4

Linearly Acting External and Internal Drum Brakes

Linearly acting drum brakes are those fitted with shoes which, when activated, approach the drum by moving parallel to a radius through the center of the shoe. Typical linearly acting drum brakes are illustrated in Figures 1–3.

Analysis of linearly acting brakes includes those in which the centrally pivoted shoes are attached to pivoted levers, as in Figure 1. Including brakes of this design within the category of linearly acting brakes is justified if they are designed so that the applied forces on the shoes and linings act along the radii of the shafts that they grip when the brakes are applied. Brakes of this type may act either upon brake drums or directly upon rotating shafts and are suitable for use in heavy-duty applications, such as found in mining and construction equipment and in materials-handling machinery.

Internal linearly acting drum brakes, such as used on trucks in Europe, may be designed as in Figure 2. Either pneumatic or hydraulic cylinders or cams may be used to force the shoes outwardly against the drum. The cylinders or cams also serve as anchors to prevent rotation and react against the vehicle frame. The springs shown are to retract the shoes when the brake is released.

A collection of segmented brake pads (backing plate plus lining) along the entire circumference of the drum may be arranged as in Figure 3 to move outwardly against a drum, as in Figure 3a, or inwardly against a drum, as in Figure 3b. The brake pads, or shoes, are themselves constrained against

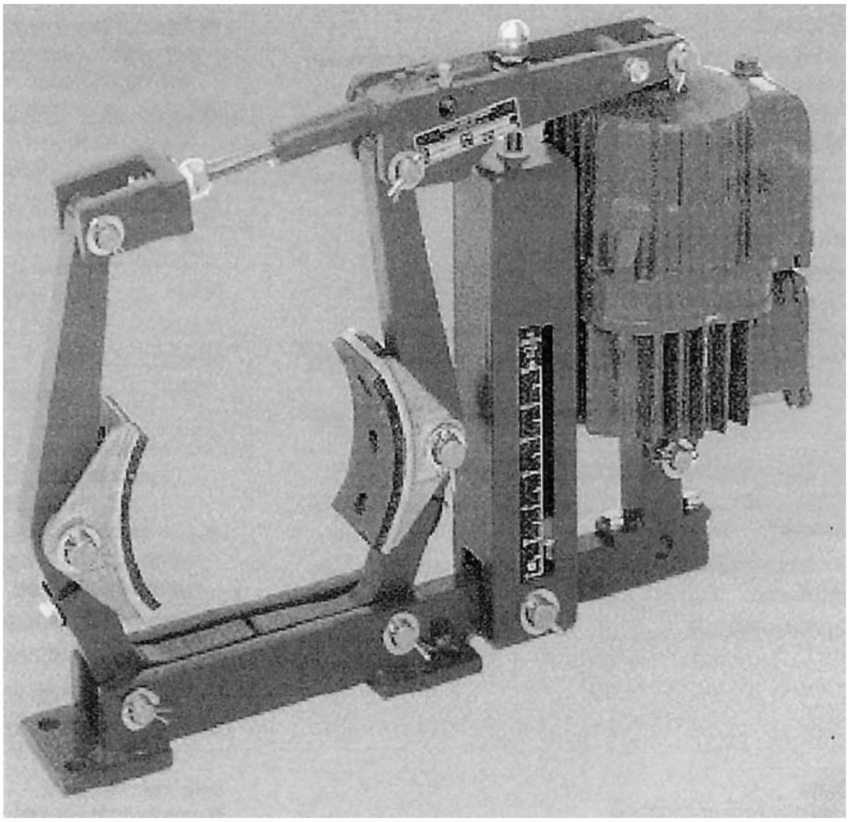

FIGURE 1 Linearly acting, centrally pivoted shoe brake. (Courtesy of the Hindon Corp., Charleston, SC)

rotation by anchor pins that fit into short radial slots between the shoes and are attached to the rim of the circular frame, as shown in Figure 3a. Brake actuation is accomplished by using air to expand the normally flat elastomeric-fabric annular tube shown in that figure between the brake pads and the circular frame. When designed to move inwardly against a drum, as in Figure 3b, the brake lining is riveted to a differently contoured backing plate which has shoulders at each end to resist a twisting torque and which is fitted with a central slot that accepts the anchor pin to the outer frame at each side of the shoe. This radial slot allows the pad to move rapidly inward but prevents tangential motion.

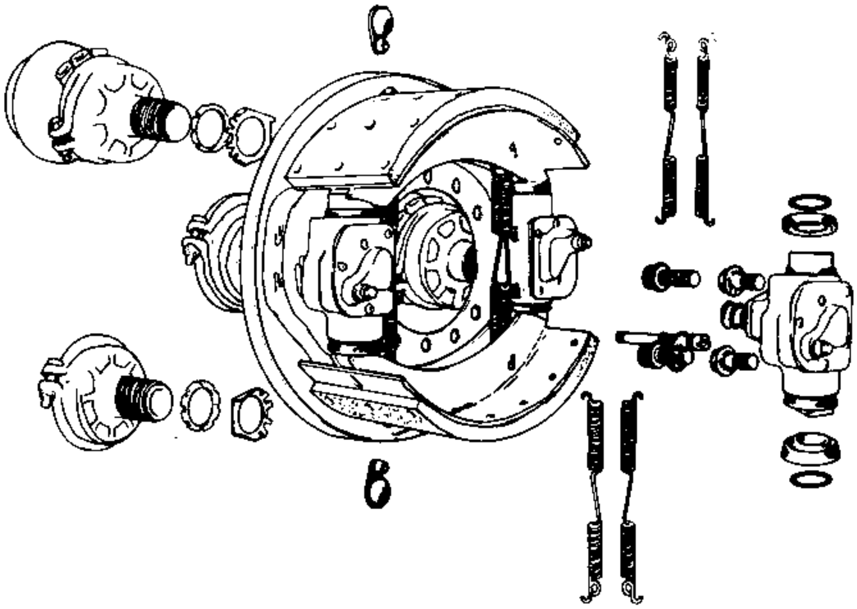

FIGURE 2 Linearly acting, twin-shoe, internal drum brake with pneumatic activation (Girling Twinstop). (Reprinted with permission. ©1977 Society of Automotive Engineers, Inc.)

I. BRAKING TORQUE AND MOMENTS FOR CENTRALLY PIVOTED EXTERNAL SHOES

To calculate the torque, we must first find an expression for the lining pressure. Guided by the geometry shown in Figure 4, we see that the lining pressure will be given by

$$p = k\Delta \cos\theta \tag{1-1}$$

in terms of the lining deformation Δ if the shoe and drum are assumed to be absolutely rigid. Maximum pressure occurs when $\theta \simeq 0$, so that $p_{\max} = k\Delta$. Thus equation (1-1) becomes

$$p = p_{\max} \cos\theta \tag{1-2}$$

and the incremental tangential friction force is given by

$$dF = \mu p_{\max} \cos\theta \; rw \; d\theta \tag{1-3}$$

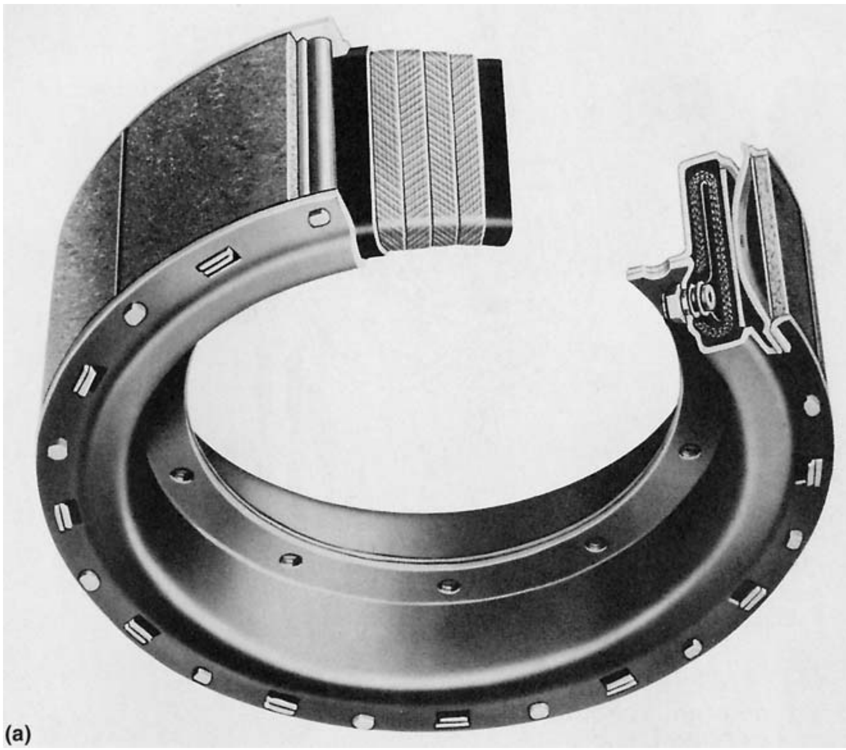

FIGURE 3 Rim brakes with pneumatic activation (also used as rim clutches). (Courtesy of Eaton Corp., Airflex Division, Cleveland, Ohio.)

so the braking torque becomes

$$T = \mu p_{\max} r^2 w \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \mu p_{\max} r^2 w (\sin\theta_2 - \sin\theta_1) \tag{1-4}$$

In designs different from those shown in Figure 1 it may prove convenient to have the shoe pivoted about a point at a radial distance R on the axis of symmetry, such as point A in Figure 4. The moment M_p due to the pressure on the lining is zero about point A because of the symmetry of the shoe about this point. No such symmetry exists for the friction moment M_f, however, so from the incremental moment due to friction

$$dM_f = (\mu p_{\max} r w \cos\theta \, d\theta)(R\cos\theta - r)$$

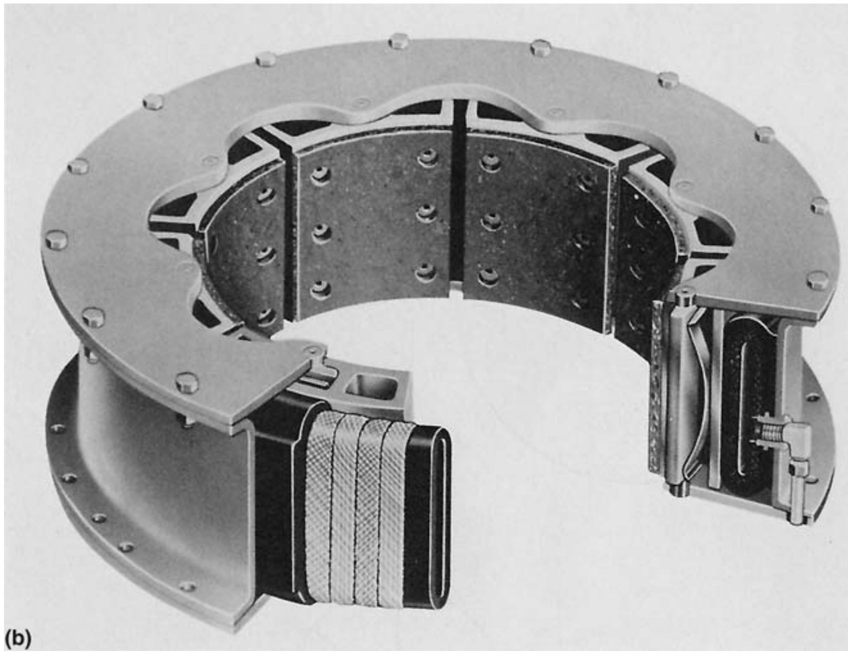

FIGURE 3 Continued.

it follows that

$$M_f = \mu p_{\max} r w \int_{\theta_1}^{\theta_2} \left(R \cos^2\theta - r \cos\theta\right) d\theta$$

$$= \mu p_{\max} r w \left\{ R\left[\frac{\phi_0}{2} + \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right] - r(\sin\theta_2 - \sin\theta_1) \right\} \qquad (1\text{-}5)$$

where $\phi_0 = \theta_2 - \theta_1$. The expression in equation (1-5) may be simplified by observing that the symmetry of the shoe about A requires that

$$-\theta_1 = \theta_2 = \frac{\varphi_0}{2} \qquad (1\text{-}6)$$

where φ_0 is the angle subtended by the lining. Substitution of these values into equation (1-5) leads to

$$M_f = \mu p_{\max} r w \left[\frac{R}{2}(\phi_0 + \sin\phi_0) - 2r \sin\frac{\phi_0}{2}\right] \qquad (1\text{-}5a)$$

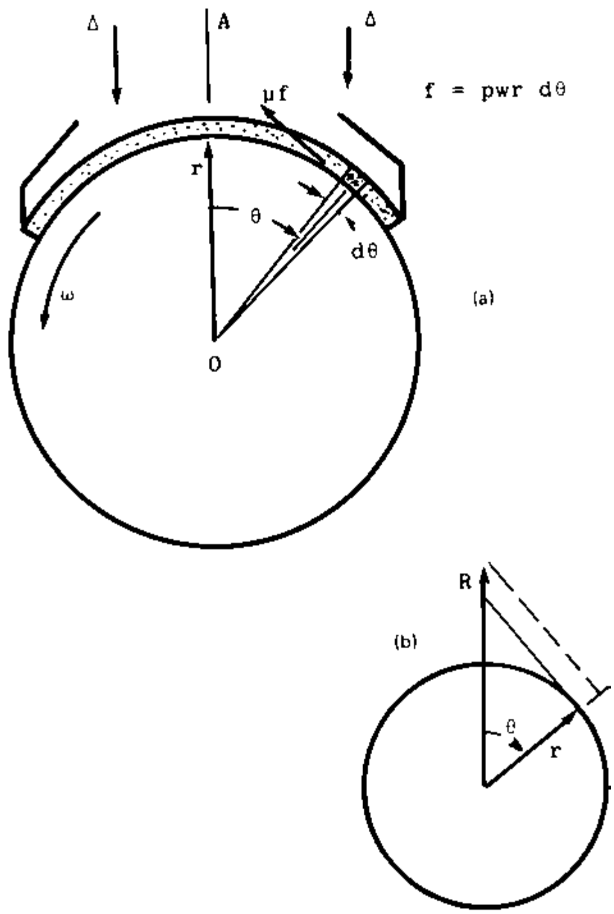

FIGURE 4 Geometry used for the analysis of a linearly acting external shoe.

which suggests that moment M_f will vanish if the shoe is pivoted at

$$R = r\frac{4\sin(\phi_0/2)}{\phi_0 + \sin\phi_0} \tag{1-7}$$

Upon plotting R/r we obtain Figure 5, wherein the ratio increases smoothly from 1.0 at $\phi_0 = 0$ to 1.273 at $\phi_0 = \pi$ rad. $= 180°$. This clearly indicates that it is impossible to find a pivot point for which $M_f = 0$ for an internal linearly acting shoe. This conclusion is, of course, unaffected by the

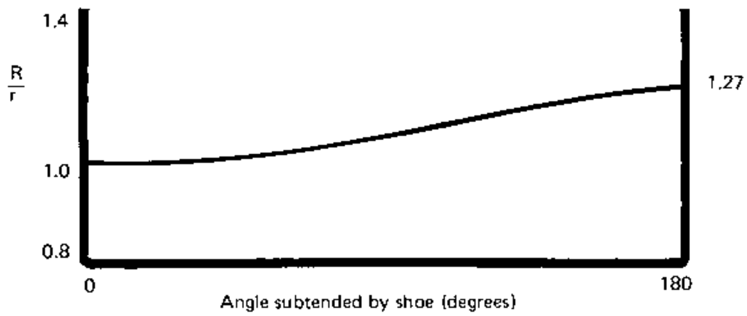

FIGURE 5 Variation of *R/r* with angle ϕ_0.

sign reversal found in the expression ($R \cos \theta - r$) when equation (1-5a) is applied to an internal shoe. The sign reversal simply changes the direction of rotation implied by a positive value of M_f, as was discussed in an earlier section.

The nearly horizontal portion from 0° to about 30° implies that for external shoes which subtend an angle less than 30°, any changes in the length of the shoe that do not increase the subtended angle beyond 30° will have a negligible effect on the R/r ratio. This correlates with the short-shoe segments used in the brakes shown in Figure 3. Moreover, the value of M_f caused by a deviation from the R/r ratio that yields a zero value of M_f will be small if ϕ_0 remains small. In particular, if the R value that yields $M_f = 0$ is replaced by $R + \delta R$ in equation (1-5a), the moment due to friction will increase to only

$$\mu p_{\max} rw \frac{\delta R}{2} (\phi_0 + \sin \phi_0)$$

which is small enough to be easily resisted by the shoulders shown on the shoes in Figure 3.

Activation force F_s and tangential force F_t on a symmetrically placed pivot are given by the relations

$$F_s = 2p_{\max} rw \int_0^{\phi_0/2} \cos^2\theta \, d\theta = \frac{1}{2} p_{\max} rw(\phi_0 + \sin \phi_0) \qquad (1\text{-}8)$$

and

$$F_t = 2\mu p_{\max} rw \int_0^{\phi_0/2} \cos^2\theta \, d\theta = \mu F_s \qquad (1\text{-}9)$$

Let us define the efficiency of a brake as the ratio of the torque provided by the brake to the torque that could be had by applying the force directly to the drum, or shaft. According to this definition, the efficiency becomes

$$\frac{T}{F_s r} = \frac{\mu p_{\max} r^2 w(\sin\theta_2 - \sin\theta_1)}{(1/2)p_{\max} r^2 w(\phi_0 + \sin\phi_0)} = 2\mu\frac{\sin\theta_2 - \sin\theta_1}{\phi_0 + \sin\phi_0} \tag{1-10}$$

Upon substituting for θ_1 and θ_2 in equation (1-8) and recalling equations (1-6) and (1-7) we find that

$$\frac{T}{F_s r} = \frac{4\mu\sin(\phi_0/2)}{\phi_0 + \sin\phi_0} = \mu\frac{R}{r} \tag{1-11}$$

where the right-hand side has already been plotted in Figure 5. From that figure we find that although maximum efficiency may be achieved only if each shoe and lining extend over half of the drum, or shaft, relatively little efficiency is lost if the lining extends over only 160° instead of 180°. This, together with the near impossibility of maintaining good contact between the lining and the drum near the ends of a shoe subtending 180° at the center of the drum, accounts for the angular dimensions of the brake linings shown in Figure 1.

Finally, it follows from equation (1-11) that if the shoe is symmetrically pivoted and if equation (1-7) holds, the applied torque is given by

$$T = \mu R F_s \tag{1-12}$$

II. BRAKING TORQUE AND MOMENTS FOR SYMMETRICALLY SUPPORTED INTERNAL SHOES

Pressure p and braking torque are again given by equations (1-2) and (1-4), respectively, for an internal shoe moved against a rotating drum along a line parallel to its axis of symmetry, line OB, in Figure 6. In the following analysis it may be more descriptive to measure the angle along the shoe from the end rather than from the middle because the activation forces are now applied at the ends. Denote this angle by ϕ. Since the expression for the torque is unaffected by this choice of angle, substitution of equation (1-6) into equation (1-4) shows it can be given by

$$T = 2\mu p_{\max} r^2 w \sin\frac{\phi_0}{2} \tag{2-1}$$

The pressure distribution described by equation (1-2) may be rewritten in terms of ϕ according to

$$p = p_{\max}\cos(\phi - \alpha) \tag{2-2}$$

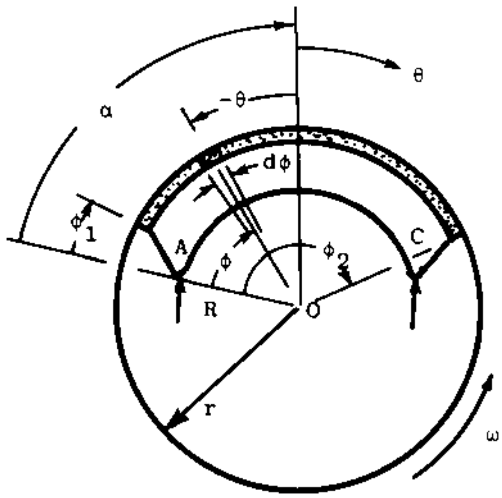

FIGURE 6 Geometry used in the analysis of a linearly acting internal shoe drum brake.

Let the shoe be restrained at *A* to prevent it from rotating with the drum, with the restraint moving with the shoe. This may be accomplished using guide pins and/or plates which may also serve as anchors to transfer braking torque from the shoes to the appropriate structure.

The moment M_p about A due to pressure p is given by integration of

$$dM_p = (prw\ d\phi)R \sin\phi = p_{\max} Rrw \cos(\phi - \alpha) \sin\phi\ d\phi \tag{2-3}$$

to obtain

$$M_p = p_{\max} wRr \int_{\phi_1}^{\phi_2} (\cos\alpha \cos\phi + \sin\alpha \sin\phi) \sin\phi\ d\phi \tag{2-4}$$

which may be integrated directly to give

$$M_p = \frac{p_{\max}}{4} wRr \big[2 \cos\alpha\ (\sin^2\phi_2 - \sin^2\phi_1) + \sin\alpha(2\phi_0 - \sin 2\phi_2 + \sin 2\phi_1)\big] \tag{2-5}$$

Let α represent the central angle from the R vector to the middle of the shoe (i.e., from R to the transverse plane of symmetry through radius OB in Figure 6), so that

$$\phi_1 = \alpha - \frac{\phi_0}{2} \qquad \phi_2 = \alpha + \frac{\phi_0}{2} \tag{2-6}$$

After substitution for ϕ_1 and ϕ_2 from equations (2-6) and using common trigonometric identities, M_p may be written as

$$M_p = \frac{p_{\max}}{2} rRw(\phi_0 + \sin\phi_0)\sin\alpha \tag{2-7}$$

Similarly, the moment about A due to friction may be found from

$$dM_f = (aprw\ d\phi)(r - R\cos\phi) \tag{2-8}$$

which with the aid of equation (2-2) leads to the integral

$$M_f = p_{\max} rw\mu \int_{\phi_1}^{\phi_2} (\cos\alpha\cos\phi + \sin\alpha\sin\phi)(r - R\cos\phi)\ d\phi \tag{2-9}$$

which, upon integration, produces

$$\begin{aligned} M_f = \frac{\mu}{4} rwp_{\max}[&4r(\sin\phi_2 - \sin\phi_1)\cos\alpha \\ &+ 4r(\cos\phi_1 - \cos\phi_2)\sin\alpha - R(2\phi_0 + \sin 2\phi_2 \\ &- \sin 2\phi_1)\cos\alpha - 2R(\sin^2\phi_2 \sim \sin^2\phi_1)\sin\alpha] \end{aligned} \tag{2-10}$$

Substitution for ϕ_1 and ϕ_2 from equations (2-6) into equation (2-10) and use of common trigonometric identities enables equation (2-10) to be simplied to read

$$M_f = \frac{p_{\max}}{2} \mu rw \left[4r \sin\frac{\phi_0}{2} - R(\phi_0 + \sin\phi_0)\cos\alpha \right] \tag{2-11}$$

According to the geometry shown in Figure 6, a positive M_p corresponds to clockwise rotation of the shoe about point A and positive M_f corresponds to counterclockwise rotation when the drum rotation is from the opposite end of the shoe toward point A. Reversing the direction of drum rotation will not affect the implied direction of shoe rotation due to M_p but will reverse the direction by a positive M_f; that is, positive M_f will then imply clockwise rotation about point A. This last observation is of academic interest only, however, if the shoes are supported at each end, because in that case each shoe will tend to pivot about the end toward which the drum rotates, regardless of the direction of rotation of the drum.

For symmetrically supported symmetric shoes it follows that the shoes will be free of self-locking if

$$M_p - M_f > 0 \tag{2-12}$$

Substitution for M_p and M_f from equations (2-7) and (2-11) into equation (2-12) yields

$$M_p - M_f = \frac{p_{\max}}{2} Rrw\left[(\phi_0 + \sin\phi_0)\sin\alpha - 4\mu\frac{r}{R}\sin\frac{\phi_0}{2} + \mu(\phi_0 + \sin\phi_0)\cos\alpha\right] > 0 \tag{2-13}$$

Condition (2-12) is, according to equation (2-13), equivalent to the condition

$$4\mu\frac{r}{R}\sin\frac{\phi_0}{2} < (\phi_0 + \sin\phi_0)(\sin\alpha + \mu\cos\alpha) \tag{2-14}$$

for internal, linearly acting brakes. Consequently, the r/R ratio must satisfy

$$1 \geq \frac{R}{r} > \frac{4\sin(\phi_0/2)}{(\phi_0 + \sin\phi_0)[\cos\alpha + (1/\mu)\sin\alpha]} \tag{2-15}$$

to ensure that the brake will not become self-locking when it is applied.

III. DESIGN EXAMPLES

Example 4.1

Design an external, linearly acting, twin-shoe brake to provide a braking torque of 2700 N-m when acting on a flywheel hub 260 mm in diameter. The lining material to be used here has a design maximum pressure of 3.41 MPa and $\mu = 0.41$.

Since the torque on either an external or an internal shoe is given by equation (2-1), it follows from the $\sin(\phi_0/2)$ term that 90% of the maximum theoretical torque (i.e., for $\phi_0 = 180°$) may be obtained from $\phi_0 = 128.3°$, that 95% may be had from $\phi_0 = 143.6°$, and 98% may be had from $\phi_0 = 157.0°$. It we select $\phi_0 = 145°$ for each shoe, assume that each shoe will supply half of the design braking torque, and solve equation (2-1) for w, we find that

$$w = \frac{T}{2\mu p_{\max} r^2 \sin(\phi_0/2)}$$

$$= \frac{1350 \times 10^3}{0.82(3.41)(130)^2 \sin 72.5°} = 29.95 \text{ mm} \to 30 \text{ mm}$$

The required vertical force on each shoe as calculated from equation (1-8) becomes

$$F_s = \frac{p_{\max}}{2} rw(\phi_0 + \sin\phi_0) = \frac{3.41}{2} 130(29.95)(2.531 + \sin 145°)$$

$$= 20,609\,N$$

So if the brake is to be pneumatically activated, as shown in Figure 1, the pressure and diaphragm diameter are related according to

$$F_s = \pi r^2 p_{\text{dia}}$$

Upon solving this relation for the diaphragm pressure pdia, and using an active diaphragm diameter of 250 mm, we find that the line pressure to the diaphragm must be 4.20 atm.

Finally, if the shoes are to be pivoted about an axis in their planes of symmetry, the radial distance to the pins may be calculated from equation (1-7), which yields

$$\frac{R}{r} = \frac{4\sin(\phi_{0/}2)}{\phi_0 + \sin\phi_0}$$

$$= \frac{4\sin 72.5^\circ}{2.531 + \sin 145^\circ}$$

So $R = 159.74$ mm from the center of the drum, or 29.74 mm from the drum surface.

Example 4.2

Design an internal, linearly acting, twin-shoe drum brake to provide a braking torque of 413,000 in. - 1b acting on a drum whose maximum inside diameter may be 26.0 in. The lining material to be used has a maximum design pressure of 450 psi and a friction coefficient of 0.50 or greater over the design temperature range.

Substitution into the expression obtained by solving equation (2-1) for w yields, for $\phi_0 = 130^\circ$,

$$w = \frac{206,500}{2(0.5)(450)(13)^2(\sin 65^\circ)} = 2.996 \text{ in.} + 3.00 \text{ in.}$$

If self-locking is to be avoided, the pivot point for each shoe should obey the inequality (2-15), which in this case becomes, for $\alpha = 70^\circ$,

$$R > \frac{4\mu r\sin(\phi_0/2)}{(\phi_0 + \sin\phi_0)(\sin\alpha + \mu\cos\alpha)}$$

$$= \frac{4(0.5)(13)\sin 65^\circ}{(2.2689 + \sin 130^\circ)(\sin 70^\circ + 0.5\cos 70^\circ)} = 6.990 \text{ in.}$$

Equal forces that must be applied at points A and C in Figure 6 to achieve the 450 psi maximum pressure may be found from equation (1-8) after replacing F_s with $2F_s$, where F_s in equation (3-1) represents the force at A and at C. Thus

$$F_s = \frac{p_{\max}}{4} \text{rw}(\phi_0 + \sin(\phi_0))$$

$$= \frac{450}{4} 13(3)\left(130\frac{\pi}{180} + \sin\left(130\frac{\pi}{180}\right)\right) \tag{3-1}$$

$$= 13,315.9 \text{ lbs}$$

If an axial piston hydraulic pump capable of a continuous pressure of 4500 psi is used, this force may be had from a hydraulic cylinder whose piston diameter is equal to, or greater than

$$\text{d(p)} = 2\sqrt{\frac{F_s}{\pi \cdot \text{p}}} = 1.941 \text{ inches.}$$

Space available for such a cylinder may be found from the geometry in Figure 6 by finding the distance from a plane P through the center of the drum and perpendicular to the $\theta = 0$ line. The available distance will be twice this value.

Angle β between R and plane P may be found from

$$2\beta = 180° - \phi_0 - 2\phi_1 \tag{3-2}$$

so that the distance h_0 from point A to the corresponding point on the opposite shoe becomes

$$h_0 = 2\text{R} \sin \beta. \tag{3-3}$$

If we let $\text{R} = 10$ inches the result is that since $\phi_1 = 5°$

$$\beta = (180° - 130° - 10°)/2 = 20°$$

so that

$$h_0 = 2(10) \sin (20) = 6.840 \text{ inches.}$$

Distance h_1 from point A to the drum surface may be found from the relation that

$$h_1(\phi_1, R) = r \sin\left(\text{acos}\left(\frac{R}{r} \cdot \cos(\beta(\phi_1) \cdot \text{deg})\right)\right) - h_0(\phi_1, R) \tag{3-4}$$

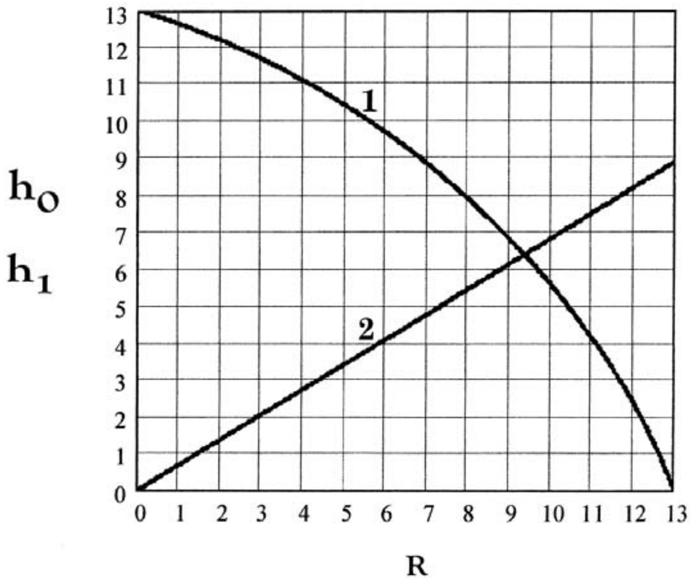

FIGURE 7 Curve 1: Distance h_1 between points A, or C, and the drum surface as a function of radius R; Curve 2: Height h_0 available for a hydraulic cylinder as a function of R. All dimensions in inches.

which for R = 10 inches yields

$$13\sin\left(\operatorname{acos}\left(\frac{10}{13}\cos\left(20\frac{\pi}{180}\right)\right)\right) - 3.4202 = 5.56299$$

which may suggest that selecting a larger value for R would give more space for the hydraulic cylinders that force the shoes against the drum and would also require less material in each shoe. Plotting h_0 and h_1 for other values of R results in the curves shown in Figure 7. Distances are measured along chords that pass through points A on opposing shoes and through points C on opposing shoes.

IV. NOTATION

- F_s Force in the transverse plane of symmetry of the shoe (mlt^{-2})
- F_t force tangential to the shoe at the transverse plane of symmetry (mlt^{-2})
- h_0 Length available for an activation mechanism

h_1	Length available for the shoe structure
k	equivalent spring constant for lining material (mt^{-2})
M_f	moment due to friction (ml^2t^{-2})
M_p	moment due to pressure (ml^2t^{-2})
p	lining pressure $(ml^{-1}t^{-2})$
$p_{\max}$	maximum lining pressure $(ml^{-1}t^{-2})$
R	radius to effective pivot point from the drum center (l)
r	drum radius (l)
T	braking torque (ml^2t^{-2})
w	shoe and lining width (l)
α	Lining half-angle (1)
Δ	Lining deflection in compression (l)
θ	Angle (1)
μ	Friction coefficient (1)
ϕ	Angle (1)

V. FORMULA COLLECTION

Pressure distribution:

$$p = p_{\max} \cos\theta = p_{\max} \cos(\phi - \alpha)$$

Lining pressure in terms of torque for external and internal shoes:

$$p_{\max} = \frac{T}{2\mu r^2 w \sin(\phi_0/2)}$$

Lining width in terms of torque for external and internal shoes:

$$w = \frac{T}{2\mu p_{\max} r^2 \sin(\phi_0/2)}$$

Moment due to friction for a symmetrically pivoted external shoe:

$$M_f = \mu p_{\max} r w \left[\frac{R}{2}(\phi_0 + \sin\phi_0) - 2r \sin\frac{\phi_0}{2}\right]$$

Moment due to friction about the trailing end of an internal shoe:

$$M_f = \frac{p_{\max}}{2} \mu r w \left[4r \sin\frac{\phi_0}{2} - R(\phi_0 + \sin\phi_0)\cos\alpha\right]$$

Moment due to pressure about the trailing end of an internal shoe:

$$M_p = \frac{p_{\max}}{2} r R w (\phi_0 + \sin\phi_0)\sin\alpha$$

Activation force

$$F_s = \frac{1}{2} p_{\max} r w (\phi_0 + \sin\phi_0)$$

Tangential force for a centrally pivoted external shoe:

$$F_t = \mu F_s$$

Anchor pin location for symmetrically pivoted external shoes:

$$\frac{R}{r} = \frac{4\ \sin(\phi_0/2)}{\phi_0 + \sin\phi_0}$$

Support point location for a linearly acting internal shoe:

$$\frac{R}{r} > \frac{4\sin(\phi_0/2)}{(\phi_0 + \sin\phi_0)[\cos\alpha + (1/\mu)\sin\alpha]}$$

Length available for an activation mechanism

$$h_0 = R\ \sin(\beta)$$

Length available for the shoe structure

$$h_1 = \mathrm{r}\ \sin\left(\mathrm{acos}\left(\frac{R}{r}\cos(\beta)\right)\right) - h_0$$