P · A · R · T · 3

GEARING

CHAPTER 9
SPUR GEARS

Joseph E. Shigley
Professor Emeritus
The University of Michigan
Ann Arbor, Michigan

9.1 DEFINITIONS

Spur gears are used to transmit rotary motion between parallel shafts. They are cylindrical, and the teeth are straight and parallel to the axis of rotation.

The *pinion* is the smaller of two mating gears; the larger is called the *gear* or the *wheel.*

The *pitch circle, B* in Fig. 9.1, is a theoretical circle upon which all calculations are based. The *operating pitch circles* of a pair of gears in mesh are tangent to each other.

The *circular pitch, p* in Fig. 9.1, is the distance, measured on the theoretical pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. The circular pitch is measured in inches or in millimeters. Note, in Fig. 9.1, that the circular pitch is the sum of the *tooth thickness t* and the *width of space.*

The *pitch diameter, d* for the pinion and D for the gear, is the diameter of the pitch circle; it is measured in inches or in millimeters.

The *module m* is the ratio of the theoretical pitch diameter to the number of teeth N. The module is the metric index of tooth sizes and is always given in millimeters.

The *diametral pitch* P_d is the ratio of the number of teeth on a gear to the theoretical pitch diameter. It is the index of tooth size when U.S. customary units are used and is expressed as teeth per inch.

The *addendum a* is the radial distance between the top land F and the pitch circle B in Fig. 9.1. The *dedendum b* is the radial distance between the pitch circle B and the *root circle D* in Fig. 9.1. The *whole depth* h_t is the sum of the addendum and dedendum.

The clearance circle C in Fig. 9.1 is tangent to the addendum circle of the mating gear. The distance from the clearance circle to the bottom land is called the *clearance c.*

Backlash is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle.

Undercutting (see distance u in Fig. 9.1) occurs under certain conditions when a small number of teeth are used in cutting a gear.

Table 9.1 lists all the relations described above. Additional terminology is shown in Fig. 9.2. Here line OP is the *line of centers* connecting the rotation axes of a pair of

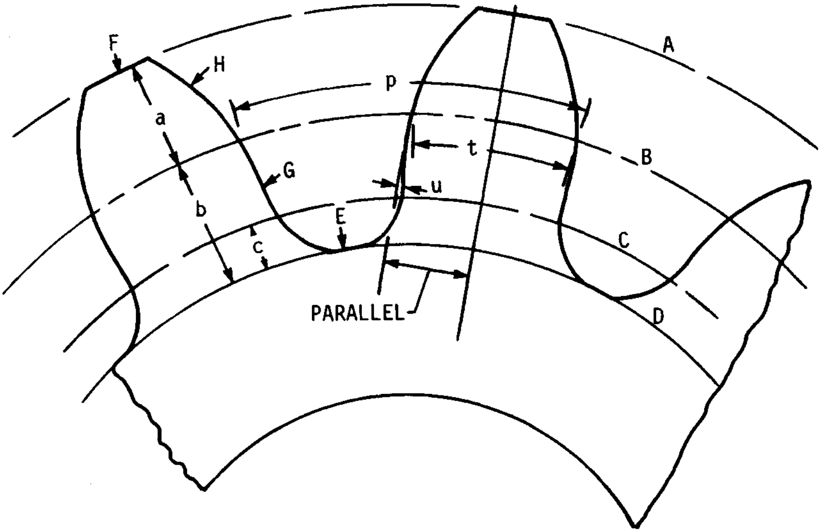

FIGURE 9.1 Terminology of gear teeth. *A*, addendum circle; *B*, pitch circle; *C*, clearance circle; *D*, dedendum circle; *E*, bottom land; *F*, top land; *G*, flank; *H*, face; a = addendum distance; b = dedendum distance; c = clearance distance; p = circular pitch; t = tooth thickness; u = undercut distance.

meshing gears. Line *E* is the *pressure line*, and the angle ϕ is the *pressure angle*. The resultant force vector between a pair of operating gears acts along this line.

The pressure line is tangent to both *base circles C* at points *F*. The operating diameters of the pitch circles depend on the center distance used in mounting the gears, but the base circle diameters are constant and depend only on how the tooth forms were generated, because they form the *base* or the starting point of the involute profile.

TABLE 9.1 Basic Formulas for Spur Gears

Quantity desired	Formula	Equation number
Diametral pitch P_d	$P_d = \dfrac{N}{d}$	(9.1)
Module m	$m = \dfrac{d}{N}$	(9.2)
Circular pitch p	$p = \dfrac{\pi d}{N} = \pi m$	(9.3)
Pitch diameter, d or D	$d = \dfrac{N}{P_d} = mN$	(9.4)

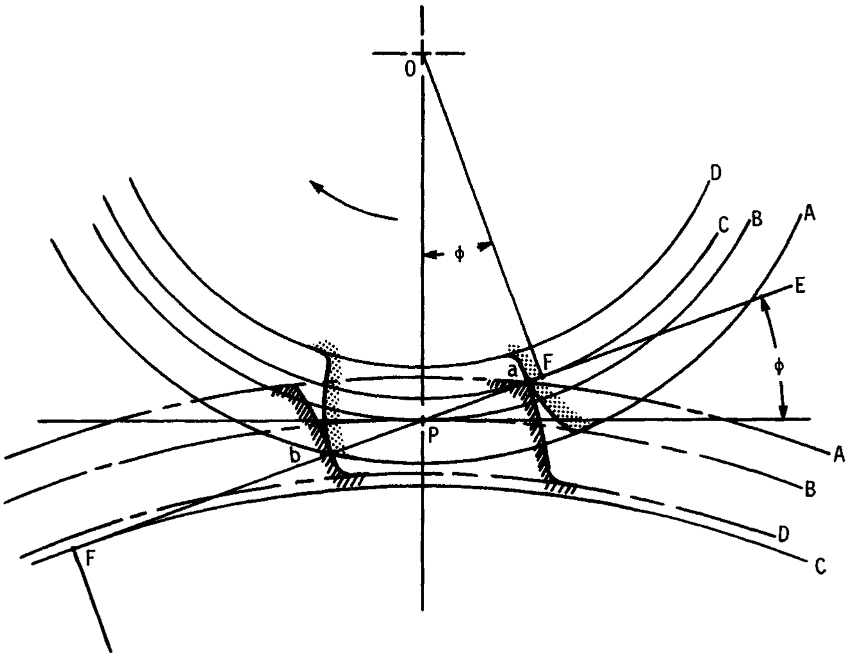

FIGURE 9.2 Layout drawing of a pair of spur gears in mesh. The pinion is the driver and rotates clockwise about the axis at O. A, addendum circles; B, pitch circles; C, base circles; D, dedendum circles; E, pressure line; F, tangent points; P, pitch point; a, initial point of contact; b, final point of contact.

Line aPb is the *line of action.* Point a is the *initial point of contact.* This point is located at the intersection of the addendum circle of the gear with the pressure line. Should point a occur on the other side of point F on the pinion base circle, the pinion flank would be *undercut* during generation of the profile.

Point b of Fig. 9.2 is the *final point of contact.* This point is located at the intersection of the addendum circle of the pinion with the pressure line. For no undercutting of the gear teeth, point b must be located between the pitch point P and point F on the base circle of the gear.

Line aP represents the *approach* phase of tooth contact; line Pb is the *recess* phase. Tooth contact is a sliding contact throughout the line of action except for an instant at P when contact is pure rolling. The nature of the sliding is quite different during the approach action and the recess action; bevel-gear teeth, for example, are generated to obtain more recess action, thus reducing wear.

Instead of using the theoretical pitch circle as an index of tooth size, the base circle, which is a more fundamental distance, can be used. The result is called the *base pitch* p_b. It is related to the circular pitch p by the equation

$$p_b = p \cos \phi \tag{9.5}$$

If, in Fig. 9.2, the distance from a to b exactly equals the base pitch, then, when one pair of teeth are just beginning contact at a, the preceding pair will be leaving contact at b. Thus, for this special condition, there is never more or less than one pair of teeth in contact. If the distance *ab* is greater than the base pitch but less than twice as much, then when a pair of teeth come into contact at *a*, another pair of teeth will still be in contact somewhere along the line of action *ab*. Because of the nature of this tooth action, usually one or two pairs of teeth in contact, a useful criterion of tooth action, called the *contact ratio* m_c, can be defined. The formula is

$$m_c = \frac{L_{ab}}{p_b} \tag{9.6}$$

where L_{ab} = distance *ab*, the length of the line of action. Do not confuse the contact ratio m_c with the module m.

9.2 TOOTH DIMENSIONS AND STANDARDS

The American Gear Manufacturer's Association (AGMA) publishes much valuable reference data.[†] The details on nomenclature, definitions, and tooth proportions for spur gears can be found in ANSI/AGMA 1012-F90. Table 9.2 contains the most used tooth proportions. The hob tip radius r_f varies with different cutters; $0.300/P_d$ or $0.300m$ is the usual value. Tables 9.3 and 9.4 list the modules and pitches in general use. Cutting tools can be obtained for all these sizes.

[†] See Chap. 10 for a special note on AGMA.

TABLE 9.2 Standard and Commonly Used Tooth Systems for Spur Gears

Tooth system	Pressure angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$ $1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$ $1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$ $1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

TABLE 9.3 Diametral Pitches in General Use

Coarse pitch	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine pitch	20, 24, 32, 40, 48, 64, 96, 120, 150, 200

TABLE 9.4 Modules in General Use

Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

9.3 FORCE ANALYSIS

In Fig. 9.3 a gear, not shown, exerts force W against the pinion at pitch point P. This force is resolved into two components, a radial force W_r, acting to separate the gears, and a tangential component W_t, which is called the *transmitted load.*

Equal and opposite to force W is the shaft reaction F, also shown in Fig. 9.3. Force F and torque T are exerted by the shaft on the pinion. Note that torque T opposes the force couple made up of W_t and F_x separated by the distance $d/2$. Thus

$$T = \frac{W_t d}{2} \tag{9.7}$$

where T = torque, lb · in (N · m)
W_t = transmitted load, lb (N)
d = operating pitch diameter, in (m)

The *pitch-line velocity* v is given by

$$v = \frac{\pi d n_P}{12} \text{ ft/min} \qquad v = \frac{\pi d n_P}{60} \text{ m/s} \tag{9.8}$$

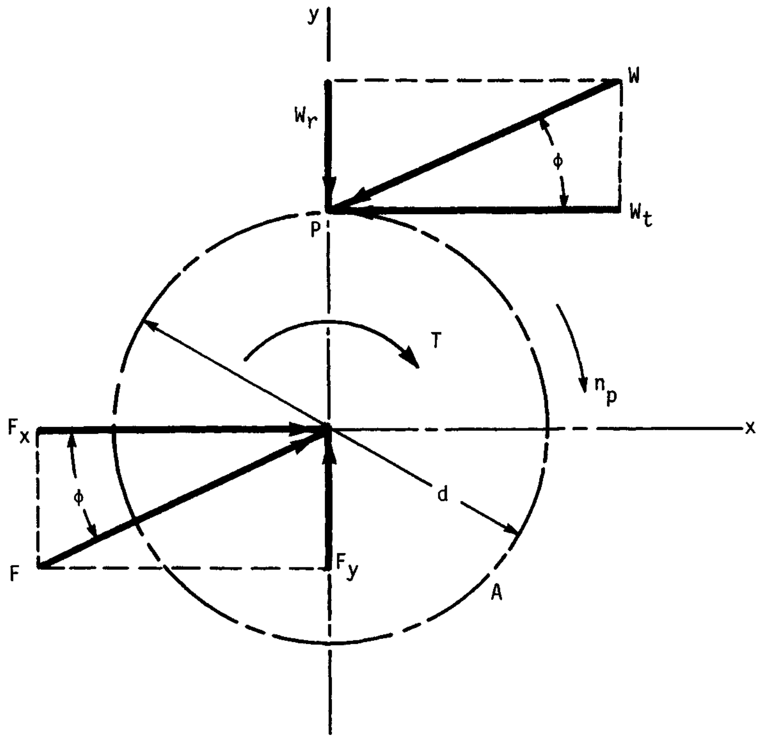

FIGURE 9.3 Force analysis of a pinion. A, operating pitch circle; d, operating pitch diameter; n_p, pinion speed; ϕ, pressure angle; W_t, transmitted tangential load; W_r, radial tooth load; W, resultant tooth load; T, torque; F, shaft force reaction.

where n_P = pinion speed in revolutions per minute (r/min). The power transmitted is

$$P = \begin{cases} \dfrac{W_t v}{33\,000} & \text{hp} \\ W_t v & \text{kW} \end{cases} \tag{9.9}$$

9.4 FUNDAMENTAL AGMA RATING FORMULAS†

Many of the terms in the formulas that follow require lengthy discussions and considerable space to list their values. This material is considered at length in Chap. 10 and so is omitted here.

9.4.1 Pitting Resistance

The basic formula for *pitting resistance,* or *surface durability,* of gear teeth is

$$s_c = C_p \left(\frac{W_t C_a}{C_v} \frac{C_s}{dF} \frac{C_m C_f}{I} \right)^{1/2} \tag{9.10}$$

where
- s_c = contact stress number, lb/in^2 (MPa)
- C_p = elastic coefficient, $(\text{lb/in}^2)^{1/2}$ $[(\text{MPa})^{1/2}]$; see Eq. (10.77) and Table 10.4
- W_t = transmitted tangential load, lb (N)
- C_a = application factor for pitting resistance; see Table 10.3
- C_s = size factor for pitting resistance; use 1.0 or more until values are established
- C_m = load distribution factor for pitting resistance; use Tables 9.5 and 9.6
- C_f = surface condition factor; use 1.0 or more until values are established
- C_v = dynamic factor for pitting resistance; use Fig. 10.4; multiply v in meters per second by 197 to get feet per minute
- d = operating pitch diameter of pinion, in (mm)
  = $2C/(m_G + 1.0)$ for external gears
  = $2C/(m_G - 1.0)$ for internal gears
- C = operating center distance, in (mm)
- m_G = gear ratio (never less than 1.0)
- F = net face width of narrowest member, in (mm)
- I = geometry factor for pitting resistance; use Eq. (10.24) with $C_\psi = 1.0$

Allowable Contact Stress Number. The contact stress number s_c, used in Eq. (9.10), is obtained from the *allowable contact stress number* s_{ac} by making several adjustments as follows:

$$s_c \le s_{ac} \frac{C_L C_H}{C_T C_R} \tag{9.11}$$

† See Ref. [10.1].

TABLE 9.5 Load-Distribution Factors C_m and K_m for Spur Gears Having a Face Width of 6 in (150 mm) and Greater†

Face-diameter ratio F/d	Contact	C_m, K_m
1 or less	95% face width contact at one-third torque	1.4 at one-third torque
	95% face width contact at full torque	1.1 at full torque
	75% face width contact at one-third torque	1.8 at one-third torque
	95% face width contact at full torque	1.3 at full torque
	35% face width contact at one-third torque	3.0 at one-third torque
	95% face width contact at full torque	1.9 at full torque
	20% face width contact at one-third torque	5.0 at one-third torque
	75% face width contact at full torque	2.5 at full torque
	Teeth are crowned:	
	35% face width contact at one-third torque	2.5 at one-third torque
	85% face width contact at full torque	1.7 at full torque
Over 1 and less than 2	Calculated combined twist and bending of pinion not over 0.001 in (0.025 mm) over entire face:	
	Pinion not over 250 bhn hardness:	
	75% face width contact at one-third torque	2.0 at one-third torque
	95% face width contact at full torque	1.4 at full torque
	30% face width contact at one-third torque	4.0 at one-third torque
	75% face width contact at full torque	3.0 at full torque

†For an alternate approach see Eq. (10.21).

SOURCE: ANSI/AGMA 2001-C95.

where s_{ac} = allowable contact stress number, lb/in^2 (MPa); see Fig.10.40
C_L = life factor for pitting resistance; use Fig.10.49
C_H = hardness ratio factor; use Figs. 10.47 and 10.48
C_T = temperature factor for pitting resistance; use 1.0 or more, but see Sec.10.5.1
C_R = reliability factor for pitting resistance; use Table 10.6

TABLE 9.6 Load-Distribution Factors C_m and K_m for Spur Gears

Condition of support	Face width: Up to 2 in (50 mm)	6 in (150 mm)	9 in (225 mm)	Over 16 in (400 mm)
Accurate mounting, low bearing clearances, minimum elastic deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across full face	1.6	1.7	1.8	2.0
Accuracy and mounting such that less than full-face contact exists	Over 2.0			

SOURCE: ANSI/AGMA 2001-C95. For an alternate approach see Eq. (10.21).

Pitting Resistance Power Rating. The allowable power rating P_{ac} for pitting resistance is given by

$$P_{ac} = \begin{cases} \dfrac{n_P F}{126\,000} \dfrac{IC_v}{C_s C_m C_f C_a} \left(\dfrac{d s_{ac}}{C_p} \dfrac{C_L C_H}{C_T C_R} \right)^2 & \text{hp} \\[2ex] \dfrac{n_P F}{1.91(10^7)} \dfrac{IC_v}{C_s C_m C_f C_a} \left(\dfrac{d s_{ac}}{C_p} \dfrac{C_L C_H}{C_T C_R} \right)^2 & \text{kW} \end{cases} \tag{9.12}$$

9.4.2 Bending Strength

The basic formula for the bending stress number in a gear tooth is

$$S_t = \begin{cases} \dfrac{W_t K_a}{K_v} \dfrac{P_d}{F} \dfrac{K_s K_m}{J} & \text{lb/in}^2 \\[2ex] \dfrac{W_t K_a}{K_v} \dfrac{1.0}{Fm} \dfrac{K_s K_m}{J} & \text{MPa} \end{cases} \tag{9.13}$$

where s_t = bending stress number, lb/in^2 (MPa)
K_a = application factor for bending strength; use Table 10.3
K_s = size factor for bending strength; use 1.0 or more until values are established
K_m = load distribution factor for bending strength; use Tables 9.5 and 9.6
K_v = dynamic factor for bending strength; use Fig.10.4; multiply v in meters per second by 197 to get feet per minute
J = geometry factor for bending strength; use Eq. (10.46) with $C_\psi = 1.0$ and Figs.10.11 to 10.22
m = module, mm
P_d = nominal diametral pitch, teeth per inch

Allowable Bending Stress Number. The bending stress number s_t in Eq. (9.13) is related to the *allowable bending stress number* s_{at} by

$$s_t \leq \frac{s_{at}K_L}{K_T K_R} \tag{9.14}$$

where
s_{at} = allowable bending stress number, lb/in^2 (MPa); use Fig. 10.41
K_L = life factor for bending strength; use Figs. 10.49 and 10.50
K_T = temperature factor for bending strength; use 1.0 or more; see Sec. 10.5.1
K_R = reliability factor for bending strength; use Table 10.6

Bending Strength Power Rating. The allowable power rating P_{at} for bending strength is given by

$$P_{at} = \begin{cases} \dfrac{n_P d K_v}{126\,000 K_a} \dfrac{FJ}{P_d K_s K_m} \dfrac{s_{at} K_L}{K_R K_T} & \text{hp} \\[2ex] \dfrac{n_P d K_v}{1.91(10)^7 K_a} Fm \dfrac{J}{K_s K_m} \dfrac{S_{at} K_L}{K_R K_T} & \text{kW} \end{cases} \tag{9.15}$$