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Cone Brakes and Clutches

These brakes have the advantage of greater torque for a smaller axial force than either type of disk brake discussed in [Chapter 5](#). The magnitude of the improvement is limited, however, by the observation that for small cone angles a disengagement force may be required, depending on the friction coefficient, because the inner and outer cones may tend to wedge together. This is because on engagement the inner cone is radially compressed and the outer cone is radially enlarged as the brake is engaged. For small cone angles the induced friction force dominates the normal force, which tends to expel the inner cone, so that an external force is required for separation. This characteristic, however, may be useful in those applications where a brake is to remain engaged in the presence of disengagement forces.

I. TORQUE AND ACTIVATION FORCE

The pertinent geometry of the cone brake is shown in [Figure 1](#). If the inner and outer cones are concentric and rigid, the amount worn from the lining during engagement will be given by

$$\delta = kpr \quad (1-1)$$

where p denotes the pressure and r is the radius to the point where p acts. Proportionality constant k may be evaluated by observing that the form of relation (1-1) demands that the maximum pressure occur at the minimum radius. Hence

$$\delta = kp_{\max}r_i \quad (1-2)$$

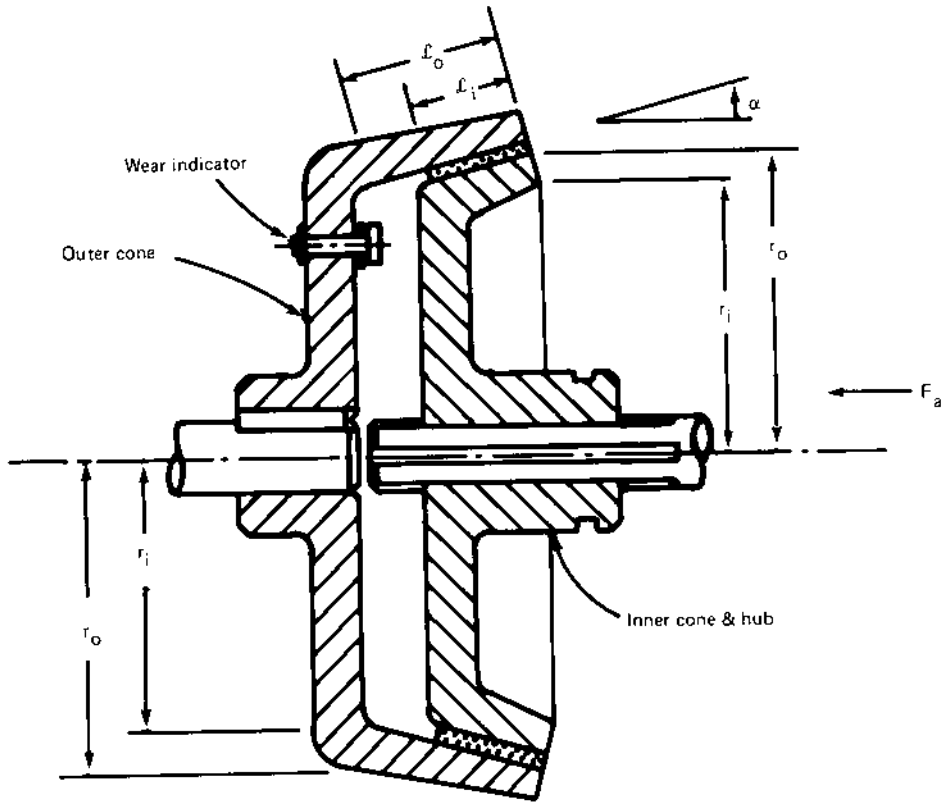


FIGURE 1 Cone brake and its geometry (partially worn lining).

Upon equating equations (1-1) and (1-2), we find that

$$p = p_{\max} \frac{r_i}{r} \quad (1-3)$$

Although the brake lining is more easily attached to the inner cone, with the torque acting at the inner surface of the outer cone, we shall derive formulas on the assumption that the torque acts on the outer surface of the inner cone because this will give a torque capacity that the brake can equal or exceed until the lining is destroyed. Thus

$$T = \mu \int_A p r \, da = \mu p_{\max} r_i \int_A da = \frac{2\mu\pi p_{\max} r_i}{\sin \alpha} \int_{r_i}^{r_o} r \, dr \quad (1-4)$$

where the element of area on the outside of the inner cone is given by

$$da = 2\pi r d\ell = 2\pi r \frac{dr}{\sin \alpha} \quad (1-5)$$

and where we have used $d\ell \sin \alpha = dr$ and the Pappus theorem for the area of a surface of revolution. Upon integration the expression for the torque becomes

$$T = \frac{\mu \pi p_{\max}}{\sin \alpha} r_i (r_o^2 - r_i^2) \quad (1-6)$$

Since this expression vanishes for $r_i = 0$ and for $r_i = r_o$ but not for intermediate values, we may set the derivative of T with respect to r_i equal to zero to find that the maximum torque may be obtained when

$$r_i = \frac{1}{\sqrt{3}} r_o \quad (1-7)$$

for which the torque is given by

$$T = \frac{2}{3\sqrt{3}} \mu \pi \frac{p_{\max}}{\sin \alpha} r_o^3 \quad (1-8)$$

To find the activation force, we return to [Figure 1](#) to discover that it is given by

$$\begin{aligned} F_a &= \int_A (p \sin \alpha + \mu p \cos \alpha) da \\ &= (\sin \alpha + \mu \cos \alpha) p_{\max} r_i \int_A \frac{1}{r} 2\pi r \frac{dr}{\sin \alpha} \\ &= 2\pi p_{\max} \left(1 + \frac{\mu}{\tan \alpha} \right) r_i (r_o - r_i) \end{aligned} \quad (1-9)$$

When $\alpha = \pi/2$, equations (1-6) and (1-9) reduce to the correct expressions for the torque and activation force for an annular contact disk brake with a single friction surface.

Unlike plate clutch and brakes, it may take a retraction force to disengage a cone clutch or brake, just as it takes a force to remove a cork from a bottle. The magnitude of the retraction force, which we shall denote by F_r , may be derived from the force equilibrium condition in the axial direction for the forces shown in [Figure 1](#). After replacing $\mu p da$ with $-\mu p da$, we find that the incremental retraction force dF_r is given by

$$dF_r = 2\pi r_i \frac{dr}{\sin \alpha} (\mu p \cos \alpha - p \sin \alpha) \quad (1-10)$$

where we again use the pressure p and element of area da as defined by equations (1-3) and (1-5), respectively. After performing the integration, we have

$$F_r = 2\pi p_{\max} r_i (r_o - r_i) \left(\frac{\mu}{\tan \alpha} - 1 \right) \quad (1-11)$$

Clearly, a retraction force is necessary only when $(\mu/\tan \alpha - 1)$ is greater than zero. F_r vanishes if

$$\frac{\mu}{\tan \alpha} = 1 \quad (1-12)$$

that is, if

$$\mu = \tan \alpha$$

The ratio of torque to activation force for a cone clutch or brake may be obtained by dividing equation (1-6) by equation (1-9) to get

$$\frac{T}{F_a} = \frac{r_o + r_i}{2} \frac{\mu}{\sin \alpha + \mu \cos \alpha} \quad (1-13)$$

in which the ratio $(r_o + r_i)/2$ may be considered a magnification factor that operates upon the ratio

$$f(\mu, \alpha) = \frac{\mu}{\sin \alpha + \mu \cos \alpha} \quad (1-14)$$

To find an extreme value of $f(\mu, \alpha)$ with respect to the cone angle, differentiate it with respect to α to get

$$\frac{df}{d\alpha} = -\mu \frac{\cos \alpha - \mu \sin \alpha}{(\sin \alpha + \mu \cos \alpha)^2} = 0 \quad \text{whenever} \quad \cos \alpha = \mu \sin \alpha \quad (1-15)$$

Since the second derivative $d^2f/d\alpha^2$ is positive whenever equation (1-15) holds, $f(\mu, \alpha)$ is minimum along the curve

$$\mu = \frac{1}{\tan \alpha} \quad (1-16)$$

Because points on this curve represent the minimum torque that can be had from a cone brake or clutch, it is clear that a design for such a unit should not lie along this curve if it can be avoided.

Upon comparison of equation (3-3) with equation (1-8) we find that equation (1-8) reduced to equation (3-3) when $\alpha = \pi/2$. Consequently, we may find what configuration of a cone brake or clutch can equal or exceed the T/R ratio of a plate clutch or brake by solving

$$f(\mu, \alpha) = \mu \quad (1-17)$$

From equation (1-14) we find that equation (1-17) holds whenever $\sin \alpha + \mu \cos \alpha = 1$. Hence, designs for which μ is greater than

$$\mu = \frac{1 - \sin \alpha}{\cos \alpha} \quad (1-18)$$

usually should be avoided because a plate clutch having the same inner and outer radii will provide the same torque, but with smaller axial dimensions.

The last relation that is of interest in the design of a cone brake or clutch is the condition for which the retraction force is zero. From equation (1-11) it is clear that F_r vanishes when

$$\mu = \tan \alpha \quad (1-19)$$

Curves given by these last three relations are plotted in Figure 4. The dashed curve in this figure is the plot of relation (1-18), the dotted curve is the plot of equation (1-16), and the solid curve is the plot of equation (1-19). The surface described by equation (1-14) is shown in Figure 1, contour lines that depict elevations on that surface itself are shown in Figure 2. Upon

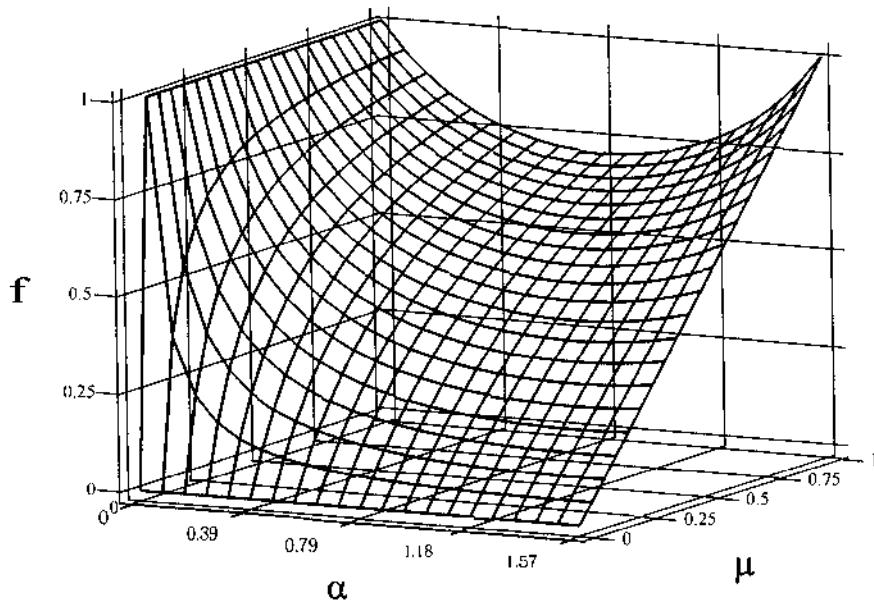


FIGURE 2 Surface defined by $f(\mu, \alpha)$ for $0 \leq \mu \leq 1$ and $0 \leq \alpha \leq \pi/2$.

comparison of the three figures, the minimum described by equation (1-16) and plotted in Figure 4 is qualitatively evident in Figures 3 and 4.

It is Figure 4 that is directly useful in the design of cone brakes and clutches, because we find from equation (1-19) that the regions to the left of the solid curve (regions 2 and 4) is where a retraction force is required; this is where $\mu \geq \tan \alpha$. Designs where μ and α are coordinates of points to the right of the solid curve that fall within regions 3 and 5 generally should be avoided because a greater torque-to-activation-force ratio (T/Fd) may be had with a plate clutch or brake. This leaves region 1, which lies below both the dotted curve and the dashed curve and to the right of the solid curve, as the only region where either a cone clutch or a cone brake is superior to either a single-plate clutch or to a single-plate brake, respectively, and where no retraction force is required.

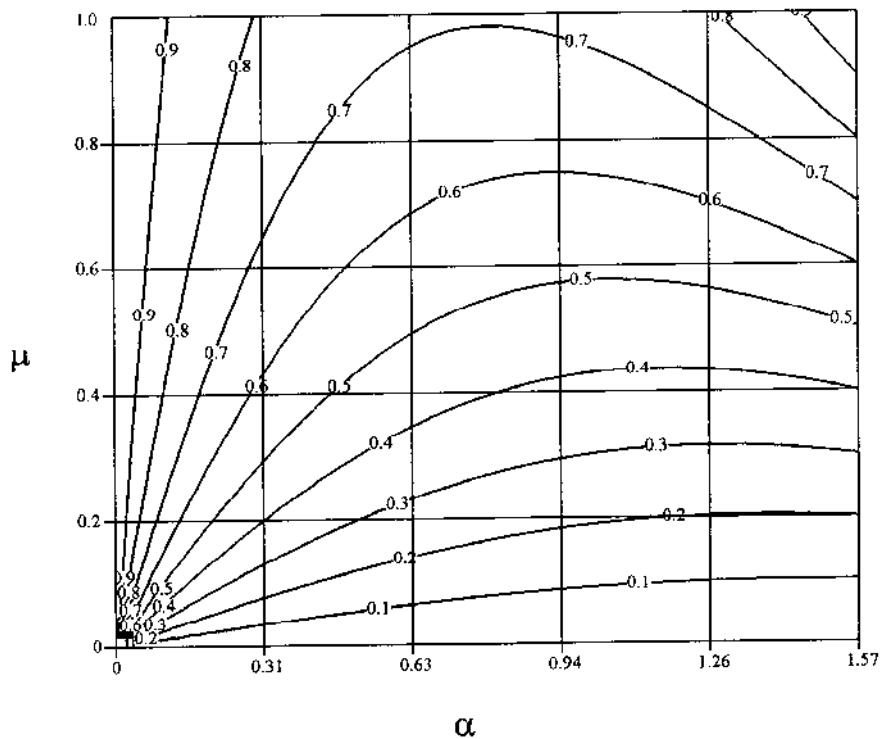


FIGURE 3 Contour plot of the surface $f(\mu, \alpha) = 2T/[(r_o + r_i)F_a]$.

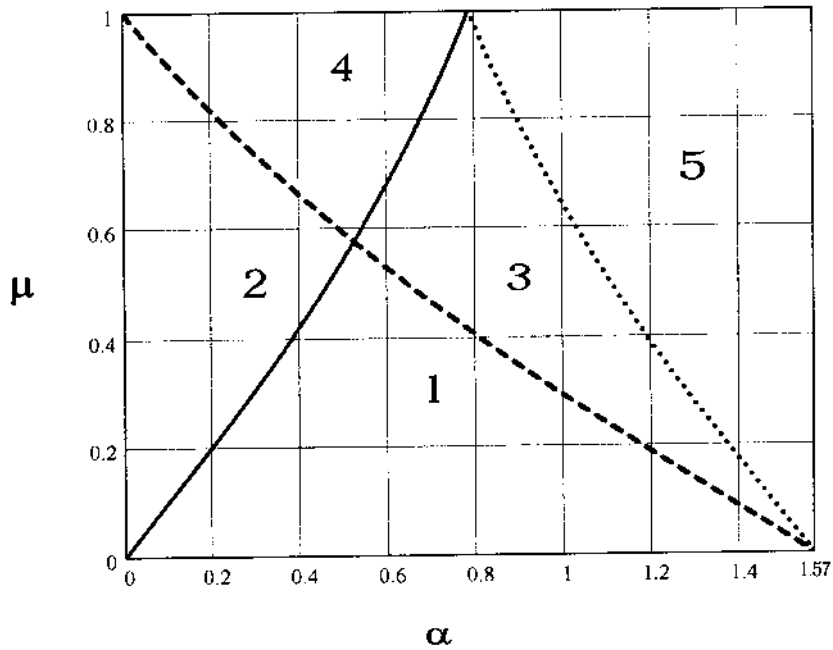


FIGURE 4 Design regions in the μ , α plane for cone clutches/brakes.

II. FOLDED CONE BRAKE

Prototype cone brakes have been designed and tested for a range of vehicle sizes, from tractors and trailers to subcompact automobiles [1]. Both the large and small sizes used a folded cone design, as illustrated in [Figures 5 and 6](#), each with $\alpha = 27^\circ$. Although the cone brake has fewer parts than drum brakes, this advantage must be balanced against the disadvantage of requiring an outboard wheel bearing.

Analysis of the folded cone brake with a sector shoe, shown in [Figure 5](#), to obtain design formulas for the torque capability and the required activation force is quite similar to that used for simple cone brakes and clutches. Since the brakes illustrated in [Figures 5 and 6](#) use a sector pad, we begin the analysis by observing from [Figure 7\(a\)](#) that an element of area on the conical surface may be written as

$$da = r d\theta \frac{dr}{\sin \alpha} \quad (2-1)$$

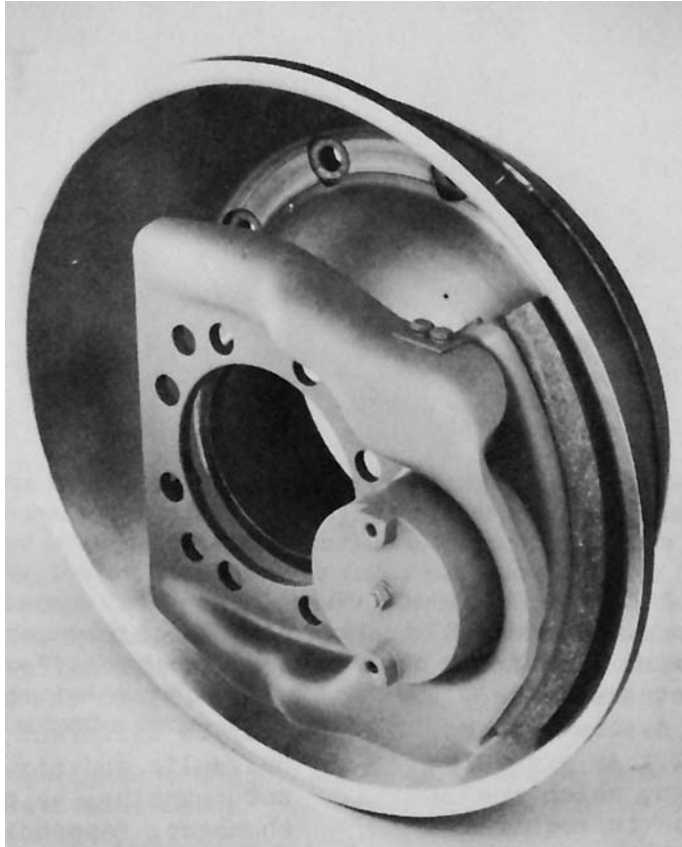


FIGURE 5 Truck cone brake and rotor (drum). (From reference 1. Reprinted with permission, © 1978 Society of Automotive Engineers, Inc.)

So the torque obtained due to a conical sector pad may be calculated from

$$\begin{aligned}
 T &= \mu p_{\max} r_i \int_A da = \frac{\mu p_{\max} r_i}{\sin \alpha} \int_0^\theta d\phi \int_{r_i}^{r_o} r dr \\
 &= \frac{\mu p_{\max} r_i}{\sin \alpha} \theta \frac{r_o^2 - r_i^2}{2}
 \end{aligned}
 \tag{2-2}$$



FIGURE 6 Cone brake on front-wheel-drive subcompact and the cone brake components. (From reference 1. Reprinted with permission, © Society of Automotive Engineers, Inc.)

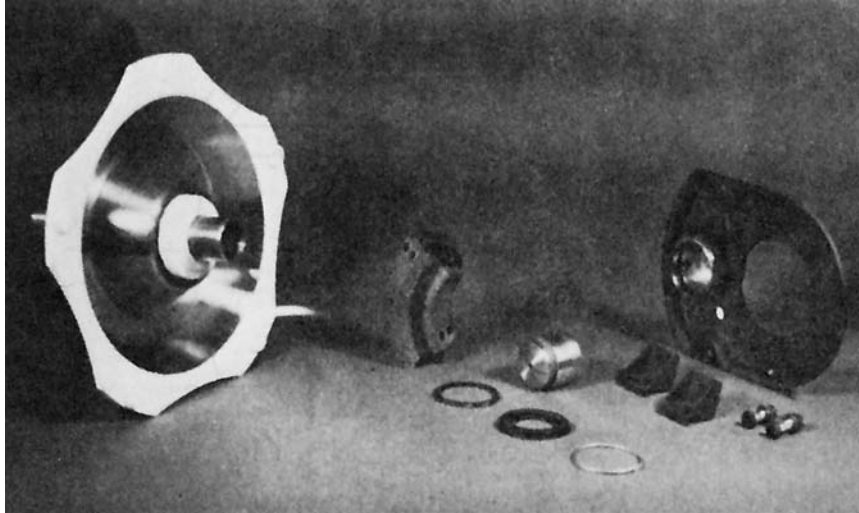


FIGURE 6 Continued.

and the corresponding activating force on the sector pad may be calculated from

$$F_a = p_{\max} r_i \frac{\sin \alpha + \mu \cos \alpha}{\sin \alpha} \int_0^\theta d\theta \int_{r_i}^{r_o} dr \quad (2-3)$$

$$= p_{\max} r_i (1 + \mu \cot \alpha) \theta (r_o - r_i)$$

Since the folded cone, shown by solid lines in [Figure 7\(b\)](#), is equivalent to two conical brakes, indicated by the dashed lines in that figure, it follows that the total torque and activating force may be found from

$$T = \frac{\mu p_{\max}}{\sin \alpha} \frac{\theta}{2} \left[r_{i1} (r_{o1}^2 - r_{i1}^2) + r_{i2} (r_{o2}^2 - r_{i2}^2) \right] \quad (2-4)$$

and

$$F_a = p_{\max} \theta r_i \left(1 + \frac{\mu}{\tan \alpha} \right) (r_{o1} - r_{i1} + r_{o2} - r_{i2}) \quad (2-5)$$

where θ is the angle subtended at the centerline by the lining sector.

III. DESIGN EXAMPLES

Example 3.1

Design a cone clutch to transmit a torque of 9050 N-mm or greater when fitted with a lining material having $\mu = 0.40$ and capable of supporting a maximum

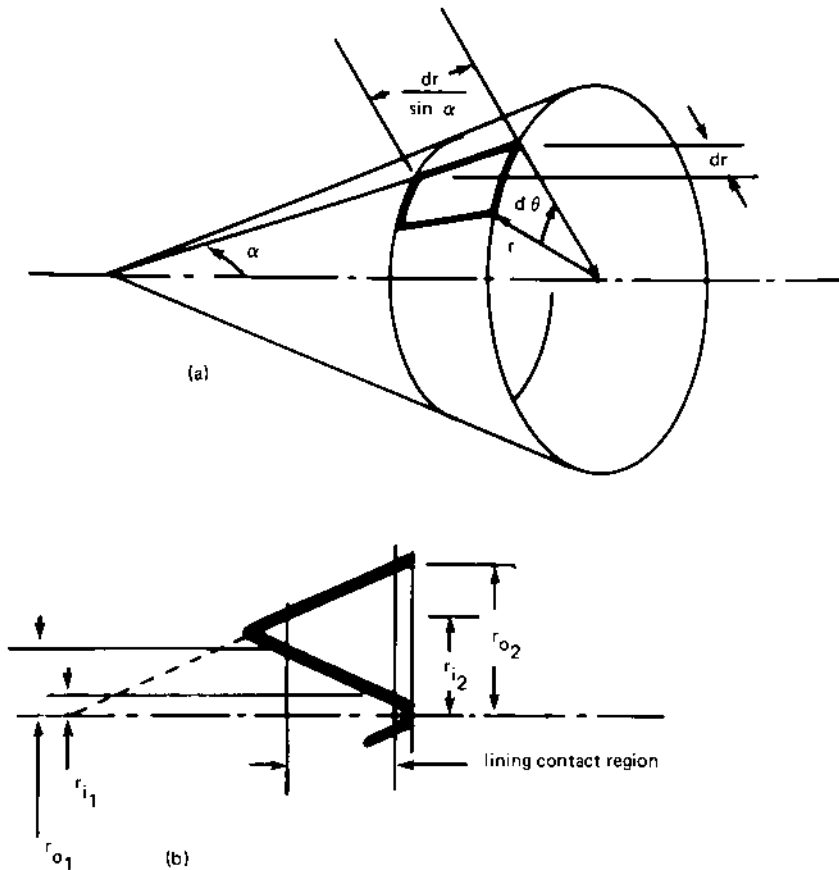


FIGURE 7 Cone geometry.

pressure of 4.22 MPa. The r_o value should be no larger than 35 mm and the clutch should release freely.

We shall begin by turning to [Figure 4](#) and find that at $\mu = 0.40$, region 1 extends from $\alpha = 0.38485$ radians $= 22.051^\circ$ to $\alpha = 0.79482$ radians $= 45.540^\circ$ (as read with the aid of the Trace feature provided by Mathcad). From [Figure 3](#) we note that the torque is greater at $\alpha = 22.051^\circ$ than it is at $\alpha = 45.54^\circ$, which suggests that a smaller α would be preferred. Hence, we shall initially consider two designs, one for $\alpha = 24^\circ$ and one for $\alpha = 45^\circ$. A slightly larger α was selected for the smaller of the two angles to ensure that no retraction force will be needed even with a manufacturing error of -0.5° .

Radius r_o was found by solving equation (1-8) for r_o . Activation force F_a was found from equation (1-9) after radius r_i was eliminated from it by using equation (1-7). Input data to these formulas was

$$T = 9050 \quad \mu = 0.40 \quad p_{\max} = 4.22 \quad \beta(\alpha) = \alpha \frac{\pi}{180}$$

Here the variable β is introduced as the radian measure of angle α that is given in degrees, to avoid entering trigonometric arguments in the form $(\alpha \cdot \text{deg})$ that would otherwise be required by Mathcad. Thus,

$$r_o(\alpha) = \left[\frac{(T 3r^{3/2} \sin(\beta(\alpha)))}{2\mu\pi p_{\max}} \right]^{1/3}$$

$$F_a(\alpha) = 2\pi p_{\max} r_o(\beta(\alpha))^2 \left(\frac{1}{\sqrt{3}} - \frac{1}{3} \right) \left(1 + \frac{\mu}{\tan(\beta(\alpha))} \right)$$

$$d_o(\alpha) = 2r_o(\alpha)$$

$$d_o(24) = 24.344 \quad d_o(45) = 29.272$$

$$F_a(24) = 124.872 \quad F_a(45) = 140.022$$

Select the smaller diameter because of its smaller activation force.

Example 3.2

Examine the possibility of designing a cone brake that is to serve as a holding brake having a torque capacity of 40 ft-lb that can be released by a retraction force greater than 3 lb but no more than 10 lb if possible. The lining material characteristics are $\mu = 0.35$ and $p_{\max} = 220$ psi.

Begin by turning to [Figure 4](#) and reading α at the intersection of the solid curve and grid line $\mu = 0.34941$ (error in μ of 0.00059). We find that the maximum α that will support a retraction force is 0.33615 rad = 19.260°. Select this value for our first trial and calculate the radius r_o from equation (1-8) and the retraction force from equation (1-10). The results are shown next in the Mathcad format, in which the base radius of the conical contact surface and the activation and the retraction forces are written as functions of the cone angle α and the coefficient of friction μ to facilitate considering a range of values for each of these variables. From their initial values we have

$$r_o(\alpha, \mu) = \left[\frac{(T 3\sqrt{3} \sin(\beta(\alpha)))}{2\mu\pi p_{\max}} \right]^{1/3}$$

$$F_a(\alpha, \mu) = 2\pi p_{\max} \frac{r_o(\alpha, \mu)^2}{\sqrt{3}} \left(1 - \frac{1}{\sqrt{3}} \right) \left(1 + \frac{\mu}{\tan(\beta(\alpha))} \right)$$

$$F_r(\alpha, \mu) = 2\pi p_{\max} \frac{r_o(\alpha, \mu)^2}{\sqrt{3}} \left(1 - \frac{1}{\sqrt{3}}\right) \left(\frac{\mu}{\tan(\beta(\alpha))} - 1\right)$$

These relations yield

$$d_o(19, 0.35) = 2.377$$

$$F_o(19, 0.35) = 960.605$$

$$F_d(19, 0.35) = 7.848$$

Guided by the steep slope of the surface shown in Figure 2 in this region, a plot of the retraction force as a function of the cone angle for friction coefficients near 0.35 is shown in Figure 8. The extreme sensitivity of this cone brake to the cone angle and especially to the value of the friction coefficient requires that the friction coefficient of the material selected be independent of temperature over the temperature range expected during the operation of this brake. Moreover, the cone angle must be held within the range from 18.924° to 19.172° to meet the retraction force requirements.

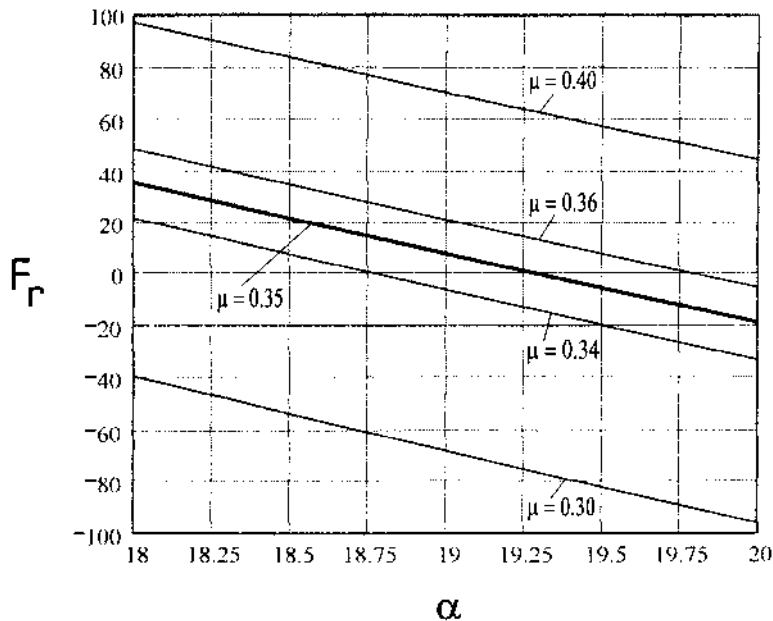


FIGURE 8 Retraction force F_r (lb) as a function of the cone angle ($^\circ$) for the friction coefficients, μ , indicated.

Example 3.3

Calculate the change in torque and in the lining pressure due to wear for the clutch in Example 3.1 and the brake in Example 3.2 for lining thicknesses of 0.125 in. and lining wear of 0.05 in. Let δ in Figure 9 represent the thickness that has been worn away. Consider that lining wear may be as large as 0.5 mm for the clutch in Example 3.1 and as large as 0.02 in. for the brake in Example 3.2

Lining wear has an effect upon the torque limits for cone clutches and brakes because the reduced lining thickness due to wear affects the values of r_o and r_i by allowing the inner cone to move farther into the outer cone. Implicit in the previous analysis has been the notion that radii r_o and r_i , as illustrated in Figures 1 and 9, were the radii to the contacting surface between the inner and outer cones. Addition of a lining merely means that these radii pertain to the contact surface between one cone and the lining on the other.

In what follows we shall consider the case where the lining material is placed on the inside of the outer cone, as in Figure 9. Furthermore, let the inner cone dimensions be designed so that the inner cone will project beyond the outer cone when the lining is new and the clutch/brake is engaged. As the lining wears, the bases will approach one another and become even when the lining is so thin that it must be replaced. Thus, the entire lining surface will always be in contact with the inner cone when the clutch or brake is engaged.

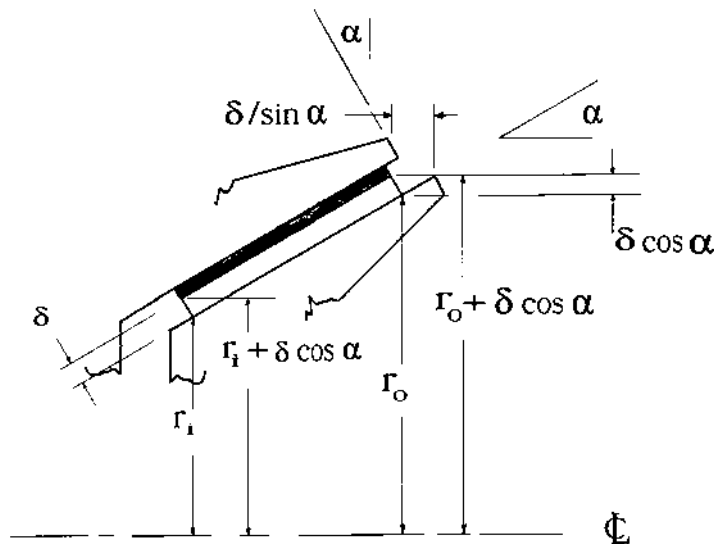


FIGURE 9 Geometry associated with lining wear in a cone clutch or brake.

When the lining has worn an amount δ , the inner cone will advance by the amount $\delta/(\sin \alpha)$, and radii r_i and r_o , measured on the conical surface that contacts the lining, will each increase by the amount $(\delta \cos \alpha)$. Consequently the smaller radius, which was initially given by $r_i = r_o / \sqrt{3}$, increases to

$$r_i = r_o / \sqrt{3} + \delta \cos \alpha \quad (3-1)$$

in terms of the lining wear δ and the cone half-angle α . The larger radius increases by the same amount, so

$$r_o \rightarrow r_o + \delta \cos \alpha \quad (3-2)$$

The maximum activation force that imposes pressure $p_{\max}$ on a new lining will impose a smaller maximum pressure on the worn lining because of its increased area. This smaller maximum pressure, denoted by p_{mw} , may be found by equating the activation force given by equation (1-9), here rewritten as

$$F_o = 2\pi p_{\max} \frac{r_o^2}{\sqrt{3}} \left(1 + \frac{\mu}{\tan \alpha}\right) \left(1 - \frac{1}{\sqrt{3}}\right) \quad (3-3)$$

with that obtained by replacing r_o and r_i in equation (1-9) with the values given by equations (3-1) and (1-2) to get

$$F_w = 2\pi p_{mw} r_o \left(1 + \frac{\mu}{\tan \alpha}\right) \left(\frac{r_o}{\sqrt{3}} + \delta \cos \alpha\right) \left(1 - \frac{1}{\sqrt{3}}\right) \quad (3-4)$$

in which $F_o(\alpha)$ represents the activation force as a function of cone angle α when the lining is new and $F_w(\alpha)$ represents an activation force of the same magnitude but one that now induces a maximum lining pressure of p_{mw} . Upon solving for p_{mw} we have

$$p_{mw} = \frac{p_{\max}}{1 + \frac{\delta \sqrt{3}}{r_o} \cos \alpha} \quad (3-5)$$

So the torque delivered by a cone clutch or brake with a worn lining may be written as

$$T_w = \mu \pi \frac{p_{mw}}{\sin \alpha} r_1 (r_2^2 - r_1^2) \quad (3-6)$$

where

$$r_1 = \frac{r_o}{\sqrt{3}} + \delta \cos \alpha \quad r_2 = r_o + \delta \cos \alpha \quad (3-7)$$

The increase in length of the interior cone needed for it to contact the full length of the lining at it moves farther into the exterior cone as the lining wears

is given by $\delta/\sin \alpha$, so the axial length of the interior cone measured from the plane containing r_i should be

$$l_o = r_o \left(1 - \frac{1}{\sqrt{3}} \right) \cot \alpha + \frac{\delta}{\sin \alpha} \quad (3-8)$$

according to the geometry displayed in [Figure 9](#).

The notation used in Examples 3.1 and 3.2, such as $\sin(\beta(\alpha))$, is due to the Mathcad requirement that trigonometric arguments be in radian measure. This requirement may be satisfied by preceding trigonometric expressions with the relation $\beta(\alpha) = \alpha\pi/180$. The functional notation such as $p_{mw}(\alpha, \beta)$, is to allow new values for α and β to be entered directly rather than at a less convenient place elsewhere in the program.

Because torque varies as the radius cubed and the pressure change due to wear varies inversely with δ , the torque capability of cone clutches and brakes increases slightly with lining wear and the maximum lining pressure decreases slightly.

Turning first to Example 3.1, substitution into the preceding equations for the cone whose half-angle is 24° shows that the torque will increase to 10,097 N-mm after the lining thickness is reduced by 0.5 mm. The maximum lining pressure will be reduced to 3.96 MPa, and the interior cone's axial length, measured from the transverse plane containing r_i , should be 12.8 mm.

Torque increases to only 9717 N-mm for the cone having a 45° half-angle and the maximum lining pressure decreases to 4.05 MPa. That interior cone's axial length, measured as before, should be 6.9 mm.

Turning now to Example 3.2, substitution as before into equations (3-5) through (3-8) results in finding that the torque capability has increased to 489.6 in.-lb, or to 40.80 ft-lb, and the maximum lining pressure has decreased to 214.1 psi. The length of the interior cone should be increased from 1.44 in. to 1.50 in. to ensure that the interior cone contacts the full length of the lining after the lining thickness has decrease by 0.02 in.

Similar comments hold for a lining attached to the inner cone. The differences are that r_i and r_o would be measured to the surface of the lining at the outer cone and these radii would become $r_i - \delta$ and $r_o - \delta$ as the lining wears. Placing lining on the inner cone results in slightly less lining contact area as wear progresses, with a correspondingly slight increase heat per unit area to be dissipated for a given torque capacity.

IV. NOTATION

A	area (ℓ^2)
da	element of area (ℓ^2)
F_a	Activation force ($m\ell t^{-2}$)

F_r	retraction, or release, force ($m\ell t^{-2}$)
f	friction function (1)
p	pressure ($m\ell^{-1}t^{-2}$)
$p_{\max}$	maximum pressure ($m\ell^{-1}t^{-2}$)
r_i	inner radius, inner cone (ℓ)
r_o	outer radius, inner cone (ℓ)
r_i	inner radius, outer cone (ℓ)
r_o	outer radius, outer cone (ℓ)
T	torque ($m\ell^2 t^{-2}$)
α	cone half-angle (1)
μ	friction coefficient between lining and cone (1)

V. FORMULA COLLECTION

Pressure distribution over lining:

$$p = p_{\max} \frac{r_i}{r}$$

Torque in terms of r_o and r_i :

$$T = \frac{\mu \pi p_{\max}}{\sin \alpha} r_i (r_o^2 - r_i^2)$$

Maximum torque:

$$T = \frac{2}{3\sqrt{3}} \mu \pi \frac{p_{\max}}{\sin \alpha} r_o^3$$

Activation force in terms of r_o and r_i :

$$F_a = 2\pi p_{\max} \left(1 + \frac{\mu}{\tan \alpha} \right) r_i (r_o - r_i)$$

Release force in terms of r_o and r_i :

$$F_r = 2\pi p_{\max} \left(\frac{\mu}{\tan \alpha} - 1 \right) r_i (r_o - r_i)$$

Pressure maximum on a worn lining:

$$p_{mw} = \frac{p_{\max}}{1 + \frac{\sqrt{3}\delta}{r_o} \cos \alpha}$$

Radius associated with torque T and angle α :

$$r_o = \left(\frac{3\sqrt{3}T \sin \alpha}{2\mu \pi p_{\max}} \right)^{1/3}$$

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