5

Dry and Wet Disk Brakes and Clutches

This chapter on disk brakes and clutches will consider annular contact disk clutches and both caliper and annular contact disk brakes, as illustrated in Figures 1, 2, and 3.

Caliper disk brakes, as shown in Figure 1, are used on aircraft, automotive, industrial, and mining equipment. Their two main advantages compared to drum brakes are greater heat dissipation, and hence less fading, because of their open construction, and a more uniform braking action, due to self-cleaning by brake pad abrasion. Their main disadvantage is that they require a larger activation force than is required for drum brakes because they have neither a friction moment nor servo action to aid in brake application.

Annular contact disk brakes and clutches are available as either dry or wet brakes, as shown in Figure 2 and 3. These units may be used as either a brake or as a clutch because the only differences between the two are whether one side of the unit is fastened to a stationary frame or to a rotating shaft and whether the unit has the necessary fittings for it to be controlled while in rotation. For example, both of these functions are combined in Minster combination dry clutch and brake units, illustrated in Figure 2, which are pneumatically controlled using air passages in the shaft to the combination unit.

Wet multiple-disk brakes and clutches, illustrated in Figure 3, have similar multiple-disk construction, but operate in an oil bath. Thus these brakes are isolated from dirt and water, and the circulation of the oil through a heat exchanger usually provides greater heat dissipation than can be had from direct air cooling. Because of these advantages, wet brakes have been

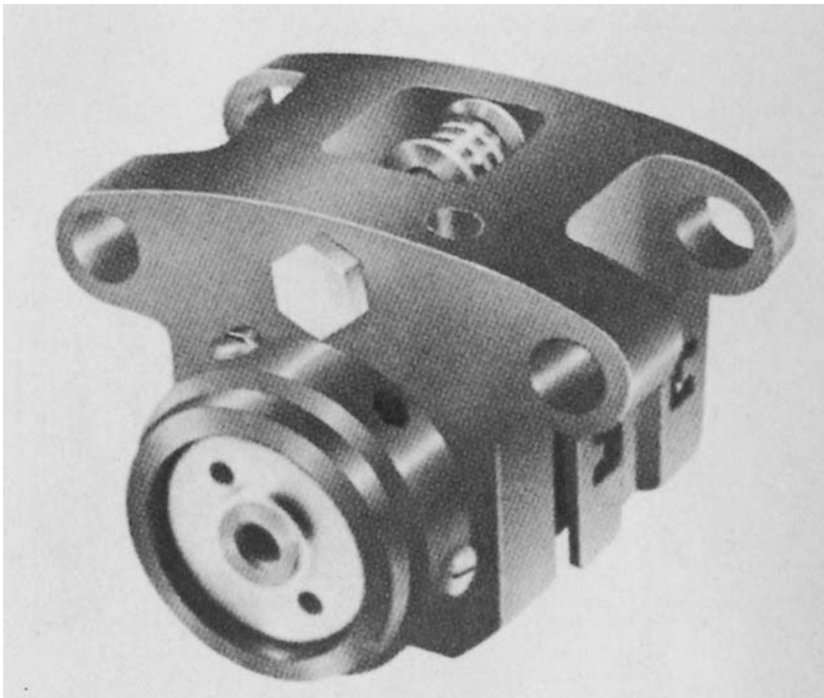

FIGURE 1 Floating, or sliding, caliper disk brake. (Courtesy of Misco, Inc., North Mankato, MN.)

used on large earth-moving equipment, on mine shuttle cars, and similar equipment which may require large braking torque and which may be designed to operate in a dirty environment.

I. CALIPER DISK BRAKES

From the moment of contact until the disk is stopped, the velocity of the disk relative to the brake pads will vary linearly with the disk radius. If the thickness of the lining material removed is denoted by δ and if δ is dependent on the relative velocity and the pressure, as is commonly assumed, then according to the uniform wear assumption,

$$\delta = kpr \tag{1-1}$$

where k is a constant of proportionality. Since the caliper brake pads are usually small enough for their supports to be considered rigid, we shall assume

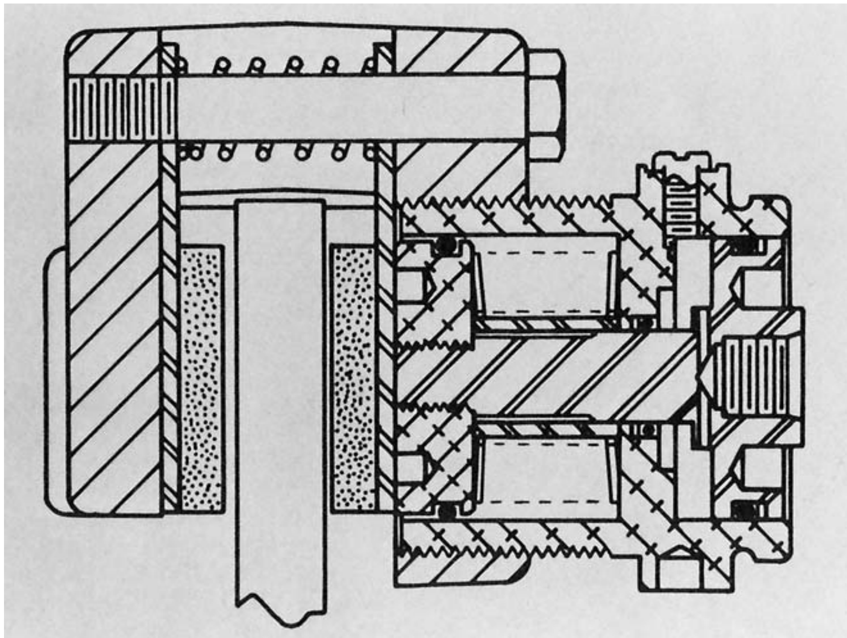

FIGURE 1 Continued.

that δ is constant over the brake pad (i.e., the wear is uniform). Whenever these conditions hold, equation (1-1) implies that the pressure increases as the radius decreases, so the maximum pressure is found at the inner radius, r_i. Thus

$$\delta = k p_{\max} r_i \tag{1-2}$$

Elimination of k and δ from equations (1-1) and (1-2) yields

$$p = p_{\max} \frac{r_j}{r} \tag{1-3}$$

With the lining pressure known, we may now calculate the required axial force from

$$F = \int_A p \; da \tag{1-4}$$

and the resulting braking torque from

$$T = \mu \int_A pr \; da \tag{1-5}$$

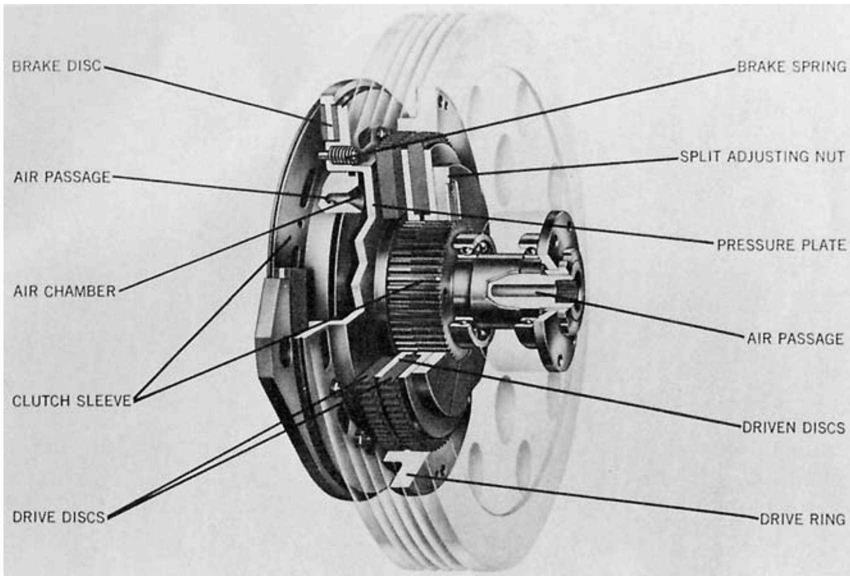

FIGURE 2 Combination disk brake and disk clutch, both dry. (Courtesy of Minster Machine Co., Minster, OH.)

Evaluation of these integrals is easiest for brake pads with radial and circular boundaries, as in Figure 4, for which equation (1-4) and (1-5) may be written using a dummy variable ϕ as

$$F = p_{\max} r_i \int_A \frac{1}{r} da = p_{\max} r_i \int_{r_i}^{r_0} \int_0^{\theta} d\phi dr \qquad (1\text{-}6)$$

$$= p_{\max} r_i \theta (r_0 - r_i)$$

and

$$T = \mu p_{\max} r_i \int_A da = \mu p_{\max} r_i \int_{r_i}^{r_o} r \ dr \int_0^{\theta} d\phi$$

$$= \mu p_{\max} r_i \frac{\theta}{2} \left(r_o^2 - r_i^2 \right) \qquad (1\text{-}7)$$

From equation (1-7) we find that for the pressure distribution given by relation (1-3) the torque may be easily calculated for any brake pad whose area is

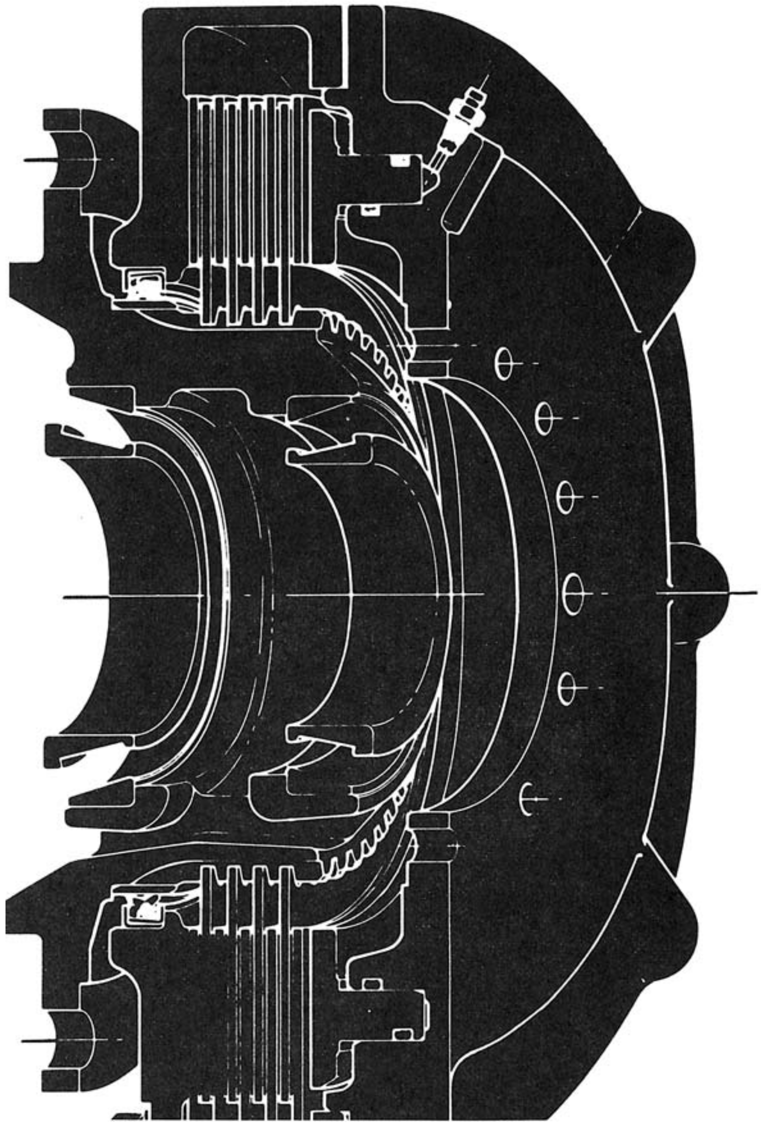

FIGURE 3 Wet multiple-disk brake. (Courtesy D. A. B. Industries, Inc., Troy, MI.)

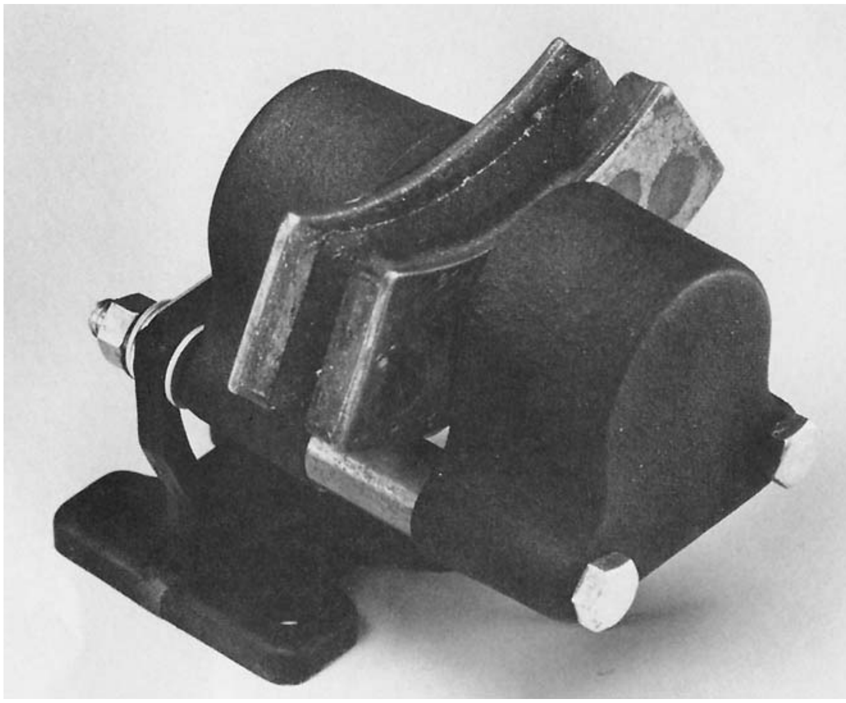

FIGURE 4 Annular sector caliper disk brake. (Courtesy of Horton Manufacturing Co., Inc., Minneapolis, MN.)

known or simply calculated. For a circular pad of diameter d, for example, the torque is given by

$$T = \mu p_{\max} r_i \frac{\pi}{4} d^2 \tag{1-8}$$

According to equation (1-7), the torque provided by a caliper brake having pads similar to those in Figure 4 usually will be greater than that provided by circular pads of equal area, as shown in Figure 5, when acting on disks of equal outside diameter because the proportions of the pads in the brake shown in Figure 4 generally place the center of pressure at a larger radius from the center of the disk. (See also Figure 6.)

Circular pads are often used, nevertheless, in hydraulically activated caliper brakes whenever the hydraulic pressure may be increased relatively cheaply because the pads themselves are supported entirely by the piston face and are therefore cheaper to produce because no additional supporting

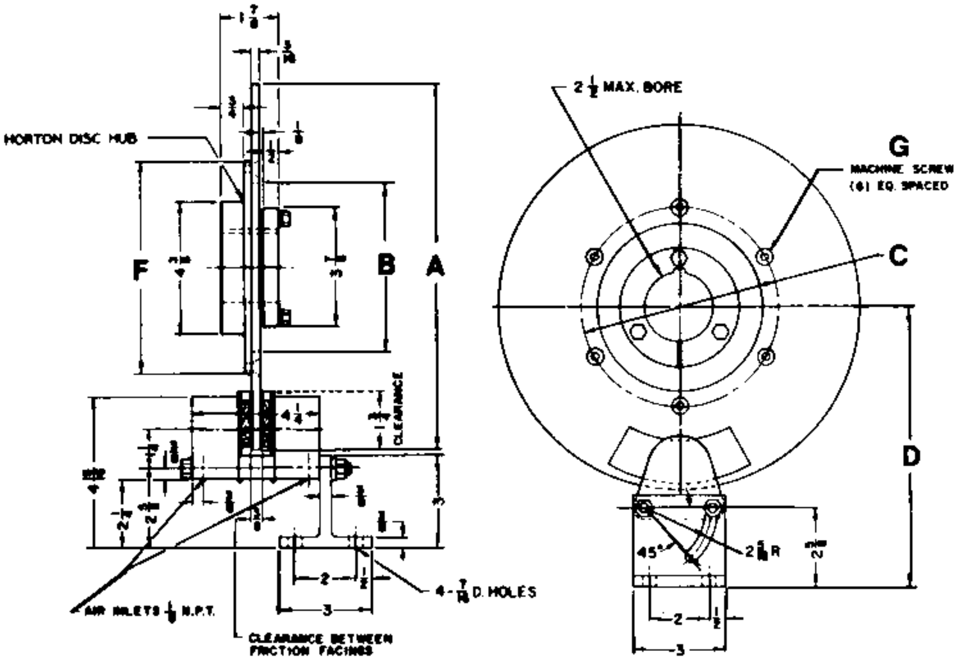

FIGURE 4 Continued.

structure is required. Noncircular pads are used where increasing the pressure may be relatively expensive and where the maximum performance is required for the pressure that is available, as in aircraft brakes.

If we replace $d^2/4$ in equation (1-8) with r_p^2, where r_p is the pad radius ($r_p = d/2$), and also replace r_i in equation (1-8) according to the relation $r_i = r_o - 2r_p$, we have

$$T = \mu\pi p_{\max}(r_o - 2r_p)r_p^2 \tag{1-9}$$

Upon differentiating equation (1-9) with respect to r_p we obtain

$$\frac{dT}{dr_p} = \mu\pi p_{\max} 2r_p(r_o - 3r_p) \tag{1-10}$$

which is equal to zero when $r_p = r_o/3$, indicating an extreme value of T for that pad radius. Since dT^2/dr_p^2 is negative at this value of r_p, it follows that T has its maximum value at $r_p = r_o/3$.

Calculating the activation force for a circular pad is more involved than it is for an annular sector pad because radius r remains in the denominator of

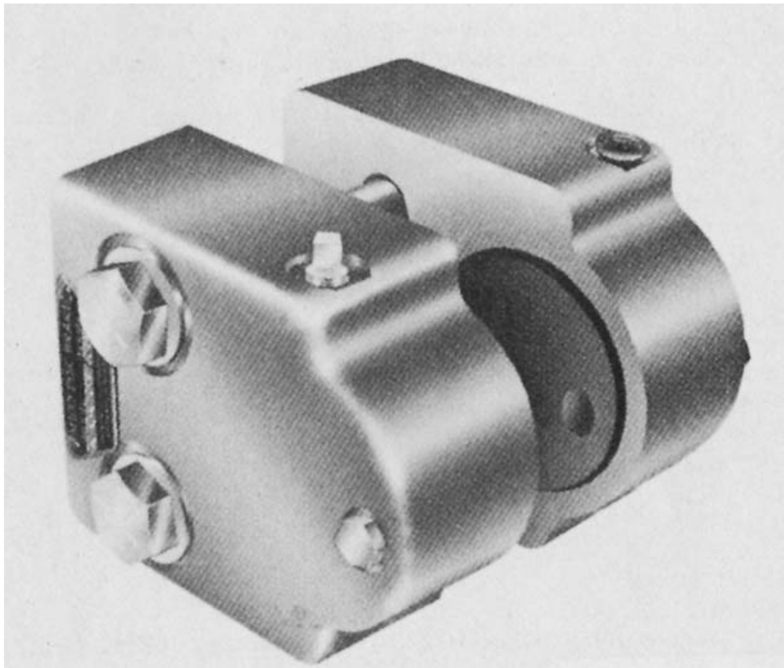

FIGURE 5 Caliper disk brake with circular pads, two pistons. (Courtesy of Misco, Inc., North Mankato, MN.)

the integrand. An element of the pad area may be written as $da = \rho \, d\rho \, d\theta$, where radius ρ is measured from the center of the pad, as shown in upcoming Figure 13, associated with later Example 4.1. Complexity arises from the requirement that the expression for the radius r from the center of the disk to the element of area of the circular pad must now be written in terms of ρ and θ.

From the law of cosines we have

$$r = (r_c^2 + \rho^2 - 2r_c\rho \cos \theta)^{1/2} \tag{1-11}$$

where r_c is the radius from the center of the brake pad to elemental area da, as shown in Figure 13 (see later Example 4.1).

Substitution of equation (1-11) into the first integral in equation (1-6) and writing the element of area as $\rho \, d\rho \, d\theta$ allows the activation force to be written as

$$F = p_{\max} r_i \int_0^{2\pi} \int_0^{r_p} \frac{\rho}{(\rho^2 + r_c^2 - 2\rho_i r_c \cos \theta)^{1/2}} \, d\rho d\theta \tag{1-12}$$

FIGURE 6 Typical caliper brake pad of sintered material for heavy aircraft brakes. (Note contour of the pad to place lining material toward the outer periphery of the disk.) (Courtesy Friction Products, Medina, OH.)

Since analytical evaluation of the integrals in equation (1-12) is somewhat tedious, it is easier to turn to numerical methods. Evaluation using a numerical program, such as Mathcad, may provide graphical data that displays the dependence of force F on the pad radius r_p, as will be demonstrated later in Example 4.1.

The Mathcad manual specifies the integration method used in its program and the references used in writing the program. They may be consulted for the details of mathematical analysis.

II. VENTILATED DISK BRAKES

Although disk brakes are less susceptible to fade than drum brakes, they will be heated by friction, which may lead to brake fade in situations requiring heavy and frequent braking. This heating may be reduced by using ventilated disk brakes, which consist of two disks separated by radial vanes, so that additional cooling surface is provided, as shown in Figure 7.

Ventilation also increases brake life, as implied by the representative brake pad life as a function of the surface temperature as given in Figure 8, where the longest life is realized for that pad and caliper combination which provides the largest heat sink, shown in Figure 9 and the shortest life for that with the smallest heat sink, shown in Figure 10.

III. ANNULAR CONTACT DISK BRAKES AND CLUTCHES

Annular contact, or face contact, disk brakes are available either as dry multiple-plate disk brakes, as shown in Figure 2, or as wet multiple-plate disk brakes, as shown in Figure 3. Their construction is similar to that of multiple disk cluthes to the extent that many manufacturers produce both multiple disk cluthes and brakes that have many components in common.

Conventional design formulas for these brakes are predicated on one of two assumptions: uniform wear or uniform pressure. Although the first of these assumptions may be a better approximation of brake behavior, it involves more calculation than the second. Following established practice, we shall consider the consequences of both of these assumptions.

A. Uniform Wear

The uniform wear assumption employed in the derivation of the force and torque relations given by equations (1-6) and (1-7) may be applied to disk brakes if the plates and the clamping structure tend to maintain uniform lining thickness. Application of equations (1-6) and (1-7) to annular contact disk brakes requires only that θ be replaced by 2π in both relations to get

$$T = \mu\pi p_{\max} r_i (r_o^2 - r_i^2) \tag{3-1}$$

and

$$F = 2\pi p_{\max} r_i (r_o - r_i) \tag{3-2}$$

So the ratio T/F of the torque to the activating force is given by

$$\frac{T}{F} = \mu \frac{r_o + r_i}{2} \tag{3-3}$$

Examination of equation (3-1) yields the somewhat surprising result that if we cover the entire face of a single-plate brake or clutch with lining material, the brake or clutch will soon become ineffective. In other words, the braking torque predicted by equation (3-1) will be zero whenever $r_i = r_o$, as reasonably expected, or whenever $r_i = 0$ and $r_o > 0$, as may not be expected.

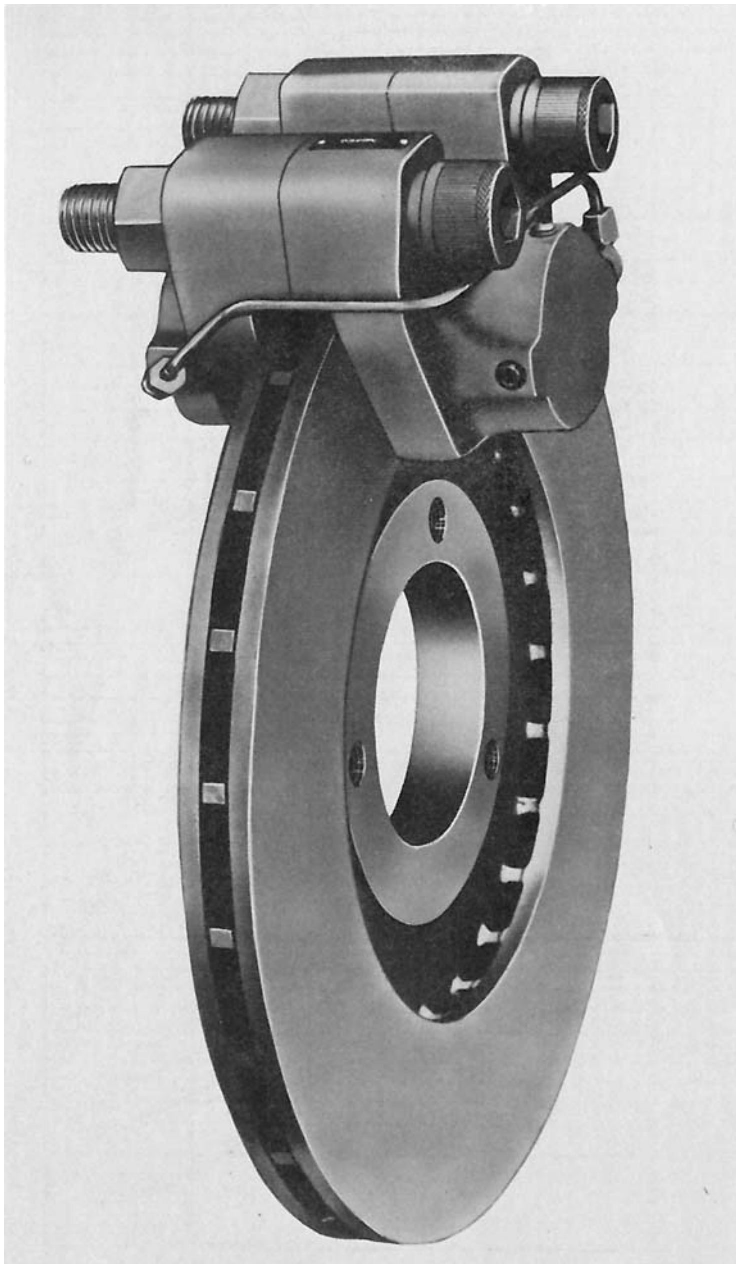

FIGURE 7 Ventilated caliper disk brake. (Courtesy of Eaton Power Transmission Systems, Airflex Division, Cleveland, OH.)

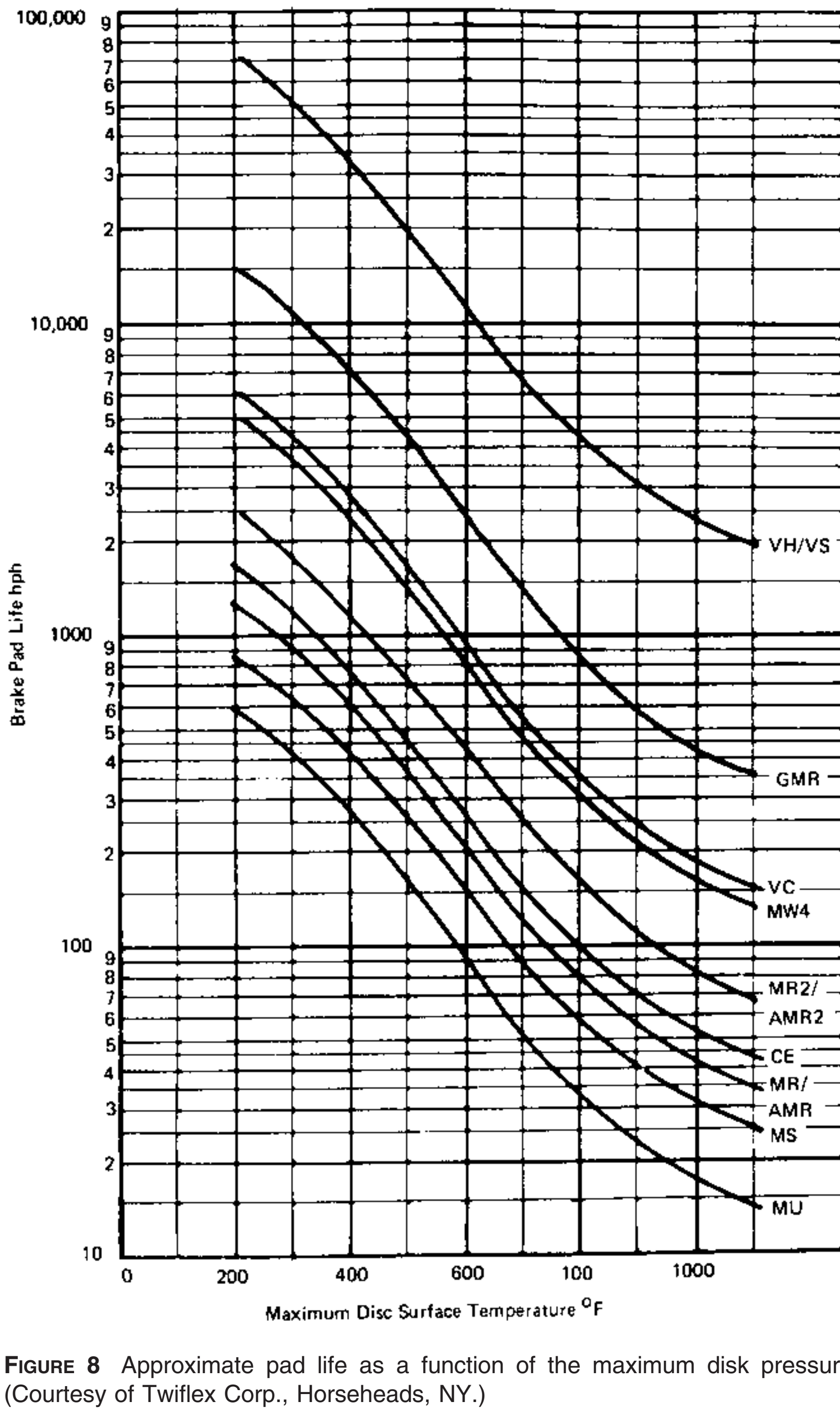

FIGURE 8 Approximate pad life as a function of the maximum disk pressure. (Courtesy of Twiflex Corp., Horseheads, NY.)

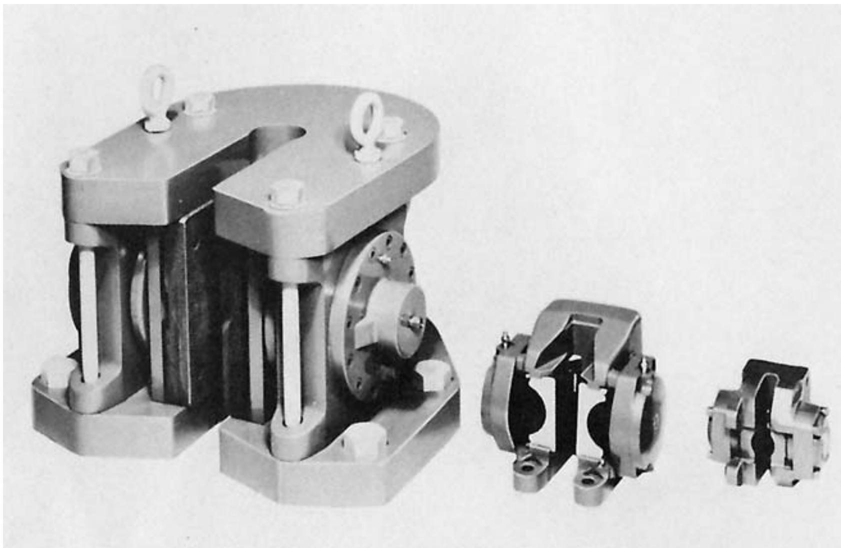

FIGURE 9 Large calipers for disk brakes–model vs. referenced in Figure 8. (Courtesy of Twiflex Corp., Horseheads, NY.)

This was unintentionally demonstrated by a winch manufacturer between 1970 and 1980, as will be described later. Because of these observations we shall turn our attention to finding the r_i that will produce the maximum torque before designing a face contact disk brake or clutch. Differentiation of equation (3-1) with respect to r_i and setting the derivative to zero yields

$$r_i = \frac{r_o}{\sqrt{3}} \tag{3-4}$$

as the theoretically optimum value of r_i, corresponding to torque and activating force given by

$$T = \frac{2}{3\sqrt{3}} \mu p_{\max} \pi r_o^3 \tag{3-5}$$

and

$$F = \frac{1}{\sqrt{3}} \left(1 - \frac{1}{\sqrt{3}}\right) 2\pi r_o^2 p_{\max} \tag{3-6}$$

for a single-face annular contact brake. Actual brake lining dimensions may differ somewhat from this inner radius because of concentric grooves in the

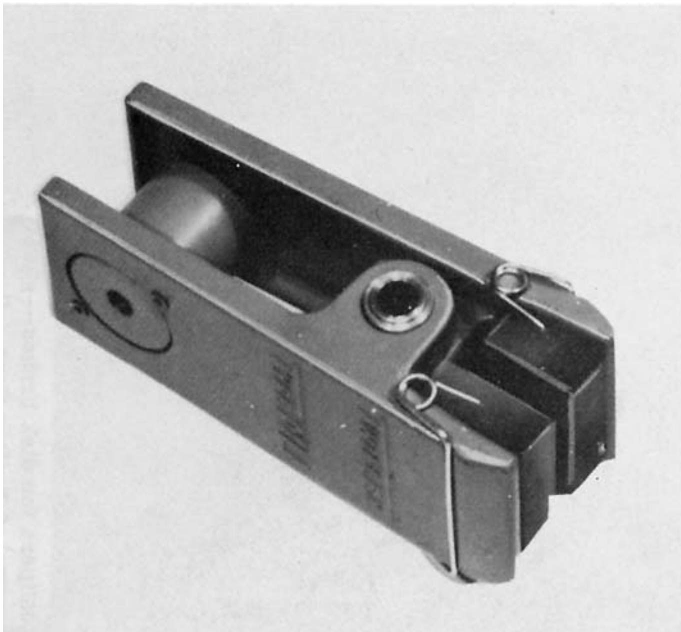

FIGURE 10 Small caliper for disk brakes–model MO referenced in Figure 8. (Courtesy of Twinflex, Corp., Horseheads, NY.)

lining and/or experimental data which may imply an effective pressure distribution different from that given in equation (1-3).

One advantage of multiple-plate brakes is that the torque increases in direct proportion to the number of plates added while the activation force theoretically remains unchanged. In mathematical terms,

$$T = \frac{2n}{3\sqrt{3}} \mu p_{\max} \pi r_o^3 \tag{3-7}$$

where n is the number of friction interfaces (8 in Figure 3, 4 in Figure 11). Adding springs to separate the plates when the brake is released will increase the activation force by the amount of the spring forces plus the friction forces generated by the motion of the plates along their lubricated splines.

B. Uniform Pressure

This assumption implies that either the disks or the lining or both are flexible enough to allow the deformation necessary for δ in equation (1-1) to vary with

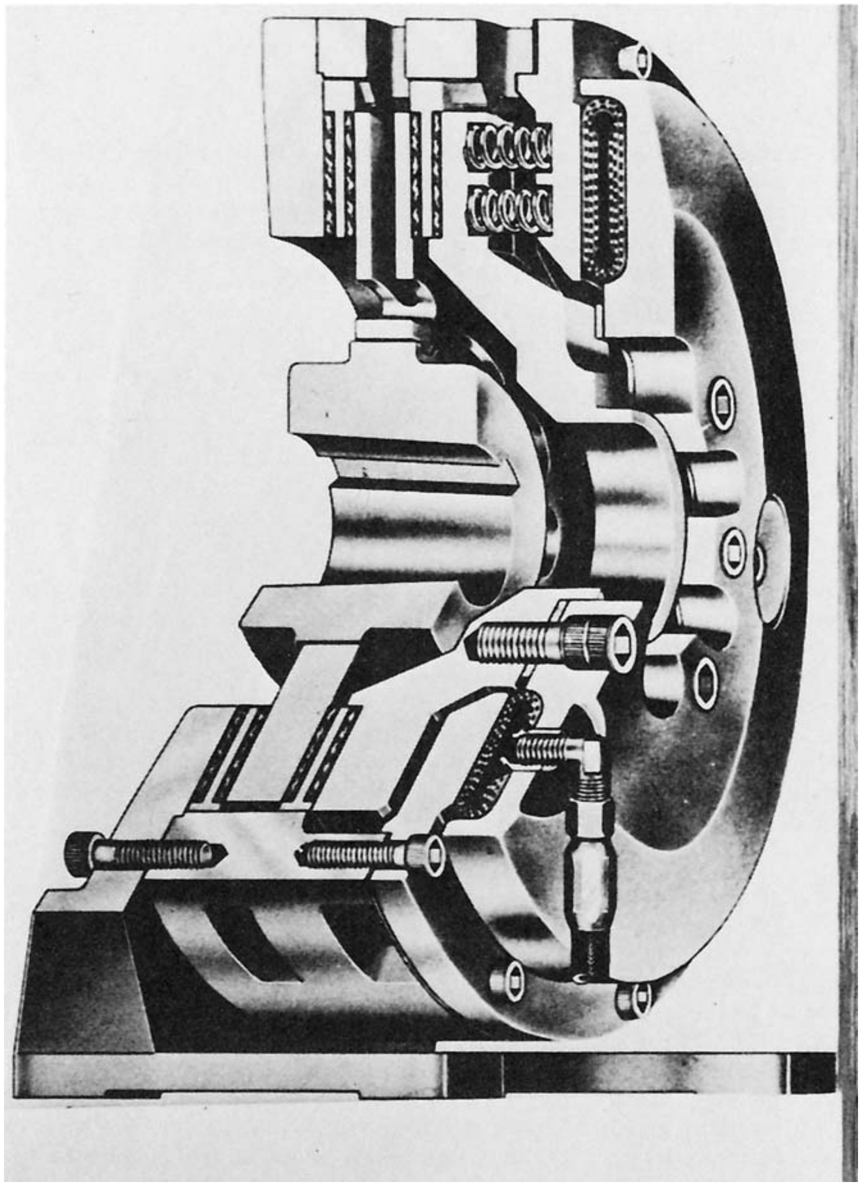

FIGURE 11 Dry multiple-disk brake, pneumatically activated. (Courtesy of Wichita Clutch Co., Dana Corp., Power Transmission Div., Toledo, OH.)

the radius such that the pressure can become constant. Whenever the pressure is uniform, equations (1-5) and (1-4) hold and may be easily integrated to give

$$T = \frac{2}{3}\pi\mu p(r_o^3 - r_i^3) \tag{3-8}$$

for the braking torque and

$$F = \pi p(r_o^2 - r_i^2) \tag{3-9}$$

as the activation force.

Since uniform pressure may require spring-loaded plates, plates of varying thickness, or some other mechanism to ensure no pressure variation, relations (3-7) and (3-8) may be restricted to single-plate brakes, where the additional mechanism may be added.

If an annular disc brake or clutch is replaced by one with full-faced rigid discs on both input and output shafts and with a lining, or facing, material that covers the entire face of one of the discs so that $r_i = 0$, the torque capability of the clutch, or brake, may be given initially by equation (3-8). Its torque capacity, however, will decrease with each application of the brake or clutch until it fails to transfer useful torque. This is because, according to equation (1-1), negligible wear will occur at and near $r_i = 0$. Consequently the lining will maintain its original thickness near the center of the disc while the lining beyond this region wears away. Eventually there will be negligible contact, and hence negligible pressure, outside of what has become a small raised circular region, or hump, centered at $r_i = 0$. The sharp peak expected at the center of the facing, or lining, material because of zero wear at that point will usually not be seen because the compressibility of the friction material will allow the peak to be mashed down by the mating plate. This compressibility of the lining material will extend the effective life of such a clutch or brake until the activating force is unable to compress the resulting central hump enough for the lining to contact the mating plate beyond this small central hump. Removing this small central region, however, will allow the brake or clutch to again transmit torque.

As noted earlier, this was unintentionally demonstrated by at least one winch manufacturer in the 1970–1980 period. The manufacturer's winch incorporated a clutch as described earlier in which one face was covered entirely by the facing material. When the clutch ultimately failed to transmit a useful torque, the manufacturer recommended replacing the facing material. Instead, the life of the facing could be, and was, more than doubled simply by removing the central region to produce an inner radius $r_i > 0$. If inner radius r_i, were made equal to that given by equation (3-4), then the torque capability would be restored to that given by equation (3-5).

Since greater torque enhancement can be obtained from multiple disk brakes that provide torque multiplication equal to the number of contacting friction surfaces, as indicated by equation (3-7), it follows that there is no motivation to try to devise some mechanism to assure uniform pressure between contacting annular plates.

IV. DESIGN EXAMPLES

Example 4.1

Estimate the torque and activation force for a floating, or sliding, caliper disk brake having circular pads 1 in. in diameter acting on a disk 11 in. in diameter, as shown in Figure 12. The expected friction coefficient is 0.32 and the maximum design pressure for the lining material is 300 psi. (A sliding caliper brake is held by a slide which allows its brake pads to be forced against opposite sides of the disk when its single piston is activated, as in Figure 1.)

From Figure 12 it is evident that $r_i = 4.5$ in., so from equation (1-8),

$$T = \pi \rho p_{\max} r_i \frac{d^2}{4} = \pi(0.32)(300)\frac{1}{4} = 339.292 \text{ in.} - \text{lb}$$

per caliper pad. Hence the total torque is 678.584 in.-lb.

According to Figures 12 and 13 and pad radius r_p it is evident that

$$r_i = r_o - 2r_p \qquad \text{and} \qquad r_c = r_o - r_p \qquad 0 \le p \le r_p$$

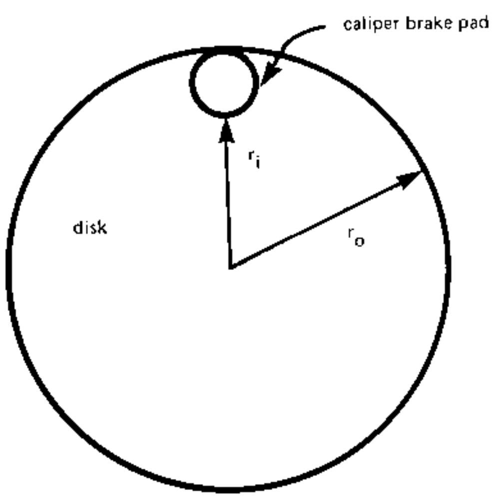

Figure 12 Circular lining pad of a caliper disk brake.

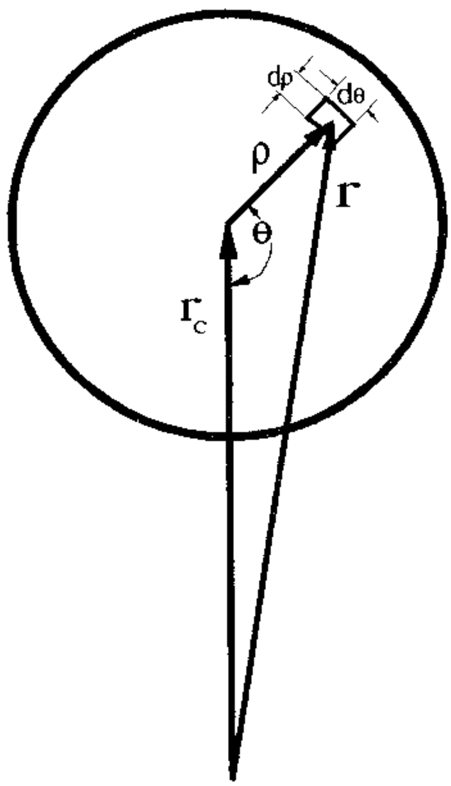

FIGURE 13 Geometric relations between ρ, θ, r, and r_c. Arc length at radius r over an angular incremet $d\theta$ is $r\,d\theta$.

Also see p. 90. Use of these relations along with substitution of

$$p_{\max} = 300\ psi \quad \text{and} \quad r_p = 0.5 \text{ in.}$$

into equation (1-12) yields, after numerical integration,

$$F = 212.324 \text{ lb}$$

on each of the two opposing brake pads, corresponding to a hydraulic pressure of 270.339 psi.

Plot both the torque and the required activation force against the radius of the brake pad in order to answer the present question or any future questions of increasing the brake pad diameter. The resulting torque and the associated force on the brake pads as a function of the brake pad radius is shown in Figure 14.

Doubling the torque to 1357.168 in -lb by adding another caliper doubles the fluid flow volume but maintains the same pressure. Increasing the pad diameter to 1.50 in. provides a torque of 678.584 in -lb from each pad for the required total torque of 1357.168 in.-lb. The caliper frame must be strengthened to support a force of 447.841 lb on each pad, but the hydraulic system pressure may be reduced to 253.426 psi.

Existence of a maximum torque within the boundaries of the disk is consistent with the existence of a similar maximum found for annular disk brakes and clutches. In this particular case the maximum torque, of approximately $T = 1858.4$ in-lb, occurs in the vicinity of $r_p = 1.836$ in., as found with the aid of the Trace routine supplied by Mathcad. The corresponding force is close to 1639 lb. A plot of the force as a function of the brake pad radius, as in Figure 14, shows that it too reaches a slightly larger maximum of about

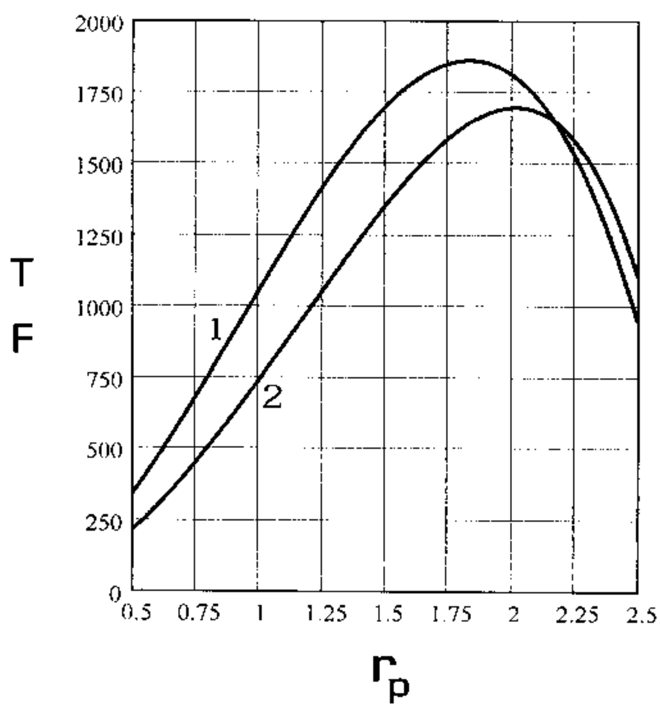

FIGURE 14 Torque in inch-pounds, curve 1, and force in pounds, curve 2, as functions of brake pad radius r_p in inches acting on a disk 11.0 inches in diameter for p_{max} = 300 psi.

1691.9 lb at a different value of r_p, at $r_p = 2.016$ in. Increased piston area will allow the line pressure to drop if the brake pad force is provided by a hydraulically driven piston whose diameter is equal to the pad diameter. The nature of this falloff in pressure with increased piston radius is illustrated in Figure 15.

Example 4.2

Estimate the torque and activation force for a caliper brake whose pad is a sector of an annular ring subtending the same angle at the center of the disk as subtended by the circular pad described in Example 4.1

According to the geometry of Figure 16, half of the subtended angle is given by

$$\frac{\theta}{2} = \sin^{-1}\frac{0.5}{5} = 0.1002 \text{ rad}$$

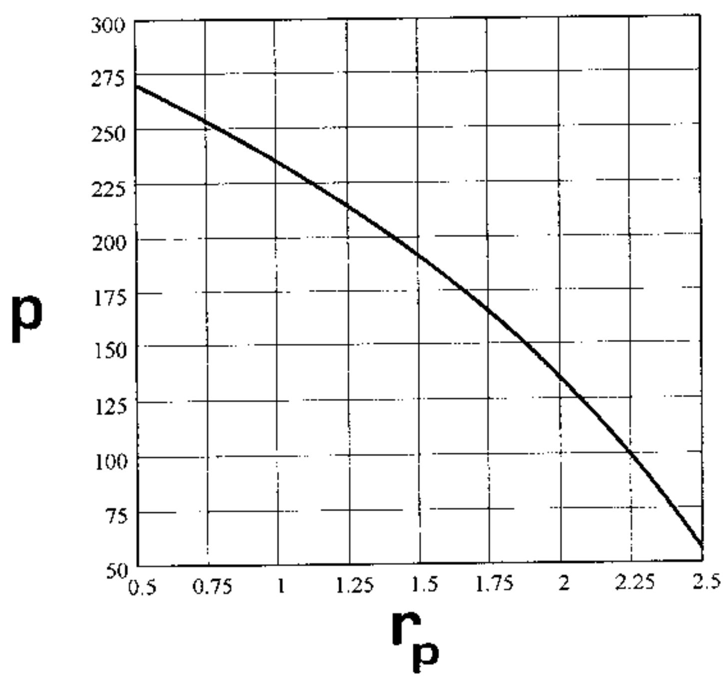

FIGURE 15 Hydraulic line pressure (psi) to a piston of radius r_p to provide the force shown in Figure 14.

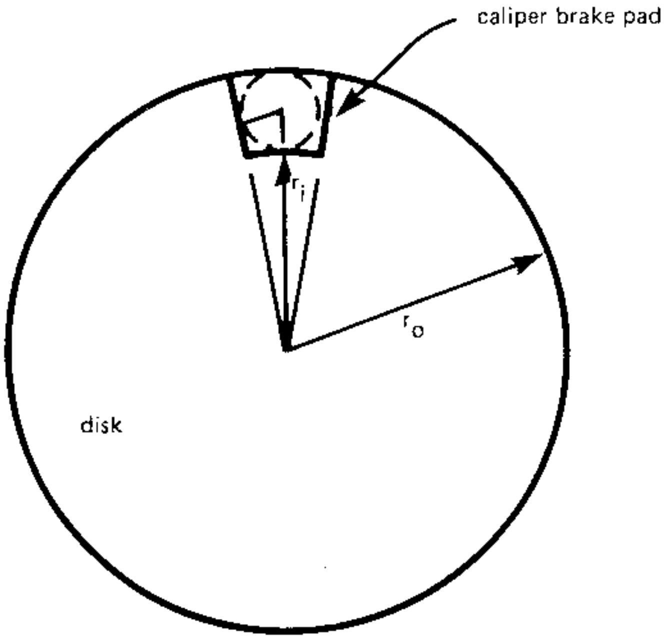

Figure 16 Caliper brake lining that is a sector of an annular ring.

So substitution into equation (1-7) yields a torque per pad of

$$T = \mu p_{\max} r_i \frac{\theta}{2}(r_o^2 - r_i^2) = 0.32\ (300)(4.50)(0.1002)(5.5^2 - 4.5^2)$$
$$= 432.862 \text{ in.} - \text{lb}$$

and substitution into equation (1-6) yields an activation force of

$$F = p_{\max} r_i \theta (r_o - r_i) = 300(4.5)(0.2004)$$
$$= 270.540 \text{ lb}$$

for a 28% increase in torque capacity and a 27% increase in the activation force.

Example 4.3

Compare the braking torque obtained from the caliper brake in Example 4.2 with that obtained from an annular, or face contact, disk brake for which r_i is determined from relation (3-4).

Substitution of $r_o = 5.50$ in. into equation (3-5) along with $p_{\max}$ and μ from Example 4.1 leads to

$$T = 19\ 313.536 \text{ in.-lb}$$

which is greater than the torque found in Example 4.2 by a factor of 44.6. A manufacturer of truck brakes has recently introduced a series of annular disk brakes to realize this advantage.

V. NOTATION

a	area (l^2)
d	diameter (l)
F	force (ml/t^2)
k	constant of proportionality
p	pressure (m/lt^2)
R, r	radius (l)
T	torque (ml^2/t^2)
δ	thickness (l)
θ	angle (1)
μ	friction coefficient (1)
ρ	radius (l)
φ	angle (1)

VI. FORMULA COLLECTION

Pressure distribution for uniform wear:

$$p = p_{\max}\frac{r}{r_i}$$

Activation force, caliper disk brake, annular sector pad:

$$F = p_{\max} r_i \theta (r_o - r_i)$$

Torque, caliper disk brake, annular sector pad:

$$T = \mu p_{\max} r_i \frac{\theta}{2}(r_D^2 - r_{-i}^2)$$

Activation force, caliper brake, circular pad:

$$F = p_{\max} r_i \int_0^{2\pi}\int_0^{r_p} \frac{\rho}{(\rho^2 + r_c^2 - 2\rho r_c \cos\theta)^{1/2}}\, d\rho\, d\theta$$

Torque, caliper brake, circular pad:

$$T = \mu p_{\max} r_i \frac{\pi}{4} d^2$$

Activation force, annular contact disk brake, uniform wear:

$$F = 2\pi p_{\max} r_i (r_o - r_i)$$

Torque, annular contact disk brake, uniform wear:

$$T = \mu p_{\max} \pi r_i (r_o^2 - r_i^2)$$

Activation force, annular contact disk brake, uniform pressure:

$$F = \pi p (r_o^2 - r_i^2)$$

Torque, annular contact disk brake, uniform pressure:

$$T = \frac{2}{3} \pi \mu p (r_o^3 - r_i^3)$$