12

Rolling-Element Bearings

12.1 INTRODUCTION

Rolling-element bearings, or, in short, rolling bearings, are commonly used in machinery for a wide range of applications. In the past, rolling bearings were referred to as *antifriction bearings*, since they have much lower friction in comparison to sliding bearings. Many types of rolling-element bearings are available in a variety of designs that can be applied for most arrangements in machinery for supporting radial and thrust loads. The rolling elements can be balls, cylindrical rollers, spherical rollers, and conical rollers.

12.1.1 Advantages of Rolling-Element Bearings

One important advantage of rolling-element bearings is their low friction. It is well known that the rolling motion has lower friction in comparison to that of sliding. In addition to friction, the rolling action causes much less wear in comparison to sliding. For most applications, rolling-element bearings require less maintenance than hydrodynamic bearings. To minimize maintenance cost in certain cases, prepacked rolling-element bearings are available with grease that is permanently sealed inside the bearing. Ultrahigh-precision rolling bearings are available for precision machinery, such as precision machine tools and measuring equipment. It is possible to completely eliminate the clearance and even to prestress the bearings. This results in a higher stiffness of the bearings, and the shaft centerline is held tightly in its concentric position. Prestressing the bearings would minimize vibrations as well as reduce any undesired radial displacement of the shaft.

12.1.2 Fatigue Life

A major limitation of rolling-element bearings is that they are subjected to very high alternating stresses at the rolling contacts. High-speed rotation involves a large number of stress cycles per unit of time, which leads to a limited fatigue life. In fact, prestressing and centrifugal forces at high-speed operation significantly increase the contact stresses and further reduce the fatigue life.

12.1.3 Terminology

A standard rolling-element bearing has two rings, an *outer ring* and *inner ring*, which enclose the rolling elements, such as *balls*, *cylindrical rollers*, and *tapered rollers*. The rolling areas on the rings are referred to as *raceways*. An example is a *deep-groove ball bearing*, which has concave raceways (an *outer ring raceway* and an *inner ring raceway*) that form the rolling areas. A *cage* holds the rolling elements at equal distance from one another and prevents undesired contact and rubbing friction among them. The terminology of bearing parts is shown in Fig. 12-1 for various bearing types. This terminology has been adopted by the Anti-Friction Bearing Manufacturers Association (AFBMA).

12.1.4 Rolling Contact Stresses

There is a theoretical point or line contact between the rolling elements and races. But due to elastic deformation, the contact areas are actually of elliptical or rectangular shape. In machinery that involves severe shocks and vibrations, the contact stresses can be very high.

In the United States, the standard bearing material is SAE 52100 steel hardened to 60 RC. This steel has a high content of carbon and chromium. It is manufactured by an induction vacuum melting process, which minimizes porosity due to gas released during the casting process.

Stainless steel AISI 440C hardened to 58 RC is the standard rolling bearing material for corrosive environments. The allowed limit of rolling contact stress for SAE 52100 is 4.2 GPa (609,000 psi). For rolling bearings made of AISI 440C stainless steel, the allowed limit of compression stress is only 3.5 GPa (508,000 psi). Discussion of other rolling bearing materials and manufacturing processes is included at the end of Chapter 13.

The theory of elasticity indicates that the maximum shear stress of rolling contact is below the surface. Due to repeated cyclic stresses, scaly particles eventually separate from the rolling surfaces. Fatigue failure is evident in the form of metal removal, often referred to as *flaking* (or *spalling*), at the rolling contact surfaces of raceways and rolling elements.

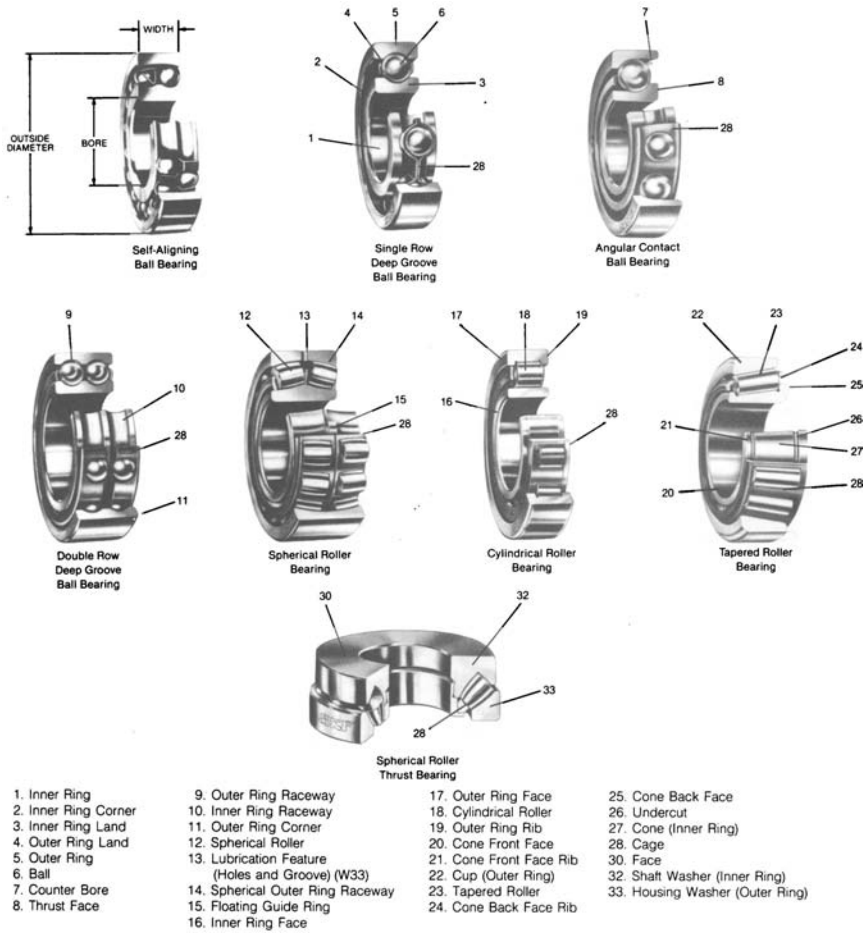

FIG. 12-1 Bearing types and terminology (with permission from SKF).

In order to make the bearing durable to the high stress levels, the contact surfaces and subsurfaces of the rolling elements and raceways are hardened by heat treatment to a minimum hardness of Rockwell C58. During bearing operation, the friction elevates the temperature of the contact surfaces. Therefore, the yield stress and hardness must be retained to elevated temperatures up to 120°C (250°F). For applications at higher temperature, rolling bearings are available that are made of better materials that can retain the desired properties at higher temperatures, The need for fatigue-resistant and hard materials at elevated temperature for rolling bearings has been the motivation for continual research and development, which has resulted in improved materials for high-speed and heavy-duty operation.

Due to the rolling action, the rolling elements and races are subjected to periodic stress cycles that can result in material failure due to fatigue. The fatigue life of rolling bearings is statistically distributed. The data for these statistics must be obtained only by many experiments for each bearing type over a long period of time. Fatigue life depends on the material and its processing methods, such as heat treatment. Fatigue life is also a function of the magnitude of the maximum stress and temperature at the contacts between the raceways and the rolling elements during operation. If stresses are low, fatigue life can be practically unlimited.

The stresses in dry contacts can be calculated via the theory of elasticity (Hertz equations). In addition to fatigue, the high stresses result in considerable wear. However, the surfaces are usually lubricated, and under favorable conditions there is a very thin lubrication film at very high pressure that separates the rolling surfaces. Whenever this film is thicker than the surface asperities, it would prevent any direct contact. In this way, this fluid film plays an important role in reducing wear. The analysis of this film is based on elastohydrodynamic (EHD) theory. The analysis of the fluid flow and pressure wave inside this thin film is performed in a similar way to that for hydrodynamic bearings. But in addition, EHD analysis considers the elastic deformation near the contact area and the increasing function of viscosity versus pressure.

Recent developments include investigation of the thermal effects in the EHD film. This analysis is referred to as *thermoelastohydrodynamic* (TEHD) analysis. This thermal analysis is quite complex because it solves for the temperature distribution by considering the dissipation of heat due to viscous friction, heat transfer, and finally the viscosity dependence on temperature and pressure distribution. Dedicated computer programs have been developed that assist in better understanding the phenomena involved in rolling contact.

Although there has been considerable progress in the analysis of stresses and fluid films, for design purposes the life of the rolling-element bearings must be estimated by means of empirical equations based on experiment. Due to the statistical nature of bearing life, bearings are selected to have a very high probability of operation without failure for a certain reasonable period of the life of the machine (such as 5 or 10 years). The life-period requirement is usually determined before the design of a machine is initiated.

Failure due to fatigue is only one possible failure mode among other, more frequent failure modes, which have a variety of causes. Proper lubrication, mounting, and maintenance of the bearing can prevent most of them. It is interesting to note that although most rolling bearings are selected by considering their fatigue life, only 5–10% of the bearings actually fail by fatigue. The causes for most bearing failures are misalignment, improper mounting, corrosion, penetration of dust or other hard particles into the bearing and lack of proper lubrication (oil starvation or not using an appropriate lubricant).

In addition to fatigue and the other reasons just mentioned, overheating can be a frequent cause of rolling bearing failure. Bearing overheating can be caused by heat sources outside the bearing, such as in the case of a steam turbine or aircraft engine. Also, friction-energy losses are converted to heat, which is dissipated in the bearing. In most cases overheating is due to heavy load and high speed. Higher bearing temperatures have an adverse effect on the lubricant. As the temperature increases, the oil oxidation process is accelerated. A rule of thumb is that the lubrication life is halved for every 10°C increase in temperature.

Thermal analyses as well as measurements have indicated that during operation there is a temperature gradient inside the bearing. Heat is transferred better from the outer ring through the housing than from the inner ring. In most practical cases of moderate load and speed, the outer ring temperature is lower than that of the inner ring by 5–10°C. This difference in temperature results in uneven thermal expansion. If the bearing has a small internal clearance or no clearance, it would result in extra thermal stresses. The thermal stresses are in the form of a tight fit and higher contact stress between the rolling elements and the races. During high-speed operation, the additional stresses further increase the temperature. There is a risk that this sequence of events can result in an unstable closed-loop process of rising temperature and stress that can lead to failure in the form of thermal seizure. For this reason, standard rolling-element bearings are manufactured with sufficient internal clearance to reduce the risk of thermal seizure.

At high speeds, the centrifugal forces of the rolling elements combine with the external load and thermal stresses to increase the maximum total contact stress between a rolling element and the outer ring race. Therefore, a combination of heavy load and high speed reduces the bearing fatigue life. In extreme cases, this combination can cause a catastrophic failure in the form of bearing seizure. The risk of failure is high whenever the product of bearing load and speed is high, because the amount of heat generated by friction in the bearing is proportional to this product. In conclusion, the load and speed are two important factors that must be considered in the selection of the proper bearing type in order to achieve reliable operation during the expected bearing life.

Developments in aircraft turbine engines resulted in a requirement for increasing power output at higher shaft speed. As discussed earlier (see Chap. 1), rolling bearings are used for aircraft engines because of the high risk of an interruption in the oil supply of hydrodynamic or hydrostatic bearings. At the very high speed required for gas turbines, the centrifugal force of the rolling elements is a major factor in limiting the fatigue life of the bearings. The centrifugal forces of the rotating rolling elements increase the contact stresses at the outer race and shorten the bearing fatigue life.

The contact force on the outer race increases due to the centrifugal force of a rolling element. The centrifugal force F_c [N] is

$$F_c = m_r \omega_c^2 R_c \tag{12-1}$$

Here, m_r is the mass of the rolling element [kg] and ω_c is the angular speed [rad/s] of the center of a rolling element in its circular orbit (equivalent to the cage angular speed, which is lower than the shaft speed). The radius R_c [m] is of the circular orbit of the rolling-element center. The units indicated are SI, but other unit systems, such as the Imperial unit system, can be applied. In a deep-groove bearing, centrifugal force directly increases the contact force on the outer race. But in an angular contact ball bearing, which is often used in high-speed turbines, the contact angle results in a higher resultant reaction force on the outer raceway.

Equation (12-1) indicates that centrifugal force, which is proportional to the second power of the angular speed, will become more significant in the future in view of the ever-increasing speeds of gas turbines in aircraft and other applications. Similarly, bearing size increases the centrifugal force, because rolling elements of larger bearings have more mass as well as larger-orbit radius. Therefore, the centrifugal forces are approximately proportional to the second power of the DN value (rolling bearing bore in millimeters times shaft speed in RPM). The centrifugal force of the rolling elements is one important consideration for limiting aircraft turbine engines to 2 million DN. A future challenge will be the development of the technology in order to break through the DN limit of 2 million.

12.1.5 Misalignment

Bearings in machines are subjected to a certain degree of angular misalignment between the shaft and bearing centerlines. A bearing misalignment can result from inaccuracy of assembly and machining (within the tolerance limits). Even if the machining and assembly are very precise, there is a certain misalignment due to the shaft's bending under load. Certain bearing types are more sensitive to angular misalignment than others. For example, cylindrical and tapered roller bearings are very sensitive to excessive misalignment, which results in uneven pressure over the roller length.

In applications where there is a relatively large degree of misalignment, the designer can select a self-aligning roller-element bearing. In most cases, it is more economical to use a self-aligning bearing than to specify close tolerances that involve the high cost of precision manufacturing. Self-aligning bearings allow angular errors in machining and assembly and reduce the requirement for very close tolerances. Self-aligning bearings include self-aligning ball bearings and spherical roller bearings. The design of a self-aligning bearing is such that the shape of the cross section of the outer raceway is circular, which allows the rolling elements to have an angular degree of freedom and self-alignment between the inner and outer rings.

12.2 CLASSIFICATION OF ROLLING-ELEMENT BEARINGS

Ball bearings can operate at higher speed in comparison to roller bearings because they have lower friction. In particular, the balls have less viscous resistance when rolling through oil or grease. However, ball bearings have lower load capacity compared with roller bearings because of the high contact pressure of point contact. There are about 50 types of ball bearings listed in manufacturer catalogues. Each one has been designed for specific applications and has its unique characteristics. The following is a description of the most common types.

12.2.1 Ball Bearings

12.2.1.1 Deep-Groove Ball Bearing

The deep-groove ball bearing (Fig. 12-2) is the most common type, since it can be used for relatively high radial loads. Deep-groove radial ball bearings are the most widely used bearings in industry, and their market share is about 80% of industrial rolling-element bearings. Owing to the deep groove in the raceways, they can support considerable thrust loads (in the axial direction of the shaft) in

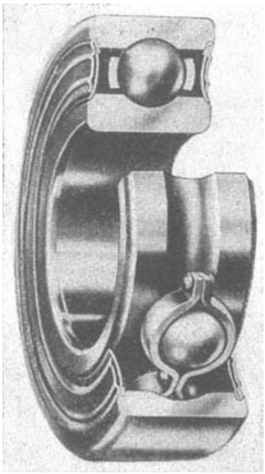

FIG. 12-2 Deep-groove ball bearing.

addition to radial loads. A deep-groove bearing can support a thrust load of about 70% of its radial load. The radial and axial load capacity increases with the bearing size and number of balls.

For maximum load capacity, a filling-notch type of bearing can be used that has a larger number of balls than the standard bearing. In this design, there is a notch on one shoulder of the race. The circular notch makes it possible to insert more balls into the deep groove between the two races. The maximum number of balls can be inserted if the outer ring is split. However, in that case, external means must be provided to hold and tighten the two ring halves together.

12.2.1.2 Self-Aligning Ball Bearings

It is very important to compensate for angular machining and assembly errors between the centerlines of the bearing and the shaft. The elastic deflection of the shaft is an additional cause of misalignment. In the case of a regular deep-groove ball bearing, the misalignment causes a bending moment in the bearing and additional severe contact stresses between the balls and races. However, in the self-aligning bearing (Fig. 12-3), the spherical shape of the outer race allows an additional angular degree of freedom (similar to that of a universal joint) that prevents the transfer of any bending moment to the bearing and prevents any additional contact stresses.

Self-aligning ball bearings have two rows of balls, and the outer ring has a common spherical raceway that allows for the self-aligning characteristic. The

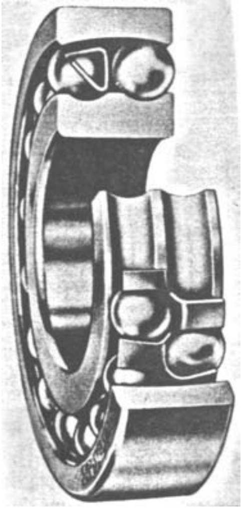

FIG. 12-3 Self-aligning ball bearing.

inner ring is designed with two *restraining ribs* (also known as *lips*), one at each side of the roller element, for accurately locating the rolling elements' path on the inner raceway. But the outside ring has no ribs, in order to allow for self-alignment. A wide spherical outer race allows for a higher degree of self-alignment.

Self-aligning ball bearings are widely used in applications where misalignment is expected due to the bending of the shaft, errors in the manufacture of the shaft, or mounting errors. The design engineer must keep in mind that there are always tolerances due to manufacturing errors. Self-aligning bearings can be applied for radial loads combined with moderate thrust loads. The feature that self-aligning bearings do not exert any bending moment on the shaft is particularly important in applications that require high precision (low radial run-out) at high speeds, because shaft bending causes imbalance and vibrations. The concept of self-alignment is useful in all types of bearings, including sleeve bearings.

12.2.1.3 Double-Row Deep-Groove Ball Bearing

This bearing type (Fig. 12-4) is used for relatively high radial loads. It is more sensitive to misalignment errors than the single row and should be used only for applications where minimal misalignment is expected. Otherwise, a self-alignment bearing should be selected.

The design of double-row ball bearings is similar to that of single-row ball bearings. Since double-row ball bearings are wider and have two rows, they can

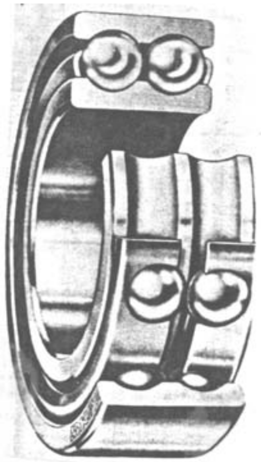

FIG. 12-4 Double-row deep-groove ball bearing.

carry higher radial loads. Unlike the deep-groove bearing, designs of split rings (for the maximum number of balls) are not used, and each ring is made from one piece. However, double-row bearings include groups with larger diameters and a larger number of balls to further improve the load capacity.

12.2.1.4 Angular Contact Ball Bearing

This bearing type (Fig. 12-5) is used to support radial and thrust loads. Contact angles of up to 40° (from the radial direction) are available from some bearing manufacturers, but 15° and 25° are the more standard contact angles. The contact angle determines the ratio of the thrust to radial load.

Angular contact bearings are widely used for adjustable arrangements, where they are mounted in pairs against each other and preloaded. In this way, clearances in the bearings are eliminated or even preload is introduced in the rolling contacts. This is often done to stiffen the bearings for a rigid support of the shaft. This is important for reducing the amplitude of shaft vibrations under oscillating forces. This type of design has significant advantages whenever precision is required (e.g., in machine tools), and it reduces vibrations due to imbalance. This is particularly important in high-speed applications. An adjustable arrangement is also possible in tapered bearings; however, angular contact ball bearings have lower friction than do tapered bearings. However, the friction of angular contact ball bearings is somewhat higher than that of radial ball bearings. Angular contact ball bearings are the preferred choice in many important applications, such as high-speed turbines, including jet engines.

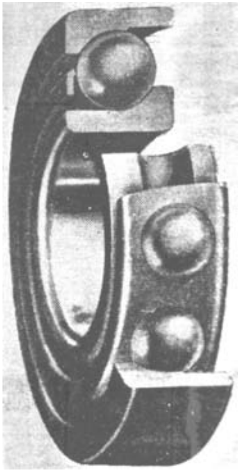

FIG. 12-5 Angular contact ball bearing.

Single-row angular contact ball bearings can carry considerable radial loads combined with thrust loads in one direction. Prefabricated mountings of two or more single-row angular contact ball bearings are widely used for two-directional thrust loads. Two bearings in series can be used for heavy unidirectional thrust loads, where two single-row angular contact ball bearings share the thrust load. Precise axial internal clearance and high-quality surface finish are required to secure load sharing of the two bearings in series. The bearing arrangement of two or more angular contact bearings facing the same direction is referred to as *tandem arrangement*. The bearings are mounted adjacent to each other to increase the thrust load carrying capacity.

12.2.2 Roller Bearings

Roller bearings have a theoretical line contact between the unloaded cylindrical rollers and races. This is in comparison to ball bearings, which have only a theoretical point contact with the raceways. Under load, there is elastic deformation, and line contact results in a larger contact area than that of a point contact in ball bearings. Therefore, roller bearings can support higher radial loads. At the same time, the friction force and friction-energy losses are higher for a line contact; therefore, roller bearings are usually not used for high-speed applications.

Roller bearings can be classified into four categories: cylindrical roller bearings, tapered roller bearings, needle roller bearings and spherical roller bearings.

12.2.2.1 Cylindrical Roller Bearings

The cylindrical roller bearing (Fig. 12-6) is used in applications where high radial load is present without any thrust load. Various types of cylindrical roller bearings are manufactured and applied in machinery. In certain applications where diameter space is limited, these bearings are mounted directly on the shaft, which serves as the inner race. For direct mounting, the shaft must be hardened to high Rockwell hardness, similar to that of the bearing race. For direct mounting, the radial load must be high in order to prevent slipping between the rollers and the shaft during the start-up. It is important to keep in mind that cylindrical roller bearings cannot support considerable thrust loads. Thus, for applications where both radial and thrust loading are present, it is preferable to use ball bearings.

12.2.2.2 Tapered Roller Bearing

The tapered roller bearing is used in applications where a high thrust load is present that can be combined with a radial load. The bearing is shown in Fig. 12-7. The races of inner and outer rings have a conical shape, and the rolling elements between them have a conical shape as well. In order to have a rolling motion, the contact lines formed by each of the various tapered roller elements

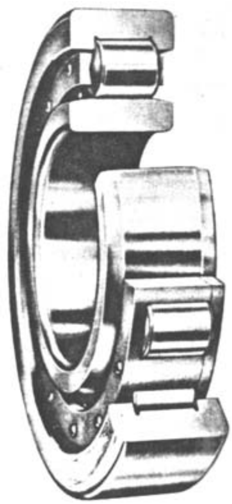

FIG. 12-6 Cylindrical roller bearing.

and the two races must intersect at a common point on the bearing axis. This intersection point is referred to as an *apex point*. The apex point is closer to the bearing when the cone angle is steeper. A steeper cone angle can support a higher thrust load relative to a radial load.

The inner ring is referred to as *cone*, while the outer ring is referred to as *cup*. The cone is designed with two retaining *ribs* (also known as *lips*) to confine

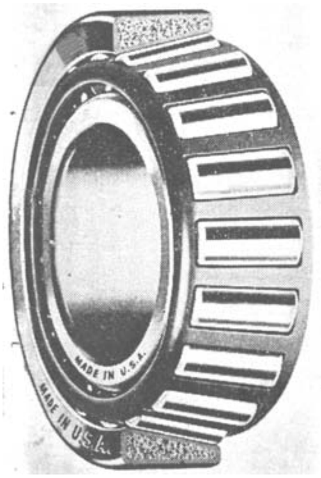

FIG. 12-7 Tapered roller bearing.

the tapered rollers as shown in Fig. 12-7. The ribs also align the rollers between the races. In addition, the larger rib has an important role in supporting the axial load. A cage holds the cone and rollers together as one unit, but the cup (outer ring) can be pulled apart.

A single-row tapered roller bearing can support a thrust load in only one direction. Two tapered roller bearings are usually mounted in opposition, to allow for thrust support in both directions (in a similar way to opposing angular contact ball bearings). Moreover, double or four-row tapered roller bearings are applied in certain applications to support a high bidirectional thrust load as well as radial load.

The reaction force on the cup acts in the direction normal to the line of contact of the rolling elements with the cup race (normal to the cup surface). This force can be divided into axial and radial load components. The intersection of the resultant reaction force (which is normal to the cup angle) with the bearing centerline is referred to as the effective center. The location of the effective center is useful in bearing load calculations.

For example, when a radial load is applied on the bearing, this produces both radial and thrust reactions. The thrust force component, which acts in the direction of the shaft centerline, can separate the cone from the cup by sliding the shaft in the axial direction through the cone or by the cup's sliding axially in its seat. To prevent such undesired axial motion, a single-row tapered bearing should be mounted with another tapered bearing in the opposite direction. This arrangement is also very important for adjusting the clearance.

One major advantage of the tapered roller bearing is that it can be applied in adjustable arrangement where two tapered roller bearings are mounted in opposite directions (in a similar way to the adjustable arrangement of the angular contact ball bearing that was discussed earlier). This arrangement allows one to eliminate undesired clearance and to provide a preload (interference or negative clearance). Bearing preload increases the bearing stiffness, resulting in reduced vibrations as well as a lower level of run-out errors in precision machining. However, the disadvantage of bearing preloading is additional contact stresses and higher friction. Preload results in lowering the speed limit because the higher friction causes overheating at high speeds.

The adjustment of bearing clearance can be done during assembly and even during steady operation of the machine. The advantage of adjustment during operation is the precise elimination of the clearance after the thermal expansion of the shaft.

12.2.2.3 Multirow Tapered Roller Bearings

The multirow tapered roller bearing (Fig. 12-8) is manufactured with a predetermined adjustment that enables assembly into a machine without any further adjustment. The multirow arrangement includes spacers and is referred to as a

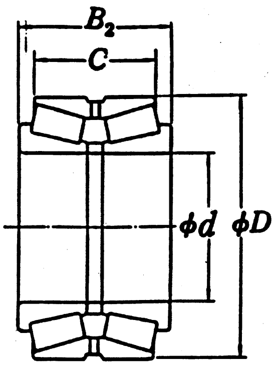

FIG. 12-8 Multirow tapered roller bearings.

spacer assembly. The spacer is matched with a specific bearing assembly during manufacturing. It is important to note that components of these assemblies are not interchangeable. Other types, without spacers, are manufactured with predetermined internal adjustment, and their components are also not interchangeable.

12.2.2.4 Needle Roller Bearing

These bearings (Fig. 12-9) are similar to cylindrical roller bearings, in the sense that they support high radial load. This type of bearing has a needlelike appearance because of its higher length-to-diameter ratio.

The objective of a needle roller bearing is to save space. This is advantageous in applications where bearing space is limited. Furthermore, in certain applications needle roller bearings can also be mounted directly on the

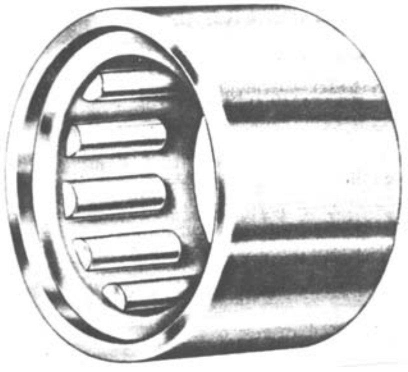

FIG. 12-9 Needle roller bearing.

shaft. For a direct mounting, the shaft must be properly hardened to a similar hardness of a bearing ring.

Two types of needle roller bearings are available. The first type, referred to as *full complement*, does not include a cage; the second type has a cage to separate the needle rollers in order to prevent them from sliding against each other. The full-complement bearing has more rollers and can support higher radial load. The second type has a lower number of rollers because it has a cage to separate the needle rollers to prevent them from rubbing against each other. The speed of a full-complement bearing is limited because it has higher friction between the rollers. A full-complement needle bearing may comprise a maximum number of needle rollers placed between a hardened shaft and a housing bore. An outer ring may not be required in certain situations, resulting in further saving of space.

12.2.2.5 Self-Aligning Spherical Roller Bearing

This bearing has barrel-shaped rollers (Fig. 12-10). It is designed for applications that involve misalignments due to shaft bending under heavy loads and due to manufacturing tolerances or assembly errors (in a similar way to the self-aligning ball bearing). The advantage of the spherical roller bearing is its higher load capacity in comparison to that of a self-aligning ball bearing, but it has higher frictional losses.

Spherical roller bearings are available as single-row, double-row, and thrust types. The single-row thrust spherical roller bearing is designed to support only thrust load, and it is not recommended where radial loads are present. Double-row

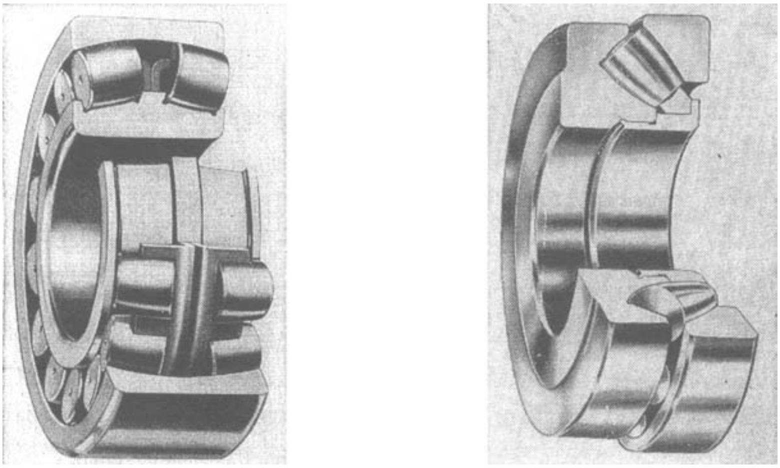

FIG. 12-10 (a) Spherical roller bearing, (b) spherical roller thrust bearing.

spherical roller bearings are commonly used when radial as well as thrust loads are present.

The double-row spherical roller bearing has the highest load capacity of all rolling bearings. This is due to the relatively large radius of contact of the rolling element. It can resist impact and other dynamic forces. It is used in heavy-duty applications such as ship shafts, rolling mills, and stone crushers.

12.3 HERTZ CONTACT STRESSES IN ROLLING BEARINGS

Hertz theory considers the elastic deformation and stress distribution near the contact of the rolling elements and races. Under load, due to an elastic deformation, the line or point contact becomes a contact area, This area is very small, resulting in a very high maximum contact pressure of the order of 1–5 GPa (1 GPa = 145,040 psi).

The calculations of the maximum contact pressure and deformation at the contact area of the rolling clement and raceways are according to Hertz's equations, which are based on the following assumptions.

1. The materials of the two bodies in contact are homogeneous and isotropic.
2. The yield point of the material is not exceeded, so plastic deformation is negligible and only elastic deformation is considered for Hertz's theory. Elastic deformation is recoverable after the load is removed.

In fact, assumption 2 is not completely accurate in practice. In heavily loaded rolling bearings there is a small plastic deformation at the contacts. However, experiments have verified Hertz's analysis. The actual stresses do not have any significant deviation from the values predicted by Hertz's theory.

3. In the contact area, only normal stresses are transmitted. Shear stresses due to friction on the surface are not considered in Hertz's theory.
4. The contact area is flat. The effect of any actual curvature can be disregarded for the analysis of stress distribution.

Concerning the last assumption, Hertz's theory is less accurate for bearings with a small radius of curvature, such as at the contact of a deep-groove ball bearing. The theory is more accurate for bearings with a large radius of curvature, e.g., at the contact of the outer ring of a self-aligning ball bearing. However, in all applications, deviations from the actual stresses are not significant for practical purposes of bearing design.

The ISO 281 standard refers to the limiting static load, C_0, of rolling-element bearings. Manufacturers' catalogues include this limit for rolling bearing static radial capacity, C_{or}. In Manufacturers' catalogues, this value is based on a limit stress of 4.2 GPa (609,000 psi) for 52100 steel and 3.5 GPa (508,000 psi) for 440C stainless steel. The static radial capacity, C_{or}, is based on the peak load, W_{max}, on one rolling element as well as additional transient and momentary overload on the same rolling element during start-up and steady operation.

At these ultimate pressure levels, the assumption of pure elastic deformation is not completely correct, because a minute plastic (irreversible) deformation occurs. For most applications, the microscopic plastic depression does not create a noticeable effect, and it does not cause a significant microcracking that can reduce the fatigue life. However, in applications that require extremely quiet or uniform rotation, a lower stress limit is usually imposed. For example, for bearings in satellite antenna tracking actuators, a static stress limit of only 2.2 GPa (320,000 psi,) is allowed on bearings made of 440C steel. This limit is because plastic deformation must be minimized for accurate functioning of the mechanism.

12.4 THEORETICAL LINE CONTACT

If a load is removed, there is only a line contact between a cylinder and a plane. However, under load, there is an elastic deformation at the contact, and the line contact becomes a rectangular contact area. The width of the contact is $2a$, as shown in Fig. 12-11. The magnitude of a (half-contact width) can be determined by the equation

$$a = R_x \left(\frac{8\bar{W}}{\pi} \right)^{1/2} \tag{12-2a}$$

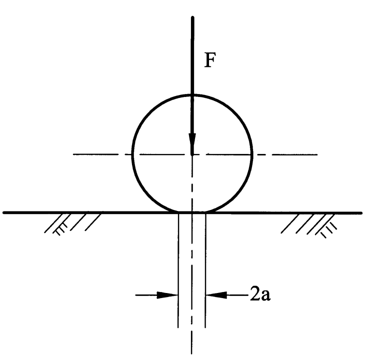

FIG. 12-11 Contact area of a cylinder and a plane.

Here, the dimensionless load, $\bar{W}$, is defined by

$$\bar{W} = \frac{W}{LE_{eq}R_x} \tag{12-2b}$$

The load W acts on the contact area. The effective length of the cylinder is L, and E_{eq} is the equivalent modulus of elasticity. In this case, R_x is an equivalent contact radius, which will be discussed in Sec. 12.4.2. For a contact of two different materials, the equivalent modulus of elasticity, E_{eq}, is determined by the following expression:

$$\frac{2}{E_{eq}} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \tag{12-3a}$$

Here ν_1 and ν_2 are Poisson's ratio and E_1, E_2 are the moduli of elasticity of the two materials in contact, respectively. If the two surfaces are made of identical materials, such as in standard rolling bearings, the equation is simplified to the form

$$E_{eq} = \frac{E}{1-\nu^2} \tag{12-3b}$$

For a contact of a cylinder and plane, R_x is the cylinder radius. The subscript x defines the direction of the coordinate x along the cylinder axis. In the case of a line contact between two cylinders, R_x is an equivalent radius (defined in Sec. 12.4.2) that replaces the cylinder radius.

The bearing load is distributed unevenly on several rolling elements. The maximum load on a single cylindrical roller, W_{max}, can be approximated by the following equation, which is based on the assumption of zero radial clearance in the bearing:

$$W_{max} \approx \frac{4W_{bearing}}{n_r} \tag{12-4}$$

Here, n_r is the number of cylindrical rolling elements around the bearing and $W_{bearing}$ is the total radial load on a bearing. For design purposes, the maximum load, W_{max}, is substituted in Eq. (12-2b) for the calculation of the contact width, which is used later in the equations of the maximum deformation and maximum contact pressure.

12.4.1 Effective Length

The actual line contact is less than the length of the cylindrical rolling element because the corners are rounded. The rounded part on each side is of an approximate length equal to the cylinder radius. For determining the effective length, the cylindrical roller diameter is subtracted from the actual length of the cylindrical rolling element, $L = L_{\text{actual}} - d$.

12.4.2 Equivalent Radius

Equations (12-2) are for a contact of a cylinder and a plane, as shown in Fig. 12-12. However, in cylindrical roller bearings, there is always a contact between two cylinders of different curvature. A theoretical line contact can be between convex or concave curvatures. In all these cases, an equivalent radius, R_x, of contact curvature can be used that replaces the cylinder radius in Eq. (12-2a) and (12-2b).

Case 1: Roller on a Plane. As stated earlier, for the simple example of a contact between a plane and cylinder of radius R (roller on a plane), the equivalent contact radius is $R_x = R$.

Case 2: Convex Contact. The second case is that of a convex line contact of two cylinders, as shown in Fig. 12-13. An example of this type of contact is that between a cylindrical roller and the inner ring race. The equivalent contact radius, R_x, that is substituted in Eqs. (12.2a and b) is derived from the following expression:

$$\frac{1}{R_x} = \frac{1}{R_1} + \frac{1}{R_2} \tag{12-5}$$

Here, R_1 and R_2 are the radii of two curvatures of the convex contact, and the subscript x defines the direction of the axis of the two cylinders.

Case 3: Concave Contact. A concave contact is shown in Fig. 12-14. An example of this type of contact is that between a cylindrical rolling element and the outer ring race. For a concave contact, radius R_1 is negative, because the contact is inside this circle. The result is that the equivalent radius is derived

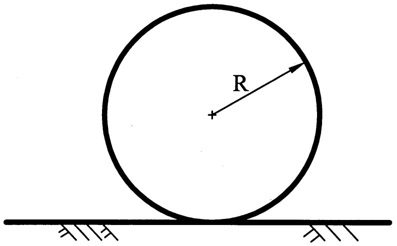

FIG. 12-12 Case 1: roller on a plane.

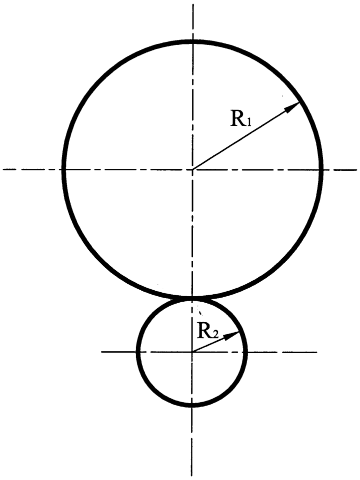

FIG. 12-13 Case 2: convex contact.

according to the following equation:

$$\frac{1}{R_x} = \frac{1}{R_2} - \frac{1}{R_1} \tag{12-6}$$

In a concave contact, the equivalent radius of contact is larger than each of the two radii in contact.

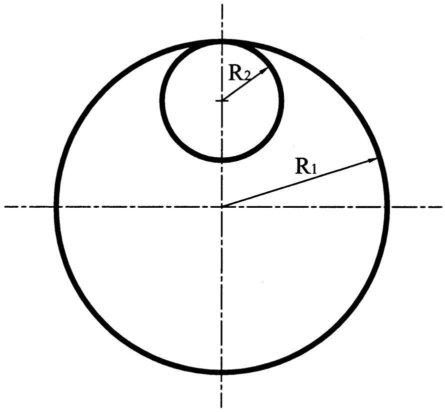

FIG. 12-14 Case 3: concave contact.

12.4.3 Deformation and Stresses in Line Contact

For a line contact, the maximum deformation of the roller in the direction normal to the contact area (vertical direction in Fig. 12-11) is

$$\delta_m = \frac{2\bar{W}R_x}{\pi}\left[\ln\left(\frac{2\pi}{\bar{W}}\right) - 1\right] \tag{12-7}$$

According to Hertz's theory, there is a parabolic pressure distribution at the contact area, as shown in Fig. 12-15. The maximum contact pressure is at the center of the contact area, and it is equal to

$$p_{\max} = E_{\mathrm{eq}}\left(\frac{\bar{W}}{2\pi}\right)^{1/2} \tag{12-8}$$

12.4.4 Subsurface Stress Distribution

An important feature in contact stresses is that the maximum shear stress is below the surface. In many cases, this is the reason for the development of subsurface fatigue cracks and eventually fatigue failure in rolling bearings. Three curves of dimensionless stress distributions below the surface and below the center of contact are shown in Fig. 12-16. The maximum pressure at the contact area center normalizes the stresses. The maximum shear, $\tau_{\max}$, is considered an important cause of failure. The ratio of the maximum shear stress to the maximum surface pressure ($\tau_{\max}/p_{\max}$) is plotted (maximum shear, $\tau_{\max}$, is at an angle of 45° to the z axis). The maximum value of this ratio is at a depth of $z = 0.78a$, and its magnitude is $\tau_{\max} = 0.3p_{\max}$.

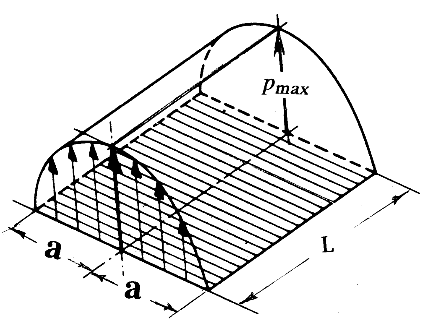

FIG. 12-15 Pressure distribution in a rectangular contact area.

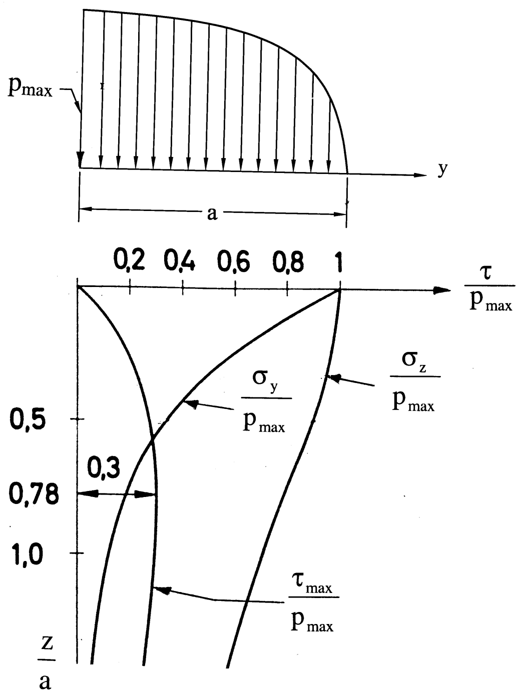

FIG. 12-16 Subsurface stresses in a rectangular contact area under p_{max}.

Example Problem 12-1

A cylindrical rolling bearing has an external load of $F = 11{,}000$ N. The bearing has the following dimensions: The diameter of the inner raceway D_{in} is 120 mm, and the diameter of the outer raceway D_{out} is 160 mm. The diameter of the cylindrical roller d_{roller} is 20 mm, and its effective length is $L = 10$ mm. There are 14 rolling elements around the bearing. The bearing (rollers and rings) is made of steel. The modulus of elasticity of the steel is $E = 2.05 \times 10^{11}$ N/m^2, and its Poisson ratio is $\nu = 0.3$.

Find the maximum pressure at the contact of the rolling elements and the raceways. Compare the maximum pressure where the rolling element contacts the inner and the outer raceways.

Solution

The radii of the contacting curvatures are

$$R_{roller} = 0.01 \text{ m}, \qquad R_{outer\ raceway} = 0.08 \text{ m}, \qquad R_{inner\ raceway} = 0.06 \text{ m}$$

The contact between the rolling elements and the inner raceway is convex, and the equivalent contact curvature, $R_{x,\mathrm{in}}$ is derived according to the equation

$$\frac{1}{R_{x,\mathrm{in}}} = \frac{1}{R_{\mathrm{roller}}} + \frac{1}{R_{\mathrm{inner\ raceway}}}$$

$$\frac{1}{R_{x,\mathrm{in}}} = \frac{1}{0.01} + \frac{1}{0.06} \qquad \Rightarrow R_{\mathrm{x,in}} = 0.0085 \text{ m}$$

However, the contact between the rolling elements and the outer raceway is concave, and the equivalent contact curvature, $R_{x,\mathrm{out}}$ is derived according to the equation

$$\frac{1}{R_{x,\mathrm{out}}} = \frac{1}{R_{\mathrm{roller}}} - \frac{1}{R_{\mathrm{outer\ raceway}}}$$

$$\frac{1}{R_{x,\mathrm{out}}} = \frac{1}{0.01} - \frac{1}{0.08}; \qquad R_{x,\mathrm{out}} = 0.0114 \text{ m}$$

If we assume that the radial clearance between rolling elements and raceways is zero, the maximum load on one cylindrical rolling element can be approximated by Eq. (12-4):

$$W_{\max} = \frac{4W_{\mathrm{bearing}}}{n} = \frac{4(11{,}000)}{14} = 3142 \text{ N}$$

Here, n is the number of cylindrical rolling elements in the bearing, W_{bearing} is the total bearing load capacity, and $W_{\max}$ is the maximum load capacity of one rolling element. The shaft and the bearing are made of identical material, so the equivalent modulus of elasticity is calculated as follows:

$$E_{\mathrm{eq}} = \frac{E}{1-\nu^2} \Rightarrow E_{\mathrm{eq}} = \frac{2.05 \times 10^{11}}{1-0.3^2} = 2.25 \times 10^{11} \text{ N/m}^2$$

The dimensionless maximum force at the contact between one rolling element and the inner ring race is

$$\text{Inner ring:} \quad \bar{W} = \frac{1}{E_{\mathrm{eq}} R_{x,\mathrm{in}} L} W_{\max} = \frac{1}{2.25 \times 10^{11} \times 0.0085 \times 0.01} \times 3142$$
$$= 1.64 \times 10^{-4}$$

In comparison, the dimensionless maximum force at the contact between one rolling element and the outer ring race is

$$\text{Outer ring:} \quad \bar{W} = \frac{1}{E_{\mathrm{eq}} R_{\mathrm{eq,out}} L} W_{\max} = \frac{1}{2.25 \times 10^{11} \times 0.0114 \times 0.01}$$
$$\times 3142 = 1.22 \times 10^{-4}$$

Comparison of the inner and outer dimensionless loads indicates a higher value for the inner contact. This results in higher contact stresses, including maximum pressure at the convex contact with the inner ring. The maximum pressure at the contact with the inner ring race is obtained via Eq. (12-8).

$$p_{\max,\text{in}} = E_{\text{eq}}\left(\frac{\bar{W}}{2\pi}\right)^{1/2} = 2.25 \times 10^{11} \times \left(\frac{1.64 \times 10^{-4}}{2\pi}\right)^{1/2} = 1.15 \times 10^{9}\ \text{Pa}$$
$$= 1.15\ \text{GPa}$$

At the same time, the maximum pressure at the contact with the outer ring race is lower:

$$p_{\max,\text{in}} = E_{\text{eq}}\left(\frac{\bar{W}}{2\pi}\right)^{1/2} = 2.25 \times 10^{11} \times \left(\frac{1.22 \times 10^{-4}}{2\pi}\right)^{1/2} = 0.99 \times 10^{9}\ \text{Pa}$$
$$= 0.99\ \text{GPa}$$

For regular-speed operation, it is sufficient to calculate the maximum pressure at the inner contact, because the stresses at the outer contact are lower (due to the concave contact). However, at high speed the centrifugal force of the rolling element increases the maximum pressure at the contact with the outer ring race relative to that of the inner ring. Therefore, at high speed, the centrifugal force is considered and the maximum pressure at the inner and outer contact should be calculated.

12.5 ELLIPSOIDAL CONTACT AREA IN BALL BEARINGS

If there is no load, there is a point contact between a sphere and a flat plane. Under load, the point contact becomes a circular contact area. However, in ball bearings the races have different curvatures in the direction of rolling and in the axial direction of the bearing. Therefore, the two bodies form an elliptical contact area. The elliptical contact area has radii a and b, as shown in Fig. 12-17.

12.5.1 Race and Ball Conformity

In a deep-groove radial ball bearing, the radius of the deep groove is always a little larger than that of the ball. A race conformity is the ratio R_r, defined as (see Hamrock and Anderson, 1973)

$$R_r = \frac{r}{d} \tag{12-9a}$$

Here, r and d are the deep-groove radius and ball diameter, respectively, as shown in Fig. 12-18a. A perfect conformity is $R_r = 0.5$. However, in order to reduce the

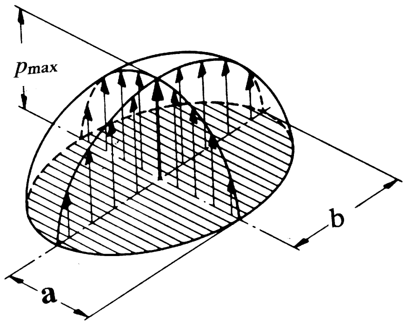

FIG. 12-17 Pressure distribution in an elliptical contact area.

friction, bearings are manufactured with a conformity ratio in the range $0.51 \leq R_r \leq 0.54$. On the other hand, if the conformity ratio is too high, it results in higher maximum contact stresses.

12.5.2 Equivalent Radius in Ball Bearing Contacts

In Fig. 12-18b, two orthogonal cross sections are shown. Each cross section shows a plane of contact of two curvatures. The left-hand cross section is the *x-z* plane, which is referred to as the *y* plane, because it is normal to the *y* coordinate. The left-hand cross section is of the inner ring in contact with a rolling ball. The contact between a ball and deep-groove curvature of the inner ring is a concave contact.

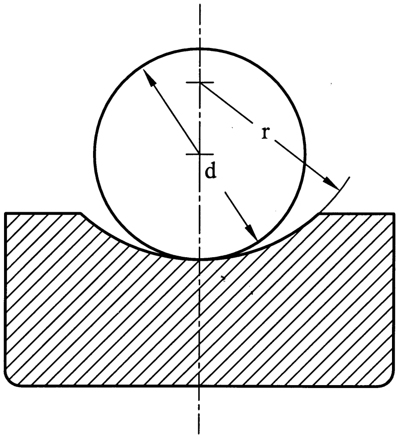

FIG. 12-18a Race and ball conformity.

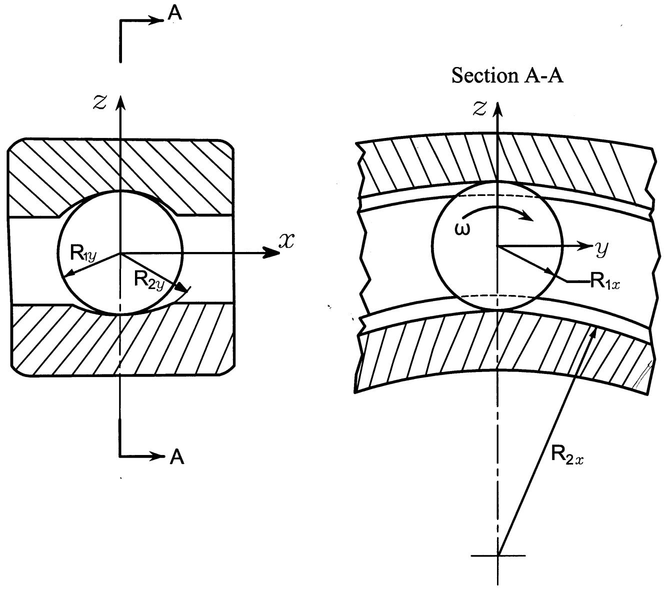

FIG. 12-18b Curvatures in contact in two orthogonal cross sections of a ball bearing (x-z and y-z planes).

In the y plane (left-hand cross section), the two radii of contact have a notation of subscript y. The radii of the curvatures in contact are R_{1y} and R_{2y}, respectively. The small radius, R_{1y}, is of the rolling ball:

$$R_{1y} = \frac{d}{2} \tag{12-9b}$$

Here, d is the rolling ball diameter. The concave curvature of the deep groove is of the somewhat larger radius R_{2y}. The equivalent contact radius in the y plane is R_y (equal to r in Fig. 12-18a). For a concave contact, the equivalent radius of contact between the ball and the deep groove of the inner ring race is calculated by the equation

$$\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} \tag{12-10}$$

The right-hand cross section in Fig. 12-18b is in the y-z plane. This plane is referred to as the x plane, because it is normal to the x direction. It shows a cross section of the rolling plane where a ball is rolling around the inner ring. This is a convex contact between the ball and the inner ring race. The ball has a rolling contact at the bottom diameter of the deep groove of the inner ring race.

In the x plane, the radii of the curvatures in convex contact are R_{1x} and R_{2x}, which are of the ball and of the lowest point of the inner ring deep groove, respectively. The rolling ball radius is $R_{1x} = R_{1y} = d/2$, while the inner ring race radius at the bottom of the deep groove is R_{2x}, as shown on the right-hand side of Fig. 12-18b. The equivalent contact radius, R_x, is in the x plane of the convex contact with the inner ring (the subscript x indicates that the equivalent radius is in the x plane). The equivalent contact radius, R_x, is derived from the equation

$$\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}} \tag{12-11}$$

The radius ratio, α_r, is defined as

$$\alpha_r = \frac{R_y}{R_x} \tag{12-12}$$

This is the ratio of the larger radius to the smaller radius, $\alpha_r > 1$. The equivalent contact radius R_{eq} is obtained from combining the equivalent radius in the two orthogonal planes, as follows:

$$\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{R_y} \tag{12-13}$$

The combined equivalent radius of curvature, R_{eq}, is derived from the contact radii of curvature R_x and R_y, in the two orthogonal cross sections shown in Fig. 12-18b. The equivalent radius R_{eq} is used in Sec. 12.5.4 to calculate the deformation and pressure distribution in the contact area.

12.5.3 Stresses and Deformation in an Ellipsoidal Contact

According to Hertz's theory, the equation of the pressure distribution in an ellipsoidal contact area in a ball bearing is

$$p = \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{1/2} p_{max} \tag{12-14}$$

Here, a and b are the small radius and the large radius, respectively, of the ellipsoidal contact area, as shown in Fig. 12-17. The maximum pressure at the center of an ellipsoidal contact area is given by the following Hertz equation:

$$p_{max} = \frac{3}{2} \frac{W}{\pi ab} \tag{12-15}$$

Here, $W = W_{max}$ is the maximum load on one spherical rolling element. The maximum pressure is proportional to the load, and it is lower when the contact area is larger. The contact area is proportional to the product of a and b of the ellipsoidal contact area. The contact area is inversely proportional to the modulus of elasticity of the material. For example, soft materials such as rubbers have a large contact area and the maximum pressure is relatively low. In contrast, steel has high elasticity modulus, resulting in a small area and high stresses. The equations for calculating a and b are given in Sec. 12.5.4.

12.5.4 Ellipsoidal Contact Area Radii

The ellipticity parameter, k, is defined as the ratio of the large radius to the small radius:

$$k = \frac{b}{a} \tag{12-16}$$

The exact solution for the ellipsoid radii a and b is quite complex. For design purposes, Hamrock and Brewe (1983) suggested an approximate solution. The equations allow a simplified solution for the deformation and pressure distribution in the contact area. The ellipticity parameter, k, is estimated by the equation

$$k \approx \alpha_r^{2/\pi} \tag{12-17}$$

The ratio α_r is defined in Eq. 12-12. The following parameter, q_a, is used to estimate the dimensionless variable $\hat{E}$ that is used for the approximate solution of the ellipsoid radii:

$$q_a = \frac{\pi}{2} - 1 \tag{12-18}$$

$$\hat{E} \approx 1 + \frac{q_a}{\alpha_r} \qquad \text{for } \alpha_r \geq 1 \tag{12-19}$$

The ellipsoid radii a and b can now be estimated

$$a = \left(\frac{6\hat{E}WR_{\text{eq}}}{\pi k E_{\text{eq}}} \right)^{1/3} \tag{12-20a}$$

$$b = \left(\frac{6k^2\hat{E}WR_{\text{eq}}}{\pi E_{\text{eq}}} \right)^{1/3} \tag{12-20b}$$

Here, W is the load on one rolling element. The rolling elements do not share the load equally. At any time there is one rolling element that carries the maximum load, W_{max}. The maximum pressure is at the contact of the rolling element of maximum load.

The maximum deformation in the direction normal to the contact area is calculated by means of the following expression, which includes estimated terms:

$$\delta_m = \hat{T}\left[\frac{9}{2\hat{E}R_{\text{eq}}}\left(\frac{W}{\pi k E_{\text{eq}}}\right)^2\right]^{1/3} \tag{12-21}$$

Here, the following estimation for $\hat{T}$ is used:

$$\hat{T} \approx \frac{\pi}{2} + q_a \ln \alpha_r \qquad \text{for } \alpha_r \geq 1 \tag{12-22}$$

For ball bearings, the maximum load, $W_{\max}$, on one rolling element can be estimated by the equation

$$W_{\max} \approx \frac{5W_{\text{bearing}}}{n_r} \tag{12-23}$$

Here, n_r is the number of balls in the bearing. The maximum load, $W_{\max}$, on one ball is substituted for the load W in Eqs. (12-15), (12-20a), (12-20b), and (12-21).

12.5.5 Subsurface Shear

Fatigue failure develops from subsurface cracks. These cracks propagate whenever there are alternating stresses and the maximum shear stress is high. It is important to evaluate the shear stresses below the surface that can cause fatigue failure.

The following is the maximum value of the shear, τ_{yz}, in the orthogonal direction (acting below the surface on a vertical plane y-z; see Fig. 12-18b. In fact, the maximum shear is in a plane inclined 45° to the vertical plane. However, the classical work of Lundberg and Palmgren (1947) on estimation of rolling bearing fatigue life is based on the maximum value of the orthogonal shear stress, τ_{yz}, acting on a vertical plane:

$$\tau_{xy} = p_{\max}\frac{(2t^* - 1)^{1/2}}{2t^*(t^* + 1)} \tag{12-24}$$

where the following estimation can be applied:

$$t^* \approx 1 + 0.16 \operatorname{csc}h\left(\frac{k}{2}\right) \tag{12-25}$$

12.5.6 Comment on Precise Solution by Elliptical Integrals

The calculation in Sec. 12.5.4 approximates a complex solution procedure that is based on the following equations [see Harris (1966)]:

$$k = \left[\frac{2\hat{T} - \hat{E}(1 + R_d)}{\hat{E}(1 - R_d)} \right]^{1/2} \tag{12-26}$$

where R_d is the curvature difference defined by the equation

$$R_d = R_{\mathrm{eq}} \left(\frac{1}{R_x} - \frac{1}{R_y} \right) \tag{12-27}$$

The two elliptical integrals are defined as follows:

$$\hat{T} = \int_0^{\pi/2} \left[1 - \left(1 - \frac{1}{k^2} \right) \sin^2 \phi \right]^{-1/2} d\phi \tag{12-28}$$

$$\hat{E} = \int_0^{\pi/2} \left[1 - \left(1 - \frac{1}{k^2} \right) \sin^2 \phi \right]^{1/2} d\phi \tag{12-29}$$

An iteration method has developed by Hamrock and Anderson (1973) to solve for the two elliptical integrals and k. The solutions of the elliptical integrals and k are presented by graphs as a function of the radius ratio α_r. The use of the graphs or tables allows a precise solution. However, the approximate solution in Sec. 12.5.4 is sufficient for design purposes.

Example Problem 12-2

Maximum Contact Pressure in a Deep-Groove Ball Bearing

Find the maximum contact pressure and maximum deformation, δ_m, of a deep-groove ball bearing in the direction normal to the contact area. The bearing speed is low, so the centrifugal forces of the rolling elements are negligible. Therefore, calculate only the maximum values at the contact with the inner ring race.

The radial load on the bearing is $W = 10{,}500$ N. The bearing has 14 balls of diameter $d = 19.04$ mm. The radius of curvature of the inner deep groove (in cross section x-z in Fig. 12-18b) is 9.9 mm. The inner race diameter (at the bottom of the deep groove) is $d_i = 76.5$ mm (cross section y-z).

The rolling elements and rings are made of steel. The modulus of elasticity of the steel is $E = 2 \times 10^{11}$ N/m^2, and its Poisson ratio is $\nu = 0.3$.

Compare the maximum pressure to the allowed compression stress of 3.5 GPa (3.5×10^9 N/m^2) for a bearing made of 52100 steel.

Solution

Referring to the right-hand side of Fig. 12-18b, the two radii of contact curvatures at the inner ring race in the y-z plane (referred to as the x plane) are:

Inner ring radius: $R_{2x} = \dfrac{d_i}{2} = 38.25$ mm

Ball radius: $R_{1x} = \dfrac{d}{2} = 9.52$ mm

Equivalent Radius of Contact, R_x, in the x Plane at the Inner Ring

$$\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}} \Rightarrow \frac{1}{R_x} = \frac{1}{9.52} + \frac{1}{38.25}; \qquad R_x = 7.62 \text{ mm}$$

On the left-hand side of Fig. 12.18b, the two radii of contact curvatures at the inner ring race in the x-z plane (referred to as the y plane) are:

Ball radius: $R_{1y} = 9.52$ mm
Deep-groove radius: $R_{2y} = 9.9$ mm

Equivalent Radius of Contact, R_y, in the y Plane at the Inner Ring

$$\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} \Rightarrow \frac{1}{R_y} = \frac{1}{9.52} - \frac{1}{9.9}; \qquad R_y = 248.0 \text{ mm}$$

The combined equivalent contact radius of curvature, R_{eq}, of the inner ring and ball contact is derived from the equivalent contact radii in the x plane and y plane according to the equation

$$\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{R_y} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{7.62} + \frac{1}{248}; \qquad R_{eq} = 7.4 \text{ mm}$$

and the ratio α_r becomes

$$\alpha_r = \frac{R_y}{R_x} = \frac{248}{7.62} = 32.55$$

The dimensionless coefficient k is derived directly from the ratio α_r:

$$k = \alpha_r^{2/\pi} = 32.55^{2/\pi} = 9.18$$

It is also necessary to calculate $\hat{E}$, which will be used for the calculation of the ellipsoid radii. For that purpose, q_a is required:

$$q_a = \frac{\pi}{2} - 1 = 0.57$$

$$\hat{E} \approx 1 + \frac{q_a}{\alpha_r} \Rightarrow 1 + \frac{0.57}{32.55} = 1.02$$

The shaft and the bearing are made of identical material, so the equivalent modulus of elasticity is calculated as follows:

$$E_{\mathrm{eq}} = \frac{E}{1 - \nu^2} \Rightarrow E_{\mathrm{eq}} = \frac{2 \times 10^{11}}{1 - 0.3^2} = 2.2 \times 10^{11}\ \mathrm{N/m^2}$$

The maximum load at the contact of one rolling element can be estimated by the following equation:

$$W_{\max} \approx \frac{5W_{\mathrm{shaft}}}{n_r} = \frac{5(10{,}500)}{14} = 3750\ \mathrm{N}$$

Here, $n_r = 14$ is the number of balls in the bearing.

The ellipsoid radii a and b can now be determined by substitution of the values already calculated (in SI units):

$$a = \left(\frac{6\hat{E}W_{\max}R_{\mathrm{eq}}}{\pi \mathrm{k} E_{\mathrm{eq}}}\right)^{1/3} = \left(\frac{6 \times 1.02 \times 3750 \times 0.0074}{\pi \times 9.18 \times 2.2 \times 10^{11}}\right)^{1/3}$$
$$= 0.3 \times 10^{-3}\mathrm{m} = 0.3\ \mathrm{mm}$$
$$b = \left(\frac{6k^2\hat{E}W_{\max}R_{\mathrm{eq}}}{\pi E_{\mathrm{eq}}}\right)^{1/3} = \left(\frac{6 \times 9.18^2 \times 1.02 \times 3750 \times 0.0074}{\pi \times 2.2 \times 10^{11}}\right)^{1/3}$$
$$= 2.75 \times 10^{-3}\mathrm{m} = 2.75\ \mathrm{mm}$$

The contact load $W_{\max}$ taken here is the maximum load on one rolling element. The maximum pressure at the contact with the inner ring race can now be determined from Eq. (12-15):

$$p_{\max} = \frac{3}{2}\frac{W_{\max}}{\pi ab} = \frac{3}{2}\frac{3750}{\pi \times 0.3 \times 2.87} = 2170.3\ \mathrm{N/mm^2} = 2.17 \times 10^9\ \mathrm{N/m^2}$$

or

$$p_{\max} = 2.17\ \mathrm{GPa}$$

In this case, the calculated maximum pressure is below the allowed compression stress of 3.5 GPa (3.5×10^9 N/m^2) for rolling bearings made of 52100 steel.

Deformation Normal to the Contact Area

For the purpose of calculating the maximum deformation, δ_m, the approximation for $\hat{T}$ is determined from the following equation:

$$\hat{T} \approx \frac{\pi}{2} + q_a \ln \alpha_r = \frac{\pi}{2} + 0.57 \times \ln 32.55 = 3.56$$

All the variables are now known and can be substituted in Eq. (12-21) to solve for the maximum elastic deformation at the contact (of one rolling element) in the direction normal to the contact area:

$$\delta_m = \hat{T}\left[\frac{9}{2\hat{E}R_{eq}}\left(\frac{W_{max}}{\pi k E_{eq}}\right)^2\right]^{1/3}$$

$$\delta_m = 3.56\left[\frac{9}{2 \times 1.02 \times 0.0074} \times \left(\frac{3750}{\pi \times 9.18 \times 2.2 \times 10^{11}}\right)^2\right]^{1/3}$$

$$= 2.12 \times 10^{-5}\ \mathrm{m} = 21.2\ \mu\mathrm{m}$$

12.6 ROLLING-ELEMENT SPEED

The velocity of a rolling-element center, U_r, is important for the calculation of the centrifugal forces. In addition, the rolling angular speed is required for elastohydrodynamic fluid film computations. The rolling speed is the velocity at which the rolling-element contact is progressing relative to a fixed point on the race.

12.6.1 Velocity of the Rolling-Element Center

The velocity diagram of a rolling element is shown in Fig. 12-19. It is for a stationary outer ring and rotating inner ring. The inner ring rotates together with the shaft at an angular speed ω. The velocity of a rolling-element center is shown

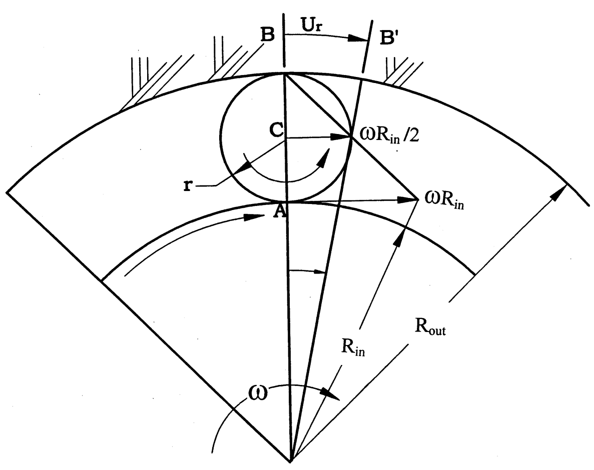

FIG. 12-19 Velocity diagram of a rolling element.

as a vector in the tangential direction; see point C. The rolling element has pure rolling (no slip over the races). The inner ring radius (at the contact) is R_{in}, and that of the outer ring is R_{out}. The velocity of the inner ring at contact point A is

$$U_A = \omega R_{in} \tag{12-30}$$

If the outer ring is stationary, point B is an instantaneous center of rotation. There is a linear velocity distribution along the line AB of the rolling element. For pure rolling, the velocity of point A on the inner ring is equal to that of point A on the rolling element U_A because there is no relative sliding between the two.

The velocity U_c of the rolling element center, point C, is half of that of point A, being at half the distance from the center of rotation B. The velocity of point C is

$$U_C = \frac{\omega R_{in}}{2} \tag{12-31}$$

12.6.2 Angular Velocity of the Rolling-Element Center

The angular velocity of the rolling-element center ω_C, together with the cage, is lower than the bearing (or shaft) speed. The angular velocity ω_C of the rolling element center, point C, is equal to the velocity U_C divided by the distance $(R_{in} + r)$ of point C from the bearing center O. The angular velocity ω_C of the rolling-element center is given by

$$\omega_C = \frac{R_{in}}{2(R_{in} + r)}\omega = \frac{R_{in}}{R_{in} + R_{out}}\omega \tag{12-32}$$

12.6.3 Rolling Velocity

Contact point B is moving due to the rolling action. Point B moves around the outer ring race at a rolling speed U_r. The rolling speed of point B can be determined by the angular motion of point C, because the contact point B is always on the line OC; therefore,

$$U_r = U_{\text{rolling of point } B} = \omega_C \overline{OB} = \omega_C R_{out} \tag{12-33}$$

Substituting the value at ω_C from Eq. (12-32) into Eq. (12-33), the expression for the rolling speed becomes

$$U_{\text{rolling}} = \frac{R_{in} R_{out}}{2(R_{in} + r)}\omega = \frac{R_{in} R_{out}}{R_{in} + R_{out}}\omega \tag{12-34}$$

Here, R_{out} is the radius of the outer ring raceway and R_{in} is the radius of the inner ring raceway; see Fig. 12-19. The angular speed ω (rad/s) is of a stationary outer ring and rotating inner ring, or vice versa.

The rolling speed U_r can also be written as a function of the inside and outside diameters, d_i and d_o respectively,

$$U_r = \frac{1}{2}\frac{d_i d_o}{d_i + d_o}\omega \tag{12-35}$$

Equations (12-32) and (12-35) also apply to the case where the inner ring is stationary and the outer ring rotates at angular speed ω. In addition, the rolling speeds at points A and B are equal, because there is no slip.

12.6.4 Centrifugal Forces of Rolling Elements

The angular velocity, ω_C, is used for solving for the centrifugal force, F_c, of an individual rolling element:

$$F_c = m_r \omega_C^2 (R_{\text{in}} + r) \tag{12-36}$$

The angular speed of the rolling-element center, ω_C is determined from Eq. (12-32). The volume of a rolling ball and its material density, ρ, determine the ball mass, m_r:

$$m_r = \frac{\pi d^3}{6}\rho \tag{12-37}$$

Here, d is the ball diameter. For a standard bearing, the density of steel is about $\rho = 7800\ \text{kg/m}^3$. In comparison, silicon nitride has a much lower density, $\rho = 3200\ \text{kg/m}^3$.

12.7 ELASTOHYDRODYNAMIC LUBRICATION IN ROLLING BEARINGS

Elastohydrodynamic (EHD) lubrication theory is concerned with the formation of a thin fluid film at the contact area of a rolling element and a raceway (see Dowson and Higginson, 1966). Under favorable conditions of speed, load, and fluid viscosity, the elastohydrodynamic fluid film can be of sufficient thickness to separate the rolling surfaces. Rolling bearings operating with a full EHD film have significant reduction of wear. However, even mixed lubrication would be beneficial in wear reduction, and much longer bearing life is expected in comparison to dry bearings.

As has been discussed in the previous sections, in cylindrical rolling bearings there is a theoretical line contact between rolling elements and raceways, whereas in ball bearings there is a point contact. However, due to elastic deformation, there is a small contact area where a thin fluid film is generated due to the rotation of the rolling elements. In a similar way to the formation of fluid film in plain bearings, the oil adheres to the surfaces, resulting in a squeeze-film effect between the rolling surfaces.

If the film thickness exceeds the size of the surface asperities, it can completely separate the rolling surfaces and thus eliminate wear due to a direct contact. Theoretically, the stresses under an EHD fluid film are similar to the Hertz stresses of dry contact, and the fluid film is not expected to improve the fatigue life. However, in practice the fluid-film acts as a damper and reduces dynamic stresses due to impact and vibrations. In this way, it improves the fatigue life as well as wear resistance. Tests indicated (Tallian, 1967) that the fatigue life is significantly improved for rolling bearings operating with a full EHD film.

In hydrodynamic journal bearings, the minimum film thickness depends only on one fluid property, the viscosity of the lubricant. In comparison, the formation of a elastohydrodynamic fluid film is more complex and depends on several physical properties of the fluid as well as of the solid material in contact. In addition to the effect of viscosity, two additional important effects are involved in the formation of a elastohydrodynamic fluid film: elastic deformation of the surfaces at the contact and a rise of viscosity with pressure. In summary, the film thickness depends on the following properties of the rolling bearing material and lubricant.

The elastohydrodynamic film thickness depends on the elastic properties of the two materials in contact, namely, their elastic modulus E and Poisson's ratio ν.

The pressure at the contact area is high. The viscosity of lubricant increases significantly with pressure. Therefore, the elastohydrodynamic film thickness depends on the pressure–viscosity coefficient $\alpha[m^2/N]$ as well as the absolute viscosity at atmospheric pressure.

The fluid film and the hydrodynamic pressure wave are shown in Fig. 12-20. The fluid film thickness increases with the rolling speed. The clearance thickness, h_0, is nearly constant along the fluid film, and it reduces only near the outlet (right side), where the minimum film thickness is h_{min}.

The left-hand side of Fig. 12-20 is a comparison between the EHD pressure wave and the Hertz pressure of a dry contact. The fluid film pressure wave increases from the inlet and reaches a peak equal to the maximum Hertz pressure, p_{max}, at the contact area center. After that, the pressure decreases, but rises again with a sharp spike near the outlet side, where the gap narrows to h_{min}. Under high loads and low speeds, the EHD pressure distribution is similar to that of Hertz theory, because the influence of the elastic deformation dominates the pressure distribution. However, at high speed the hydrodynamic effect prevails, and the EHD pressure spike is relatively high.

Generally, the minimum thickness of the lubricant film is of the order of a few tenths to 1 micrometer; however optimal conditions of high speed and adequate viscosity, a film thickness of several micrometers can be generated. For critical applications, such as high-speed turbines, it is very important to minimize

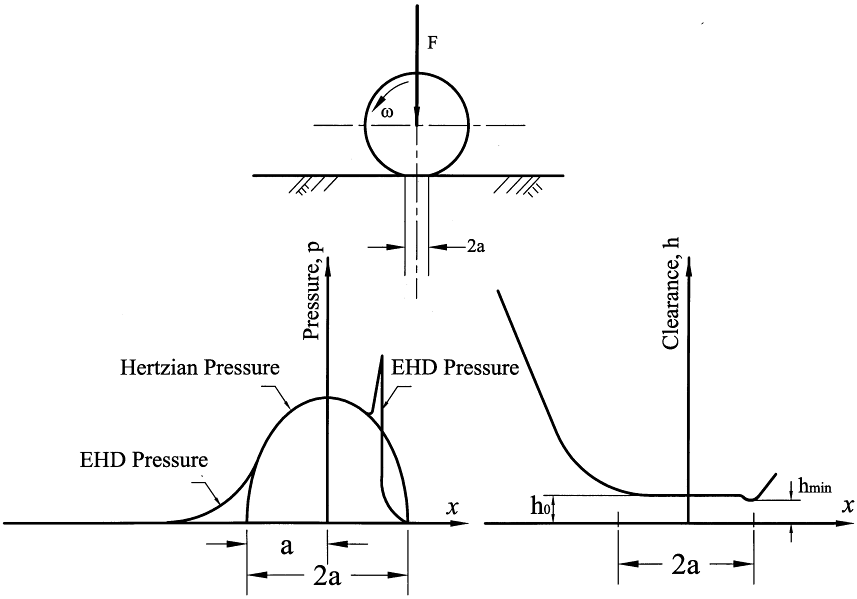

FIG. 12-20 Pressure wave and film thickness distribution in elastohydrodynamic lubrication.

wear and optimize the bearing life. In such cases, it is possible to have an optimal design that operates with a full EHD fluid film. For this purpose, the designer can calculate the minimum film thickness, $h_{\min}$, according to the equations discussed in the following sections.

For optimal conditions, the minimum film thickness, $h_{\min}$, must be greater than the surface roughness. The equivalent surface roughness at the contact, R_s (RMS), is obtained from the roughness of the two individual surfaces in contact, R_{s1} and R_{s2}, from the equation (see Hamroch, 1994)

$$R_s = (R_{s1}^2 + R_{s2}^2)^{1/2} \tag{12-38}$$

The surface roughness is measured by a profilometer, often referred to as *stylus measurement*. Microscope devices are used when higher-precision measurements are needed. The ratio Λ of the film thickness to the size of surface asperities, R_s, is:

$$\Lambda = \frac{h_{\min}}{R_s} \tag{12-39}$$

For a full EHD lubrication of rolling bearings, Λ is usually between 3 and 5. The desired ratio Λ is determined according to the expected level of vibrations and other disturbances in the machine. Although h_{min} is very important for successful bearing operation, calculation of h_0 is often required for determining the viscous shear force, referred to as *traction force*, which is the resistance to relative sliding.

12.8 ELASTOHYDRODYNAMIC LUBRICATION OF A LINE CONTACT

A theoretical line contact is formed between two cylinders, such as in a cylindrical rolling bearing. For this case, Pan and Hamrock (1989) introduced the following empirical equation for the minimum film thickness, h_{min}:

$$\frac{h_{min}}{R_x} = \frac{1.714 \bar{U}_r^{0.694} (\alpha E_{eq})^{0.568}}{\bar{W}^{0.128}} \tag{12-40}$$

Here, α is the viscosity–pressure coefficient. Although theoretical equations were derived earlier, Eq. (12-40) is more accurate, because it is based on actual measurements. The equation is a function of dimensionless terms. The advantage in using dimensionless terms is that any system of units can be used, as long as the units are consistent and result in dimensionless terms.

The dimensionless terms $\bar{U}_r$ and $\bar{W}$ are the dimensionless rolling velocity and load per unit of cylinder length, respectively. The dimensionless terms are defined by:

$$\bar{U}_r = \frac{\mu_0}{E_{eq} R_x} U_r \qquad \bar{W} = \frac{1}{E_{eq} R_x L} W \tag{12-41}$$

Here, U_r is the rolling velocity, η_0 is the viscosity of the lubricant at atmospheric pressure, W is the load on one rolling element, E_{eq} is the equivalent modulus of elasticity, and R_x is an equivalent radius of contact in the x plane (direction of axis of rotation of rolling element), as defined in Sec. 12.5.2.

Jones et al. (1975) measured the viscosity–pressure coefficient α [m^2/N] for various lubricants. The data is summarized in Table 12-1.

The rolling velocity $\bar{U}_r$ in Eq. (12-41) is for pure rolling (no relative sliding) (Eq. (12-34)). However, in many other problems, such as in gears and cams, there is a combination of rolling and sliding. If the ratio of the rolling to the sliding is ξ and U_s is the sliding velocity, then the dimensionless rolling velocity $\bar{U}_r$ used in Eq. (12-41) is replaced by

$$\bar{U}_r = \frac{\mu_0}{2 E_{eq} R_x} U_s (1 + \xi) \tag{12-42}$$

Equation (12-40) indicates that the viscosity and rolling speed have a more significant effect on the minimum film thickness, h_{min}, in comparison to the load

TABLE 12-1 Viscosity–Pressure Coefficient (α [m^2/N]) for Various Lubricants

Fluid	Temperature t_m 38°C	99°C	149°C
Ester	1.28×10^{-8}	0.987×10^{-8}	0.851×10^{-8}
Formulated ester	1.37	1.00	0.874
Polyalkyl aromatic	1.58	1.25	1.01
Synthetic paraffinic oil	1.77	1.51	1.09
Synthetic paraffinic oil	1.99	1.51	1.29
Synthetic paraffinic oil plus antiwear additive	1.81	1.37	1.13
Synthetic paraffinic oil plus antiwear additive	1.96	1.55	1.25
C-ether	1.80	0.980	0.795
Superrefined naphthenic mineral oil	2.51	1.54	1.27
Synthetic hydrocarbon (traction fluid)	3.12	1.71	0.937
Fluorinated polyether	4.17	3.24	3.02

Source: Jones et al. (1975).

(the viscosity and speed are of higher power in this equation). This means that a high rolling velocity, U_r, is essential for a full EHD lubrication.

On the right-hand side of Fig. 12-20, we can see that the fluid film thickness is nearly uniform along the contact, except where the clearance narrows at the exit. The film thickness h_0 at the center of the contact is used for calculation of the traction force (resistance to relative sliding), and it can be derived from the following equation of Pan and Hamrock (1989):

$$\frac{h_o}{R_x} = \frac{2.922\bar{U}_r^{0.694}(\alpha E_{eq})^{0.470}}{\bar{W}^{0.166}} \tag{12-43}$$

Example Problem 12-3

Calculation of Oil Film Thickness in a Cylindrical Roller Bearing

The radial load on a cylindrical rolling bearing is $W = 11{,}000$ N, and the inner ring speed is $N = 5000$ RPM. The rolling bearing has the following dimensions: The diameter of the inner raceway, d_{in}, is 120 mm, the diameter of the outer raceway, d_{out}, is 160 mm, and the diameter of the cylindrical roller, d_{roller}, is 20 mm. The effective length of the cylindrical rolling element is $L = 10$ mm. There are 14 rolling elements around the bearing. The bearing (rollers and rings) is made of steel. The modulus of elasticity of the steel is $E = 2.05 \times 10^{11}$ N/m^2, and its Poisson ratio is $\nu = 0.3$. The bearing is lubricated by oil having an absolute viscosity of $\mu_0 = 0.01$ N-s/m^2 at atmospheric pressure and bearing operating temperature. The viscosity–pressure coefficient is $\alpha = 2.2 \times 10^{-8}$ m^2/N.

Find the minimum elastohydrodynamic film thickness at the following points:

a. At the contact of the rolling elements with the inner raceway
b. At the contact of the rolling elements with the outer raceway

Solution

The radii of the contacting curvatures are:

Roller radius: $R_{\text{roller}} = 0.01$ m,
Outer ring raceway: $R_{\text{outer raceway}} = 0.08$ m,
Inner ring raceway: $R_{\text{inner raceway}} = 0.06$ m

The contact between the rolling elements and the inner raceway is convex, and the equivalent contact curvature, $R_{x,\text{in}}$, is derived according to the equation

$$\frac{1}{R_{x,\text{in}}} = \frac{1}{R_{\text{roller}}} + \frac{1}{R_{\text{inner raceway}}}$$

$$\frac{1}{R_{x,\text{in}}} = \frac{1}{0.01} + \frac{1}{0.06} \Rightarrow R_{\text{eq,in}} = 0.0085 \text{ m}$$

However, the contact between the rolling elements and the outer raceway is concave, and the equivalent contact curvature, $R_{x,\text{out}}$, is derived according to the equation

$$\frac{1}{R_{x,\text{out}}} = \frac{1}{R_{\text{roller}}} - \frac{1}{R_{\text{outer raceway}}}$$

$$\frac{1}{R_{x,\text{out}}} = \frac{1}{0.01} - \frac{1}{0.08}; \qquad R_{x,\text{out}} = 0.0114 \text{ m}$$

If we assume that there is no radial clearance, then the maximum load on one cylindrical rolling element can be calculated from the following formula:

$$W_{\max} = \frac{4W_{\text{shaft}}}{n_r} = \frac{4 \times 11{,}000}{14} = 3142 \text{ N}$$

where n_r is the number of rollers in the bearing. The shaft and the bearing are made of identical material, so the equivalent modulus of elasticity is

$$E_{\text{eq}} = \frac{E}{1 - \nu^2} \Rightarrow E_{\text{eq}} = \frac{2.05 \times 10^{11}}{1 - 0.3^2} = 2.25 \times 10^{11} \text{ N/m}^2$$

The dimensionless load on the inner race is

$$\bar{W} = \frac{1}{E_{eq} R_{x,\mathrm{in}} L} W_{max} = \frac{1}{2.25 \times 10^{11} \times 0.0085 \times 0.01} 3142 = 1.64 \times 10^{-4}$$

In comparison, the dimensionless load on the outer race is

$$\bar{W} = \frac{1}{E_{eq} R_{x,\mathrm{out}} L} W_{max} = \frac{1}{2.25 \times 10^{11} \times 0.0114 \times 0.01} 3142 = 1.22 \times 10^{-4}$$

Comparison of the inner and outer dimensionless loads indicates a higher value for the inner contact. This results in higher contact stresses, including maximum pressure, at the inner contact. The rolling velocity, U_r, for a cylindrical roller is calculated via Eq. (12-34).

$$U_r = \frac{R_{\mathrm{in}} R_{\mathrm{out}}}{R_{\mathrm{out}} + R_{\mathrm{in}}} \omega$$

where the angular shaft speed, ω, is equal to

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 5000}{60} = 523 \text{ rad/s}$$

Hence, the rolling speed is

$$U_r = \frac{R_{\mathrm{in}} R_{\mathrm{out}}}{R_{\mathrm{out}} + R_{\mathrm{in}}} \omega = \frac{0.06 \times 0.08}{0.06 + 0.08} 523 = 17.93 \text{ m/s}$$

The dimensionless rolling velocity of the inner surface is

$$\bar{U}_r = \frac{\mu_0}{E_{eq} R_{x,\mathrm{in}}} U_r = \frac{0.01}{2.25 \times 10^{11} \times 0.0085} 17.93 = 9.375 \times 10^{-11}$$

In comparison, the dimensionless rolling velocity at the outer ring race is

$$\bar{U}_r = \frac{\mu_0}{E_{eq} R_{x,\mathrm{out}}} U_r = \frac{0.01}{2.25 \times 10^{11} \times 0.0114} 17.93 = 6.99 \times 10^{-11}$$

The elastohydrodynamic minimum film thickness is derived from Eq. (12.40) for a fully lubricated ball bearing.

$$\frac{h_{min}}{R_x} = \frac{1.714 \bar{U}^{0.694} (\alpha E_{eq})^{0.568}}{\bar{W}^{0.128}}$$

The minimum oil film thickness at the contact with the inner raceway is

$$\frac{h_{min}}{R_{x,\mathrm{in}}} = \frac{1.714 (9.3 \times 10^{-11})^{0.694} (2.2 \times 10^{-8} \times 2.25 \times 10^{11})^{0.568}}{(1.64 \times 10^{-4})^{0.128}}$$

$$h_{min,\mathrm{in}} = 0.609\ \mu\text{m}$$

The minimum oil film thickness at the contact with the outer raceway is

$$\frac{h_{\min}}{R_{x,\mathrm{out}}} = \frac{1.714(6.99 \times 10^{-11})^{0.694}(2.2 \times 10^{-8} \times 2.25 \times 10^{11})^{0.568}}{(1.22 \times 10^{-4})^{0.128}}$$

$$h_{\min,\mathrm{out}} = 0.695\ \mu\mathrm{m}$$

The minimum film thickness at the contact with the inner ring race is lower because the equivalent radius of convex curvatures is lower. Therefore, it is sufficient to calculate the minimum film thickness at the contact with the inner ring race. However, for a rolling bearing operating at high speed, the centrifugal force of the rolling element is added to the contact force at the contact with the outer raceway. In such cases, $h_{\min,\mathrm{out}}$ may be lower than $h_{\min,\mathrm{in}}$, and the EHD fluid film should be calculated at the inner and outer ring races.

Example Problem 12-4

Elastohydrodynamic Fluid Film in a Cam and a Follower

A cam and a follower operate in a car engine as shown in Fig. 12-21. The cam radius at the tip of the cam is $R = 20$ mm, the distance of this radius center from

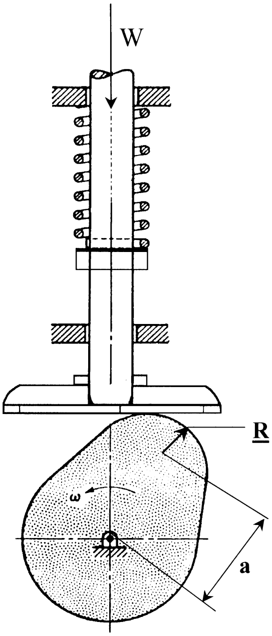

FIG. 12-21 Cam and follower.

the cam center of rotation is $a = 40$ mm, and the width of the cam (effective length of contact) is 10 mm. There is a maximum reaction force $W = 1200$ N between the cam and the follower when the follower reaches its maximum height. The rotation speed of the cam is $N = 600$ RPM. The cam and the follower are made of steel. The steel modulus of elasticity is $E = 2.05 \times 10^{11}$ N/m^2, and its Poisson ratio ν is 0.3. The contact between the cam and follower is fully lubricated. The absolute viscosity at atmospheric pressure and engine temperature is $\mu_0 = 0.01$ N-s/m^2, and the viscosity–pressure coefficient is $\alpha = 2.2 \times 10^{-8}$ m^2/N.

Find the minimum oil film thickness at the contact between the cam and the shaft.

Solution

The contact is between a plane and a curvature of radius $R = 20$ mm at the tip of the cam. In that case, the equivalent radius is R. The equation for the equivalent modulus is

$$E_{eq} = \frac{E}{1 - \nu^2} = \frac{2.05 \times 10^{11}}{1 - 0.3^2} = 2.2 \times 10^{11}\ \text{N/m}^2$$

The dimensionless load becomes

$$\bar{W} = \frac{1}{E_{eq} R L} W_{max} = \frac{1}{2.2 \times 10^{11} \times 0.020 \times 0.01} 1200 = 2.727 \times 10^{-5}$$

The sliding velocity U is equal to the tangential velocity at the tip of the cam:

$$U = \omega(a + R)$$

Here, ω is equal to

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 600}{60} = 62.83\ \text{rad/s}$$

This problem is one of pure sliding, $\xi = 0$, and the sliding velocity is

$$U_s = (0.04 + 0.02)62.83 = 3.76\ \text{m/s}$$

For pure sliding, $\xi = 0$, the dimensionless equivalent rolling velocity is [Eq. (12-42)]

$$\bar{U}_r = \frac{\mu_0}{2 E_{eq} R} U_s(1 + \xi) = \frac{0.01}{2 \times 2.2 \times 10^{11} \times 0.02} 3.76 = 4.27 \times 10^{-12}$$

For line contact and in the presence of sufficient lubricant, the elastohydrodynamic minimum film thickness is derived according to the equation

$$\frac{h_{\min}}{R} = \frac{1.714\bar{U}^{0.694}(\alpha E_{\text{eq}})^{0.568}}{\bar{W}^{0.128}}$$

The oil film thickness in the inner surface is

$$\frac{h_{\min}}{0.02} = \frac{1.714(4.27 \times 10^{-12})^{0.694}(2.2 \times 10^{-8} \times 2.22 \times 10^{11})^{0.568}}{(2.727 \times 10^{-5})^{0.128}}$$

$$h_{\min} = 0.21\ \mu\text{m}$$

12.9 ELASTOHYDRODYNAMIC LUBRICATION OF BALL BEARINGS

Under load, the theoretical point contact between a rolling ball and raceways becomes an elliptical contact area. The elliptical contact area has radii a and b, as shown in Fig. 12-17.

In a similar way to the theoretical line contact, there is a minimum elastohydrodynamic film thickness, $h_{\min}$, near the exit from a uniform film thickness, h_0. For hard surfaces, such as steel in rolling bearings, and sufficient lubricant, Hamrock and Dowson (1977) obtained the following formula for the minimum film thickness, $h_{\min}$:

$$\frac{h_{\min}}{R_x} = 3.63\frac{\bar{U}_r^{0.68}(\alpha E_{\text{eq}})^{0.49}}{\bar{W}^{0.073}}(1 - e^{-0.68k}) \qquad (12\text{-}44)$$

Here, α is the viscosity–pressure coefficient and $\bar{U}_r$ and $\bar{W}$ are dimensionless velocity and load, respectively, defined by:

$$\bar{U}_r = \frac{\mu_o U_r}{E_{\text{eq}} R_x} \qquad \text{and} \qquad \bar{W} = \frac{W}{E_{\text{eq}} R_x^2} \qquad (12\text{-}45)$$

Here, U_r is the rolling velocity, μ_0 is the viscosity of the lubricant at atmospheric pressure and bearing operating temperature, W is the load on one rolling element, and E_{eq} is the equivalent modulus of elasticity.

The equations for the equivalent modulus of elasticity and equivalent contact radius are used for calculating the Hertz stresses at a point contact. These equations were discussed in Secs. 12.4 and 12.5. For convenience, these equations are repeated here.

Recall that E_{eq} is determined from equation

$$\frac{2}{E_{\text{eq}}} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

Here, ν is poisson's ratio and E is the modulus of elasticity of the respective two materials. For identical materials, the equation becomes

$$E_{eq} = \frac{E}{1 - \nu^2}$$

The equivalent radius of curvature in the plane of rotation, R_x, for the contact with the inner ring race is

$$\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}}$$

where R_{1x} and R_{2x} are as shown on the right-hand side of Fig. 12-18b for the contact at the inner ring race and $R_{1x} = d/2$, where d is the ball diameter.

On the left-hand side of Fig. 12-18b, the contact at the inner ring race is concave, and the equivalent contact curvature radius in this plane is

$$\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}}$$

The radius ratio, α_r (the ratio of the larger radius to the smaller radius, $\alpha_r > 1$) is defined as

$$\alpha_r = \frac{R_y}{R_x}$$

The ellipticity parameter, k, is the ratio

$$k = \frac{b}{a}$$

The parameter k can be estimated from

$$k \approx \alpha_r^{2/\pi}$$

For hard surfaces, sufficient lubricant, and for pure rolling, Hamrock and Dowson (1981) obtained the following formula for the central film thickness, h_c (at the center of the fluid film):

$$\frac{h_c}{R_x} = 2.69 \frac{\bar{U}_r^{0.67} (\alpha E_{eq})^{0.53}}{\bar{W}^{0.067}} (1 - 0.61 e^{-0.73k}) \qquad (12\text{-}46)$$

This equation is useful for the calculation of the traction force (resistance to relative sliding) where an EHD film is separating the surfaces.

For soft surfaces, such as rubber, the contact area is relatively large, resulting in a lower contact pressure. In addition, the viscosity does not increase as much as predicted by the preceding equations.

Example Problem 12-5

Find the minimum film thickness for a rolling contact of a deep-groove ball bearing having the following dimensions: The bearing has 14 balls of diameter $d = 19.04$ mm. The radius of curvature of the inner-deep groove (in cross section x-z on the left-hand side of Fig. 12-18b) is 9.9 mm. The inner race diameter, R_{2x} (at the bottom of the deep groove), is $d_i = 76.5$ mm (cross section y-z on the right-hand side of Fig. 12-18b). The radial load on the bearing is $W = 10{,}500$ N, and the bearing speed is $N = 5000$ RPM. The rolling elements and rings are made of steel. The modulus of elasticity of the steel for rollers and rings is $E = 2 \times 10^{11}$ N/m^2, and Poisson's ratio is $\nu = 0.3$. The properties of the lubricant are: The absolute viscosity at ambient pressure and bearing operating temperature is $\mu_0 = 0.04$ N-s/m^2, and the viscosity–pressure coefficient is $\alpha = 2.3 \times 10^{-8}$ m^2/N.

Solution

Referring to Fig. 12-18b, the radius of curvature in the y-z plane is

$$R_{2x} = 38.25 \text{ mm} \qquad \text{and } R_{1x} = \frac{d}{2} = 9.52 \text{ mm}$$

Equivalent Radius for Inner Raceway Convex Contact in y-z Plane (x Plane)

$$\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}} \Rightarrow \frac{1}{R_x} = \frac{1}{9.52} + \frac{1}{38.25}; \qquad R_x = 7.62 \text{ mm}$$

Equivalent Inner Radius in y Plane (Concave Contact)

The curvatures in this plane are: $R_{1y} = 9.52$ mm and $R_{2y} = 9.9$ mm. The equivalent inner radius in the y plane is

$$\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} \Rightarrow \frac{1}{R_y} = \frac{1}{9.52} - \frac{1}{9.9}; \qquad R_y = 248.0 \text{ mm}$$

Equivalent Curvature of Inner Ring and Ball Contact

$$\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{R_y} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{7.62} + \frac{1}{248}; \qquad R_{eq} = 7.4 \text{ mm}$$

$$\alpha_r = \frac{R_y}{R_x} = 32.55 \qquad \text{and} \qquad k = \alpha_r^{2/\pi} = 32.54^{2/\pi} = 9.18$$

The shaft and the bearing are made of identical steel, so the equivalent modulus of elasticity is:

$$E_{eq} = \frac{E}{1-\nu^2} \Rightarrow E_{eq} = \frac{2 \times 10^{11}}{1-0.3^2} = 2.2 \times 10^{11}\ \mathrm{N/m^2}$$

For a ball bearing without radial clearance, the maximum load at the contact of one rolling element can be estimated with the following equation:

$$W_{max} \approx \frac{5W_{shaft}}{n_r} = \frac{5(10{,}500)}{14} = 3750\ \mathrm{N}$$

where $n_r = 14$ is the number of balls in the bearing.

The angular velocity, ω, is

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 5000}{60} = 523.6\ \mathrm{rad/s}$$

The rolling speed is derived according to Eq. (12-34):

$$U_{rolling} = \frac{R_{in}R_{out}}{2(R_{in}+r)}\omega = \frac{R_{in}R_{out}}{R_{in}+R_{out}}\omega$$

or

$$U_r = \frac{R_{2x}(R_{2x}+d)}{2R_{2x}+d}\omega$$

$$U_r = \frac{38.25 \times 10^{-3}(38.25 \times 10^{-3} + 19.05 \times 10^{-3})}{(2 \times 38.25 \times 10^{-3}) + 19.04 \times 10^{-3}} 523.6 = 12\ \mathrm{m/s}$$

The dimensionless rolling velocity at the inner ring race is

$$\bar{U}_r = \frac{\mu_0}{E_{eq}R_x}U_r = \frac{0.01}{2.2 \times 10^{11} \times 0.00762} 12 = 7.17 \times 10^{-11}$$

The dimensionless load on the inner ring race is

$$\bar{W} = \frac{1}{E_{eq}R_x^2}W_{max} = \frac{1}{2.2 \times 10^{11} \times 0.00762^2} 3750 = 2.94 \times 10^{-4}$$

Finally, the oil film thickness between the inner race and the roller is

$$\frac{h_{min}}{R_x} = 3.63 \frac{\bar{U}_r^{0.68}(\alpha E_{eq})^{0.49}}{\bar{W}^{0.073}}(1 - e^{-0.68k}) \Rightarrow$$

$$\frac{h_{min}}{0.00762} = 3.63 \frac{(7.17 \times 10^{-11})^{0.68}(2.3 \times 10^{-8} \times 2.2 \times 10^{11})^{0.49}}{(2.94 \times 10^{-4})^{0.073}} \times (1 - e^{-0.68 \times 9.18})$$

Thus, at the inner ring race

$$h_{\min} = 0.412\ \mu\text{m}$$

Example Problem 12-6

Centrifugal Force

The deep-groove ball bearing in Example Problem 12-5 is used in a high-speed turbine where the average shaft speed is increased to $N = 30{,}000$ RPM. The radial bearing load is equal to that in Example Problem 12-5, $W = 10,500$ N. The lubricant is also equivalent to that in Example Problem 12-5. The properties of the lubricant are: The absolute viscosity at ambient pressure and bearing operating temperature is $\mu_0 = 0.04$ N-s/m^2, and the viscosity–pressure coefficient is $\alpha = 2.3 \times 10^{-8}$ m^2/N.

Consider the centrifugal force at this high-speed operation, and

a. find the maximum contact pressure at the inner and outer ring raceways.
b. calculate the minimum film thickness at the contact with the outer ring.

Solution

a. Maximum Contact Pressure

For the contact with the inner ring race, the maximum pressure and minimum film thickness was calculated in Example Problems 12-2 and 12-5. In this problem, the maximum pressure will be compared with that at the outer ring race in the presence of a centrifugal force.

Contact with Outer Ring Race

The contact radii in the x plane (y-z plane) in Fig. 12-18b are:

$$R_{2x} = 57.29\ \text{mm} \qquad \text{and} \qquad R_{1x} = \frac{d}{2} = 9.52\ \text{mm}$$

The equivalent radius for the outer raceway concave contact in the x plane (y-z plane) is

$$\frac{1}{R_x} = \frac{1}{R_{1x}} - \frac{1}{R_{2x}} \Rightarrow \frac{1}{R_x} = \frac{1}{9.52} - \frac{1}{57.29}; \qquad R_x = 11.42\ \text{mm}$$

Equivalent Outer Radius in y Plane

The curvatures in this plane are $R_{1y} = 9.52$ mm and $R_{2y} = 9.9$ mm. The equivalent inner radius in the y (or x-z) plane is

$$\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} \Rightarrow \frac{1}{R_y} = \frac{1}{9.52} - \frac{1}{9.9}; \qquad R_y = 248.0 \text{ mm}$$

The equivalent radius of the outer ring and ball contact is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_x} + \frac{1}{R_y} \Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{11.42} + \frac{1}{248}; \qquad R_{\text{eq}} = 10.92 \text{ mm}$$

$$\alpha_r = \frac{R_y}{R_x} = 21.72$$

and

$$k = \alpha_r^{2/\pi} = 21.72^{2/\pi} = 7.1$$

It is also necessary to calculate $\hat{E}$, which will be used to calculate the ellipsoid radii. First q_a will be determined

$$q_a = \frac{\pi}{2} - 1 = 0.57$$

$$\hat{E} \approx 1 + \frac{q_a}{\alpha_r} \Rightarrow 1 + \frac{0.57}{21.72} = 1.03$$

From Example Problem 12-5, the equivalent modulus of elasticity is

$$E_{\text{eq}} = 2.2 \times 10^{11} \text{ N/m}^2$$

The load on the bearing is divided unevenly between the rolling elements. The approximate equation for zero bearing clearance is used. The maximum load, W_{max}, at a contact of one rolling element can be estimated by the following equation:

$$W_{\text{max (one ball)}} \approx \frac{5W_{\text{shaft}}}{n_r} = \frac{5(10{,}500)}{14} = 3750 \text{ N}$$

where $n_r = 14$ is the number of balls in the bearing.

The total maximum load at the contact of one-roller and the outer raceway is equal to the transmitted shaft load plus the centrifugal force generated.

The angular velocity ω is

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 30{,}000}{60} = 3141.59 \text{ rad/s}$$

The angular velocity of the rolling-element center, point C, is given by

$$\omega_C = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{out}}}\omega = \frac{38.25}{38.25 + 57.29}3141.59 = 1257.75 \text{ rad/s}$$

This angular velocity is used to calculate the centrifugal force, F_c:

$$F_c = m_r R_c \omega_C^2$$

The density of steel and the ball volume determine its mass:

$$m_r = \frac{\pi}{6}d^3\rho = \frac{\pi}{6} \times 0.01904^3 \times 7800 = 0.028 \text{ kg}$$

After substituting these values in the equation for the centrifugal force, we obtain

$$F_c = 0.028 \times 0.04777 \times 1257.75^2 = 2115.93 \text{ N}$$

The maximum force is at the outer raceway. It is equal to the sum of the transmitted shaft load and the centrifugal force:

$$W_{\text{max}} = W_{\text{max,load}} + F_c = 3750 + 2115.93 = 5865.93 \text{ N}$$

The ellipsoid radii a and b can now be determined by substituting the values already calculated:

$$\begin{aligned} a &= \left(\frac{6\hat{E}W_{\text{max}}R_{\text{eq}}}{\pi k E_{\text{eq}}}\right)^{1/3} \\ &= \left(\frac{6 \times 1.03 \times 5865.93 \times 0.01092)}{\pi \times 7.1 \times 2.2 \times 10^{11}}\right)^{1/3} = 0.43 \text{ mm} \\ b &= \left(\frac{6k^2\hat{E}W_{\text{max}}R_{\text{eq}}}{\pi E_{\text{eq}}}\right)^{1/3} \\ &= \left(\frac{6 \times 7.1^2 \times 1.03 \times 5865.93 \times 0.01092}{\pi \times 2.2 \times 10^{11}}\right)^{1/3} = 3.07 \text{ mm} \end{aligned}$$

The maximum pressure at the outer contact is

$$p_{\text{max}} = \frac{3}{2}\frac{W_{\text{max}}}{\pi ab} = \frac{3}{2}\frac{5865.93}{\pi \times 0.43 \times 3.07} = 2121.64 \text{ N/mm}^2 = 2.12 \text{ GPa}$$

The maximum pressure is very close to that at the inner ring contact, $p_{max} = 2.17$ GPa (see Example Problem 12.2).

b. Minimum Film Thickness at the Outer Race Contact

Rolling occurs only in the x plane, so

$$U_r = \frac{R_{2x}(R_{2x} + d)}{2R_{2x} + d}\omega$$

$$U_r = \frac{57.29 \times 10^{-3}(57.29 \times 10^{-3} + 19.04 \times 10^{-3})}{(2 \times 57.29 \times 10^{-3}) + 19.04 \times 10^{-3}} 3141.59 = 102.81 \text{ m/s}$$

The dimensionless velocity for the outer surface is

$$\bar{U}_r = \frac{\mu_0}{E_{eq}R_x} U_r = \frac{0.01}{2.2 \times 10^{11} \times 0.01142} 102.81 = 4.09 \times 10^{-10}$$

where U_r is the rolling velocity of the ball on the race and ω is the angular velocity of the shaft, in rad/s. The dimensionless load on the outer race is

$$\bar{W} = \frac{1}{E_{eq}R_x^2} W_{max} = \frac{1}{2.2 \times 10^{11} \times (0.01142)^2} \times 5865.93 = 2.04 \times 10^{-4}$$

Finally, the oil film thickness between the outside race and the ball is

$$\frac{h_{min}}{R_x} = 3.63 \frac{\bar{U}_r^{0.68}(\alpha E_{eq})^{0.49}}{\bar{W}^{0.073}} (1 - e^{-0.68k})$$

$$\frac{h_{min}}{0.01142} = 3.63 \frac{(4.09 \times 10^{-10})^{0.68}(2.3 \times 10^{-8} \times 2.2 \times 10^{11})^{0.49}}{(2.04 \times 10^{-4})^{0.073}} \times (1 - e^{-0.68 \times 7.1})$$

$$h_{min} = 2.06 \ \mu\text{m}$$

Example Problem 12-7

Ceramic Rolling Elements

For the high-speed turbine given in Example Problem 12-6, the bearing is replaced by a deep-groove ball bearing with equivalent geometry. However, the bearing is hybrid and the rolling elements are made of silicone nitride. The rings are made of steel. The lubricant is also equivalent to that in Example Problems 12-5 and 12-6. The shaft speed is $N = 30{,}000$ RPM. The radial bearing load is equal to that in Example Problem 12-5, $W = 10{,}500$ N.

a. Find the maximum pressure at the outer race contact
b. Compare the maximum pressure to that of all steel bearings in Example Problem 12-6.

The properties of silicon nitride are:

$E = 3.14$ GPa (3.14×10^{11} Pa), in comparison to steel, with $E = 2.00$ GPa.

$\rho = 3200$ kg/m^3, in comparison to steel, with $\rho = 7800$ kg/m^3.

$\nu = 0.24$, in comparison to steel, with $\nu = 0.3$.

Solution

a. Maximum Pressure at the Outer Race

It is possible to decrease the centrifugal force by lowering the density of the rolling elements. One important advantage of a hybrid bearing where the rolling elements are made of silicone nitride (and the rings are made of steel) is that it lowers the density of the rolling elements. However, we have to keep in mind that the modulus of elasticity of silicone nitride is higher than that of steel, and this may result in a higher maximum pressure. In this problem, the maximum pressure is calculated and compared with those of a conventional steel bearing.

Radius of Curvature in the x Plane

$$R_{2x} = 57.29 \text{ mm} \qquad R_{1x} = \frac{d}{2} = 9.52 \text{ mm}$$

The equivalent radius for the outer raceway contact in the x plane is

$$\frac{1}{R_x} = \frac{1}{R_{1x}} - \frac{1}{R_{2x}} \Rightarrow \frac{1}{R_x} = \frac{1}{9.52} - \frac{1}{57.29}; \qquad R_x = 11.42 \text{ mm}$$

Equivalent Outer Radius in y Plane

The curvatures in this plane are $R_{1y} = 9.52$ mm and $R_{2y} = 9.9$ mm. The equivalent inner radius in the y-z plane is

$$\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} \Rightarrow \frac{1}{R_y} = \frac{1}{9.52} - \frac{1}{9.9}; \qquad R_y = 248.0 \text{ mm}$$

The equivalent radius of the curvature of the outer ring and ball contact is

$$\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{R_y} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{11.42} + \frac{1}{248}; \qquad R_{eq} = 10.92 \text{ mm}$$

$$\alpha_r = \frac{R_y}{R_x} = 21.72$$

and

$$k = \alpha_r^{2/\pi} = 21.72^{2/\pi} = 7.1$$

It is also necessary to calculate $\hat{E}$, which will be used to calculate the ellipsoid radii. First, q_a will be determined:

$$q_a = \frac{\pi}{2} - 1 = 0.57$$

$$\hat{E} \approx 1 + \frac{q_a}{\alpha_r} \Rightarrow 1 + \frac{0.57}{21.72} = 1.03$$

For a hybrid bearing, we must consider the equivalent modulus of elasticity of two different materials (steel and silicon nitride), given by

$$\frac{2}{E_{\text{eq}}} = \frac{1 - \nu_{21}^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \Rightarrow$$

$$\frac{2}{E_{\text{eq}}} = \frac{1 - (0.24)^2}{3.14 \times 10^{11}} + \frac{1 - (0.3)^2}{2.0 \times 10^{11}} \Rightarrow E_{\text{eq}} = 2.65 \times 10^{11} \text{ Pa}$$

The maximum load transmitted by one rolling element at the contact with the ring (assuming zero clearance) is estimated by the following equation:

$$W_{\max} \approx \frac{5W_{\text{shaft}}}{n_r} = \frac{5(10{,}500)}{14} = 3750 \text{ N}$$

The number of balls in the bearing is $n_r = 14$.

The total maximum load at the contact of one roller and the outer raceway is equal to the transmitted shaft load plus the centrifugal force generated. The angular velocity ω is equal to

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 30{,}000}{60} = 3141.59 \text{ rad/s}$$

The angular velocity of the rolling element center, point C, is given by

$$\omega_C = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{out}}} \omega = \frac{38.25}{38.25 + 57.29} 3141.59 = 1257.75 \text{ rad/s}$$

This angular velocity is used to calculate the centrifugal force, F_c:

$$F_c = m_r (R_i + r) \omega_C^2$$

The density (silicon nitride) and volume determine the mass of the ball:

$$m_r = \frac{\pi}{6} d^3 \rho \Rightarrow \frac{\pi}{6} \times 0.01904^3 \times 3200 = 0.012 \text{ kg}$$

Substituting these values in the equation for centrifugal force we obtain

$$F_c = 0.012 \times 0.04777 \times 1257.75^2 = 906.76 \text{ N}$$

Therefore, the maximum force occurs at the outer raceway. It is equal to the sum of the shaft load and the centrifugal force:

$$W_{\max} = W_{\max,\text{load}} + F_c \Rightarrow 3750 + 906.76 = 4656.76\ \text{N}$$

In comparison, $w_{max} = 5866\ N$, for all steel-bearing and identical conditions (Example Problem 12-6).

The ellipsoid radii a and b can now be determined by substitution of the values already calculated:

$$a = \left(\frac{6\hat{E}W_{\max}R_{\text{eq}}}{\pi k E_{\text{eq}}}\right)^{1/3} = \left(\frac{6 \times 1.03 \times 4656.76 \times 0.01092}{\pi \times 7.1 \times 2.65 \times 10^{11}}\right)^{1/3} = 0.38\ \text{mm}$$

$$b = \left(\frac{6k^2\hat{E}W_{\max}R_{\text{eq}}}{\pi E_{\text{eq}}}\right)^{1/3}$$

$$= \left(\frac{6 \times 7.1^2 \times 1.03 \times 4656.76 \times 0.01092}{\pi \times 2.65 \times 10^{11}}\right)^{1/3} = 2.67\ \text{mm}$$

The maximum pressure at the outer race contact is

$$p_{\max} = \frac{3}{2}\frac{W_{\max}}{\pi ab} = \frac{3}{2}\frac{4656.76}{\pi \times 0.38 \times 2.67} = 2190\ \text{N/mm}^2 = 2.19\ \text{GPa}$$

b. Comparison with All-Steel Bearing

The bearing made of steel has a maximum pressure of 2.12 GPa on the outer raceway, while the hybrid bearing has a maximum pressure of 2.19 GPa. By using rollers made of silicon nitride, it is possible to reduce the centrifugal force by lowering the density of the rolling element. Although the centrifugal force is reduced, the modulus of elasticity of the ceramic rolling element is higher. The combination of these two factors results in a slight increase in pressure on the outer raceway, which was unexpected. In conclusion, for this specific application there is no reduction in the maximum pressure by replacing the steel with silicon nitride ceramic rolling elements.

12.10 FORCE COMPONENTS IN AN ANGULAR CONTACT BEARING

In an angular contact bearing, the resultant contact forces of a ball with the inner and outer ring races, W_i and W_o, are shown in Fig. 12-22. At low speed, the centrifugal forces of the rolling element are small and can be neglected in comparison to the external load. In that case, the contact angle of the inner ring, α_i is equal to the contact angle of the outer ring, α_o; see Fig. 12-22. However, at high speed the centrifugal forces are significant and should be considered. In that

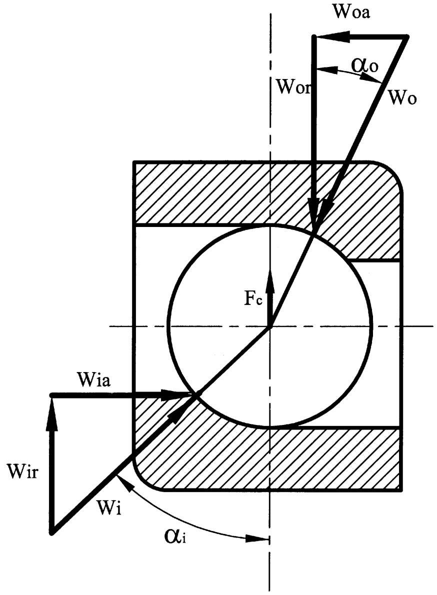

FIG. 12-22 Force components in an angular contact bearing.

case, the contact angle of the inner ring, α_i, is no longer equal to the contact angle of the outer ring, α_o.

For design purposes, the maximum contact stresses at the inner and outer ring races should be calculated. For this purpose, the first step is to solve for the resultant contact forces with the inner and outer rings, W_i and W_o, in the direction normal to the contact area.

Let us consider an example where the following bearing data is given:

a. Inner ring contact angle is α_i.
b. Thrust load on each rolling element is W_{ia}.
c. The centrifugal force of each rolling element is F_c.

We solve for the contact angle of the outer ring, α_o, and the resultant normal contact forces of the inner and outer ring races, W_i and W_o, respectively. The resultant contact force at the inner ring, W_i, can be directly solved from the thrust load and inner contact angle:

$$\frac{W_{ia}}{W_i} = \sin \alpha_i \qquad W_i = \frac{W_{ia}}{\sin \alpha_i} \tag{12-47}$$

The two unknowns, the contact angle with the outer ring, α_o, and the reaction force, W_o, are solved from the two equations of balance of forces on a rolling element. The balance of forces in the x (horizontal) direction is

$$\begin{aligned} \Sigma F_x &= 0 \\ W_{oa} &= W_{ia} \end{aligned} \tag{12-48}$$

In the y (vertical) direction, the balance of forces is

$$\begin{aligned} \Sigma F_y &= 0 \\ W_{ir} + F_c &= W_{or} \end{aligned} \tag{12-49}$$

Example Problem 12-8

An angular contact ball bearing has a contact angle with the inner ring of $a_i = 40°$. The thrust load is 2400 N divided exactly evenly on 12 balls (thrust load of 200 N on each ball). The ball has a diameter of $d_r = 14$ mm. The shaft speed is $N = 30{,}000$ RPM. The diameter of the outer race at the contact point with the balls is 78 mm, and the radius of the race groove, in an axial cross section, is 8 mm. The balls and rings are made of steel having the following properties:

$$E = 2 \times 10^{11}\ \mathrm{N/m^2}$$
$$\nu = 0.3$$
$$\rho = 7870\ \mathrm{kg/m^3}$$

Find the contact angle α_o and the contact force with the inner and outer rings.

Solution

Given:

$$\begin{aligned} \alpha_i &= 40^\circ \\ N &= 30{,}000\ \mathrm{RPM} \\ W_a &= 200\ \mathrm{N} \\ d_r &= 14\ \mathrm{mm} = 0.014\ \mathrm{m} \\ R_{\mathrm{out}} &= 39\ \mathrm{mm} = 0.039\ \mathrm{m} \\ R_{\mathrm{in}} &= 25\ \mathrm{mm} = 0.025\ \mathrm{m} \\ R_{c(\text{ball center})} &= 32\ \mathrm{mm} = 0.032\ \mathrm{m} \end{aligned}$$

Volume of a Sphere

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$
$$V_{\text{sphere}} = \frac{4}{3}\pi(0.7)^3 = 1.43\ \text{cm}^3$$

Mass of the Ball

$$m_r = \rho V = 7.87\ \text{g/cm}^3 \times 1.43\ \text{cm}^3 = 11.25\ \text{g} = 0.011\ \text{kg}$$

The shaft and inner ring angular speed is

$$\omega = \frac{2\pi N}{60}$$
$$\omega = 30{,}000\frac{\text{rev}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{2\pi\text{rad}}{1\ \text{rev}} = 3142\ \text{rad/s}$$

Angular Speed of Rolling Elements and Cage

$$\omega_C = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{out}}}\omega_{\text{shaft}} = \frac{0.025\ \text{m}}{0.025\ \text{m} + 0.039\ \text{m}} \times 3142\ \text{rad/s} = 1227\ \text{rad/s}$$

Centifugal Force, F_c

$$F_c = m_r\omega_C^2 R_r = 0.011\ \text{kg} \times (1227\ \text{rad/s})^2 \times 0.032\ \text{m} = 530\ \text{N}$$

Thrust Force Component at Outer Ring Contact

$$\Sigma F_x = 0$$
$$W_{oa} = W_{ia} = 200\ \text{N}$$

Radial Component of Inner Ring Contact

$$\frac{W_{ir}}{W_{ia}} = c\tan\alpha_i$$
$$W_{ir} = 200 \times c\tan 40^\circ = 238.4\ \text{N}$$

Radial Component of Outer Ring Contact

$$\Sigma F_y = 0$$
$$W_{or} = W_{ir} + F_c$$
$$W_{or} = 238.4 + 530 = 768.4 \text{ N}$$

Outer Ring Contact Angle

$$\tan \alpha_o = \frac{W_{oa}}{W_{or}}$$
$$\tan \alpha_o = \frac{200 \text{ N}}{768.4 \text{ N}} = 0.26$$
$$\alpha_o = 14.5^\circ$$

Resultant Force on Outer Ring Contact

The resultant force on the outer ring race, W_o, is in the direction normal to surface, as shown in Fig. 12-22. It has two components in the axial and radial directions:

$$\frac{W_{oa}}{W_o} = \sin 14.5^\circ$$
$$W_o = \frac{200 \text{ N}}{\sin 14.5^\circ} = 798.8 \text{ N}$$

Normal Contact Force on Inner Ring Race

The resultant force on the inner ring race, W_i, is in the direction normal to surface, as shown in Fig. 12-22. It has two components in the axial and radial directions:

$$\frac{W_{ia}}{W_o} = \sin 40^\circ$$
$$W_i = \frac{200 \text{ N}}{\sin 40^\circ} = 311.2 \text{ N}$$

Example Problem 12-9

An angular contact ball bearing has a contact angle with the inner ring of $\alpha_i = 30^\circ$. The thrust load of $W = 11{,}500$ N is divided evenly on 14 balls. The ball has a diameter of $d_r = 18$ mm. The shaft speed is $N = 33{,}000$ RPM. The conformity ratio $R_r = 0.52$. The diameter of the outer race at the contact point with the balls is 118 mm. The balls and rings are made of steel having the following properties: modulus of elasticity $E = 2 \times 10^{11}$ N/m^2, Poisson's ratio $\nu = 0.3$, density $\rho = 7870$ kg/m^3.

a. Find contact angle, α_o, with the outer ring.
b. Find the contact force with the inner and outer rings.
c. Find the centrifugal force of each rolling element.
d. Find the maximum pressure at the contact with the outer ring.
e. Find the maximum pressure at the contact with the outer ring, given rings made of steel and balls made of silicone nitride (hybrid bearing).

The properties of silicone nitride are: modulus of elasticity $E = 3.14 \times 10^{11}$ N/m^2, Poisson's ration $\nu = 0.24$, density $\rho = 3200$ kg/m^3.

Solution

a. Contact Angle, α_o, with Outer Ring

The first step is to find the centrifugal force, $F_c = m_r \omega_C^2 R_c$, where m_r is the mass of a ball, ω_C is the angular speed of the ball center (or cage), and R_c is the radius of the ball center circular orbit. The volume and mass of a ball in the bearing are:

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 0.009^3 \text{ m}^3 = 3.05 \times 10^{-6} \text{ m}^3$$
$$m_r = \rho V = 7870 \text{ kg/m}^3 \times 3.05 \times 10^{-6} \text{ m}^3 = 0.024 \text{ kg}$$

The shaft and inner ring angular speed is

$$\omega = \frac{2\pi \times 33{,}000}{60} = 3456 \text{ rad/s}$$

The outer diameter is

$$R_{\text{out}} = \frac{118}{2} \text{ mm} = 59 \times 10^{-3} \text{ m}$$

In order to find the inner diameter, we assume that $\alpha_o \sim \alpha_i$ and that the difference between the outer and inner radius is $d_r \cos\alpha_i$. In that case,

$$R_{\text{in}} = (R_{\text{out}} - d_r)\cos\alpha_i = (59 \text{ mm} - 18 \text{ mm})\cos 30^\circ = 43.4 \text{ mm}$$
$$= 43.4 \times 10^{-3} \text{ m}$$

Now it is possible to find the angular speed of a rolling-element center (or cage), ω_c:

$$\omega_c = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{out}}}\omega_{\text{shaft}} = \frac{43.4}{43.4 + 59} \times 3456 \text{ rad/s} = 1465 \text{ rad/s}$$

The distance between the ball center and the bearing center as R_c, is required for the calculation of the centrifugal force.

$$R_c = \frac{R_{\text{in}} + R_{\text{out}}}{2} = 51.2 \text{ mm}$$

The centrifugal force, F_c, of each rolling element is in the radial direction:

$$F_c = m_r \omega_c^2 R_c = 0.024 \text{ kg} \times 1465^2 \times 1/s^2 \times 0.0512 \text{ m} = 2637 \text{ N}$$

The thrust force component of each ball at the outer ring contact is equal to that of the inner ring:

$$W_{oa} = W_{ia} = \frac{11{,}500 \text{ N}}{14} = 821 \text{ N}$$

The radial component of inner ring contact is

$$\frac{W_{ir}}{W_{ia}} = c\tan \alpha_i$$

$$W_{ir} = 821 \times c\tan 30^\circ = 1422 \text{ N}$$

The radial component of outer ring contact force is

$$W_{or} = W_{ir} + F_c = 1421 \text{ N} + 2637 \text{ N} = 4059 \text{ N}$$

The outer ring contact angle is solved as follows:

$$\tan \alpha_{\text{o}} = \frac{W_{oa}}{W_{or}} = \frac{821 \text{ N}}{4058 \text{ N}} = 0.202, \qquad \alpha_o = 11.43^\circ$$

b. Contact Force with Inner and Outer Rings

The resultant (normal) component of the outer ring contact is

$$\frac{W_{oa}}{W_o} = \sin 11.43^\circ$$

$$W_o = \frac{821 \text{ N}}{\sin 11.43^\circ} = 4143 \text{ N}$$

The resultant (normal) contact force at the inner ring race is

$$\frac{W_{ia}}{W_i} = \sin 30^\circ$$

$$W_i = \frac{W_{ia}}{\sin 30^\circ} = 1642 \text{ N}$$

c. Rolling-Element Centrifugal Force

From part (a) the centrifugal force is 2637 N, in the radial direction.

d. Maximum Pressure at Contact of Outer Ring

The maximum force is at the race of the outer ring, $W_o = 4143\,\text{N}$. The radius of the contact curvatures is

$$R_{1x} = R_{1y} = 9 \text{ mm}$$
$$R_{2y} = R_r d_r = 18 \text{ mm} \times 0.52 = 9.36\,\text{mm}$$
$$R_{2x} = 59 \text{ mm}$$

The equivalent radius of the outer raceway contact in the y-z plane (referred to as the x plane) is

$$\frac{1}{R_x} = \frac{1}{R_{1x}} - \frac{1}{R_{2x}} = \frac{1}{9 \text{ mm}} - \frac{1}{59 \text{ mm}}$$
$$R_x = \frac{9 \times 59 \text{ mm}}{43.4 - 9} = 10.62 \text{ mm}$$

The equivalent inner radius in the y plane is

$$\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} = \frac{1}{9 \text{ mm}} - \frac{1}{9.36 \text{ mm}}$$
$$R_y = \frac{R_{2y} \cdot R_{1y}}{R_{2y} - R_{1y}} = \frac{9 \times 9.36 \text{ mm}}{9.36 - 9} = 234 \text{ mm}$$

The equivalent curvature of the inner ring and ball contact is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{10.62 \text{ mm}} + \frac{1}{234 \text{ mm}}$$
$$R_{\text{eq}} = \frac{R_x \cdot R_y}{R_x + R_y} = \frac{10.62 \times 234 \text{ mm}}{234 + 10.62} = 10.16 \text{ mm}$$

The ratio α_r becomes

$$\alpha_r = \frac{R_y}{R_x} = \frac{234 \text{ mm}}{10.62 \text{ mm}} = 22.03$$

The dimensionless coefficient k is estimated from the ratio α_r:

$$k = \alpha_r^{2/\pi} = 7.16$$

For the calculation of the ellipsoid radii, the following values are required:

$$q_a = \frac{\pi}{2} - 1 = 0.57$$

$$\hat{E} \approx 1 + \frac{q_a}{\alpha_r}$$

$$\hat{E} \approx 1 + \frac{0.57}{22.03} = 1.025$$

The shaft and the bearing are made of identical material, so the equivalent modulus of elasticity is

$$E_{eq} = \frac{E}{1 - \nu^2} = \frac{2 \times 10^{11} \mathrm{N/m^2}}{1 - 0.3^2} = 2.2 \times 10^{11}\ \mathrm{N/m^2}$$

Now the ellipsoid radii a and b can be determined:

$$a = \left(\frac{6 \cdot \hat{E} \cdot W_{\max} \cdot R_{eq}}{\pi \cdot k \cdot E_{eq}} \right)^{1/3}$$

$$= \left(\frac{6 \times 1.025 \times 4143\ \mathrm{N} \times 10.82 \times 10^{-3}\ \mathrm{m}}{\pi \times 7.16 \times 2.2 \times 10^{11}\ \mathrm{N/m^2}} \right)^{1/3} = 0.37\ \mathrm{mm}$$

$$b = \left(\frac{6 \cdot k^2 \cdot \hat{E} \cdot W_{\max} \cdot R_{eq}}{\pi \cdot E_{eq}} \right)^{1/3}$$

$$= \left(\frac{6 \times 7.16^2 \times 1.025 \times 4143\ \mathrm{N} \times 10.16 \times 10^{-3}\ \mathrm{m}}{\pi \times 2.2 \times 10^{11}\ \mathrm{N/m^2}} \right)^{1/3} = 2.68\ \mathrm{mm}$$

The maximum pressure at the contact with outer ring is

$$p_{\max} = \frac{3}{2} \cdot \frac{W_{\max}}{\pi \cdot ab} = \frac{3}{2} \cdot \frac{4134\ \mathrm{N}}{\pi \times 0.37 \times 2.68 \times 10^{-6}\ \mathrm{m^2}} = 1.99\ \mathrm{GPa}$$

e. Maximum stress of hybrid bearing

The rings are made of steel and the balls are made of silicone nitride, which has the following properties:

Modulus of elasticity $E = 3.14 \times 10^{11}\ \mathrm{N/m^2}$
Poisson's ratio $\nu = 0.24$
Density $\rho = 3200\ \mathrm{kg/m^3}$

The major advantage of a hybrid bearing is a lower centrifugal force due to lower density of the rolling element. However, the modulus of elasticity of silicone nitride is higher than that of steel, and this can result in a higher maximum pressure.

For a hybrid bearing, it is necessary to consider the equivalent modulus of elasticity of silicone nitride on steel:

$$\frac{2}{E_{\text{eq}}} = \frac{1-\nu_{21}^2}{E_1} + \frac{1-\nu_2^2}{E_2} = \frac{1-0.24^2}{3.14 \times 10^{11}\ \text{N/m}^2} + \frac{1-0.3^2}{2.0 \times 10^{11}\ \text{N/m}^2}$$
$$= 0.755 \times 10^{-11}\ \text{m}^2/N$$
$$E_{\text{eq}} = 2.65 \times 10^{11}\ \text{N/m}^2$$

Due to the low density of the balls, the centrifugal force is lower:

$$m_r = \rho V = 3200 \frac{kg}{m^3} \times 3.05 \times 10^{-6}\ \text{m}^3 = 0.0098\ \text{kg}$$

The centrifugal force is

$$F_c = m_r \omega_c^2 R_c = 0.0098\ \text{kg} \times 1465^2\ 1/s^2 \times 0.0512\ \text{m} = 1077\ \text{N}$$

The radial component of the inner ring contact is the same as for a steel bearing:

$$W_{ir} = 1422\ \text{N}$$

The radial component on the outer ring contact becomes

$$W_{or} = W_{\text{ir}} + F_c = 1422\ \text{N} + 1077\ \text{N} = 2499\ \text{N}$$

The outer ring contact angle is

$$\tan \alpha_{\text{o}} = \frac{W_{oa}}{W_{or}} = \frac{821\ \text{N}}{2499\ \text{N}} = 0.329, \qquad \alpha_{\text{o}} = 18.19^\circ$$

The resultant (normal) component on the outer ring contact becomes

$$\frac{W_{oa}}{W_o} = \sin 18.19^\circ \qquad \text{and} \qquad W_o = \frac{821\ \text{N}}{\sin 18.19^\circ} = 2630\ \text{N}$$

The ellipsoid radii a and b can now be determined by substituting the values already calculated:

$$a = \left(\frac{6\,E\,W_{\max}\,R_{eq}}{\pi \times k \times E_{eq}}\right)^{1/3}$$

$$= \left(\frac{6 \times 1.025 \times 2630\ \text{N} \times 10.16 \times 10^{-3}\ \text{m}}{\pi \times 6.86 \times 2.65 \times 10^{11}\ \text{N/m}^2}\right)^{1/3} = 0.30\ \text{mm}$$

$$b = \left(\frac{6\,k^2\,E\,W_{\max}\,R_{eq}}{\pi \times E_{eq}}\right)^{1/3}$$

$$= \left(\frac{6 \times 6.86^2 \times 1.028 \times 2630\ \text{N} \times 10.82 \times 10^{-3}\ \text{m}}{\pi \times 2.65 \times 10^{11}\ \text{N/m}^2}\right)^{1/3}$$

$$= 2.16\ \text{mm}$$

The maximum pressure at the contact is

$$p_{\max} = \frac{3}{2}\frac{W_{\max}}{\pi ab} = \frac{3}{2}\frac{2630\ \text{N}}{\pi \times 0.3 \times 2.16 \times 10^{-6}\ \text{m}^2} = 1.94\ \text{GPa}$$

Conclusion

In this example of a high-speed bearing, the maximum stress is only marginally lower for the hybrid bearing. The maximum pressure at the contact with outer ring is

For an all-steel bearing: $p_{\max} = 1.99$ GPa
For a hybrid bearing: $p_{\max} = 1.94$ GPa

Example Problem 12-10

For the bearing in Example Problem 12-9, find $h_{\min}$ at the contact with the outer race and the inner race for both (a) an all-steel bearing and (b) a hybrid bearing. Use oil SAE 10 at 70°C and a viscosity–pressure coefficient $\alpha = 2.2 \times 10^{-8}\ \text{m}^2/\text{N}$.

Solution

The analysis of the forces is identical to that of Example Problem 12-9. In this problem, the EHD minimum film thickness is calculated.

Bearing Data from Example Problem 12-9:

Inner contact diameter, $R_{in} = 43.4$ mm
Outer contact diameter, $R_{out} = 59$ mm

Outer ring equivalent radius in the x plane, $R_x = 10.62$ mm (contact at outer race)
Shaft speed, $\omega = 3456$ rad/s
$R_x = 9$ mm, $R_{2x} = 43.4$ mm, $R_y = 234$ mm

Data for Steel Bearing from Example Problem 12.9:

Equivalent modulus of elasticity: $E_{\text{eq}} = 2.2 \times 10^{11}\ \text{N/m}^2$
Resultant component of outer ring contact: $W_o = 4143$ N
Normal contact force at inner ring race: $W_i = 1642$ N

Data for Hybrid Bearing:

Equivalent modulus of elasticity: $E_{\text{eq}} = 2.65 \times 10^{11}\ \text{N/m}^2$
Resultant component of outer ring contact: $W_o = 2630$ N
Normal contact force at inner ring race: $W_i = 1642$ N

For hard surfaces, such as steel in rolling bearings, the equation for calculating the minimum film thickness, $h_{\min}$, is presented in dimensionless form, as follows:

$$\frac{h_{\min}}{R_x} = 3.63 \frac{\bar{U}_r^{0.68} (\alpha \cdot E_{\text{eq}})^{0.49}}{\bar{W}^{0.073}} (1 - e^{-0.68k})$$

Here, α is the viscosity–pressure coefficient and $\bar{U}_r$ and $\bar{W}$ are dimensionless velocity and load, respectively, defined by the following equations:

$$\bar{U}_r = \frac{\mu_o U_r}{E_{\text{eq}} R_x} \qquad \text{and} \qquad \bar{W} = \frac{W}{E_{\text{eq}} R_x^2}$$

Here, U_r is the rolling velocity, μ_o is the viscosity of the lubricant at atmospheric pressure and bearing operating temperature, W is the reaction force of one rolling element, and E_{eq} is the equivalent modulus of elasticity.

All steel and hybrid bearings have the same rolling velocity, which is calculated from the shaft speed via Eq. (12-34):

$$U_{\text{rolling}} = \frac{R_{\text{in}} R_{\text{out}}}{2(R_{\text{in}} + r)} \omega = \frac{R_{\text{in}} R_{\text{out}}}{R_{\text{in}} + R_{\text{out}}} \omega$$

$$U_r = \frac{0.0434 \times 0.059\ \text{m}^2}{0.0434\ \text{m} + 0.059\ \text{m}} \times 3456\ \text{rad/s} = 86.42\ \text{m/s}$$

The first step is to calculate R_x and k for the inner and outer contacts. The radius of curvature at the inner contact is

$$\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}} = \frac{1}{9 \text{ mm}} + \frac{1}{43.4 \text{ mm}}$$

$$R_x = \frac{9 \times 43.4 \text{ mm}}{43.4 + 9} = 7.45 \text{ mm}$$

The ratio α_r is

$$\alpha_r = \frac{R_y}{R_x} = \frac{234 \text{ mm}}{7.45 \text{ mm}} = 31.2$$

The dimensionless coefficient k is derived directly from the ratio α_r:

$$k = \alpha_r^{2/\pi} = 8.94$$

For the outer contact, R_x and k can be taken from Example Problem 12-9:

$$R_x = 10.62 \text{ mm} \qquad \text{and} \qquad k = 7.16$$

a. All-Steel Bearing

Equivalent modulus of elasticity: $E_{eq} = 2.2 \times 10^{11}$ N/m^2
Resultant component of outer ring contact is: $W_o = 4143$ N
Normal contact force at inner ring race: $W_i = 1642$ N

Inner Race Contact: The dimensionless rolling velocity is

$$\bar{U}_r = \frac{0.01 \text{ N-s/m}^2 \times 86.42 \text{ m/s}}{2.2 \times 10^{11} \text{ N/m}^2 \times 7.45 \times 10^{-3} \text{ m}^2} = 52.7 \times 10^{-11}$$

The dimensionless load at the inner race is

$$\bar{W} = \frac{1642 \text{ N}}{2.2 \times 10^{11} \text{ N/m}^2 \times 7.45^2 \times 10^{-6} \text{ m}^2} = 13.44 \times 10^{-5}$$

Substituting these values in the formula for the minimum thickness, we get

$$\frac{h_{min}}{7.45 \times 10^{-3}} = 3.63 \times \frac{(52.7 \times 10^{-11})^{0.68}(2.2 \times 10^{-8} \times 2.2 \times 10^{11})^{0.49}}{(13.44 \times 10^{-5})^{0.073}}$$
$$\times (1 - e^{-0.68 \times 8.94)} = 217.8 \times 10^{-6}$$

The minimum thickness for the inner race is

$$h_{min} = 217.8 \cdot 10^{-6} \times 7.45 \cdot 10^{-3} \text{ m} = 1.62 \cdot 10^{-6} \text{ m}$$
$$h_{min} = 1.62 \ \mu\text{m}$$

Outer Race Contact: The dimensionless rolling velocity is

$$\bar{U}_r = \frac{0.01 \text{ N-s/m}^2 \times 86.42 \text{ m/s}}{2.2 \times 10^{11} \text{ N/m}^2 \times 10.62 \times 10^{-3} \text{ m}^2} = 37 \times 10^{-11}$$

The dimensionless load at the outer race is

$$\bar{W} = \frac{4134 \text{ N}}{2.2 \times 10^{11} \text{ N/m}^2 \times 10.62^2 \times 10^{-6} \text{ m}^2} = 16.7 \times 10^{-5}$$

Substituting these values in the formula for the minimum thickness, we get

$$\frac{h_{\min}}{10.62 \times 10^{-3}} = 3.63 \times \frac{(37 \times 10^{-11})^{0.68}(2.2 \times 10^{-8} \times 2.2 \times 10^{11})^{0.49}}{16.7 \times 10^{-5})^{0.073}} \times (1 - e^{-0.68 \times 7.16}) = 167.1 \times 10^{-6}$$

The minimum thickness at the outer race contact is

$$h_{\min} = 167.1 \times 10^{-6} \times 10.62 \times 10^{-3} \text{ m} = 1.78 \times 10^{-6} \text{ m}$$
$$h_{\min} = 1.78 \ \mu\text{m}$$

b. Hybrid Bearing

Equivalent modulus of elasticity: $E_{\text{eq}} = 2.65 \times 10^{11}$ N/m^2
Resultant force component on outer race contact: $W_o = 2630$ N
Resultant force component on inner ring contact: $W_i = 1642$ N

Inner Race: The value of dimensionless velocity is

$$\bar{U}_r = \frac{0.01 \text{ N-s/m}^2 \times 86.42 \text{ m/s}}{2.65 \times 10^{11} \text{ N/m}^2 \times 7.45 \times 10^{-3} \text{ m}^2} = 43.8 \times 10^{-11}$$

The dimensionless load on the inner race is

$$\bar{W} = \frac{1642 \text{ N}}{2.65 \times 10^{11} \text{ N/m}^2 \times 7.45^2 \times 10^{-6} \text{ m}^2} = 11.16 \times 10^{-5}$$

Substituting these values in the equation for the minimum film thickness, we get

$$\frac{h_{\min}}{7.45 \times 10^{-3}} = 3.63 \cdot \frac{(43.8 \times 10^{-11})^{0.68}(2.2 \times 10^{-8} \times 2.65 \times 10^{11})^{0.49}}{(11.16 \times 10^{-5})^{0.073}} \times (1 - e^{-0.68 \times 8.94}) = 213.24 \times 10^{-6}$$

The minimum fluid film thickness at the inner race contact is

$$h_{\min} = 213.24 \times 10^{-6} \times 7.45 \times 10^{-3} \text{ m} = 1.59 \times 10^{-6} \text{ m}$$
$$h_{\min} = 1.59 \ \mu\text{m}$$

Outer Race: The value of dimensionless velocity is

$$\bar{U}_r = \frac{0.01 \text{ N-s/m}^2 \times 86.42 \text{ m/s}}{2.65 \times 10^{11} \text{ N/m}^2 \times 10.62 \times 10^{-3}} \text{m}^2 = 30.7 \times 10^{-11}$$

The dimensionless load on the outer race is

$$\bar{W} = \frac{2630 \text{ N}}{2.65 \times 10^{11} \text{ N/m}^2 \times 10.62^2 \times 10^{-6} \text{ m}^2} = 8.8 \times 10^{-5}$$

Substituting these values in the equation for the minimum thickness, we get

$$\frac{h_{\min}}{10.62 \times 10^{-3}} = 3.63 \times \frac{(30.7 \times 10^{-11})^{0.68}(2.2 \times 10^{-8} \times 2.65 \times 10^{11})^{0.49}}{(8.8 \times 10^{-5})^{0.073}}$$
$$\times (1 - e^{-0.68 \times 7.16}) = 169.4 \times 10^{-6}$$

The minimum fluid-film thickness at the outer race is

$$h_{\min} = 169.4 \times 10^{-6} \times 10.62 \times 10^{-3} \text{ m} = 1.80 \times 10^{-6} \text{ m}$$

Conclusion

The following is a summary of the results.

Steel bearing

Minimum thickness for inner race: $h_{\min} = 1.62\,\mu\text{m}$
Minimum thickness for outer race: $h_{\min} = 1.78\,\mu\text{m}$

Hybrid Bearing

Minimum thickness for inner race: $h_{\min} = 1.59\,\mu\text{m}$
Minimum thickness for outer race: $h_{\min} = 1.80\,\mu\text{m}$

In this case, the results show only a marginal difference. The minimum thickness for the inner race is a little thinner. It means that the hybrid bearing has only a marginal adverse effect on the EHD lubrication. In this problem, the shaft speed is $N = 30{,}000$ RPM. A significant effect of the hybrid bearing is apparent only at much higher speed.

Problems

12-1 A deep-groove ball bearing has the following dimensions: The bearing has 12 balls of diameter $d = 16$ mm. The radius of curvature of the inner groove (in cross section x-z) is 9 mm. The inner race diameter (at the bottom of the deep groove) is $d_i = 62$ mm (in cross section y-z). The radial load on the bearing is $W = 15{,}000$ N, and the bearing speed is $N = 8000$ RPM. The bearing, rolling elements, and rings are made of steel. The modulus of elasticity of the steel for rollers and rings is $E = 2 \times 10^{11}$ N/m^2, and Poisson's ratio for the steel is $\nu = 0.3$. The properties of the lubricant are: The absolute viscosity at ambient pressure and bearing operating temperature is $\mu_o = 0.01$ N-s/m^2, and the viscosity–pressure coefficient is $\alpha = 2.31 \times 10^{-8}$ m^2/N.

Find the minimum film thickness at the contact with the inner ring.

12-2 A cam and follower are shown in Fig. 12-21. For the same cam and follower, find the maximum stress when there is no rotation. The cam and follower are made of steel. The steel modulus of elasticity is $E = 2.05 \times 10^{11}$ N/m^2 and its Poisson ratio ν is 0.3. The follower is in contact with the tip radius, under the load of $W = 1200$ N.

12-3 For the cam and follower shown in Fig. 12-21, find the viscous torque when the follower is in contact with the tip radius under the same load and fluid viscosity as in Example Problem 12-4. Hint: Find h_o and the contact area, and consider it as a simple shear problem.

12-4 A deep-groove ball radial bearing has 10 balls of diameter $d = 20$ mm. The radius of curvature of the deep grooves (in cross section y-z) is determined by the conformity ratio of $R_r = 0.54$. The inner race diameter (at the bottom of the deep groove) is $d_i = 80$ mm (cross section x-z). The radial load on the bearing is $W = 20{,}000$ N, and the bearing speed is $N = 3000$ RPM. The bearing, rolling elements, and rings are made of steel. The modulus of elasticity of the steel for rollers and rings is $E = 2 \times 10^{11}$ N/m^2, and Poisson ratio for the steel is $\nu = 0.3$.

a. Find the maximum rolling contact pressure at the deep-groove contact.
b. Suppose this bearing is to be used in a high-speed turbine where the average shaft speed is increased to $N = 40{,}000$ RPM. Find the maximum contact pressure.
c. For the preceding two cases, given rolling elements made of silicone nitride, find the maximum pressure in each case of low and high speed. Find the maximum pressure at the inner and outer raceways.

Note: Centrifugal force must be considered at this high rotational speed.

12-5 A deep-groove ball radial bearing has 10 balls of diameter

$d = 20$ mm. The radius of curvature of the deep grooves (in cross section y-z) is determined by the conformity ratio of $R_r = 0.54$. The inner race diameter (at the bottom of the deep groove) is $d_i = 80$ mm (cross section x-z). The radial load on the bearing is $W = 20{,}000$ N, and the bearing speed is $N = 3000$ RPM. The bearing, rolling elements, and rings are made of steel. The modulus of elasticity of the steel for rollers and rings is $E = 2 \times 10^{11}$ N/m^2, and Poisson ratio for the steel is $\nu = 0.3$. The absolute viscosity of the lubricant at ambient pressure and bearing operating temperature is $\mu_o = 0.015$ N-s/m^2. The viscosity–pressure coefficient is $\alpha = 2.2 \times 10^{-8}$ m^2/N.

a. Find the minimum fluid film thickness of a rolling contact at the deep-groove contact.
b. Suppose this bearing is to be used in a high-speed turbine where the average shaft speed is increased to $N = 40{,}000$ RPM. Find the minimum film thickness (consider centrifugal force).
c. For the preceding two cases, given rolling elements made of silicone nitride, find the maximum pressure in each case of low and high speed. Find the minimum film thickness at the inner and outer raceways.