (1992), and Decker and Shapiro (1968).

The fluid film in the clearance separates the two surfaces of the journal and the bearing and thus reduces significantly the friction and wear. At the same time, the thin clearance at the land forms a resistance to the outlet flow from each recess. This flow resistance is essential for maintaining high pressure in the recess. The hydrodynamic load capacity that carries the external load is the resultant force of the pressure around the bearing.

There is also a fluid film in a hydrodynamic bearing. However, unlike the hydrodynamic bearing, where the pressure wave is generated by the hydrodynamic action of the rotation of the journal, hydrostatic bearing pressure is generated by an external pump.

There are certain designs of hydrodynamic bearings where the oil is also supplied under pressure from an external oil pump. However, the difference is that in hydrostatic bearings the design entails recesses, and the operation does not depend on the rotation of the journal for generating the pressure wave that supports the load. For example, the hydrodynamic journal bearing does not generate hydrodynamic pressure and load capacity when the journal and sleeve are stationary. In contrast, the hydrostatic bearing maintains pressure and load capacity when it is stationary; this characteristic is important for preventing wear during the bearing start-up. In fact most hydrostatic journal bearings are hybrid, in the sense that they combine hydrostatic and hydrodynamic action.

An important advantage of a hydrostatic bearing, in comparison to the hydrodynamic bearing, is that it maintains complete separation of the sliding surfaces at low velocities, including zero velocity. The hydrostatic bearing requires a high-pressure hydraulic system to pump and circulate the lubricant. The hydraulic system involves higher initial cost; in addition, there is an extra-long-term cost for the power to pump the fluid through the bearing clearances. Although hydrostatic bearings are more expensive, they have important advantages. For many applications, the extra expenses are justified, because these bearings have improved performance characteristics, and, in addition, the life of the machine is significantly extended. The following is a summary of the most important advantages of hydrostatic bearings in comparison to other bearings.

Advantages of Hydrostatic Bearings

1. The journal and bearing surfaces are completely separated by a fluid film at all times and over the complete range of speeds, including zero speed. Therefore, there is no wear due to direct contact between the surfaces during start-up. In addition, there is a very low sliding friction, particularly at low sliding speeds.
2. Hydrostatic bearings have high stiffness in comparison to hydrodynamic bearings. High stiffness is important for the reduction of journal radial and axial displacements. High stiffness is important in high-speed applications in order to minimize the level of vibrations. Also, high stiffness is essential for precise operation in machine tools and measurement machines. Unlike in hydrodynamic bearings, the high stiffness of hydrostatic bearings is maintained at low and high loads and at all speeds. This is a desirable characteristic for precision machines (Rowe, 1989) and high-speed machinery such as turbines.
3. Hydrostatic bearings operate with a thicker fluid film, which reduces the requirement for high-precision manufacture of the bearing (in comparison to hydrodynamic bearings). This means that it is possible to get more precision (lower journal run-out) in comparison to other bearings made with comparable manufacturing.
4. The continuous oil circulation prevents overheating of the bearing.
5. The oil pumped into the bearing passes through an oil filter and then through the bearing clearance. In this way, dust and other abrasive

particles are removed and do not damage the bearing surface. This is an important advantage in a dusty environment.

A hydrostatic bearing has one or several recesses where a uniformly high pressure is maintained by an external pump. Hydrostatic bearings are used for radial and thrust loads. Various types of recess geometry are used, such as circular or rectangular recesses in hydrostatic pads. The following is an example of a hydrostatic circular pad for a thrust load.

10.2 HYDROSTATIC CIRCULAR PADS

A circular pad is a hydrostatic thrust bearing that has a load-carrying capacity in the axial (vertical) direction, as shown in Fig. 10-1. This hydrostatic thrust bearing comprises two parallel concentric disks having a small clearance, h_0, between them. The radius of the disk is R, and the radius of the round recess is R_0. The bearing under a thrust load. Fluid is fed from an external pump into the round recess. The recess pressure is p_r above atmospheric pressure. The circular pad has load capacity W while maintaining complete separation between the surfaces. The external pump pressurizes the fluid in the recess, at a uniform pressure p_r. The clearance outside the recess, h_0, is thin relative to that of the recess. The thin clearance forms resistance to the outlet flow from the recess, in the radial direction. In this way, high pressure in the circular recess is continually maintained.

In the area of thin clearance, h_0 (often referred to as the *land*) the pressure reduces gradually in the radial direction, due to viscous friction. The uniform pressure in the recess combined with the radial pressure distribution in the land carry the external load. The recess pressure, p_r, increases with the flow rate, Q, that the pump feeds into the bearing. In turn, the thin clearance, h_0, adjusts its thickness to maintain the pressure and load capacity to counterbalance the thrust load. The clearance h_0 increases with the flow rate, Q, and the designer can control the desired clearance by adjusting the flow rate.

10.3 RADIAL PRESSURE DISTRIBUTION AND LOAD CAPACITY

Example Problem 10-1

Derive the equation for the radial pressure distribution, p, at the land and the load capacity, W, for a stationary circular pad under steady load as shown in Fig. 10-1. The fluid is incompressible, and the inlet flow rate, Q (equal to the flow in the radial direction), is constant. The clearance is h_0, and the radius of the circular pad and of the recess are R and R_0, respectively.

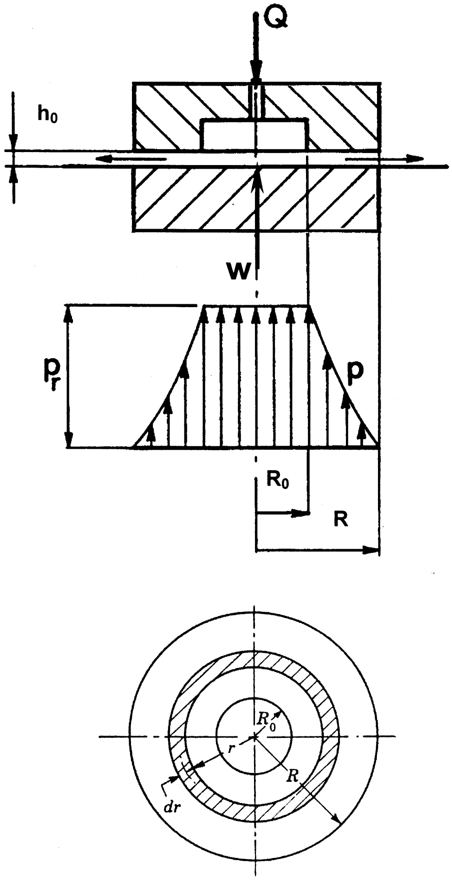


FIG. 10-1 Hydrostatic circular pad made up of two parallel disks and a round recess.

Solution

In Sec. 4.9, Example Problem 4-3 is solved for the pressure gradient in a parallel flow between two stationary parallel plates. The pressure gradient for a constant clearance, h_0, and rate of flow Q is

$$\frac{dp}{dx} = -\frac{12\mu}{bh_0^3} Q \tag{10-1}$$

The flow is in the direction of x, and b is the width (perpendicular to the flow direction). In a circular pad, there are radial flow lines, having radial symmetry.

The flow in the radial direction is considered between the round recess, at $r = R_0$, to the pad exit at $r = R$.

Pressure Distribution

In order to solve the present problem, the flow is considered in a thin ring, of radial thickness dr, as shown in Fig. 10-1. Since dr is small in comparison to the radius, r, the curvature can be disregarded. In that case, the flow along dr is assumed to be equal to a unidirectional flow between parallel plates of width $b = 2\pi r$. The pressure gradient dp is derived from Eq. (10-1):

$$\frac{dp}{dr} = -\frac{12\mu Q}{2\pi r h_0^3} \qquad \text{or} \qquad dp = \frac{12\mu Q}{2\pi h_0^3}\frac{dr}{r} \tag{10-2}$$

Integrating of Eq. (10-2) yields

$$p = -\frac{6\mu Q}{\pi h_0^3}\ln r + C \tag{10-3}$$

The constant of integration, C, is determined by the following boundary condition:

$$p = 0 \qquad \text{at} \qquad r = R \tag{10-4}$$

After solving for C, the expression for the radial pressure distribution in the radial clearance as a function of r is

$$p = \frac{6\mu Q}{\pi h_0^3}\ln\frac{R}{r} \tag{10-5}$$

The expression for the pressure at the recess, p_r, at $r = R_0$ is

$$p_r = \frac{6\mu Q}{\pi h_0^3}\ln\frac{R}{R_0} \tag{10-6}$$

Load Capacity

The load capacity is the integration of the pressure over the complete area according to the following equation:

$$W = \int_{(A)} p\,dA \tag{10-7}$$

The area dA of a ring thickness dr is (see Fig. 10-1)

$$dA = 2\pi r\,dr \tag{10-8}$$

The pressure in the recess, p_r, is constant, and the load capacity of the recess is derived by integration:

$$W = \int_0^{R_0} 2\pi r p_r dr \tag{10-9}$$

For the total load capacity, the pressure is integrated in the recess and in the thin clearance (land) according to the following equation:

$$W = \int_0^{R_0} p_r(2\pi r\, dr) + \int_{R_0}^{R} p(2\pi r dr) \tag{10-10}$$

After substituting the pressure equation into Eq. (10-10) and integrating, the following expression for the load capacity of a circular hydrostatic pad is obtained:

$$W = \frac{\pi}{2} \frac{R^2 - R_0^2}{\ln(R/R_0)} p_r \qquad \text{(10-11}$$

Equation (10-11) can be rearranged as a function of the recess ratio, R_0/R, and the expression for load capacity of a hydrostatic pad is

$$W = \frac{\pi R^2}{2} \frac{1 - (R_0/R)^2}{\ln(R/R_0)} p_r \tag{10-12}$$

The expression for the flow rate Q is obtained by rearranging Eq. (10-6) as follows:

$$Q = \frac{\pi}{6\mu} \frac{h_0^3}{\ln(R/R_0)} p_r \tag{10-13}$$

Equations (10-12) and (10-13) are for two stationary parallel disks. These equations can be extended to a hydrostatic pad where one disk is rotating, because according to the assumptions of classical lubrication theory, the centrifugal forces of the fluid due to rotation can be disregarded. In fact, the centrifugal forces are negligible (in comparison to the viscous forces) if the clearance h_0 is small and the viscosity is high (low Reynolds number). Equations (10-12) and (10-13) are used for the design of this thrust bearing. Whenever the fluid is supplied at constant flow rate Q, Eq. (10-13) is used to determine the flow rate. It is necessary to calculate the flow rate Q, which results in the desired clearance, h_0.

Unlike hydrodynamic bearings, the desired clearance, h_0, is based not only on the surface finish and vibrations, but also on the minimization of the power losses for pumping the fluid and for rotation of the pad. In general, hydrostatic bearings operate with larger clearances in comparison to their hydrodynamic counterparts. Larger clearance has the advantage that it does not require the high-precision machining that is needed in hydrodynamic bearings.

10.4 POWER LOSSES IN THE HYDROSTATIC PAD

The fluid flows through the clearance, h_0, in the radial direction, from the recess to the bearing exit. The pressure, p_r, is equal to the pressure loss due to flow resistance in the clearance (pressure loss p_r results from viscous friction loss in the thin clearance). In the clearance, the pressure has a negative slope in the radial directions, as shown in Fig. 10-1. The bearing is loaded by a thrust force in the vertical direction (direction of the centerline of the disks). Under steady conditions, the resultant load capacity, W, of the pressure distribution is equal to the external thrust load.

The upper disk rotates at angular speed ω, driven by an electrical motor, and power is required to overcome the viscous shear in the clearance. All bearings require power to overcome friction; however, hydrostatic bearings require extra power in order to circulate the fluid. An important task in hydrostatic bearing design is to minimize the power losses.

The following terms are introduced for the various components of power consumption in the hydrostatic bearing system.

$\dot{E}_h$—is the hydraulic power required to pump the fluid through the bearing and piping system. The flow resistance is in the clearance (land) and in the pipes. In certain designs there are flow restrictors at the inlet to each recess that increase the hydraulic power. The hydraulic power is dissipated as heat in the fluid.

$\dot{E}_f$—is the mechanical power provided by the drive (electrical motor) to overcome the friction torque resulting from viscous shear in the clearance due to relative rotation of the disks. This power is also dissipated as heat in the oil. This part of the power of viscous friction is present in hydrodynamic bearings without an external pump.

$\dot{E}_t$—is the total hydraulic power and mechanical power required to maintain the operation of the hydrostatic bearing, $\dot{E}_t = \dot{E}_h + \dot{E}_f$.

The mechanical torque of the motor, T_f, overcomes the viscous friction of rotation at angular speed ω. The equation for the motor-driving torque is (see Problem 2-1)

$$T_f = \frac{\pi}{2}\mu\frac{R^4}{h_0}\left(1 - \frac{R_0^4}{R^4}\right)\omega \tag{10-14}$$

The mechanical power of the motor, $\dot{E}_f$, that is required to overcome the friction losses in the pad clearance is

$$\dot{E}_f = T_f\omega = \frac{\pi}{2}\mu\frac{R^4}{h_0}\left(1 - \frac{R_0^4}{R^4}\right)\omega^2 \tag{10-15}$$

The power of a hydraulic pump, such as a gear pump, is discussed in Sec. (10.14). For hydrostatic pads where each recess is fed by a constant flow rate Q, there is no need for flow restrictors at the inlet to the recess. In that case, it is possible to simplify the calculations, since the pressure loss in the piping system is small and can be disregarded in comparison to the pressure loss in the bearing clearance:

$$\dot{E}_h \approx Qp_r \tag{10-16}$$

Here, Q is the flow rate through the pad and p_r is the recess pressure. The total power consumption $\dot{E}_t$ is the sum of the power of the drive for turning the bearing and the power of the pump for circulating the fluid through the bearing resistance. The following equation is for net power consumption. In fact, the pump and motor have power losses, and their efficiency should be considered for the calculation of the actual power consumption:

$$\dot{E}_{t(\mathrm{net})} = Qp_r + \frac{\pi}{2}\mu\frac{R^4}{h_0}\left(1 - \frac{R_0^4}{R^4}\right)\omega^2 \tag{10-17}$$

Substituting the value of Q and dividing by the efficiency of the motor and drive η_1 and the efficiency of the pump η_2, the following equation is obtained for the total power consumption (in the form of electricity consumed by the hydrostatic system) for the operation of a bearing:

$$\dot{E}_t = \frac{1}{\eta_2}\frac{1}{6}\frac{\pi h_0^3}{\mu\ln(R/R_0)}p_r^2 + \frac{1}{\eta_1}\frac{\pi}{2}\mu\frac{R^4}{h_0}\left(1 - \frac{R_0^4}{R^4}\right)\omega^2 \tag{10-18}$$

The efficiency η_2 of a gear pump is typically low, about 0.6–0.7. The motor-drive system has a higher efficiency η_1 of about 0.8–0.9.

10.5 OPTIMIZATION FOR MINIMUM POWER LOSS

The total power consumption, $\dot{E}_t$, is a function of the clearance h_0. In general, hydrostatic bearings operate with larger clearance in comparison to hydrodynamic bearings. In order to optimize the clearance for minimum power consumption, it is convenient to rewrite Eq. (10-18) as a function of the clearance while all the other terms are included in a constant coefficient in the following form:

$$\dot{E}_t = C_1 h_0^3 + \frac{C_2}{h_0} \tag{10-19}$$

where

$$C_1 = \frac{1}{6\eta_2}\frac{\pi}{\mu \ln(R/R_0)} p_r^2 \qquad \text{and} \qquad C_2 = \frac{1}{2\eta_1}\pi\mu R^4\left(1 - \frac{R_0^4}{R^4}\right)\omega^2 \tag{10-20}$$

Plotting the curve of power consumption versus clearance according to Eq. (10-19) allows optimization of the clearance, h_0, for minimum power loss in the bearing system. Hydrostatic bearings should be designed to operate at this optimal clearance.

Example Problem 10-2

Optimization of a Circular Hydrostatic Pad

A circular hydrostatic pad, as shown in Fig. 10-1, is supporting a load of $W = 1000$ N, and the upper disk has rotational speed of 5000 RPM. The disk diameter is 200 mm, and the diameter of the circular recess is 100 mm. The oil is SAE 10 at an operating temperature of 70°C, having a viscosity of $\mu = 0.01$ N-s/m^2. The efficiency of the hydraulic pump system is 0.6 and that of the motor and drive system is 0.9. Optimize the clearance, h_0, for minimum total power consumption.

Solution

The radius of the circular pad is $R = 100$ mm. The recess ratio is $R/R_0 = 2$. The angular speed is

$$N = 5000 \text{ RPM} \Rightarrow \omega\frac{2\pi N}{60} = \frac{2\pi 5000}{60} = 523.6 \text{ rad/s}$$

The first step is to find p_r by using the following load capacity equation:

$$W = R^2\left(\frac{\pi}{2}\frac{1 - R_0^2/R^2}{\ln(R/R_0)}\right)p_r$$

Substituting the known values, the following equation is obtained, with p_r as unknown:

$$1000 = 0.1^2\left(\frac{\pi}{2}\frac{1 - 0.5^2}{\ln 2}\right)p_r$$

Solving gives

$$p_r = 58{,}824 \text{ N/m}^2$$

Substituting p_r in Eq. (10-20), the constant C_1, which is associated with the pump, is solved for:

$$C_1 = \frac{1}{0.6} \times \frac{1}{6} \times \frac{\pi}{0.01 \ln(1/0.5)} (58{,}824)^2 = 4.35 \times 10^{11} N/\text{s-m}^2$$

In a similar way, the second constant, C_2, which is associated with the motor, is calculated from Eq. (10-20):

$$C_2 = \frac{1}{0.9} \frac{\pi}{2} (0.01)(0.1^4)(1 - 0.5^4) 523.6^2 = 0.448 \text{ N-m}^2/\text{s}$$

Substituting these values of C_1 and C_2 into Eq. (10-19), the total power as a function of the clearance becomes

$$\dot{E}_t = (4.35 \times 10^{11}) h_0^3 + \frac{0.448}{h_0}$$

In this equation, the power of the pump for circulating the fluid is the first term, which is proportional to h_0^3, while the second term, which is proportional to h_0^{-1}, is the power of the motor for rotating the disk.

The powers of the pump and of the drive are the two power components required to maintain the operation of the hydrostatic bearing. In Fig. 10-2, the curves of the hydraulic power and mechanical power are plotted as a function of the clearance, h_0. The curve of the mechanical power, $\dot{E}_f$, that is provided by the motor points down with increasing clearance, h_0. The power, $\dot{E}_f$, is for rotating one disk relative to the other and overcoming the viscous friction in the clearance between the two disks. The second curve is of the hydraulic power, $\dot{E}_h$, which is provided by the pump to maintain hydrostatic pressure in the recess. This power is rising with increasing clearance, h_0. The hydraulic power, $\dot{E}_h$, is for overcoming the flow resistance in the thin clearance at the outlet from the recess. The total power, $\dot{E}_t$, is the sum of these curves. For the hydrostatic pad in this problem, the optimal point (minimum power) is for a clearance of about 0.75 mm, and the total power consumption of the bearing in this problem is below 0.8 kW.

The result of 0.8 kW is too high for a hydrostatic bearing. It is possible to reduce the power consumption by using lower-viscosity oil.

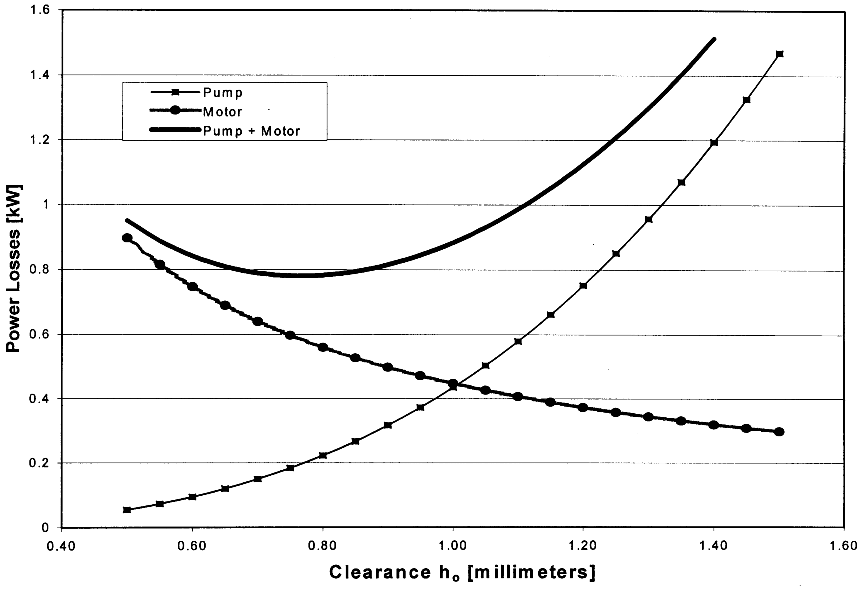


FIG. 10-2 Optimization of clearance for minimum power consumption.

10.6 LONG RECTANGULAR HYDROSTATIC BEARINGS

There are several applications where a long rectangular hydrostatic pad is used. An important example is the hydrostatic slideway in machine tools, as well as other machines where slideways are applied.

A long rectangular pad is shown in Fig. 10-3. The pad, as well as the recess, is long in comparison to its width, $L \gg B$ and $l \gg b$. Therefore, the flow through the clearance of width b is negligible relative to that through length l. The flow is considered one dimensional, because it is mostly in the x direction, while the flow in the y direction is negligible. For solving the pressure distribution, unidirectional flow can be assumed.

The pressure in the recess is constant, and it is linearly decreasing along the land of clearance, h_0, in the x direction. Integration of the pressure distribution results in the fluid load capacity W. At the same time, the flow rate in the two directions of the two sides of the recess is a flow between two parallel plates according to Eq. (10-1). The pressure gradient is linear, $dp/dx =$ constant. The total flow rate Q in the two directions is

$$Q = 2\frac{h_0^3}{12}\frac{l}{\mu}\frac{dp}{dx} = 2\frac{h_0^3}{12}\frac{l}{\mu}\frac{p_r}{(B-b)/2} = \frac{l\,h_0^3}{3\mu(B-b)}p_r \qquad (10\text{-}21)$$

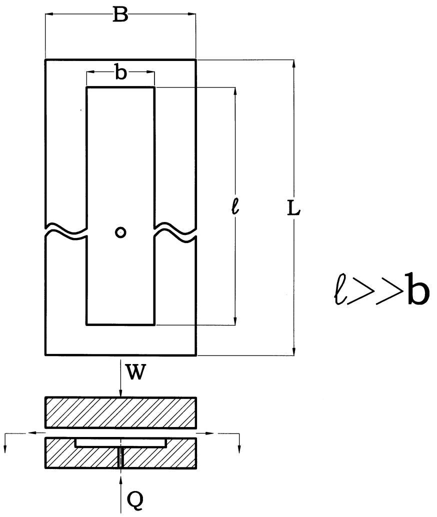


FIG. 10-3 Long rectangular pad.

Here, $B - b$ is the clearance (land) dimension along the direction of flow and l is the dimension of clearance (land) normal to the flow direction.

10.7 MULTIDIRECTIONAL HYDROSTATIC SUPPORT

Slideways are used in machinery for accurate linear motion. Pressurized fluid is fed into several recesses located at all contact surfaces, to prevent any direct metal contact between the sliding surfaces. Engineers already recognized that friction has an adverse effect on precision, and it is important to minimize friction by providing hydrostatic pads.

In machine tools, multirecess hydrostatic bearings are used for supporting the slideways as well as the rotating spindle. Hydrostatic slideways make the positioning of the table much more accurate, because it reduces friction that limits the precision of sliding motion.

A slideway supported by constant-flow-rate pads is shown in Fig. 10-4. Whenever there is a requirement for high stiffness in the vertical direction, the preferred design is of at least two hydrostatic pads with vertical load capacity in two opposite directions. Bidirectional support is also necessary when the load is changing its direction during operation. Additional pads are often included with horizontal load capacity in opposite directions for preventing any possible direct contact.

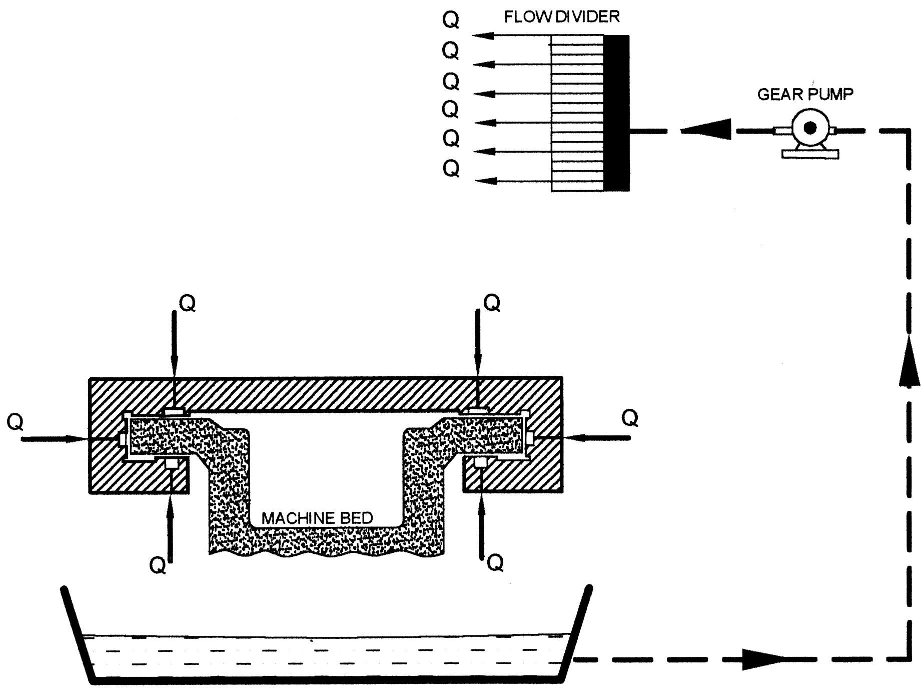


FIG. 10-4 Slideway supported by constant-flow-rate pads.

In a multipad support, one of the following two methods for feeding the oil into each recess is used.

1. Constant-flow-rate system, where each recess is fed by a constant flow rate Q.
2. Constant pressure supply, where each recess is fed by a constant pressure supply p_s. The oil flows into each recess through a flow restrictor (such as a capillary tube). The flow restrictor causes a pressure drop, and the recess pressure is reduced to a lower level, $p_r < p_s$. The flow restrictor makes the bearing stiff to displacement due to variable load.

In the case of the constant-flow-rate system, the fluid is fed from a pump to a flow divider that divides the flow rate between the various recesses. The flow divider is essential for the operation because it ensures that the flow will be evenly distributed to each recess and not fed only into the recesses having the least resistance.

High stiffness is obtained whenever each pad is fed by a constant flow rate Q. The explanation for the high stiffness lies in the relation between the clearance and recess pressure. For a bearing with given geometry, the constant flow rate Q is proportional to

$$Q \propto \left(\frac{h_0^3}{\mu} p_r\right) \tag{10-22}$$

A vertical displacement, Δh, of the slide will increase and decrease the clearance h_0 at the lands of the opposing hydrostatic pads. For constant flow rate Q and viscosity, Eq. (10-22) indicates that increase and decrease in the clearance h_0 would result in decrease and increase, respectively, of the recess pressure (the recess pressure is inversely proportional to h_0^3). High stiffness means that only a very small vertical displacement of the slide is sufficient to generate a large difference of pressure between opposing recesses. The force resulting from these pressure differences acts in the direction opposite to any occasional additional load on the thrust bearing.

Theoretically, the bearing stiffness can be very high for a hydrostatic pad with a constant flow rate to each recess; but in practice, the stiffness is limited by the hydraulic power of the motor and its maximum flow rate and pressure. This theoretical explanation is limited in practice because there is a maximum limit to the recess pressure, p_r. The hydraulic power of the pump and the strength of the complete system limit the recess pressure. A safety relief valve is installed to protect the system from exceeding its allowable maximum pressure. In addition, the fluid viscosity, μ, is not completely constant. When the clearance, h_0, reduces, the viscous friction increases and the temperature rises. In turn, the viscosity is lower in comparison to the opposing side, where the clearance, h_0, increases.

10.8 HYDROSTATIC PAD STIFFNESS FOR CONSTANT FLOW RATE

In this system, each recess is fed by a constant flow rate, Q. This system is also referred to as *direct supply system*. For this purpose, each recess is fed from a separate positive-displacement pump of constant flow rate. Another possibility, which is preferred where there are many hydrostatic pads, is to use a *flow divider*. A flow divider is designed to divide the constant flow rate received from one pump into several constant flow rates that are distributed to several recesses. Each recess is fed by constant flow rate directly from the divider. (The design of a flow divider is discussed in this chapter.) The advantage of using flow dividers is that only one pump is used. If properly designed, the constant-flow-rate system would result in high stiffness.

The advantage of this system, in comparison to the constant pressure supply with restrictors, is that there are lower viscous friction losses. In the flow restrictors there is considerable resistance to the flow (pressure loss), resulting in high power losses. In turn, the system with flow restrictors requires a pump and motor of higher power. However, the flow divider is an additional component, which also increases the initial cost of the system.

An example of a constant-flow-rate system is the machine tool slideway shown in Fig. 10-4. The areas of the two opposing recesses, in the vertical direction, are not equal. The purpose of the larger recess area is to support the weight of the slide, while the small pad recess is for ensuring noncontact sliding and adequate stiffness.

10.8.1 Constant-Flow-Rate Pad Stiffness

The bearing stiffness, k, is the rate of rise of the load capacity, W, as a function of incremental reduction of the clearance, h_0, by a small increment dh_0. It is equivalent to the rise of the load capacity with a small downward vertical displacement dh_0 of the upper surface in Fig. 10-1, resulting in lower clearance. The bearing stiffness is similar to a spring constant:

$$k = -\frac{dW}{dh_0} \tag{10-23}$$

The meaning of the negative sign is that the load increases with a reduction of the clearance. High stiffness is particularly important in machine tools where any displacement of the slide or spindle during machining would result in machining errors. The advantage of the high-stiffness bearing is that it supports any additional load with minimal displacement.

For the computation of the stiffness with a constant flow rate, it is convenient to define the bearing clearance resistance, R_c, at the land (resistance to flow through the bearing clearance) and the effective bearing area, A_e. The flow resistance to flow through the bearing clearance, R_c, is defined as

$$Q = \frac{p_r}{R_c} \qquad \text{or} \qquad R_c = \frac{p_r}{Q} \tag{10-24}$$

The effective bearing area, A_e, is defined by the relation

$$A_e p_r = W \qquad \text{or} \qquad A_e = \frac{W}{p_r} \tag{10-25}$$

For a constant flow rate, the load capacity, in terms of the effective area and bearing resistance, is

$$W = A_e R_c Q \tag{10-26}$$

Comparison with the equations for the circular pad indicates that the resistance is proportional to h_0^{-3} or

$$R_c = \kappa h_0^{-3}; \qquad \text{and} \qquad W = \kappa A_e Q \frac{1}{h_0^3} \tag{10-27}$$

Here, κ is a constant that depends on bearing geometry, flow rate, and fluid viscosity

$$\text{Stiffness } k = -\frac{dW}{dh_0} = K \frac{1}{h_0^4} \qquad \text{where } K = 3\kappa A_e Q \tag{10-28a}$$

$$\text{Stiffness } k = -\frac{dW}{dh_0} = 3\kappa A_e Q \frac{1}{h_0^4} \tag{10-28b}$$

Equation (10-28b) indicates that stiffness increases very fast with reduction in the bearing clearance. This equation can be applied as long as the flow rate Q to the recess is constant. As discussed earlier, deviation from this can occur in practice if the pressure limit is reached and the relief valve of the hydraulic system is opened. In that case, the flow rate is no longer constant.

Equation (10-28a) can be used for any hydrostatic bearing, after the value of K is determined. For a circular pad:

$$K = 9\mu Q (R^2 - R_0^2) \tag{10-29}$$

The expression for the stiffness of a circular pad becomes

$$k = \frac{9\mu Q(R^2 - R_0^2)}{h_0^4} \tag{10-30}$$

Whenever there are two hydrostatic pads in series (bidirectional hydrostatic support), the stiffnesses of the two pads are added for the total stiffness.

Example Problem 10-3

Stiffness of a Constant Flow Rate Pad

A circular hydrostatic pad, as shown in Fig. 10-1, has a constant flow rate Q. The circular pad is supporting a load of $W = 5000$ N. The outside disk diameter is 200 mm, and the diameter of the circular recess is 100 mm. The oil viscosity is $\mu = 0.005$ N-s/m^2. The pad is operating with a clearance of 120 μm.

a. Find the recess pressure, p_r.
b. Calculate the constant flow rate Q of the oil through the bearing to maintain the clearance.
c. Find the effective area of this pad.
d. Find the stiffness of the circular pad operating under the conditions in this problem.

Solution

Given:

$$W = 5000 \text{ N}$$
$$R = 0.1 \text{ m}$$
$$R_0 = 0.05 \text{ m}$$
$$\mu = 0.005 \text{ N-s/m}^2$$
$$h_0 = 120\ \mu\text{m}$$

a. Recess Pressure

In order to solve for the flow rate, the first step is to determine the recess pressure. The recess pressure is calculated from Eq. (10-12) for the load capacity:

$$W = R^2\left(\frac{\pi}{2}\frac{1 - R_0^2/R^2}{\ln(R/R_0)}\right)p_r$$

After substitution, the recess pressure is an unknown in the following equation:

$$5000 = 0.1^2 \left(\frac{\pi}{2} \times \frac{1 - 0.25}{\ln(2)} \right) p_r$$

Solving for the recess pressure p_r yields:

$$p_r = 294.12 \text{ kPa}$$

b. Flow Rate

The flow rate Q can now be determined from the recess pressure. It is derived from Eq. (10-13):

$$Q = \frac{\pi}{6\mu} \frac{h_0^3}{\ln(R/R_0)} p_r; \qquad Q = \left(\frac{1}{6} \times \frac{\pi (120 \times 10^{-6})^3}{0.005 \times \ln(0.1/0.05)} \times 294{,}120 \right)$$

The result for the flow rate is

$$Q = 76.8 \times 10^{-6} \text{ m}^3/\text{s}$$

c. Pad Effective Area

The effective area is defined by

$$W = A_e p_r$$

Solving for A_e as the ratio of the load and the recess pressure, we get

$$A_e = \frac{5000}{294{,}120}$$

$$A_e = 0.017 \text{ m}^2$$

d. Bearing Stiffness

Finally, the stiffness of the circular pad fed by a constant flow rate can be determined from Eq. (10-30):

$$k = \frac{9\mu Q (R^2 - R_0^2)}{h_0^4}$$

Substituting the values in this stiffness equation yields

$$k = \frac{9 \times 0.005 \times 76.8 \times 10^{-6} \times (0.1^2 - 0.05^2)}{(120 \times 10^{-6})^4}$$

$$k = 125 \times 10^6 \text{ N/m}$$

This result indicates that the stiffness of a constant-flow-rate pad is quite high. This stiffness is high in comparison to other bearings, such as hydrodynamic bearings and rolling-element bearings. This fact is important for designers of machine tools and high-speed machinery. The high stiffness is not obvious. The bearing is supported by a fluid film, and in many cases this bearing is not selected because it is mistakenly perceived as having low stiffness.

Example Problem 10-4

Bidirectional Hydrostatic Pads

We have a machine tool with four hydrostatic bearings, each consisting of two bidirectional circular pads that support a slider plate. Each recess is fed by a constant flow rate, Q, by means of a flow divider. Each bidirectional bearing is as shown in Fig. 10-4 (of circular pads). The weight of the slider is 20,000 N, divided evenly on the four bearings (5000-N load on each bidirectional bearing). The total manufactured clearance of the two bidirectional pads is $(h_1 + h_2) = 0.4$ mm. Each circular pad is of 100-mm diameter and recess diameter of 50 mm. The oil viscosity is 0.01 N-s/m^2.

In order to minimize vertical displacement under load, the slider plate is prestressed. The pads are designed to have 5000 N reaction from the top, and the reaction from the bottom is 10,000 N (equivalent to the top pad reaction plus weight).

a. Find the flow rates Q_1 and Q_2 in order that the top and bottom clearances will be equal, $(h_1 = h_2)$.
b. Given that the same flow rate applies to the bottom and top pads, $Q_1 = Q_2$, find the magnitude of the two clearances, h_1 and h_2. What is the equal flow rate, Q, into the two pads?
c. For the first case of equal clearances, find the stiffness of each bidirectional bearing.
d. For the first case of equal clearances, if an extra vertical load of 120 N is placed on the slider (30 N on each pad), find the downward vertical displacement of the slider.

Solution

a. Flow rates Q_1 and Q_2

Given that $h_1 = h_2 = 0.2$ mm, the flow rate Q can be obtained via Eq. (10-13):

$$Q = \frac{1}{6}\frac{\pi h_0^3}{\mu \ln(R/R_0)} p_r$$

The load capacity of a circular hydrostatic pad is obtained from Eq. (10-12):

$$W = \frac{\pi R^2}{2} \frac{1 - (R_0/R)^2}{\ln(R/R_0)} p_r$$

The first step is to find p_r by using the load capacity equation (for a top pad). Substituting the known values, the recess pressure is the only unknown:

$$5000 = 0.05^2 \frac{\pi}{2} \frac{1 - (0.025^2/0.05^2)}{\ln(0.05/0.025)} p_{r1}$$

The result for the recess pressure at the upper pad is

$$p_{r1} = 1.176 \times 10^6 \text{ Pa}$$

Substituting this recess pressure in Eq. (10-13), the following flow rate, Q_1, is obtained:

$$Q_1 = \frac{1}{6} \frac{\pi\, 0.0002^3}{0.01 \times \ln(0.05/0.025)} \times 1.176 \times 10^6 = 7.1 \times 10^{-4} \text{ m}^3/\text{s}$$

The second step is to find p_{r2} by using the load capacity equation (for the bottom pad), substituting the known values; the following equation is obtained, with P_{r2} as unknown:

$$10{,}000 = 0.05^2 \frac{\pi}{2} \frac{1 - (0.025^2/0.05^2)}{\ln(0.05/0.025)} p_{r2}$$

The recess pressure at the lower pad is

$$p_{r2} = 2.352 \times 10^6 \text{ Pa}$$

Substituting the known values, in Eq. (10-13) the flow rate Q_2 is:

$$Q_2 = \frac{1}{6} \frac{\pi(0.0002^3)}{0.01 \ln(0.05/0.025)} \times 2.352 \times 10^6 = 14.2 \times 10^{-4} \text{ m}^3/\text{s}$$

b. Upper and Lower Clearances h_1 and h_2, for $Q_1 = Q_2$

The flow rate equation (10-13) is

$$Q = \frac{\pi}{6\mu} \frac{h_0^3}{\ln(R/R_0)} p_r$$

For $Q_1 = Q_2$ the following two equations with two unknowns, h_1 and h_2, are obtained:

$$Q = \frac{\pi}{6\mu} \frac{h_1^3}{\ln R/R_0)} p_{r1} = \frac{\pi}{6\mu} \frac{h_2^3}{\ln(R/R_0)} p_{r2}$$

$$h_1 + h_2 = 0.0004 \text{ m}$$

Substituting yields

$$\frac{1}{6} \frac{\pi h_1^3}{0.01 \ln 2} 1.176 \times 10^6 = \frac{1}{6} \frac{\pi(0.0004 - h_1)^3}{0.01 \ln 2} 2.352 \times 10^6$$

The equation can be simplified to the following:

$$1.176 \times h_1^3 = 2.352 \times (0.0004 - h_1)^3$$

Converting to millimeters, the solution for h_1 and h_2 is

$$h_1 = 0.223 \text{ mm} \qquad \text{and} \qquad h_2 = 0.177 \text{ mm}$$

c. Stiffness of Each Pad

Equation (10-30) yields the stiffness of a constant-flow rate circular pad:

$$k = \frac{9\mu Q(R^2 - R_0^2)}{h_0^4}$$

Substitute in Eq. (10-30) (for the top pad):

$$k \text{ (top pad)} = \frac{9\,(0.01)(7.07 \times 10^{-4})(0.05^2 - 0.025^2)}{0.0002^4} = 74.56 \times 10^6 \text{ N/m}$$

Substitute in Eq. (10-30) (for the bottom pad):

$$k \text{ (lower pad)} = \frac{9\,(0.01)(0.00142((0.05^2 - 0.025^2)}{0.0002} = 149.76 \times 10^6 \text{ N/m}$$

The total bidirectional bearing stiffness is obtained by adding the top and bottom stiffnesses, as follows:

$$k \text{ (bearing)} = k \text{ (top pad)} + k \text{ (lower pad)} = 224.32 \times 10^6 \text{ N/m}$$

d. Vertical Downward Displacement Δh of the Slider

$$k = \frac{\Delta W}{\Delta h} \qquad \text{where } \Delta W = 30 \text{ N}$$

$$\Delta h = \frac{\Delta W}{k}$$

$$\Delta h = \frac{30 \text{ N}}{224.32 \times 10^6} = 1.33 \times 10^{-7} \text{ m} = 0.133 \text{ µm}$$

This example shows that under extra force, the displacement is very small.

10.9 CONSTANT-PRESSURE-SUPPLY PADS WITH RESTRICTORS

Hydrostatic pads with a constant flow rate have the desirable characteristic of high stiffness, which is important in machine tools as well as many other applications. However, it is not always practical to supply a constant flow rate to each of the many recesses, because each recess must be fed from a separate positive-displacement pump or from a flow divider. For example, in designs involving many recesses, such as machine tool spindles, a constant flow rate to each of the many recesses requires an expensive hydraulic system that may not be practical.

An alternative arrangement is to use only one pump that supplies a constant pressure to all the recesses. This system is simpler, because it does not require many pumps or flow dividers. Unlike in the constant-flow-rate system, in this system each recess is fed from a *constant supply pressure*, p_s. The oil flows into each recess through a flow restrictor (such as a capillary tube). The flow restrictor causes a pressure drop, and the recess pressure is reduced to a lower level, p_r. The important feature of the flow restrictor is that it is making the bearing stiff to displacement under variable load.

Although hydraulic pumps are usually of the positive-displacement type, such as a gear pump or a piston pump, and have a constant flow rate, the system can be converted to a constant pressure supply by installing a relief valve that returns the surplus flow into the oil sump. The relief valve makes the system one of constant pressure supply. The preferred arrangement is to have an adjustable relief valve so that the supply pressure, p_s, can be adjusted for optimizing the bearing performance. In order to have the desired high bearing stiffness, constant-pressure-supply systems operate with flow restrictors at the inlet to each recess.

10.9.1 Flow Restrictors and Bearing Stiffness

A system of bidirectional hydrostatic pads with a constant pressure supply is presented in Fig. 10-5. The oil flows from a pump, through a flow restrictor, and into each recess on the two sides of this thrust bearing. From the recesses, the fluid flows out, in the radial direction, through the thin clearances, h_1 and h_2 along the lands (outside the recesses). This thin clearance forms a resistance to the outlet flow from each recess. This resistance at the outlet is subject to variations resulting from any small vertical displacement of the slider due to load variations. The purpose of feeding the fluid to the recesses through flow restrictors is to make the bearing stiffer under thrust force; namely, it reduces vertical displacement of the slider when extra load is applied.

When the vertical load on the slider rises, the slider is displaced downward in the vertical direction, and under constant pressure supply a very small displacement results in a considerable reaction force to compensate for the load rise. After a small vertical displacement of the slider, the clearances at the lands of the opposing pads are no longer equal. In turn, the resistances to the outlet flow from the opposing recesses decrease and increase, respectively. It results in unequal flow rates in the opposing recesses. The flow increases and decreases, respectively (the flow is inversely proportional to h_0^3). An important

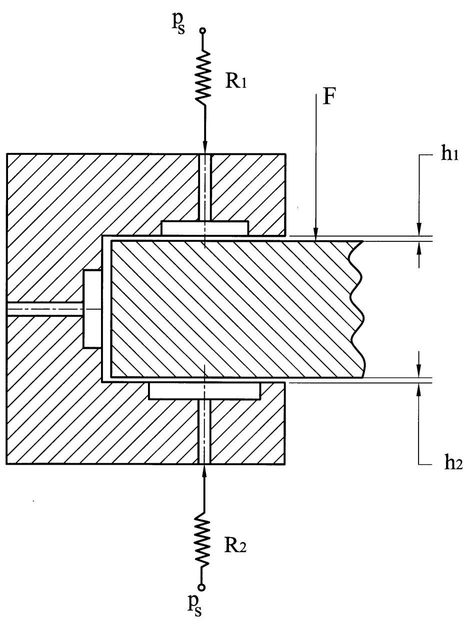


FIG. 10-5 Bidirectional hydrostatic pads with flow restrictors.

characteristic of a flow restrictor, such as a capillary tube, is that its pressure drop increases with the flow rate. In turn, this causes the pressures in the opposing recesses to decrease and increase, respectively. The bearing load capacity resulting from these pressure differences acts in the direction opposite to the vertical load on the slider. In this way, the bearing supports the slider with minimal vertical displacement, Δh. In conclusion, the introduction of inlet flow restrictors increases the bearing stiffness, because only a very small vertical displacement of the slider is sufficient to generate a large difference of pressure between opposing recesses.

10.10 ANALYSIS OF STIFFNESS FOR A CONSTANT PRESSURE SUPPLY

Where the fluid is fed to each recess through a flow restrictor, the fluid in the recesses is bounded between the inlet and outlet flow resistance. The following equations are for derivation of the expression for the stiffness of one hydrostatic pad with a constant pressure supply.

In general, flow resistance causes a pressure drop. Flow resistance R_f is defined as the ratio of pressure loss, Δp (along the resistance), to the flow rate, Q. Flow resistance is defined, similar to Ohm's law in electricity, as

$$R_f = \frac{\Delta p}{Q} \tag{10-31}$$

For a given resistance, the flow rate is determined by the pressure difference:

$$Q = \frac{\Delta p}{R_f} \tag{10-32}$$

The resistance of the inlet flow restrictor is R_{in}, and the resistance to outlet flow through the bearing clearance is R_c (resistance at the clearance). The pressure at the recess, p_r, is bounded between the inlet and outlet resistances; see a schematic representation in Fig. 10-6. The supply pressure, p_s, is constant; therefore, any change in the inlet or outlet resistance would affect the recess pressure.

From Eq. (10-32), the flow rate into the recess is

$$Q = \frac{p_s - p_r}{R_{in}} \tag{10-33}$$

The flow rate through the clearance resistance is given by

$$Q = \frac{p_r}{R_c} \tag{10-34}$$

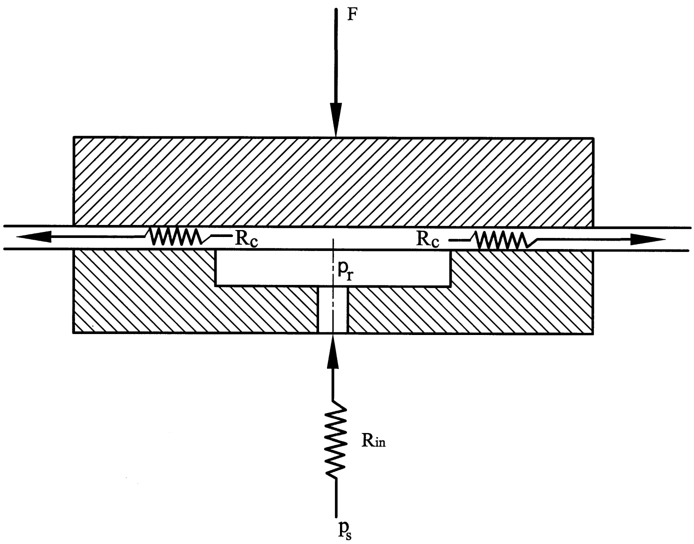


FIG. 10-6 Recess pressure bounded between inlet and outlet resistances.

The fluid is incompressible, and the flow rate Q is equal through the inlet resistance and clearance resistance (continuity). Equating the preceding two flow rate expressions yields

$$\frac{p_r}{R_c} = \frac{p_s - p_r}{R_{\text{in}}} \tag{10-35}$$

The recess pressure is solved for as a function of the supply pressure and resistances:

$$p_r = \frac{1}{(1 + R_{\text{in}}/R_c)} p_s \tag{10-36}$$

The load capacity is [see Eq. (10-25)]

$$W = A_e p_r \tag{10-37}$$

In terms of the supply pressure and the effective pad area, the load capacity is

$$W = A_e \frac{1}{(1 + R_{\text{in}}/R_c)} p_s \tag{10-38}$$

The inlet resistance of laminar flow through a capillary tube is constant, and the pressure drop is proportional to the flow rate. However, the flow resistance through the variable pad clearance is proportional to h_0^{-3} see Eq. (10-27). The clearance resistance can be written as

$$R_c = \kappa h_0^{-3} \tag{10-39}$$

Here, κ is a constant that depends on the pad geometry and fluid viscosity. Equation (10-38) can be written in the form,

$$W = A_e \frac{1}{(1 + K_1 h_0^3)} p_s \tag{10-40}$$

where K is defined as

$$K_1 = \frac{R_{\text{in}}}{\kappa} \tag{10-41}$$

In Eq. (10-40) for the load capacity, all the terms are constant except the clearance thickness. Let us recall that the expression for the stiffness is

$$\text{Stiffness } k = -\frac{dW}{dh_0} \tag{10-42}$$

Differentiating Eq. (10-40) for the load capacity W by h_0 results in

$$\text{Stiffness } k = -\frac{dW}{dh_0} = A_e \frac{3K_1 h_0^2}{(1 + K_1 h_0^3)^2} p_s \tag{10-43}$$

Equation (10-43) is for the stiffness of a hydrostatic pad having a constant supply pressure p_s. If the inlet flow is through a capillary tube, the pressure drop is

$$\Delta p = \frac{64\mu l_c}{\pi d_i^4} Q \tag{10-44}$$

Here, d_i is the inside diameter of the tube and l_c is the tube length. The inlet resistance by a capillary tube is

$$R_{\text{in}} = \frac{64\mu l_c}{\pi d_i^4} \tag{10-45}$$

For calculating the pad stiffness in Eq. (10-43), the inlet resistance is calculated from Eq. (10-45), and the value of κ is determined from the pad equations.

Equation (10-43) can be simplified by writing it as a function of the ratio of the recess pressure to the supply pressure, β, which is defined as

$$\beta = \frac{p_r}{p_s} \tag{10-46}$$

Equations (10-43) and (10-46) yield a simplified expression for the stiffness as a function of β:

$$k = -\frac{dW}{dh_0} = \frac{3}{h_0} A_e(\beta - \beta^2) p_s \tag{10-47}$$

Equation (10-47) indicates that the maximum stiffness is when $\beta = 0.5$, or

$$\frac{p_r}{p_s} = 0.5 \tag{10-48}$$

For maximum stiffness, the supply pressure should be twice the recess pressure. This can be obtained if the inlet resistance were equal to the recess resistance. This requirement will double the power of the pump that is required to overcome viscous friction losses. The conclusion is that the requirement for high stiffness in constant-supply-pressure systems would considerably increase the friction losses and the cost of power for operating the hydrostatic bearings.

Example Problem 10-5

Stiffness of a Circular Pad with Constant Supply Pressure

A circular hydrostatic pad as shown in Fig. 10-1 has a constant supply pressure, p_s. The circular pad is supporting a load of $W = 5000$ N. The outside disk diameter is 200 mm, and the diameter of the circular recess is 100 mm. The oil viscosity is $\mu = 0.005$ N-s/m^2. The pad is operating with a clearance of 120 μm.

a. Find the recess pressure, p_r.
b. Calculate the flow rate Q of the oil through the bearing to maintain the clearance.
c. Find the effective area of the pad.
d. If the supply pressure is twice the recess pressure, $p_s = 2p_r$, find the stiffness of the circular pad.
e. Compare with the stiffness obtained in Example Problem 10-3 for a constant flow rate.
f. Find the hydraulic power required for circulating the oil through the bearing. Compare to the hydraulic power in a constant-flow-rate pad.

Solution

a. Recess Pressure

Similar to Example Problem 10-3 for calculating the flow rate Q, the first step is to solve for the recess pressure. This pressure is derived from the equation of the load:

$$W = R^2\left(\frac{\pi}{2}\frac{1 - R_0^2/R^2}{\ln(R/R_0)}\right)p_r$$

After substitution, the recess pressure is only unknown in the following equation:

$$5000 = 0.1^2\left(\frac{\pi}{2} \times \frac{1 - 0.25}{\ln(2)}\right)p_r$$

Solving for the recess pressure, p_r yields

$$p_r = 294.18 \text{ kPa}$$

b. Flow Rate

The flow rate Q can now be determined. It is derived from the following expression [see Eq. (10-13)] for Q as a function of the clearance pressure:

$$Q = \frac{\pi}{6\mu}\frac{h_0^3}{\ln(R/R_0)}p_r$$

Similar to Example Problem 10-3, after substituting the values, the flow rate is

$$Q = 76.8 \times 10^{-6} \text{ m}^3/\text{s}$$

c. Pad Effective Area

The effective area is defined by

$$W = A_e p_r$$

Solving for A_e as the ratio of the load and the recess pressure, we get

$$A_e = \frac{5000}{294.180} A_e = 0.017 \text{ m}^2$$

d. Pad Stiffness

Supply Pressure: Now the supply pressure can be solved for as well as the stiffness for constant supply pressure:

$$p_s = 2p_r$$
$$= 2 \times 294.18 \text{ kPa}$$
$$p_s = 588.36 \text{ kPa}$$

Pad Stiffness of Constant Pressure Supply: The stiffness is calculated according to Eq. (10-47):

$$k = \frac{3}{h_0} A_e(\beta - \beta^2)p_s \quad \text{where } \beta = 0.5$$

$$k = \frac{3}{120 \times 10^{-6}} \times 0.017(0.5 - 0.5^2)588.36 \times 10^3,$$

and the result is

$$k = 62.5 \times 10^6 \text{ N/m} \quad \text{(for constant pressure supply)}$$

e. Stiffness Comparison

In comparison, for a constant flow rate (see Example Problem 10-3) the stiffness is

$$k = 125 \times 10^6 \text{ N/m} \quad \text{(for constant flow rate}$$

For the bearing with a constant pressure supply in this problem, the stiffness is about half of the constant-flow-rate pad in Example Problem 10-3.

f. Hydraulic Power

The power for circulating the oil through the bearing for constant pressure supply is twice of that for constant flow rate. Neglecting the friction losses in the pipes, the equation for the net hydraulic power for circulating the oil through the bearing in a constant-flow-rate pad is

$$\dot{E}_h \approx Qp_r \quad \text{(for constant-flow-rate pad)}$$
$$= 76.8 \times 10^{-6} \times 294.18 \times 10^3 = 22.6 \text{ W} \quad \text{(constant-flow-rate pad).}$$

In comparison, the equation for the net hydraulic power for constant pressure supply is

$$\dot{E}_h \approx Qp_s \quad \text{(For a constant pressure supply pad).}$$

Since $p_s = 2p_r$, the hydraulic power is double for constant pressure supply:

$$\dot{E}_h \approx Qp_r = 76.8 \times 10^{-6} \times 588.36 \times 10^3 = 45.20 \text{ W}$$
$$\text{(for a constant-pressure-supply pad)}$$

Example Problem 10-6

Constant-Supply-Pressure Bidirectional Pads

A bidirectional hydrostatic bearing (see Fig. 10-5) consists of two circular pads, a constant supply pressure, p_s, and flow restrictors. If there is no external load, the two bidirectional circular pads are prestressed by an equal reaction force, $W = 21{,}000$ N, at each side.

The clearance at each side is equal, $h_1 = h_2 = 0.1$ mm. The upper and lower circular pads are each of 140-mm diameter and circular recess of 70-mm diameter. The oil is SAE 10, and the operation temperature of the oil in the clearance is 70°C. The supply pressure is twice the recess pressure, $p_s = 2p_r$.

a. Find the recess pressure, p_r, and the supply pressure, p_s, at each side to maintain the required prestress.
b. Calculate the flow rate Q of the oil through each pad.
c. Find the stiffness of the bidirectional hydrostatic bearing.
d. The flow restrictor at each side is a capillary tube of inside diameter $d_i = 1$ mm. Find the length of the capillary tube.
e. If there is no external load, find the hydraulic power required for circulating the oil through the bidirectional hydrostatic bearing.

Solution

a. Recess Pressure and Supply Pressure

The recess pressure is derived from the equation of the load capacity:

$$W = R^2 \frac{\pi}{2} \cdot \frac{1 - (R_0/R)^2}{\ln(R/R_0)} \cdot p_r$$

After substitution, the recess pressure is the only unknown in the following equation:

$$21{,}000 \text{ N} = 0.07^2 \frac{\pi}{2} \frac{1 - (0.035/0.07)^2}{\ln(0.07/0.035)} p_r$$

The solution for the recess pressure yields

$$p_r = 2.52 \text{ MPa}$$

The supply pressure is

$$p_s = 2p_r = 5.04 \text{ MPa}$$

b. Flow Rate Through Each Pad

The flow rate Q can now be derived from the following expression as a function of the recess pressure:

$$Q = \frac{\pi}{6\mu} \cdot \frac{h_0^3}{\ln(R/R_0)} \cdot p_r$$

Substituting the known values gives

$$Q = \frac{\pi}{6 \times 0.01} \times \frac{10^{-12}}{\ln 2} \times 2.52 \times 10^6 = 190.4 \times 10^{-6} \frac{m^3}{s}$$

c. Stiffness of the Bidirectional Hydrostatic Pad

In order to find the stiffness of the pad, it is necessary to find the effective area:

$$W = A_e p_r$$

$$21{,}000 \text{ N} = A_e \times 2.52 \text{ MPa}$$

Solving for A_e as the ratio of the load capacity and the recess pressure yields

$$A_e = \frac{21{,}000 \text{ N}}{2.52 \text{ MPa}} = 0.0083 \text{ m}^2$$

The ratio of the pressure to the supply pressure, β, is

$$\beta = \frac{p_r}{p_s} = 0.5$$

The stiffness of the one circular hydrostatic pad is

$$k = \frac{3}{h_0} A_e(\beta - \beta^2) p_s$$

$$k = \frac{3}{0.1 \times 10^{-3}} \times 0.0083 \times (0.5 - 0.5^2) \times 5.04 \times 10^6 = 315 \times 10^6 \text{ N/m}$$

and the stiffness of the bidirectional bearing is

$$K = 2 \times 315 \times 10^6 = 630 \times 10^6 \text{ N/m}$$

d. Length of the Capillary Tube

The internal diameter of the tube is $d_i = 1$ mm.

The equation for the flow rate in the recess is

$$Q = \frac{p_s - p_r}{R_{\text{in}}}$$

After substituting the known values for p_s, p_r, and Q, the inlet resistance becomes

$$R_{\text{in}} = \frac{p_s - p_r}{Q} = \frac{5.04 \times 10^6 - 2.52 \times 10^6}{190.4 \times 10^{-6}} = 1.32 \times 10^{10} \text{ N-s/m}^4$$

The inlet resistance of capillary tube is given by the following tube equation:

$$R_{\text{in}} = \frac{64\ \mu l_c}{\pi d_i^4}$$

Here, d_i is the inside diameter of the tube and l_c is the tube length. The tube length is

$$l_c = \frac{R_{\text{in}} \pi d_i^4}{64\mu}$$

$$= \frac{1.32 \times 10^{10} \times \pi 0.001^4}{64 \times 0.01} = 65 \times 10^{-3} \text{ m},$$

$$I_c = 65 \text{ mm}$$

e. Hydraulic Power for Circulating Oil Through the Bidirectional Hydrostatic Bearing

Neglecting the friction losses in the pipes, the equation for the net hydraulic power for one pad is

$$\dot{E}_h \approx Q p_s$$

Substituting the values for Q and P_s results in

$$\dot{E}_h \approx 190.4 \times 10^{-6} \times 5.04 \times 10^6 = 960 \text{ W}$$

Hydraulic power for bidirectional bearing is

$$\dot{E}_h \text{ (bidirectional bearing)} = 2 \times 960 = 1920\ W$$

10.11 JOURNAL BEARING CROSS-STIFFNESS

The hydrodynamic thrust pad has its load capacity and the stiffness in the same direction. However, for journal bearings the stiffness is more complex and involves four components. For most designs, the hydrostatic journal bearing has hydrodynamic as well as hydrostatic effects, and it is referred to as a hybrid bearing. The hydrodynamic effects are at the lands around the recesses. The displacement is not in the same direction as the force W. In such cases, the journal bearing has cross-stiffness (see Chapter 7). The stiffness components are presented as four components related to the force components and the displacement component.

Similar to the hydrodynamic journal bearing, the load of the hydrostatic journal bearing is also divided into two components, W_x and W_y, and the displacement of the bearing center e is divided into two components, e_x and e_y. In Chapter 7, the two components of the journal bearing stiffness are defined [Eq. (7-31)], and the cross-stiffness components are defined in Eq. (7-32). Cross-stiffness components can result in bearing instability, which was discussed in Chapter 9.

10.12 APPLICATIONS

An interesting application is the hydrostatic pad in machine tool screw drives (Rumberger and Wertwijn, 1968). For high-precision applications, it is important to prevent direct metal contact, which results in stick-slip friction and limits the machining precision. Figure 10-7 shows a noncontact design that includes hydrostatic pads for complete separation of the sliding surfaces of screw drive. Another important application is in a friction testing machine, which will be described in Chapter 14.

10.13 HYDRAULIC PUMPS

An example of a positive-displacement pump that is widely used for lubrication is the gear pump. The use of gear pumps is well known in the lubrication system of automotive engines. Gear pumps, as well as piston pumps, are positive-displacement pumps; i.e., the pumps deliver, under ideal conditions, a fixed quantity of liquid per cycle, irrespective of the flow resistance (head losses in the system). However, it is possible to convert the discharge at a constant flow rate to discharge at a constant pressure by installing a pressure relief valve that maintains a constant pressure and returns the surplus flow.

A cross section of a simple gear pump is shown in Fig. 10-8a. A gear pump consists of two spur gears (or helix gears) meshed inside a pump casing, with one of the gears driven by a constant-speed electric motor. The liquid at the suction side is trapped between the gear teeth, forcing the liquid around the casing and finally expelling it through the discharge. The quantity of liquid discharged per revolution of the gear is known as *displacement*, theoretically equal to the sum of the volumes of all the spaces between the gear teeth and the casing. However, there are always tolerances and small clearance for a free fit between the gears and casing. The presence of clearance in pump construction makes it practically impossible to attain the theoretical displacement.

The advantages of the gear pump, in comparison to other pumps, are as follows.

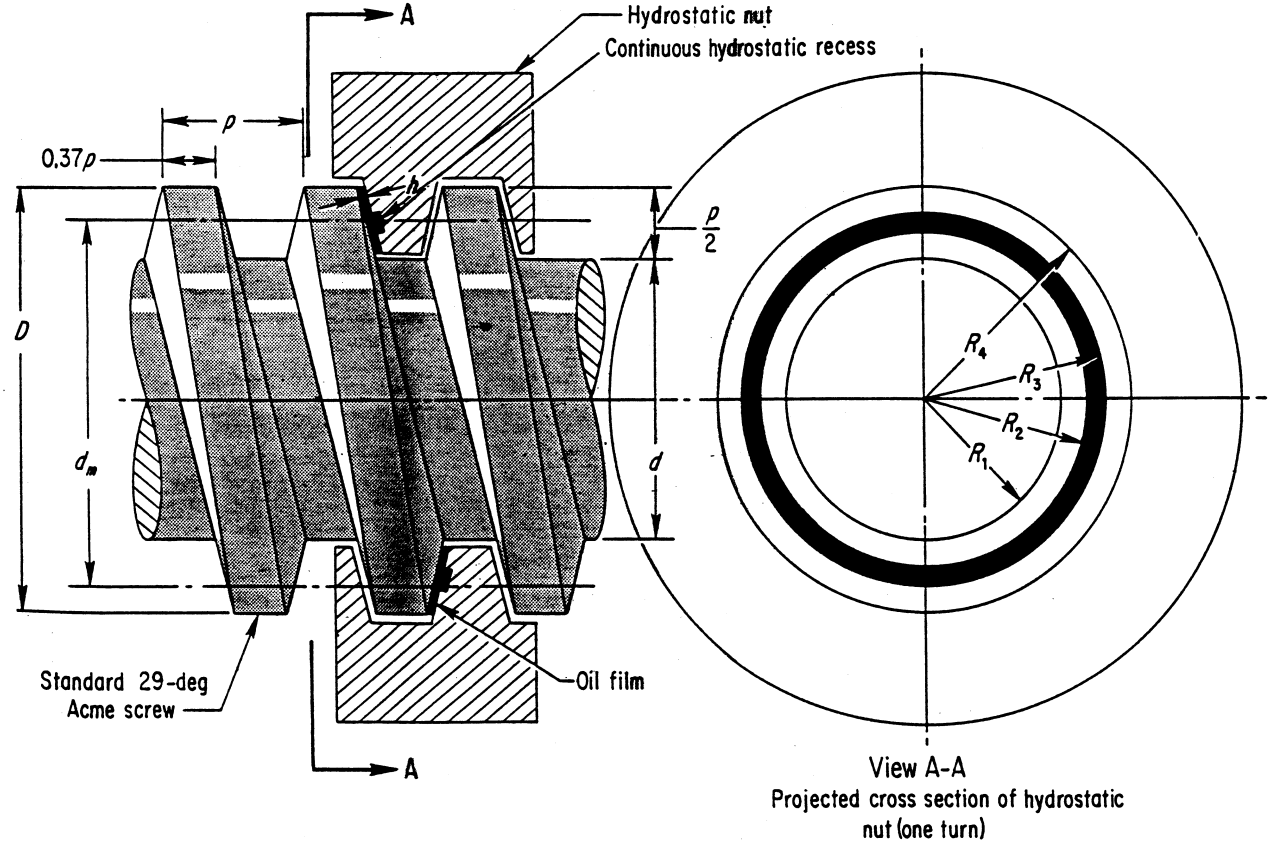
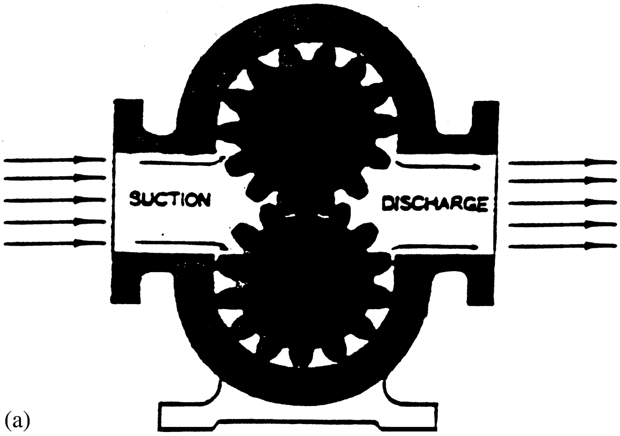
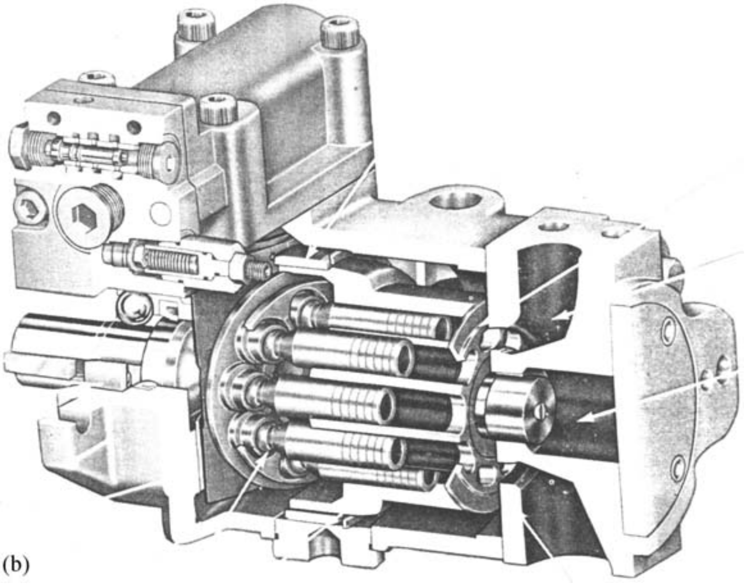


FIG. 10-7 Noncontact screw drive with hydrostatic pads. From Rumberger and Wertwijn (1968), reprinted with permission from *Machine Design*.



(a)



(b)

FIG. 10-8 (a) Cross section of a gear pump. (b) Multipiston hydraulic pump. (Reprinted with permission from The Oilgear Company.)

1. It is a simple and compact pump, and does not need inlet and outlet valves, such as in the piston pump. However, gear pumps require close running clearances.
2. It involves continuous flow (unlike positive-displacement reciprocating pumps).

3. It can handle very high-viscosity fluid.
4. It can generate very high heads (or outlet pressure) in comparison to centrifugal pumps.
5. It is self-priming (unlike the centrifugal pump). It acts like a compressor and pumps out trapped air or vapors.
6. It has good efficiency at very high heads.
7. It has good efficiency over a wide speed range.
8. It requires relatively low suction heads.

The flow rate of a gear pump is approximately constant, irrespective of its head losses. If we accidentally close the discharge valve, the discharge pressure would rise until the weakest part of the system fails. To avoid this, a relief valve should be installed in parallel to the discharge valve.

When a small amount of liquid escapes backward from the discharge side to the suction side through the gear pump clearances, this is referred to as *slip*. The capacity (flow rate) lost due to slip in the clearances increases dramatically with the clearance, h_0, between the housing and the gears (proportional to h_0^3) and is inversely proportional to the fluid viscosity. An idea about the amount of liquid lost in slippage can be obtained via the equation for laminar flow between two parallel plates having a thin clearance, h_0, between them:

$$Q = \frac{l h_0^3}{12 \mu b} \Delta p \tag{10-49}$$

where

Q = flow rate of flow in the clearance (slip flow rate)
Δp = differential pressure (between discharge and suction)
b = width of fluid path (normal to fluid path)
h_0 = clearance between the two plates
μ = fluid viscosity
l = length along the fluid path

This equation is helpful in understanding the parameters affecting the magnitude of slip. It shows that slip is mostly dependent on clearance, since it is proportional to the cube of clearance. Also, slip is proportional to the pressure differential Δp and inversely proportional to the viscosity μ of the liquid. Gear pumps are suitable for fluids of higher viscosity, for minimizing slip, and are widely used for lubrication, since lubricants have relatively high viscosity (in comparison to water). Fluids with low viscosity, such as water and air, are not suitable for gear pumps.

Piston pumps are also widely used as high-pressure positive-displacement (constant-flow-rate) hydraulic pumps. An example of the multipiston pump is shown in Fig. 10-8b. The advantage of the piston pump is that it is better sealed and the slip is minimized. In turn, the efficiency of the piston pump is higher, compared to that of a gear pump, but the piston pump requires valves, and it is more expensive.

10.14 GEAR PUMP CHARACTERISTICS

The actual capacity (flow rate) and theoretical displacement versus pump head are shown in Fig. 10-9. The constant theoretical displacement is a straight horizontal line. The actual capacity (flow rate) reduces with the head because the "slip" is proportional to the head of the pump (discharge head minus suction head). When the head approaches zero, the capacity is equivalent to the theoretical displacement.

10.14.1 Hydraulic Power and Pump Efficiency

The SI unit of power $\dot{E}$ is the watt. Another widely used unit is the Imperial unit, horse power [HP]. Brake power, $\dot{E}_b$ (BHP in horsepower units), is the mechanical shaft power required to drive the pump by means of electric motor. In the pump, this power is converted into two components: the useful hydraulic power, $\dot{E}_h$, and the frictional losses, $\dot{E}_f$. The useful hydraulic power can be converted back to work done by the fluid. A piston or hydraulic motor can do this energy conversion. In the pump, the friction losses are dissipated as heat. Friction

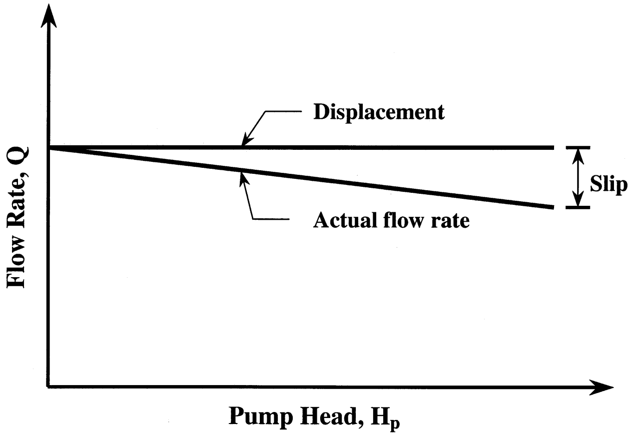


FIG. 10-9 Gear pump *Q–H* characteristics.

losses result from friction in the bearings, the stuffing box (or mechanical seal), and the viscous shear of the fluid in the clearances.

In Fig. 10-10, the curves of the various power components $\dot{E}$ versus pump head H_p are presented in horsepower [HP] units. The frictional horsepower [FHP] does not vary appreciably with increased head; it is the horizontal line in Fig. 10-10. The other useful component is the hydraulic horsepower [HHP]. This power component is directly proportional to the pump head and is shown as a straight line with a positive slope. This component is added to constant FHP, resulting in the total brake horsepower [BHP].

The BHP curve in Fig. 10-10 is a straight line, and at zero pump head there is still a definite brake horsepower required, due to friction in the pump. In a gear pump, the friction horsepower, FHP, is a function of the speed and the viscosity of the fluid, but not of the head of the pump. Because FHP is nearly constant versus the head, it is a straight horizontal line in Fig. 10-10. On the other hand, HHP is an increasing linear function of H_p (see equation for hydraulic power). (This is approximation, since Q is not constant because it is reduced by the slip.) The sum of the friction power and the hydraulic power is the brake horsepower. The brake horsepower increases nearly linearly versus H_p, as shown in Fig. 10-10. Since FHP is constant, the efficiency η is an increasing function versus H_p. The result is that gear pumps have a higher efficiency at high heads.

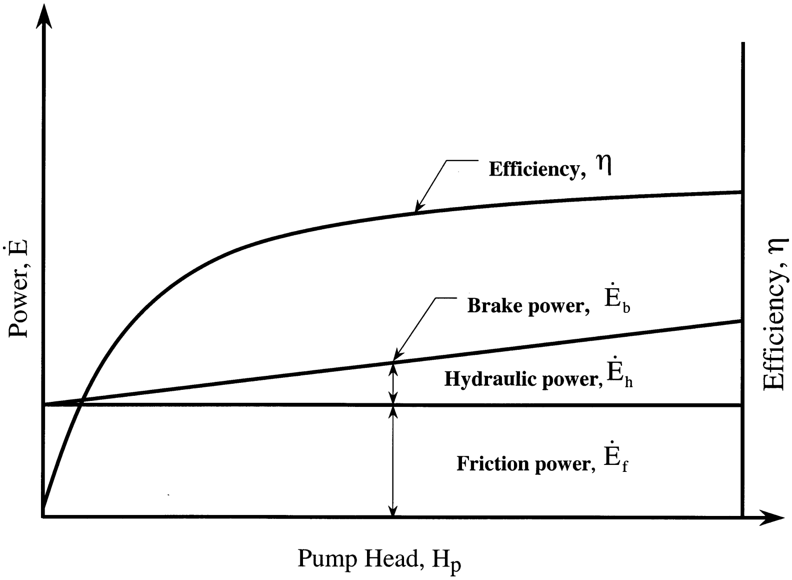


FIG. 10-10 Power and efficiency characteristics of the gear pump.

The head of the pump, H_p, generated by the pump is equal to the head losses in a closed-loop piping system, such as in the hydrostatic bearing system. If the fluid is transferred from one tank to another at higher elevation, the head of the pump is equal to the head losses in the piping system plus the height difference ΔZ.

The head of the pump, H_p, is the difference of the heads between the two points of discharge and suction:

$$H_p = H_d - H_s \tag{10-50}$$

Pump head units are of length (m, ft). Head is calculated from the Bernoulli equation. The expression for discharge and suction heads are:

$$H_d = \frac{p_d}{\gamma} + \frac{V_d^2}{2g} + Z_d \tag{10-51}$$

$$H_s = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + Z_s \tag{10-52}$$

where

H_d = head at discharge side of pump (outlet)
H_s = head at suction side of pump (inlet)
p_d = pressure measured at discharge side of pump (outlet)
p_s = pressure measured at suction side of pump (inlet)
γ = specific weight of fluid (for water, $\gamma = 9.8 \times 10^3$ [N/m^3])
V = fluid velocity
g = gravitational acceleration
Z = height

The pump head, H_p, is the difference between the discharge head and suction head. In a closed loop, H_p is equal to the head loss in the loop. The expression for the pump head is

$$H_p = \frac{p_d - p_s}{\gamma} + \frac{V_d^2 - V_s^2}{2g} + (Z_d - Z_s) \tag{10-53}$$

The velocity of the fluid in the discharge and suction can be determined from the rate of flow and the inside diameter of the pipes. In most gear pumps, the pipe inside the diameters on the discharge and suction sides are equal. In turn, the discharge velocity is equal to that of the suction. Also, there is no significant difference in height between the discharge and suction. In such cases, the last two terms can be omitted, and the pump is determined by a simplified equation that considers only the pressure difference:

$$H_p = \frac{p_d - p_s}{\gamma} \tag{10-54}$$

10.14.2 Hydraulic Power

The hydraulic power of a pump, $\dot{E}_h$, is proportional to the pump head, H_p, according to the following equation:

$$\dot{E}_h = Q\gamma H_p \tag{10-55}$$

The SI units for Eq. (10-53), (10-54), and (10-55) are

$\dot{E}_h[w]$
Q [m^3/s]
γ [N/m^3]
H [m]
p [N/m^2 or pascals]

The pump efficiency is the ratio of hydraulic power to break power:

$$\eta = \frac{\dot{E}_h}{\dot{E}_b} \tag{10-56}$$

The conversion to horsepower units is 1 HP = 745.7 W. In most gear pumps, the inlet and outlet pipes have the same diameter and the inlet and outlet velocities are equal.

In Imperial units, the hydraulic horsepower (HHP) is given by

$$HHP = \frac{Q\Delta p}{1714} \tag{10-57}$$

Here, the units are as follows:

Δp [psi] $= \gamma(H_p - H_s)$ and Q [GPM]

In imperial units, the efficiency of the pump is:

$$\eta = \frac{HHP}{BHP} \tag{10-58}$$

The BHP can be measured by means of a motor dynamometer. If we are interested in the efficiency of the complete system of motor and pump, the input power is measured by the electrical power, consumed by the electric motor that drives the pump (using a wattmeter). The horsepower lost on friction in the pump, FHP, cannot be measured but can be determined from

$$\text{FHP} = \text{BHP} - \text{HHP} \tag{10-59}$$

10.15 FLOW DIVIDERS

Using many hydraulic pumps for feeding the large number of recesses of hydrostatic pads in machines is not practical. A simple solution of this problem is to use constant pressure and flow restrictors. However, flow restrictors increase the power losses in the system. Therefore, this method can be applied only with small machines or machines that are operating for short periods, and the saving in the initial cost of the machine is more important than the long-term power losses.

Another solution to this problem is to use flow dividers. Flow dividers are also used for distributing small, constant flow rates of lubricant to rolling-element bearings. It is designed to divide the constant flow rate of one hydraulic pump into several constant flow rates. In hydrostatic pads, each recess is fed at a constant flow rate from the flow divider. The advantage of using flow dividers is that only one hydraulic pump is needed for many pads and a large number of recesses.

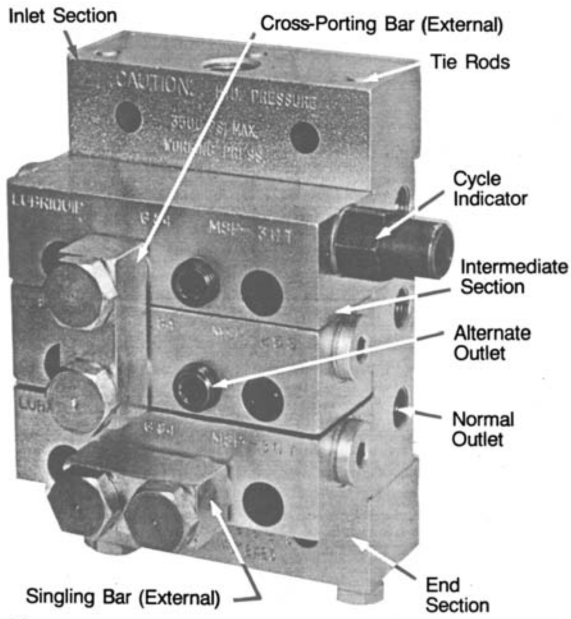
The design concept of a flow divider is to use the hydraulic power of the main pump to activate many small pistons that act as positive-displacement pumps (constant-flow-rate pumps), and thus the flow of one hydraulic pump is divided into many constant flow rates. A photo of a flow divider is shown in Fig. 10-11a. Figure 10-11b presents a cross section of a flow divider made up of many rectangular blocks connected together for dividing the flow for feeding a large number of bearings. The contact between the blocks is sealed by O-rings. The intricate path of the inlet and outlet of one piston is shown in this drawing.

For a large number of bearings, the flow divider outlets are divided again. An example of such a combination is shown in Fig. 10-12.

10.16 CASE STUDY: HYDROSTATIC SHOE PADS IN LARGE ROTARY MILLS

Size reduction is an important part of the process of the enrichment of ores. Ball-and-rod rotary mills are widely used for grinding ores before the enrichment process in the mines. Additional applications include the reduction of raw-material particle size in cement plants and pulverizing coal in power stations.

In rotary mills, friction and centrifugal forces lift the material and heavy balls against the rotating cylindrical internal shell and liners of the mill, until they fall down by gravity. The heavy balls fall on the material, and reduce the particle size by impact. For this operation, the rotation speed of the mills must be slow,



(a)

FIG. 10-11a Flow divider. (Reprinted with permission from Lubriquip, Inc.)

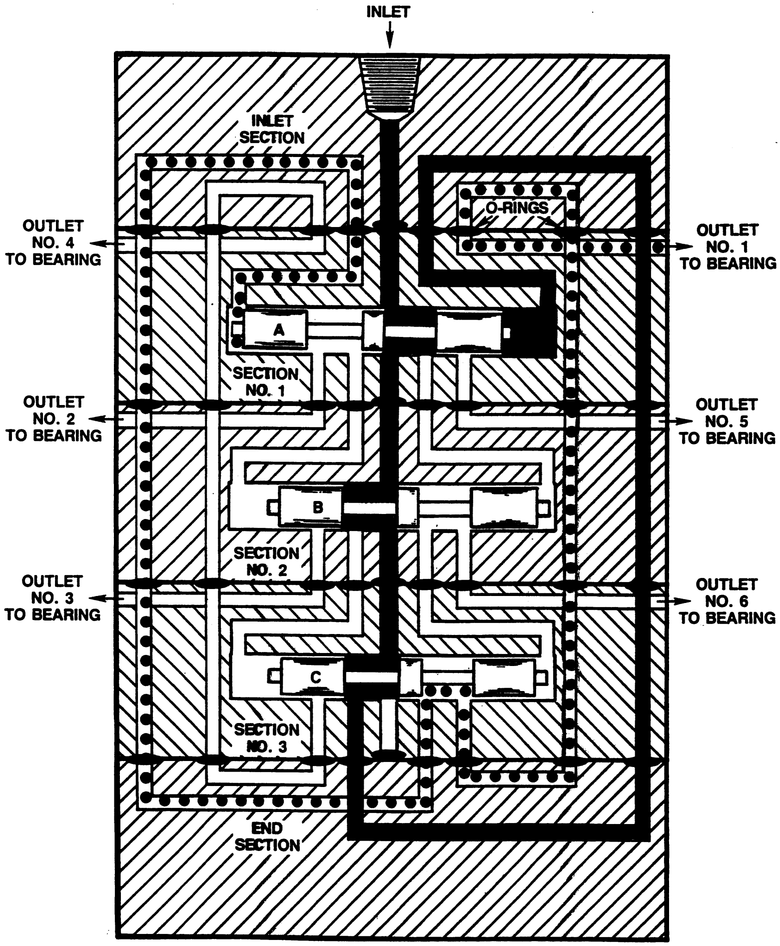
about 12–20 RPM. In most cases, the low speed of rotation is not sufficient for adequate fluid film thickness in hydrodynamic bearings.

There has been continual trend to increase the diameter, D, of rotary mills, because milling output is proportional approximately to $D^{2.7}$ and only linearly proportional to the length. In general, large rotary mills are more economical for the large-scale production of ores. Therefore, the outside diameter of a rotary mill, D, is quite large; many designs are of about 5-m diameter, and some rotary mills are as large as 10 m in diameter.

Two bearings on the two sides support the rotary mill, which is rotating slowly in these bearings. Although each bearing diameter is much less than the rotary mill diameter, it is still very large in comparison to common bearings in machinery. The *trunnion* on each side of the rotary mill is a hollow shaft (large-diameter sleeve) that is turning in the bearings; at the same time, it is used for feeding the raw material and as an outlet for the reduced-size processed material. The internal diameter of the trunnion must be large enough to accommodate the high feed rate of ores. The trunnion outside diameter is usually more than 1.2 m.

In the past, as long as the trunnion outside diameter was below 1.2 m, large rolling-element bearings were used to support the trunnion. However, rolling

SERIES PROGRESSIVE DIVIDER VALVE OPERATION



POSITION NO. 1 — LUBE PRESSURE TO INLET MOVES PISTON A TO LEFT FORCING A MEASURED AMOUNT OF LUBRICANT TO OUTLET NO. 1 BEARING.

(b)

FIG. 10-11b Cross section of a flow divider. (Reprinted with permission from Lubriquip, Inc.)

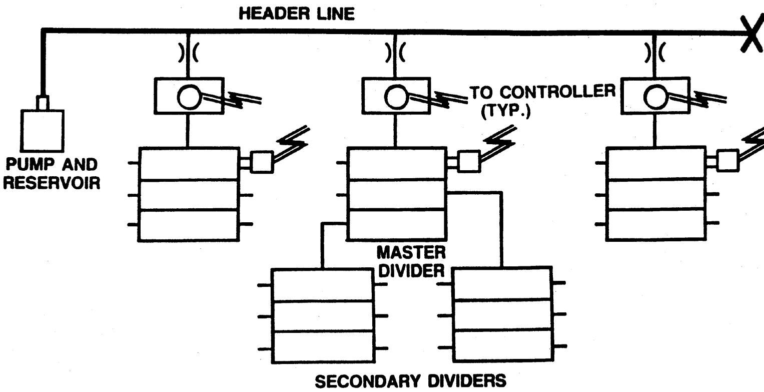


FIG. 10-12 Combination of flow dividers. (Reprinted with permission from Lubriquip, Inc.)

bearings are not practical any more for the larger trunnion diameters currently in use in rotary mills. Several designs of self-aligning hydrodynamic bearings are still in use in many rotary mills. These designs include a hydrostatic lift, of high hydrostatic pressure from an external pump, only during start-up. This hydrostatic lift prevents the wear and high friction torque during start-up. Due to the low speed of rotation, these hydrodynamic bearings operate with very low minimum film thickness. Nevertheless, these hydrodynamic bearings have been operating successfully for many years in various rotary mills. The hydrodynamic bearings are designed with a thick layer of white metal bearing material (babbitt), and a cooling arrangement is included in the bearing. However, ever-increasing trunnion size makes the use of continuous hydrostatic bearings the preferred choice.

Large-diameter bearings require special design considerations. A major problem is the lack of manufacturing precision in large bearings. A large-diameter trunnion is less accurate in comparison to a regular, small-size journal, for the following reasons.

1. Machining errors of round parts, in the form of out-of-roundness, are usually proportional to the diameter.
2. The trunnion supports the heavy load of the mill, and elastic deformation of the hollow trunnion causes it to deviate from its ideal round geometry.
3. Many processes require continuous flow of hot air into the rotary mill through the trunnion, to dry the ores. This would result in thermal distortion of the trunnion; in turn, it would cause additional out-of-roundness errors.

For successful operation, rolling bearings as well as hydrodynamic bearings require precision machining. For rolling-element bearings, any out-of-roundness of the trunnion or the bearing housing would deform the inner or outer rings of the rolling bearing. This undesired deformation would adversely affect the performance of the bearing and significantly reduce its life. Moreover, large-diameter rolling-element bearings are expensive in comparison to other alternatives. For a hydrodynamic bearing, the bearing and journal must be accurately round and fitted together for sustained performance of a full hydrodynamic fluid film. Any out-of-roundness in the bearing or journal results in a direct contact and excessive wear. In addition, rotary mills rotate at relatively low speed, which is insufficient for building up a fluid film of sufficient thickness to support the large trunnion.

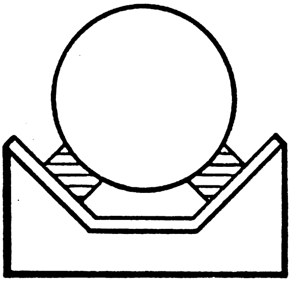
An alternative that is often selected is the hydrostatic bearing system. As mentioned earlier, hydrostatic bearings can operate with a thicker fluid film and therefore are less sensitive to manufacturing errors and elastic deformation.

10.16.1 Self-Aligning and Self-Adjusted Hydrostatic Shoe Pads

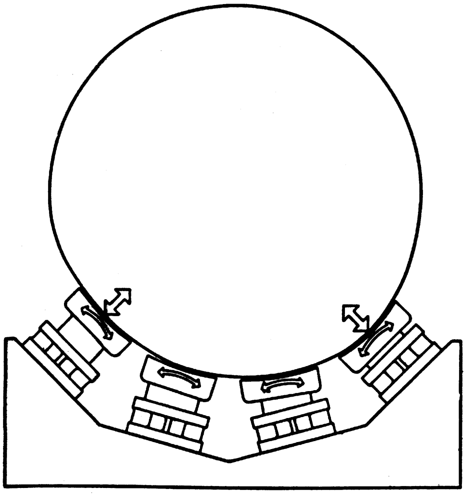
A working solution to the aforementioned problems of large bearings in rotary mills has been in practice for many years, patented by Arsenius, from SKF (see Arsenius and Goran, 1973) and Trygg and McIntyre (1982). It is in the form of self-aligning hydrostatic shoe pads that support the trunnion as shown in Fig. 10-13. These shoe pads can pivot to compensate for aligning errors, in all directions. Hydrostatic pads that pivot on a sphere for universal self-aligning are also used in small bearings.

When two hydrostatic shoe pads support a circular trunnion (Fig. 10-13a), the load is distributed evenly between these two pads. In fact, the location of the two pads determines the location of the trunnion center. However, whenever three or more pads are supporting the trunnion, the load is no longer distributed evenly, and the design must include radial adjustment of the pads, as shown in Fig. 10-13b.

The load capacity is inversely proportional to h_0^3, where h_0 is the radial clearance between the face of the hydrostatic pad and the trunnion running surface. Due to limitations in precision in the mounting of the pads, the clearance h_0 is never equal in all the pads. Therefore, the design must include adjustment of the pad height to ensure that the load is distributed evenly among all the pads. Adjustment is required only for the extra pads above the first two pads, which do not need adjustment. Therefore, each of the extra hydrostatic pads must be designed to move automatically in the radial direction of the trunnion until the load is divided evenly among all pads. This way, the pads always keep a constant clearance from the trunnion surface.



(a)



(b)

FIG. 10-13 Hydrostatic shoe pads: (a) Two-pad support. (b) Four-pad support: All pads are self-aligning, two have radial adjustment. From Trygg and McIntyre (1982), reprinted with permission from CIM Bulletin.

In addition, the large trunnion becomes slightly oval under the heavy load of the mill. For a large trunnion, out-of-roundness errors due to elastic deformation are of the order of 6 mm (one-quarter inch). In addition, mounting errors, deflection of the mill axis, and out-of-roundness errors in the machining of the trunnion surface all add up to quite significant errors that require continuous clearance adjustment by means of radial motion of the pads. Also, it is impossible to construct the hydrostatic system precisely so that all pads will have equal clearance, h_0, for equally sharing the load among the pads. Therefore, the hydrostatic pads must be designed to be self-adjusting; namely they must move automatically in the radial direction of the trunnion until the load is equalized among all the pads.

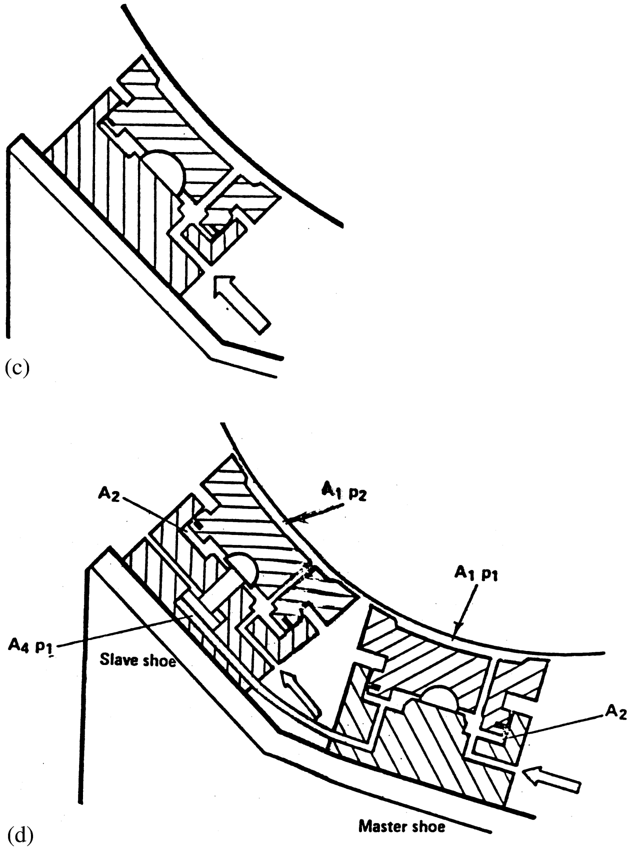


FIG. 10-13 Hydrostatic shoe pads: (c) Self-aligning ball support with pressure relief. (d) Master and slave shoe pads. (From Trygg and McIntyre (1982), reprinted with permission from CIM Bulletin.)

Since the pads are self-aligning and self-adjusting, the foundation's construction does not have to be precise, and a relatively low-cost welded structure can be used as a bed to support the set of hydrostatic pads.

The design concept is as follows: The surface of the pads is designed with the same radius of curvature as the trunnion outside surface. The clearance is adjusted, by pad radial displacement, which requires additional lower piston and hydraulic oil pressure for radial displacement. Explanation of the control of the pad radial motion will follow shortly.

If sufficient constant-flow-rate of oil is fed into each pad from external pumps, it is then possible to build up appropriate pressure in each of the pad recesses for separating completely the mating surfaces by means of a thin oil film. A major advantage of hydrostatic pads is that the fluid film thickness is independent of the trunnion speed. The fluid film is formed when the trunnion is stationary or rotating, and the mating surfaces are completely separated by oil film during start-up as well as during steady operation.

All pads have universal angular self-aligning (see Fig. 10-13b). This is achieved by supporting each pad on a sphere (hard metal ball), as shown in Fig. 10-13c, where the pad has a spheroid recess with its center coinciding with the sphere center. In this way, it can tilt in all directions, and errors in alignment with the trunnion outside surface are compensated.

However, the spheroid pivot arrangement under high load has a relatively high friction torque. This friction torque, combined with the inertia of the pad, would result in slow movement and slow reaction to misalignment. In fact, in large hydrostatic pads the reaction is too slow to adequately compensate the variable misalignment during the rotation of the trunnion.

To improve the self-alignment performance, part of the load on the metal ball is relieved by hydrostatic pressure. The bottom part of the pad has been designed as a piston and is pressurized by oil pressure. The oil pressure relieves a portion of the load on the metal ball, and in turn the undesired friction torque is significantly reduced, as shown schematically in Fig. 10-13c.

In Fig. 10-13b, the radial positions of two inner pads determine the location of the axis of rotation of the trunnion; therefore, these two pads do not require radial adjustments, and they are referred to as *master shoe pads*. Any additional shoe pads require radial adjustment and are referred to as *slave shoes*. The design of the master and slave shoe pads with the hydraulic connections is shown in Fig. 10-13d.

In the slave shoe, there is radial adjustment of the pad clearance with the trunnion surface. The radial adjustment requires an additional lower piston, as shown in Fig. 10-13d. The radial motion of the lower piston is by means of hydrostatic oil pressure. The oil is connected by an additional duct to the space beneath the lower pad. There is a hydraulic duct connection, and the pressures are equalized in the two spaces below the two pads and in the pad recess (in contact with the trunnion surface) of the master and slave shoes. Since there is a constant flow rate, this equal pressure is a load-dependent pressure. If the area of the lower piston is larger than the effective pad area, the lower piston will push the piston and shoe pad in the radial direction (in the slave shoe) and adjust the radial clearance with the trunnion until equal load capacity is reached in all pads.

The recess pressure is a function of the load and the pad effective area. As the load increases, the film thickness diminishes and the pressure rises. It is desirable to limit the pressure and the size of the pad. This can be achieved by increasing the number of pads.

When the oil pump is turned off, the pads with the pistons return to the initial position, where the pistons rest completely on the metal ball. The pistons of the slave shoes must have sufficient freedom of movement in the radial direction of the trunnion; therefore, only the master shoes carry the load when the hydraulic system is not under pressure. To minimize this load, the master shoes are placed in center positions between the slave shoes when four or six shoes are used.

The combination of a master shoe and slave shoe operates as follows: The effective areas of the two pads are equal. If the clearance is the same in both shoes, the hydrostatic recess pressures must also be equal in the two pads. In this case, the load on both shoe pads is equalized.

There is hydraulic connection between the lower piston cylinders of the master and slave recesses (both are supplied by one pump). In this way, the load-dependent pressure in the piston cylinder of the slave shoe will be the same as that in the master shoe, resulting in equal load capacity of each shoe pad at all times.

This design can operate with certain deviation from roundness of the trunnion. For example, if there is a depression (reduced radius) in the trunnion surface, when this depression passes the pads of the slave shoe, the pressure at the recess of this pad drops. At the same time, the master shoe pad has not yet been affected by the depression and the pad of the master shoe will carry most of the load for a short duration. After this disturbance, the pressure at the slave pad would rise and lift the piston until there are equal recess pressures and load capacity in the two pads. Similarly, if there is a bulge (increased radius) on the surface of the trunnion, the process is reversed. In this way, the radial loads on the master and slave shoes are automatically controlled to be equal (with a minimal delay time). In conclusion, the clearances between the pads and trunnion surface are automatically controlled to be equal even if the trunnion is not perfectly round.

Cross-sectional views of the slave and master shoes and an isometric view of the slave shoe are shown in Fig. 10-14. In the master pad, there is one oil inlet and there is a hydraulic connection to the slave shoe. The pad recess of the slave and master shoes is of a unique design. For stable operation, it is important that the pad angular misalignment be immediately corrected. Each pad has one large circular recess and four additional recesses at each corner, all hydraulically connected. The purpose of this design is to have higher hydrostatic restoring torque for fast correction of any misalignment error. Oil is supplied at equal pressure to all the recesses (see Fig. 10-15).

For the large hydrostatic pads in rotary mills, each recess is fed at a constant flowrate, and flow restrictors are not used. Large hydrostatic pads consume a lot of power for the circulation of oil, and flow restrictors considerably increase power losses. The preferred design is to use one central pump with flow dividers. A standby pump in parallel is usually provided, to prevent loss of production in

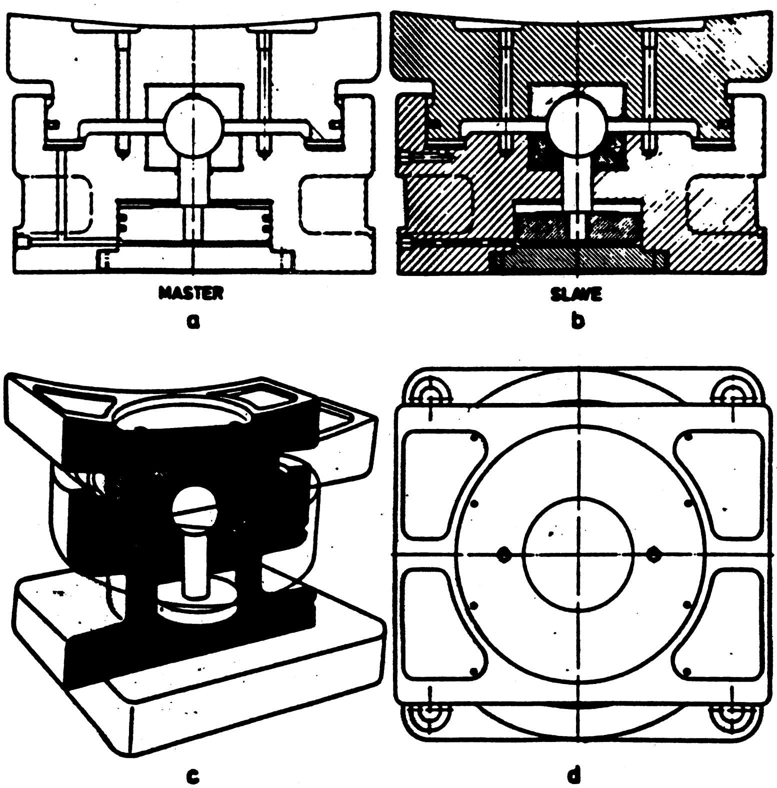
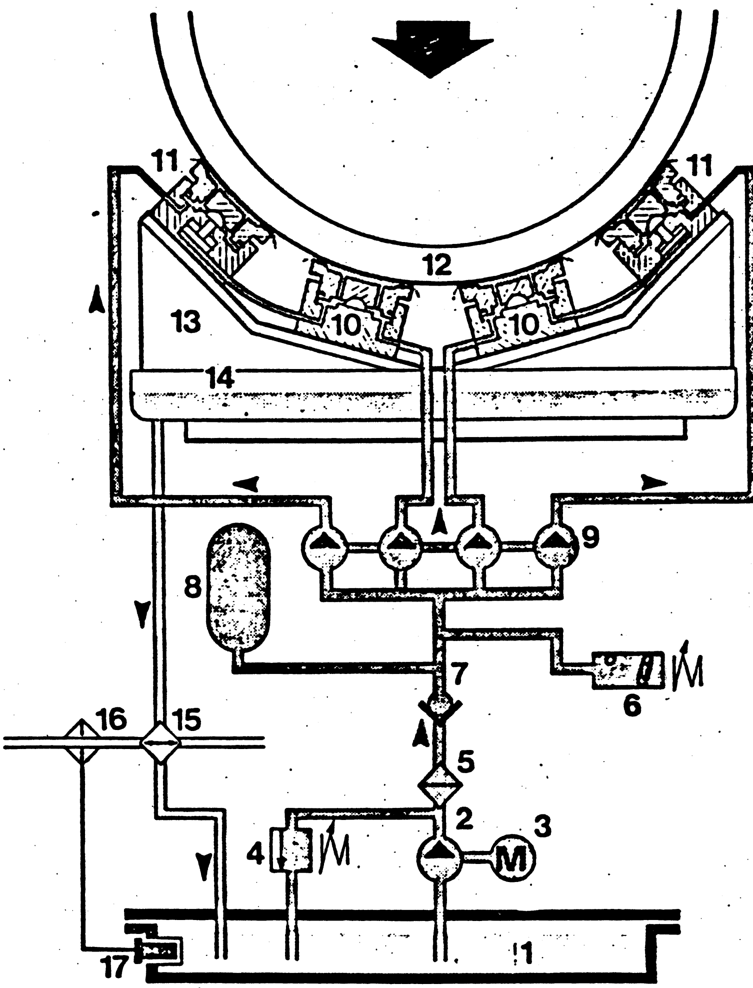


FIG. 10-14 Cross sections and isometric views of master and slave shoe pads. (From Trygg and McIntyre (1982), reprinted with permission from CIM Bulletin.)

case of oil pump failure. The trunnion bearings are continually monitored, to prevent early failure due to unexpected conditions. The bearing temperature and the hydrostatic pressure are continually recorded, and a warning system is set off whenever these values exceed an acceptable limit. In addition, the operation of the mill is automatically cut off when the hydrostatic bearing loses its pressure.

The hydraulic supply system is shown in Fig. 10-15. Each shoe is fed at a constant flow rate by four flow dividers. One positive-displacement pump is used to pump the oil from the oil tank. The oil is fed into the flow dividers through an oil filter, relief valve, and check valve. An accumulator is used to reduce the pressure fluctuations involved in positive-displacement pumps. Four pumps can be used as well. The oil returns to the sump and passes to the oil tank. Additional safety devices are pressure sensors provided to ensure that if the supply pressure



1. Oil tank
2. Pump
3. Electric motor
4. Relief valve
5. Oil filter
6. Pressure switch monitoring the pump motor
7. Check valve
8. Accumulator
9. Flow divider
10. Master shoe
11. Slave shoe
12. Girth ring
13. Pedestal for the shoes
14. Oil sump
15. Oil cooler
16. Valve for cooling water
17. Thermostat

FIG. 10-15 Hydraulic system for hydrostatic pad shoes. (From Trygg and McIntyre (1982), reprinted with permission from CIM Bulletin.)

drops, the mill rotation is stopped. Temperature monitoring is included to protect against overheating of the oil.

10.16.2 Advantages of Self-Aligning Hydrostatic Shoe Pads

Several publications related to the manufacture of these self-adjusting hydrostatic pads claim that there are major advantages in this design: It made it possible to significantly reduce the cost and to reduce the weight of the bearing and trunnion as well as the length of the complete mill in comparison to hydrodynamic bearings. Most important, it improved the bearing performance, namely, it reduced significantly the probability of bearing failure or excessive wear.

The concept of this design is to apply self-aligning hydrostatic shoes, preferably four shoes for each bearing. One important advantage of the design is that the length of the trunnion is much shorter in comparison to that in hydrodynamic bearing design. The shortening of the trunnion results in several advantages.

1. It reduces the weight of the trunnion and thus reduces the total weight of the mill.
2. It simplifies the feed into and from the mill.
3. It reduces the total length of the mill and its weight, resulting in reduced bending moment, and thus the mill can be designed to be lighter. It will, in fact, reduce the cost of the materials and labor for construction of the mill.
4. It reduces the elastic deformation, in the radial direction, of the trunnion.
5. Stiffer trunnion has significant advantages in bearing operation, because it reduces roundness errors; namely, it reduces elastic deformation to an elliptical shape.

In addition to shorter trunnion length, this design eliminates expensive castings followed by expensive precise machining, which are involved in the manufacturing process of the conventional hydrodynamic design. In this case, the casting can be replaced by a relatively low-cost welded construction. The hydrostatic shoes are relatively small, and their machining cost is much lower in comparison to that of large bearings. Moreover, the hydrostatic design operates with a thicker oil film and provides self-aligning bearings in all directions. These improvements prevent unexpected failures due to excessive wear or seizure. This aspect is important because of the high cost involved in rotary mill repair as well as loss of production.

Problems

10-1 A circular hydrostatic pad as shown in Fig. 10-1 has a constant supply pressure, p_s. The circular pad is supporting a load of $W = 1000$ N. The outside disk diameter is 200 mm, and the diameter of the circular recess is 100 mm. The oil is SAE 10 at an operating temperature of 70°C, having a viscosity of $\mu = 0.01$ N-s/m^2. The pad is operating with a clearance of 120 μm.

a. Calculate the flow rate Q of oil through the bearing to maintain the clearance of 120 μm.
b. Find the recess pressure, p_r.
c. Find the effective area of this pad.
d. If the supply pressure is twice the recess pressure, $p_s = 2p_r$, find the stiffness of the circular pad.

10-2 A circular hydrostatic pad, as shown in Fig. 10-1, has a constant flow rate Q. The circular pad is supporting a load of $W = 1000$ N. The outside disk diameter is 200 mm, and the diameter of the circular recess is 100 mm. The oil is SAE 10 at an operating temperature of 70°C, having a viscosity of $\mu = 0.01$ N-s/m^2. The pad is operating with a clearance of 120 μm.

a. Calculate the constant flow rate Q of oil through the bearing to maintain the clearance of 120 μm.
b. Find the recess pressure, p_r.
c. Find the effective area of this pad.
d. For a constant flow rate, find the stiffness of the circular pad operating under the conditions in this problem.

10-3 A long rectangular hydrostatic pad, as shown in Fig. 10-3, has constant flow rate Q. The pad is supporting a load of $W = 10{,}000$ N. The outside dimensions of the rectangular pad are: length is 300 mm and width is 60 mm. The inside dimensions of the central rectangular recess are: length is 200 mm and width is 40 mm. The pad is operating with a clearance of 100 μm. The oil is SAE 20 at an operating temperature of 60°C. Assume that the leakage in the direction of length is negligible in comparison to that in the width direction (the equations for two-dimensional flow of a long pad apply).

a. Calculate the constant flow rate Q of oil through the bearing to maintain the clearance of 100 μm.
b. Find the recess pressure, p_r.

c. Find the effective area of this pad.
d. For a constant flow rate, find the stiffness of the rectangular long pad operating under the conditions in this problem.

10-4 A slider-plate in a machine tool is supported by four bidirectional hydrostatic circular pads. Each recess is fed by a separate pump and has a constant flow rate. Each bidirectional pad is as shown in Fig. 10-3 (but it is a circular and not a rectangular pad). The weight of the slider is 20,000 N, or 5000 N on each pad. The total manufactured clearance between the two pads $(h_1 + h_2)$ is 0.4 mm. Each circular pad is of 100-mm diameter and recess diameter of 50 mm. R = 50 mm and $R_0 = 25$ mm. The oil viscosity is 0.01 N-s/m^2. In order to minimize vertical displacement, the slider plate is prestressed. The reaction force at the top is $W_1 = 5000$ N, and the reaction at the bottom is $W_2 = 10{,}000$ N (reaction to the top bearing reaction plus weight).

a. Find Q_1 and Q_2 in order that the two clearances will be equal $(h_1 = h_2)$.
b. If the flow rate is the same at the bottom and top pads, find the magnitude of the two clearances, h_1 and h_2. What is the equal flow rate, Q, into the two pads?
c. For the first case of equal clearances, find the stiffness of each pad. Add them together for the stiffness of the slider.
d. For the first case of equal clearances, if we place an extra vertical load of 40 N on the slider (10 N on each pad), find the downward vertical displacement of the slider.

10-5 In a machine tool, hydrostatic bearings support the slide plate as shown in Fig. 10-4. The supply pressure reaches each recess through a flow restrictor. The hydrostatic bearings are long rectangular pads. Two bidirectional hydrostatic pads are positioned along the two sides of the slider plate. The weight of the slider is 10,000 N, or 5000 N on each pad. The total manufactured clearance between the two pads $(h_1 + h_2) = 0.4$ mm. The oil viscosity is 0.05 N-s/m^2. For minimizing vertical displacement, the slider plate is prestressed. The reaction of each pad from the top is 5000 N, and the reaction from the bottom of each pad is 10,000 N (reaction to the top bearing reaction plus half slider weight). In order to have equal recess pressure in all pads, the dimensions of the widths of the bottom pad are double those in the top pad. The dimensions of each rectangular pad are as follows.

Top rectangular pad dimensions: 400 mm long and 30 mm wide. A rectangular recess is centered inside the rectangular pad, and its dimensions are 360-mm length and 10-mm width.

Bottom rectangular pad dimensions: 400-mm length and 60-mm width. A rectangular recess is centered inside the rectangular pad and its dimensions are 360-mm length and 20-mm width.

a. Find the recess pressure, p_r, at the bottom and top recesses.
b. The supply pressure from one pump is twice the recess pressure, $p_s = 2p_r$. Find the supply pressure.
c. Find the flow resistance, R_{in}, at the inlet of the bottom and top recesses in order that the two clearances will be equal ($h_1 = h_2$).
d. The flow resistance is made of a capillary tube of 1-mm ID. Find the length, l_c, of the capillary tube at the inlet of the bottom and top recesses.
e. Find the flow rates Q_1 and Q_2 into the bottom and top recesses.
f. For equal clearances, find the stiffness of each pad. Add them together for the stiffness of the bi-directional pad.
g. If we place an extra vertical load of 60 N on the slider (30 N on each bidirectional pad), find the vertical displacement (down) of the slider.

10-6 A long rectangular hydrostatic pad, as shown in Fig. 10-3, has a constant supply pressure, p_s. The pressure is fed into the recess through flow restrictors. The pad supports a load of $W = 20{,}000$ N. The outside dimensions of the rectangular pad are: length is 300 mm and width is 60 mm. The inside dimensions of the central rectangular recess are: length is 200 mm and width is 40 mm. The pad is designed to operate with a minimum clearance of 100 μm. The oil is SAE 30 at an operating temperature of 60°C. Assume that the equations for two-dimensional flow of a long pad apply.

a. Calculate the flow rate Q of oil through the bearing to maintain the clearance of 100 μm.
b. Find the recess pressure, p_r.
c. Find the effective area of this pad.
d. If the supply pressure is twice the recess pressure, $p_s = 2p_r$, find the stiffness of the pad.