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MACHINE ELEMENTS THAT ABSORB AND STORE ENERGY

CHAPTER 6
SPRINGS

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GENERAL NOMENCLATURE†

A	Area, mm^2 (in^2)
b	Width, mm (in)
C	Spring index, D/d
d	Wire diameter, mm (in)
D	Mean diameter (OD minus wire diameter), mm (in)
E	Modulus of elasticity in tension or Young's modulus, MPa (psi)
f	Deflection, mm (in)
g	Gravitational constant, 9.807 m/s^2 (386.4 in/s^2)
G	Shear modulus or modulus of rigidity, MPa (psi)
I	Moment of inertia, mm^4 (in^4)
ID	Inside diameter, mm (in)
k	Spring rate, N/mm (lb/in) or N·mm/r (lb·in/r)

† The symbols presented here are used extensively in the spring industry. They may differ from those used elsewhere in this Handbook.

K	Design constant
K_w	Stress correction factor for helical springs
L	Length, mm (in)
L_f	Free length, mm (in)
L_s	Length at solid, mm (in)
M	Moment or torque, N·mm (lb·in)
n	Frequency, Hz
N_a	Number of active coils or waves
N_t	Total number of coils
OD	Outside diameter, mm (in)
P	Load, N (lbf)
r	Radius, mm (in)
S	Stress, MPa (psi)
TS	Tensile strength, MPa (psi)
t	Thickness, mm (in)
YS	Yield strength, MPa (psi)
ρ	Density, g/cm^3 (lb/in^3)
θ	Angular deflection, expressed in number of revolutions
μ	Poisson's ratio

6.1 INTRODUCTION

Spring designing is a complex process. It is an interactive process which may require several iterations before the best design is achieved. Many simplifying assumptions have been made in the design equations, and yet they have proved reliable over the years. When more unusual or complex designs are required, designers should rely on the experience of a spring manufacturer.

The information in this chapter is offered for its theoretical value and should be used accordingly.

6.2 GLOSSARY OF SPRING TERMINOLOGY

active coils: those coils which are free to deflect under load.

baking: heating of electroplated springs to relieve hydrogen embrittlement.

buckling: bowing or lateral displacement of a compression spring; this effect is related to slenderness ratio *L/D*.

closed and ground ends: same as *closed ends,* except that the first and last coils are ground to provide a flat bearing surface.

closed ends: compression spring ends with coil pitch angle reduced so that they are square with the spring axis and touch the adjacent coils.

close-wound: wound so that adjacent coils are touching.

deflection: motion imparted to a spring by application or removal of an external load.

elastic limit: maximum stress to which a material may be subjected without permanent set.

endurance limit: maximum stress, at a given stress ratio, at which material will operate in a given environment for a stated number of cycles without failure.

free angle: angular relationship between arms of a helical torsion spring which is not under load.

free length: overall length of a spring which is not under load.

gradient: see *rate.*

heat setting: a process to prerelax a spring in order to improve stress-relaxation resistance in service.

helical springs: springs made of bar stock or wire coiled into a helical form; this category includes compression, extension, and torsion springs.

hooks: open loops or ends of extension springs.

hysteresis: mechanical energy loss occurring during loading and unloading of a spring within the elastic range. It is illustrated by the area between load-deflection curves.

initial tension: a force that tends to keep coils of a close-wound extension spring closed and which must be overcome before the coils start to open.

loops: formed ends with minimal gaps at the ends of extension springs.

mean diameter: in a helical spring, the outside diameter minus one wire diameter.

modulus in shear or torsion (modulus of rigidity G): coefficient of stiffness used for compression and extension springs.

modulus in tension or bending (Young's modulus E): coefficient of stiffness used for torsion or flat springs.

moment: a product of the distance from the spring axis to the point of load application and the force component normal to the distance line.

natural frequency: lowest inherent rate of free vibration of a spring vibrating between its own ends.

pitch: distance from center to center of wire in adjacent coils in an open-wound spring.

plain ends: end coils of a helical spring having a constant pitch and with the ends not squared.

plain ends, ground: same as *plain ends,* except that wire ends are ground square with the axis.

rate: spring gradient, or change in load per unit of deflection.

residual stress: stress mechanically induced by such means as set removal, shot peening, cold working, or forming; it may be beneficial or not, depending on the spring application.

set: permanent change of length, height, or position after a spring is stressed beyond material's elastic limit.

set point: stress at which some arbitrarily chosen amount of set (usually 2 percent) occurs; set percentage is the set divided by the deflection which produced it.

set removal: an operation which causes a permanent loss of length or height because of spring deflection.

solid height: length of a compression spring when deflected under load sufficient to bring all adjacent coils into contact.

spiral springs: springs formed from flat strip or wire wound in the form of a spiral, loaded by torque about an axis normal to the plane of the spiral.

spring index: ratio of mean diameter to wire diameter.

squared and ground ends: see *closed and ground ends.*

squared ends: see *closed ends.*

squareness: angular deviation between the axis of a compression spring in a free state and a line normal to the end planes.

stress range: difference in operating stresses at minimum and maximum loads.

stress ratio: minimum stress divided by maximum stress.

stress relief: a low-temperature heat treatment given springs to relieve residual stresses produced by prior cold forming.

torque: see *moment.*

total number of coils: the sum of the number of active and inactive coils in a spring body.

6.3 SELECTION OF SPRING MATERIALS

6.3.1 Chemical and Physical Characteristics

Springs are resilient structures designed to undergo large deflections within their elastic range. It follows that the materials used in springs must have an extensive elastic range.

Some materials are well known as spring materials. Although they are not specifically designed alloys, they do have the elastic range required. In steels, the medium- and high-carbon grades are suitable for springs. Beryllium copper and phosphor bronze are used when a copper-base alloy is required. The high-nickel alloys are used when high strength must be maintained in an elevated-temperature environment.

The selection of material is always a cost-benefit decision. Some factors to be considered are costs, availability, formability, fatigue strength, corrosion resistance, stress relaxation, and electric conductivity. The right selection is usually a compromise among these factors. Table 6.1 lists some of the more commonly used metal alloys and includes data which are useful in material selection.

Surface quality has a major influence on fatigue strength. This surface quality is a function of the control of the material manufacturing process. Materials with high surface integrity cost more than commercial grades but must be used for fatigue applications, particularly in the high cycle region.

6.3.2 Heat Treatment of Springs

Heat treatment is a term used in the spring industry to describe both low- and high-temperature heat treatments. Low-temperature heat treatment, from 350 to 950°F (175 to 510°C), is applied to springs after forming to reduce unfavorable residual stresses and to stabilize parts dimensionally.

When steel materials are worked in the spring manufacturing process, the yield point is lowered by the unfavorable residual stresses. A low-temperature heat treatment restores the yield point. Most heat treatment is done in air, and the minor oxide that is formed does not impair the performance of the springs.

When hardened high-carbon-steel parts are electroplated, a phenomenon known as *hydrogen embrittlement* occurs, in which hydrogen atoms diffuse into the metallic lattice, causing previously sound material to crack under sustained stress. Low-temperature baking in the range of 375 to 450°F (190 to 230°C) for times ranging from 0.5 to 3 h, depending on the type of plating and the degree of embrittlement, will reduce the concentration of hydrogen to acceptable levels.

High-temperature heat treatments are used to strengthen annealed material after spring forming. High-carbon steels are austenitized at 1480 to 1652°F (760 to 900°C), quenched to form martensite, and then tempered to final hardness. Some nickel-base alloys are strengthened by high-temperature aging. Oxidation will occur at these temperatures, and it is advisable to use a protective atmosphere in the furnace.

Heat treatments for many common materials are listed in Table 6.2. Unless otherwise noted, 20 to 30 min at the specified temperature is sufficient. Thin, flimsy cross-sectional springs can be distorted by the heat-treatment operation. Pretempered materials are available for use in such cases.

6.3.3 Relaxation

The primary concern in elevated-temperature applications is stress relaxation. *Stress relaxation* is the loss of load or spring length that occurs when a spring is held at load or cycled under load. Heat affects modulus and tensile strength. In addition to the factors of stress, time, and temperature which affect relaxation, other controllable factors are

1. Alloy type—the highly alloyed materials are generally more temperature-resistant.
2. Residual stresses—such stresses remaining from forming operations are detrimental to relaxation resistance. Use the highest practical stress-relief temperature.
3. Heat setting—procedures employed to expose springs under some load to stress and heat to prepare them for a subsequent exposure. The effect is to remove the first stage of relaxation.

6.3.4 Corrosion

The specific effect of a corrosive environment on spring performance is difficult to predict. In general, if the environment causes damage to the spring surface, the life and the load-carrying ability of the spring will be reduced.

The most common methods of combating corrosion are to use materials that are resistant or inert to the particular corrosive environment or to use coatings that slow

TABLE 6.1 Typical Properties of Common Spring Materials

Common Name	Young's Modulus E (1) MPa 10^3	Young's Modulus E (1) psi 10^6	Modulus of Rigidity G (1) MPa 10^3	Modulus of Rigidity G (1) psi 10^6	Density (1) g/cm^3 (lb/in^3)	Electrical Conductivity (1) % IACS	Sizes Normally Available (2) Min. mm (in.)	Sizes Normally Available (2) Max. mm (in.)	Typical Surface Quality (3)	Maximum Service Temperature (4) °C	Maximum Service Temperature (4) °F
Carbon Steel Wires:											
Music (5)	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.10 (0.004)	6.35 (0.250)	a	120	250
Hard Drawn (5)	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.13 (0.005)	16 (0.625)	c	150	250
Oil Tempered	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.50 (0.020)	16 (0.625)	c	150	300
Valve Spring	207	(30)	79.3	(11.5)	7.86 (0.284)	7	1.3 (0.050)	6.35 (0.250)	a	150	300
Alloy Steel Wires:											
Chrome Vanadium	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.50 (0.020)	11 (0.435)	a,b	220	425
Chrome Silicon	207	(30)	79.3	(11.5)	7.86 (0.284)	5	0.50 (0.020)	9.5 (0.375)	a,b	245	475
Stainless Steel Wires:											
Austenitic Type 302	193	(28)	69.0	(10.)	7.92 (0.286)	2	0.13 (0.005)	9.5 (0.375)	b	260	500
Precipitation Hardening 17-7 PH	203	(29.5)	75.8	(11)	7.81 (0.282)	2	0.08 (0.002)	12.5 (0.500)	b	315	600
NiCr A286	200	(29)	71.7	(10.4)	8.03 (0.290)	2	0.40 (0.016)	5 (0.200)	b	510	950
Copper Base Alloy Wires:											
Phosphor Bronze (A)	103	(15)	43.4	(6.3)	8.86 (0.320)	15	0.10 (0.004)	12.5 (0.500)	b	95	200
Silicon Bronze (A)	103	(15)	38.6	(5.6)	8.53 (0.308)	7	0.10 (0.004)	12.5 (0.500)	b	95	200
Silicon Bronze (B)	117	(17)	44.1	(6.4)	8.75 (0.316)	12	0.10 (0.004)	12.5 (0.500)	b	95	200
Beryllium Copper	128	(18.5)	48.3	(7.0)	8.26 (0.298)	21	0.08 (0.003)	12.5 (0.500)	b	205	400
Spring Brass, CA260	110	(16)	42.0	(6.0)	8.53 (0.308)	17	0.10 (0.004)	12.5 (0.500)	b	95	200
Nickel Base Alloys:											
Inconel® Alloy 600	214	(31)	75.8	(11)	8.43 (0.304)	1.5	0.10 (0.004)	12.5 (0.500)	b	320	700
Inconel Alloy X750	214	(31)	79.3	(11.5)	8.25 (0.298)	1	0.10 (0.004)	12.5 (0.500)	b	595	1100
Ni-Span-C®	186	(27)	62.9	(9.7)	8.14 (0.294)	1.6	0.10 (0.004)	12.5 (0.500)	b	95	200
Monel® Alloy 400	179	(26)	66.2	(9.6)	8.83 (0.319)	3.5	0.05 (0.002)	9.5 (0.375)	b	230	450
Monel Alloy K500	179	(26)	66.2	(9.6)	8.46 (0.306)	3	0.05 (0.002)	9.5 (0.375)	b	260	500

Carbon Steel Strip:											
AISI 1050	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.25 (0.010)	3 (0.125)	b	95	200
1065	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.08 (0.003)	3 (0.125)	b	95	200
1074, 1075	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.08 (0.003)	3 (0.125)	b	120	250
1095	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.08 (0.003)	3 (0.125)	b	120	250
Stainless Steel Strip:											
Austenitic Types 301, 302	193	(28)	69.0	(10)	7.92 (0.286)	2	0.08 (0.003)	1.5 (0.063)	b	315	600
Precipitation Hardening 17-7 PH	203	(29.5)	75.8	(11)	7.81 (0.282)	2	0.08 (0.003)	3 (0.125)	b	370	700
Copper Base Alloy Strip:											
Phosphor Bronze (A)	103	(15)	43	(6.3)	8.86 (0.320)	15	0.08 (0.003)	5 (0.188)	b	95	200
Beryllium Copper	128	(18.5)	48	(7.0)	8.26 (0.298)	21	0.08 (0.003)	9.5 (0.375)	b	205	400

(1) Elastic moduli, density and electrical conductivity can vary with cold work, heat treatment and operating stress. These variations are usually minor but should be considered if one or more of these properties is critical.
(2) Sizes normally available are diameters for wire; thicknesses for strip.
(3) Typical surface quality ratings. (For most materials, special processes can be specified to upgrade typical values.)
a. Maximum defect depth: 0 to 0.5% of d or t.
b. Maximum defect depth: 1.0% of d or t.
c. Defect depth: less than 3.5% of d or t.
(4) Maximum service temperatures are guidelines and may vary due to operating stress and allowable relaxation.
(5) Music and hard drawn are commercial terms for patented and cold-drawn carbon steel spring wire.

SOURCE: Associated Spring, Barnes Group Inc.

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TABLE 6.2 Typical Heat Treatments for Springs after Forming

Materials	Heat Treatment °C	°F
Patented and Cold-Drawn Steel Wire	190–230	375–450
Tempered Steel Wire:		
Carbon	260–400	500–750
Alloy	315–425	600–800
Austenitic Stainless Steel Wire	230–510	450–950
Precipitation Hardening Stainless Wire (17–7 PH):		
Condition C	480/1 hour	900/1 hour
Condition A to TH 1050	760/1 hour cool to 15°C followed by 565/1 hour	1400/1 hour, cool to 60°F followed by 1050/1 hour
Monel:		
Alloy 400	300–315	575–600
Alloy K500, Spring Temper	525/4 hours	980/4 hours
Inconel:		
Alloy 600	400–510	750–950
Alloy X-750:		
#1 Temper	730/16 hours	1350/16 hours
Spring Temper	650/4 hours	1200/4 hours
Copper Base, Cold Worked (Brass, Phosphor Bronze, etc.)	175–205	350–400
Beryllium Copper:		
Pretempered (Mill Hardened)	205	400
Solution Annealed, Temper Rolled or Drawn	315/2-3 hours	600/2-3 hours
Annealed Steels:		
Carbon (AISI 1050 to 1095)	800–830*	1475–1525*
Alloy (AISI 5160H 6150, 9254)	830–885*	1525–1625*

*Time depends on heating equipment and section size. Parts are austenitized then quenched and tempered to the desired hardness.

SOURCE: Associated Spring, Barnes Group Inc.

down the rate of corrosion attack on the base metal. The latter approach is most often the most cost-effective method.

Spring Wire. The tensile strength of spring wire varies inversely with the wire diameter (Fig. 6.1).

Common spring wires with the highest strengths are ASTM A228 (music wire) and ASTM A401 (oil-tempered chrome silicon). Wires having slightly lower tensile strength and with surface quality suitable for fatigue applications are ASTM A313 type 302 (stainless steel), ASTM A230 (oil-tempered carbon valve-spring-quality steel), and ASTM A232 (oil-tempered chrome vanadium). For most static applica-

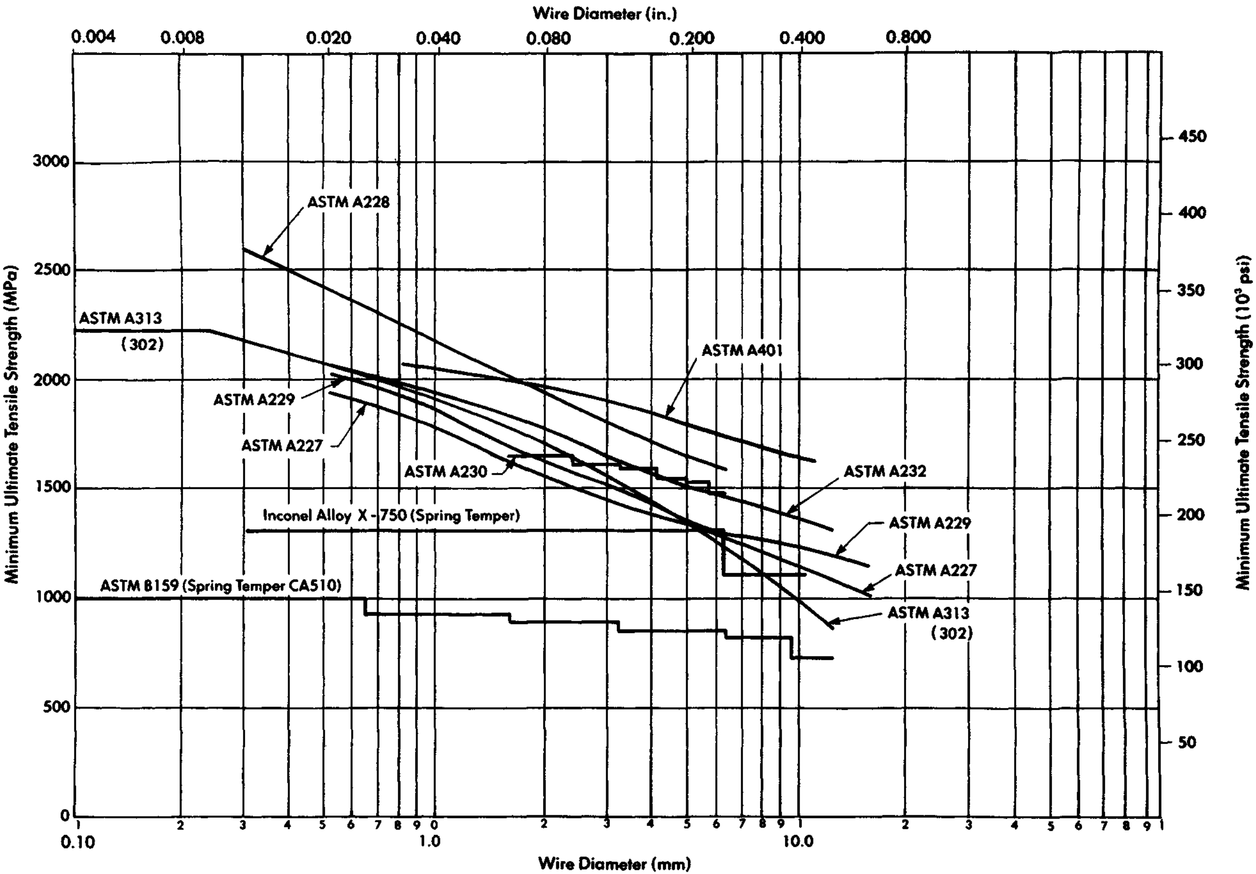


FIGURE 6.1 Minimum tensile strengths of spring wire. *(Associated Spring, Barnes Group Inc.)*

tions ASTM A227 (hard-drawn carbon steel) and ASTM A229 (oil-tempered carbon steel) are available at lower strength levels. Table 6.3 ranks the relative costs of common spring materials based on hard-drawn carbon steel as 1.0.

Spring Strip. Most "flat" springs are made from AISI grades 1050, 1065, 1074, and 1095 steel strip. Strength and formability characteristics are shown in Fig. 6.2, covering the range of carbon content from 1050 to 1095. Since all carbon levels can be obtained at all strength levels, the curves are not identified by composition. Figure 6.3 shows the tensile strength versus Rockwell hardness for tempered carbon-steel strip. Edge configurations for steel strip are shown in Fig. 6.4.

Formability of annealed spring steels is shown in Table 6.4, and typical properties of various spring-tempered alloy strip materials are shown in Table 6.5.

6.4 HELICAL COMPRESSION SPRINGS

6.4.1 General

A helical compression spring is an open-pitch spring which is used to resist applied compression forces or to store energy. It can be made in a variety of configurations and from different shapes of wire, depending on the application. Round, high-carbon-steel wire is the most common spring material, but other shapes and compositions may be required by space and environmental conditions.

Usually the spring has a uniform coil diameter for its entire length. Conical, barrel, and hourglass shapes are a few of the special shapes used to meet particular load-deflection requirements.

TABLE 6.3 Ranking of Relative Costs of Common Spring Wires

Wire	Specification	Relative Cost of 2 mm (0.079") Dia.	
		Mill Quantities	Ware-House Lots
Patented and Cold Drawn	ASTM A227	1.0	1.0
Oil Tempered	ASTM A229	1.3	1.3
Music	ASTM A228	2.6	1.4
Carbon Valve Spring	ASTM A230	3.1	1.9
Chrome Silicon Valve	ASTM A401	4.0	3.9
Stainless Steel (Type 302)	ASTM A313 (302)	7.6	4.7
Phosphor Bronze	ASTM	8.0	6.7
Stainless Steel (Type 631) (17–7 PH)	ASTM A 313 (631)	11	8.7
Beryllium Copper	ASTM B197	27	17
Inconel Alloy X–750		44	31

SOURCE: Associated Spring, Barnes Group Inc.

Helical compression springs are stressed in the torsional mode. The stresses, in the elastic range, are not uniform about the wire's cross section. The stress is greatest at the surface of the wire and, in particular, at the inside diameter (ID) of the spring.

In some circumstances, residual bending stresses are present as well. In such cases, the bending stresses become negligible after set is removed (or the elastic limit is exceeded) and the stresses are redistributed more uniformly about the cross section.

6.4.2 Compression Spring Terminology

The definitions that follow are for terms which have evolved and are commonly used in the spring industry. Figure 6.5 shows the relationships among the characteristics.

Wire Diameter d. Round wire is the most economical form. Rectangular wire is used in situations where space is limited, usually to reduce solid height.

Coil Diameter. The outside diameter (OD) is specified when a spring operates in a cavity. The inside diameter is specified when the spring is to operate over a rod. The mean diameter D is either OD minus the wire size or ID plus the wire size.

The coil diameter increases when a spring is compressed. The increase, though small, must be considered whenever clearances could be a problem. The diameter increase is a function of the spring pitch and follows the equation

$$\mathrm{OD}_{\text{at solid}} = \sqrt{D^2 + \frac{p^2 - d^2}{\pi^2}} + d \tag{6.1}$$

where p = pitch and d = wire size.

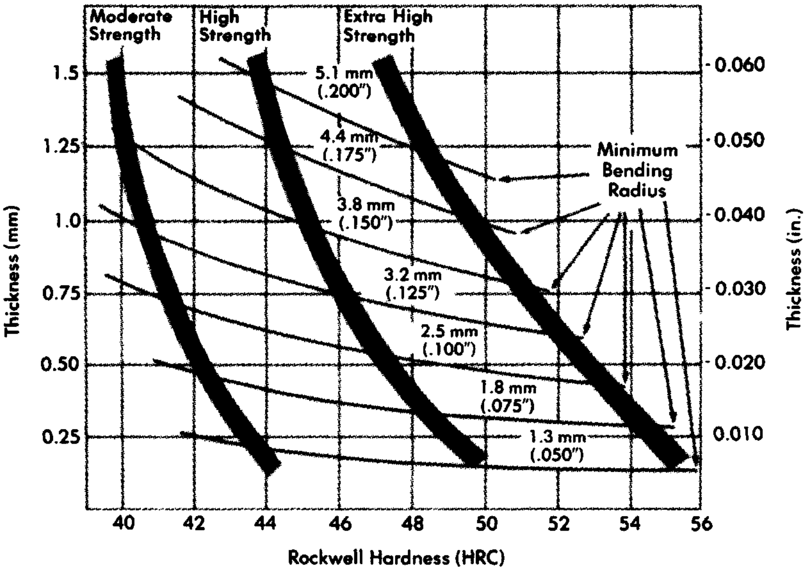


FIGURE 6.2 Minimum transverse bending radii for various tempers and thicknesses of tempered spring steel. *(Associated Spring, Barnes Group Inc.)*

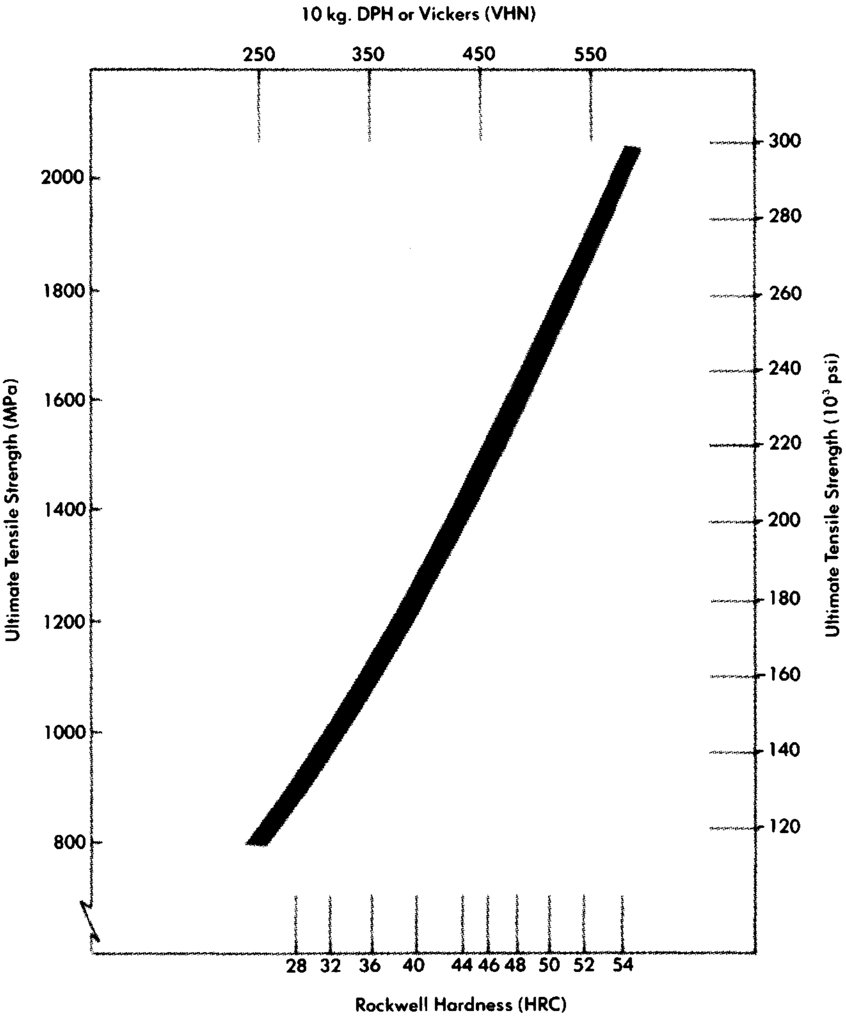


FIGURE 6.3 Tensile strength versus hardness of quenched and tempered spring steel. *(Associated Spring, Barnes Group Inc.)*

Spring Index. Spring index C is the ratio of the mean diameter to the wire diameter (or to the radial dimension if the wire is rectangular). The preferred range of index is 5 to 9, but ranges as low as 3 and as high as 15 are commercially feasible. The very low indices are hard to produce and require special setup techniques. High indices are difficult to control and can lead to spring tangling.

Free Length. Free length L_f is the overall length measured parallel to the axis when the spring is in a free, or unloaded, state. If loads are not given, the free length should be specified. If they are given, then free length should be a reference dimension which can be varied to meet the load requirements.

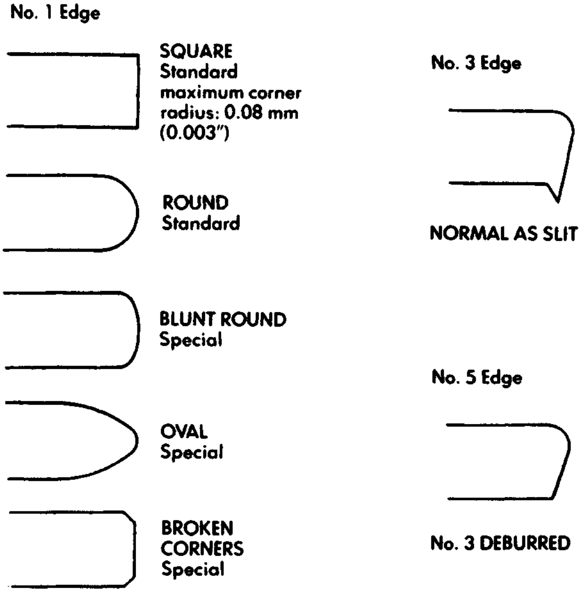


FIGURE 6.4 Edges available on steel strip. *(Associated Spring, Barnes Group Inc.)*

Types of Ends. Four basic types of ends are used: closed (squared) ends, closed (squared) ends ground, plain ends, and plain ends ground. Figure 6.6 illustrates the various end conditions. Closed and ground springs are normally supplied with a ground bearing surface of 270 to 330°.

Number of Coils. The number of coils is defined by either the total number of coils N_t or the number of active coils N_a. The difference between N_t and N_a equals the number of inactive coils, which are those end coils that do not deflect during service.

Solid Height. The solid height L_s is the length of the spring when it is loaded with enough force to close all the coils. For ground springs, $L_s = N_t d$. For unground springs, $L_s = (N_t + 1)d$.

Direction of the Helix. Springs can be made with the helix direction either right or left hand. Figure 6.7 illustrates how to define the direction. Springs that are nested one inside the other should have opposite helix directions. If a spring is to be assembled onto a screw thread, the direction of the helix must be opposite to that of the thread.

Spring Rate. Spring rate k is the change in load per unit deflection. It is expressed as

$$k = \frac{P}{f} = \frac{Gd^4}{8D^3N_a} \tag{6.2}$$

where G = shear modulus.

TABLE 6.4 Formability of Annealed Spring Steels

Thickness (t) mm (in.)	Direction of Bend	AISI 1050 N_t/t		AISI 1065 N_t/t		AISI 1074 N_t/t	AISI 1095 N_t/t
		Annealed (standard lowest max.)	WBS* Barco-Form®	Annealed (standard lowest max.)	WBS Barco-Form	Annealed (standard lowest max.)	Annealed (standard lowest max.)
1.9 mm (0.076)-over	⊥ ‖	2 4	0 3	2 4	0 3	2 4	3 5
0.9–1.89 mm (0.036–0.075″)	⊥ ‖	1 2	0 1	1 2	0 1	1 2	2 3
0.37–0.89 mm (0.015–0.035″)	⊥ ‖	0 1	0 0	0 1 1/2	0 1	1 1 1/2	1 2
0.2–0.36 mm (0.008–0.014″)	⊥ ‖	0 0	0 0	0 0	0 0	1 1	1 1

Formability is determined by slowly bending a sample over 180° until its ends are parallel. The measured distance between the ends is N_t. For example, if $N_t = 4$ and $t = 2$, then $N_t/t = 2$

*Wallace Barnes Steel.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.5 Typical Properties of Spring-Tempered Alloy Strip

Material	Tensile Strength MPa (10^3 psi)	Rockwell Hardness	Elongation(1) Percent	Bend Factor (1) (2r/t trans. bends)	Modulus of Elasticity 10^4 MPa (10^6 psi)	Poisson's Ratio
Steel, spring temper	1700 (246)	C50	2	5	20.7 (30)	0.30
Stainless 301	1300 (189)	C40	8	3	19.3 (28)	0.31
Stainless 302	1300 (189)	C40	5	4	19.3 (28)	0.31
Monel 400	690 (100)	B95	2	5	17.9 (26)	0.32
Monel K500	1200 (174)	C34	40	5	17.9 (26)	0.29
Inconel 600	1040 (151)	C30	2	2	21.4 (31)	0.29
Inconel X-750	1050 (152)	C35	20	3	21.4 (31)	0.29
Copper-Beryllium	1300 (189)	C40	2	5	12.8 (18.5)	0.33
Ni- Span-C	1400 (203)	C42	6	2	18.6 (27)	—
Brass CA 260	620 (90)	B90	3	3	11 (16)	0.33
Phosphor Bronze	690 (100)	B90	3	2.5	10.3 (15)	0.20
17–7 PH RH950	1450 (210)	C44	6	flat	20.3 (29.5)	0.34
17–7 PH Condition C	1650 (239)	C46	1	2.5	20.3 (29.5)	0.34

(1) Before heat treatment.

SOURCE: Associated Spring, Barnes Group Inc.

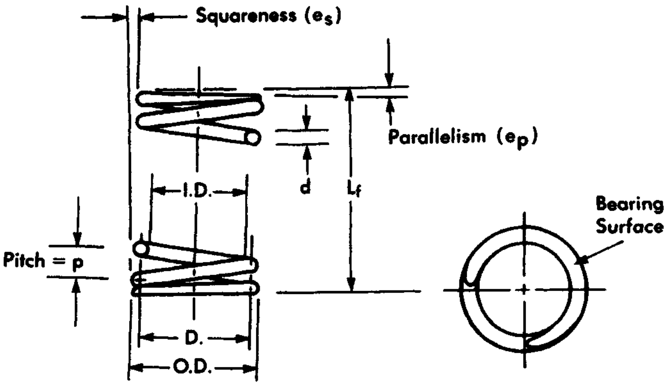


FIGURE 6.5 Dimensional terminology for helical compression springs. *(Associated Spring, Barnes Group Inc.)*

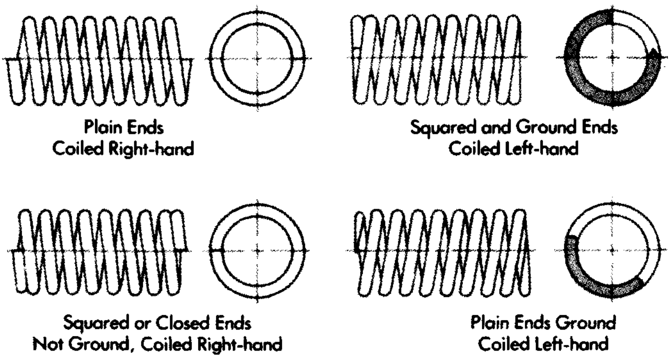


FIGURE 6.6 Types of ends for helical compression springs. *(Associated Spring, Barnes Group Inc.)*

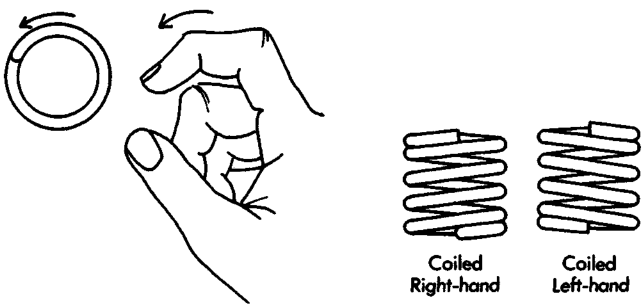


FIGURE 6.7 Direction of coiling of helical compression springs. *(Associated Spring, Barnes Group Inc.)*

The rate equation is accurate for a deflection range between 15 and 85 percent of the maximum available deflection. When compression springs are loaded in parallel, the combined rate of all the springs is the sum of the individual rates. When the springs are loaded in series, the combined rate is

$$k = \frac{1}{1/k_1 + 1/k_2 + 1/k_3 + \cdots + 1/k_n} \tag{6.3}$$

This relationship can be used to design a spring with variable diameters. The design method is to divide the spring into many small increments and calculate the rate for each increment. The rate for the whole spring is calculated as in Eq. (6.3).

Stress. Torsional stress S is expressed as

$$S = \frac{8K_w PD}{\pi d^3} \tag{6.4}$$

Under elastic conditions, torsional stress is not uniform around the wire's cross section because of the coil curvature and direct shear loading.

The highest stress occurs at the surface in the inside diameter of the spring, and it is computed by using the stress factor K_w. In most cases, the correction factor is expressed as

$$K_{w_1} = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \tag{6.5}$$

The stress-concentration factor K_{w_1} becomes K_{w_2} after a spring has been set out because stresses become more uniformly distributed after subjecting the cross section to plastic flow during set-out:

$$K_{w_2} = 1 + \frac{0.5}{C} \tag{6.6}$$

The appropriate stress correction factor is discussed in Sec. 6.4.3.

Loads. If deflection is known, the load is found by multiplying deflection by the spring rate. When the stress is either known or assumed, loads can be obtained from the stress equation.

Loads should be specified at a test height so that the spring manufacturer can control variations by adjustments of the free length. The load-deflection curve is not usually linear at the start of deflection from free position or when the load is very close to solid height. It is advisable to specify loads at test heights between 15 and 85 percent of the load-deflection range.

Loads can be conveniently classified as static, cyclic, and dynamic. In static loading, the spring will operate between specified loads only a few times. In other instances, the spring may remain under load for a long time. In cyclic applications, the spring may typically be required to cycle between load points from 10^4 to more than 10^9 times. During dynamic loading, the rate of load application is high and causes a surge wave in the spring which usually induces stresses higher than calculated from the standard stress equation.

Buckling. Compression springs with a free length more than 4 times the mean coil diameter may buckle when compressed. Guiding the spring, either in a tube or over a rod, can minimize the buckling but can result in additional friction which will affect loads, especially when the L_f/D ratio is high.

Buckling conditions are shown in Figs. 6.8 and 6.9 for springs loaded axially and with squared and ground ends. Buckling occurs at points above and to the right of the curves. Curve A is for the springs with one end on a fixed, flat surface and the other end free to tip. Curve B is for springs with both ends on fixed, flat surfaces. The tendency to buckle is clearly less for curve B springs.

6.4.3 Choice of Operating Stress

The choice of operating stress depends on whether the application is static or cyclic. For static applications, yield strength or stress-relaxation resistance of the material limits the load-carrying ability of the springs. The required cycles are few, if any, and the velocity of the end coils is so low as to preclude surging or impact conditions.

The maximum allowable torsional stresses for static applications are shown in Table 6.6 as percentages of tensile strengths for common spring materials. To calculate the stress before set removal, use the K_{w_1} correction factor. If the calculated stress is greater than the indicated percentage of the tensile strength, then the spring will take a permanent set when deflected to solid. The amount of set is a function of the amount by which the calculated stress exceeds the tabular percentage.

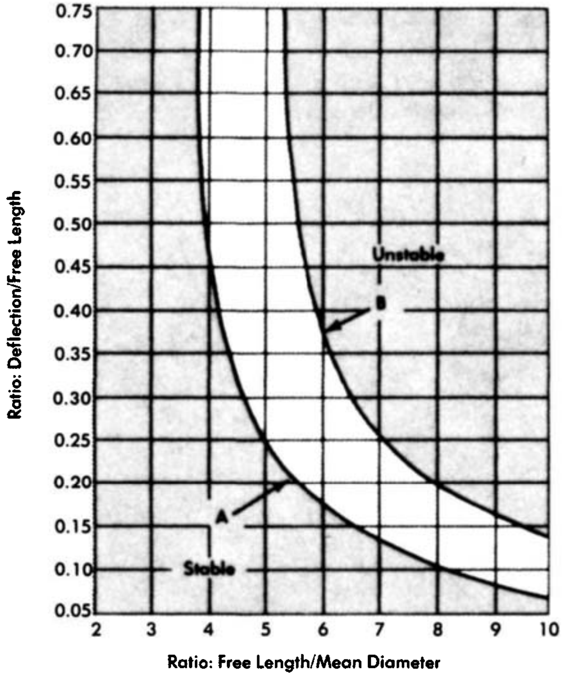


FIGURE 6.8 Critical buckling curves. *(Associated Spring, Barnes Group Inc.)*

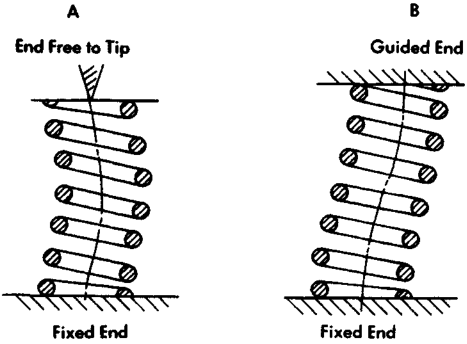


FIGURE 6.9 End conditions used to determine critical buckling. *(Associated Spring, Barnes Group Inc.)*

It is common practice, in static applications, to increase the load-carrying capability of a spring by making it longer than the desired free length and then compressing it to solid. The spring *sets* to its final desired length. This procedure is called *removing set.* It induces favorable residual stresses which allow for significantly higher stresses than in springs not having the set removed. The loss of the length should be at least 10 percent to be effective (see Fig. 6.10).

Note that set removal causes stresses to be more uniformly distributed about the cross section. Therefore, stress after set removal is calculated by using the K_{w_2} correction factor. If the stress calculated by using the K_{w_2} correction factor exceeds the percentage of tensile strength shown in Table 6.6, the spring cannot be made. It is then necessary either to lower the design stress or to select a higher-strength material.

For cyclic applications, the load-carrying ability of the spring is limited by the fatigue strength of the material. To select the optimum stress level, spring costs must be balanced against reliability. The designer should know the operating environ-

TABLE 6.6 Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Materials	Maximum % of Tensile Strength	
	Before Set Removed (K_{w1})	After Set Removed (K_{w2})
Patented and cold drawn carbon steel	45%	
Hardened and tempered carbon and low alloy steel	50%	65–75%
Austenitic stainless steels	35%	
Nonferrous alloys	35%	

SOURCE: Associated Spring, Barnes Group Inc.

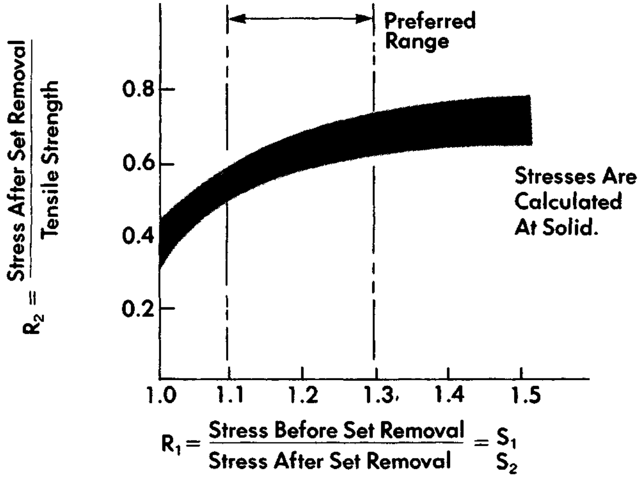


FIGURE 6.10 Spring load-carrying ability versus amount of set removed. *(Associated Spring, Barnes Group Inc.)*

ment, desired life, stress range, frequency of operation, speed of operation, and permissible levels of stress relaxation in order to make a cost-reliability decision.

Fatigue life can be severely reduced by pits, seams, or tool marks on the wire surface where stress is at a maximum. Shot peening improves fatigue life, in part, by minimizing the harmful effects of surface defects. It does not remove them. Additionally, shot peening imparts favorable compression stresses to the surface of the spring wire.

Maximum allowable stresses for fatigue applications should be calculated by using the K_{w_1} stress correction factor. Table 6.7 shows the estimated fatigue life for common spring materials. Note the significant increase in fatigue strength from shot peening.

TABLE 6.7 Maximum Allowable Torsional Stress for Round-Wire Helical Compression Springs in Cyclic Applications

Fatigue Life (cycles)	Percent of Tensile Strength			
	ASTM A228, Austenitic Stainless Steel and Nonferrous		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened	Not Shot-Peened	Shot-Peened
10^5	36	42	42	49
10^6	33	39	40	47
10^7	30	36	38	46

This information is based on the following conditions: no surging, room temperature and noncorrosive environment.

$$\text{Stress ratio in fatigue} = \frac{\text{S minimum}}{\text{S maximum}} = 0$$

SOURCE: Associated Spring, Barnes Group Inc.

The fatigue life estimates in Table 6.7 are guideline values which should be used only where specific data are unavailable. The values are conservative, and most springs designed using them will exceed the anticipated lives.

6.4.4 Dynamic Loading under Impact

When a spring is loaded or unloaded, a surge wave is established which transmits torsional stress from the point of load along the spring's length to the point of restraint. The surge wave will travel at a velocity approximately one-tenth that of a normal, torsional-stress wave. The velocity of the torsional-stress wave V_T, in meters per second (m/s) [inches per second (in/s)], is given by

$$V_T = 10.1\sqrt{\frac{Gg}{\rho}} \quad \text{m/s} \qquad \text{or} \qquad V_T = \sqrt{\frac{Gg}{\rho}} \quad \text{in/s} \tag{6.7}$$

The velocity of the surge wave V_s varies with material and design but is usually in the range of 50 to 500 m/s. The surge wave limits the rate at which a spring can absorb or release energy by limiting the impact velocity V. *Impact velocity* is defined as the spring velocity parallel to the spring axis and is a function of stress and material as shown:

$$V \simeq 10.1S\sqrt{\frac{g}{2\rho G}} \quad \text{m/s} \qquad \text{or} \qquad V \simeq S\sqrt{\frac{g}{2\rho G}} \quad \text{in/s} \tag{6.8}$$

For steel, this reduces to

$$V = \frac{S}{35.5} \quad \text{m/s} \qquad \text{or} \qquad V = \frac{S}{131} \quad \text{in/s} \tag{6.9}$$

If a spring is compressed to a given stress level and released instantaneously, the maximum spring velocity is the stress divided by 35.5. Similarly, if the spring is loaded at known velocity, the instantaneous stress can be calculated. At very high load velocities, the instantaneous stress will exceed the stress calculated by the conventional equation. This will limit design performance. Since the surge wave travels the length of the spring, springs loaded at high velocity often are subject to resonance.

6.4.5 Dynamic Loading—Resonance

A spring experiences resonance when the frequency of cyclic loading is near the natural frequency or a multiple of it. Resonance can cause an individual coil to deflect to stress levels above those predicted by static stress analysis. Resonance can also cause the spring to bounce, resulting in loads lower than calculated. To avoid these effects, the natural frequency should be a minimum of 13 times the operating frequency.

For a compression spring with both ends fixed and no damper, the natural frequency in International System (SI) units is

$$n = \frac{1.12(10^3)d}{D^2N_a}\sqrt{\frac{Gg}{\rho}} \tag{6.10}$$

For steel, this equation becomes

$$n = \frac{3.5(10^5)d}{D^2 N_a} \tag{6.11}$$

where n = frequency in hertz (Hz). The corresponding equation in U.S. Customary System (USCS) units is

$$n = \frac{d}{9D^2 N_a}\sqrt{\frac{Gg}{\rho}} \tag{6.12}$$

and for steel we have

$$n = \frac{14(10^3)d}{D^2 N_a} \tag{6.13}$$

If the spring cannot be designed to have a natural frequency more than 13 times the operating frequency, energy dampers may be employed. They are generally friction devices which rub against the coils. Often, variable-pitch springs are used to minimize resonance effects.

6.4.6 Rectangular-Wire Springs

In applications where high loads and relatively low stresses are required but solid height is also restricted, rectangular wire can be used to increase the material volume while maintaining the maximum solid-height limitation.

Springs made of rectangular wire with the long side of the wire cross section perpendicular to the axis of the coils can store more energy in a smaller space than an equivalent, round-wire spring.

When rectangular wire is coiled, it changes from a rectangular to a keystone shape, as shown in Fig. 6.11. Similarly, if the wire is made to the keystone shape, it will become rectangular after coiling. The cross-sectional distortion can be approximated by

$$t_1 = t\,\frac{C + 0.5}{C} \tag{6.14}$$

where t_1 = wider end of keystone section and t = original, smaller dimension of rectangle.

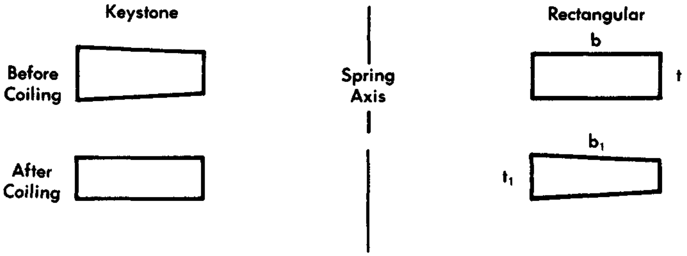


FIGURE 6.11 Wire cross section before and after coiling. *(Associated Spring, Barnes Group Inc.)*

The spring rate for a rectangular-wire spring is

$$k = \frac{P}{f} = \frac{K_2 G b t^3}{N_a D^3} \tag{6.15}$$

Since the wire is loaded in torsion, it makes no difference whether the wire is wound on the flat or on edge. See Fig. 6.12.

Stress is calculated by

$$S = \frac{K_E P D}{K_1 b t^2} \quad \text{or} \quad S = \frac{K_F P D}{K_1 b t^2} \tag{6.16}$$

Values for K_1 and K_2 are found in Fig. 6.13, and those for K_E and K_F are found in Figs. 6.15 and 6.14, respectively.

When a round wire cannot be used because the solid height exceeds the specification, the approximate equivalent rectangular dimensions are found from

$$t = \frac{2d}{1 + b/t} \tag{6.17}$$

where d = round-wire diameter.

6.4.7 Variable-Diameter Springs

Conical, hourglass, and barrel-shaped springs, shown in Fig. 6.16, are used in applications requiring a low solid height and an increased lateral stability or resistance to surging. Conical springs can be designed so that each coil nests wholly or partly within an adjacent coil. Solid height can be as low as one wire diameter. The rate for conical springs usually increases with deflection (see Fig. 6.17) because the number of active coils decreases progressively as the spring approaches solid. By varying the pitch, conical springs can be designed to have a uniform rate. The rate for conical springs is calculated by considering the spring as many springs in series. The rate for

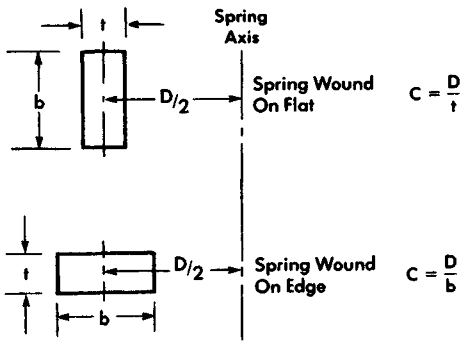


FIGURE 6.12 Rectangular-wire compression spring wound on flat or edge. *(Associated Spring, Barnes Group Inc.)*

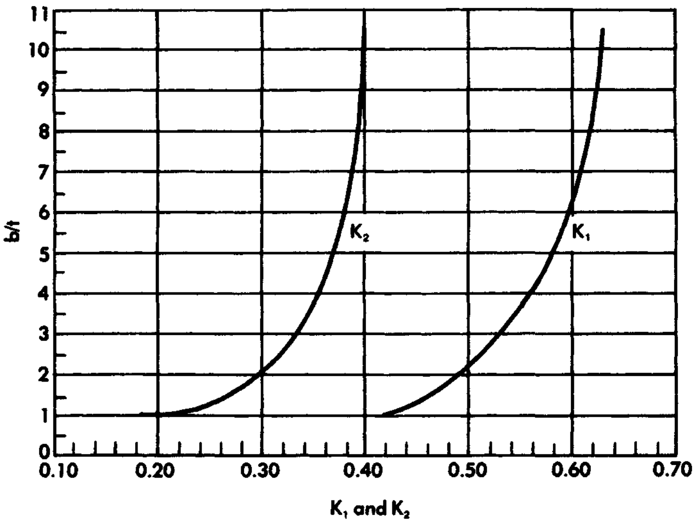


FIGURE 6.13 Constants for rectangular wire in torsion. *(Associated Spring, Barnes Group Inc.)*

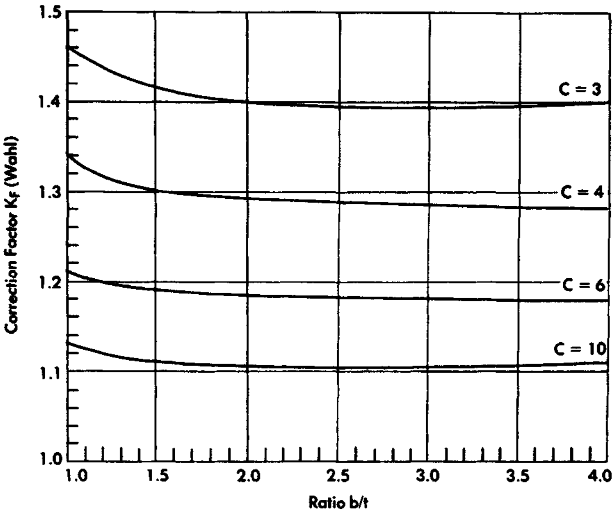


FIGURE 6.14 Stress correction factors for rectangular-wire compression springs wound on flat. *(Associated Spring, Barnes Group Inc.)*

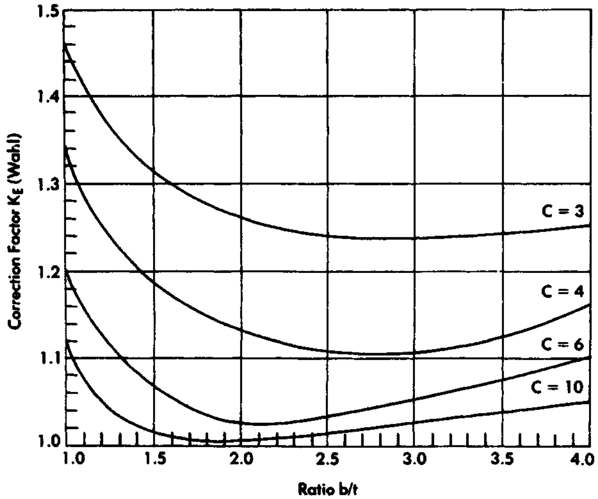


FIGURE 6.15 Stress correction factors for rectangular-wire compression springs wound on edge. *(Associated Spring, Barnes Group Inc.)*

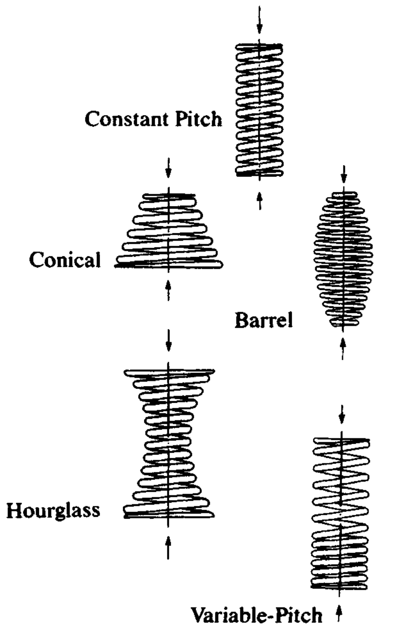


FIGURE 6.16 Various compression-spring body shapes. *(Associated Spring, Barnes Group Inc.)*

each turn or fraction of a turn is calculated by using the standard rate equation. The rate for a complete spring is then determined, given that the spring rate follows the series relationship in Eq. (6.3).

To calculate the highest stress at a given load, the mean diameter of the largest active coil at load is used. The solid height of a uniformly tapered, but not telescoping, spring with squared and ground ends made from round wire can be estimated from

$$L_S = N_a \sqrt{d^2 - u^2} + 2d \qquad (6.18)$$

where u = OD of large end minus OD of small end, divided by $2N_a$.

Barrel- and hourglass-shaped springs are calculated as two conical springs in series.

6.4.8 Commercial Tolerances

Standard commercial tolerances are presented in Tables 6.8, 6.9, and 6.10 for free length, coil diameter, and load tolerances, respectively. These tolerances represent the best tradeoffs between manufacturing costs and performance.

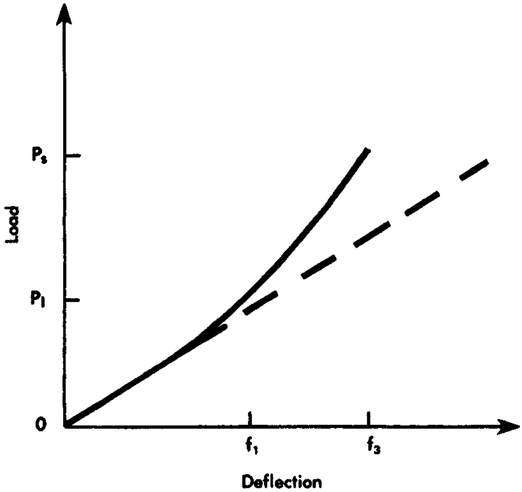


FIGURE 6.17 Typical load-deflection curve for variable-diameter springs (solid line). *(Associated Spring, Barnes Group Inc.)*

TABLE 6.8 Free-Length Tolerances of Squared and Ground Helical Compression Springs

Number of Active coils per mm(in.)	Tolerances: ±mm/mm (in./in.) of Free Length						
	Spring Index (D/d)						
	4	6	8	10	12	14	16
0.02 (0.5)	0.010	0.011	0.012	0.013	0.015	0.016	0.016
0.04 (1)	0.011	0.013	0.015	0.016	0.017	0.018	0.019
0.08 (2)	0.013	0.015	0.017	0.019	0.020	0.022	0.023
0.2 (4)	0.016	0.018	0.021	0.023	0.024	0.026	0.027
0.3 (8)	0.019	0.022	0.024	0.026	0.028	0.030	0.032
0.5 (12)	0.021	0.024	0.027	0.030	0.032	0.034	0.036
0.6 (16)	0.022	0.026	0.029	0.032	0.034	0.036	0.038
0.8 (20)	0.023	0.027	0.031	0.034	0.036	0.038	0.040

For springs less than 12.7 mm (0.500″) long, use the tolerances for 12.7 mm (0.500″). For closed ends not ground, multiply above values by 1.7.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.9 Coil Diameter Tolerances of Helical Compression and Extension Springs

Wire Dia., mm(in.)	Tolerances: ±mm (in.)						
	Spring Index (D/d)						
	4	6	8	10	12	14	16
0.38 (0.015)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.13 (0.005)	0.15 (0.006)	0.18 (0.007)
0.58 (0.023)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.20 (0.008)	0.25 (0.010)
0.89 (0.035)	0.05 (0.002)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.23 (0.009)	0.28 (0.011)	0.33 (0.013)
1.30 (0.051)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.25 (0.010)	0.30 (0.012)	0.38 (0.015)	0.43 (0.017)
1.93 (0.076)	0.10 (0.004)	0.18 (0.007)	0.25 (0.010)	0.33 (0.013)	0.41 (0.016)	0.48 (0.019)	0.53 (0.021)
2.90 (0.114)	0.15 (0.006)	0.23 (0.009)	0.33 (0.013)	0.46 (0.018)	0.53 (0.021)	0.64 (0.025)	0.74 (0.029)
4.34 (0.171)	0.20 (0.008)	0.30 (0.012)	0.43 (0.017)	0.58 (0.023)	0.71 (0.028)	0.84 (0.033)	0.97 (0.038)
6.35 (0.250)	0.28 (0.011)	0.38 (0.015)	0.53 (0.021)	0.71 (0.028)	0.90 (0.035)	1.07 (0.042)	1.24 (0.049)
9.53 (0.375)	0.41 (0.016)	0.51 (0.020)	0.66 (0.026)	0.94 (0.037)	1.17 (0.046)	1.37 (0.054)	1.63 (0.064)
12.70 (0.500)	0.53 (0.021)	0.76 (0.030)	1.02 (0.040)	1.57 (0.062)	2.03 (0.080)	2.54 (0.100)	3.18 (0.125)

SOURCE: Associated Spring, Barnes Group Inc.

6.5 HELICAL EXTENSION SPRINGS

6.5.1 General

Helical extension springs store energy and exert a pulling force. They are usually made from round wire and are close-wound with initial tension. They have various types of end hooks or loops by which they are attached to the loads.

Like compression springs, extension springs are stressed in torsion in the body coils. The design procedures for the body coil are similar to those discussed in Sec. 6.4 except for the initial tension and the hook stresses.

Most extension springs are made with the body coils held tightly together by a force called *initial tension.* The measure of initial tension is the load required to overcome the internal force and start coil separation.

Extension springs, unlike compression springs, seldom have set removed. Furthermore, they have no solid stop to prevent overloading. For these reasons, the design stresses are normally held to lower values than those for compression springs.

The pulling force exerted by an extension spring is transmitted to the body coils through hooks or loops. Careful attention must be given to the stresses in the hooks. The hook ends must be free of damaging tool marks so that spring performance will not be limited by hook failure.

TABLE 6.10 Load Tolerances of Helical Compression Springs

Length Tolerance ± mm (in.)	Tolerances: ±% of Load. Start with Tolerance from Table 6-8 Multiplied by L_F. Deflection from Free Length to Load, mm (in.)														
	1.27 (0.050)	2.54 (0.100)	3.81 (0.150)	5.08 (0.200)	6.35 (0.250)	7.62 (0.300)	10.2 (0.400)	12.7 (0.500)	19.1 (0.750)	25.4 (1.00)	38.1 (1.50)	50.8 (2.00)	76.2 (3.00)	102 (4.00)	152 (6.00)
0.13 (0.005)	12.	7.	6.	5.	—	—	—	—	—	—	—	—	—	—	—
0.25 (0.010)	—	12.	8.5	7.	6.5	5.5	5.	—	—	—	—	—	—	—	—
0.51 (0.020)	—	22.	15.5	12.	10.	8.5	7.	6.	5.	—	—	—	—	—	—
0.76 (0.030)	—	—	22.	17.	14.	12.	9.5	8.	6.	5.	—	—	—	—	—
1.0 (0.040)	—	—	—	22.	18.	15.5	12.	10.	7.5	6.	5.	—	—	—	—
1.3 (0.050)	—	—	—	—	22.	19.	14.5	12.	9.	7.	5.5	—	—	—	—
1.5 (0.060)	—	—	—	—	25.	22.	17.	14.	10.	8.	6.	5.	—	—	—
1.8 (0.070)	—	—	—	—	—	25.	19.5	16.	11.	9.	6.5	5.5	—	—	—
2.0 (0.080)	—	—	—	—	—	—	22.	18.	12.5	10.	7.5	6.	5.	—	—
2.3 (0.090)	—	—	—	—	—	—	25.	20.	14.	11.	8.	6.	5.	—	—
2.5 (0.100)	—	—	—	—	—	—	—	22.	15.5	12.	8.5	7.	5.5	—	—
5.1 (0.200)	—	—	—	—	—	—	—	—	—	22.	15.5	12.	8.5	7.	5.5
7.6 (0.300)	—	—	—	—	—	—	—	—	—	—	22.	17.	12.	9.5	7.
10.2 (0.400)	—	—	—	—	—	—	—	—	—	—	—	21.	15.	12.	8.5
12.7 (0.500)	—	—	—	—	—	—	—	—	—	—	—	25.	18.5	14.5	10.5

First load test at not less than 15% of available deflection.
Final load test at not more than 85% of available deflection.

SOURCE: Associated Spring, Barnes Group Inc.

6.5.2 Initial Tension

Initial tension is illustrated in Fig. 6.18. The point of intersection on the ordinate is initial tension P_I. The amount of initial tension is governed by the spring index, material, method of manufacture, and the post-stress-relief heat treatment temperature. Note that a high stress-relief temperature can reduce the initial tension. This is sometimes used as a means to control initial tension in low-stress, low-index springs. It follows that an extension spring requiring no initial tension can be made either by removing the initial tension with heat treatment or by keeping the coils open during coiling. The levels of initial tension obtainable are shown in Fig. 6.19.

6.5.3 Types of Ends

Extension springs require a means of attachment to the system which is to be loaded. A variety of end configurations have been developed over the years. The configurations most commonly used are shown in Fig. 6.20. Loops or hooks longer than recommended will require special setup and are more expensive. Specifying an angular relationship for the loops may also add to the cost. Allow a random relationship of loops whenever possible.

Stresses in the loops are often higher than those in the body coils. In such cases, the loops are the performance limiters, particularly in cyclic applications. Generous bend radii, elimination of tool marks, and a reduced diameter of end coils are methods used to reduce loop stresses. In a full-twist loop, stress reaches a maximum in bending at point A (Fig. 6.21) and a maximum in torsion at point B. The stresses at these locations are complex, but useful approximations are, for bending,

$$S_A = \frac{16K_1DP}{\pi d^3} + \frac{4P}{\pi d^2} \tag{6.19}$$

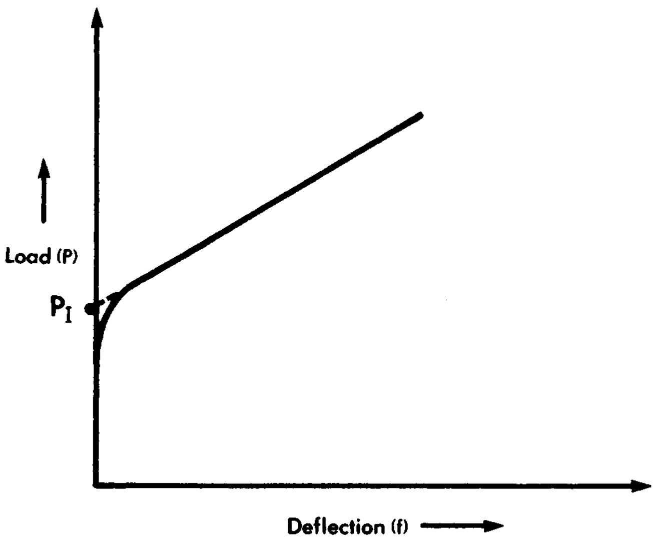


FIGURE 6.18 Load-deflection curve for a helical extension spring with initial tension. *(Associated Spring, Barnes Group Inc.)*

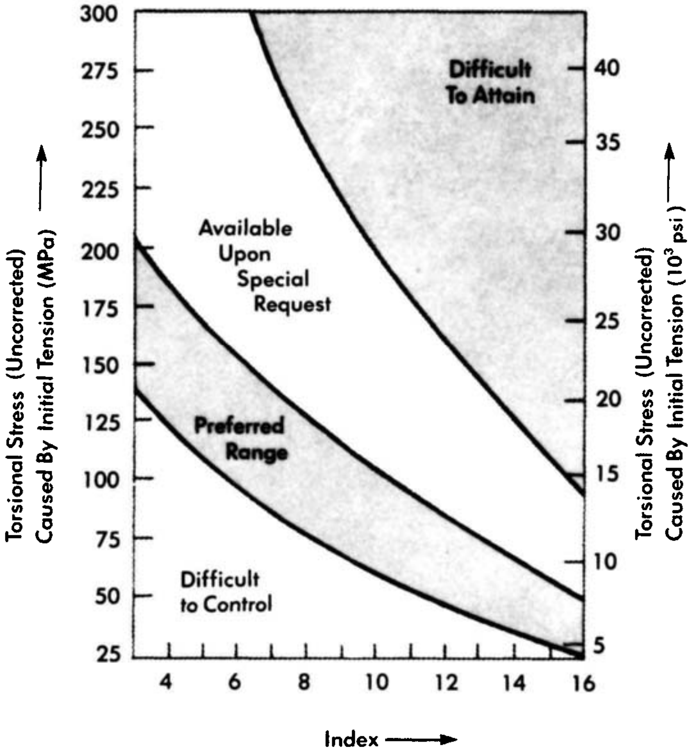


FIGURE 6.19 Torsional stress resulting from initial tension as a function of index in helical extension springs. *(Associated Spring, Barnes Group Inc.)*

where the constants are

$$K_1 = \frac{4C^2 - C_1 - 1}{4C_1(C_1 - 1)} \tag{6.20}$$

and

$$C_1 = \frac{2R_1}{d} \tag{6.21}$$

The torsional stresses are

$$S_B = \frac{8DP}{\pi d^3} \frac{4C_2 - 1}{4C_2 - 4} \tag{6.22}$$

where

$$C_2 = \frac{2R_2}{d} \tag{6.23}$$

General practice is to make C_2 greater than 4.

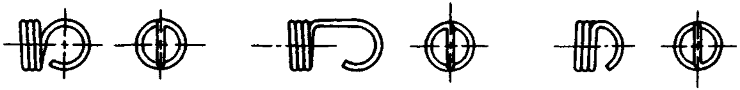
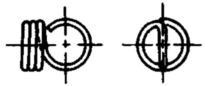
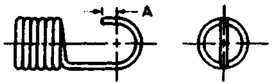
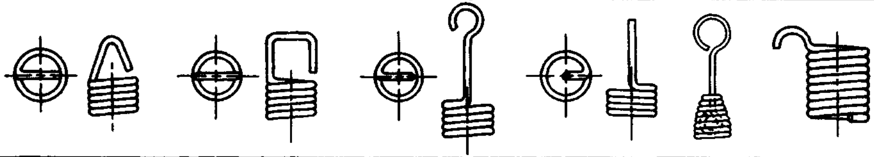
Type	Configurations	Recommended Length Min.- Max.
Twist Loop or Hook		0.5-1.7 I.D.
Cross Center Loop or Hook		I.D.
Side Loop or Hook		0.9-1.0 I.D.
Extended Hook		1.1 I.D. and up. as required by design
Special Ends		As required by design

FIGURE 6.20 Common end configurations for helical extension springs. Recommended length is distance from last body coil to inside of end. ID is inside diameter of adjacent coil in spring body. *(Associated Spring, Barnes Group Inc.)*

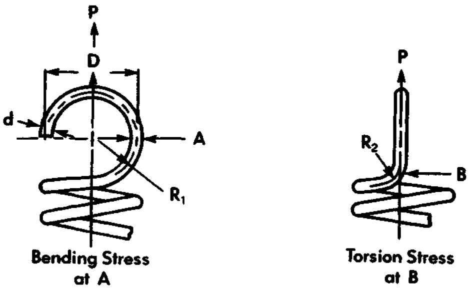


FIGURE 6.21 Location of maximum bending and torsional stresses in twist loops. *(Associated Spring, Barnes Group Inc.)*

6.5.4 Extension Spring Dimensioning

The dimensioning shown in Fig. 6.22 is generally accepted for extension springs. The free length is the distance between the inside surfaces of the loops. The body length is $L_B = d(N + 1)$. The loop opening, or gap, can be varied. The number of active coils is equal to the number of coils in the body of the spring. However, with special ends such as threaded plugs or swivel hooks, the number of active coils will be less than the number of body coils.

6.5.5 Design Equations

The design equations are similar to those for compression springs with the exception of initial tension and loop stresses. The rate is given by

$$k = \frac{P - P_I}{f} = \frac{Gd^4}{8D^3N_a} \tag{6.24}$$

where P_I is initial tension. Stress is given by

$$S = \frac{K_w 8PD}{\pi d^3} \tag{6.25}$$

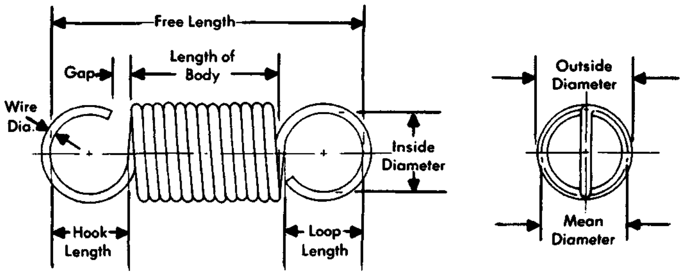


FIGURE 6.22 Typical extension-spring dimensions. *(Associated Spring, Barnes Group Inc.)*

Dynamic considerations discussed previously are generally applicable to extension springs. Natural frequency with one end fixed, in SI units, is

$$n = \frac{560d}{D^2 N_a}\sqrt{\frac{Gg}{\rho}} \tag{6.26}$$

For steel, this equation becomes

$$n = \frac{176\,000d}{N_a D^2} \tag{6.27}$$

where n = frequency in hertz. The corresponding equation in USCS units is

$$n = \frac{d}{18D^2 N_a}\sqrt{\frac{Gg}{\rho}} \tag{6.28}$$

And for steel we have

$$n = \frac{7000d}{N_a D^2} \tag{6.29}$$

6.5.6 Choice of Operating Stress—Static

The maximum stresses recommended for extension springs in static applications are given in Table 6.11. Note that extension springs are similar to compression springs without set removed. For body coil stresses in springs that cannot be adequately stress-relieved because of very high initial-tension requirements, use the maximum recommended stress in torsion, given for the end loops.

6.5.7 Choice of Operating Stress—Cyclic

Table 6.12 presents the maximum stresses for extension springs used in cyclic applications. The data are for stress-relieved springs with initial tension in the preferred range.

TABLE 6.11 Maximum Allowable Stresses (K_{w_1} Corrected) for Helical Extension Springs in Static Applications

Materials	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
Patented, cold-drawn or hardened and tempered carbon and low alloy steels	45–50	40	75
Austenitic stainless steel and nonferrous alloys	35	30	55

This information is based on the following conditions: set not removed and low temperature heat treatment applied.
For springs that require high initial tension, use the same percent of tensile strength as for end.

SOURCE: Associated Spring, Barnes Group Inc.

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TABLE 6.12 Maximum Allowable Stresses for ASTM A228 and Type 302 Stainless-Steel Helical Extension Springs in Cyclic Applications

Number of Cycles	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
10^5	36	34	51
10^6	33	30	47
10^7	30	28	45

This information is based on the following conditions: not shot-peened, no surging and ambient environment with a low temperature heat treatment applied. Stress ratio = 0.

SOURCE: Associated Spring, Barnes Group Inc.

6.5.8 Tolerances

Extension springs do not buckle or require guide pins when they are deflected, but they may vibrate laterally if loaded or unloaded suddenly. Clearance should be allowed in these cases to eliminate the potential for noise or premature failure. The load tolerances are the same as those given for compression springs. Tolerances for free length and for angular relationship of ends are given in Tables 6.13 and 6.14.

6.6 HELICAL TORSION SPRINGS

Helical springs that exert a torque or store rotational energy are known as *torsion springs.* The most frequently used configuration of a torsion spring is the single-body type (Fig. 6.23). Double-bodied springs, known as double-torsion springs, are sometimes used where dictated by restrictive torque, stress, and space requirements. It is often less costly to make a pair of single-torsion springs than a double-torsion type.

TABLE 6.13 Commercial Free-Length Tolerances for Helical Extension Springs with Initial Tension

Spring Free Length (inside hooks) mm (in.)	Tolerance ± mm (in.)
Up to 12.7 (0.500)	0.51 (0.020)
Over 12.7 to 25.4 (0.500 to 1.00)	0.76 (0.030)
Over 25.4 to 50.8 (1.00 to 2.00)	1.0 (0.040)
Over 50.8 to 102 (2.00 to 4.00)	1.5 (0.060)
Over 102 to 203 (4.00 to 8.00)	2.4 (0.093)
Over 203 to 406 (8.00 to 16.0)	4.0 (0.156)
Over 406 to 610 (16.0 to 24.0)	5.5 (0.218)

SOURCE: Associated Spring, Barnes Group Inc.