TABLE 6.12 Maximum Allowable Stresses for ASTM A228 and Type 302 Stainless-Steel Helical Extension Springs in Cyclic Applications

Number of Cycles	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
10^5	36	34	51
10^6	33	30	47
10^7	30	28	45

This information is based on the following conditions: not shot-peened, no surging and ambient environment with a low temperature heat treatment applied. Stress ratio = 0.

SOURCE: Associated Spring, Barnes Group Inc.

6.5.8 Tolerances

Extension springs do not buckle or require guide pins when they are deflected, but they may vibrate laterally if loaded or unloaded suddenly. Clearance should be allowed in these cases to eliminate the potential for noise or premature failure. The load tolerances are the same as those given for compression springs. Tolerances for free length and for angular relationship of ends are given in Tables 6.13 and 6.14.

6.6 HELICAL TORSION SPRINGS

Helical springs that exert a torque or store rotational energy are known as *torsion springs*. The most frequently used configuration of a torsion spring is the single-body type (Fig. 6.23). Double-bodied springs, known as double-torsion springs, are sometimes used where dictated by restrictive torque, stress, and space requirements. It is often less costly to make a pair of single-torsion springs than a double-torsion type.

TABLE 6.13 Commercial Free-Length Tolerances for Helical Extension Springs with Initial Tension

Spring Free Length (inside hooks) mm (in.)	Tolerance ± mm (in.)
Up to 12.7 (0.500)	0.51 (0.020)
Over 12.7 to 25.4 (0.500 to 1.00)	0.76 (0.030)
Over 25.4 to 50.8 (1.00 to 2.00)	1.0 (0.040)
Over 50.8 to 102 (2.00 to 4.00)	1.5 (0.060)
Over 102 to 203 (4.00 to 8.00)	2.4 (0.093)
Over 203 to 406 (8.00 to 16.0)	4.0 (0.156)
Over 406 to 610 (16.0 to 24.0)	5.5 (0.218)

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.14 Tolerances on Angular Relationship of Extension Spring Ends

Angular Tolerance per Coil: ± Degrees									
Index									
4	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**14**	**16**
0.75	0.9	1.1	1.3	1.5	1.7	1.9	2.3	2.6	3

For example, tolerance for a 10-coil spring with an index of 8 is $10 \times \pm 1.5 = \pm 15°$.

If angular tolerance is greater than ± 45°, or if closer tolerances than indicated must be held, consult with Associated Spring.

SOURCE: **Associated Spring, Barnes Group Inc.**

Torsion springs are used in spring-loaded hinges, oven doors, clothespins, window shades, ratchets, counterbalances, cameras, door locks, door checks, and many other applications. Torsion springs are almost always mounted on a shaft or arbor with one end fixed. They can be wound either right or left hand.

In most cases the springs are not stress-relieved and are loaded in the direction that winds them up or causes a decrease in body diameter. The residual forming stresses which remain are favorable in that direction. Although it is possible to load a torsion spring in the direction to unwind and enlarge the body coils, ordinarily it is not good design practice and should be avoided. Residual stresses in the unwind direction are unfavorable. Torsion springs which are plated or painted and subsequently baked or are stress-relieved will have essentially no residual stresses and can be loaded in either direction, but at lower stress levels than springs which are not heat-treated.

Correlation of test results between manufacturer and user may be difficult because there are few, if any, standardized torsion-spring testing machines. The springs will have varying degrees of intercoil friction and friction between the mounting arbor and the body coils. Often, duplicate test fixtures must be made and test methods coordinated.

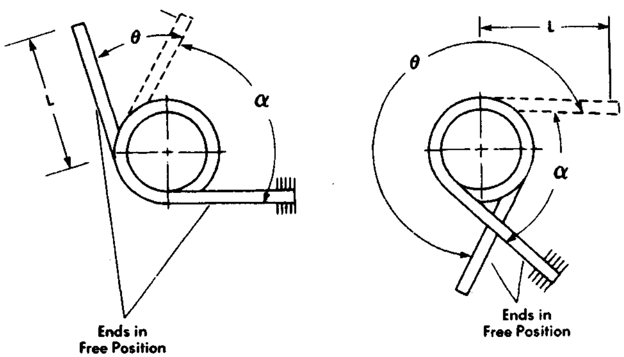

FIGURE 6.23 Specifying load and deflection requirements for torsion spring: α = angle between ends; P = load on ends at α; L = moment arm; θ = angular deflection from free position. *(Associated Spring, Barnes Group Inc.)*

Spring ends most commonly used are shown in Fig. 6.24, although the possible variations are unlimited. In considering spring mounting, it must be recognized that for each turn of windup, the overall length L of the spring body will increase as

$$L_1 = d(N_a + 1 + \theta) \tag{6.30}$$

where θ = deflection in revolutions.

Also note that the body coil diameter will be reduced to

$$D = \frac{D_I N_a}{Na + \theta} \tag{6.31}$$

where D_I = initial mean coil diameter. Experience indicates that the diameter of the arbor over which the spring operates should be approximately 90 percent of the smallest inside diameter to which the spring is reduced under maximum load. Too large an arbor will interfere with deflection, while too small an arbor will provide too little support. Both conditions lead to unexpectedly early failure. Coil diameter tolerances are given in Table 6.17.

6.6.1 Spring Rate

The spring rate, or moment per turn, is given by

$$k = \frac{M}{\theta} = \frac{Ed^4}{10.8DN_a} \tag{6.32}$$

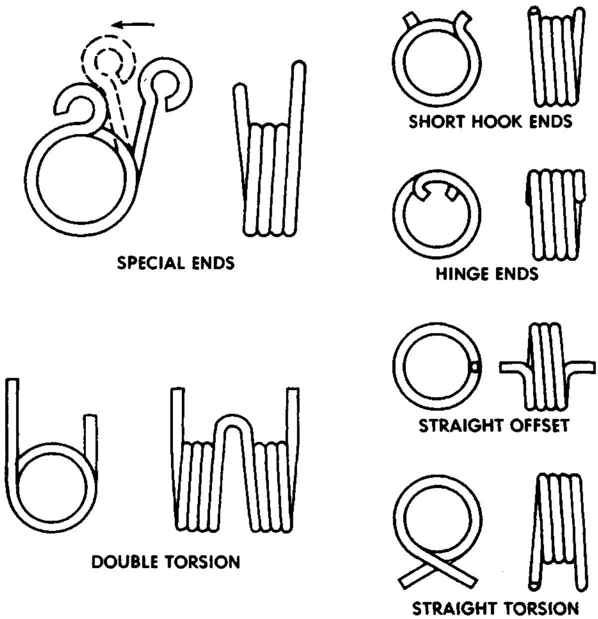

FIGURE 6.24 Common helical torsion-spring end configurations. *(Associated Spring, Barnes Group Inc.)*

The number of coils is equal to the number of body coils plus a contribution from the ends. The effect is more pronounced when the ends are long. The number of equivalent coils in the ends is

$$N_e = \frac{L_1 + L_2}{3\pi D} \tag{6.33}$$

where L_1 and L_2 = lengths of ends, and so $N_a = N_b + N_e$, where N_b = number of body coils.

The load should be specified at a fixed angular relationship of the spring ends rather than at a specific angular deflection from free or load positions. Helical torsion springs are stressed in bending. Rectangular sections are more efficient than round sections, but round sections are normally used because there is usually a premium cost for rectangular wire.

6.6.2 Stresses

Stress in round-wire torsion springs is given by

$$S = \frac{32K_B M}{\pi d^3} \tag{6.34}$$

where K_B = a stress correction factor. Stress is higher on the inner surface of the coil. A useful approximation of this factor is

$$K_B = \frac{4C - 1}{4C - 4} \tag{6.35}$$

6.6.3 Rectangular-Wire Torsion Springs

When rectangular wire is formed into coils, it approaches a keystone according to the relation

$$b_I = b\,\frac{C + 0.5}{C} \tag{6.36}$$

where b_I = axial dimension b after keystoning. The radial dimension is always t. The rate equation is

$$k = \frac{M}{\theta} = \frac{Ebt^3}{6.6DN_a} \tag{6.37}$$

Stress in rectangular-wire torsion springs is given by

$$S = \frac{6K_B M}{bt^2} \tag{6.38}$$

where $K_{B_{ID}} = 4C/(4C - 3)$ and b = axial dimension of rectangular cross section. Maximum recommended stresses are given in Table 6.15 for static applications and in Table 6.16 for cyclic applications.

TABLE 6.15 Maximum Recommended Bending Stresses for Helical Torsion Springs in Static Applications

Material	Percent of Tensile Strength	
	Stress-Relieved (1) (K_B Corrected)	With Favorable Residual Stress (2) (No Correction Factor)
Patented and Cold Drawn	80	100
Hardened and Tempered Carbon and Low Alloy Steels	85	100
Austenitic Stainless Steels and Non-Ferrous Alloys	60	80

(1) Also for springs without residual stresses.
(2) Springs that have not been stress-relieved and which have bodies and ends loaded in a direction that decreases the radius of curvature.
SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.16 Maximum Recommended Bending Stresses (K_B Corrected) for Helical Torsion Springs in Cyclic Applications

Fatigue Life (cycles)	ASTM A228 and Type 302 Stainless Steel		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened*	Not Shot-Peened	Shot-Peened*
10^5	53	62	55	64
10^6	50	60	53	62

This information is based on the following conditions: no surging, springs are in the "as-stress-relieved" condition
*Not always possible.

SOURCE: Associated Spring, Barnes Group Inc.

6.6.4 Tolerances

The tolerances for coil diameter and end position are given in Tables 6.17 and 6.18, respectively. Use them as guides.

6.7 BELLEVILLE SPRING WASHER

Belleville washers, also known as *coned-disk springs,* take their name from their inventor, Julian F. Belleville. They are essentially circular disks formed to a conical shape, as shown in Fig. 6.25. When load is applied, the disk tends to flatten. This elastic deformation constitutes the spring action.

TABLE 6.17 Commercial Tolerances for Torsion-Spring Coil Diameters

Wire Diameter mm (in.)	Tolerance: ±mm (in.)						
	Spring Index D/d						
	4	6	8	10	12	14	16
0.38 (0.015)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.08 (0.003)	0.10 (0.004)
0.58 (0.023)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.13 (0.005)	0.15 (0.006)
0.89 (0.035)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.23 (0.009)
1.30 (0.051)	0.05 (0.002)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.20 (0.008)	0.25 (0.010)	0.31 (0.012)
1.93 (0.076)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.23 (0.009)	0.31 (0.012)	0.38 (0.015)	0.46 (0.018)
2.90 (0.114)	0.10 (0.004)	0.18 (0.007)	0.25 (0.010)	0.33 (0.013)	0.46 (0.018)	0.56 (0.022)	0.71 (0.028)
4.37 (0.172)	0.15 (0.006)	0.25 (0.010)	0.33 (0.013)	0.51 (0.020)	0.69 (0.027)	0.86 (0.034)	1.07 (0.042)
6.35 (0.250)	0.20 (0.008)	0.36 (0.014)	0.56 (0.022)	0.76 (0.030)	1.02 (0.040)	1.27 (0.050)	1.52 (0.060)

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.18 End-Position Tolerances (for *D/d* Ratios up to and Including 16)

Total Coils	Tolerance: ± Degrees*
Up to 3	8
Over 3–10	10
Over 10–20	15
Over 20–30	20
Over 30	25

*Closer tolerances available

SOURCE: Associated Spring, Barnes Group Inc.

Belleville springs are used in two broad types of applications. First, they are used to provide very high loads with small deflections, as in stripper springs for punch-press dies, recoil mechanisms, and pressure-relief valves. Second, they are used for their special nonlinear load-deflection curves, particularly those with a constant-load portion. In loading a packing seal or a live center for a lathe, or in injection molding machines, Belleville washers can maintain a constant force throughout dimensional changes in the mechanical system resulting from wear, relaxation, or thermal change.

The two types of performance depend on the ratio of height to thickness. Typical load-deflection curves for various height-thickness ratios are shown in Fig. 6.26. Note that the curve for a small h/t ratio is nearly a straight line. At $h/t = 1.41$ the curve shows a nearly constant load for approximately the last 50 percent of deflection

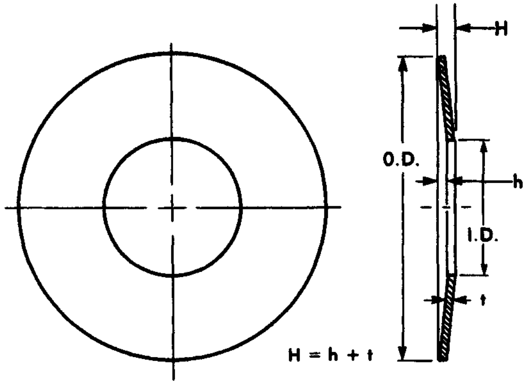

FIGURE 6.25 Belleville washer. *(Associated Spring, Barnes Group Inc.)*

before the flat position. Above $h/t = 1.41$ the load decreases after reaching a peak. When h/t is 2.83 or more, the load will go negative at some point beyond flat and will require some force to be restored to its free position. In other words, the washer will turn inside out.

The design equations given here are complex and may present a difficult challenge to the occasional designer. Use of charts and the equation transpositions presented here have proved helpful. Note that these equations are taken from the mathematical analysis by Almen and Laszlo [6.5]. The symbols used here are those originally used by the authors and may not necessarily agree with those used elsewhere in the text.

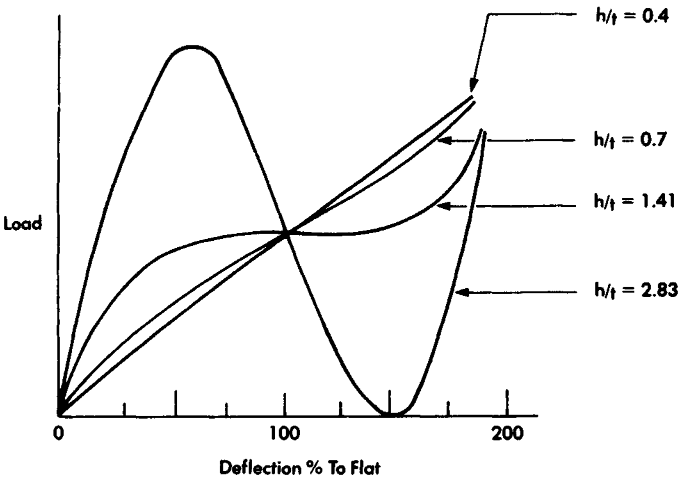

FIGURE 6.26 Load-deflection curves for Belleville washers with various h/t ratios. *(Associated Spring, Barnes Group Inc.)*

6.7.1 Nomenclature

a	OD/2, mm (in)
C_1	Compressive stress constant (see formula and Fig. 6.28)
C_2	Compressive stress constant (see formula and Fig. 6.28)
E	Modulus of elasticity (see Table 6.19), MPa (psi)
f	Deflection, mm (in)
h	Inside height, mm (in)
ID	Inside diameter, mm (in)
M	Constant
OD	Outside diameter, mm (in)
P	Load, N (lb)
P_f	Load at flat position, N (lb)
R	OD/ID
S_c	Compressive stress (Fig. 6.27), MPa (psi)
S_{T_1}	Tensile stress (Fig. 6.27), MPa (psi)
S_{T_2}	Tensile stress (Fig. 6.27), MPa (psi)
t	Thickness, mm (in)
T_1	Tensile stress constant (see formula and Fig. 6.29)
T_2	Tensile stress constant (see formula and Fig. 6.29)
μ	Poisson's ratio (Table 6.19)

6.7.2 Basic Equations

$$P = \frac{Ef}{(1-\mu^2)Ma^2}\left[(h-f)\left(h-\frac{f}{2}\right)t + t^3\right] \tag{6.39}$$

$$P_F = \frac{Eht^3}{(1-\mu^2)Ma^2} \tag{6.40}$$

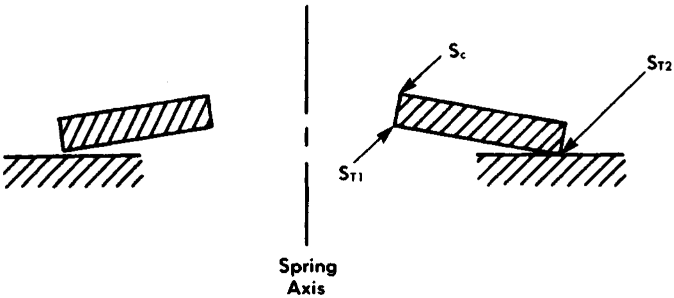

FIGURE 6.27 Highest-stressed regions in Belleville washers. *(Associated Spring, Barnes Group Inc.)*

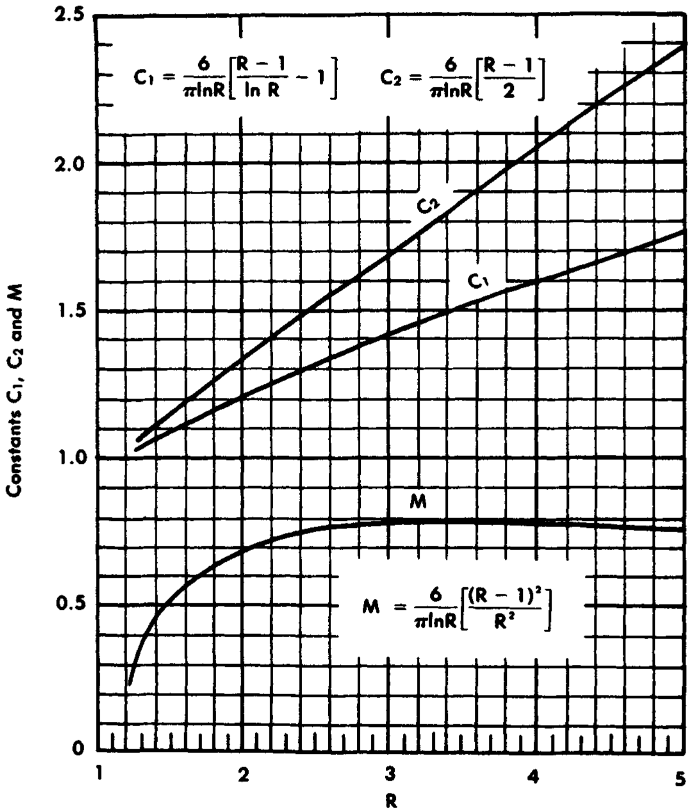

FIGURE 6.28 Compressive stress constants for Belleville washers. *(Associated Spring, Barnes Group Inc.)*

TABLE 6.19 Elastic Constants of Common Spring Materials

Material	Modulus of Elasticity E		Poisson's ratio μ
	Mpsi	GPa	
Steel	30	207	0.30
Phosphor bronze	15	103	0.20
17-7 PH stainless	29	200	0.34
302 stainless	28	193	0.30
Beryllium copper	18.5	128	0.33
Inconel	31	214	0.29
Inconel X	31	214	0.29

SOURCE: Associated Spring, Barnes Group Inc.

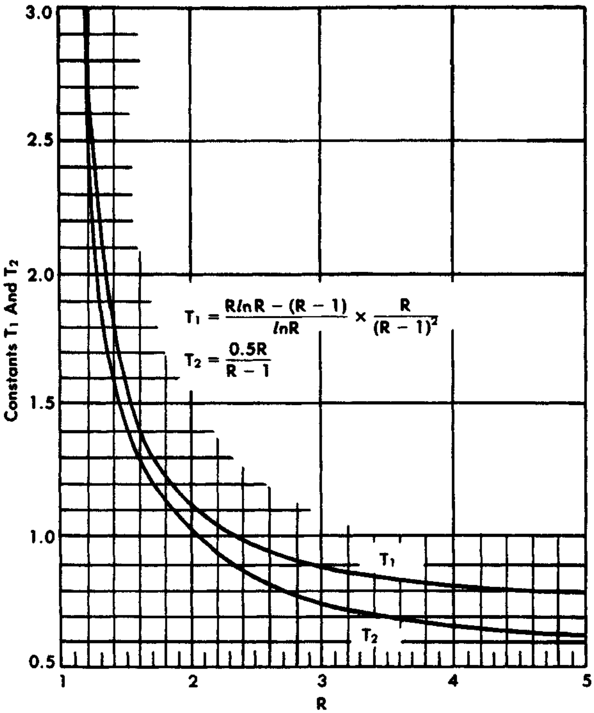

FIGURE 6.29 Tensile stress constants for Belleville washers. *(Associated Spring, Barnes Group Inc.)*

$$S_c = \frac{Ef}{(1-\mu^2)Ma^2}\left[C_1\left(h-\frac{f}{2}\right)+C_2t\right] \tag{6.41}$$

$$S_{T_1} = \frac{Ef}{(1-\mu^2)Ma^2}\left[C_1\left(h-\frac{f}{2}\right)-C_2t\right] \tag{6.42}$$

$$S_{T_2} = \frac{Ef}{(1-\mu^2)a^2}\left[T_1\left(h-\frac{f}{2}\right)+T_2t\right] \tag{6.43}$$

The design approach recommended here depends on first determining the loads and stresses at flat position, as shown in Fig. 6.30. Intermediate loads are determined from the curves in Fig. 6.31.

Figure 6.30 gives the values graphically for compressive stresses S_c at flat position. The stress at intermediate stages is approximately proportional to the deflection. For critical applications involving close tolerances or unusual proportions, stresses should be checked by using the equation before the design is finalized.

The stress level for static applications is evaluated in accordance with Eq. (6.41). This equation has been used most commonly for appraising the design of a Belleville spring because it gives the highest numerical value. It gives the compressive stress at the point shown in Fig. 6.27.

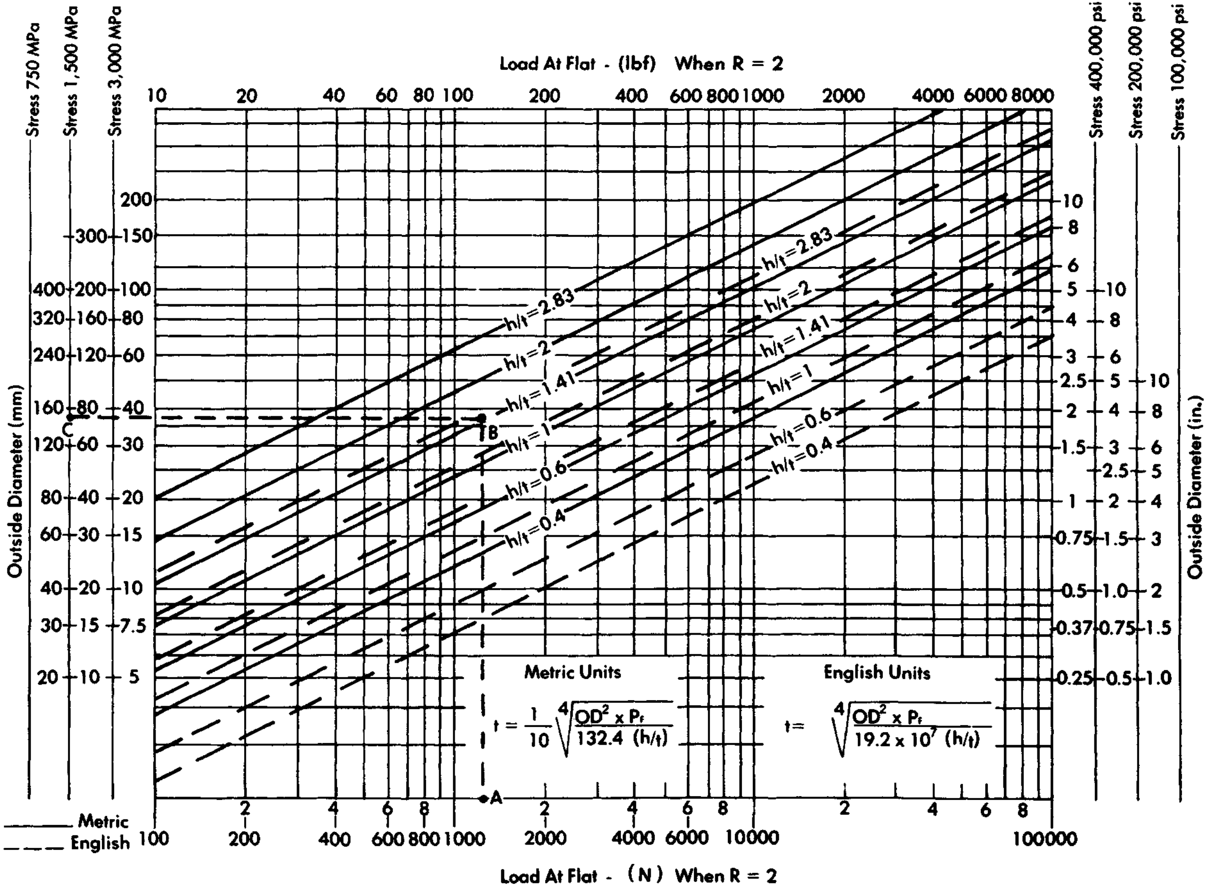

FIGURE 6.30 Loads and compressive stresses S_c for Belleville washers with various outside diameters and *h/t* ratios. *(Associated Spring, Barnes Group Inc.)*

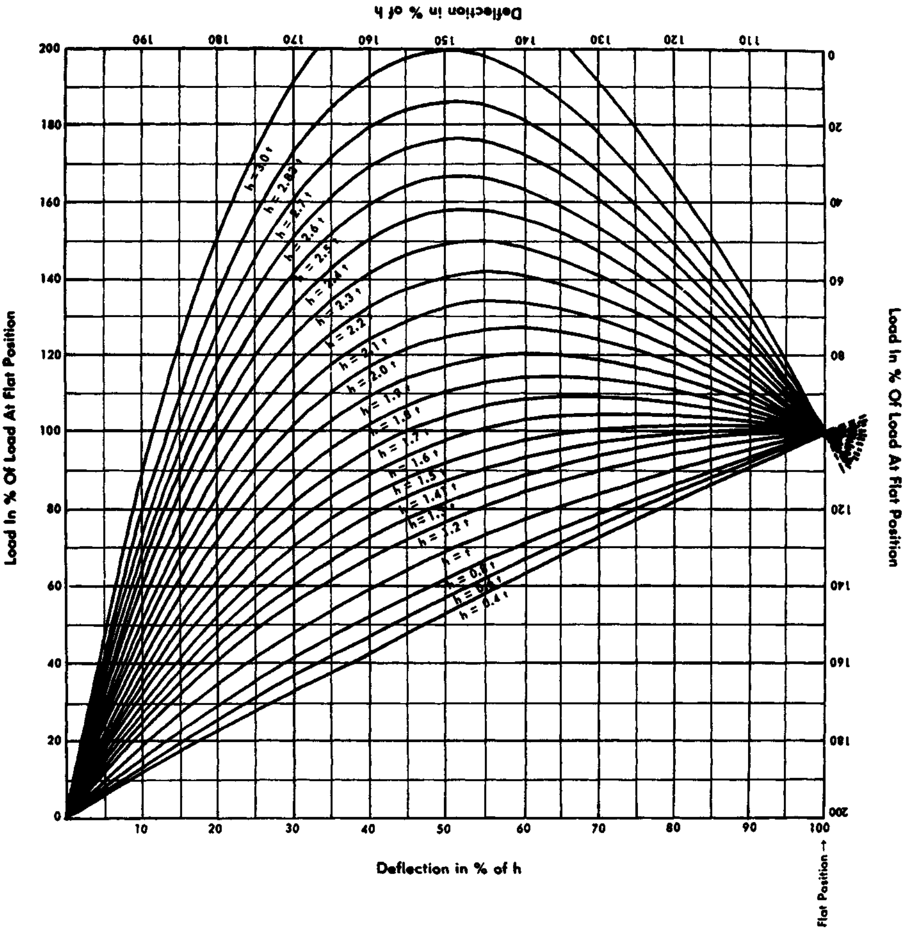

FIGURE 6.31 Load-deflection characteristics for Belleville washers. If a washer is supported and loaded at its edges so that it is deflected beyond the flat position, then the greatest possible deflection can be utilized. Since the load-deflection curve beyond the horizontal position is symmetric with the first part of the curve, this chart has been labeled at the right and top to be read upside down for deflection beyond horizontal. Dotted lines extending beyond the chart indicate continuation of curves beyond flat. *(Associated Spring, Barnes Group Inc.)*

A Belleville spring washer should be designed so that it can be compressed flat by accidental overloading, without setting. This can be accomplished either by using a stress so low that the spring will not set or by forming the spring higher than the design height and removing set by compressing flat or beyond flat (see Table 6.21). The table values should be reduced if the washers are plated or used at elevated temperatures.

For fatigue applications it is necessary to consider the tensile stresses at the points marked S_{T_1} and S_{T_2} in Fig. 6.27. The higher value of the two can occur at either the ID or the OD, depending on the proportions of the spring. Therefore, it is necessary to compute both values.

Fatigue life depends on the stress range as well as the maximum stress value. Figure 6.32 predicts the endurance limits based on either S_{T_1} or S_{T_2}, whichever is higher. Fatigue life is adversely affected by surface imperfections and edge fractures and can be improved by shot peening.

Since the deflection in a single Belleville washer is relatively small, it is often necessary to combine a number of washers. Such a combination is called a *stack.*

The deflection of a series stack (Fig. 6.33) is equal to the number of washers times the deflection of one washer, and the load of the stack is equal to that of one washer. The load of a parallel stack is equal to the load of one washer times the number of washers, and the deflection of the stack is that of one washer.

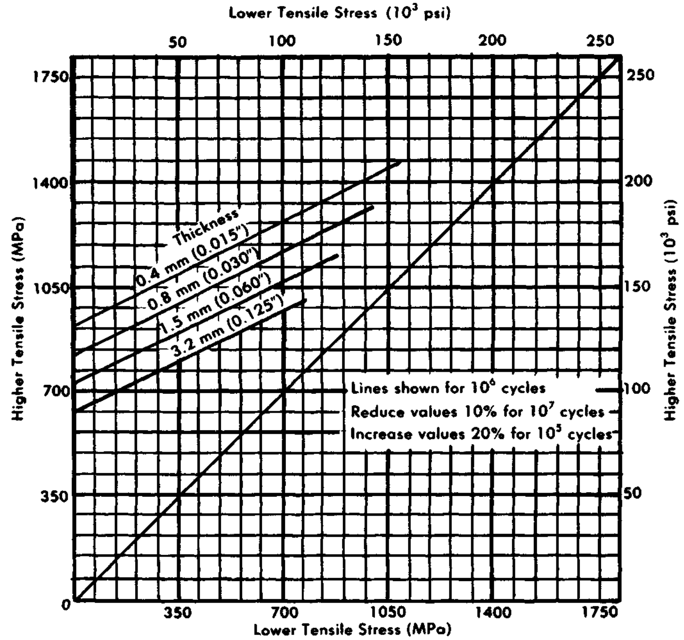

FIGURE 6.32 Modified Goodman diagram for Belleville washers; for carbon and alloy steels at 47 to 49 R_c with set removed, but not shot-peened. *(Associated Spring, Barnes Group Inc.)*

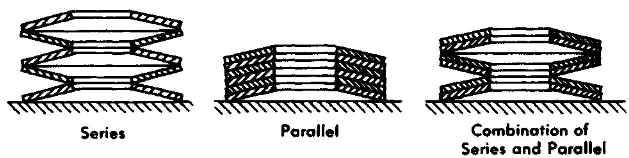

FIGURE 6.33 Stacks of Belleville washers. *(Associated Spring, Barnes Group Inc.)*

Because of production variations in washer parameters, both the foregoing statements carry cautionary notes. In the series stack, springs of the constant-load type ($h/t = 1.41$) may actually have a negative rate in some portion of their deflection range. When such a series stack is deflected, some washers will snap through, producing jumps in the load-deflection curve. To avoid this problem, the h/t ratio in a series stack design should not exceed 1.3.

In the parallel stack, friction between the washers causes a hysteresis loop in the load-deflection curve (Fig. 6.34). The width of the loop increases with each washer added to the stack but may be reduced by adding lubrication as the washers burnish each other during use.

Stacked washers normally require guide pins or sleeves to keep them in proper alignment. These guides should be hardened steel at HRC 48 minimum hardness. Clearance between the washer and the guide pin or sleeve should be about 1.5 percent of the appropriate diameter.

6.7.3 Tolerances

Load tolerances should be specified at test height. For carbon-steel washers with $h/t < 0.25$, use load tolerance of ±15 percent. For washers with $h/t > 0.25$, use ±10 percent. The recommended load tolerance for stainless steel and nonferrous washers is ±15 percent. See Table 6.20 for outside- and inside-diameter tolerances.

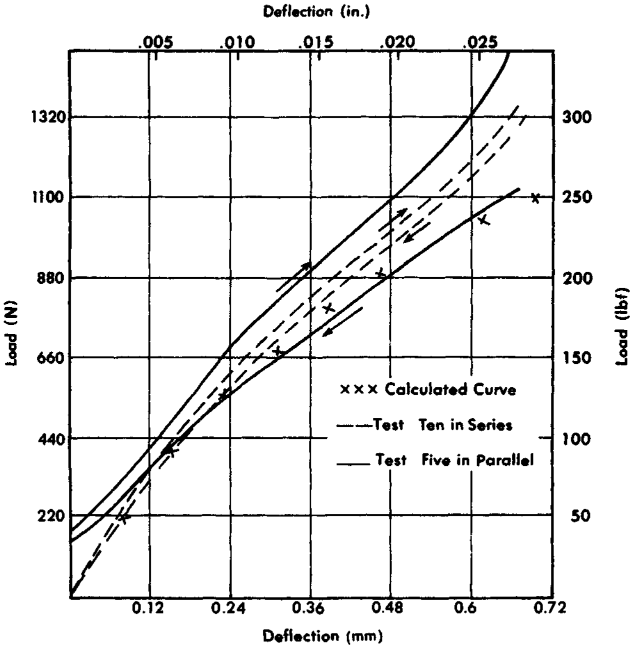

FIGURE 6.34 Hysteresis in stacked Belleville washers. *(Associated Spring, Barnes Group Inc.)*

TABLE 6.20 Belleville Washer Diameter Tolerances

Diameter, mm (in.)	O.D. mm (in.)	I.D. mm (in.)
	+0.00	−0.00
Up to 5 (0.197)	−0.20 (−0.008)	+0.20 (+0.008)
5–10 (0.197–0.394)	−0.25 (−0.010)	+0.25 (+0.010)
10–25 (0.394–0.984)	−0.30 (−0.012)	+0.30 (+0.012)
25–50 (0.984–1.969)	−0.40 (−0.016)	+0.40 (+0.016)
50–100 (1.969–3.937)	−0.50 (−0.020)	+0.50 (+0.020)

Based on R = 2, increased tolerances are required for lower R ratios.
SOURCE: Associated Spring, Barnes Group Inc.

Example. In a clutch, a minimum pressure of 202 lb (900 N) is required. This pressure must be held nearly constant as the clutch facing wears down 0.31 in (7.9 mm). The washer OD is 2.99 in (76 mm). The material washer OD is 2.99 in (76 mm). The material selected for the application is spring steel HRC 47-50.

Solution

1. Base the load on a value 10 percent above the minimum load, or 202 + 10 percent = 223 lb (998 N). Assume OD/ID = 2. From Fig. 6.31, select a load-deflection curve which gives approximately constant load between 50 and 100 percent of deflection to flat. Choose the $h/t = 1.41$ curve.
2. From Fig. 6.31, the load at 50 percent of deflection to flat is 88 percent of the flat load.
3. Flat load is $P_F = 223/0.88 = 252$ lb (1125 N).
4. From Fig. 6.30 [follow line AB from 1125 N to $h/t = 1.41$ and line BC to approximately 76-mm (2.99-in) OD], the estimated stress is 1500 MPa [218 kilopounds per square inch (kpsi)].
5. From Table 6.21 maximum stress without set removed is 120 percent of tensile strength. From Fig. 6.3, the tensile strength at HRC 48 will be approximately 239 kpsi (1650 MPa). Yield point without residual stress will be (239 kpsi)(1.20) = 287 kpsi. Therefore 218 kpsi stress is less than the maximum stress of 287 kpsi.
6. Stock thickness is

$$t = \sqrt[4]{\frac{\text{OD}^2(P_F)}{19.2(10^7)(h/t)}} = 0.054 \text{ in } (1.37 \text{ mm})$$

TABLE 6.21 Maximum Recommended Stress Levels for Belleville Washers in Static Applications

Material	Percent of Tensile Strength	
	Set Not Removed	Set Removed
Carbon or Alloy Steel	120	275
Nonferrous and Austenitic Stainless Steel	95	160

SOURCE: Associated Spring, Barnes Group Inc.

7.

$$h = 1.41t = 1.41(0.054) = 0.076 \text{ in}$$

$$H = h + t = 0.076 + 0.054 = 0.130 \text{ in}$$

8. Refer to Fig. 6.31. The load of 202 lb will be reached at f_1 = 50 percent of maximum available deflection. And $f_1 = 0.50(0.076) = 0.038$ in deflection, or the load of 223 lb will be reached at $H_1 = H - f_1 = 0.130 - 0.038 = 0.092$ in height at load. To allow for wear, the spring should be preloaded at $H_2 = H_1 - f(\text{wear}) = 0.092 - 0.032 = 0.060$ in height. This preload corresponds to a deflection $f_2 = H - H_2 = 0.130 - 0.060 = 0.070$ in. Then $f_2/h = 0.070/0.076 = 0.92$, or 92 percent of h.

9. Because 92 percent of h exceeds the recommended 85 percent (the load-deflection curve is not reliable beyond 85 percent deflection when the washer is compressed between flat surfaces), increase the deflection range to 40 to 85 percent. From Fig. 6.31, the load at 40 percent deflection is 78.5 percent, and P_F = 223/0.785 = 284 lb. Repeat previous procedures 4, 5, 6, 7, and 8, and find that $100(f_2/h)$ = 81 percent of h. The final design is as follows:

Material: AISI 1074

OD = 2.99 in (76 mm)

ID = 1.50 in (38 mm)

t = 0.055 in (1.40 mm) nominal

h = 0.078 in (1.95 mm) nominal

Tensile stress S_{T_1} = −29.5 kpsi (−203 MPa) at f_2 = 85 percent of h

Tensile stress S_{T_2} = 103 kpsi (710 MPa) at f_2 = 85 percent of h

6.8 SPECIAL SPRING WASHERS

Spring washers are being used increasingly in applications where there is a requirement for miniaturization and compactness of design. They are used to absorb vibrations and both side and end play, to distribute loads, and to control end pressure.

Design equations have been developed for determining the spring characteristics of curved, wave, and Belleville washers. There are no special design equations for slotted and finger washers. They are approximated by using Belleville and cantilever equations and then are refined through sampling and testing.

6.8.1 Curved Washers

These springs (Fig. 6.35) exert relatively light thrust loads and are often used to absorb axial end play. The designer must provide space for diametral expansion which occurs as the washer is compressed during loading. Bearing surfaces should be hard, since the washer edges tend to dig in. The spring rate is approximately linear up to 80 percent of the available deflection. Beyond that the rate will be much higher than calculated. Load tolerance should not be specified closer than ±20 percent.

Approximate equations are

$$P = \frac{4fEt^3}{\text{OD}^2(K)} \tag{6.44}$$

and

$$S = \frac{1.5KP}{t^2} \tag{6.45}$$

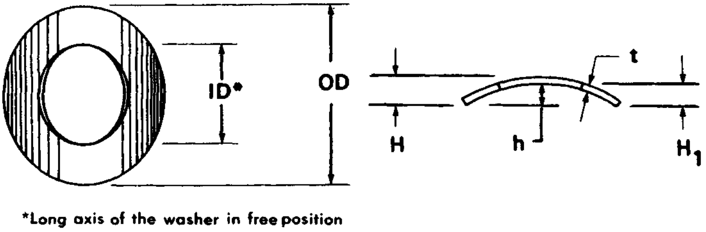

FIGURE 6.35 Curved washer. *(Associated Spring, Barnes Group Inc.)*

where K is given in Fig. 6.36 and f is 80 percent of h or less.

Maximum recommended stress levels for static operations are given in Table 6.22. Favorable residual stresses can be induced by shot peening and, to a lesser extent, by removing set. The maximum recommended stresses for cyclic applications are given in Table 6.23.

Tensile strengths for carbon steel are obtained from Fig. 6.3.

6.8.2 Wave Washer

These spring washers (Fig. 6.37) are regularly used in thrust loading applications, for small deflections, and for light to medium loads. The rate is linear between 20 and 80 percent of available deflection. Load tolerances should be no less than ±20 percent. In the most commonly used range of sizes, these washers can have three, four, or six waves.

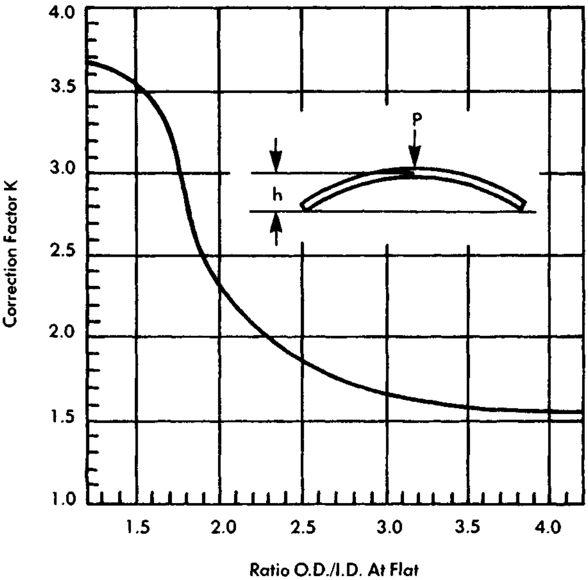

FIGURE 6.36 Empirical correction factor K for curved spring washers. *(Associated Spring, Barnes Group Inc.)*

TABLE 6.22 Maximum Recommended Operating Stress Levels for Special Spring Washers in Static Applications

Material	Percent of Tensile Strength	
	Stress-Relieved	With Favorable Residual Stresses
Steels, Alloy Steels	80	100
Nonferrous Alloys and Austenitic Steel	—	80

Finger washers are not generally supplied with favorable residual stresses.
SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.23 Maximum Recommended Operating Stress Levels for Steel Curved and Wave Washers in Cyclic Applications

Life (Cycles)	Percent of Tensile Strength
	Maximum Stress
10^4	80
10^5	53
10^6	50

This information is based on the following conditions: ambient environment, free from sharp bends, burrs, and other stress concentrations. AISI 1075
SOURCE: Associated Spring, Barnes Group Inc.

Design equations are

$$\frac{P}{f} = \frac{Ebt^3N^4(\text{OD})}{2.4D^3(\text{ID})} \tag{6.46}$$

and

$$S = \frac{3\pi PD}{4bt^2N^2} \tag{6.47}$$

where $D = \text{OD} - b$. The washer expands in diameter when compressed, according to the formula

$$D' = \sqrt{D^2 + 0.458h^2N^2} \tag{6.48}$$

Maximum recommended stress levels for static applications are given in Table 6.22. Favorable residual stresses are induced by shot peening or removing set. Table 6.23 gives the maximum recommended stress levels for cyclic applications. Figure 6.3 provides tensile strengths for carbon steel.

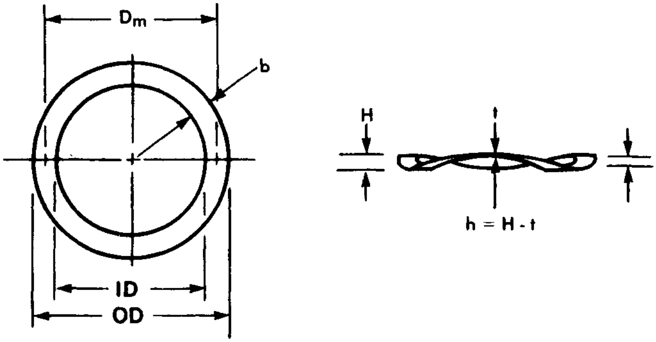

FIGURE 6.37 Typical wave spring washer. *(Associated Spring, Barnes Group Inc.)*

6.8.3 Finger Washers

Finger washers (Fig. 6.38) have both the flexibility of curved washers and the distributed points of loading of wave washers. They are calculated, approximately, as groups of cantilever springs; then samples are made and tested to prove the design. They are most frequently used in static applications such as applying axial load to ball-bearing races to reduce vibration and noise. These washers are not used in cyclic applications because of the shear cuts.

6.8.4 Slotted Washers

These are more flexible than plain dished washers but should be designed to maintain a constant pressure rather than to operate through a deflection range (see Fig. 6.39).

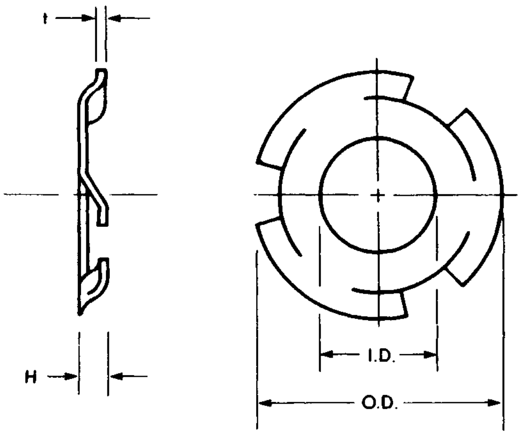

FIGURE 6.38 Finger washer. *(Associated Spring, Barnes Group Inc.)*

6.9 FLAT SPRINGS

6.9.1 Introduction

The classification *flat springs* applies to a wide range of springs made from sheet, strip, or plate material. Exceptions to this classification are power springs and washers. Flat springs may contain bends and forms. Thus the classification refers to the raw material and not to the spring itself.

Flat springs can perform functions beyond normal spring functions. A flat spring may conduct electricity, act as a latch, or hold a part in position. In some flat springs, only a portion of the part may have a spring function.

FIGURE 6.39 Slotted washers. *(Associated Spring, Barnes Group Inc.)*

Most flat springs are custom designs, and the tooling is often a major cost consideration. Flat springs can be cantilever or simple elliptical beams or combinations of both. These two elementary forms are discussed in this section. For a description of the methods used to compute complex flat-spring designs, see [6.6].

Load specification in flat springs is closely connected with the dimensioning of the form of the spring. From the equations it can be seen that the deflection and load vary in proportion to the third power of the material thickness. The important factors in load control are first, the material thickness and second, the deflection. Where close load control is required, the material may have to be selected to restricted thickness tolerance, and/or the free shape may be trued.

6.9.2 Cantilever Springs

The basic type of cantilever is a rectangular spring as shown in Fig. 6.40. The maximum bending stress occurs at the clamping point, and the stress is not uniform through the section. This stress is

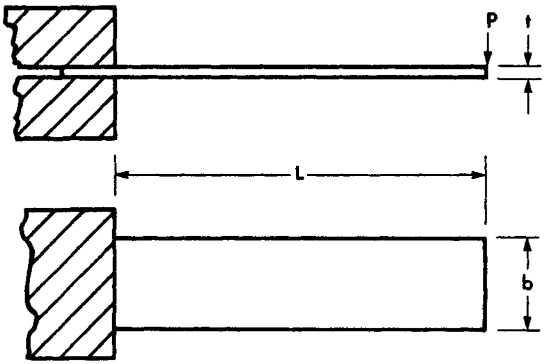

FIGURE 6.40 Rectangular cantilever spring. *(Associated Spring, Barnes Group Inc.)*

$$S = \frac{6PL}{bt^2} \tag{6.49}$$

The load is given by

$$P = \frac{fEbt^3}{4L^3} \tag{6.50}$$

These equations are satisfactory when the ratio of deflection to length f/L is less than 0.3. For larger deflections, use the method described in Fig. 6.41.

In cantilever springs with a trapezoidal or triangular configuration (Fig. 6.42), the stress is uniform throughout and is

$$S = \frac{6PL}{b_o t^2} \tag{6.51}$$

The corresponding load is

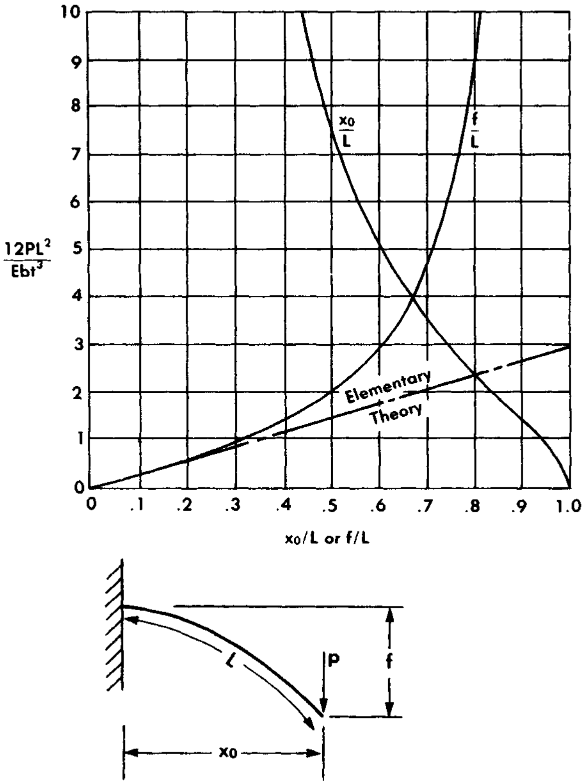

FIGURE 6.41 Calculating large deflection in cantilever beams [6.7]. To utilize this figure for any load P, first calculate the quantity $12PL^3/Ebt^3$. Using this value, from the curves find f/L and x_o/L, where x_o is the moment arm of the load P. Deflection then equals L multiplied by f/L. The maximum stress is reduced in the ratio x_o/L. *(Associated Spring, Barnes Group Inc.)*

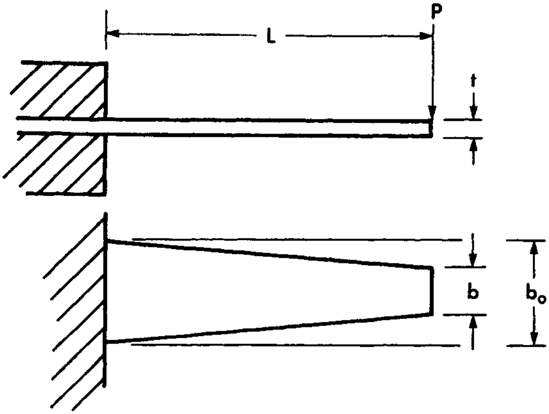

FIGURE 6.42 Trapezoidal cantilever spring. *(Associated Spring, Barnes Group Inc.)*

$$P = \frac{fEb_o t^3}{4L^3 K} \tag{6.52}$$

where K = constant based on the ratio b/b_o (Fig. 6.43). These equations are valid for f/L ratios of less than 0.3.

6.9.3 Simple Beams or Elliptical Springs

Simple beams are usually rectangular and are formed into an arc as in Fig. 6.44. If holes are introduced for clamping purposes, stress will increase at the hole and at the clamping point owing to stress concentration.

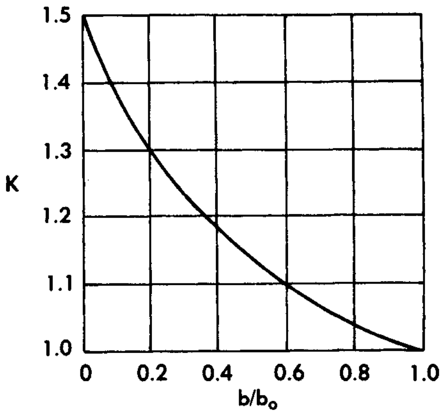

FIGURE 6.43 Correction factor for trapezoidal beam-load equation. *(Associated Spring, Barnes Group Inc.)*

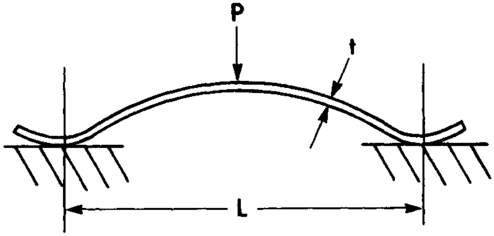

FIGURE 6.44 Simple beam spring. *(Associated Spring, Barnes Group Inc.)*

When ends are free to move laterally, the equation for load is

$$P = \frac{4fEbt^3}{L^3} \tag{6.53}$$

and stress is given by

$$S = \frac{1.5PL}{bt^2} \tag{6.54}$$

These equations apply when the ratio f/L is less than 0.15.

Stress Considerations. The maximum design stresses for cantilevers and simple beams are given in Table 6.24 for static applications and in Table 6.25 for cyclic applications. These recommendations do not apply when holes, sharp corners, notches, or abrupt changes in cross section are incorporated in the design, and should be used for guidance only.

6.10 CONSTANT-FORCE SPRINGS

A constant-force spring is a roll of prestressed material which exerts a nearly constant restraining force to resist uncoiling. Its unique characteristic is *force independent of deflection.* The force required to produce a unit deflection is the same for

TABLE 6.24 Maximum Design Stresses for Cantilever and Simple Beam Springs in Static Applications

Percent of Tensile Stength			
Ferrous Material		Nonferrous Material	
No Residual Stress	Maximum Residual Stress	No Residual Stress	Maximum Residual Stress
80	100	75	80

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.25 Maximum Design Stresses for Carbon-Steel Cantilever and Simple Beam Springs in Cyclic Applications

Number of Cycles	Percent of Tensile Strength	
	Not Shot-Peened	Shot-Peened*
10^5	53	62
10^6	50	60
10^7	48	58

*Shot peening is not recommended for thin materials and complex shapes. This information is based on an ambient environment. Stress ratio = 0.

SOURCE: Associated Spring, Barnes Group Inc.

each increment of coil because the radius of curvature of each increment is the same as any other.

Although these springs are not constant-load or constant-torque springs in the precise meaning of those terms, they produce a more nearly constant load over a greater deflection than any other spring design covered here. See Fig. 6.45. Constant-force springs are made of both type 301 stainless steel and ultra-high-strength high-carbon steels, with many of the applications using stainless steel because of its inherent resistance to corrosion.

One of the most severe limitations on the use of constant-force springs is their relatively short operating life. The most efficient use of material will produce a life of about 3000 cycles. Although life of hundreds of thousands of cycles is possible,

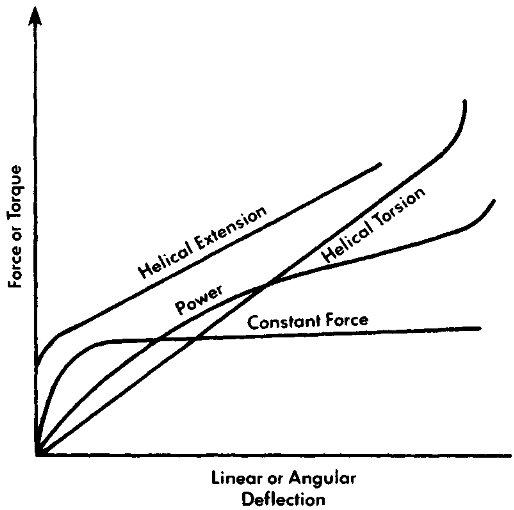

FIGURE 6.45 Load-deflection curves for various spring configurations. *(Associated Spring, Barnes Group Inc.)*

most applications fall into the range of 3000 to 30 000 cycles. Figure 6.46 shows the relationship between stress and fatigue life. These curves are derived from experimentally obtained data.

Some applications involving constant-force and constant-torque springs are simple extension springs, window sash counterbalances, camera motors, toys, machine carriage returns, constant-pressure electric-motor brush springs, space vehicle applications, and retraction devices.

6.10.1 Extension Type

This type of spring is a spiral spring made of strip material wound on the flat with an inherent curvature such that, in repose, each coil wraps tightly on its inner neighbor. In use the strip is extended with the free end loaded and the inner end supported on a drum or arbor. Very long deflections are possible, but the strip becomes unstable in long deflections and must be guided or supported to avoid kinking or snarling on the return stroke.

The rated load is not reached until after an initial deflection of 1.25 times the drum diameter, as shown in Fig. 6.47. Idler pulleys can be used but should be no smaller in diameter than the natural diameter of the coils and should never be used in a direction to cause backbending against the strip curvature.

6.10.2 Design Equations

$$P = \frac{Ebt^3}{6.5D_n^2} \qquad \text{for } N \leq 10 \tag{6.55}$$

$$P = \frac{Ebt^3}{6.5D_1}\left(\frac{2}{D_n} - \frac{1}{D_1}\right) \qquad \text{for } N > 10 \tag{6.56}$$

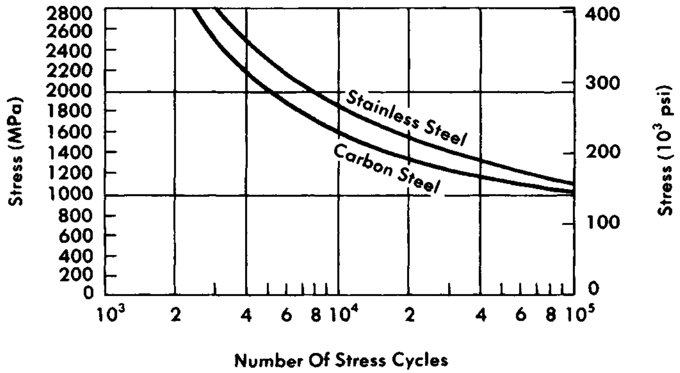

FIGURE 6.46 Maximum bending stress versus number of stress cycles for constant-force springs. These curves are based on no. 1 round-edge strip. *(Associated Spring, Barnes Group Inc.)*

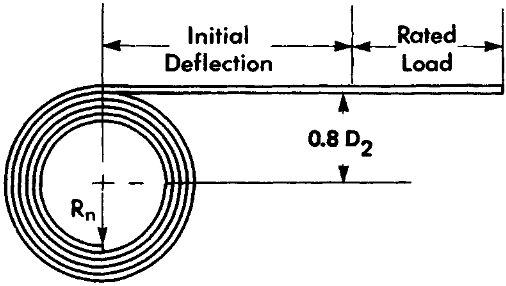

FIGURE 6.47 Typical constant-force extension spring (extension form). *(Associated Spring, Barnes Group Inc.)*

If unknown, let $b/t = 100/1$, $D_2 = 1.2\ D_n$,

$$S = \frac{Et}{D_n} \tag{6.57}$$

and

$$L = 1.57N(D_1 + D_2) \qquad \text{or} \qquad L \approx f + 5D_2$$

where N = number of turns
D_1 = outside coil diameter
D_2 = drum (arbor) diameter
D_n = natural diameter
E = modulus of elasticity

6.10.3 Spring Motor Type

When a constant-force spring is mounted on two drums of different diameters and the spring is backbent onto the larger diameter, the result is a constant-force spring motor. The strip is in repose on the smaller (storage) drum and is backbent onto the larger (output) drum. Torque is taken from the output drum shaft as shown in Fig. 6.48.

Note here that constant torque does not mean constant speed. Constant torque implies uniform acceleration, and the mechanism so driven will continue to speed up unless restrained by a governor mechanism. Load tolerances are normally held within ±10 percent.

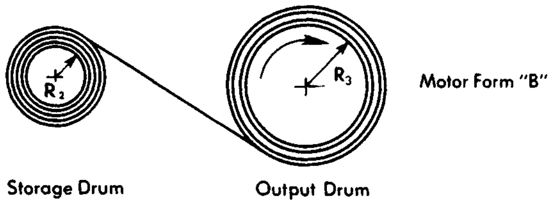

FIGURE 6.48 Typical constant-torque motor spring. *(Associated Spring, Barnes Group Inc.)*

6.10.4 Design Equations

$$M = \frac{Ebt^3 D_3}{13}\left(\frac{1}{D_n} + \frac{1}{D_3}\right)^2 \tag{6.58}$$

$$S = Et\left(\frac{1}{D_n} + \frac{1}{D_3}\right) \tag{6.59}$$

$$L = \pi N(D_3 + Nt) + 10D_3 \tag{6.60}$$

$$R_c = R_n \sqrt{4 + \frac{4R_3}{R_n} + \frac{R_n}{R_3} + \left(\frac{R_3}{R_n}\right)^2} \tag{6.61}$$

Design Suggestions. Let

$$\frac{b}{t} = 100 \qquad \frac{D_n}{t} = 250 \qquad \frac{D_3}{D_n} = 2 \qquad \frac{D_3}{D_2} = 1.6$$

where D_n = natural diameter
R_n = natural radius
D_2 = storage-drum diameter
D_3 = output-drum diameter
R_3 = output-drum radius
N = number of revolutions
R_c = minimum center-to-center distance of drums

6.11 TORSION BARS

Torsion bars used as springs are usually straight bars of spring material to which a twisting couple is applied. The stressing mode is torsional. This type of spring is very efficient in its use of material to store energy. The major disadvantage with the torsion bar is that unfavorable stress concentrations occur at the point where the ends are fastened.

Although both round and rectangular bar sections are used, the round section is used more often.

6.11.1 Design Equations: Round Sections

$$\phi = \frac{584ML}{d^4 G} \tag{6.62}$$

$$S = \frac{16M}{\pi d^3} \tag{6.63}$$

where ϕ = rotation angle in degrees
S = shear stress
L = active length

6.11.2 Design Equations: Rectangular Sections

$$\phi = \frac{57.3ML}{K_1 bt^3 G} \quad (6.64)$$

$$S = \frac{M}{K_2 bt^2} \quad (6.65)$$

where factors K_1 and K_2 are taken from Table 6.26.

The assumptions used in deriving these equations are (1) the bar is straight, (2) the bar is solid, and (3) loading is in pure torsion.

Torsion-bar springs are often preset in the direction in which they are loaded by twisting the bar beyond the torsional elastic limit. Care must be taken in the use of a preset bar: It must be loaded in the same direction in which it was preset; otherwise, excessive set will occur.

6.12 POWER SPRINGS

Power springs, also known as clock, motor, or flat coil springs, are made of flat strip material which is wound on an arbor and confined in a case. Power springs store and release rotational energy through either the arbor or the case in which they are retained. They are unique among spring types in that they are almost always stored in a case or housing while unloaded. Figure 6.49 shows typical retainers, a case, and various ends.

6.12.1 Design Considerations

Power springs are stressed in bending, and stress is related to torque by

$$S = \frac{6M}{bt^2} \quad (6.66)$$

Load-deflection curves for power springs are difficult to predict. As a spring is wound up, material is wound onto the arbor. This material is drawn from that which

TABLE 6.26 Factors for Computing Rectangular Bars in Torsion

b/t	K_1	K_2
1.0	0.140	0.208
1.5	0.196	0.231
2.0	0.229	0.246
2.5	0.249	0.258
3.0	0.263	0.267
5.0	0.291	0.291

SOURCE: A. M. Wahl, *Mechanical Springs*, 2d ed., McGraw-Hill Book Company, New York, 1963.

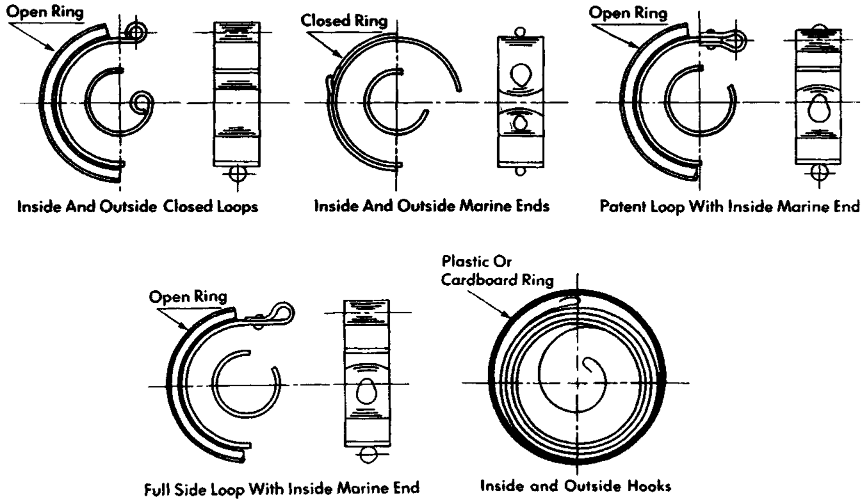

FIGURE 6.49 Typical power spring retainers and ends. *(Associated Spring, Barnes Group Inc.)*

was at rest against the case. Thus, the length of active material is constantly changing, which makes it difficult to develop a workable expression for the spring rate. For these reasons, ratios, tables, and graphical presentations are used to develop the design criteria.

The ratio of arbor diameter to thickness D_a/t is sometimes called the *life factor.* If it is too small, fatigue life will suffer. The life factor is usually maintained from 15 to 25. The ratio of active strip length to thickness L/t determines the flatness of the spring-gradient (torque-revolution) curve. The curve is flatter when L is longer. The usual range of the L/t ratio is from 5000 to 10 000. The ratio of the inside diameter of cup (case or housing) to thickness D_c/t is the *turns factor.* This determines the motion capability of the spring or indicates how much space is available between the arbor and the material lying against the inside of the case.

6.12.2 Design Procedure

In order to design a power spring that will deliver a given torque and number of turns, first determine its maximum torque in the fully wound condition. If a spring is required to deliver a minimum torque of 0.5 N·m for 10 revolutions (r) of windup and 10 r equals 80 percent unwound from solid, then from Fig. 6.50 we see that the torque at that point is 50 percent of the fully wound. Thus the fully wound torque is 1.0 N·m. Table 6.27 shows that a strip of steel 0.58 mm thick and 10 mm wide will provide 1.0 N·m of torque at the fully wound position per 10 mm of strip width.

Figure 6.51 shows that the average maximum solid stress for 0.58-mm-thick stock is about 1820 MPa. At the hardness normally supplied in steel strip for power springs, this is about 95 percent of tensile strength.

In Fig. 6.52, 10 turns relate to a length-to-thickness L/t ratio of 4300. With $t = 0.58$, L equals 2494 mm. Similarly, 4300 L/t relates to a D_c/t ratio of 107. Then $D_c = 62.06$ mm. If

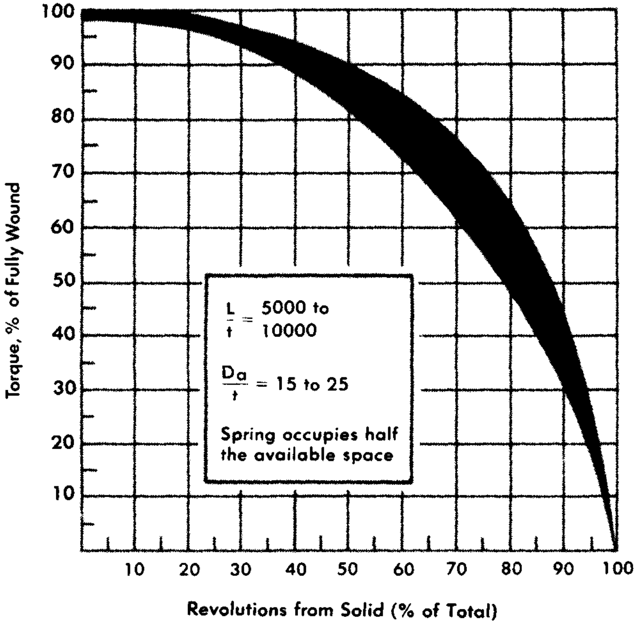

FIGURE 6.50 Typical normalized torque-revolution curve for power springs. *(Associated Spring, Barnes Group Inc.)*

$$L = \frac{D_c^2 - D_a^2}{2.55t} \tag{6.67}$$

then $D_a = \sqrt{D_c^2 - 2.55Lt} = 12.72$ mm and $D_a/t = 22$.

The equation for the number of turns a power spring will deliver, when it occupies half the space between arbor and case, is

$$\theta = \frac{\sqrt{2(D_c^2 + D_a^2)} - (D_c + D_a)}{2.55t} \tag{6.68}$$

In this example $\theta = 10$ r.

Experience shows that highly stressed power springs, made from pretempered AISI 1095 steel with a hardness of HRC 50 to 52 and stressed to 100 percent of tensile strength, could be expected to provide approximately 10 000 full-stroke life cycles. If the maximum stress were 50 percent of tensile strength at full stroke, then a life of about 100 000 cycles could be expected.

The final design is as follows:

$t = 0.023$ in (0.58 mm)
1095 carbon steel, HRC 51, no. 1 round edge
$b = 0.394$ in (10 mm)
$L = 98.188$ in (2494 mm)
$D_a = 0.501$ in (12.72 mm)
$D_c = 2.443$ in (62.06 mm)

TABLE 6.27 Torque per Unit of Width at Maximum Allowable Stress for Steel; L/t Range Is 5000 to 10 000

Thickness t		Unit Torque M		Thickness t		Unit Torque M	
mm	in	N·m/10 mm of width	lb·in/in of width	mm	in	N·m/10 mm of width	lb·in/in of width
0.127	0.005	0.0587	1.32	1.30	0.051	4.132	92.90
0.152	0.006	0.0841	1.89	1.37	0.054	4.541	102.1
0.178	0.007	0.1094	2.46	1.45	0.057	4.991	112.2
0.203	0.008	0.1419	3.19	1.60	0.063	5.947	133.7
0.229	0.009	0.1775	3.99	1.70	0.067	6.619	148.8
0.254	0.010	0.2171	4.88	1.83	0.072	7.504	168.7
0.279	0.011	0.2620	5.89	1.93	0.076	8.282	186.2
0.305	0.012	0.3074	6.91	2.03	0.080	8.981	201.9
0.330	0.013	0.3567	8.02	2.18	0.086	10.37	233.2
0.356	0.014	0.4101	9.22	2.34	0.092	11.74	264.0
0.381	0.015	0.4679	10.52	2.49	0.098	13.12	295
0.406	0.016	0.5271	11.85	2.67	0.105	14.86	334
0.432	0.017	0.5876	13.21	2.84	0.112	16.59	373
0.457	0.018	0.6530	14.68	3.05	0.120	18.82	423
0.483	0.019	0.7215	16.22	3.18	0.125	20.24	455
0.508	0.020	0.7953	17.88	3.43	0.135	23.35	525
0.584	0.023	1.025	23.05	3.58	0.141	25.35	570
0.635	0.025	1.189	26.72	3.76	0.148	27.76	624
0.711	0.028	1.452	32.65	3.96	0.156	30.69	690
0.813	0.032	1.841	41.40	4.11	0.162	33.00	742
0.889	0.035	2.144	48.20	4.50	0.177	39.23	882
1.041	0.041	2.824	63.50	4.75	0.187	43.81	985
1.19	0.047	3.585	80.60				

SOURCE: Associated Spring, Barnes Group Inc.

6.13 HOT-WOUND SPRINGS

6.13.1 Introduction

Springs are usually cold-formed when bar or wire diameters are less than 10 mm (approximately ⅜ in). When the bar diameter exceeds 16 mm (approximately ⅝ in), cold forming becomes impractical and springs are hot-wound.

Hot winding involves heating the steel into the austenitic range, winding hot, quenching to form martensite, and then tempering to the required properties. Although the most common types of hot-wound springs are compression springs for highway, off-highway, and railroad-vehicle suspension applications, torsion and extension springs can also be hot-wound.

6.13.2 Special Design Considerations

Design equations for hot-wound springs are the same as those for cold-formed springs except for the use of an empirical factor K_H which adjusts for effects related to hot-winding springs. Multiply the spring rate by K_H.

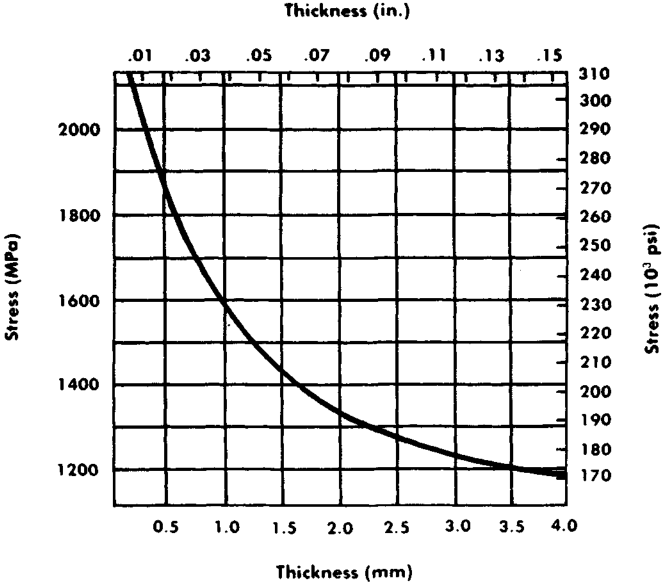

FIGURE 6.51 Average maximum solid stress in carbon-steel power springs. *(Associated Spring, Barnes Group Inc.)*

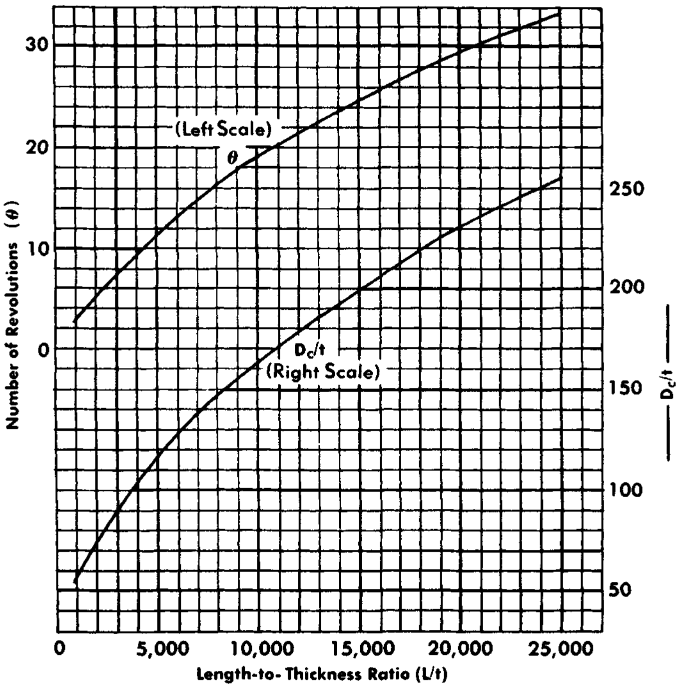

FIGURE 6.52 Relationships among number of revolutions, case diameter, strip length, and thickness for power springs. *(Associated Spring, Barnes Group Inc.)*

The values for factor K_H are 0.91 for springs made from hot-rolled carbon or low-alloy steel, *not* centerless ground; 0.96 for springs made from hot-rolled carbon or low-alloy steel, centerless ground; and 0.95 for torsion springs made from carbon or low-alloy steel.

The ends of hot-wound springs can be open or squared or either ground or not ground. Solid height is calculated in the same way as for cold-wound springs; but when space is limited, L_s can be reduced to $(N_t - 0.5)d$ by using a heavy grind.

The end configurations of extension or torsion springs must be formed hot at the same time as the spring is wound. If the configuration is complex, they may become cool in the process, and the whole spring may have to be reheated into the austenitic range. Note that hot-wound extension springs cannot have initial tension.

6.13.3 Materials

The common hot-wound alloys are AISI 5160, 5160H, and 1095 steels. The normal range of hardness is from HRC 44 to 48. Corresponding tensile strengths are 1430 to 1635 MPa.

The hot-rolled wire used in hot-wound springs is produced in standard sizes. Bar diameter variation and bar out-of-roundness tolerances are approximated in Table 6.28.

6.13.4 Choice of Operating Stress

Static Applications. The stress is calculated as in cold-wound springs. Use Table 6.29 for set-point information.

Cyclic Applications. Hot-wound springs made from hot-rolled wire are used in cyclic applications because rolled bars are subject to a variety of characteristic material defects mostly related to the bar surface condition. Therefore Table 6.30 can be

TABLE 6.28 Diameter and Out-of-Roundness Tolerances for Hot-Rolled Carbon-Steel Bars

Diameter mm(in.) Over	Diameter mm(in.) Through	Tolerance ± mm(in.)	Out-of-Roundness mm(in.)
	8 (0.315)	0.13 (0.005)	0.20 (0.008)
8 (0.315)	10 (0.394)	0.15 (0.006)	0.22 (0.009)
10 (0.394)	15 (0.591)	0.18 (0.007)	0.27 (0.011)
15 (0.591)	20 (0.787)	0.20 (0.008)	0.30 (0.012)
20 (0.787)	25 (0.984)	0.23 (0.009)	0.34 (0.013)
25 (0.984)	30 (1.181)	0.25 (0.010)	0.38 (0.015)
30 (1.181)	35 (1.378)	0.30 (0.012)	0.45 (0.018)
35 (1.378)	40 (1.575)	0.35 (0.014)	0.52 (0.020)
40 (1.575)	60 (2.362)	0.40 (0.016)	0.60 (0.024)
60 (2.362)	80 (3.150)	0.60 (0.024)	0.90 (0.035)

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.29 Maximum Allowable Torsional Stress for Hot-Wound Helical Compression Springs in Static Applications

Before Set Removal	After Set Removal
50% of TS	65–75% of TS

Torsional stress after set removal depends on material size and amount of set removed.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.30 Maximum Allowable Torsional Stress for Hot-Wound Helical Compression Springs in Cyclic Applications

Fatigue Life (Cycles)	Percent of Tensile Strength	
	Not Shot-Peened	Shot-Peened
10^5	40	48
10^6	38	46
10^7	35	43

This information is based on centerless ground AISI 5160, 5160H and 1095, HRC 44 to 48, 25 mm (1") diameter. Set has not been removed. Conditions are: no surging, room temperature and non-corrosive environment.

$$\text{Stress ratio in fatigue} = \frac{\text{S Minimum}}{\text{S Maximum}} = 0.$$

SOURCE: Associated Spring, Barnes Group Inc.

used only for centerless ground alloy bars. Practical manufacturing tolerances for hot-wound springs can be found in ASTM A125.

REFERENCES

6.1 Leonard G. Johnson, *The Statistical Treatment of Fatigue Experiments,* Elsevier Publishing Co., New York, 1964.

6.2 R. E. Little and E. H. Jebe, *Statistical Design of Fatigue Experiments,* John Wiley & Sons, New York, 1975.

6.3 A. M. Wahl, *Mechanical Springs,* 2d ed., McGraw-Hill, New York, 1963.

6.4 W. R. Berry, *Spring Design: A Practical Treatment,* Emmot & Co., London, 1961.

6.5 J. O. Almen and A. Laszlo, "The Uniform Section Disc Spring," *ASME Transactions,* vol. 58, no. 4, May 1936, pp. 305–314.

6.6 A. Blake, *Design of Curved Members for Machines,* Industrial Press, New York, 1966.

6.7 K. E. Bisshopp and D. C. Drucker, "Large Deflections of Cantilever Beams," *Quarterly of Applied Mathematics,* vol. 3, no. 3, 1945, p. 272.

6.8 J. H. Maker, "Steel Springs," *Metals Handbook,* 9th ed., vol. 1, American Society for Metals, Metals Park, Ohio, 1978, pp. 283–313.

6.9 *Design and Manufacture of Volute Springs,* SAE, New York, 1945.

6.10 R. E. Peterson, *Stress Concentration Design Factors,* John Wiley & Sons, New York, 1953, p. 102.

6.11 Karl W. Maier, "Dynamic Loading of Compression Springs," *Product Engineering,* January 1954, pp. 162–167; "Dynamic Loading of Compression Springs," *Product Engineering,* March 1955, pp. 162–172; "Surge Waves in Compression Springs," *Product Engineering,* August 1957, pp. 167–174.