2

Band Brakes

Band brakes are simpler and less expensive than most other braking devices, with shoe brakes, as perhaps their nearest rival. Because of their simplicity, they may be produced easily by most equipment manufacturers without having to purchase special equipment and without having to use foundry or forging facilities. Only the lining must be purchased from outside sources.

Band brakes are used in many applications such as in automatic transmissions (Figure 1) and as backstops (Figure 5—devices designed to prevent reversal of rotation), for bucket conveyors, hoists, and similar equipment. They are especially desirable in the last-mentioned application because their action can be made automatic without additional controls.

I. DERIVATION OF EQUATIONS

Figure 2 shows the quantities involved in the derivation of the force relations used in the design of a band brake. Consistent with the direction of rotation of the drum, indicated by ω, the forces acting on an element of the band are as illustrated in the lower right section of Figure 2. In this figure, r is the outer radius of the brake drum and F_1 and F_2 are the forces applied to the ends of the brake band. Because of the direction of drum rotation, F_1 is greater than F_2. Equilibrium of forces in directions parallel and perpendicular to the tangent to a typical brake-band element at its midpoint requires that

$$(F + dF)\cos\frac{d\theta}{2} - F\cos\frac{d\theta}{2} - \mu\, pwr\, d\theta = 0 \qquad (1\text{-}1)$$

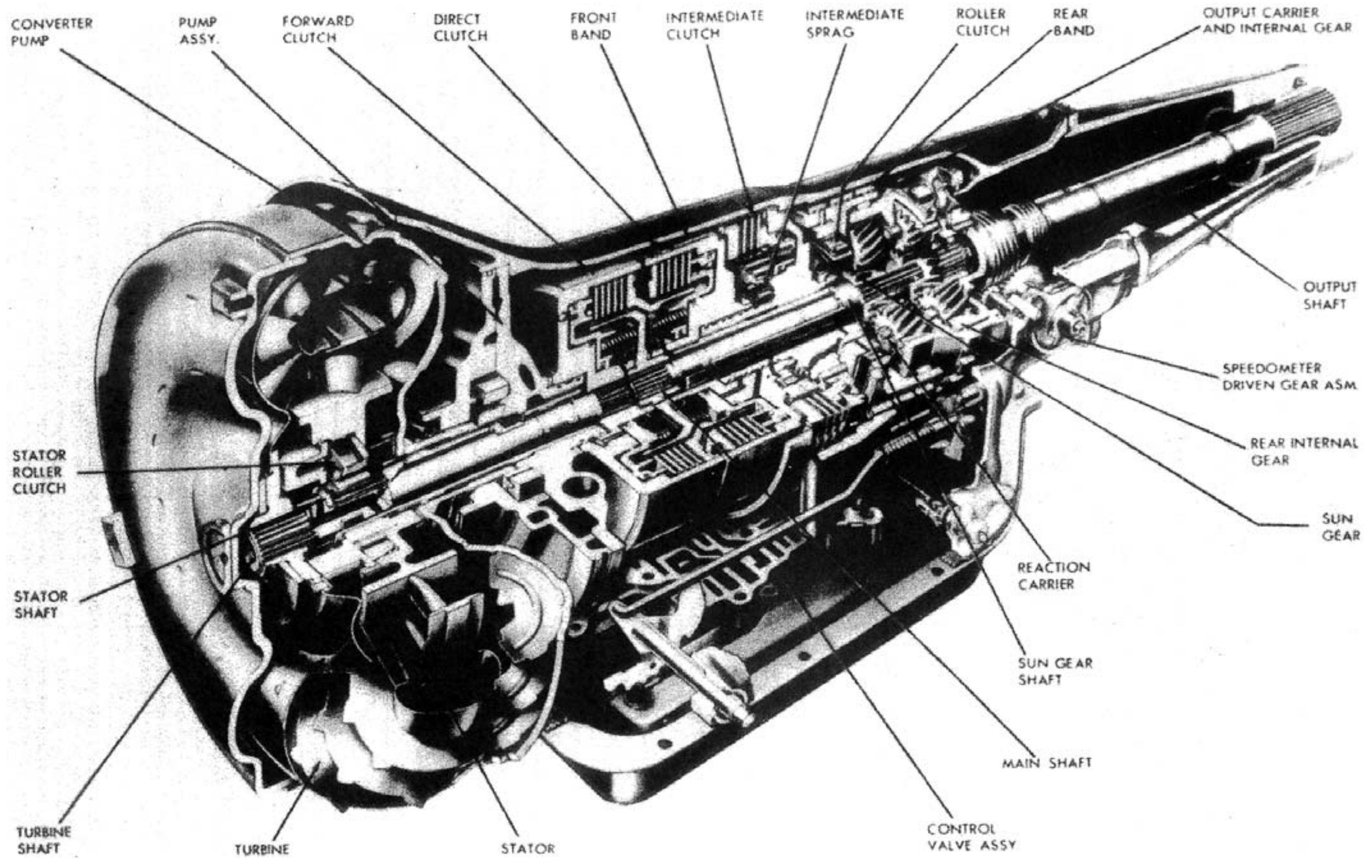

Figure 1 Band brakes used in an automatic transmission system.

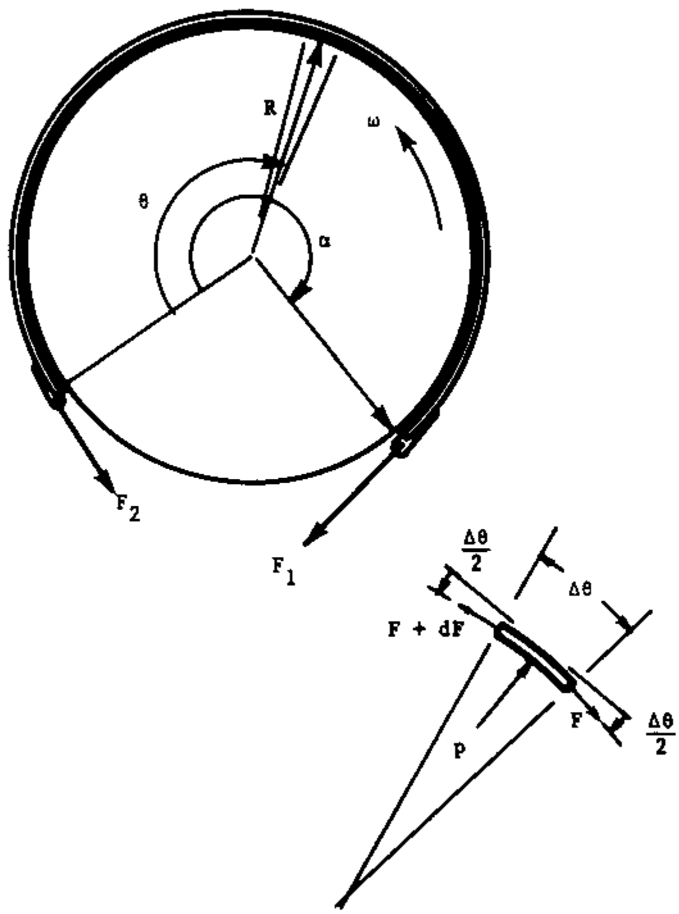

FIGURE 2 Quantities and geometry used in the derivation of the band-brake design relations.

$$(F + dF) \sin \frac{d\theta}{2} + F \sin \frac{d\theta}{2} - pwr\, d\theta = 0 \qquad (1\text{-}2)$$

when the brake lining and the supporting brake band together are assumed to have negligible flexural rigidity, where μ represents the coefficient of friction between the lining material and the drum, p represents the pressure between the drum and the lining, and w represents the width of the band. Upon simplifying equations (1-1) and (1-2) and remembering that as the element of band length approaches zero, $\sin(d\theta/2)$ approaches $d\theta/2$, $\cos(d\theta/2)$

approaches 1, and the product $dF(d\theta/2)$ becomes negligible compared to $F\,dt$, we find that these two equations reduce to

$$dF = \mu pwr\, d\theta \tag{1-3}$$

so that

$$F = pwr \tag{1-4}$$

Substitution for pwr from equation (1-4) into equation (1-3) yields an expression that may be integrated to give

$$\ln F - \ln F_2 = \ln \frac{F}{F_2} = \mu\theta \tag{1-5}$$

where θ is taken to be zero at the end of the band where F_2 acts. It is usually more convenient to write this relation in the form

$$\frac{F}{F_2} = e^{\mu\theta} \tag{1-6}$$

which expresses the tangential force in the band brake as a function of position along the brake.

We may find F_1 from equation (1-6) by simply setting $\theta = \alpha$ to obtain

$$\frac{F_1}{F_2} = e^{\mu\alpha} \qquad (\alpha = \text{wrap angle}) \tag{1-7}$$

Since this equation shows that the maximum force occurs at $\theta = \alpha$, it follows from equation (1-4) that

$$F_1 = wrp_{\max} \tag{1-8}$$

in terms of the radius r of the drum and the width w of the band. This equation points out a disadvantage of a band brake: The lining wear is greater at the high-pressure end of the band. Because of this the lining must be discarded when it is worn out at only one end, or it must be reversed approximately halfway through its life, or the brake must have two, or perhaps even three, different lining materials with different coefficients of friction so that the lining does not need to be changed as frequently.

The torque exerted by the brake is related to the band force according to

$$T = (F_1 - F_2)r \tag{1-9}$$

Upon factoring out F_1 by referring to equation (1-7) and then replacing F_1 by the right-hand side of equation (1-8), we get

$$T = F_1 r(1 - e^{-\mu\alpha}) = p_{\max} wr^2 (1 - e^{-\mu\alpha}) \tag{1-10}$$

which gives the brake's maximum restraining torque as a function of its dimensions and its maximum compressive pressure. This equation may be applied if the leading link can withstand the force $F_1 = rwp_{max}$ and if the band is strong enough to support the force given by equation (1-6) for $0 \leqq \theta \leqq \alpha$.

A measure of the efficiency of a band brake is the ratio of the torque applied by the brake to the torque that could be obtained if the force were applied directly to the drum itself:

$$\frac{T}{F_1 r} = 1 - e^{-\mu\alpha} \tag{1-11}$$

The maximum value of this ratio for a single-turn band brake is 0.998 when $\mu = 1.00$. From the plot of this ratio, Figure 3, it is apparent that reductions in the angle of wrap from 360° to 270° has relatively little effect on the efficiency for $\mu = 0.5$ or greater. We also see that the brake should subtend an arc of 270° or more if degradation of the friction coefficient, perhaps due to a dirty environment and infrequent maintenance, is to be expected.

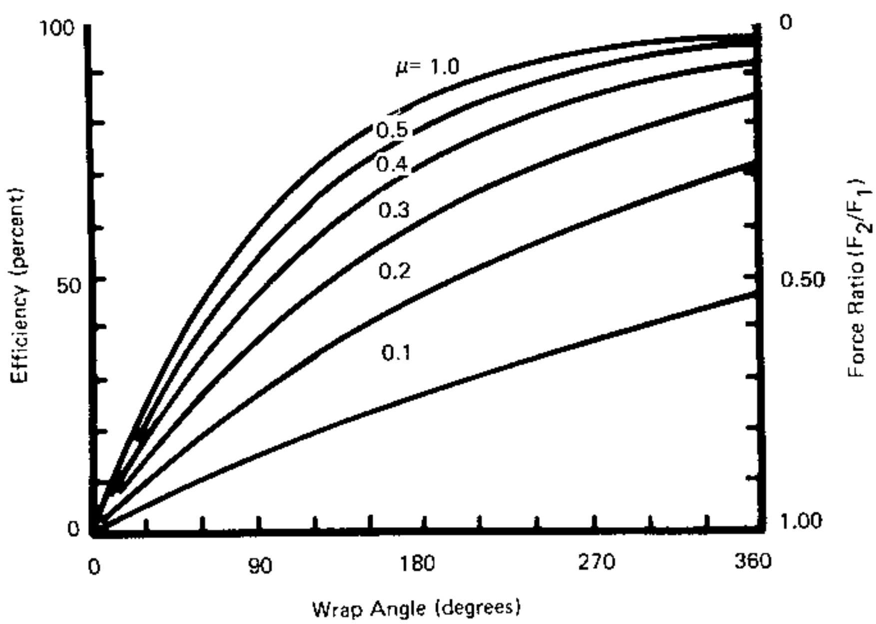

FIGURE 3 Efficiency (T/F_1r) and force ratio (F/F_1) as a function of angle from the leading end of the brake band.

Since reinforcement of the band near its leading end depends on the force decay as a function of angle along the band, it may be of interest to display how F decreases with ϕ, measured from the leading end of the band. To do this we simply replace F_2 with F and replace α with ϕ in equation (1-7) to obtain

$$\frac{F}{F_1} = e^{-\mu\phi} \qquad \phi = \alpha - \theta \tag{1-12}$$

For a brake band extending over an angle ϕ from F_1.

$$T = (F_1 - F)r = F_1 r\left(1 - \frac{F}{F_1}\right) = F_1 r(1 - e^{-\mu\phi}) \tag{1-13}$$

Thus the decay of the band force from its maximum at the leading end of the band may be found from Figure 3 using the scales shown on the right-hand ordinate and associating the abscissa with ϕ.

It is because of the low coefficient of friction for wet friction material that the brake bands in an automatic transmission are relatively thick and curved to fit the drum with only a small clearance. The thickness is required to support the large band force necessary to deliver a relatively large torque when operating at low efficiency and the small clearance is necessary to minimize the required activation force to bend the band and lining to the drum radius.

II. APPLICATION

In this section we consider the design of a band brake to exert a torque of 9800.0 N-m subject to the conditions that the drum width be no greater than 100 mm and that the drum diameter be no greater than 750 mm. To complete the design we should also specify the necessary link strength for a safety factor of 3.5 when using a steel that has a working stress of 410 MPa. Other mechanisms require that the angle of wrap not exceed 290°. Lining temperature is not expected to rise above 300°F (148°C) during the most severe conditions. Select a lining material that can sustain a maximum pressure of 1.10 MPa.

Return to Chapter 1 to find that the lining represented by Figure 4 is one of several that is flexible enough for use in a band brake and has the limiting temperature and pressure capability. Thus, use $\mu = 0.4$ and equation (1-10) to find that at the maximum radius the band width should be given by

$$w(r) = \frac{T}{p_{\max} r^2 (1 - e^{-\mu\alpha})} \tag{2-1}$$

where lining width w is written as a function of r in a numerical analysis program. Likewise, the lining area is given by

$$A(r) = \alpha r w(r) \tag{2-2}$$

where α is in radians. Similarly, substitution for $w(r)$ from equation (2-1) into equation (1-8) gives

$$F(r) = p_{\max} w(r) r \tag{2-3}$$

which enables calculation of the link diameter for a safety factor ζ and maximum operating stress σ from the relation.

$$d_1(r) = 2\sqrt{\frac{F(r)}{\pi\sigma}\zeta} \tag{2-4}$$

For the largest drum diameter, which is 375 mm, turn to equation (2-1) to find that for this drum the lining width should be

$$w(375) = 72.992 \text{ mm}$$

which is within the width limits. The corresponding lining area and link diameter $d_1(r)$ as given by equations (2-2) and (2-4) are

$$A(375) = 1.385\times 10^5 \text{ mm}^2 = 1385 \text{ cm}^2 \qquad d_1(r) = 2r_1(r) = 18.09 \text{ mm}$$

For the largest lining width, solve equation (2-1) for the drum radius and find the drum diameter as a function of the lining width from

$$d(w) = 2\left[\frac{T}{p_{\max} w(1 - e^{-\mu\alpha})}\right]^{1/2} \tag{2-5}$$

which yields that the drum diameter for a 100-mm lining width should be

$$d(100) = 640.77 \text{ mm}$$

According to equations (2.2) and (2.4), the corresponding lining area and link diameter are

$$A(320.38) = 1622 \text{ cm}^2 \qquad \text{and} \qquad d_1 = 2r_1 = 19.57 \text{ mm}$$

Select the design with the larger lining area in order to reduce the energy dissipation per unit area, lower the operating temperature, and thereby decrease lining wear. Selecting a convenient drum diameter slightly larger than 640.77mm, namely, 641 mm, while retaining the lining width of 100 mm will only increase the brake's torque capability for a negligibly smaller link force while reducing the pressure upon the lining.

III. LEVER-ACTUATED BAND BRAKE: BACKSTOP DESIGN

This type of brake may be represented as shown in Figure 4(a). Moment equilibrium about the pivot point of the lever requires that

$$F_1 a - F_2 b + P(b + c) = 0 \tag{3-1}$$

so that substitution for F_2 from equation (1-7) yields

$$-F_1(a - be^{-\mu\alpha}) = P(b + c) \tag{3-2}$$

as the force P required to activate the brake. Substitution for F_1 in equation (3-2) from relation (1-11) yields

$$P = \frac{be^{-\mu\alpha} - a}{r(1 - e^{-\mu\alpha})} \frac{T}{b + c} \tag{3-3}$$

Note that not only is the force related to the lever arm length, as is to be expected from elementary statics, but a braking torque may be exerted with no activating force if

$$a = be^{-\mu\alpha} \tag{3-4}$$

In other words, the lever portion of length c could be removed and the mechanism would stop rotation in the direction shown [Figure 4(b)]. The brake is then termed "self-locking in one direction."

Mechanisms of this sort, illustrated in Figure 4(c), are known as backstops. Their function is to permit rotation is one direction and prevent rotation in the other direction.

If the direction of rotation is reversed, the brake will loosen because a slight rotation in the counterclockwise direction of the lever will cause a larger motion at B than at A. Brake-band sag should be sufficient to provide enough friction force to activate the brake whenever the rotation reverses direction.

A backstop using the linkage shown in Figure 4(c) is shown in Figure 5. The two small tabs on the brake band are to prevent it from slipping off the drum. A relatively close fit (with a slight increase in power dissipation) is intended between the band and the drum to maintain sufficient frictional force to assure quick response whenever the direction of rotation is reversed.

IV. EXAMPLE: DESIGN OF A BACKSTOP

Design a backstop similar to that shown in Figure 2.4(c) to prevent gravity unloading of a bucket elevator similar to that shown in Figure 6 that has 41 buckets on each side. For design purposes assume that all buckets on the downward-moving side are empty and that all of the buckets on the upward-

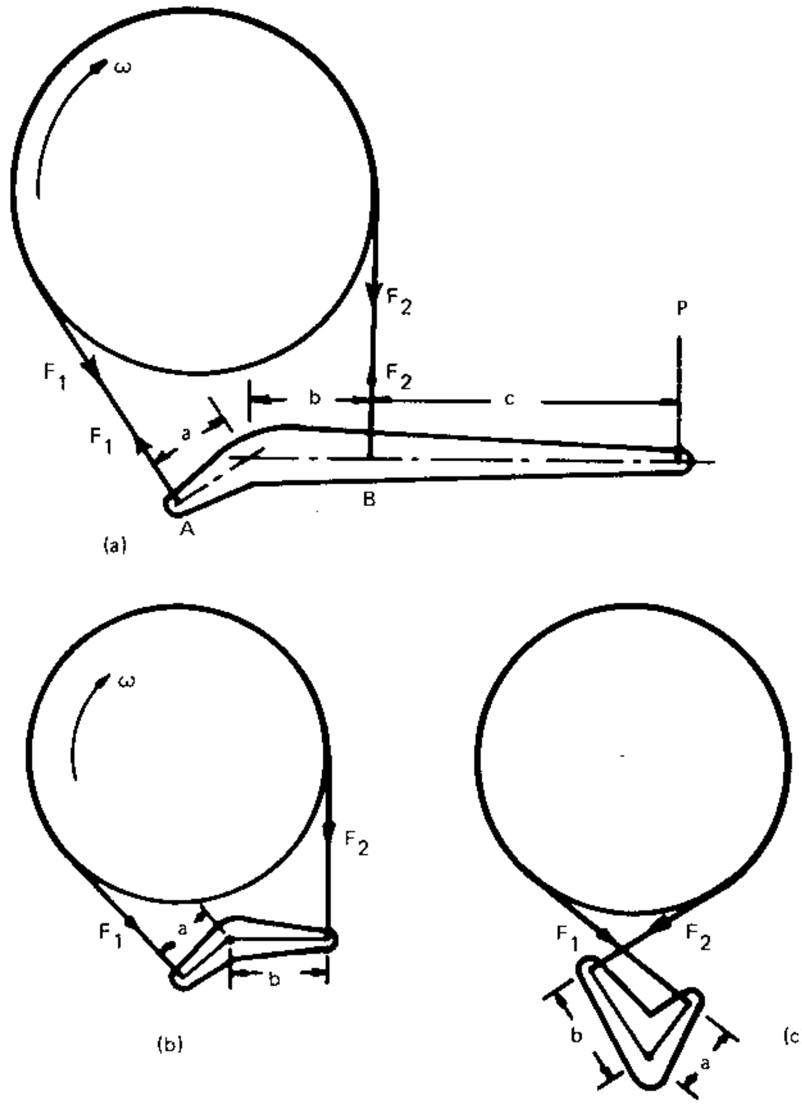

FIGURE 4 (a) Lever-activated band brake; (b) backstop configuration with $a = be^{-\mu\alpha}$; (c) backstop with levers a and b rearranged to provide a greater wrap angle.

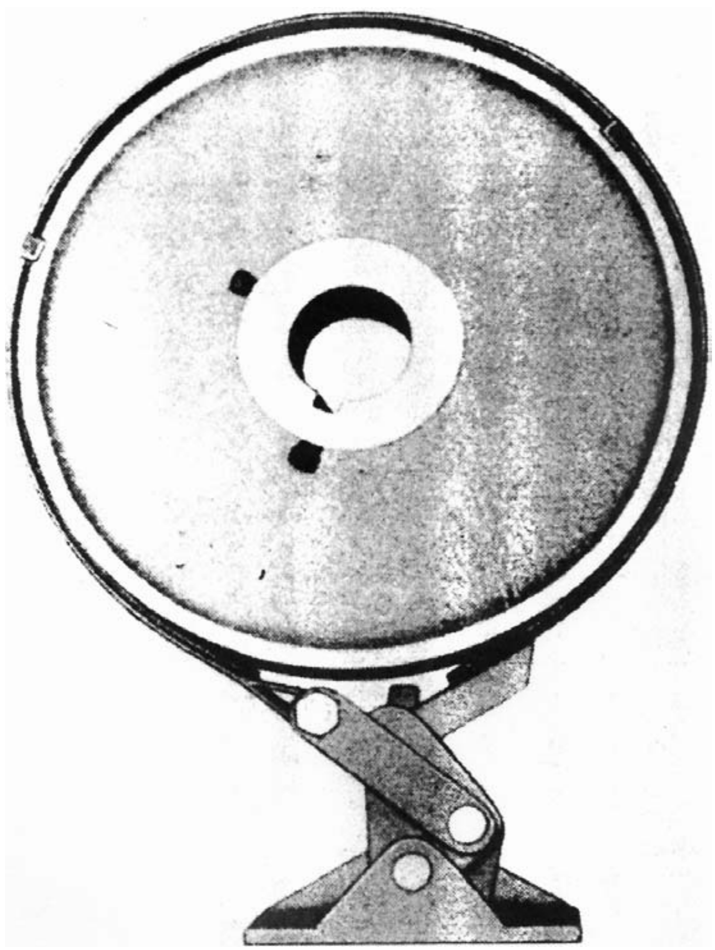

Figure 5 Backstop.

moving side are filled when the power is turned off, with each bucket containing 129 lb of material. The pitch diameter d_s of the sprocket is 34 inches.

Assume that the friction coefficient of the lining will always be 0.4 and that the minimum value of p_{max} is 275 psi. Housing requirements demand that the backstop drum diameter be no larger than 33 in. Use a safety factor of 1.5 in sizing the drum band, which is to be made from spring steel having a yield stress of 102,000 psi.

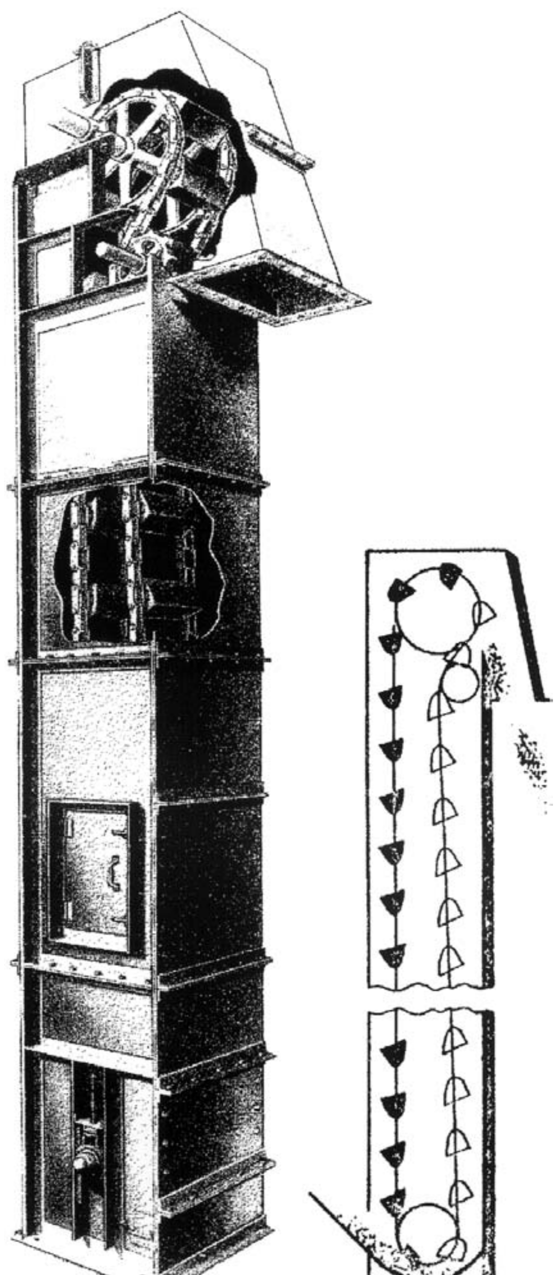

FIGURE 6 Positive discharge bucket conveyor cutaway and cross section. (Courtesy American Chain Association, Washington, DC.)

To ensure clearance, let the drum diameter be 32 in., and design for a wrap angle, α, of 300°. From the sprocket pitch diameter and the bucket weights, find

$$T = (d_s/2)WN = (34/2)129(41) = 89{,}913 \text{ in.-lb}$$

as the maximum expected value of the torque. Here W denotes the expected maximum weight of material in a bucket and N denotes the number of buckets one each side. Chain and empty bucket weights were ignored because the chain and empty bucket assembly is in equilibrium due to the symmetry of the conveyor system about its vertical axis.

After solving equation (1-10) for w, we have

$$w = \frac{T}{p_{\max} r^2 (1 - e^{-\mu\alpha})} \tag{4-1}$$

so substitution of $\alpha = 300\pi/180 = 5.2360$ radians along with the given values into this expression yields

$$w = 1.457 \text{ in.}$$

Force F_1 may be calculated for this width from equation (1-8), to get the maximum force as

$$F_1 = 6409 \text{ lb}$$

The thickness of the spring steel band to which the lining is attached may be calculated from

$$t = \frac{\zeta F}{w\sigma} \tag{4-2}$$

in which ζ represents the safety factor and σ represents the yield stress of the steel band. Substitution of these values along with w and F into equation (4-2) yields $t = 0.065$ in. Finally, from equation (3-4), we have

$$b/a = e^{\mu\alpha} = e^{0.4(5.236)} = 8.121$$

Although relation (3-4) may be derived from the backstop configuration using the equilibrium equation for the backstop lever, which is

$$F_1 a = F_2 b$$

together with equation (1-7), use of equation (3-3) has the advantage of showing that when b/a is less than $e^{\mu\alpha}$, the direction of force P on the lever reverses. This implies that the backstop lever proportions should obey the inequality

$$a/b = e^{-\mu\alpha} \tag{4-3}$$

to function properly. In particular, the equality follows by setting $P = 0$ and the inequality follows by setting $P < 0$ in equation (3-3).

Return to equation (4-1) and substitute that expression for w into equation (1-8) to find that F_1 may be written as

$$F_1 = \frac{T}{r(1 - e^{-\mu\alpha})} \tag{4-4}$$

to show that F_1 is independent of p_{max}. Therefore w may be increased to 1.50 in. without affecting F_1. It merely slightly reduces the p_{max} required to provide the design torque capability.

V. NOTATION

a	lever length (l)
b	lever length (l)
c	lever length (l)
d	drum diameter (l)
d_1	link diameter (l)
F	band force (mlt^{-2})
F_1	maximum band force (mlt^{-2})
F_2	minimum band force (mlt^{-2})
P	force applied to brake lever (mlt^{-2})
p	lining pressure ($ml^{-1}t^{-2}$)
p_{max}	maximum lining pressure ($ml^{-1}t^{-2}$)
r	drum radius (l)
T	torque (ml^2t^{-2})
t	band thickness (l)
w	band width (l)
α	brake wrap angle (1) $0 \leqq \theta \leqq \alpha$
θ, ϕ	Angle subtended at the center of the drum (1)
μ	Friction coefficient (1)
σ	yield stress ($ml^{-1}t^{-2}$)
ζ	safety factor (1)

VI. FORMULA COLLECTION

Torque:

$$T = F_1 r(1 - e^{-\mu\alpha}) = (F_1 - F_2)r = p_{max} w r^2 (1 - e^{-\mu\alpha})$$

Activation force in terms of torque:

$$F_1 = \frac{T}{r(1 - e^{-\mu\alpha})}$$

Activation force in terms of maximum pressure:

$$F_1 = p_{\max} wr$$

Link diameter:

$$d_1 = 2\sqrt{\frac{F}{\pi\sigma}\zeta}$$

Maximum pressure in terms of torque:

$$p_{\max} = \frac{T}{r^2 w(1 - e^{-\mu\alpha})}$$

Band thickness:

$$t = \frac{F\zeta}{w\sigma}$$

Minimum wrap angle:

$$\alpha = \frac{1}{\mu}\ln\frac{1}{1 - T/(F_1 r)}$$

Drum diameter in terms of activation force:

$$d = \frac{2F_1}{wp_{\max}}$$

Drum diameter in term of torque:

$$d = \frac{2T}{F_1(1 - e^{-\mu\alpha})} = 2\left[\frac{T}{p_{\max} w(1 - e^{-\mu\alpha})}\right]^{1/2}$$

Band brake lever force:

$$P = \frac{be^{-\mu\alpha} - a}{r(1 - e^{-\mu\alpha})}\,\frac{T}{b + c}$$

Band width in terms of torque:

$$w = \frac{T}{p_{\max} r^2(1 - e^{-\mu\alpha})}$$

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