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Classification and Selection of Bearings

1.1 INTRODUCTION

Moving parts in machinery involve relative sliding or rolling motion. Examples of relative motion are linear sliding motion, such as in machine tools, and rotation motion, such as in motor vehicle wheels. Most bearings are used to support rotating shafts in machines. Rubbing of two bodies that are loaded by a normal force (in the direction normal to the contact area) generates energy losses by friction and wear. Appropriate bearing design can minimize friction and wear as well as early failure of machinery. The most important objectives of bearing design are to extend bearing life in machines, reduce friction energy losses and wear, and minimize maintenance expenses and downtime of machinery due to frequent bearing failure. In manufacturing plants, unexpected bearing failure often causes expensive loss of production. Moreover, in certain cases, such as in aircraft, there are very important safety considerations, and unexpected bearing failures must be prevented at any cost.

During the past century, there has been an ever-increasing interest in the friction and wear characteristics of various bearing designs, lubricants, and materials for bearings. This scientific discipline, named *Tribology*, is concerned with the friction, lubrication, and wear of interacting surfaces in relative motion. Several journals are dedicated to the publication of original research results on this subject, and several books have been published that survey the vast volume of research in tribology. The objectives of the basic research in tribology are similar to those of bearing design, focusing on the reduction of friction and wear. These efforts resulted in significant advances in bearing technology during the past century. This improvement is particularly in lubrication, bearing materials, and the introduction of rolling-element bearings and bearings supported by lubrication films. The improvement in bearing technology resulted in the reduction of friction, wear, and maintenance expenses, as well as in the longer life of machinery.

The selection of a proper bearing type for each application is essential to the reliable operation of machinery, and it is an important component of machine design. Most of the maintenance work in machines is in bearing lubrication as well as in the replacement of damaged or worn bearings. The appropriate selection of a bearing type for each application is very important to minimize the risk of early failure by wear or fatigue, thereby to secure adequate bearing life. There are other considerations involved in selection, such as safety, particularly in aircraft. Also, cost is always an important consideration in bearing selection—the designer should consider not only the initial cost of the bearing but also the cost of maintenance and of the possible loss of production over the complete life cycle of the machine.

Therefore, the first step in the process of bearing design is the selection of the bearing type for each application. In most industries there is a tradition concerning the type of bearings applied in each machine. However, a designer should follow current developments in bearing technology; in many cases, selection of a new bearing type can be beneficial. Proper selection can be made from a variety of available bearing types, which include rolling-element bearings, dry and boundary lubrication bearings, as well as hydrodynamic and hydrostatic lubrication bearings. An additional type introduced lately is the electromagnetic bearing. Each bearing type can be designed in many different ways and can be made of various materials, as will be discussed in the following chapters.

It is possible to reduce the size and weight of machines by increasing their speed, such as in motor vehicle engines. Therefore, there is an increasing requirement for higher speeds in machinery, and the selection of an appropriate bearing type for this purpose is always a challenge. In many cases, it is the limitation of the bearing that limits the speed of a machine. It is important to select a bearing that has low friction in order to minimize friction-energy losses, equal to the product of friction torque and angular speed. Moreover, friction-energy losses are dissipated in the bearing as heat, and it is essential to prevent bearing overheating. If the temperature of the sliding surfaces is too close to the melting point of the bearing material, it can cause bearing failure. In the following chapters, it will be shown that an important task in the design process is the prevention of bearing overheating.

1.1.1 Radial and Thrust Bearings

Bearings can also be classified according to their geometry related to the relative motion of elements in machinery. Examples are journal, plane-slider, and spherical bearings. A journal bearing, also referred to as a sleeve bearing, is widely used in machinery for rotating shafts. It consists of a bushing (sleeve) supported by a housing, which can be part of the frame of a machine. The shaft (journal) rotates inside the bore of the sleeve. There is a small clearance between the inner diameter of the sleeve and the journal, to allow for free rotation. In contrast, a plane-slider bearing is used mostly for linear motion, such as the slides in machine tools.

A bearing can also be classified as a radial bearing or a thrust bearing, depending on whether the bearing load is in the radial or axial direction, respectively, of the shaft. The shafts in machines are loaded by such forces as reactions between gears and tension in belts, gravity, and centrifugal forces. All the forces on the shaft must be supported by the bearings, and the force on the bearing is referred to as a *bearing load*. The load on the shaft can be divided into radial and axial components. The axial component (also referred to as *thrust load*) is in the direction of the shaft axis (see Fig. 1-1), while the *radial load* component is in the direction normal to the shaft axis. In Fig. 1-1, an example of a loaded shaft is presented. The reaction forces in helical gears have radial and axial components. The component F_a is in the axial direction, while all the other components are radial loads. Examples of solved problems are included at the end of this chapter. Certain bearings, such as conical roller bearings, shown in Fig. 1-1, or angular ball bearings, can support radial as well as thrust forces. Certain other bearings, however, such as hydrodynamic journal bearings, are applied only for radial loads, while the hydrodynamic thrust bearing supports

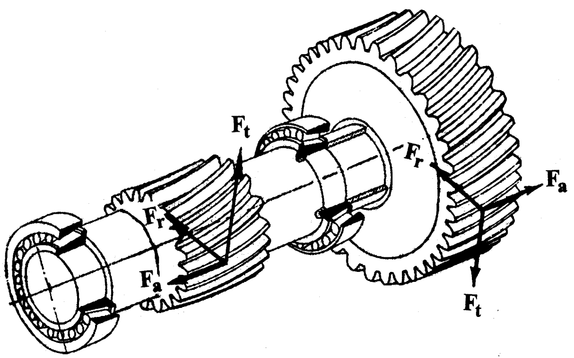

FIG. 1-1 Load components on a shaft with helical gears.

only axial loads. A combination of radial and thrust bearings is often applied to support the shaft in machinery.

1.1.2 Bearing Classification

Machines could not operate at high speed in their familiar way without some means of reducing friction and the wear of moving parts. Several important engineering inventions made it possible to successfully operate heavily loaded shafts at high speed, including the rolling-element bearing and hydrodynamic, hydrostatic, and magnetic bearings.

1. *Rolling-element bearings* are characterized by rolling motion, such as in ball bearings or cylindrical rolling-element bearings. The advantage of rolling motion is that it involves much less friction and wear, in comparison to the sliding motion of regular sleeve bearings.
2. The term *hydrodynamic bearing* refers to a sleeve bearing or an inclined plane-slider where the sliding plane floats on a thin film of lubrication. The fluid film is maintained at a high pressure that supports the bearing load and completely separates the sliding surfaces. The lubricant can be fed into the bearing at atmospheric or higher pressure. The pressure wave in the lubrication film is generated by hydrodynamic action due to the rapid rotation of the journal. The fluid film acts like a viscous wedge and generates high pressure and load-carrying capacity. The sliding surface floats on the fluid film, and wear is prevented.
3. In contrast to hydrodynamic bearing, *hydrostatic bearing* refers to a configuration where the pressure in the fluid film is generated by an external high-pressure pump. The lubricant at high pressure is fed into the bearing recesses from an external pump through high-pressure tubing. The fluid, under high pressure in the bearing recesses, carries the load and separates the sliding surfaces, thus preventing high friction and wear.
4. A recent introduction is the *electromagnetic bearing*. It is still in development but has already been used in some unique applications. The concept of operation is that a magnetic force is used to support the bearing load. Several electromagnets are mounted on the bearing side (stator poles). The bearing load capacity is generated by the magnetic field between rotating laminators, mounted on the journal, and stator poles, on the stationary bearing side. Active feedback control keeps the journal floating without any contact with the bearing surface. The advantage is that there is no contact between the sliding surfaces, so wear is completely prevented as long as there is magnetic levitation.

Further description of the characteristics and applications of these bearings is included in this and the following chapters.

1.2 DRY AND BOUNDARY LUBRICATION BEARINGS

Whenever the load on the bearing is light and the shaft speed is low, wear is not a critical problem and a sleeve bearing or plane-slider lubricated by a very thin layer of oil (boundary lubrication) can be adequate. Sintered bronzes with additives of other elements are widely used as bearing materials. Liquid or solid lubricants are often inserted into the porosity of the material and make it self-lubricated. However, in heavy-duty machinery—namely, bearings operating for long periods of time under heavy load relative to the contact area and at high speeds—better bearing types should be selected to prevent excessive wear rates and to achieve acceptable bearing life. Bearings from the aforementioned list can be selected, namely, rolling-element bearings or fluid film bearings.

In most applications, the sliding surfaces of the bearing are lubricated. However, bearings with dry surfaces are used in unique situations where lubrication is not desirable. Examples are in the food and pharmaceutical industries, where the risk of contamination by the lubricant forbids its application. The sliding speed, V, and the average pressure in the bearing, P, limit the use of dry or boundary lubrication. For plastic and sintered bearing materials, a widely accepted limit criterion is the product PV for each bearing material and lubrication condition. This product is proportional to the amount of friction-energy loss that is dissipated in the bearing as heat. This is in addition to limits on the maximum sliding velocity and average pressure. For example, a self-lubricated sintered bronze bearing has the following limits:

Surface velocity limit, V, is 6 m/s, or 1180 ft/min
Average surface-pressure limit, P, is 14 MPa, or 2000 psi
PV limit is 110,000 psi-ft/min, or 3.85×10^6 Pa-m/s

In comparison, bearings made of plastics have much lower PV limit. This is because the plastics have a low melting point; in addition, the plastics are not good conductors of heat, in comparison to metals. For these reasons, the PV limit is kept at relatively low values, in order to prevent bearing failure by overheating. For example, Nylon 6, which is widely used as a bearing material, has the following limits as a bearing material:

Surface velocity limit, V, is 5 m/s
Average surface-pressure limit, P, is 6.9 MPa
PV limit is 105×10^3 Pa-m/s

Remark. In hydrodynamic lubrication, the symbol for surface velocity of a rotating shaft is U, but for the PV product, sliding velocity V is traditionally used.

Conversion to SI Units.

$$1 \text{ lbf/in.}^2 \text{ (psi)} = 6895 \text{ N/m}^2 \text{ (Pa)}$$
$$1 \text{ ft/min} = 0.0051 \text{ m/s}$$
$$1 \text{ psi-ft/min} = 6895 \times 0.0051 = 35 \text{ Pa-m/s} = 35 \times 10^{-6} \text{ MPa-m/s}$$

An example for calculation of the PV value in various cases is included at the end of this chapter. The PV limit is much lower than that obtained by multiplying the maximum speed and maximum average pressure due to the load capacity. The reason is that the maximum PV is determined from considerations of heat dissipation in the bearing, while the average pressure and maximum speed can be individually of higher value, as long as the product is not too high. If the maximum PV is exceeded, it would usually result in a faster-than-acceptable wear rate.

1.3 HYDRODYNAMIC BEARING

An inclined plane-slider is shown in Fig. 1-2. It carries a load F and has horizontal velocity, U, relative to a stationary horizontal plane surface. The plane-slider is inclined at an angle α relative to the horizontal plane. If the surfaces were dry, there would be direct contact between the two surfaces, resulting in significant friction and wear. It is well known that friction and wear can be reduced by lubrication. If a sufficient quantity of lubricant is provided and the

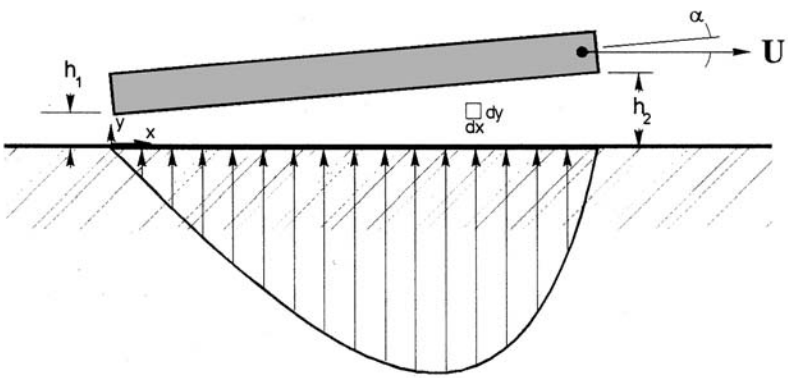

FIG. 1-2 Hydrodynamic lubrication of plane-slider.

sliding velocity is high, the surfaces would be completely separated by a very thin lubrication film having the shape of a fluid wedge. In the case of complete separation, full hydrodynamic lubrication is obtained. The plane-slider is inclined, to form a converging viscous wedge of lubricant as shown in Fig. 1-2. The magnitudes of h_1 and h_2 are very small, of the order of only a few micrometers. The clearance shown in Fig. 1-2 is much enlarged.

The lower part of Fig. 1-2 shows the pressure distribution, p (pressure wave), inside the thin fluid film. This pressure wave carries the slider and its load. The inclined slider, floating on the lubricant, is in a way similar to water-skiing, although the physical phenomena are not identical. The pressure wave inside the lubrication film is due to the fluid viscosity, while in water-skiing it is due to the fluid inertia. The generation of a pressure wave in hydrodynamic bearings can be explained in simple terms, as follows: The fluid adheres to the solid surfaces and is dragged into the thin converging wedge by the high shear forces due to the motion of the plane-slider. In turn, high pressure must build up in the fluid film in order to allow the fluid to escape through the thin clearances.

A commonly used bearing in machinery is the hydrodynamic journal bearing, as shown in Fig. 1-3. Similar to the inclined plane-slider, it can support a radial load without any direct contact between the rotating shaft (journal) and the bearing sleeve. The viscous fluid film is shaped like a wedge due to the eccentricity, e, of the centers of the journal relative to that of bearing bore. As with the plane-slider, a pressure wave is generated in the lubricant, and a thin fluid

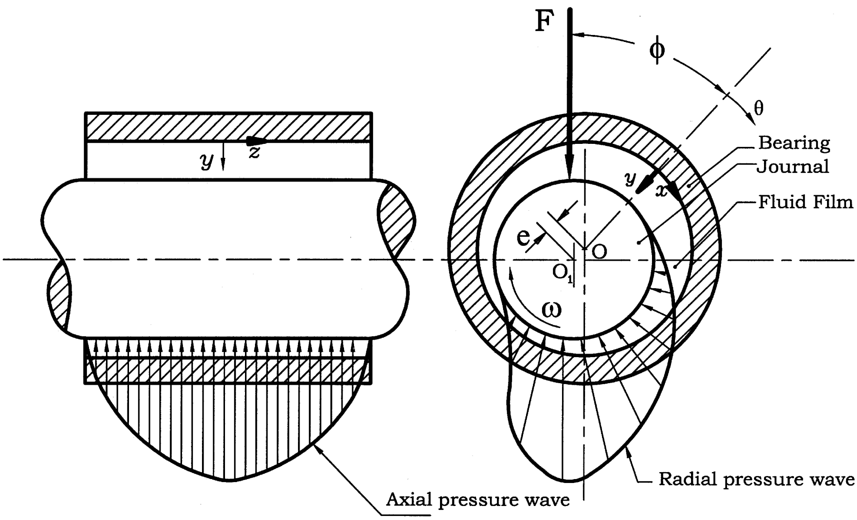

FIG. 1-3 Hydrodynamic journal bearing.

film completely separates the journal and bearing surfaces. Due to the hydrodynamic effect, there is low friction and there is no significant wear as long as a complete separation is maintained between the sliding surfaces.

The pressure wave inside the hydrodynamic film carries the journal weight together with the external load on the journal. The principle of operation is the uneven clearance around the bearing formed by a small eccentricity, e, between the journal and bearing centers, as shown in Fig. 1-3. The clearance is full of lubricant and forms a thin fluid film of variable thickness. A pressure wave is generated in the converging part of the clearance. The resultant force of the fluid film pressure wave is the load-carrying capacity, W, of the bearing. For bearings operating at steady conditions (constant journal speed and bearing load), the load-carrying capacity is equal to the external load, F, on the bearing. But the two forces of action and reaction act in opposite directions.

In a hydrodynamic journal bearing, the load capacity (equal in magnitude to the bearing force) increases with the eccentricity, e, of the journal. Under steady conditions, the center of the journal always finds its equilibrium point, where the load capacity is equal to the external load on the journal. Figure 1-3 indicates that the eccentricity displacement, e, of the journal center, away from the bearing center, is not in the vertical direction but at a certain attitude angle, ϕ, from the vertical direction. In this configuration, the resultant load capacity, due to the pressure wave, is in the vertical direction, opposing the vertical external force. The fluid film pressure is generated mostly in the converging part of the clearance, and the attitude angle is required to allow the converging region to be below the journal to provide the required lift force in the vertical direction and, in this way, to support the external load.

In real machinery, there are always vibrations and disturbances that can cause occasional contact between the surface asperities (surface roughness), resulting in severe wear. In order to minimize this risk, the task of the engineer is to design the hydrodynamic journal bearing so that it will operate with a minimum lubrication-film thickness, h_n, much thicker than the size of the surface asperities. Bearing designers must keep in mind that if the size of the surface asperities is of the order of magnitude of 1 micron, the minimum film thickness, h_n, should be 10–100 microns, depending on the bearing size and the level of vibrations expected in the machine.

1.3.1 Disadvantages of Hydrodynamic Bearings

One major disadvantage of hydrodynamic bearings is that a certain minimum speed is required to generate a full fluid film that completely separates the sliding surfaces. Below that speed, there is mixed or boundary lubrication, with direct contact between the asperities of the rubbing surfaces. For this reason, even if the bearing is well designed and successfully operating at the high rated speed of the machine, it can be subjected to excessive friction and wear at low speed, such as during starting and stopping of journal rotation. In particular, hydrodynamic bearings undergo severe wear during start-up, when they accelerate from zero speed, because static friction is higher than dynamic friction.

A second important disadvantage is that hydrodynamic bearings are completely dependent on a continuous supply of lubricant. If the oil supply is interrupted, even for a short time for some unexpected reason, it can cause overheating and sudden bearing failure. It is well known that motor vehicle engines do not last a long time if run without oil. In that case, the hydrodynamic bearings fail first due to the melting of the white metal lining on the bearing. This risk of failure is the reason why hydrodynamic bearings are never used in critical applications where there are safety concerns, such as in aircraft engines. Failure of a motor vehicle engine, although it is highly undesirable, does not involve risk of loss of life; therefore, hydrodynamic bearings are commonly used in motor vehicle engines for their superior performance and particularly for their relatively long operation life.

A third important disadvantage is that the hydrodynamic journal bearing has a low stiffness to radial displacement of the journal (low resistance to radial run-out), particularly when the eccentricity is low. This characteristic rules out the application of hydrodynamic bearings in precision machines, e.g., machine tools. Under dynamic loads, the low stiffness of the bearings can result in dynamic instability, particularly with lightly loaded high-speed journals. The low stiffness causes an additional serious problem of bearing whirl at high journal speeds. The bearing whirl phenomenon results from instability in the oil film, which often results in bearing failure.

Further discussions of the disadvantages of journal bearing and methods to overcome these drawbacks are included in the following chapters.

1.4 HYDROSTATIC BEARING

The introduction of externally pressurized hydrostatic bearings can solve the problem of wear at low speed that exists in hydrodynamic bearings. In hydrostatic bearings, a fluid film completely separates the sliding surfaces at all speeds, including zero speed. However, hydrostatic bearings involve higher cost in comparison to hydrodynamic bearings. Unlike hydrodynamic bearings, where the pressure wave in the oil film is generated inside the bearing by the rotation of the journal, an external oil pump pressurizes the hydrostatic bearing. In this way, the hydrostatic bearing is not subjected to excessive friction and wear rate at low speed.

The hydrostatic operation has the advantage that it can maintain complete separation of the sliding surfaces by means of high fluid pressure during the starting and stopping of journal rotation. Hydrostatic bearings are more expensive than hydrodynamic bearings, since they require a hydraulic system to pump and circulate the lubricant and there are higher energy losses involved in the circulation of the fluid. The complexity and higher cost are reasons that hydrostatic bearings are used only in special circumstances where these extra expenses can be financially justified.

Girard introduced the principle of the hydrostatic bearing in 1851. Only much later, in 1923, did Hodgekinson patent a hydrostatic bearing having wide recesses and fluid pumped into the recesses at constant pressure through flow restrictors. The purpose of the flow restrictors is to allow bearing operation and adequate bearing stiffness when all the recesses are fed at constant pressure from one pump. The advantage of this system is that it requires only one pump without flow dividers for distributing oil at a constant flow rate into each recess.

Whenever there are many recesses, the fluid is usually supplied at constant pressure from one central pump. The fluid flows into the recesses through flow restrictors to improve the radial stiffness of the bearing. A diagram of such system is presented in Fig. 1-4. From a pump, the oil flows into several recesses around the bore of the bearing through capillary flow restrictors. From the recesses, the fluid flows out in the axial direction through a thin radial clearance, h_o, between the journal and lands (outside the recesses) around the circumference of the two ends of the bearing. This thin clearance creates a resistance to the outlet flow from each bearing recess. This outlet resistance, at the lands, is essential to maintain high pressure in each recess around the bearing. This resistance at the outlet varies by any small radial displacement of the journal due to the bearing load. The purpose of supplying the fluid to the recesses through flow restrictors is to make the bearing stiffer under radial force; namely, it reduces radial displacement (radial run-out) of the journal when a radial load is applied. The following is an explanation for the improved stiffness provided by flow restrictors.

When a journal is displaced in the radial direction from the bearing center, the clearances at the lands of the opposing recesses are no longer equal. The resistance to the flow from the opposing recesses decreases and increases, respectively (the resistance is inversely proportional to h_o^3). This results in unequal flow rates in the opposing recesses. The flow increases and decreases, respectively. An important characteristic of a flow restrictor, such as a capillary tube, is that its pressure drop increases with flow rate. In turn, this causes the pressures in the opposing recesses to decrease and increase, respectively. The bearing load capacity resulting from these pressure differences acts in the opposite direction to the radial load on the journal. In this way, the bearing supports the journal with minimal radial displacement. In conclusion, the introduction of inlet flow restrictors increases the bearing stiffness because only a very small radial displacement of the journal is sufficient to generate a large pressure difference between opposing recesses.

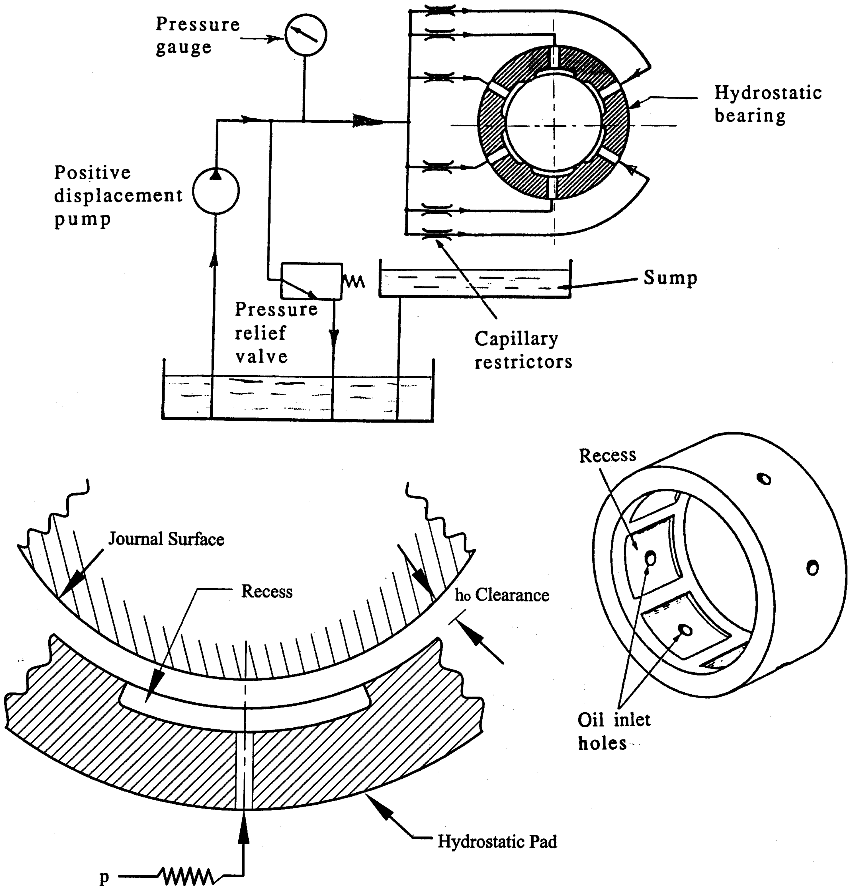

FIG. 1-4 Hydrostatic bearing system.

In summary, the primary advantage of the hydrostatic bearing, relative to all other bearings, is that the surfaces of the journal and bearing are separated by a fluid film at all loads and speeds. As a result, there is no wear and the sliding friction is low at low speeds. A second important advantage of hydrostatic bearings is their good stiffness to radial loads. Unlike hydrodynamic bearings, high stiffness is maintained under any load, from zero loads to the working loads, and at all speeds, including zero speed.

The high stiffness to radial displacement makes this bearing suitable for precision machines, for example, precise machine tools. The high bearing stiffness is important to minimize any radial displacement (run-out) of the journal. In addition, hydrostatic journal bearings operate with relatively large clearances (compared to other bearings); and therefore, there is not any significant run-out that results from uneven surface finish or small dimensional errors in the internal bore of the bearing or journal.

1.5 MAGNETIC BEARING

A magnetic bearing is shown in Fig. 1-5. The concept of operation is that a magnetic field is applied to support the bearing load. Several electromagnets are

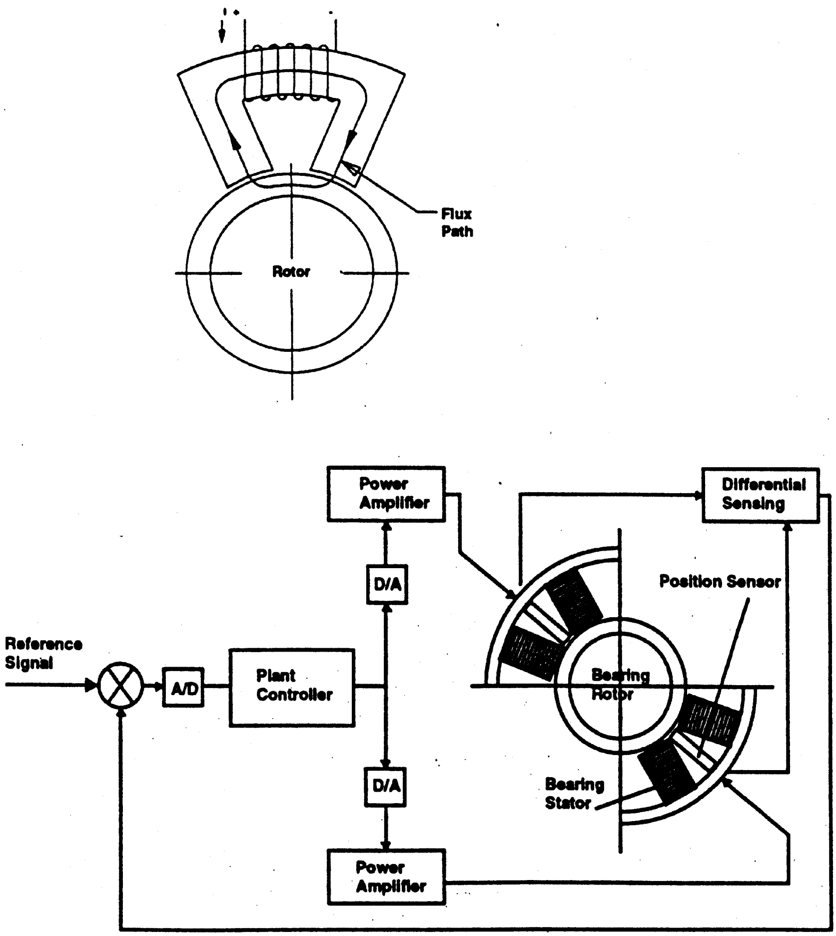

FIG. 1-5 Concept of magnetic bearing. Used by permission of Resolve Magnetic Bearings.

mounted on the bearing side (stator poles). Electrical current in the stator poles generates a magnetic field. The load-carrying capacity of the bearing is due to the magnetic field between the rotating laminators mounted on the journal and the coils of the stator poles on the stationary bearing side.

Active feedback control is required to keep the journal floating without its making any contact with the bearing. The control entails on-line measurement of the shaft displacement from the bearing center, namely, the magnitude of the eccentricity and its direction. The measurement is fed into the controller for active feedback control of the bearing support forces in each pole in order to keep the journal close to the bearing center. This is achieved by varying the magnetic field of each pole around the bearing. In this way, it is possible to control the magnitude and direction of the resultant magnetic force on the shaft. This closed-loop control results in stable bearing operation.

During the last decade, a lot of research work on magnetic bearings has been conducted in order to optimize the performance of the magnetic bearing. The research work included optimization of the direction of magnetic flux, comparison between electromagnetic and permanent magnets, and optimization of the number of magnetic poles. This research work has resulted in improved load capacity and lower energy losses. In addition, research has been conducted to improve the design of the control system, which resulted in a better control of rotor vibrations, particularly at the critical speeds of the shaft.

1.5.1 Disadvantages of Magnetic Bearings

Although significant improvement has been achieved, there are still several disadvantages in comparison with other, conventional bearings. The most important limitations follow.

a. Electromagnetic bearings are relatively much more expensive than other noncontact bearings, such as the hydrostatic bearing. In most cases, this fact makes the electromagnetic bearing an uneconomical alternative.
b. Electromagnetic bearings have less damping of journal vibrations in comparison to hydrostatic oil bearings.
c. In machine tools and other manufacturing environments, the magnetic force attracts steel or iron chips.
d. Magnetic bearings must be quite large in comparison to conventional noncontact bearings in order to generate equivalent load capacity. An acceptable-size magnetic bearing has a limited static and dynamic load capacity. The magnetic force that supports static loads is limited by the saturation properties of the electromagnet core material. The maximum magnetic field is reduced with temperature. In addition, the dynamic

load capacity of the bearing is limited by the available electrical power supply from the amplifier.

e. Finally, electromagnetic bearings involve complex design problems to ensure that the heavy spindle, with its high inertia, does not fall and damage the magnetic bearing when power is shut off or momentarily discontinued. Therefore, a noninterrupted power supply is required to operate the magnetic bearing, even at no load or at shutdown conditions of the system. In order to secure safe operation in case of accidental power failure or support of the rotor during shutdown of the machine, an auxiliary bearing is required. Rolling-element bearings with large clearance are commonly used. During the use of such auxiliary bearings, severe impact can result in premature rolling-element failure.

1.6 ROLLING-ELEMENT BEARINGS

Rolling-element bearings, such as ball, cylindrical, or conical rolling bearings, are the bearings most widely used in machinery. Rolling bearings are often referred to as *antifriction bearings*. The most important advantage of rolling-element bearings is the low friction and wear of rolling relative to that of sliding.

Rolling bearings are used in a wide range of applications. When selected and applied properly, they can operate successfully over a long period of time. Rolling friction is lower than sliding friction; therefore, rolling bearings have lower friction energy losses as well as reduced wear in comparison to sliding bearings. Nevertheless, the designer must keep in mind that the life of a rolling-element bearing can be limited due to fatigue. Ball bearings involve a point contact between the balls and the races, resulting in high stresses at the contact, often named *hertz stresses*, after Hertz (1881), who analyzed for the first time the stress distribution in a point contact.

When a rolling-element bearing is in operation, the rolling contacts are subjected to alternating stresses at high frequency that result in metal fatigue. At high speed, the centrifugal forces of the rolling elements, high temperature (due to friction-energy losses) and alternating stresses all combine to reduce the fatigue life of the bearing. For bearings operating at low and medium speeds, relatively long fatigue life can be achieved in most cases. But at very high speeds, the fatigue life of rolling element bearings can be too short, so other bearing types should be selected. Bearing speed is an important consideration in the selection of a proper type of bearing. High-quality rolling-element bearings, which involve much higher cost, are available for critical high-speed applications, such as in aircraft turbines.

Over the last few decades, a continuous improvement in materials and the methods of manufacturing of rolling-element bearings have resulted in a significant improvement in fatigue life, specifically for aircraft applications. But the trend in modern machinery is to increase the speed of shafts more and more in order to reduce the size of machinery. Therefore, the limitations of rolling-element bearings at very high speeds are expected to be more significant in the future.

The fatigue life of a rolling bearing is a function of the magnitude of the oscillating stresses at the contact. If the stresses are low, the fatigue life can be practically unlimited. The stresses in dry contact can be calculated by the theory of elasticity. However, the surfaces are usually lubricated, and there is a very thin lubrication film at very high pressure separating the rolling surfaces. This thin film prevents direct contact and plays an important role in wear reduction. The analysis of this film is based on the elastohydrodynamic (EHD) theory, which considers the fluid dynamics of the film in a way similar to that of hydrodynamic bearings.

Unlike conventional hydrodynamic theory, EHD theory considers the elastic deformation in the contact area resulting from the high-pressure distribution in the fluid film. In addition, in EHD theory, the lubricant viscosity is considered as a function of the pressure, because the pressures are much higher than in regular hydrodynamic bearings. Recent research work has considered the thermal effects in the elastohydrodynamic film. Although there has been much progress in the understanding of rolling contact, in practice the life of the rolling-element bearing is still estimated by means of empirical equations. One must keep in mind the statistical nature of bearing life. Rolling bearings are selected to have a very low probability of premature failure. The bearings are designed to have a certain predetermined life span, such as 10 years. The desired life span should be determined before the design of a machine is initiated.

Experience over many years indicates that failure due to fatigue in rolling bearings is only one possible failure mode among many other, more frequent failure modes, due to various reasons. Common failure causes include bearing overheating, misalignment errors, improper mounting, corrosion, trapped hard particles, and not providing the bearing with proper lubrication (oil starvation or not using the optimum type of lubricant). Most failures can be prevented by proper maintenance, such as lubrication and proper mounting of the bearing. Fatigue failure is evident in the form of spalling or flaking at the contact surfaces of the races and rolling elements. It is interesting to note that although most rolling bearings are selected by considering their fatigue life, only 5% to 10% of the bearings actually fail by fatigue.

At high-speed operation, a frequent cause for rolling bearing failure is overheating. The heat generated by friction losses is dissipated in the bearing, resulting in uneven temperature distribution in the bearing. During operation, the temperature of the rolling bearing outer ring is lower than that of the inner ring. In turn, there is uneven thermal expansion of the inner and outer rings, resulting in thermal stresses in the form of a tight fit and higher contact stresses between the rolling elements and the races. The extra contact stresses further increase the level of friction and the temperature. This sequence of events can lead to an unstable closed-loop process, which can result in bearing failure by seizure. Common rolling-element bearings are manufactured with an internal clearance to reduce this risk of thermal seizure.

At high temperature the fatigue resistance of the metal is deteriorating. Also, at high speed the centrifugal forces increase the contact stresses between the rolling elements and the outer race. All these effects combine to reduce the fatigue life at very high speeds. Higher risk of bearing failure exists whenever the product of bearing load, F, and speed, n, is very high. The friction energy is dissipated in the bearing as heat. The power loss due to friction is proportional to the product Fn, similar to the product PV in a sleeve bearing. Therefore, the temperature rise of the bearing relative to the ambient temperature is also proportional to this product. In conclusion, load and speed are two important parameters that should be considered for selection and design purposes. In addition to friction-energy losses, bearing overheating can be caused by heat sources outside the bearing, such as in the case of engines or steam turbines.

In aircraft engines, only rolling bearings are used. Hydrodynamic or hydrostatic bearings are not used because of the high risk of a catastrophic (sudden) failure in case of interruption in the oil supply. In contrast, rolling bearings do not tend to catastrophic failure. Usually, in case of initiation of damage, there is a warning noise and sufficient time to replace the rolling bearing before it completely fails. For aircraft turbine engines there is a requirement for ever increasing power output and speed. At the very high speed required for gas turbines, the centrifugal forces of the rolling elements become a major problem. These centrifugal forces increase the hertz stresses at the outer-race contacts and shorten the bearing fatigue life.

The centrifugal force is proportional to the second power of the angular speed. Similarly, the bearing size increases the centrifugal force because of its larger rolling-element mass as well as its larger orbit radius. The DN value (rolling bearing bore, in millimeters, times shaft speed, in revolutions per minute, RPM) is used as a measure for limiting the undesired effect of the centrifugal forces in rolling bearings. Currently, the centrifugal force of the rolling elements is one important consideration for limiting aircraft turbine engines to 2 million DN.

Hybrid bearings, which have rolling elements made of silicon nitride and rings made of steel, have been developed and are already in use. One important advantage of the hybrid bearing is that the density of silicon nitride is much lower than that of steel, resulting in lower centrifugal force. In addition, hybrid bearings have better fatigue resistance at high temperature and are already in use for many industrial applications. Currently, intensive tests are being conducted in hybrid bearings for possible future application in aircraft turbines. However, due to the high risk in this application, hybrid bearings must pass much more rigorous tests before actually being used in aircraft engines.

Thermal stresses in rolling bearings can also be caused by thermal elongation of the shaft. In machinery such as motors and gearboxes, the shaft is supported by two bearings at the opposite ends of the shaft. The friction energy in the bearings increases the temperature of the shaft much more than that of the housing of the machine. It is important to design the mounting of the bearings with a free fit in the housing on one side of the shaft. This bearing arrangement is referred to as a *locating/floating* arrangement; it will be explained in Chapter 13. This arrangement allows for a free thermal expansion of the shaft in the axial direction and elimination of the high thermal stresses that could otherwise develop.

Rolling-element bearings generate certain levels of noise and vibration, in particular during high-speed operation. The noise and vibrations are due to irregular dimensions of the rolling elements and are also affected by the internal clearance in the bearing.

1.7 SELECTION CRITERIA

In comparison to rolling-element bearings, limited fatigue life is not a major problem for hydrodynamic bearings. As long as a full fluid film completely separates the sliding surfaces, the life of hydrodynamic bearings is significantly longer than that of rolling bearings, particularly at very high speeds.

However, hydrodynamic bearings have other disadvantages that make other bearing types the first choice for many applications. Hydrodynamic bearings can be susceptible to excessive friction and wear whenever the journal surface has occasional contact with the bearing surface and the superior fluid film lubrication is downgraded to boundary or mixed lubrication. This occurs at low operating speeds or during starting and stopping, since hydrodynamic bearings require a certain minimum speed to generate an adequate film thickness capable of completely separating the sliding surfaces.

According to the theory that is discussed in the following chapters, a very thin fluid film is generated inside a hydrodynamic bearing even at low journal speed. But in practice, due to surface roughness or vibrations and disturbances, a certain minimum speed is required to generate a fluid film of sufficient thickness that occasional contacts and wear between the sliding surfaces are prevented. Even at high journal speed, surface-to-surface contact may occur because of unexpected vibrations or severe disturbances in the system.

An additional disadvantage of hydrodynamic bearings is a risk of failure if the lubricant supply is interrupted, even for a short time. A combination of high speed and direct contact is critical, because heat is generated in the bearing at a very fast rate. In the case of unexpected oil starvation, the bearing can undergo a catastrophic (sudden) failure. Such catastrophic failures are often in the form of bearing seizure (welding of journal and bearing) or failure due to the melting of the bearing lining material, which is often a white metal of low melting temperature. Without a continuous supply of lubricant, the temperature rises because of the high friction from direct contact.

Oil starvation can result from several causes, such as failure of the oil pump or the motor. In addition, the lubricant can be lost due to a leak in the oil system. The risk of a catastrophic failure in hydrodynamic journal bearings is preventing their utilization in important applications where safety is involved, such as in aircraft engines, where rolling-element bearings with limited fatigue life are predominantly used.

For low-speed applications and moderate loads, plain sleeve bearings with boundary lubrication can provide reliable long-term service and can be an adequate alternative to rolling-element bearings. In most industrial applications, these bearings are made of bronze and lubricated by grease or are self-lubricated sintered bronze. For light-duty applications, plastic bearings are widely used. As long as the product of the average pressure and speed, PV, is within the specified design values, the two parameters do not generate excessive temperature.

If plain sleeve bearings are designed properly, they wear gradually and do not pose the problem of unexpected failure, such as fatigue failure in rolling-element bearings. When they wear out, it is possible to keep the machine running for a longer period before the bearing must be replaced. This is an important advantage in manufacturing machinery, because it prevents the financial losses involved in a sudden shutdown. Replacement of a plain sleeve bearing can, at least, be postponed to a more convenient time (in comparison to a rolling bearing). In manufacturing, unexpected shutdown can result in expensive loss of production. For sleeve bearings with grease lubrication or oil-impregnated porous metal bearings, the manufacturers provide tables of maximum speed and load as well as maximum PV value, which indicate the limits for each bearing material. If these limits are not exceeded, the temperature will not be excessive, resulting in a reliable operation of the bearing. A solved problem is included at the end of this chapter.

Sleeve bearings have several additional advantages. They can be designed so that it is easier to mount and replace them, in comparison to rolling bearings. Sleeve bearings can be of split design so that they can be replaced without removing the shaft. Also, sleeve bearings can be designed to carry much higher loads, in comparison to rolling bearings, where the load is limited due to the high "hertz" stresses. In addition, sleeve bearings are usually less sensitive than rolling bearings to dust, slurry, or corrosion caused by water infiltration.

However, rolling bearings have many other advantages. One major advantage is their relatively low-cost maintenance. Rolling bearings can operate with a minimal quantity of lubrication. Grease-packed and sealed rolling bearings are very convenient for use in many applications, since they do not require further lubrication. This significantly reduces the maintenance cost.

In many cases, machine designers select a rolling bearing only because it is easier to select from a manufacturer's catalogue. However, the advantages and disadvantages of each bearing type must be considered carefully for each application. Bearing selection has long-term effects on the life of the machine as well as on maintenance expenses and the economics of running the machine over its full life cycle. In manufacturing plants, loss of production is a dominant consideration. In certain industries, unplanned shutdown of a machine for even 1 hour may be more expensive than the entire maintenance cost or the cost of the best bearing. For these reasons, in manufacturing, bearing failure must be prevented without consideration of bearing cost. In aviation, bearing failure can result in the loss of lives; therefore, careful bearing selection and design are essential.

1.8 BEARINGS FOR PRECISION APPLICATIONS

High-precision bearings are required for precision applications, mostly in machine tools and measuring machines, where the shaft (referred to as the *spindle* in machine tools) is required to run with extremely low radial or axial run-out. Therefore, precision bearings are often referred to as *precision spindle bearings*. Rolling bearings are widely used in precision applications because in most cases they provide adequate precision at reasonable cost.

High-precision rolling-element bearings are manufactured and supplied in several classes of precision. The precision is classified by the maximum allowed tolerance of spindle run-out. In machine tools, spindle run-out is undesirable because it results in machining errors. Radial spindle run-out in machine tools causes machining errors in the form of deviation from roundness, while axial run-out causes manufacturing errors in the form of deviation from flat surfaces. Rolling-element bearing manufacturers use several tolerance classifications, but the most common are the following three tolerance classes of precision spindle bearings (FAG 1986):

Precision class	Maximum run-out (μm)
1. High-precision rolling-element bearings	2.0
2. Special-precision bearings	1.0
3. Ultraprecision bearings	0.5

Detailed discussion of rolling-element bearing precision is included in Chapter 13. Although rolling-element bearings are widely used in high-precision machine tools, there is an increasing requirement for higher levels of precision. Rolling-element bearings always involve a certain level of noise and vibrations, and there is a limit to their precision. The following is a survey of other bearing types, which can be alternatives for high precision applications

1.9 NONCONTACT BEARINGS FOR PRECISION APPLICATIONS

Three types of noncontact bearings are of special interest for precision machining, because they can run without any contact between the sliding surfaces in the bearing. These noncontact bearings are hydrostatic, hydrodynamic, and electromagnetic bearings. The bearings are noncontact in the sense that there is a thin clearance of lubricant or air between the journal (spindle in machine tools) and the sleeve. In addition to the obvious advantages of low friction and the absence of wear, other characteristics of noncontact bearings are important for ultra-high-precision applications. One important characteristic is the isolation of the spindle from vibrations. Noncontact bearings isolate the spindle from sources of vibrations in the machine or even outside the machine. Moreover, direct contact friction can induce noise and vibrations, such as in stick-slip friction; therefore, noncontact bearings offer the significant advantage of smooth operation for high-precision applications. The following discussion makes the case that hydrostatic bearings are the most suitable noncontact bearing for high-precision applications such as ultra-high-precision machine tools.

The difference between hydrodynamic and hydrostatic bearings is that, for the first, the pressure is generated inside the bearing clearance by the rotation action of the journal. In contrast, in a hydrostatic bearing, the pressure is supplied by an external pump. Hydrodynamic bearings have two major disadvantages that rule them out for use in machine tools: (a) low stiffness at low loads, and (b) at low speeds, not completely noncontact, since the fluid film thickness is less than the size of surface asperities.

In order to illustrate the relative advantage of hydrostatic bearings, it is interesting to compare the nominal orders of magnitude of machining errors in the form of deviation from roundness. The machining errors result from spindle run-out. Higher precision can be achieved by additional means to isolate the spindle from external vibrations, such as from the driving motor. In comparison to rolling-element bearings, experiments in hydrostatic-bearings indicated the following machining errors in the form of deviation from roundness by machine tools with a spindle supported by hydrostatic bearings (see Donaldson and Patterson, 1983 and Rowe, 1967):

Precison class	Machining error (μm)
1. Regular hydrostatic bearing	0.20
2. When vibrations are isolated from the drive	0.05

Experiments indicate that it is important to isolate the spindle from vibrations from the drive. Although a hydrostatic bearing is supported by a fluid film, the film has relatively high stiffness and a certain amount of vibrations can pass through, so additional means for isolation of vibrations is desirable. The preceding figures illustrate that hydrostatic bearings can increase machining precision, in comparison to precision rolling bearings, by one order of magnitude. The limits of hydrostatic bearing technology probably have not been reached yet.

1.10 BEARING SUBJECTED TO FREQUENT STARTS AND STOPS

In addition to wear, high start-up friction in hydrodynamic journal bearings increases the temperature of the journal much more than that of the sleeve, and there is a risk of bearing seizure. There is uneven thermal expansion of the journal and bearing, and under certain circumstances the clearance can be completely eliminated, resulting in bearing seizure. Bearing seizure poses a higher risk than wear, since the failure is catastrophic. This is the motivation for much research aimed at reducing start-up friction.

According to hydrodynamic theory, a very thin fluid film is generated even at low journal speed. But in practice, due to surface roughness, vibrations, and disturbances, a certain high minimum speed is required to generate an adequate film thickness so that occasional contacts and wear between the sliding surfaces are prevented. The most severe wear occurs during starting because the journal is accelerated from zero velocity, where there is relatively high static friction. The lubricant film thickness increases with speed and must be designed to separate the journal and sleeve completely at the rated speed of the machine. During starting, the speed increases, the fluid film builds up its thickness, and friction is reduced gradually.

In applications involving frequent starts, rolling element bearings are usually selected because they are less sensitive to wear during start-up and stopping. But this is not always the best solution, because rolling bearings have a relatively short fatigue-life when the operating speed is very high. In Chapter 18, it is shown that it is possible to solve these problems by using a "*composite* *bearing*," which is a unique design of hydrodynamic and rolling bearings in series (Harnoy and Rachoor, 1993).

Manufacturers continually attempt to increase the speed of machinery in order to reduce its size. The most difficult problem is a combination of high operating speed with frequent starting and stopping. At very high speed, the life of the rolling-element bearing is short, because fatigue failure is partly determined by the number of cycles, and high speed results in reduced life (measured in hours). In addition to this, at high speed the centrifugal forces of the rolling elements (balls or rollers) increase the fatigue stresses. Furthermore, the temperature of the bearing rises at high speeds; therefore, the fatigue resistance of the material deteriorates. The centrifugal forces and temperature exacerbate the problem and limit the operating speed at which the fatigue life is acceptable. Thus the two objectives, longer bearing life and high operating speed, are in conflict when rolling-element bearings are used. Hydrodynamic bearings operate well at high speeds but are not suitable for frequent-starting applications.

Replacing the hydrodynamic bearing with an externally pressurized hydrostatic bearing can eliminate the wear and friction during starting and stopping. But a hydrostatic bearing is uneconomical for many applications, since it requires an oil pump system, an electric motor, and flow restrictors in addition to the regular bearing system. An example of a unique design of a composite bearing—hydrodynamic and rolling bearings in series—is described in chapter 18. This example is a low-cost solution to the problem involved when high-speed machinery are subjected to frequent starting and stopping.

In conclusion, the designer should keep in mind that the optimum operation of the rolling bearing is at low and moderate speeds, while the best performance of the hydrodynamic bearing is at relatively high speeds. Nevertheless, in aviation, high-speed rolling-element bearings are used successfully. These are expensive high-quality rolling bearings made of special steels and manufactured by unique processes for minimizing impurity and internal microscopic cracks. Materials and manufacturing processes for rolling bearings are discussed in Chapter 13.

1.11 EXAMPLE PROBLEMS

Example Problem 1-1

PV Limits

Consider a shaft supported by two bearings, as shown in Fig. 1-6. The two bearings are made of self-lubricated sintered bronze. The bearing on the left side is under radial load, $F_r = 1200$ lbf, and axial load, $F_a = 0.5F_r$. (The bearing on the right supports only radial load). The journal diameter is $D = 1$ inch, and the

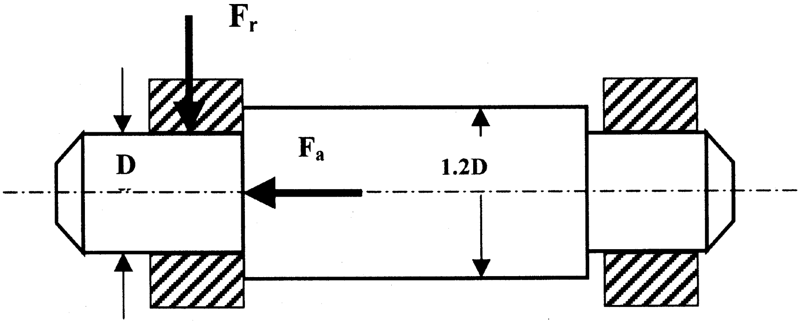

FIG. 1-6 Journal bearing under radial and thrust load.

bearing length $L = D$. The thrust load is supported against a shaft shoulder of diameter $D_1 = 1.2D$. The shaft speed is $N = 1000$ RPM.

Sintered bronze has the following limits:

Surface velocity limit, V, is 6 m/s, or 1180 ft/min.
Surface pressure limit, P, is 14 MPa, or 2000 psi.
PV limit is 110,000 psi-ft/min, or 3.85×10^6 Pa-m/s

a. For the left-side bearing, find the P, V, and PV values for the thrust bearing (in imperial units) and determine if this thrust bearing can operate with a sintered bronze bearing material.
b. For the left-side bearing, also find the P, V, and PV values for the radial bearing (in imperial units) and determine if the radial bearing can operate with sintered bronze bearing material.

Summary of data for left bearing:

$$F_r = 1200 \text{ lbf}$$
$$F_a = 0.5F_r = 600 \text{ lbf}$$
$$D = 1 \text{ in.} = 0.083 \text{ ft (journal diameter)}$$
$$D_1 = 1.2 \text{ in.} = 0.1 \text{ ft (shoulder diameter)}$$
$$N = 1000 \text{ RPM}$$

Solution

a. Thrust Bearing

Calculation of Average Pressure, P. The average pressure, P, in the axial direction is,

$$P = \frac{F_a}{A}$$

where

$$A = \frac{\pi}{4}(D_1^2 - D^2)$$

This is the shoulder area that supports the thrust load. Substituting yields

$$P = \frac{4F_a}{\pi(D_1^2 - D^2)} = \frac{4 \times 600}{\pi(1.2^2 - 1^2)} = 1736 \text{ psi}$$

This is within the allowed limit of $P_{\text{allowed}} = 2000$ psi.

b. Calculation of Average Surface Velocity of Thrust Bearing, V_{th}. The average velocity of a thrust bearing is at the average diameter, $(D_1 + D)/2$:

$$V_{\text{th}} = \omega R_{\text{av}} = \omega \frac{D_{\text{av}}}{2}$$

where $\omega = 2\pi N$ rad/min. Substituting yields

$$V_{\text{th}} = 2\pi N R_{\text{av}} = \frac{2\pi N(D_1 + D)}{4} = 0.5\pi N(D_1 + D)$$

Substitution in the foregoing equation yields

$$V_{\text{th}} = 0.5\pi \times 1000 \text{ rev/min } (0.1 + 0.083) \text{ ft} = 287.5 \text{ ft/min}$$

This is well within the allowed limit of $V_{\text{allowed}} = 1180$ ft/min.

c. Calculation of Actual Average PV Value for the Thrust Bearing:

$$PV = 1736 \text{ psi} \times 287.5 \text{ ft/min} = 500 \times 10^3 \text{ psi-ft/min}$$

Remark. The imperial units for PV are of pressure, in psi, multiplied by velocity, in ft/min.

Conclusion. Although the limits of the velocity and pressure are met, the PV value exceeds the allowed limit for self-lubricated sintered bronze bearing material, where the PV limit is 110,000 psi-ft/min.

b. Radial Bearing

Calculation of Average Pressure

$$P = \frac{F_r}{A}$$

where $A = LD$ is the projected area of the bearing. Substitution yields

$$P = \frac{F_r}{LD} = \frac{1200 \text{ lbf}}{1 \text{ in.} \times 1 \text{ in.}} = 1200 \text{ psi}$$

Calculation of Journal Surface Velocity. The velocity is calculated as previously; however, this time the velocity required is the velocity at the surface of the 1-inch shaft, $D/2$.

$$V_r = \omega \frac{D}{2} = 2\pi n \frac{D}{2} = \pi \times 1000 \text{ rev/min} \times 0.083 \text{ ft} = 261 \text{ ft/min}$$

Calculation of Average PV Value:

$$PV = 1200 \text{ psi} \times 261 \text{ ft/min} = 313 \times 10^3 \text{ psi-ft/min}$$

In a similar way to the thrust bearing, the limits of the velocity and pressure are met; however, the PV value exceeds the allowed limit for sintered bronze bearing material, where the PV limit is 110,000 psi-ft/min.

Example Problem 1-2

Calculation of Bearing Forces

In a gearbox, a spur gear is mounted on a shaft at equal distances from two supporting bearings. The shaft and gear turn together at a speed of 600 RPM. The gearbox is designed to transmit a maximum power of 5 kW. The gear pressure angle is $\phi = 20°$. The diameter of the gear pitch circle is $d_p = 5$ in.

Remark. The gear pressure angle ϕ (PA) is the angle between the line of force action (normal to the contact area) and the direction of the velocity at the pitch point (see Fig. 1-7). Two standard pressure angles ϕ for common involute gears are $\phi = 20°$ and $\phi = 14.5°$. Detailed explanation of the geometry of gears is included in many machine design textbooks, such as *Machine Design*, by Deutschman et al. (1975), or *Machine Design*, by Norton (1996).

a. Find the reaction force on each of the two bearings supporting the shaft.
b. The ratio of the two bearings' length and bore diameter is $L/D = 0.5$. The bearings are made of sintered bronze material ($PV = 110{,}000$ psi-ft/min). Find the diameter and length of each bearing that is required in order not to exceed the PV limit.

Solution

a. Reaction Forces

Given:

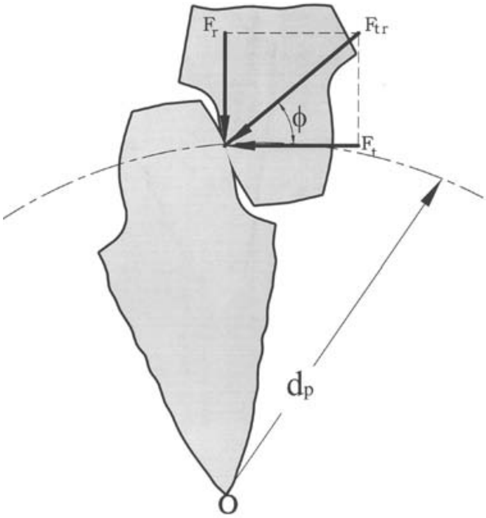

FIG. 1-7 Gear pressure angle.

Rotational speed $N = 600\,\text{RPM}$
Power $\dot{E} = 5000$ W
Diameter of pitch circle $d_p = 5\,\text{in.}$
Pressure angle $\phi = 20^\circ$
$PV_{\text{allowed}} = 110{,}000\,\text{psi-ft/min}$
$L/D = 0.5$ (the bore diameter of the bearing, D, is very close to that of the journal, d)

Conversion Factors:

$$1\text{psi} = 6895\ \text{N/m}^2$$
$$1\ \text{ft/min} = 5.08 \times 10^{-3}\ \text{m/s}$$
$$1\ \text{psi-ft/min} = 35\ \text{N/m}^2\text{-m/s}$$

The angular velocity, ω, of the journal is:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 600}{60} = 52.83\ \text{rad/s}$$

Converting the diameter of the pitch circle to SI units,

$$d_p = 5\ \text{in.} \times 0.0254\ \text{m/in.} = 0.127\ \text{m}$$

The tangential force, F_t, acting on the gear can now be derived from the power, $\dot{E}$:

$$\dot{E} = T\omega$$

where the torque is

$$T = \frac{F_t d_p}{2}$$

Substituting into the power equation:

$$\dot{E} = \frac{F_t d_p \omega}{2}$$

and solving for F_t and substituting yields

$$F_t = \frac{2\dot{E}}{d_p \omega} = \frac{2 \times 5000 \text{ Nm/s}}{0.127 \text{ m} \times 62.83 \text{ rad/s}} = 1253.2 \text{ N}$$

In spur gears, the resultant force acting on the gear is $F = F_{tr}$ (Fig. 1-7)

$$\cos\phi = \frac{F_t}{F}$$

so

$$F = \frac{F_t}{\cos\phi} = \frac{1253.2}{\cos 20^\circ} = 1333.6 \text{ N}$$

The resultant force, F, acting on the gear is equal to the radial component of the force acting on the bearing. Since the gear is equally spaced between the two bearings supporting the shaft, each bearing will support half the load, F. Therefore, the radial reaction, W, of each bearing is

$$W = \frac{F}{2} = \frac{1333.6}{2} = 666.8 \text{ N}$$

b. Bearing Dimensions

The average bearing pressure, P, is

$$P = \frac{F}{A}$$

Here, $A = LD$, where D is the journal diameter and A is the projected area of the contact surface of journal and bearing surface,

$$P = \frac{F}{LD}$$

The velocity of shaft surface, V, is

$$V = \frac{D}{2}\omega$$

Therefore,

$$PV = \frac{W}{LD}\frac{D}{2}\omega$$

Since $L/D = 0.5$, $L = 0.5D$, substituting and simplifying yields

$$PV = \frac{W}{0.5D^2}\frac{\omega D}{2} = \frac{W}{D}\omega$$

The PV limit for self-lubricated sintered bronze is given in English units, converted to SI units, the limit is

$$110{,}000 \text{ psi-ft/min} \times 35 \text{ N/m}^2\text{-m/s} = 3{,}850{,}000 \text{ Pa-m/s}$$

Solving for the journal diameter, D, and substituting yields the diameter of the bearing:

$$D = \frac{W\omega}{PV} = \frac{666.8 \text{ N} \times 62.83 \text{ rad/s}}{3.85 \times 10^6 \text{ Pa} \cdot \text{m/s}} = 0.011 \text{ m}, \qquad \text{or} \qquad \text{D} = 11 \text{ mm}$$

The length of the bearing, L, is

$$L = 0.5D = 0.5 \times 11 \text{ mm} = 5.5 \text{ mm}$$

The resulting diameter, based on a PV calculation, is very small. In actual design, the journal is usually of larger diameter, based on strength-of-material considerations, because the shaft must have sufficient diameter for transmitting the torque from the drive.

Example Problem 1-3

Calculation of Reaction Forces

In a gearbox, one helical gear is mounted on a shaft at equal distances from two supporting bearings. The *helix angle* of the gear is $\psi = 30^\circ$, and the pressure angle (PA) is $\phi = 20^\circ$. The shaft speed is 3600 RPM. The gearbox is designed to transmit maximum power of 20 kW. The diameter of the pitch circle of the gear is equal to 5 in. The right-hand-side bearing is supporting the total thrust load. Find the axial and radial loads on the right-hand-side bearing and the radial load on the left-side bearing.

Solution

The angular velocity of the shaft, ω, is:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 3600}{60} = 377 \text{ rad/s}$$

Torque produced by the gear is $T = F_t d_p/2$. Substituting this into the power equation, $\dot{E} = T\omega$, yields:

$$\dot{E} = \frac{F_t d_p}{2}\omega$$

Solving for the tangential force, F_t, results in

$$F_t = \frac{2\dot{E}}{d_p\omega} = \frac{2 \times 20{,}000 \text{ N-m/s}}{0.127 \text{ m} \times 377 \text{ rad/s}} = 836 \text{ N}$$

Once the tangential component of the force is solved, the resultant force, F, and the thrust load (axial force), F_a, can be calculated as follows:

$$F_a = F_t \tan\psi$$
$$F_a = 836 \text{ N} \times \tan 30^\circ = 482 \text{ N}$$

and the radial force component is:

$$\begin{aligned} F_r &= F_t \tan\phi \\ &= 836 \text{ N} \times \tan 20^\circ \\ &= 304 \text{ N} \end{aligned}$$

The force components, F_t and F_r, are both in the direction normal to the shaft centerline. The resultant of these two gear force components, F_{tr}, is cause for the radial force component in the bearings. The resultant, F_{tr}, is calculated by the equation (Fig. 1-7)

$$F_{tr} = \sqrt{F_t^2 + F_r^2} = \sqrt{836^2 + 304^2} = 890 \text{ N}$$

The resultant force, F_{tr}, on the gear is supported by the two bearings. It is a radial bearing load because it is acting in the direction normal to the shaft centerline. Since the helical gear is mounted on the shaft at equal distances from both bearings, each bearing will support half of the radial load,

$$W_r = \frac{F_{tr}}{2} = \frac{890 \text{ N}}{2} = 445 \text{ N}$$

However, the thrust load will act only on the right-hand bearing:

$$F_a = 482 \text{ N}$$

Example Problem 1-4

Calculation of Reaction Forces

In a gearbox, two helical gears are mounted on a shaft as shown in Fig. 1-1. The helix angle of the two gears is $\psi = 30^\circ$, and the pressure angle (PA) is $\phi = 20^\circ$. The shaft speed is 3600 RPM. The gearbox is designed to transmit a maximum power of 10 kW. The pitch circle diameter of the small gear is equal to 5 in. and that of the large gear is of 15 in.

a. Find the axial reaction force on each of the two gears and the resultant axial force on each of the two bearings supporting the shaft.
b. Find the three load components on each gear, F_t, F_r, and F_a.

Solution

Given:

Helix angle	$\psi = 30^\circ$
Pressure angle	$\phi = 20^\circ$
Rotational speed	$N = 3600$ RPM
Power	10 kW
Diameter of pitch circle (small)	$d_{P1} = 5$ in.
Diameter of pitch circle (large)	$d_{P2} = 15$ in.

Small Gear

a. Axial Reaction Forces.

The first step is to solve for the tangential force acting on the small gear, F_t. It can be derived from the power, E, and shaft speed:

$$\dot{E} = T\omega$$

where the torque is $T = F_t d_p/2$. The angular speed ω in rad/s is

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3600}{60} = 377 \text{ rad/s}$$

Substituting into the power equation yields

$$\dot{E} = \frac{F_t d_p \omega}{2}$$

The solution for F_t acting on the small gear is given by

$$F_t = \frac{2\dot{E}}{d_p \omega}$$

The pitch diameter is 5 in., or $d_p = 0.127$ m. After substitution, the tangential force is

$$F_t = \frac{2 \times 10{,}000 \text{ W}}{(0.127 \text{ m}) \times (377 \text{ rad/s})} = 418 \text{ N}$$

The radial force on the gear (F_r in Fig 1-1) is:

$$F_r = F_t \tan\phi$$

$$F_r = 418 \text{ N} \times \tan 20^\circ$$

$$F_r = 152 \text{ N}$$

b. Calculation of the Thrust Load, F_a:

The axial force on the gear is calculated by the equation,

$$F_a = F_t \tan\psi$$

$$= 418 \text{ N} \times \tan 30^\circ$$

$$= 241 \text{ N}$$

$$F_t = 418 \text{ N}$$

$$F_r = 152 \text{ N}$$

$$F_a = 241 \text{ N}$$

Large gear

The same procedure is used for the large gear, and the results are:

$$F_t = 140 \text{ N}$$

$$F_r = 51 \text{ N}$$

$$F_a = 81 \text{ N}$$

Thrust Force on a Bearing

One bearing supports the total thrust force on the shaft. The resultant thrust load on one bearing is the difference of the two axial loads on the two gears, because the thrust reaction forces in the two gears are in opposite directions (see Fig. 1-1):

$$F_a \text{ (bearing)} = 241 - 81 = 180 \text{ N}$$

Problems

1-1 Figure 1-4 shows a drawing of a hydrostatic journal bearing system that can support only a radial load. Extend this design and sketch a hydrostatic bearing system that can support combined radial and thrust loads.

1-2 In a gearbox, a spur gear is mounted on a shaft at equal distances from two supporting bearings. The shaft and mounted gear turn together at a speed of 3600 RPM. The gearbox is designed to transmit a maximum power of 3 kW. The gear contact angle is $\phi = 20^\circ$. The pitch diameter of the gear is $d_p = 30$ in. Find the radial force on each of the two bearings supporting the shaft.

The ratio of the two bearings' length and diameter is $L/D = 0.5$. The bearings are made of acetal resin material with the following limits:

Surface velocity limit, V, is 5 m/s.
Average surface-pressure limit, P, is 7 MPa.
PV limit is 3000 psi-ft/min.

Find the diameter of the shaft in order not to exceed the stated limits.

1-3 A bearing is made of Nylon sleeve. Nylon has the following limits as a bearing material:

Surface velocity limit, V, is 5 m/s.
Average surface-pressure limit, P, is 6.9 MPa.
PV limit is 3000 psi-ft/min.

The shaft is supported by two bearings, as shown in Fig. 1-6. The bearing on the left side is under a radial load $F_r = 400\ N$ and an axial load $F_a = 200\ N$. (The bearing on the left supports the axial force.) The journal diameter is d, and the bearing length $L = d$. The thrust load is supported against a shaft shoulder of diameter $D = 1.2d$. The shaft speed is $N = 800$ RPM. For the left-side bearing, find the minimum journal diameter d that would result in P, V, and PV below the allowed limits, in the radial and thrust bearings.

1-4 In a gearbox, one helical gear is mounted on a shaft at equal distances from two supporting bearings. The helix angle of the gear is $\psi = 30°$, and the pressure angle (PA) is $\phi = 20°$. The shaft speed is 1800 RPM. The gearbox is designed to transmit a maximum power of 12 kW. The diameter of the pitch circle of the gear is 5 in. The right-hand-side bearing is supporting the total thrust load. Find the axial and radial load on the right-hand-side bearing and the radial load on the left-side bearing.

1-5 In a gearbox, two helical gears are mounted on a shaft as shown in Fig. 1-1. The helix angle of the two gears is $\phi = 30°$, and the pressure angle (PA) is $\phi = 20°$. The shaft speed is 3800 RPM. The gearbox is designed to transmit a maximum power of 15 kW. The pitch circle diameters of the two gears are 5 in. and 15 in. respectively.

a. Find the axial reaction force on each of the two gears and the resultant axial force on each of the two bearings supporting the shaft.
b. Find the three load components on each gear, F_t , F_r, and F_a.