CHAPTER 10
HELICAL GEARS

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The following is quoted from the Foreword of Ref. [10.1]:

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This reference is cited because numerous American Gear Manufacturer's Association (AGMA) tables and figures are used in this chapter. In each case, the appropriate publication is noted in a footnote or figure caption.

10.1 INTRODUCTION

Helical gearing, in which the teeth are cut at an angle with respect to the axis of rotation, is a later development than spur gearing and has the advantage that the action is smoother and tends to be quieter. In addition, the load transmitted may be somewhat larger, or the life of the gears may be greater for the same loading, than with an equivalent pair of spur gears. Helical gears produce an end thrust along the axis of the shafts in addition to the separating and tangential (driving) loads of spur gears. Where suitable means can be provided to take this thrust, such as thrust collars or ball or tapered-roller bearings, it is no great disadvantage.

Conceptually, helical gears may be thought of as stepped spur gears in which the size of the step becomes infinitely small. For external parallel-axis helical gears to mesh, they must have the same helix angle but be of different hand. An external-internal set will, however, have equal helix angle with the same hand.

Involute profiles are usually employed for helical gears, and the same comments made earlier about spur gears hold true for helical gears.

Although helical gears are most often used in a parallel-axis arrangement, they can also be mounted on nonparallel noncoplanar axes. Under such mounting conditions, they will, however, have limited load capacity.

Although helical gears which are used on crossed axes are identical in geometry and manufacture to those used on parallel axes, their operational characteristics are quite different. For this reason they are discussed separately at the end of this chapter. All the forthcoming discussion therefore applies only to helical gears operating on parallel axes.

10.2 TYPES

Helical gears may take several forms, as shown in Fig. 10.1:

1. Single
2. Double conventional
3. Double staggered
4. Continuous (herringbone)

Single-helix gears are readily manufactured on conventional gear cutting and grinding equipment. If the space between the two rows of a double-helix gear is wide enough, such a gear may also be cut and ground, if necessary, on conventional equipment. Continuous or herringbone gears, however, can be cut only on a special shaping machine (Sykes) and usually cannot be ground at all.

Only single-helix gears may be used in a crossed-axis configuration.

10.3 ADVANTAGES

There are three main reasons why helical rather than straight spur gears are used in a typical application. These are concerned with the noise level, the load capacity, and the manufacturing.

10.3.1 Noise

Helical gears produce less noise than spur gears of equivalent quality because the total contact ratio is increased. Figure 10.2 shows this effect quite dramatically. However, these results are measured at the mesh for a specific test setup; thus, although the trend is accurate, the absolute results are not.

Figure 10.2 also brings out another interesting point. At high values of helix angle, the improvement in noise tends to peak; that is, the curve flattens out. Had data been obtained at still higher levels, the curve would probably drop drastically. This is due to the difficulty in manufacturing and mounting such gears accurately enough to take full advantage of the improvement in contact ratio. These effects at

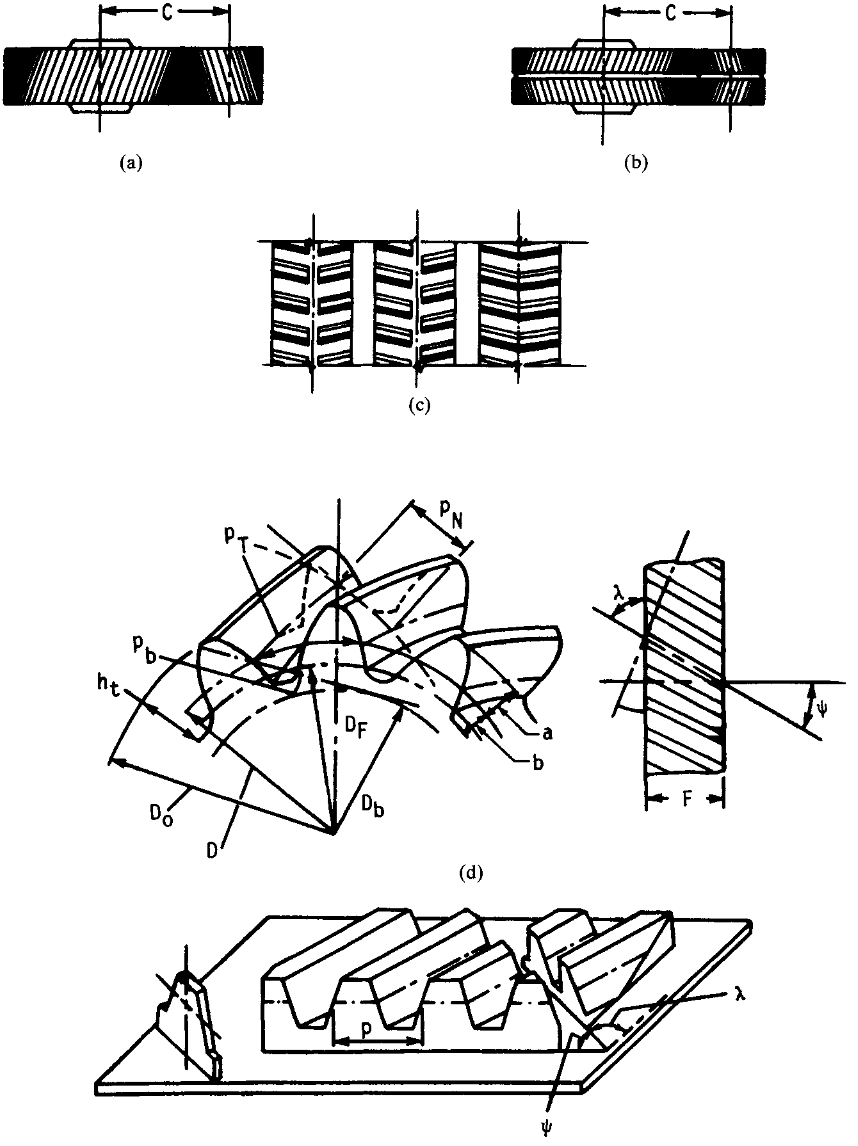

FIGURE 10.1 Terminology of helical gearing. (*a*) Single-helix gear. (*b*) Double-helix gear. (*c*) Types of double-helix gears: left, conventional; center, staggered; right, continous or herringbone. (*d*) Geometry. (*e*) Helical rack.

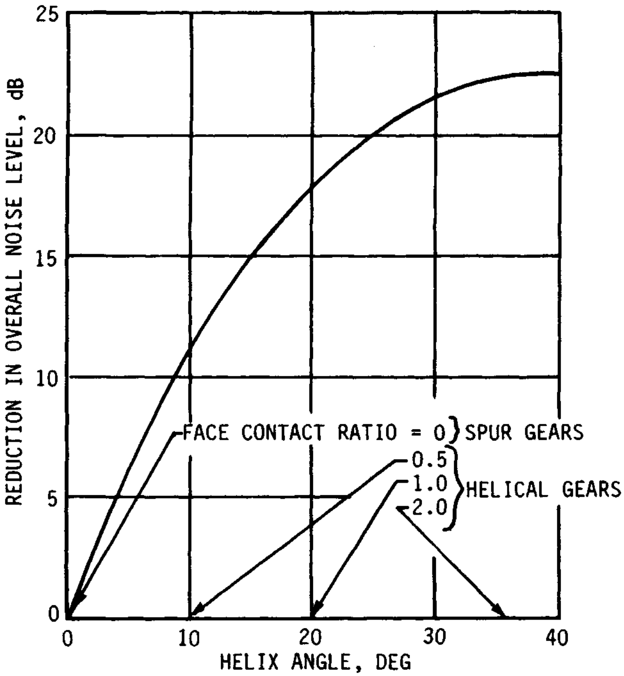

FIGURE 10.2 Effect of face-contact ratio on noise level. Note that increased helix angles lower the noise level.

very high helix angles actually tend to reduce the effective contact ratio, and so noise increases. Since helix angles greater than 45° are seldom used and are generally impractical to manufacture, this phenomenon is of academic interest only.

10.3.2 Load Capacity

As a result of the increased total area of tooth contact available, the load capacity of helical gears is generally higher than that of equivalent spur gears. The reason for this increase is obvious when we consider the contact line comparison which Fig. 10.3 shows. The most critical load condition for a spur gear occurs when a single tooth carries all the load at the highest point of single-tooth contact (Fig. 10.3*c*). In this case, the total length of the contact line is equal to the face width. In a helical gear, since the contact lines are inclined to the tooth with respect to the face width, the total length of the line of contact is increased (Fig. 10.3*b*), so that it is greater than the face width. This lowers unit loading and thus increases capacity.

10.3.3 Manufacturing

In the design of a gear system, it is often necessary to use a specific ratio on a specific center distance. Frequently this results in a diametral pitch which is nonstandard. If

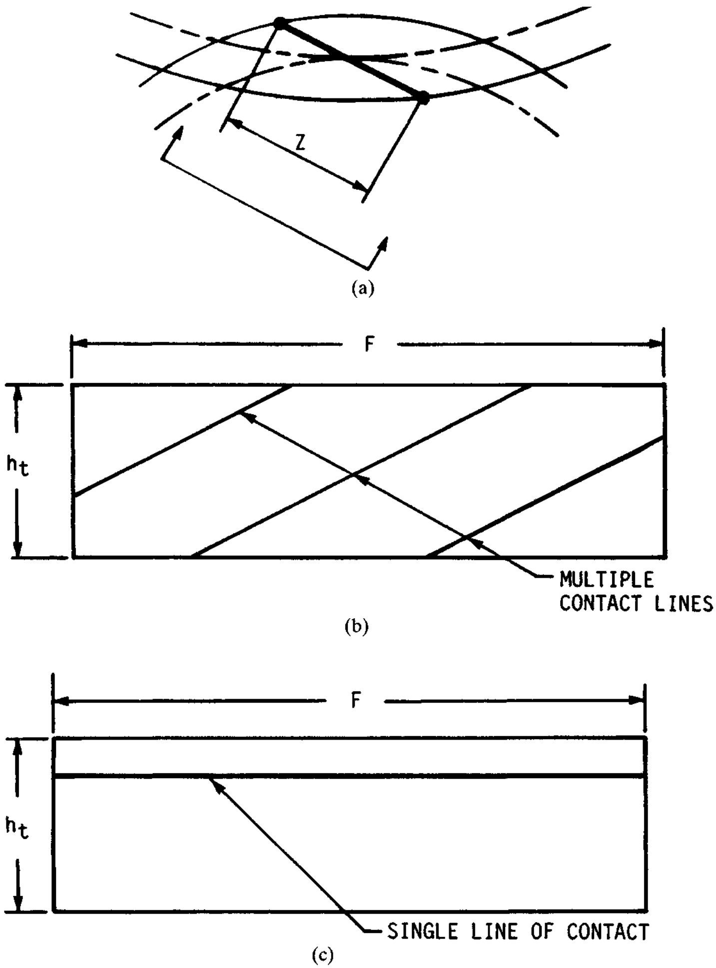

FIGURE 10.3 Comparison of spur and helical contact lines. (*a*) Transverse section; (*b*) helical contact lines; (*c*) spur contact line.

helical gears are employed, a limited number of standard cutters may be used to cut a wide variety of transverse-pitch gears simply by varying the helix angle, thus allowing virtually any center-distance and tooth-number combination to be accommodated.

10.4 GEOMETRY

When considered in the transverse plane (that is, a plane perpendicular to the axis of the gear), all helical-gear geometry is identical to that for spur gears. Standard tooth proportions are usually based on the normal diametral pitch, as shown in Table 10.1.

TABLE 10.1 Standard Tooth Proportions for Helical Gears

Quantity†	Formula	Quantity†	Formula
Addendum	$\dfrac{1.00}{P_N}$	External gears:	
Dedendum	$\dfrac{1.25}{P_N}$	Standard center distance	$\dfrac{D+d}{2}$
Pinion pitch diameter	$\dfrac{N_P}{P_N \cos\psi}$	Gear outside diameter	$D + 2a$
Gear pitch diameter	$\dfrac{N_G}{P_N \cos\psi}$	Pinion outside diameter	$d + 2a$
Normal arc tooth thickness	$\dfrac{\pi}{P_N} - \dfrac{B_N}{2}$	Gear root diameter	$D - 2b$
Pinion base diameter	$d \cos \phi_T$	Pinion root diameter	$d - 2b$
Gear base diameter	$D \cos \phi_T$	Internal gears:	
		Center distance	$\dfrac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan\psi \cos\phi_T)$	Inside diameter	$d - 2a$
		Root diameter	$D + 2b$

†All dimensions in inches, and angles are in degrees.

It is frequently necessary to convert from the normal plane to the transverse plane and vice versa. Table 10.2 gives the necessary equations. All calculations previously defined for spur gears with respect to transverse or profile-contact ratio, top land, lowest point of contact, true involute form radius, nonstandard center, etc., are valid for helical gears if only a transverse plane section is considered.

For spur gears, the profile-contact ratio (ratio of contact to the base pitch) must be greater than unity for uniform rotary-motion transmission to occur. Helical gears, however, provide an additional overlap along the axial direction; thus their profile-contact ratio need not necessarily be greater than unity. The sum of both the profile-

TABLE 10.2 Conversions between Normal and Transverse Planes

Parameter (normal/ transverse)	Normal to transverse	Transverse to normal
Pressure angle (ϕ_N/ϕ_T)	$\phi_T = \tan^{-1}\dfrac{\tan\phi_N}{\cos\psi}$	$\phi_N = \tan^{-1}(\tan\phi_T \cos\psi)$
Diametral pitch (P_N/P_d)	$P_d = P_N \cos\psi$	$P_N = \dfrac{P_d}{\cos\psi}$
Circular pitch (p_N/p_T)	$P_T = \dfrac{P_N}{\cos\psi}$	$P_N = P_T \cos\psi$
Arc tooth thickness (T_N/T_T)	$T_T = \dfrac{T_N}{\cos\psi}$	$T_N = T_T \cos\psi$
Backlash (B_N/B_T)	$B_T = \dfrac{B_N}{\cos\psi}$	$B_N = B_T \cos\psi$

contact ratio and the axial overlap must, however, be at least unity. The axial overlap, also often called the *face-contact ratio,* is the ratio of the face width to the axial pitch. The face-contact ratio is given by

$$m_F = \frac{P_{do} F \tan \psi_o}{\pi} \tag{10.1}$$

where P_{do} = operating transverse diametral pitch
ψ_o = helix angle at operating pitch circle
F = face width

Other parameters of interest in the design and analysis of helical gears are the base pitch p_b and the length of the line of action Z, both in the transverse plane. These are

$$p_b = \frac{\pi}{P_d} \cos \phi_T \tag{10.2}$$

and

$$Z = (r_o^2 - r_b^2)^{1/2} + (R_o^2 - R_b^2)^{1/2} - C_o \sin \phi_o \tag{10.3}$$

This equation is for an external gear mesh. For an internal gear mesh, the length of the line of action is

$$Z = (R_i^2 - R_b^2)^{1/2} - (r_o^2 - r_b^2)^{1/2} + C_o \sin \phi_o \tag{10.4}$$

where P_d = transverse diametral pitch as manufactured
ϕ_T = transverse pressure angle as manufactured, degrees (deg)
r_o = effective pinion outside radius, inches (in)
R_o = effective gear outside radius, in
R_i = effective gear inside radius, in
ϕ_o = operating transverse pressure angle, deg
r_b = pinion base radius, in
R_b = gear base radius, in
C_o = operating center distance, in

The operating transverse pressure angle ϕ_o is

$$\phi_o = \cos^{-1}\left(\frac{C}{C_o} \cos \phi_T\right) \tag{10.5}$$

The manufactured center distance C is simply

$$C = \frac{N_P + N_G}{2P_d} \tag{10.6}$$

for external mesh; for internal mesh, the relation is

$$C = \frac{N_G - N_P}{2P_d} \tag{10.7}$$

The contact ratio m_P in the transverse plane (profile-contact ratio) is defined as the ratio of the total length of the line of action in the transverse plane Z to the base pitch in the transverse plane p_b. Thus

$$m_P = \frac{Z}{p_b} \tag{10.8}$$

The diametral pitch, pitch diameters, helix angle, and normal pressure angle at the operating pitch circle are required in the load-capacity evaluation of helical gears. These terms are given by

$$P_{do} = \frac{N_P + N_G}{2C_o} \tag{10.9}$$

for external mesh; for internal mesh,

$$P_{do} = \frac{N_G - N_P}{2C_o} \tag{10.10}$$

Also,

$$d = \frac{N_P}{P_{do}} \qquad D = \frac{N_G}{P_{do}} \tag{10.11}$$

$$\psi_B = \tan^{-1} (\tan \psi \cos \phi_T) \tag{10.12}$$

$$\psi_o = \tan^{-1} \frac{\tan \psi_B}{\cos \phi_o} \tag{10.13}$$

$$\phi_{No} = \sin^{-1} (\sin \phi_o \cos \psi_B) \tag{10.14}$$

where P_{do} = operating diametral pitch
ψ_B = base helix angle, deg
ψ_o = helix angle at operating pitch point, deg
ϕ_{No} = operating normal pressure angle, deg
d = operating pinion pitch diameter, in
D = operating gear pitch diameter, in

10.5 LOAD RATING

Reference [10.1] establishes a coherent method for rating external helical and spur gears. The treatment of strength and durability provided here is derived in large part from this source.

Four factors must be considered in the load rating of a helical-gear set: strength, durability, wear resistance, and scoring probability. Although strength and durability must always be considered, wear resistance and scoring evaluations may not be required for every case. We treat each topic in some depth.