8

Acceleration Time and Heat Dissipation Calculations

Brake and clutch design or selection from a manufacturer's catalog both require that we design or select a brake or clutch which has the torque capability necessary to stop or start either a machine or a mechanical component in a specified amount of time and also has the ability to dissipate the heat generated.

Torque capability depends, as we have found, on the particular brake or clutch design. The heat to be dissipated does not; it depends only on the machinery being stopped and is, therefore, independent of the brake or clutch used.

In this chapter we are concerned with the related problems of estimating stop or startup times and the amount of heat generated. Both problems may be analyzed in terms of the energy supplied by the driving unit, the energy transmitted to the driven unit, and the energy dissipated as heat by either the brake or clutch. Although the energy considerations are independent of the particular brake/clutch design involved, the resulting formulas may be used to compare various brake/clutch design suitability for any mechanical system.

Calculation of heat dissipation by a mechanical system involving a clutch or brake may be divided into two parts: the mechanical energy converted to heat in the clutch or brake, and the rate of transfer of this heat to the surroundings. In the remainder of this chapter we shall be concerned only with the first of these two problems. Those readers who may be concerned with the second problem as well are referred to existing books devoted to the calculation of heat transfer by conduction, convection, and radiation, along with the specific heats for common cooling fluids, including air, the methods for determining the coefficients involved, and the numerical techniques required for solving practical heat transfer problems.

I. ENERGY DISSIPATED IN BRAKING

The heat dissipated in any mechanical system is equal to the energy withdrawn from the system as it is either stopped or slowed by a brake or as it is accelerated by a clutch, plus any work done on the system during the time a brake or a clutch is being applied. This equality is the foundation of the formulas to be developed and demonstrated.

Following industry practice in the United States we shall measure heat in terms of its mechanical equivalent pound feet (foot-pounds) in old english (OE) units or in joules (newton-meters) in SI units, rather than in terms of calories or Btu. This may be converted to the temperature rise in the brake components by converting to kilocalories or Btu using the joule equivalent, which is that 1.0 kilocalorie = 4186 N-m and that 1.0 Btu = 778.26 foot-pounds and using the relation that

$$\left(\frac{\partial Q}{\partial \Theta}\right)_P = C_P$$

or

$$\Theta_2 - \Theta_1 = \frac{1}{C_P}\int_{Q_1}^{Q_2} dQ$$

where Θ represents the temperature, Θ_1 and Θ_2 are the temperatures before and after the amount of heat Q is added to the system, and C_p denotes the specific heat at constant pressure for the material involved.

The mechanical equivalent of the heat, Q_m to be dissipated is given by

$$Q_m = \mathrm{KE}_2 - \mathrm{KE}_1 + W_a \tag{1-1}$$

where KE_1 and KE_2 represent the kinetic energy of the system at the beginning and at the end of the interval during which either a brake or a clutch is applied and W_a is the work added to the system during that interval. Heat Q_m is also equal to the integral of the work done on the brakes during the braking interval, so

$$Q_m = \int_{t_1}^{t_2} \frac{dW_a}{dt}\, dt \tag{1-2}$$

This last relation, in somewhat modified form, may be used to estimate the relation between the torque to be exerted by a brake or clutch, the time the brake or clutch must act, and the heat dissipated during the time the brake or clutch acts.

Before we can equate the energy in a moving mechanical system to the work done by a brake or a clutch in changing the rotational speed of a mechanical system, we must have expressions for total energy in the system and for the work done by a brake or clutch. These matters are considered in the next two sections in that order.

II. MECHANICAL ENERGY OF REPRESENTATIVE SYSTEMS

To apply equation (1-1) we need to obtain expressions for the kinetic energy for three typical mechanical systems: geared systems; translating and rotating systems, exemplified by vehicles and conveyor belts; and systems involving a change in potential energy, as exemplified by cranes and hoists. All formulas will initially be given in terms of the physical quantities involved and will subsequently be rewritten in terms of commonly used OE and SI units in the Formula Collection at the end of the chapter.

A. Geared Systems

Whenever a geared system similar to that illustrated in Figure 1(a) involving a single gear train is to be stopped, or slowed, by a brake acting on shaft 1 rotating at speed ω_1, the kinetic energy to be dissipated in reducing the rotational speed from ω_{1a} to ω_{1b} may be expressed in terms of the gear ratios n_{21} and the moments of inertia of each rotating member as

$$KE = \frac{1}{2}(I_1 + I_2 n_{21}^2)(\omega_{1_a}^2 - \omega_{1_b}^2) \tag{2-1}$$

where I_1 is the total moment of inertia of all masses rotating with shaft 1, that is, the sum of the moments of inertia of the brake drum or disk, shaft 1 itself, and gear 1. Similarly, I_2 represents the total moment of inertia of gear 2, shaft 2, and whatever mass rotates with shaft 2. The speed ratio n_{21} is defined by

$$n_{21} = \frac{\omega_2}{\omega_1} \tag{2-2}$$

where ω_1 and ω_2 denote the rational speeds of shafts 1 and 2, respectively, at any instant. In a more complicated case, as illustrated in Figure 1(b), the kinetic energy to be dissipated in slowing or stopping the rotation is given by

$$\text{KE} = \frac{1}{2}(\omega_{1_a}^2 - \omega_{1_b}^2)(I_1 + I_2 n_{21}^2 + I_3 n_{31}^2 + I_4 n_{41}^2) \tag{2-3}$$

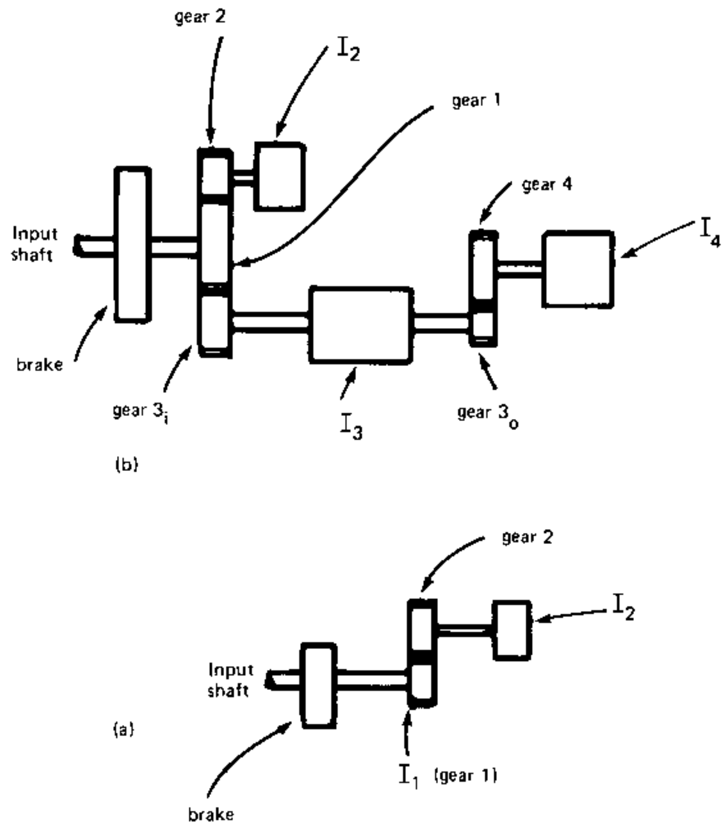

FIGURE 1 Brake and gear train schematic. Moments of inertia I_i include moments of inertia of all masses rotating with shaft *i* (i.e., gears and shaft itself).

where n_{41} may be written in terms of n_{43} and n_{31} as

$$n_{41} = n_{43}n_{31} \tag{2-3}$$

In summary, the kinetic energy to be dissipated from a geared system may be written as

$$\mathrm{KE} = \frac{1}{2}(\omega_{1_a}^2 - \omega_{1_b}^2)(I_1 + \sum_{i=2}^{k} I_i n_{i1}^2) \tag{2-4}$$

for moments of inertia I_i rotating at speeds ratios n_{i1} relative to shaft 1, where the brake is located.

For simplicity the moment of inertia of most rotating mechanical components is often given in terms of the radius of gyration r_g, which is defined by

$$I = mr_g^2 \tag{2-5}$$

where $m = W/g$ in terms of the weight of the component and the acceleration due to gravity, usually taken as 32.2 ft/sec^2 or 9.81 m/sec^2.

Returning to equation (2-1), we note that if n_{21} is less than 1, i.e., if ω_2 is less than ω_1, the contribution of I_2 to the kinetic energy is reduced by the square of n_{21}. Guided by this observation, we may conclude that it is generally advantageous to place the brake on the fastest of all of the shafts involved so that the torque requirement for the brake is reduced.

B. Combined Translation and Rotation

When translation is present, as in the case of a moving vehicle, the kinetic energy due to linear motion must also be included to obtain the total kinetic energy that must be dissipated by the brakes. In the case of a vehicle, if we take the rotation of one of the road wheels as our reference, the translational velocity is given by

$$v = r\dot{\phi} = r\omega \tag{2-6}$$

where $\dot{\phi} = d\phi/dt = \omega$ is in rad/sec so that v is in terms of the units of r per second. If the motor is not disconnected as the brakes are applied, its effect must also be included, either as a retarder, which adds to the braking effect, or as a driver, which opposes the brakes. In some vehicles and machines the motor may act as retarder for some operating conditions and as a driver in others. In either event, the contribution of the motor is usually included in the W_a term, so the energy to be dissipated in slowing from v_a to v_b may be written as

$$E = \frac{1}{2}N_w I_w \omega^2 + \frac{1}{2}mv^2 + W_a = \frac{1}{2}\left[N_w m_w \left(\frac{r_g}{r_w}\right)^2 + m\right](v_a^2 - v_b^2) + W_a \tag{2-7}$$

where m represents the total mass of the vehicle and its cargo. This relation holds if each of the N_w wheels has a mass m_w, a radius of gyration r_g, and an outside radius r_w. W_a is positive if it represents the work done by the motor during braking and negative if it represents the work dissipated either by the motor itself or by a retarder.

Although equations (2-6) and (2-7) have been discussed in terms of vehicle motion, they apply equally well to conveyors having N_w similar rollers of mass m_w, radius of gyration r_g, and radius r.

Often, the kinetic energy due to wheel rotation is negligible compared to the translational kinetic energy of the cargo, so that the rotational terms in equation (1-10) are usually omitted from the brake selection formulas found in a manufacturer's catalog.

C. Braking with Changes in Potential Energy: Cranes and Hoists

Since motion is assumed to be in the vertical direction, the energy change due to braking or clutching when a load is either raised or lowered is the sum of the changes in kinetic and potential energy and the work W_h done on the system by motors and retarders. Thus energy E may be written as

$$E = \frac{1}{2} \sum_{i=1}^{k} m_i(v_{ia}^2 - v_{ib}^2) + \frac{1}{2} \sum_{i=1}^{m} I_i(\omega_{ia}^2 - \omega_{ib}^2) + \sum_{i=1}^{n} W_i(h_{ia} - h_{ib}) + W_a \tag{2-8}$$

which is an extended version of equation (2-7) by including k masses m_i, m rotating components, each having moment of inertia I_i, n weights $\hat{W}_i$ and their elevation changes, and including nonzero values of velocity v_i, and angular rotation ω_i.

III. BRAKING AND CLUTCHING TIME AND TORQUE

Work done by a brake in slowing or stopping a mechanical system is converted to heat at the mechanical interface in friction brakes or in the inner and outer members in eddy-current, hysteresis, or magnetic particle brakes. Regardless of the particular brake design, the work done is equal to

$$W = \int_{t_1}^{t_2} \omega T \, dt = \int_{\phi_1}^{\phi_2} T \, d\phi \tag{3-1}$$

where T denotes the braking torque, $\omega = d\phi/dt$, ϕ represents the angular rotation of the active braking element (drum, disk, outer member), and t denotes time.

Preliminary design or selection of a brake is often predicted on constant torque, constant load, and therefore, constant deceleration. For this condition,

$$\omega = \omega_0 - \alpha t \tag{3-2}$$

so substitution into equation (3-1) with $t_1 = 0$ and $t_2 = t$ yields

$$\begin{aligned} W &= \int_0^t T(\omega_0 - \alpha s) ds = T(\omega_0 - \frac{\alpha t}{2})t \\ &= \Delta E = \Delta \mathrm{KE} + \Delta \mathrm{PE} + \Delta W_a \end{aligned} \tag{3-3}$$

when time is measured from that instant when the brake was first applied. If the brake is to stop or slow the rotation of a component, this work must equal the energy that must be dissipated in bringing that component to the new rotational speed. Upon substitution for αt from equation (3-2) into equation (3-3), we find that

$$W = Tt\frac{\omega_0 + \omega_1}{2} \tag{3-3a}$$

Hence equation (3-3) may be written as

$$\begin{aligned} Tt = \frac{1}{\omega_0 + \omega_1}\Bigg[& \sum_{i=1}^{k} m_i(v_{i_a}^2 - v_{i_b}^2) + \sum_{i=1}^{m} I_i(\omega_{i_a}^2 - \omega_{i_b}^2) \\ & + 2\sum_{i=1}^{n} W_{\mathrm{i}}(h_{i_a} - h_{i_b}) + 2W_{h_a}\Bigg] \end{aligned} \tag{3-4}$$

If a single rotating moment of inertia I is involved, KE $=$ (1/2) $I\,(\omega_0^2 - \omega_1^2)$ and

$$T = I\frac{\omega_0^2 - \omega_1^2}{(\omega_0 + \omega_1)t} = I\frac{\omega_0^2 - \omega_1^2}{2\omega_{\mathrm{av}}t} = \frac{I}{t}(\omega_0 - \omega_1) \tag{3-5}$$

Finally, if all rotation is to be stopped, $\omega_1 = 0$ and equation (3-5) becomes

$$T = \frac{I\omega_0}{t} \tag{3-6}$$

Moments of inertia for other than geometrically simple objects–such as a solid, homogeneous cylinder–are generally given in terms of the mass m of the rotating object when SI units are implied (i.e., kilograms) and in terms of the weight W when OE units are implied (i.e., pounds). According to this practice, I will be presented in terms of mass m and radius of gyration r_g as

$$I = mr_g^2 = \frac{W}{g}r_g^2 \tag{3-7}$$

Returning now to equation (3-6), it frequently appears in design guides in different terms. Its modified form may be found by replacing ω in rad/sec by n, the initial rotational speed in rpm, according to

$$\omega = \frac{2\pi n}{60} \tag{3-8}$$

and by replacing I by Wr^2/g according to equation (3-7). The result is

$$\begin{aligned} T &= 2\pi\frac{mr_g^2 n}{60t} \cong \frac{mr_g^2 n}{10t} \qquad \text{(SI)} \\ T &= 2\pi\frac{Wr_g^2 n}{60gt} \cong \frac{Wr_g^2 n}{307t} \cong \frac{Wr_g^2 n}{308t} \qquad \text{(OE)} \end{aligned} \tag{3-9}$$

Our previous discussion has been concerned with brake design without specific knowledge of the friction and heat dissipation characteristics of the brake as a function of the slip speed, which is the rotational speed difference between the engaging faces of the brake or clutch. When that information is known from catalog data, as represented by Figure 2, we can use it, together with the governing equation of motion, to obtain a more realistic estimate of the activation time and the heat dissipated for a viscously damped system, as shown schematically in Figure 3(a), where the viscous damping is due to the process itself, or in Figure 3(b), where the viscous damping is supplied by a retarder used to add to the energy dissipated during stopping. Except for the brake itself, Coulomb, or dry friction, damping is generally suppressed in the remainder of the system and elastic effects are generally negligible.

From this figure we find the governing equation to be

$$I\frac{d\omega}{dt} = -T(\omega) - c\omega \tag{3-10}$$

where $T(\omega)$ is negative because it acts to slow the motion (i.e., to cause $d\omega/dt$ to be negative) and where ω denotes the instantaneous angular velocity of the system as it is being stopped or retarded and I denotes the moment of inertia of all masses in the system when written in terms of the angular velocity of the shaft on which the brake acts. Integration of equation (3-10) yields

$$t_1 - t_2 = I\int_{\omega_2}^{\omega_1} \frac{d\omega}{T(\omega) + c\omega} \tag{3-11}$$

which relates the deceleration time t to:

1. The net torque $T(\omega)$, which includes the torque transferred across the brake (positive), as given by curves similar to those shown in Figure 2, as well as any torque (negative) due to motors or other drivers that may continue to supply torque while the brake is applied
2. The damping $c\omega$ supplied by a retarded (described in Chap. 11), damping in the system itself, or both.

In equation (3-11), I represents both the rotational and translational inertia, where the translational velocity is expressed in terms of ω and the appropriate radius according to $v = r\omega$.

Equation (3-11) may be used to obtain an estimate of the relation between the torque and the braking time whenever $T(\omega)$ is known from data such as that shown in Figure 2. This will be demonstrated in one of the following examples. To show that this equation produces relation (3-6) when the torque is constant, it may be integrated to give

$$t_2 - t_1 = \frac{I}{c}\ln\frac{T + c\omega_1}{T + c\omega_2} \tag{3-12}$$

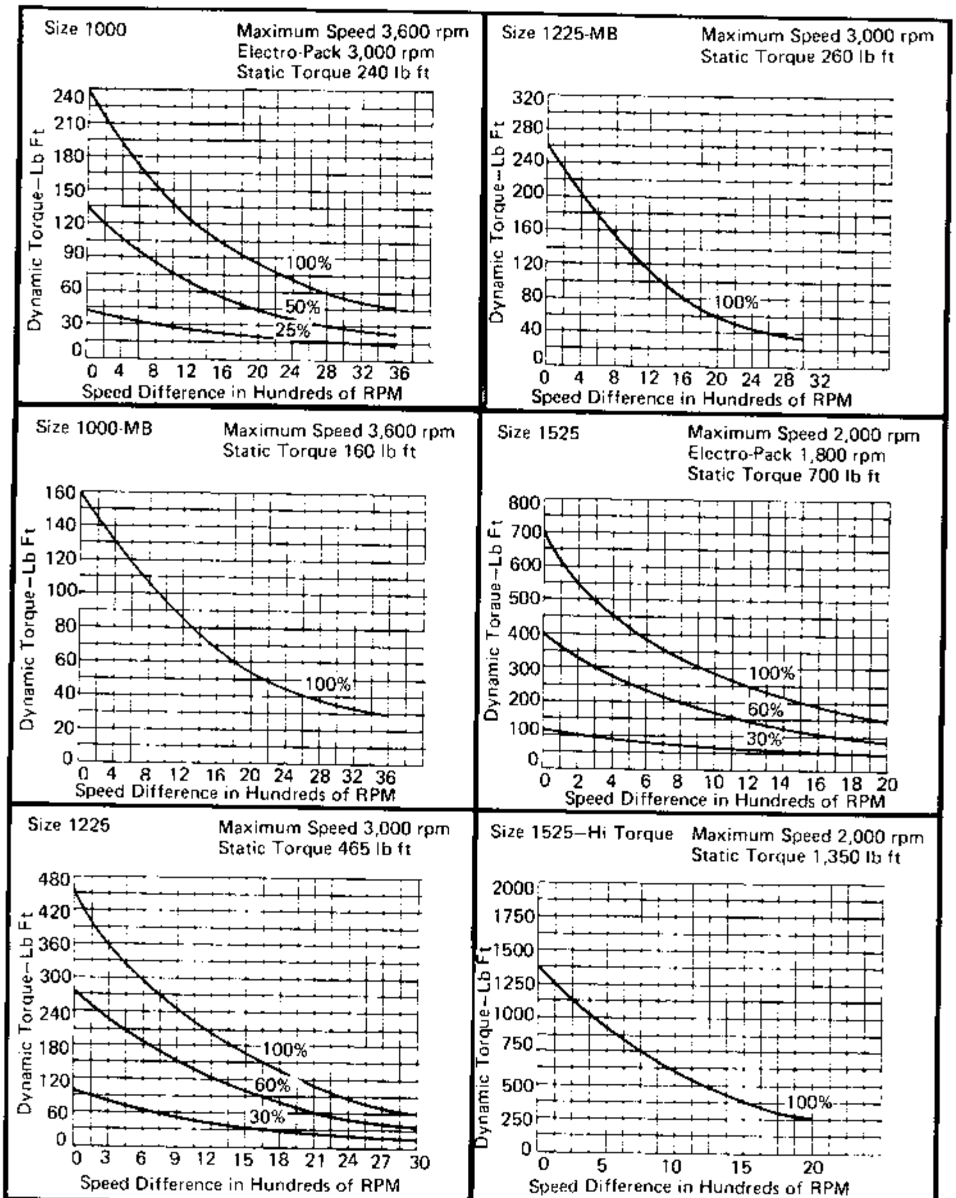

FIGURE 2 Dynamic torque as a function of the speed difference, or slip speed, between input and output shafts. (Courtesy of Warner Electric Brake & Clutch Co., South Beloit, IL.)

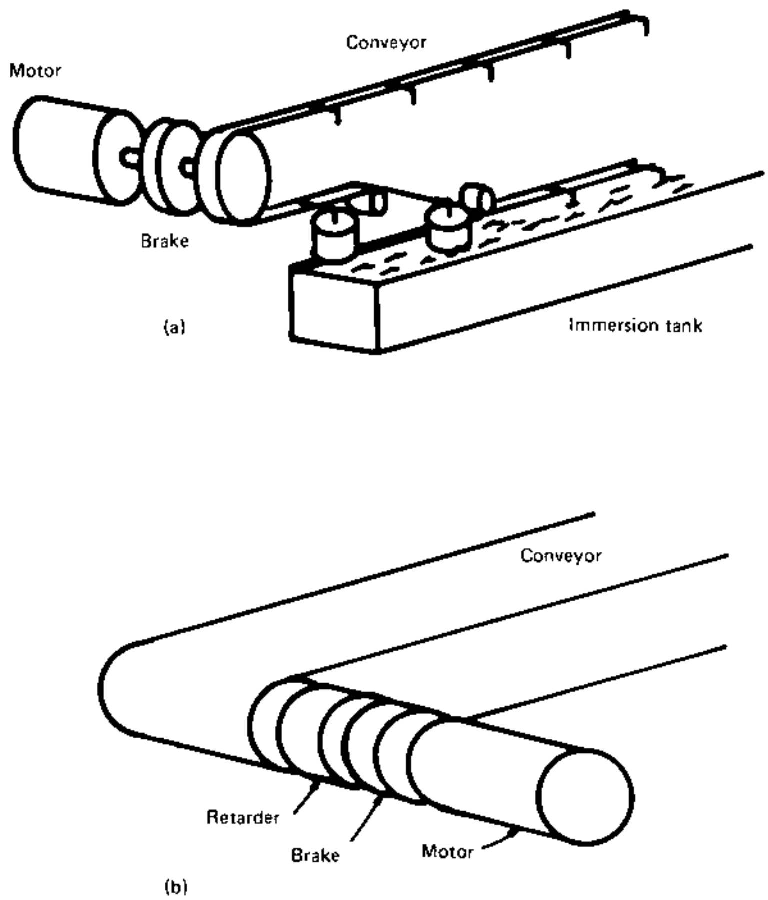

FIGURE 3 Schematic conveyor systems where viscous damping is due to (a) the process itself or (b) a retarder to aid in stopping.

which may also be written to give the required torque as

$$T = c\frac{\omega_1 - \omega_2 e^{(c/I)(t_2 - t_1)}}{e^{(c/I)(t_2 - t_1)} - 1} \tag{3-13}$$

If time is measured from the instant the brake is applied so that $t_1 = 0$ and if the system is brought to rest so that $\omega_2 = 0$, equation (3-13) simplifies to

$$T = \frac{c\omega_1}{e^{(c/I)t_2} - 1} \tag{3-14}$$

Finally, after expansion of the exponential in equation (3-14) according to

$$e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

and setting $x = ct_2/J$, we see that, if c is small enough for c^2 to be negligible compared to c, we then have

$$T = \frac{c\omega_1}{(c/I)t_2 + \ldots} \cong \frac{I\omega_1}{t_2} \tag{3-15}$$

in agreement with equation (3-6), since ω_1 and t_2 in this equation play the role of ω_0 and t in equation (3-6).

IV. CLUTCH TORQUE AND ACCELERATION TIME

Many of the formulas developed in Sections 1 and 2 apply equally well to clutch applications. Only their use differs, in that now they are used to determine the work that must be done by the clutch on the load to accelerate it to the required speed.

The equations that may be used for either a clutch or a brake are (3-4) through (3-9). In the case of a clutch, equation (3-10) is replaced by

$$I\frac{d\omega}{dt} + c\omega = T(\omega) \tag{4-1}$$

which then requires that equation (3-11) be replaced by

$$t_2 - t_1 = I\int_{\omega_1}^{\omega_2} \frac{d\omega}{T(\omega) - c\omega} \tag{4-2}$$

as the relation between the torque, the damping, and the inertia of the system, both linear and rotational. When applied to a clutch, however, the time interval $t_2 - t_1$ in equation (4-2) applies to the time interval required for the clutch to bring the load up to speed. After the load is at operating speed, $d\omega/dt$ in equation (4-2) goes to zero, so the torque $T(\omega) = c\omega$ holds as long as the operating speed and load are constant (Figure 4).

Whenever T is constant, differential equation (4-1) may be integrated to give

$$t_2 - t_1 = -\frac{I}{c}\ln\frac{T - c\omega_2}{T - c\omega_1} \tag{4-3}$$

which differs from equation (3-12) only in the algebraic sign of c. Equation (4-3) may be solved for T to get

$$T = c\frac{\omega_1 e^{(-c/I)(t_2 - t_1)} - \omega_2}{e^{(-c/I)(t_2 - t_1)} - 1} \tag{4-4}$$

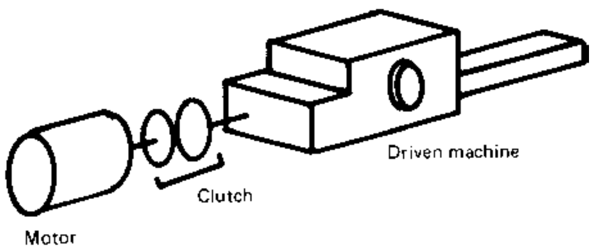

FIGURE 4 Schematic of a typical motor, clutch, machine configuration.

As a check on equation (4-4), note that if the clutch were applied at time $t_2 = 0$ when $\omega_1 = 0$, then equation (4-4) may be written as

$$T = c\frac{-\omega_2}{e^{(-c/I)t_2} - 1} \tag{4-5}$$

If we again use the series expansion for e^x given in the previous section, but with x now replaced by $-ct_2/I$ we find

$$T \cong \frac{I\omega_2}{t_2} \tag{4-6}$$

as in the case of a brake.

V. EXAMPLE 1: GRINDING WHEEL

Find the minimum torque capacity for a brake to be added to a twin-wheel motor grinder turning at 1725 rpm such that when either guard is raised the motor and two grinding wheels will stop within 0.1 sec. The moment of inertia of the motor rotor is 0.0137 slug-ft and each grinding wheel weights 10 lb and has a radius of gyration of 4.00 in.

Since all the rotating masses are on a single shaft, equation (3-6) applies, where I represents the sum of the moments of inertia for the grinding wheels and the rotor. From equation (3-7) we find that the moment of inertia for each grinding wheel is

$$I_w = \frac{w}{g}r^2 = \frac{10}{32.2}\left(\frac{4}{12}\right)^2 = 0.0345 \text{ slug-ft}^2 \tag{5-1}$$

so the total moment of inertia is

$$I = 2(0.0345) + 0.0137 = 0.0827 \text{ slug-ft}^2 \tag{5-2}$$

With the rotational speed in rad/sec given by

$$\omega = \frac{2\pi \,\text{rpm}}{60} = \frac{\pi(1725)}{30} = 180.6416 \text{ rad/sec}$$

substitution for I from equation (5-2) into equation (3-6) yields

$$T = \frac{0.0827(180.6416)}{0.1} = 149.3906 \sim 150 \text{ lb-ft} \tag{5-3}$$

as the required torque.

VI. EXAMPLE 2: CONVEYOR BRAKE

Recommend the torque requirement for a brake for the conveyor belt shown schematically in Figure 5. It is rated for a total load of 180 lb (the combined weight of all items conveyed by the conveyor). The conveyor belt weight is 50 lb, the end rollers weigh 22 lb each, and the 20 intermediate rollers weigh 4.0 lb each. The diameter of each end roller is 8.750 in. and the radius of gyration of each end roller is 4.0 in. The intermediate rollers are 2.00 in. in diameter and each has a radius of gyration of 0.8 in. The reduction ratio of the gear train is 5.488, the maximum conveyor velocity is 90 ft/min, and the brake is mounted between the driving gear motor and the gear train. The motor is disconnected from the drive line when the brake is engaged and the conveyor is to be stopped in the minimum time which will not cause the packages on the conveyor to slide along the belt. All the products to be conveyed have such a low center of gravity that tipping is not a problem. The friction coefficient is 0.30.

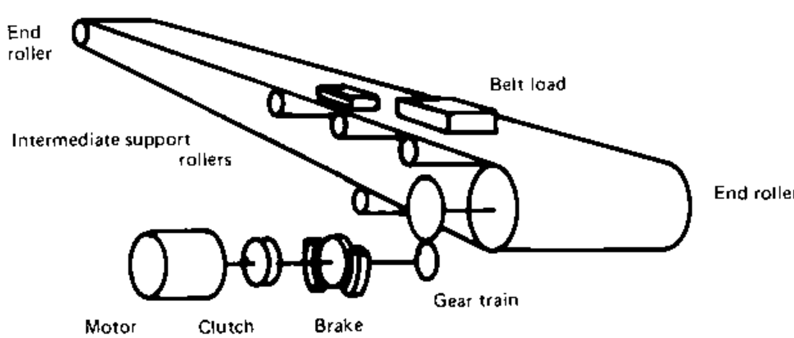

FIGURE 5 Conveyor belt schematic.

Kinetic energy due to rotation of the end and intermediate rollers, translation of the belt load, and translation of the belt itself will be considered; kinetic energy contributed by the gears and shafts in the gear train will be ignored because their combined moments of inertia is less than that of one of the intermediate rollers.

From equation (3-4) we find that the governing equation for a conveyor with k_J rotating masses and k_m translating masses is given by

$$T = \frac{\omega_0}{t}\left[\sum_{i=1}^{k_J} I_i n_i^2 + \left(\frac{d}{2}n_1\right)^2 \sum_{i=1}^{k_m} m_i\right] \tag{6-1}$$

where n_i is the ratio of rotational speed of roller i to the rotational speed of the shaft on which the brake is mounted and d is the diameter of the drive roller, whose speed ratio is represented by n_1

From equation (3-7) we find the moment of inertia of an end roller to be

$$I_e = mr_g^2 = \frac{22}{32.2}\left(\frac{4}{12}\right)^2 = 0.0759 \text{ slug-ft}^2 \tag{6-2}$$

and the moment of inertia of an intermediate roller to be

$$I_i = \frac{4.1}{32.2}\left(\frac{0.8}{12}\right)^2 = 0.0006 \text{ slug-ft}^2 \tag{6-3}$$

The rotational speed of the end rollers may be found from equation (2-6) to be

$$\omega = \frac{90}{60}\left(\frac{12}{4.375}\right) = 4.1143\,\text{rad/sec} \tag{6-4}$$

from which it follows that the speed of the input shaft to the gear train is

$$\omega_b = \omega n_1 = 22.5792 \text{ rad/sec} \tag{6-5}$$

for $n_1 = 5.488$. Since the intermediate rollers that support the belt along its length have radii of 1.00 in., their angular velocity is 18 rad/sec for an effective speed reduction factor of 1.254 relative to the input shaft to the gear train.

Since the belt moves with the same velocity as the product being conveyed, we can group them together so that $k_m = 1 + 1 = 2$. The two end rollers and the 20 intermediate rollers give $k_J = 20 + 2 = 22$. With all masses and moments of inertia known, we may substitute into equation (6-1) once we select a stopping time t. To find the minimum stopping time without slip between the product and the conveyor belt, recall that the stopping force of the product is μmg, so the maximum deceleration becomes μg. If this force is constant, the stopping time may be found from $t = v/a = 90/[60(0.3)32.2] = 0.1553$ sec. Substitution into equation (6-4) yields

$$T = \frac{22.579}{0.1553}\left[\frac{2(0.0759)}{5.488^2} + \frac{20(0.000556)}{1.254^2} + \frac{230}{32.2}\,\frac{4.375^2}{(12^2)(5.488^2)}\right]$$
$$= 6.344 \text{ lb-ft} \qquad (6\text{-}6)$$
$$= 76.130 \text{ lb-in.}$$

where $n_i = 1/5.488$ for the end rollers and $n_i = 1/1.254$ for the intermediate rollers.

If the brake had been mounted on either of the end roller shafts, equation (6-6) would have been replaced by

$$T = \frac{4.114}{0.1553}\left[2(0.0759) + 20(0.000556)(4.375)^2 + \frac{230}{32.2}\left(\frac{4.375}{12}\right)^2\right]$$
$$= 34.811 \text{ lb-ft} \qquad (6\text{-}7)$$

and the braking torque requirement would have been $n = 5.488$ times larger than that found by equation (6-6). This comparison is an example of the general rule that the brake should usually be placed in the faster shaft.

VII. EXAMPLE 3: ROTARY KILN

The curves in Figure 6 clearly imply that efficient use of a clutch by reducing the power loss due to heat generation, along with wear, requires that the speeds of its input and output shafts should be nearly equal. Accordingly, depending upon the power source (electric or hydraulic motor, turbine, or internal combustion engine), a clutch may be used to change gear ratios, to change from one power source to another when the speeds are nearly equal, or to disconnect the power source before braking.

This example will consider a load that is essentially rotational in order to concentrate on clutch and brake selection when dynamic torque and brake heating curves are available. Both clutch and brake analyses will display some of the calculation involved when the speeds of the input and output shafts are not almost equal.

A rotary kiln is to be driven by a 15-hp three-phase motor operating at 870 rpm and rated to deliver a torque of 240 lb-ft with a K factor (overload factor for starting) of 2.64. The motor, clutch, gear train with a 28.4 speed-reduction ratio, and rotary kiln are arranged as shown in Figure 7. The overall damping coefficient is approximately 0.10. The starting moment of inertia of

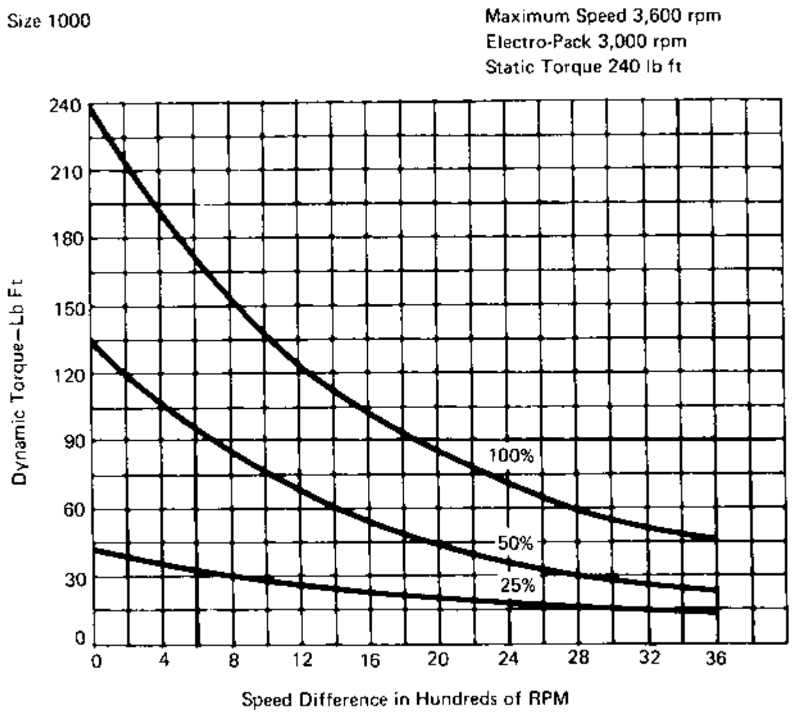

FIGURE 6 Dynamic torque as a function of the speed difference between input and output shafts. (Courtesy of Warner Electric Brake & Clutch Co., South Beloit, IL.)

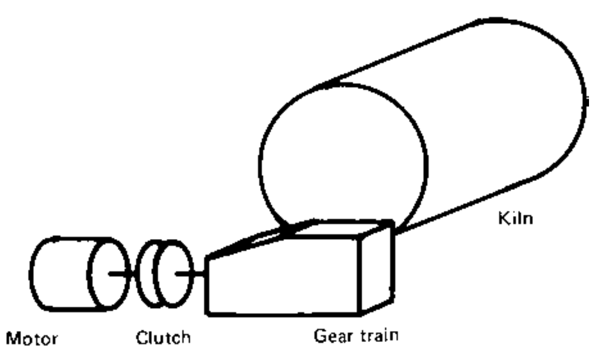

FIGURE 7 Schematic of motor, clutch, gear train, and kiln.

the kiln is equivalent to a weight of 31,832 lb and a radius of gyration of 2.8 ft, the clutch characteristics are given in Figure 6. A brake with similar characteristics will be used to stop kiln rotation.

The moments of inertia of the gears in the gear train will be neglected for simplicity. They will be considered for a different gear train in a subsequent example.

Conversion from horsepower (hp) and revolutions per minute (n) to torque (T) in ft-lb according to

$$T = \frac{(16,500\ \text{hp})K}{\pi n}$$

yields

$$T = \frac{(16,500)15(2.64)}{870\ \pi} = 239.0617\ \text{lb-ft}$$

as the required starting torque

Upon calculating the moment of inertia of the kiln according to equation (3-7), we find

$$I = \frac{31,832}{32.2}(2.8)^2 = 7750.4\ \text{slug-ft}^2$$

From equation (2-1), the equivalent moment of inertia at the clutch is given by

$$In_{21}^2 = \frac{7750.4}{28.4^2} = 9.6092\ \ \text{slug-ft}^2$$

According to equation (3-6), the approximate time for the motor to bring the kiln up to speed is

$$t = \frac{I\omega_o}{240} = \frac{9.6092}{240}\left(\frac{870\pi}{60}\right) = 1.82\ \text{sec}$$

For a more precise calculation of the time to get up to speed, we may turn to equation (4-2), which requires the input data shown in Table 1, as read and calculated from the 100% speed difference curve in Figure 6.

Upon turning to a TK Solver routine* for the numerical integration of a integral whose integrand is given as a series of data points, we find that

*Enter L in the Type column after entering a name (i.e., time) in the Function Sheet and enter the data in Table 1 in the List Function Sheet. On the Rule Sheet type "value = integral ('time, x_1, x_2" where x_1 and x_2 are the lower and upper limits of integration, respectively.

Table 1 Input Data and Intermediate Values for Integrands in Equations (3-11) and (4-2)

Δn (rpm)	ω (rad/sec)	$c\omega$ (lb-ft)	$T(\omega)$ (lb-ft)	$T(\omega) - c\omega$ (lb-ft)	$T(\omega) + c\omega$ (lb-ft)	$\frac{1000}{T(\omega) - c\omega}$	$\frac{1000}{T(\omega) + c\omega}$
870	0	0	145	145.0000	145.0000	6.8966	6.8966
800	7.3304	0.73304	153	152.2670	153.7333	6.5674	6.5048
700	17.8024	1.78024	160	158.2198	161.7802	6.3203	6.1812
600	28.2743	2.82743	170	167.1726	172.8274	5.9818	5.7861
500	38.7463	3.87463	180	176.1254	183.8763	5.6778	5.4385
400	49.2183	4.92183	190	185.0782	194.9218	5.4031	5.1303
300	59.6903	5.96903	202	196.0310	207.9690	5.1020	4.8084
200	70.1622	7.01622	213	205.9838	220.0162	4.8548	4.5451
100	80.6342	8.06342	225	216.9366	233.0643	4.6096	4.2907
0	91.1062	9.11062	240	230.8894	249.11062	4.3311	4.0143

Note: Entries in the two right-hand-most columns have been multiplied by 1000 to avoid including 10^{-3} after each entry.

evaluation of equation (3-11) for a brake and equation (4-2) for a clutch gives start-up times $\tau = (t_2 - t_1)$ of:

$\tau = 4.6396 \rightarrow 4.6$ seconds for start-up

$\tau = 4.8397 \rightarrow 4.8$ seconds for stopping

The difference between these values and the time of 1.8 seconds given by equation (3-6) is, of course, largely due to the omission of damping in equation (3-6). Heat transferred to the surroundings for the surface temperatures shown in Figure 8 may be read directly from these curves by interpolating for surface temperatures between 250°F and 300°F.

Heat dissipation in the absence of curves similar to Figure 8 may be estimated from the work dissipated according to the relation

$$W(\omega) = T_0\,\omega_i\,\tau - I \cdot \int_0^{\omega_i} \frac{T(\omega)\,\omega}{T(\omega) - c\,\omega}\,d\omega \tag{7-1}$$

where $T_0\omega_i\tau$ represents the work done on the clutch and its load by the input shaft rotating at angular velocity ω_i, where τ denotes the time required for the load speed to reach ω_i, and where the integral represents the work done by the clutch in accelerating the load. Heat generated per cycle by the clutch determines the cooling method to be used so that the

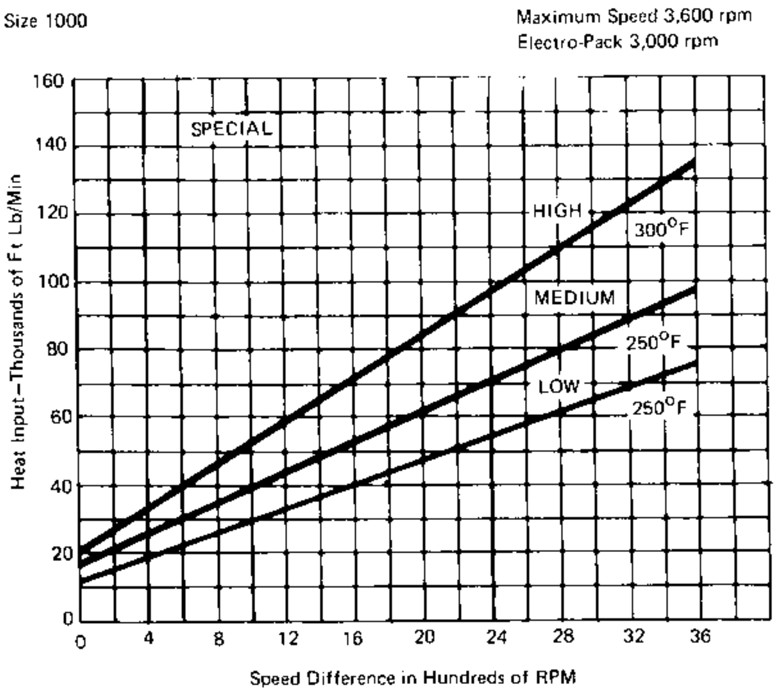

Figure 8 Heat input that can be transferred by radiation and convection for the surface temperatures shown in the rotational speed range of the rotating element. (*Speed difference* refers to the speed of the rotating element relative to the stationary element.) (Courtesy of Warner Electric Brake & Clutch Co., South Beloit, IL.)

heat can be transferred from the clutch per cycle for the expected ambient temperature.

Use of 31,832 lb for the average gross weight of the kiln instead of 31,800 lb may be justified by noting that it takes no more keystrokes to enter nonzero values. Carrying four digits to the right of the decimal point simply gives a more precise basis for the final round-off of the result to practical values than may be had when carrying fewer digits.

VIII. EXAMPLE 4: CRANE

Select a brake for a crane rated for a maximum load of 2800 kg as limited by the load rating for the 19 × 7 nonrotating wire rope used. The rope diameter is

21 mm, its weight is 2.069 kg/m, and the maximum drop of the cable is 30 m. The grooved cable drum is 0.50 m in diameter at the base of the grooves, weighs 696 kg, will accept 20 turns of wire rope, and has a radius of gyration of 0.23 m. The drum is driven by a gear train, illustrated in Figure 9 for which the gear data are as follows:

Gear number	Pitch diameter (m)	Radius of gyration (m)	Mass (k)
1	2.00	0.81	1278
2	0.40	0.17	276
3	0.80	0.32	721
4	0.25	0.09	231

Four turns remain on the drum when the load is 30 m below the top of the crane. The rope length from the drum to the top of the crane, Figure 10 is 21 m. Motor speed is 485 rpm, and a descending maximum load is to be stopped within 2.00 seconds after the brake is applied. The motor is disengaged by means of a clutch immediately before the brake is applied.

We may begin by calculating the angular velocity of each of the gears, their gear ratios relative to the input shaft on which the brake is mounted,

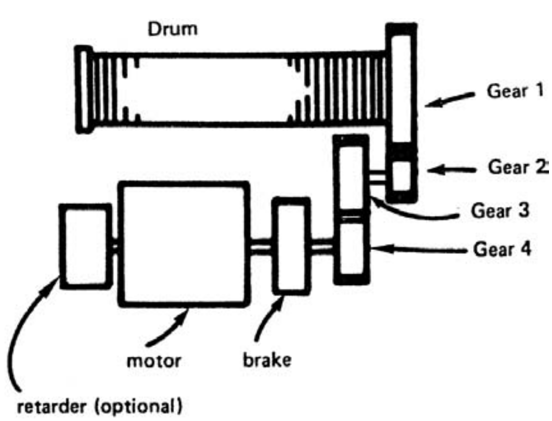

FIGURE 9 Schematic of motor, gear train, and drum for a typical crane. A retarder, if used, may be added at either end of the motor shaft.

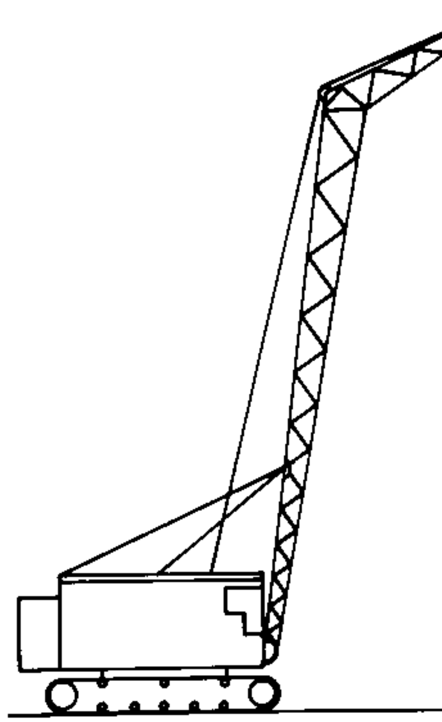

Figure 10 Sketch of a hoist showing the wire rope extension beyond the drum.

shaft 4, and the polar moment of inertia of each gear. The results are as follows:

Gear number	Speed ratio	Speed (rad/sec)	Polar moment of inertia (kg-m^2)
1	1:16.0	3.1743	838.496
2	1:3.2	15.8716	7.976
3	1:3.2	15.8716	73.830
4	1:1.0	50.7891	1.871

where all speed ratios have been calculated relative to the motor speed.

The dimensions from the drum to the top of the crane and the maximum drop include that portion of the cable over the pulley, or sheave, at the top of the crane, for a total cable length of 63 meters. Hence, the input data for the following formulas are:

$m_L = 2800\,\text{kg}$	$d_r = 0.021\,\text{m}$	$\gamma_r = 2.069\,\text{kg/m}$	$g = 9.8067\,\text{m/sec}^2$
$d_d = 0.50\,\text{m}$	$m_d = 696\,\text{kg}$	$r_{dg} = 0.23\ \text{m}$	$n = 485\ \text{rpm}$
$N_t = 4$ turns	$l_o = 63\ \text{m}$	$t = 2.0\,\text{sec}$	$y_o = 30\,\text{m}$

The moment of inertia of the drum is given by

$$I_d = m_d r_{dg}^2 = 36.818\ kg\text{-}m^2 \tag{8-1}$$

The angular velocity of the drum and the mass of the rope are given by

$$\omega_m = \pi \frac{485}{30} \qquad m_r = [4\pi(d_d + d_r) + l_o]\gamma_i \tag{8-2}$$

respectively, to give $\omega_m = 50.789$ rad/sec and $m_r = 143.893$ kg. Rope velocity is given by

$$v_r = \frac{d_d + d_r}{2}\frac{\omega_m}{16} = 0.827\ \text{m/sec} \tag{8-3}$$

Next, estimate the distance the load will descend during its deceleration due to braking by integrating $a = d^2x/dt^2$ twice, subject to the initial conditions that $x(0) = 0$ and $dx(0)/dt = 0$ under the assumption that the deceleration is constant. From the resulting formulas,

$$v = at \qquad \text{and} \qquad s = \tfrac{1}{2}\, at^2$$

it follows that if the load is to stop 2.0 seconds after the brake is applied, the values of acceleration a and distance s must be

$$a = 0.413\ \text{m/sec}^2 \qquad \text{and} \qquad s = 0.827\ \text{m}$$

Now that distance s is known, we can calculate the potential energy change for the rope as it extends from $y_1 = y_o - s$ to $y_2 = y_o$ by integrating over this length to get

$$\text{PE} = \gamma_r \int_{y_1}^{y_2} y\,dy = \frac{\gamma_r g}{2}\left[y_o^2 - (y_o - s)^2\right] = 496.405\ \text{N-m} \tag{8-4}$$

The potential energy for the load is given by

$$\text{PE}_L = m_L g s = 22{,}705.916\ \text{N-m} \tag{8-5}$$

The kinetic energies may be found from

$$n_{14} = \frac{1}{16} \qquad n_{24} = \frac{1}{3.2} \qquad n_{34} = \frac{1}{3.2}$$

$$m_1 = 1278 \qquad m_2 = 276 \qquad m_3 = 721 \qquad m_4 = 231$$

$$r_{1g} = 0.81 \qquad r_{2g} = 0.17 \qquad r_{3g} = 0.32 \qquad r_{4g} = 0.09$$

$$I_{g1} = m_1 r_{1g}^2 \qquad I_{g2} = m_2 r_{2g}^2 \qquad I_{g3} = m_3 r_{3g}^2 \qquad I_{g4} = m_4 r_{4g}^2 \qquad I_d = m_d r_{dg}^2$$

$$\text{ke}_d = \frac{1}{2} I_d n_{14}^2 \qquad \text{ke}_{g1} = \frac{1}{2} I_{g1} n_{14}^2 \qquad \text{ke}_{g2} = \frac{1}{2} I_{g2} n_{24}^2 \qquad \text{ke}_{g3} = \frac{1}{2} I_{g3} n_{34}^2$$

$$\text{ke}_{g4} = \frac{1}{2} I_{g4}$$

where m_1 through m_4 are the masses of gears 1 through 4, respectively, and r_{1g} through r_{4g} are their respective radii of gyration. Thus,

$$\text{KE}_d = \text{ke}_d \omega_m^2 = 185.5 \text{ N-m} \qquad \text{KE}_L = \frac{1}{2} m_L v_c^2 = 957.3 \text{ N-m}$$

$$\text{KE}_r = \frac{1}{2} m_r v_c^2 = 49.2 \text{ N-m} \qquad \text{KE}_{g1} = \text{ke}_{g1} \omega_m^2 = 4224.5 \text{ N-m}$$

$$\text{KE}_{g2} = \text{ke}_{g2} \omega_m^2 = 1004.7 \text{ N-m} \qquad \text{KE}_{g3} = \text{ke}_{g3} \omega_m^2 = 9299.2 \text{ N-m}$$

$$\text{KE}_{g4} = \text{ke}_{g4} \omega_m^2 = 2413.3 \text{ N-m}$$

Addition of these gives

$$\begin{aligned} \text{KE} &= (\text{ke}_{g1} + \text{ke}_{g2} + \text{ke}_{g3} + \text{ke}_{g4} + \text{ke}_d)\omega_m^2 + \text{KE}_L + \text{KE}_r \\ &= 18{,}133.6 \text{ N-m} \end{aligned}$$

So upon adding this to the total potential energy of

$$\text{PE} = 23,202.3 \text{ N-m}$$

the torque required may be found from

$$T_o = \frac{\text{PE} + \text{KE}}{(\omega_m/2)t} = 813.9 \quad \text{N-m} \tag{8-6}$$

in which the average motor speed during braking was taken to be $\omega_m/2$.

The braking requirement of 813.9 N-m may be met by using a variety of brakes, such as band, external linear, annular caliper, and annular disk brakes. To choose among these, recall equation (1-10) from Chapter 1, equation (2-1) from Chapter 4, and equations (1-7) and (3-5) from Chapter 5 corresponding to the foregoing order, and let the internal radius, r_o, for both the annular caliper and annular disk be given by equation. Accordingly, evaluate the formulas

$$T = p_{\max} w r_o^2 (1 - e^{-\mu\phi}) \tag{8-7}$$

for a band brake,

$$T = 2\mu p_{\max} w r_o^2 \sin\left(\frac{\phi_o}{2}\right) \tag{8-8}$$

for either two opposing internal or external linearly acting brake shoes,

$$T = 2\mu p_{\max} \frac{r_o^3}{3\sqrt{3}} \phi_o \tag{8-9}$$

for two opposing disc brake pads, each subtending angle φ_o, and

$$T = 4\pi\mu p_{\max} \frac{r_o^3}{3\sqrt{3}} \tag{8-10}$$

for two complete annular pads in which $\varphi_o = 2\pi$ in equation (8-9).

We shall also consider an external pivoted drum brake with a leading and trailing shoe that may be evaluated by invoking the program used in Chapter 3. In all of these calculations assume a friction coefficient of 0.3, and set the width for the band, the linearly acting drum brake, and the externally pivoted brake to 5 cm. Limit the maximum lining pressure for the band brake and for the externally pivoted brake to 2.0 MPa, and limit the pressure for the other linings to 3.0 MPa, which may be either formed or solid. Lining pressure for the externally pivoted brake was taken to be 2.0 MPa, merely for comparison with the band brake.

Figure 11(a) shows the torque capacity in newton-meters as a function of angle φ subtended by each shoe for a drum diameter of 300 mm, and Figure 11(b) shows the torque capacity in newton-meters for band, linearly acting, caliper, and annular brakes as a function of the drum or disc diameter in millimeters.

Although the linearly acting drum brake is clearly more effective than the other brakes shown in Figure 11(b), it and all of the other three brakes in that figure require more hardware than does the band brake. Therefore, select the band brake, because it can provide the necessary torque capability with mechanical simplicity.

External dual-shoe drum brakes are the next simplest. Increasing the maximum lining pressure to 3.0 MPa for an externally pivoted dual-shoe brake allows the drum diameter to be reduced to 170 mm and the radial distance to the shoe pivot to be reduced to 100 mm, from the 150 mm associated with Figure 11(a), to get a torque vs. angle curve similar in shape and magnitude to that in Figure 11(a). Thus, either an externally pivoted

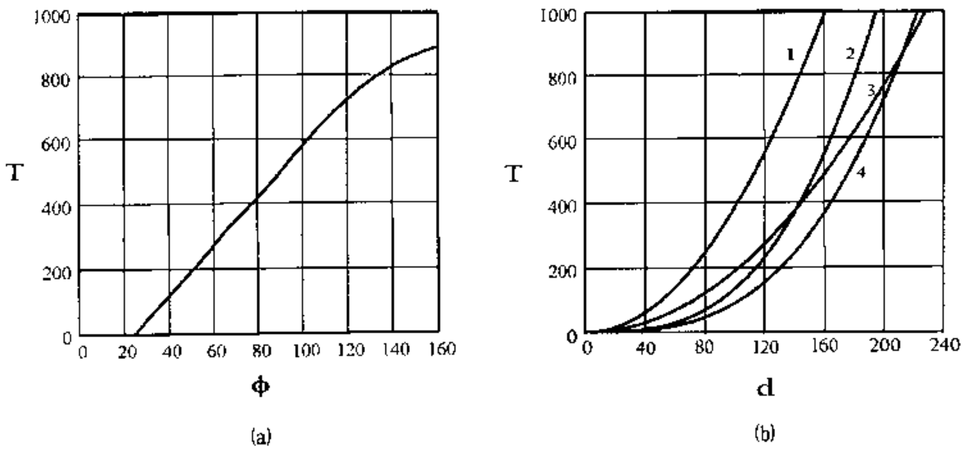

FIGURE 11 (a) Torque (N-m) as a function of shoe subtended angle for a drum diameter of 200 mm. (b) Torque (N-m) as a function of disk or drum diameter *d* (mm) (1) a linearly acting drum brake, (2) an annular disk brake, (3) a band brake, (4) a caliper brake.

dual-shoe brake or an external linearly acting dual-shoe brake might be recommended if space considerations are more important than mechanical simplicity.

IX. EXAMPLE 5: MAGNETIC PARTICLE OR HYSTERESIS BRAKE DYNAMOMETER

The dynamometer application is represented schematically in Figure 12, wherein either a magnetic particle or hysteresis clutch is used. Torque is independent of rotational speed throughout the range of a magnetic particle clutch and is independent of rotational speed to within about 0.003% per rpm for a hysteresis clutch for rotational speeds from 0 to a speed that is dependent on the cooling provided, as illustrated in Figure 13. Since the torque acts continuously, brake heating is expressed in terms of the dissipated power in units of watts, given by

$$P_d = \begin{cases} T_\omega = \dfrac{\pi T n}{30} & \text{(SI units)} \\ \dfrac{\pi T n}{22.126.5} & \text{(OE units)} \end{cases} \tag{9-1}$$

where P_d is in watts, often termed slip watts, ω is in rad/sec, and n is in rev/min. Input torque T is in kg-m in the SI system and in lb-ft in the old English system

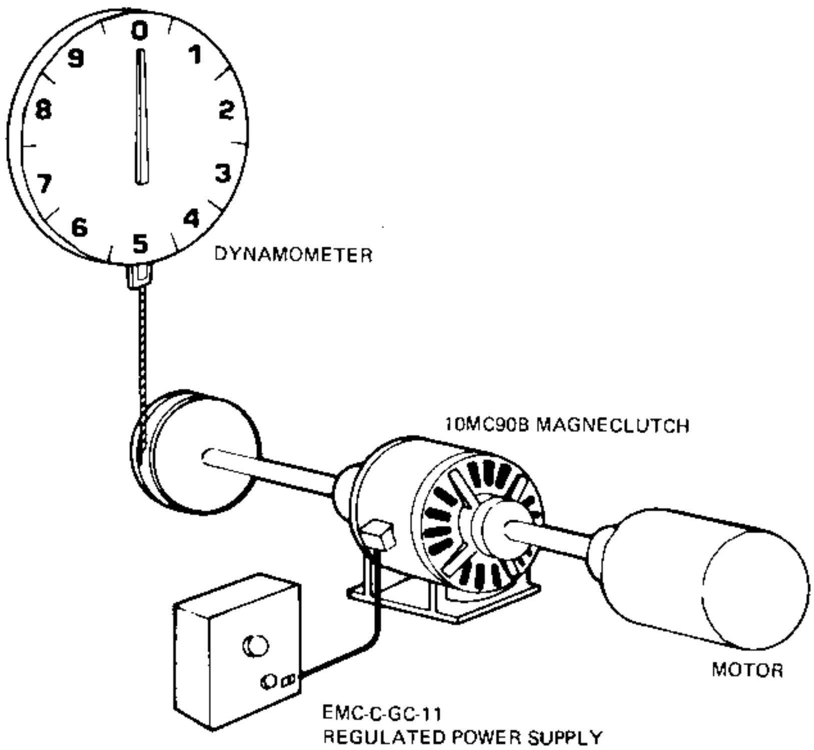

FIGURE 12 Magnetic particle brake dynamometer. (Courtesy of Sperry Electro Components, Durham, NC.)

of units. A typical slip watts–rpm curve showing the heat dissipation capability of a magnetic particle clutch is presented in Figure 14.

Equation (9-1) can also be applied to a clutch if n is redefined to be the difference in rpm between the speed of the input and output shafts. It also gives the power transmitted if T is redefined as the output torque and n is redefined as the speed of the output shaft.

Calculation of the power dissipated by either magnetic particle or hysteresis brakes is very simple. For example, consider a dynamometer as shown in Figure 12, where the motor runs at 890 rpm and the force reads 429.182 N for a 0.500-m lever arm. The torque is $431.342 \times 0.500 = 215.671$ N-m and the power dissipated, according to the first of equations (9-1), is

$$P_d = \frac{215.671(890)\pi}{30} = 20.101 \text{ kW}$$

which is, of course, equal to the power delivered by the motor at 890 rpm.

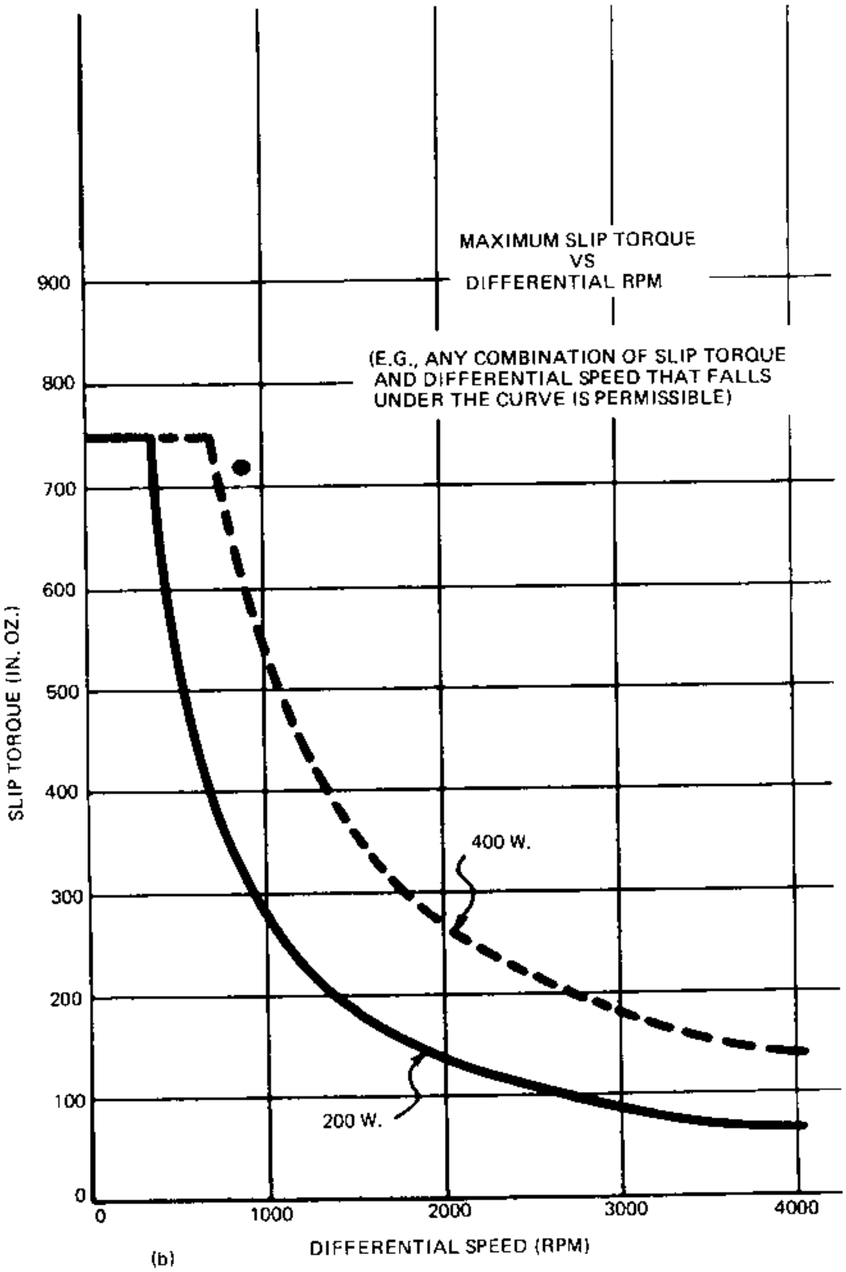

FIGURE 13 Representative torque-slip speed curve for hysteresis brake showing the effect of improved cooling. (Courtesy of General Electro-Mechanical Corp., Buffalo, NY.)

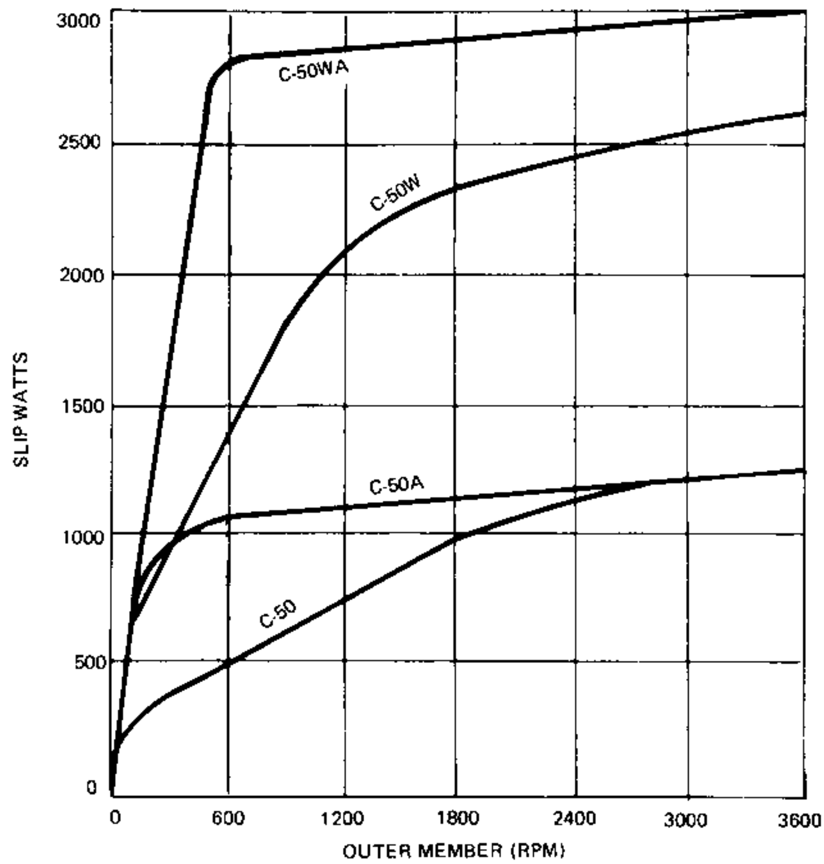

FIGURE 14 Typical slip watts-rpm curve for a magnetic particle clutch for various means of cooling. A force air; W, circulated water; otherwise, radiation and convection to ambient air. (Courtesy of Magnetic Power Systems, Inc., Fenton, MO.)

X. EXAMPLE 6: TENSION CONTROL

Tension control is often used in manufacturing processes that involve drawing, coating, slitting, printing, and winding of sheet material and in the formation of wires and filaments. Selection of magnetic particle or hysteresis brakes for such an application is usually based on the torque required and the brake's steady-state power dissipation capacity because the braking is generally continuous in these operations.

Suppose we are to select brakes to be used for the two draw rolls shown in Figure 15(a). The drive motor provides 1.5 kW at 950 rpm to drive rollers

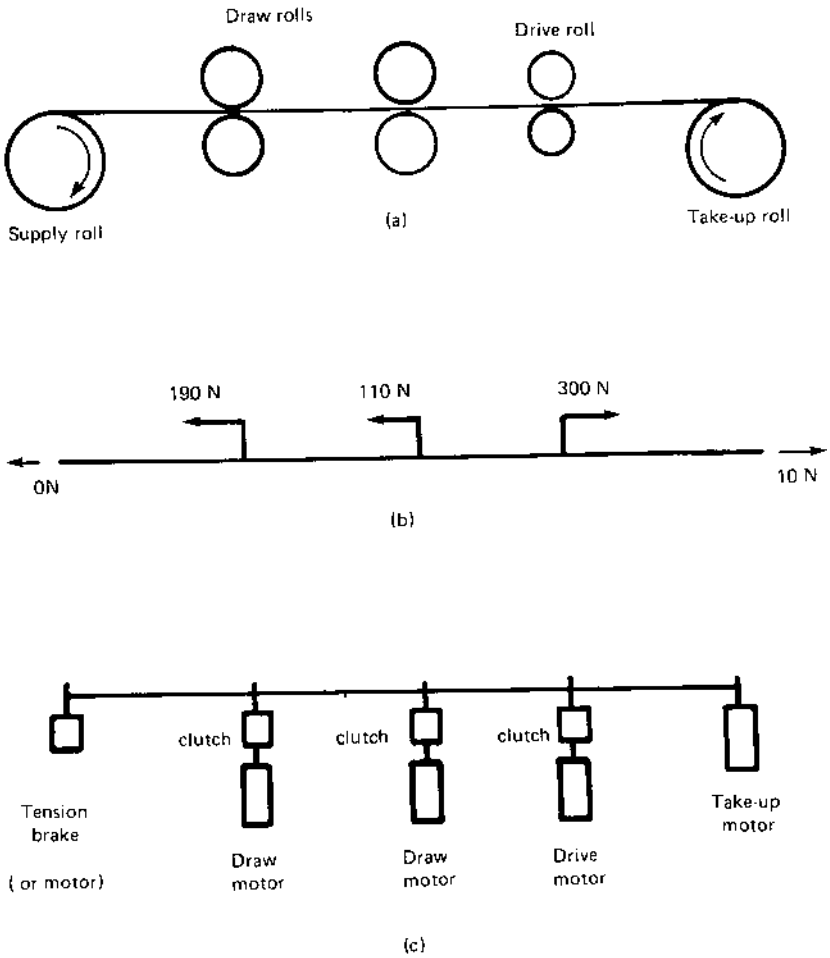

Figure 15 (a) Schematic of a tension-control drawing process; (b) the corresponding diagram of the forces acting on the web; (c) draw and tension motors used in braking.

100 mm in diameter. Draw rollers are 130 mm in diameter and web tension provided by the take-up roll motor is 10 N. Lab test results are available to aid in estimating the elongation of the web due to drawing.

From the force diagram shown in Figure 15(b) we observe that the drive rollers rotate with the speed of the web and that although both sets of draw rollers rotate in the same direction as the drive rollers, the torque on these rollers opposes the motion of the web. The clutch at the drive rolls may be selected on the basis of torque alone because it will experience only slight heating due to coll losses as long as the web moves at the design velocity.

Web velocity at the drive rolls may be calculated from

$$v = \pi dn = \pi(0.1)(950) = 298.451 \text{ m/min}$$

Based on lab results we estimate that web velocity at draw roller 1 will be 297.141 m/min, corresponding to a rotational speed of

$$n_1 = \frac{v}{\pi d} = \frac{297.141}{\pi(0.130)} = 727.561 \text{ rpm}$$

The torque requirement at draw rolls 1 is given by

$$T = rF = 0.065(110) = 7.150 \text{ N-m}$$

Cooling requirements at the brakes may be greatly reduced if the differential speed at the brakes is reduced by installing them between the draw rolls and a motor that is controlled to resist rotational speeds greater than a specified value, as illustrated in Figure 18. If these motors are to operate at 950 rpm, the power dissipated at the draw rolls may be estimated from equations (9-1), with the rotational speed replaced by the differential speed, as

$$P_d = \frac{\pi T(n_r - n_1)}{30} \tag{10-1}$$

At draw rolls 1, therefore,

$$P_{d1} = \frac{\pi(7.150)(950.000 - 727.561)}{30} = 166.550 \text{ slip watts}$$

At draw rolls 2, the web velocity is estimated to be 296.920 m/min, so $n = 296.920/0.130\pi = 727.020$ rpm, which implies that the power dissipated by the brake at draw rolls 2 may be

$$P_{d2} = \frac{\pi(12.350)(950.000 - 727.020)}{30} = 288.378 \text{ slip watts}$$

XI. EXAMPLE 7: TORQUE AND SPEED CONTROL

Control of both output torque and output speed for a constant input speed may be accomplished with a magnetic particle, eddy-current, or hysteresis clutch, simply by controlling the coil current. This capability allows us to drive a machine using a motor whose torque-speed curve would otherwise be incompatible with that of the prime mover if they were directly connected.

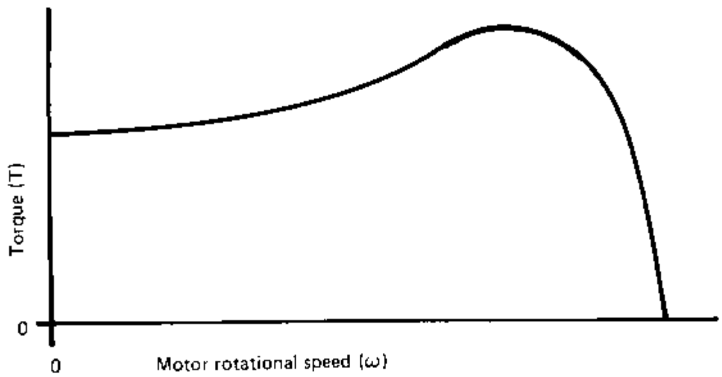

FIGURE 16 Torque-speed curve for the prime mover, an electric motor.

Supposed, for example, that the prime mover is an electric motor with the torque-speed curve shown in Figure 16 and that the desired torque-speed curve for the load is that shown in Figure 17.

In this example an eddy-current clutch will be selected because the design considerations in its use are somewhat more complicated than those associated with either a magnetic particle or a hysterests clutch.

To transfer power from the motor to the load, the eddy-current clutch must have a torque curve at 100% excitation whose maximum torque equals or exceeds the maximum torque required by the load, as illustrated in Figure 18. Selection from eddy-current clutches with curves represented by curve c, d, or e in Figure 18(a) depends on the degree of control required and the precision required for the maximum torque between points 1 and 2 in Figure 18(b).

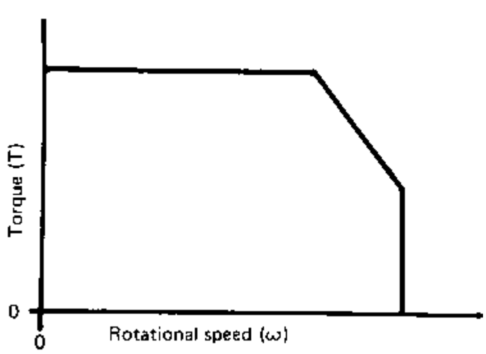

FIGURE 17 Torque-speed curve for the load.

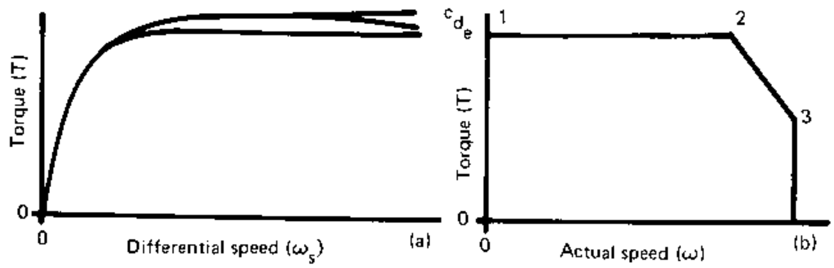

FIGURE 18 Typical eddy-current clutch curves c, d, and e in (a), which may be used to drive the load in (b). In (a) the slip speed is represented by ω_s, and in (b) the load speed is represented by ω.

In what follows we shall assume that curve e in Figure 18(a) has been selected so that the controller monitoring the speed and torque between points 1 and 2, where the slope is slightly positive, may uniquely relate speed to torque.

Minimum motor speeds at the required torques for this clutch may be found from Figure 19 by reading the minimum slip speeds at these torques from the clutch torque-slip speed curve as shown. The dashed lines represent the family of curves obtained by coil excitation less than 100%, as labeled. Thus torque and load combination at point 3 in Figure 19 requires a slip speed of ω_{s3}, while the combination at 2 requires a slip speed of ω_{s2}. Note that since we selected curve (c) in Figure 18(a), other, larger, slip speeds may also be used to achieve this torque by reducing the coil excitation current. (The implicit

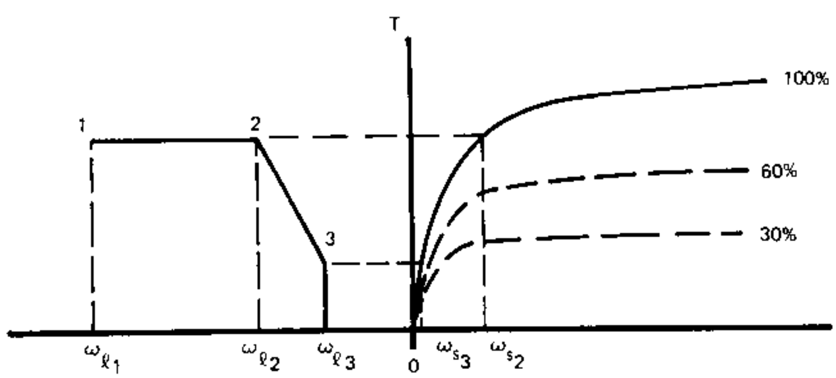

FIGURE 19 Load torque-speed and clutch torque-slip speed curves used to find minimum slip speed for load levels 1, 2, and 3 based on 100% coil excitation.

assumption that the torque-speed curves do not change character as the excitation current is reduced is not always true.)

Upon superimposing the slip speed obtained from Figure 19 to the operating speed of the load, we may find the minimum operating speed of the motor that will enable the clutch to deliver the specified torques, as has been done in Figure 20. This figure also clearly shows that by using an eddy-current clutch, we are able to operate at a higher torque at low load speeds than would have been possible with the motor alone.

In this example the load torque-speed curve was such that each torque-speed curve of the clutch crossed it only once. Where a single coil current may correspond to more than one torque-speed combination, as shown in Figure 21, it may be advisable for some applications to increase the motor speed to provide the curves shown in Figure 22 in order to reestablish a unique torque-speed relation.

This example, mentioned at the outset, was constructed to show the considerations involved in the use of an eddy-current clutch. Obviously, the controls would generally have been simpler if a magnetic particle or a hysteresis clutch had been used because the torque would have been constant

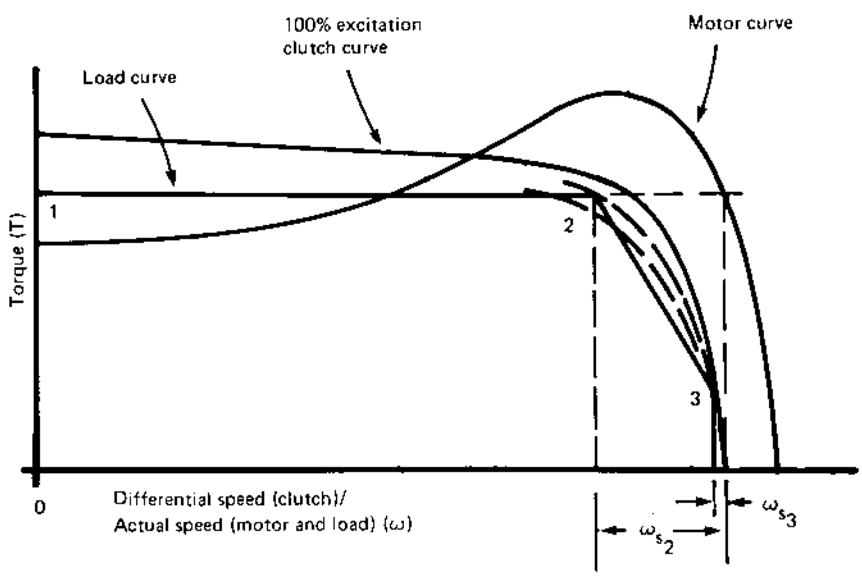

FIGURE 20 Graphical relation between the motor operating speed, the minimum eddy-current clutch slip speed, and the load curve. Less than 100% coil excitation curves are dashed.

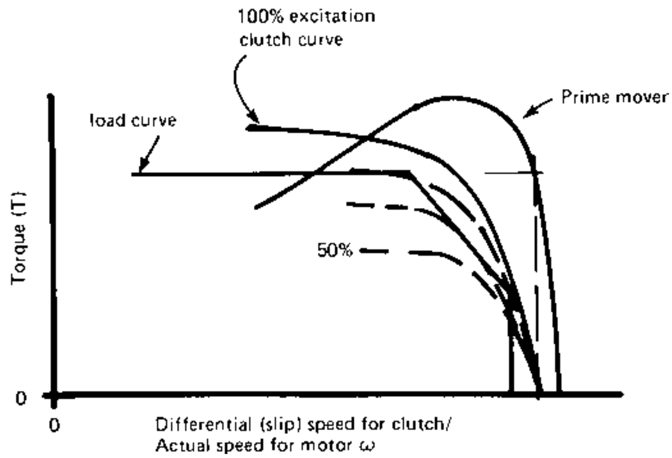

FIGURE 21 Coil current and slip speed combinations that permit more than one torque-slip speed combination for some coil excitation values.

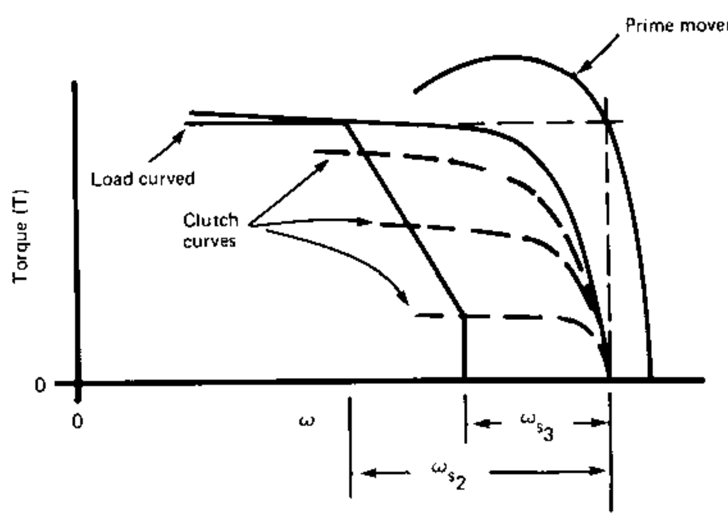

FIGURE 22 Increased slip speeds to obtain unique coil current values for each point on the torque-speed curve for the load when using an eddy-current clutch.

over the range for a given coil current. The slightly more complicated controls for eddy-current clutches are justified in those applications where the torque is to vanish whenever the driver and driven units approach equal speeds.

XII. EXAMPLE 8: SOFT START

The term *soft start* denotes starting without an initial shock, as may occur when a friction clutch is engaged too quickly. Soft starts may be had by using a torque converter, a fluid coupling, a magnetic particle clutch, a hysteresis clutch, or an eddy-current clutch. In the case of either a torque converter or a fluid coupling the torque transferred for a given input torque may be controlled by controlling the amount of fluid pumped into the converter or coupling. The same effect may be had from a magnetic particle, hysteresis, or eddy-current clutch by controlling the field current.

Generally, torque converters and fluid couplings are used in portable equipment, such as oil field drilling rigs, and in vehicles, such as trucks, buses, and automobiles, while magnetic particle, hysteresis, and eddy-current clutches are usually used in factories and mills where electrical power is available and where data from remote sensors may be processed to control brakes and clutches on machinery such as printing presses, tape transports, conveyor belts, and extrusion equipment.

Soft starts are perhaps most easily accomplished by using a clutch in which the torque is constant over a differential speed range that equals or exceeds the operating speed of the driven machine. Magnetic particle and hysteresis clutches fulfill this requirement and do not require that the motor speed exceed the driven speed by a minimum amount, as in the case of an eddy-current clutch.

Since the driving torque is constant over the operating range of the driven machine, we may in principle prescribe any coil-current versus time relation we wish to in order to prescribe the torque, and hence the acceleration as a function of time.

With these comments in mind, recommend a coil current profile for a magnetic particle clutch so that the acceleration of the take-up roll on a tape winder will increase slowly at the beginning of the acceleration period and will decrease slowly at the end of the acceleration period such that the first derivative of the acceleration, known as the jerk, will be zero at the beginning and end of the acceleration period. Assume that the damping in the system is negligible.

To provide a soft start we may consider providing a torque to the driven load that varies as the load torque versus time curve shown in the upper right-hand panel in Figure 23, in which the torque increases smoothly from zero to a maximum and then decreases to the steady-state torque when the machine is up to speed.

Before writing a program to find the required variation of coil current with time to transmit this torque profile, it may be instructive to demonstrate the procedure graphically. Upon entering the load torque versus time curve at time t_1, say, we project upward to the curve to read to corresponding torque. By projecting this torque to the clutch torque versus coil current curve we may read downward from the intersection to find the required current, say, i_1. If we now plot coil current and time axes as shown in the lower left-hand panel in Figure 23, we may locate the corresponding time on this second time axis by projecting downward from the time axis for the load torque curve to a 45° line and then project horizontally to the left from the 45° line as shown. The intersection of this projection with the vertical projection from the coil current

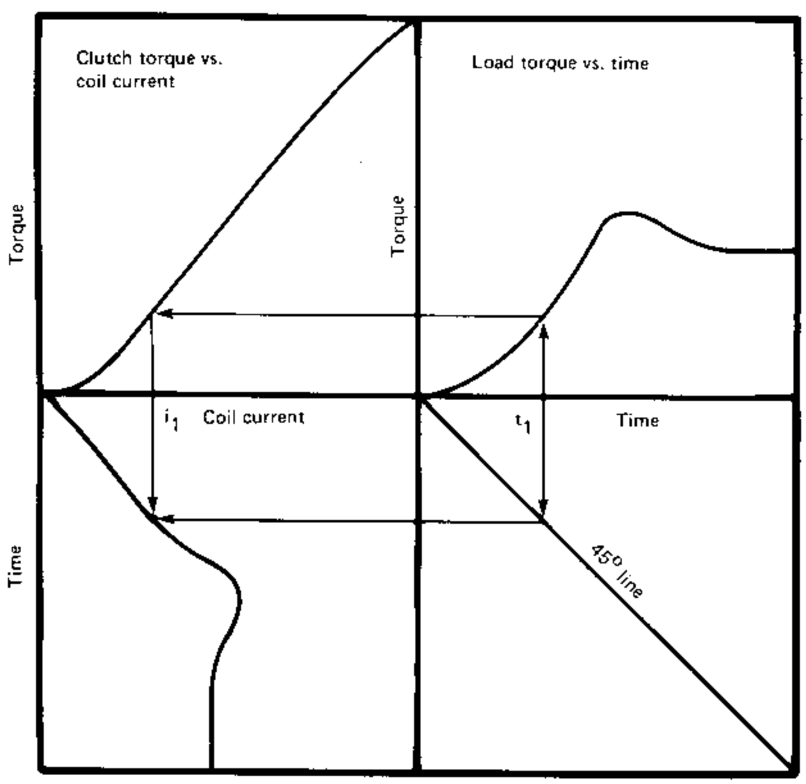

FIGURE 23 Graphical determination of the control current as a function of time to produce a prescribed soft start.

axis locates point (i_1,t_1) on the desired coil current versus time curve. Continuing in this manner for a sequence of points enables us to find sufficient points to complete the coil current versus time curve as shown.

Our program may be written in a parallel manner. After entering tabular data CTCC describing the clutch torque versus coil current and tabular data LTT describing the load torque versus time, we select a sequence of times $t(i)$. For each of these $t(i)$ values we use the LTT data to interpolate to find corresponding torques $TQ(i)$. We then use the CTCC data to interpolate to find the coil current $I(i)$ associated with torque $TQ(i)$. Thus we have tabulated $TQ(i)$ as a function of $I(i)$. These data, if plotted, would yield the coil current versus time curve used to control the soft start.

XIII. NOTATION

a	linear acceleration or deceleration (lt^{-2})
C_p	specific heat at constant pressure
c	damping coefficient (mt^{-2})
d	diameter (l)
E	energy (ml^2t^{-2})
F	force (mlt^{-2})
g	acceleration due to gravity (lt^{-2})
h	height (l)
I	moment of inertia (ml^2)
KE	kinetic energy (ml^2t^{-2})
k	integer (l)
m	mass (m)
N	integer (1)
n	revolutions/minute (rpm) (t^{-1})
n_{ij}	speed ratio of gear i relative to gear j (l)
p	pressure ($ml^{-1}\ t^{-2}$)
Q	heat ($mt^2\ t^{-2}$)
r	radius (l)
r_g	radius of gyration (l)
t	time (t)
v	velocity (lt^{-1})
W	work ($ml^2\ t^{-2}$)
w	weight (mlt^{-2})
α	angular acceleration or deceleration (t^{-2})
γ	mass/length (ml^{-1})
Δ	increment of the quantity that follows
Θ	temperature (θ)
θ	angular position (1)

ϕ angular position (l)
ω angular velocity (t^{-1})

XIV. FORMULA COLLECTION

Braking time, variable torque

$$t_2 - t_1 = -I\int_{\omega_2}^{\omega_1} \frac{d\omega}{T(\omega) + c\omega}$$

Braking time, constant torque

$$t_2 - t_1 = \frac{I}{c}\ln\frac{T + c\omega_1}{T + c\omega_2}$$

Braking time, full stop, constant torque

$$t = \frac{I}{c}\ln\left(1 + \frac{c}{T}\omega\right)$$

Braking time or clutch, acceleration time, negligible damping, constant torque

$$T \approx \frac{I\omega}{t} = \frac{mr_g^2 n}{10t}\text{(SI units)} = \frac{Wr_g^2 n}{307t}\text{(OE units)}$$

Constant braking torque, full stop

$$T = c\ \frac{\omega}{e^{\frac{c}{I}t} - 1}$$

Constant clutch torque, start from rest

$$T = c\ \frac{\omega}{1 - e^{-\frac{c}{I}t}}$$

Clutch acceleration time, variable torque

$$t_2 - t_1 = I\int_{\omega_1}^{\omega_2} \frac{d\omega}{T(\omega) + c\omega}$$

Clutch acceleration time, constant torque

$$t_2 - t_1 = \frac{I}{c}\ln\frac{T - c\omega_1}{T - c\omega_2}$$

Clutch acceleration time, constant torque, from rest

$$t = \frac{I}{c} \ln \frac{1}{1 - (c/T)\omega}$$

Clutch, heat dissipated during acceleration

$$W(\omega) = T\,\omega_i\,\tau - I \int_0^{\omega_i} \frac{T(\omega)\,\omega}{T(\omega) - c\omega}\, d\omega$$