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# Non-Newtonian Viscoelastic Effects

## 19.1 INTRODUCTION

The previous chapters focused on Newtonian lubricants such as regular mineral oils. However, non-Newtonian multigrade lubricants, also referred to as VI (viscosity index) improved oils are in common use today, particularly in motor vehicle engines. The multigrade lubricants include additives of long-chain polymer molecules that modify the flow characteristics of the base oils. In this chapter, the hydrodynamic analysis is extended for multigrade oils.

The initial motivation behind the development of the multigrade lubricants was to reduce the dependence of lubricant viscosity on temperature (to improve the viscosity index). This property is important in motor vehicle engines, e.g., starting the engine on cold mornings. Later, experiments indicated that multigrade lubricants have complex non-Newtonian characteristics. The polymer-containing lubricants were found to have other rheological properties in addition to the viscosity. These lubricants are *viscoelastic* fluids, in the sense that they have viscous as well as elastic properties.

Polymer additives modify several flow characteristics of the base oil.

1. The polymer additives increase the viscosity of the base oil.
2. The polymer additives moderate the reduction of viscosity with temperature (improve the viscosity index).

3. The viscosity becomes a decreasing function of shear rate (shear-thinning property).
4. Normal stresses are introduced. In simple shear flow, $u = u(y)$, there are normal stress differences $\sigma_x - \sigma_y$ (first difference) and $\sigma_y - \sigma_z$ (second difference). The first difference is much higher than the second difference.
5. The polymer additives introduce stress-relaxation characteristics into the fluid, exemplified by a phase lag between the shear stress and a periodic shear rate. This property is what is meant by the term *viscoelasticity*; namely, the fluid becomes elastic as well as viscous.

Although multi grade oils were developed to improve the viscosity index, later experiments revealed a significant improvement in the lubrication performance of journal bearings that cannot be explained by changes of viscosity. Dubois et al. (1960) compared the performance of mineral oils and VI improved oils in journal bearings under static load. They used high journal speeds and measured load capacity, friction and eccentricity. The results indicated a superior performance of the multigrade oils with polymer additives. Additional conclusion of this investigation (important for comparison with analytical investigations) is that the relative improvement in load capacity of the VI improved oils becomes greater as the eccentricity increases. Okrent (1961) and Savage and Bowman (1961) found less friction and wear in the connecting-rod bearing in a car engine (dynamically loaded journal bearing).

Analytical investigations showed that the improvements in the lubrication performance of VI improved oils are not due to changes in the viscosity. Horowitz and Steidler (1960) performed analytical investigation and showed that the improvement in the lubrication performance could not be accounted for by the different function of viscosity versus shear rate and temperature. In fact, they found that the non-Newtonian viscosity increases the friction coefficient (opposite trend to the experiments of Dubois et al., 1960).

A survey of the previous analytical investigation by Harnoy (1978) shows that the measured order of magnitude of the first and second normal stresses is too low to explain any significant improvement in the lubrication performance. This discussion indicates that the elasticity of the fluid (stress-relaxation effect) is the most probable explanation of the improvement in performance of *viscoelastic* lubricants.

The criterion for improvement of the lubrication performance is very important. For example, polymer additives increase the viscosity of mineral oils; in turn, the load capacity increases. However, our basis of comparison is the load capacity at equivalent viscosity and bearing geometry. Higher viscosity on its own is not considered as an improvement in the lubrication performance, because the friction losses as well as load capacity are both proportional to the viscosity. Moreover, it is possible to use higher viscosity oils without resorting to oil additives of long chain polymer molecules. An appropriate criterion for an improvement of the lubrication performance is the ratio between the friction force and the load capacity (bearing friction coefficient).

## 19.2 VISCOELASTIC FLUID MODELS[1]

For the analysis of viscoelastic fluids, various models have been developed. The models are in the form of *rheological equations*, also referred to as *constitutive equations*. An example is the Maxwell fluid equation (Sec. 2.9).

Multi-grade lubricants are predominantly viscous fluids with a small elastic effect. Therefore, in hydrodynamic lubrication, the viscosity has a dominant role in generating the pressure wave, while the fluid elasticity has only a small (*second order*) effect. In such cases, the flow of non-Newtonian viscoelastic fluids can be analyzed by using *differential type constitutive equations*. The main advantage of these equations is that the stress components are explicit functions of the strain-rate components. In a similar way to Newtonian Navier-Stokes equations, viscoelastic differential-type equations can be directly applied for solving the flow. Differential type equations were widely used in the theory of lubrication for bearings under steady and particularly unsteady conditions.

Differential type constitutive equations are restricted to a class of flow problems where the Deborah number is low, $De \ll 1$. The ratio De is of the relaxation time of the fluid, $\lambda$, to a characteristic time of the flow, $\Delta t$; $De = \lambda/\Delta t$. Here, $\Delta t$ is the time for a significant change in the flow; e.g., in a sinusoidal flow, $\Delta t$ is the oscillation period.

The early analytical work in hydrodynamics lubrication of viscoelastic fluids is based on the second-order fluid equation of Rivlin and Ericksen (1955) or on the equation of Oldroyd (1959). These early equations are referred to as *conventional*, differential-type rheological equations. Coleman and Noll (1960) showed that the Rivlin and Ericksen equation represents the first perturbation from Newtonian fluid for slow flows, but its use has been extended later to high shear rates of lubrication.

An analysis based on the conventional second order equation (Harnoy and Hanin, 1975) indicated significant improvements of the viscoelastic lubrication performance in journal bearings under steady and dynamic loads. Moreover, the improvements increase with the eccentricity (in agreement with the trends observed in the experiments of Dubois et al., 1960).

[1] This section and the following viscoelastic analysis are for advanced studies.

An important feature of these conventional equations for viscoelastic fluids is that they describe the unsteady stress-relaxation effect and the first normal-stress difference ($\sigma_x - \sigma_y$) in a steady shear flow by the same parameter. In many cases, the relaxation time that describes dynamic (unsteady) flow effects was determined by normal-stress measurements in steady shear flow between rotating plate and cone (Weissenberg rheometer).

In conventional rheological equations, the normal stresses are proportional to the second power of the shear-rate. Hydrodynamic lubrication involves very high shear-rates, and the conventional equations predict unrealistically high first-normal-stress differences. Moreover, when the actual measured magnitude of the normal stresses was considered in lubrication, its effect is negligibly small in comparison to the stress-relaxation effect. It was realized that for high shear-rate flows, the two effects of the first normal stress difference and stress relaxation must be described by means of two parameters capable of separate experimental determination.

For high shear rate flows of lubrication, the forgoing arguments indicated that there is a requirement for a different viscoelastic model that can separate the unsteady relaxation effects from the normal stresses.

### 19.2.1 Viscoelastic Model for High Shear-Rate Flows

A rheological equation that separates the normal stresses from the relaxation effect was developed and used for hydrodynamic lubrication by Harnoy (1976). For this purpose, a unique convective time derivative, $\delta/\delta t$, is defined in a coordinate system that is attached to the three principal directions of the derived tensor. This rheological equation can be derived from the Maxwell model (analogy of a spring and dashpot in series). The Maxwell model in terms of the deviatoric stresses, $\tau'$, is

$$\tau'_{ij} + \lambda \frac{\delta}{\delta t} \tau'_{ij} = \mu e_{ij} \tag{19-1}$$

Here, $\lambda$ is the relaxation time and the strain-rate components, $e_{ij}$, are

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{19-2}$$

where $v_i$ are the velocity components in orthogonal coordinates $x_i$. The deviatoric stress tensor can be derived explicitly as

$$\tau'_{ij} = \mu\left(1 + \lambda \frac{\delta}{\delta t}\right)^{-1} e_{ij} \tag{19-3}$$

Expanding the operator in terms of an infinite series of increasing powers of $\lambda$ results in

$$\tau'_{ij} = \mu\left(e_{ij} - \lambda\frac{\delta}{\delta t}e_{ij} + \lambda^2\frac{\delta^2}{\delta t^2}e_{ij} + \cdots + (-\lambda)^{n-1}\frac{\delta^{n-1}}{\delta t^{n-1}}e_{ij}\right) \tag{19-4}$$

For low-Deborah number, $De = \lambda/\Delta t$, where $\Delta t$ is a characteristic time of the flow, second-order and higher powers of $\lambda$ are negligible. Therefore, only terms with the first power of $\lambda$ are considered, and the equation gets the following simplified form:

$$\tau'_{ij} = \mu\left(e_{ij} - \lambda\frac{\delta}{\delta t}e_{ij}\right) \tag{19-5}$$

The tensor time derivative is defined as follows (see Harnoy 1976):

$$\frac{\delta e_{ij}}{\delta t} = \frac{\partial e_{ij}}{\partial t} + \frac{\partial e_{ij}}{\partial x_\alpha}v_\alpha - \Omega_{i\alpha}e_{\alpha j} + e_{i\alpha}\Omega_{\alpha j} \tag{19-6}$$

The definition is similar to that of the Jaumann time derivative (see Prager 1961). Here, however, the rotation vector $\Omega_{ij}$ is the rotation components of a rigid, rectangular coordinate system (1, 2, 3) having its origin fixed to a fluid particle and moving with it. At the same time, its directions always coincide with the three principle directions of the derived tensor. The last two terms, having the rotation, $\Omega_{ij}$, can be neglected for high-shear-rate flow because the rotation of the principal directions is very slow. Equations (19-5) and (19-6) form the viscoelastic fluid model for the following analysis.

## 19.3 ANALYSIS OF VISCOELASTIC FLUID FLOW

Similar to the analysis in Chapter 4, the following derivation starts from the balance of forces acting on an infinitesimal fluid element having the shape of a rectangular parallelogram with dimensions $dx$ and $dy$, as shown in Fig. 4-1. The following derivation of Harnoy (1978) is for two-dimensional flow in the $x$ and $y$ directions. In an infinitely long bearing, there is no flow or pressure gradient in the $z$ direction. In a similar way to that described in Chapter 4, the balance of forces results in

$$d\tau\, dx = dp\, dy \tag{19-7}$$

*Remark*: If the fluid inertia is not neglected, the equilibrium equation in the $x$ direction for two-dimensional flow is [see Eq. (5-4b)]

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \tag{19-8}$$

After disregarding the fluid inertia term on the left-hand side, the equation is equivalent to Eq. (19-7). In two-dimensional flow, the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{19-9}$$

For viscoelastic fluid, the constitutive equations (19-5) and (19-6) establishes the relation between the stress and velocity components. Substituting Eq. (19-5) in the equilibrium equation (19-8) yields the following differential equation of steady-state flow in a two-dimensional lubrication film:

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2} - \lambda\mu \frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial y\, \partial x} + \frac{\partial^2 u}{\partial y^2} v\right) \tag{19-10}$$

Converting to dimensionless variables:

$$\bar{u} = \frac{u}{U}; \qquad \bar{v} = \frac{R}{C}\frac{v}{U}; \qquad \bar{x} = \frac{x}{R}; \qquad \bar{y} = \frac{y}{C} \tag{19-11}$$

The ratio $\Gamma$, often referred to as the Deborah number, De, is defined as

$$\text{De} = \Gamma = \frac{\lambda U}{R} \tag{19-12}$$

The flow equation (19-10) becomes

$$\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \mu \frac{\partial}{\partial y}\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}\, \partial \bar{x}} \bar{u} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \bar{v}\right) = 2F(\bar{x}) \tag{19-13}$$

where

$$2F(\bar{x}) = \frac{C^2}{\eta U R} \frac{dp}{d\bar{x}} \tag{19-14}$$

In these equations, $\lambda$ is small in comparison to the characteristic time of the flow, $\Delta t$. The characteristic time $\Delta t$ is the time for a significant periodic flow to take place, such as a flow around the bearing or the period time in oscillating flow. It results that De is small in lubrication flow, or $\Gamma \ll 1$.

### 19.3.1 Velocity

The flow $\bar{u} = \bar{u}(\bar{y})$ can be divided into a Newtonian flow, $\bar{u}_0$, and a secondary flow, $\bar{u}_1$, owing to the elasticity of the fluid:

$$\bar{u} = \bar{u}_0 + \Gamma \bar{u}_1 \tag{19-15}$$

In the flow equations, the secondary flow terms include the coefficient $\Gamma$.

### 19.3.2 Solution of the Differential Equation of Flow

In order to solve the nonlinear differential equation of flow for small $\Gamma$, a perturbation method is used, expanding in powers of $\Gamma$ and retaining the first power only, as follows:

$$\bar{u} = \bar{u}_0 + \Gamma \bar{u}_1 + 0(\Gamma^2) \tag{19-16}$$

$$\bar{v} = \bar{v}_0 + \Gamma \bar{v}_1 + 0(\Gamma^2) \tag{19-17}$$

$$F(\bar{x}) = F_0(\bar{x}) + \Gamma F_1(\bar{x}) + 0(\Gamma^2) \tag{19-18}$$

Introducing Eqs. (19-16)–(19-18) into Eq. (19-13) and equating terms with corresponding powers of $\Gamma$ yields two linear equations:

$$\frac{\partial^2 \bar{u}_0}{\partial^2 \bar{y}^2} = 2F_0(\bar{x}) \tag{19-19}$$

$$\frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} - \frac{\partial}{\partial \bar{y}}\left(\frac{\partial^2 \bar{u}_0}{\partial \bar{x}\, \partial \bar{y}}\, \bar{u}_0 + \frac{\partial^2 \bar{u}_0}{\partial \bar{y}^2}\, \bar{v}_0\right) = 2F_1(\bar{x}) \tag{19-20}$$

The boundary conditions of the flow are:

$$\text{at } \bar{y} = 0, \qquad \bar{u} = 0 \tag{19-21}$$

$$\text{at } \bar{y} = \frac{h}{c}, \qquad \bar{u} = 1 \tag{19-22}$$

Because there is no side flow, the flux $q$ is constant:

$$\int_0^h u\, dy = q = \frac{h_e U}{2} \tag{19-23}$$

For the first velocity term, $\bar{u}_0$, the boundary conditions are:

$$\text{at } \bar{y} = 0 \qquad \bar{u}_0 = 0 \tag{19-24}$$

$$\text{at } \bar{y} = \frac{h}{c} \qquad \bar{u}_0 = 1 \tag{19-25}$$

Expanding the flux into powers of $\Gamma$:

$$q = q_0 + \Gamma q_1 + 0(\Gamma^2 q) = \frac{h_e U}{2} \tag{19-26}$$

and we denote

$$h_i = \frac{2q_i}{U} \qquad \text{for } i = 0 \text{ and } 1 \tag{19-27}$$

The flow rate of the zero-order (Newtonian) velocity is

$$\int_0^{h/c} \bar{u}_0 \, d\bar{y} = \frac{q_0}{CU} = \frac{h_0}{2C} \tag{19-28}$$

After integrating Eq. (19-19) twice and using the boundary conditions (19-24), (19-25), and (19-28), the zero-order equations result in the well-known Newtonian solutions:

$$\bar{u}_0 = M\bar{y}^2 + N\bar{y} \tag{19-29}$$

where

$$M = 3C^2\left(\frac{1}{h^2} - \frac{h_0}{h^3}\right) \tag{19-30}$$

$$N = C\left(\frac{3h_0}{h^2} - \frac{2}{h}\right) \tag{19-31}$$

The velocity component in the $y$ direction, $v_0$, is determined from the continuity equation. Substituting $\bar{u}_0$ and $\bar{v}_0$ in Eq. (19-20) enables solution of the second velocity, $\bar{u}_1$. The boundary conditions for $\bar{v}_1$ are:

$$\text{at } \bar{y} = 0, \qquad \bar{u}_1 = 0 \tag{19-32}$$

$$\text{at } \bar{y} = \frac{h}{c}, \qquad \bar{u}_1 = 0 \tag{19-33}$$

$$\int_0^{h/c} \bar{u}_1 \, d\bar{y} = \frac{q_1}{CU} = \frac{h_1}{2C} \tag{19-34}$$

The resulting solution for the velocity in the $x$ direction is

$$\bar{u} = \bar{u}_0 + \Gamma u_1 = \alpha\bar{y}^4 + \beta\bar{y}^3 + \gamma\bar{y}^2 + \delta\bar{y} \tag{19-35}$$

where

$$\alpha = 3\Gamma C^4\left(-\frac{2}{h^5}+\frac{5h_e}{h^6}-\frac{3h_e^2}{h^7}\right)\frac{dh}{d\bar{x}} \tag{19-36}$$

$$\beta = \Gamma C^3\left(18\frac{h_e^2}{h^6}-\frac{24h_e}{h^5}+\frac{8}{h^4}\right)\frac{dh}{d\bar{x}} \tag{19-37}$$

$$\gamma = 3C^2\left(\frac{1}{h^2}-\frac{h_e}{h^3}\right)+\Gamma C^2\left(-\frac{6}{5h^3}+\frac{9h_e}{h^4}-\frac{54h_e^2}{5h^5}\right)\frac{dh}{d\bar{x}} \tag{19-38}$$

$$\delta = C\left(\frac{3h_e}{h^2}-\frac{2}{h}\right)+\Gamma C\left(\frac{9h_e^2}{5h^4}-\frac{4}{5}\frac{1}{h^2}\right)\frac{dh}{d\bar{x}} \tag{19-39}$$

where $h_e = h_o + \Gamma h_1$ is an unknown constant.

## 19.4 PRESSURE WAVE IN A JOURNAL BEARING

In a similar way to the solution in Chapter 4, the following pressure wave equation is obtained from Eq. (19-10) and the fluid velocity:

$$p = 6R\mu U\int_0^x\left(\frac{1}{h^2}-\frac{h_e}{h^3}\right)dx + \Gamma R\mu U\left(-\frac{4}{5}\frac{1}{h^2}+\frac{9}{10}\frac{h_e^2}{h^4}\right)+k \tag{19-40}$$

The last constant, $k$, is determined by the external oil feed pressure. The constant $h_e$ is determined from the boundary conditions of the pressure $p$ around the bearing.

The analysis is limited to a relaxation time $\lambda$ that is much smaller than the characteristic time, $\Delta t$ of the flow. In this case, the characteristic time is $\Delta t = O(U/R)$, which is the order of magnitude of the time for a fluid particle to flow around the bearing. The condition becomes $\lambda \ll U/R$.

For a journal bearing, the pressure wave for a viscoelastic lubricant was solved and compared to that of a Newtonian fluid; see Harnoy (1978). The pressure wave was solved by numerical integration. Realistic boundary conditions were applied for the pressure wave [see Eq. (6-67)]. The pressure wave starts at $\theta = 0$ and terminates at $\theta_2$, where the pressure gradient also vanishes. The solution in Fig. 19-1 indicates that the elasticity of the fluid increases the pressure wave and load capacity.

### 19.4.1 Improvements in Lubrication Performance of Journal Bearings

The velocity in Eq. (19-35) allows the calculation of the shear stresses, and friction torque on the journal. The results indicated (Harnoy, 1978) that the elasticity of the fluid has a very small effect on the viscous friction losses of a

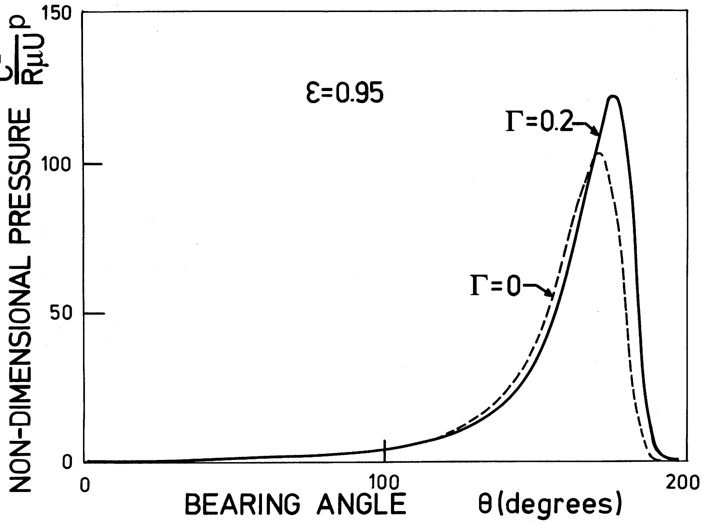


**FIG. 19-1** Journal bearing pressure wave for Newtonian and viscoelastic lubricants. (From Harnoy, 1978.)

journal bearing, and the reduction in the friction coefficient is mostly due the load capacity improvement. As mentioned in Sec. 19.1, the friction coefficient is a criterion for the improvement in the lubrication performance under static load. In short hydrodynamic journal bearings, e.g., in car engines, the elasticity of the fluid reduces the friction coefficient by a similar order of magnitude (Harnoy, 1977).

Harnoy and Zhu (1991) conducted dynamic analysis of short hydrodynamic journal bearings based on the same viscoelastic fluid model. The results show that viscoelastic lubricants play a significant role in improving the lubrication performance under heavy dynamic loads, where the eccentricity ratio is high; see Fig. 15-2. For a viscoelastic lubricant, the maximum eccentricity ratio $\varepsilon_{min}$ of the locus of the journal center is significantly reduced in comparison to that of a Newtonian lubricant. In conclusion, analytical results based on the viscoelastic fluid model of Eqs. (19-5) and (19-6) indicated significant improvements of lubrication performance under steady and dynamic loads. Moreover, the improvement increases with the eccentricity. These results are in agreement with the trends obtained in the experiments of Dubois et al. 1960.

However, similar improvements in performance were obtained by using the conventional second-order equation. Therefore, the results for journal bearings cannot indicate the appropriate rheological equation, which is in better agreement with experimentation. It is shown in Sec. 19.5 that squeeze-film flow can be used for the purpose of validation of the appropriate rheological equation, because the solutions of two theoretical models are in opposite trends.

Viscoelastic lubricants play a significant role in improving lubrication performance under heavy dynamic loads, where the eccentricity ratio is high; see Fig. 15-2. For a viscoelastic lubricant, the maximum locus eccentricity ratio $\varepsilon_{min}$ is significantly reduced in comparison to that of a Newtonian lubricant.

### 19.4.2 Viscoelastic Lubrication of Gears and Rollers

Harnoy (1976) investigated the role of viscoelastic lubricants in gears and rollers. In this application, there is a pure rolling or, more often, a rolling combined with sliding. For rolling and sliding between a cylinder and plane (see Fig. 4-4) the solution of the pressure wave for Newtonian and viscoelastic lubricants is shown in Fig. 19-2. The viscoelastic fluid model is according to Eqs. (19-5) and (19-6). The results of the numerical integration are presented for different rolling-to-sliding ratios $\xi$. The relative improvement of the pressure wave and load capacity due to the elasticity of the fluid are more pronounced for rolling than for sliding (the relative rise of the pressure wave increases with $\xi$).

## 19.5 SQUEEZE-FILM FLOW

Squeeze-film flow between two parallel circular and concentric disks is shown in Fig. 5-5 and 5-6. Unlike experiments in journal bearings, squeeze-film experiments can be used for verification of viscoelastic models. In fact, the viscoelastic fluid model described by Eqs. (19-5) and (19-6) resulted in agreement with squeeze-film experiments, while the conventional second-order equation resulted in conflict with experiments.

Two types of experiments are usually conducted:

1. The upper disk has a constant velocity $V$ toward the lower disk, and a load cell measures the upper disk load capacity versus the film thickness, $h$.
2. There is a constant load $W$ on the upper disk, and the film thickness $h$ is measured versus time. Experiments were conducted to measure the *descent time*, namely, the time for the film thickness to be reduced to half of its initial height.

For Newtonian fluids, the solution of the load capacity in the first experiment is presented in Sec. 5.7. If the upper disk has a constant velocity $V$ toward the lower disk (first experiment), the load capacity of the squeeze-film of

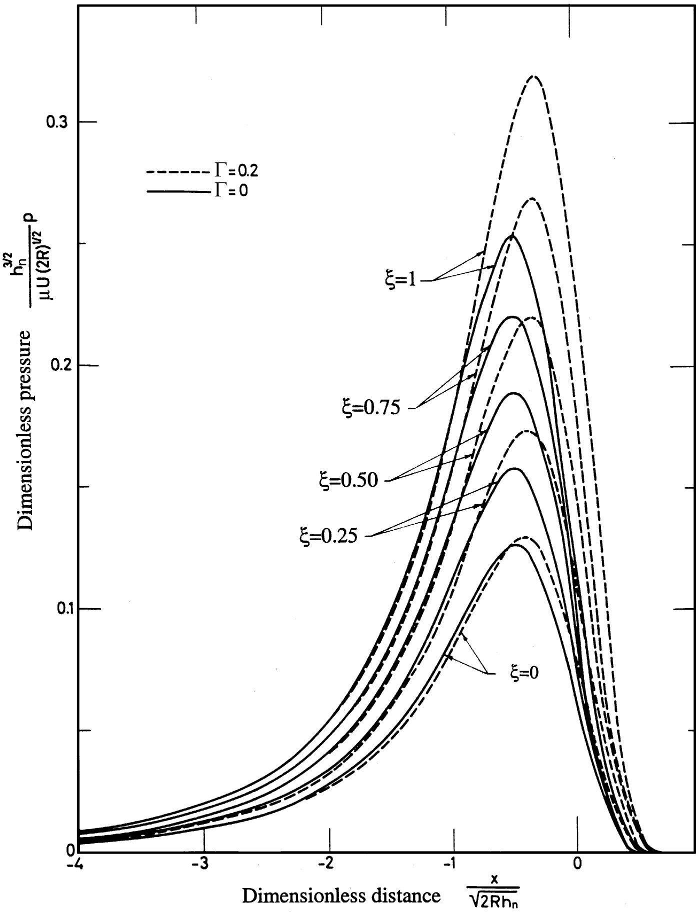


**FIG. 19-2** Comparison of Newtonian and viscoelastic pressure waves in rollers for various rolling-to-sliding ratios ξ.

viscoelastic fluid is less than its Newtonian counterpart. In Sec. 5.7, it was shown that the squeeze-film load capacity of a Newtonian fluid is

$$W_o = \frac{3\pi\mu V R^4}{2h^3} \tag{19-41}$$

Here, $W_o$ is the Newtonian load capacity, $h$ is the clearance, and $V$ is the disk velocity when the disks are approaching each other. If the fluid is viscoelastic, under constant velocity $V$, the equation for the load capacity $W$ becomes (Harnoy, 1987)

$$\frac{W}{W_o} = 1 - 2.1\ \mathrm{De} \qquad (19\text{-}42)$$

Here, De is the ratio

$$\mathrm{De} = \frac{\lambda V}{h} \qquad (19\text{-}43)$$

and $h$ is the clearance. This result is in agreement with the physical interpretation of the viscoelasticity of the fluid. In a squeeze action, the stresses increase with time, because the film becomes thinner. For viscoelastic fluid, the stresses are at an earlier, lower value. This effect is referred to as a *memory effect*, in the sense that the instantaneous stress is affected by the history of previous stress. In this case, it is affected only by the recent history of a very short time period.

For the first experiment of load under constant velocity, all the viscoelastic models are in agreement with the experiments of small reduction in load capacity. However, for the second experiment under constant load, the early conventional models (the second order fluid and other models) are in conflict with the experiments. Leider and Bird (1974) conducted squeeze-film experiments under a constant load. For viscoelastic fluids, the experiments demonstrated a longer squeezing time (*descent time*) than for a comparable viscous fluid. Grimm (1978) reviewed many previous experiments that lead to the same conclusion.

Tichy and Modest (1980) were the first to analyze the squeeze-film flow based on Harnoy rheological equations (19.5) and (19.6). Later, Avila and Binding (1982), Sus (1984), and Harnoy (1987) analyzed additional aspects of the squeeze-film flow of viscoelastic fluid according to this model. The results of all these analytical investigations show that Harnoy equation correctly predicts the trend of increasing descent time under constant load, in agreement with experimentation. In that case, the theory and experiments are in agreement that the fluid elasticity improves the lubrication performance in unsteady squeeze-film under constant load.

Brindley et al. (1976) solved the second experiment problem of squeeze-film under constant load using the second order fluid model. The result predicts an opposite trend of decreasing descent time, which is in conflict with the experiments. In this case, the second dynamic experiment can be used for validation of rheological equations.

An additional example where the rheological equations (19.5) and (19.6) are in agreement with experiments, while the conventional equations are in conflict with experiments is the boundary-layer flow around a cylinder. These experiments can also be used for similar validation of the appropriate viscoelastic models, resulting in similar conclusions for high shear rate flows (Harnoy, 1977, 1989)

### 19.5.1 Conclusions

The theory and experiments indicate that the viscoelasticity improves the lubrication performance in comparison to that of a Newtonian lubricant, particularly under dynamic loads.

Although the elasticity of the fluid increases the load capacity of a journal bearing, the bearing stability must be tested as well. The elasticity of the fluid (spring and dashpot in series) must affect the dynamic characteristics and stability of journal bearings. Mukherjee et al. (1985) studied the bearing stability based on Harnoy rheological equations [Eqs. (19.5) and (19.6)]. Their results indicated that the stability map of viscoelastic fluid is different than for Newtonian lubricant. This conclusion is important to design engineers for preventing instability, such as bearing whirl.

As mentioned earlier, these experiments were in conflict with previous rheological equations, which describe normal stresses as well as the stress-relaxation effect. However, the experiments were in agreement with the trend that is predicted by the rheological model based on Eqs. (19-5) and (19-6) which does not consider normal stresses.