CHAPTER 12
WORM GEARING

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GLOSSARY OF SYMBOLS

b_G	Dedendum of gear teeth
C	Center distance
d	Worm pitch diameter
d_o	Outside diameter of worm
d_R	Root diameter of worm
D	Pitch diameter of gear in central plane
D_b	Base circle diameter
D_o	Outside diameter of gear
D_t	Throat diameter of gear
f	Length of flat on outside diameter of worm
h_k	Working depth of tooth
h_t	Whole depth of tooth
L	Lead of worm
m_G	Gear ratio = N_G/N_W
m_o	Module, millimeters of pitch diameter per tooth (SI use)
m_p	Number of teeth in contact
n_w	Rotational speed of worm, r/min
n_G	Rotational speed of gear, r/min

N_G	Number of teeth in gear
N_W	Number of threads in worm
p_n	Normal circular pitch
p_x	Axial circular pitch of worm
P	Transverse diametral pitch of gear, teeth per inch of diameter
W	Force between worm and gear (various components are derived in the text)
λ	Lead angle at center of worm, deg
ϕ_n	Normal pressure angle, deg
ϕ_x	Axial pressure angle at center of worm, deg

12.1 INTRODUCTION

Worm gears are used for large speed reduction with concomitant increase in torque. They are limiting cases of helical gears, treated in Chap. 10. The shafts are normally perpendicular, though it is possible to accommodate other angles. Consider the helical-gear pair in Fig. 12.1*a* with shafts at 90°.

The lead angles of the two gears are described by λ (lead angle is 90° less the helix angle). Since the shafts are perpendicular, $\lambda_1 + \lambda_2 = 90°$. If the lead angle of gear 1 is made small enough, the teeth eventually wrap completely around it, giving the appearance of a screw, as seen in Fig. 12.1*b*. Evidently this was at some stage taken to resemble a *worm*, and the term has remained. The mating member is called simply the *gear*, sometimes the *wheel*. The helix angle of the gear is equal to the lead angle of the worm (for shafts at 90°).

The worm is always the driver in speed reducers, but occasionally the units are used in reverse fashion for speed increasing. Worm-gear sets are self-locking when the gear cannot drive the worm. This occurs when the tangent of the lead angle is less than the coefficient of friction. The use of this feature in lieu of a brake is not rec-

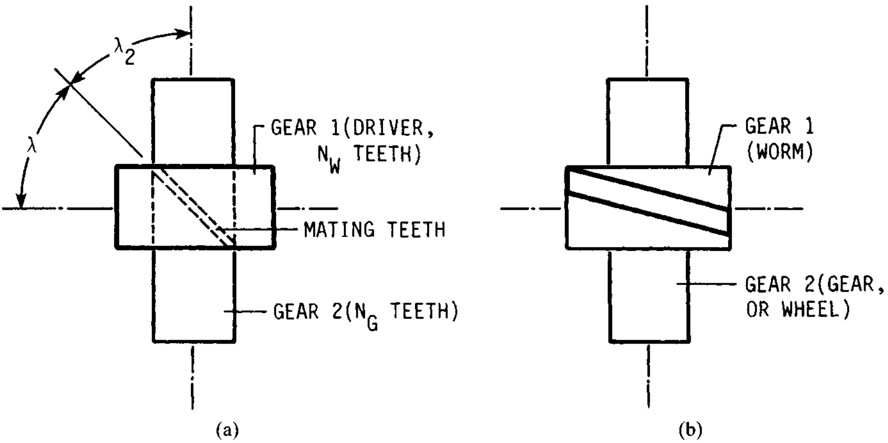

FIGURE 12.1 (*a*) Helical gear pair; (*b*) a small lead angle causes gear one to become a worm.

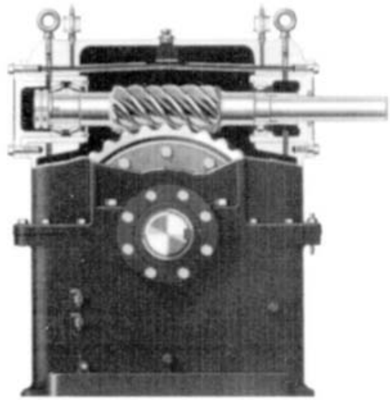

FIGURE 12.2 Photograph of a worm-gear speed reducer. Notice that the gear partially wraps, or envelopes, the worm. *(Cleveland Worm and Gear Company.)*

ommended, since under running conditions a gear set may not be self-locking at lead angles as small as 2°.

There is only point contact between helical gears as described above. Line contact is obtained in worm gearing by making the gear envelop the worm as in Fig. 12.2; this is termed a *single-enveloping gear set,* and the worm is cylindrical. If the worm and gear envelop each other, the line contact increases as well as the torque that can be transmitted. The result is termed a *double-enveloping gear set.*

The minimum number of teeth in the gear and the reduction ratio determine the number of threads (teeth) for the worm. Generally, 1 to 10 threads are used. In special cases a larger number may be required.

12.2 KINEMATICS

In specifying the pitch of worm-gear sets, it is customary to state the axial pitch p_x of the worm. For 90° shafts this is equal to the transverse circular pitch of the gear. The advance per revolution of the worm, termed the lead L, is

$$L = p_x N_W$$

This and other useful relations result from consideration of the developed pitch cylinder of the worm, seen in Fig. 12.3. From the geometry, the following relations can be found:

$$d = \frac{N_W p_n}{\pi \sin \lambda} \tag{12.1}$$

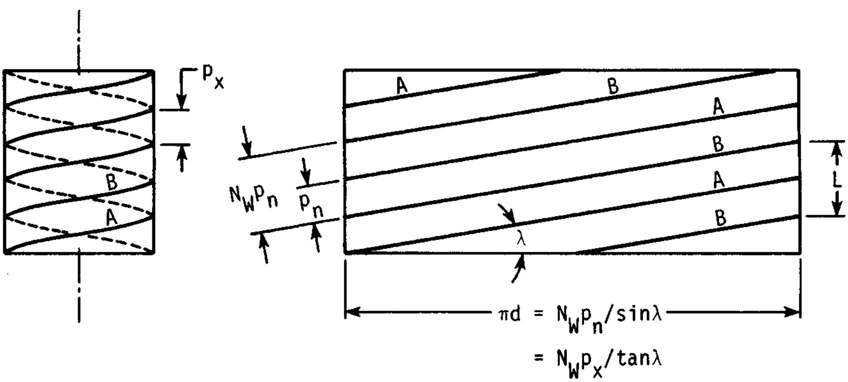

FIGURE 12.3 Developed pitch cylinder of worm.

$$d = \frac{N_W p_x}{\pi \tan \lambda} \tag{12.2}$$

$$\tan \lambda = \frac{L}{\pi d} = \frac{N_W p_x}{\pi d} \tag{12.3}$$

$$p_x = \frac{p_n}{\cos \lambda} \tag{12.4}$$

$$D = \frac{p_x N_G}{\pi} = \frac{N_G p_n}{\pi \cos \lambda} \tag{12.5}$$

From Eqs. (12.1) and (12.5), we find

$$\tan \lambda = \frac{N_W D}{N_G d} = \frac{1}{m_G} \frac{D}{d} \tag{12.6}$$

The center distance C can be derived from the diameters

$$C = \frac{p_n N_W}{2\pi} \left(\frac{m_G}{\cos \lambda} + \frac{1}{\sin \lambda} \right) \tag{12.7}$$

which is sometimes more useful in the form

$$\frac{m_G}{\cos \lambda} + \frac{1}{\sin \lambda} = \begin{cases} \dfrac{2\pi C}{p_n N_W} & \text{U.S. customary units} \\ \dfrac{2C}{m_o N_W \cos \lambda} & \text{SI units} \\ \dfrac{2C}{d \sin \lambda} & \text{either} \end{cases} \tag{12.8}$$

For use in the International System (SI), recognize that

$$\text{Diameter} = N m_o = \frac{N p_x}{\pi}$$

so that the substitution

$$p_x = \pi m_o$$

will convert any of the equations above to SI units.

The pitch diameter of the gear is measured in the plane containing the worm axis and is, as for spur gears,

$$D = \frac{N_G p_x}{\pi} \tag{12.9}$$

The worm pitch diameter is unrelated to the number of teeth. It should, however, be the same as that of the hob used to cut the worm-gear tooth.

12.3 VELOCITY AND FRICTION

Figure 12.4 shows the pitch line velocities of worm and gear. The coefficient of friction between the teeth μ is dependent on the sliding velocity. Representative values of μ are charted in Fig. 12.5. The friction has importance in computing the gear set efficiency, as will be shown.

12.4 FORCE ANALYSIS

If friction is neglected, then the only force exerted by the gear on the worm will be W, perpendicular to the mating tooth surface, shown in Fig. 12.6, and having the three components W^x, W^y, and W^z. From the geometry of the figure,

$$\begin{aligned} W^x &= W \cos \phi_n \sin \lambda \\ W^y &= W \sin \phi_n \\ W^z &= W \cos \phi_n \cos \lambda \end{aligned} \tag{12.10}$$

In what follows, the subscripts W and G refer to forces *on* the worm and the gear. The component W^y is the separating, or radial, force for both worm and gear (opposite in direction for the gear). The tangential force is W^x on the worm and W^z on the gear. The axial force is W^z on the worm and W^x on the gear. The gear forces are opposite to the worm forces:

$$\begin{aligned} W_{W_t} &= -W_{G_a} = W^x \\ W_{W_r} &= -W_{G_r} = W^y \\ W_{W_a} &= -W_{G_t} = W^z \end{aligned} \tag{12.11}$$

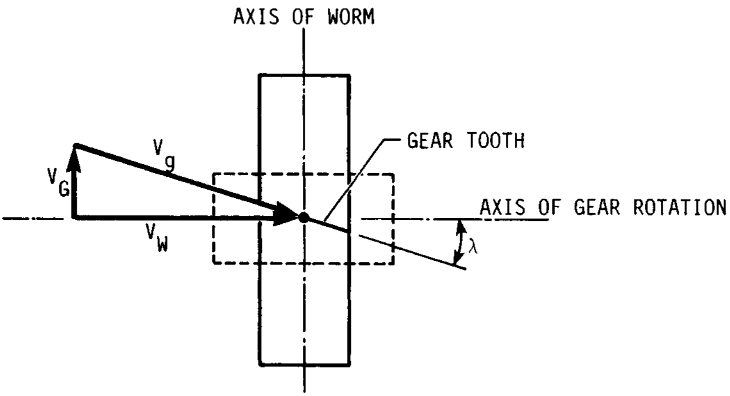

FIGURE 12.4 Velocity components in a worm-gear set. The sliding velocity is $V_S = (V_W^2 + V_G^2)^{1/2} = \dfrac{V_W}{\cos \lambda}$.

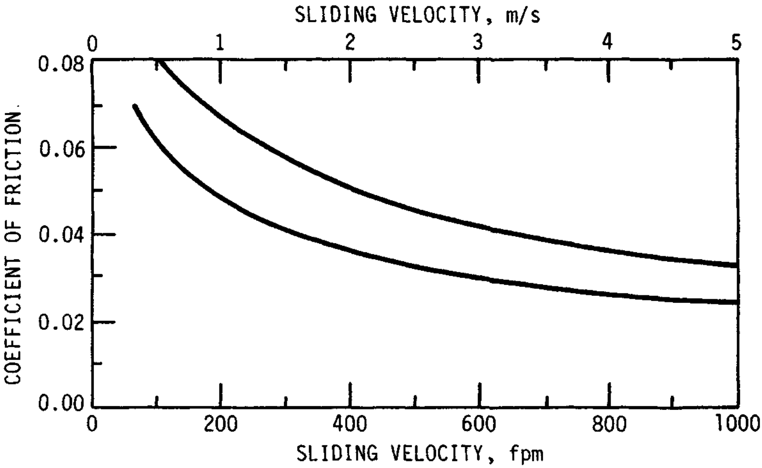

FIGURE 12.5 Approximate coefficients of sliding friction between the worm and gear teeth as a function of the sliding velocity. All values are based on adequate lubrication. The lower curve represents the limit for the very best materials, such as a hardened worm meshing with a bronze gear. Use the upper curve if moderate friction is expected.

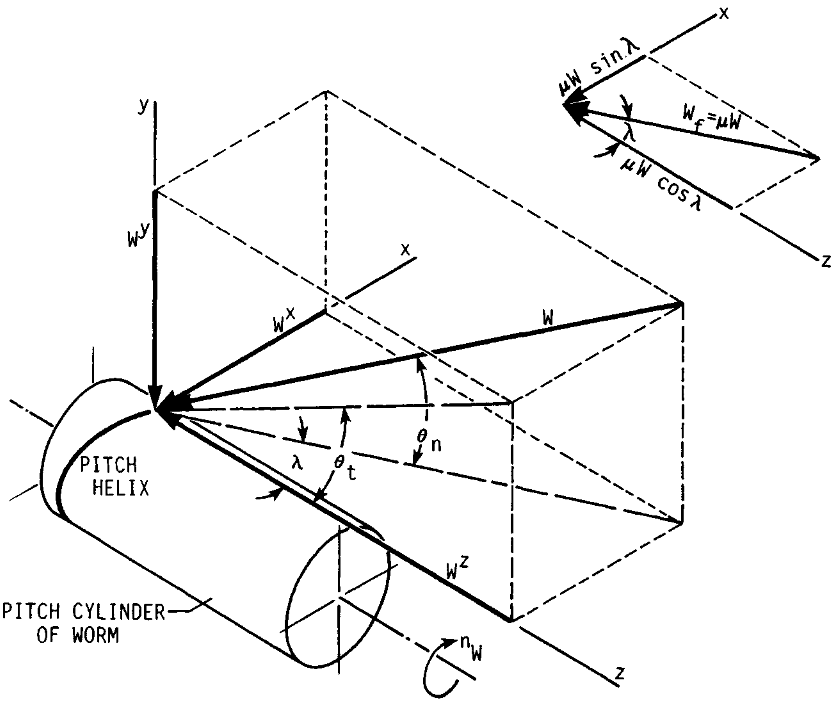

FIGURE 12.6 Forces exerted on worm.

where the subscripts are t for the tangential direction, r for the radial direction, and a for the axial direction. It is worth noting in the above equations that the gear axis is parallel to the x axis and the worm axis is parallel to the z axis. The coordinate system is right-handed.

The force W, which is normal to the profile of the mating teeth, produces a frictional force $W_f = \mu W$, shown in Fig. 12.6, along with its components $\mu W \cos \lambda$ in the negative x direction and $\mu W \sin \lambda$ in the positive z direction. Adding these to the force components developed in Eqs. (12.10) yields

$$\begin{aligned} W^x &= W(\cos \phi_n \sin \lambda + \mu \cos \lambda) \\ W^y &= W \sin \phi_n \\ W^z &= W(\cos \phi_n \cos \lambda - \mu \sin \lambda) \end{aligned} \qquad (12.12)$$

Equations (12.11) still apply. Substituting W^z from Eq. (12.12) into the third of Eqs. (12.11) and multiplying by μ, we find the frictional force to be

$$W_f = \mu W = \frac{\mu W_{G_t}}{\mu \sin \lambda - \cos \phi_n \cos \lambda} \qquad (12.13)$$

A relation between the two tangential forces is obtained from the first and third of Eqs. (12.11) with appropriate substitutions from Eqs. (12.12):

$$W_{W_t} = W_{G_t} \frac{\cos \phi_n \sin \lambda + \mu \cos \lambda}{\mu \sin \lambda - \cos \phi_n \cos \lambda} \qquad (12.14)$$

The efficiency can be defined as

$$\eta = \frac{W_{W_t}\ (\text{without friction})}{W_{W_t}\ (\text{with friction})} \qquad (12.15)$$

Since the numerator of this equation is the same as Eq. (12.14) with $\mu = 0$, we have

$$\eta = \frac{\cos \phi_n - \mu \tan \lambda}{\cos \phi_n + \mu \cot \lambda} \qquad (12.16)$$

Table 12.1 shows how η varies with λ, based on a typical value of friction $\mu = 0.05$ and the pressure angles usually used for the ranges of λ indicated. It is clear that small λ should be avoided.

Example 1. A 2-tooth right-hand worm transmits 1 horsepower (hp) at 1200 revolutions per minute (r/min) to a 30-tooth gear. The gear has a transverse diametral pitch of 6 teeth per inch. The worm has a pitch diameter of 2 inches (in). The normal pressure angle is 14½°. The materials and workmanship correspond to the lower of the curves in Fig. 12.5. Required are the axial pitch, center distance, lead, lead angle, and tooth forces.

Solution. The axial pitch is the same as the transverse circular pitch of the gear. Thus

$$p_x = \frac{\pi}{P} = \frac{\pi}{6} = 0.5236 \text{ in}$$

TABLE 12.1 Efficiency of Worm-Gear Sets for $\mu = 0.05$

Normal pressure angle ϕ_n, deg	Lead angle λ, deg	Efficiency η, percent
$14\frac{1}{2}$	1	25.2
	2.5	46.8
	5	62.6
	7.5	71.2
	10	76.8
	15	82.7
20	20	86.0
	25	88.0
	30	89.2

The pitch diameter of the gear is $D = N_G/P = 30/6 = 5$ in. The center distance is thus

$$C = \frac{D+d}{2} = \frac{2+5}{2} = 3.5 \text{ in}$$

The lead is

$$L = p_x N_W = 0.5236(2) = 1.0472 \text{ in}$$

From Eq. (12.3),

$$\lambda = \tan^{-1} \frac{L}{\pi d} = \tan^{-1} \frac{1.0472}{2\pi} = 9.46°$$

The pitch line velocity of the worm, in inches per minute, is

$$V_W = \pi d n_W = \pi(2)(1200) = 7540 \text{ in/min}$$

The speed of the gear is $n_G = 1200(2)/30 = 80$ r/min. The gear pitch line velocity is thus

$$V_G = \pi D n_G = \pi(5)(80) = 1257 \text{ in/min}$$

The sliding velocity is the square root of the sum of the squares of V_W and V_G, or

$$V_S = \frac{V_W}{\cos \lambda} = \frac{7540}{\cos 9.46} = 7644 \text{ in/min}$$

This result is the same as 637 feet per minute (ft/min); we enter Fig. 12.5 and find $\mu = 0.03$.

Proceeding now to the force analysis, we use the horsepower formula to find

$$W_W = \frac{(33\,000)(12)(\text{hp})}{V_W} = \frac{(33\,000)(12)(1)}{7540} = 52.5 \text{ lb}$$

This force is the negative x direction. Using this value in the first of Eqs. (12.12) gives

$$W = \frac{W^x}{\cos \phi_n \sin \lambda + \mu \cos \lambda}$$

$$= \frac{52.5}{\cos 14.5° \sin 9.46° + 0.03 \cos 9.46°} = 278 \text{ lb}$$

From Eqs. (12.12) we find the other components of W to be

$$W^y = W \sin \phi_n = 278 \sin 14.5° = 69.6 \text{ lb}$$

$$W^z = W(\cos \phi_n \cos \lambda - \mu \sin \lambda)$$

$$= 278(\cos 14.5° \cos 9.46° - 0.03 \sin 9.46°)$$

$$= 265 \text{ lb}$$

The components acting on the gear become

$$W_{G_a} = -W^x = 52.5 \text{ lb}$$

$$W_{G_r} = -W^y = 69.6 \text{ lb}$$

$$W_{G_t} = -W^z = -265 \text{ lb}$$

The torque can be obtained by summing moments about the x axis. This gives, in inch-pounds,

$$T = 265(2.5) = 662.5 \text{ in} \cdot \text{lb}$$

It is because of the frictional loss that this output torque is less than the product of the gear ratio and the input torque (778 lb · in).

12.5 STRENGTH AND POWER RATING

Because of the friction between the worm and the gear, power is consumed by the gear set, causing the input and output horsepower to differ by that amount and resulting in a necessity to provide for heat dissipation from the unit. Thus

$$\text{hp(in)} = \text{hp(out)} + \text{hp(friction loss)}$$

This expression can be translated to the gear parameters, resulting in

$$\text{hp(in)} = \frac{W_{G_t} D n_W}{126\,000 m_G} + \frac{V_S W_f}{396\,000} \tag{12.17}$$

The force which can be transmitted W_{G_t} depends on tooth strength and is based on the gear, it being nearly always weaker than the worm (worm tooth strength can be computed by the methods used with screw threads, as in Chap. 13). Based on material strengths, an empirical relation is used. The equation is

$$W_{G_t} = K_s D^{0.8} F_e K_m K_v \tag{12.18}$$

TABLE 12.2 Materials Factor K_s for Cylindrical Worm Gearing†

Face width of gear F_G, in	Sand-cast bronze	Static-chill-cast bronze	Centrifugal-cast bronze
Up to 3	700	800	1000
4	665	780	975
5	640	760	940
6	600	720	900
7	570	680	850
8	530	640	800
9	500	600	750

†For copper-tin and copper-tin-nickel bronze gears operating with steel worms case-hardened to 58 R_C minimum.

SOURCE: Darle W. Dudley (ed.), *Gear Handbook,* McGraw-Hill, New York, 1962, p. 13–38.

where K_s = materials and size correction factor, values for which are shown in Table 12.2
F_e = effective face width of gear; this is actual face width or two-thirds of worm pitch diameter, whichever is less
K_m = ratio correction factor; values in Table 12.3
K_v = velocity factor (Table 12.4)

Example 2. A gear catalog lists a 4-pitch, 14½° pressure angle, single-thread hardened steel worm to mate with a 24-tooth sand-cast bronze gear. The gear has a 1½-in face width. The worm has a 0.7854-in lead, 4.767° lead angle, 4½-in face width, and a 3-in pitch diameter. Find the safe input horsepower.

From Table 12.2, $K_s = 700$. The pitch diameter of the gear is

$$D = \frac{N_G}{P} = \frac{24}{4} = 6 \text{ in}$$

The pitch diameter of the worm is given as 3 in; two-thirds of this is 2 in. Since the face width of the gear is smaller (1.5 in), $F_e = 1.5$ in. Since $m_G = N_G/N_W = 24/1 =$

TABLE 12.3 Ratio Correction Factor K_m

m_G	K_m	m_G	K_m	m_G	K_m
3.0	0.500	8.0	0.724	30.0	0.825
3.5	0.554	9.0	0.744	40.0	0.815
4.0	0.593	10.0	0.760	50.0	0.785
4.5	0.620	12.0	0.783	60.0	0.745
5.0	0.645	14.0	0.799	70.0	0.687
6.0	0.679	16.0	0.809	80.0	0.622
7.0	0.706	20.0	0.820	100.0	0.490

SOURCE: Darle W. Dudley (ed.), *Gear Handbook,* McGraw-Hill, New York, 1962, p. 13–38.

TABLE 12.4 Velocity Factor K_v

Velocity V_S, fpm	K_v	Velocity V_S, fpm	K_v
1	0.649	600	0.340
1.5	0.647	700	0.310
10	0.644	800	0.289
20	0.638	900	0.269
30	0.631	1000	0.258
40	0.625	1200	0.235
60	0.613	1400	0.216
80	0.600	1600	0.200
100	0.588	1800	0.187
150	0.558	2000	0.175
200	0.528	2200	0.165
250	0.500	2400	0.156
300	0.472	2600	0.148
350	0.446	2800	0.140
400	0.421	3000	0.134
450	0.398	4000	0.106
500	0.378	5000	0.089
550	0.358	6000	0.079

SOURCE: Darle W. Dudley (ed.), *Gear Handbook*, McGraw-Hill, New York, 1962, p. 13–39.

24, from Table 12.3, $K_m = 0.823$ by interpolation. The pitch line velocity of the worm is

$$V_W = \pi d n_W = \pi(3)(1800) = 16\,965 \text{ in/min}$$

The sliding velocity is

$$V_S = \frac{V_W}{\cos\lambda} = \frac{16\,965}{\cos 4.767°} = 17\,024 \text{ in/min}$$

Therefore, from Table 12.4, $K_v = 0.215$. The transmitted load is obtained from Eq. (12.18) and is

$$W_{G_t} = K_s d^{0.8} F_e K_m K_v = 700(6^{0.8})(1.5)(0.823)(0.215)$$
$$= 779 \text{ lb}$$

To find the friction load, the coefficient of friction is needed. Converting V_S to feet per minute and using Fig. 12.5, we find $\mu = 0.023$. From Eq. (12.13) we find

$$W_f = \frac{\mu W_{G_t}}{\mu \sin\lambda - \cos\phi_n \cos\lambda}$$
$$= \frac{0.023(779)}{0.023 \sin 4.767° - \cos 14.5° \cos 4.767°}$$
$$= 18.6 \text{ lb}$$

Next, using Eq. (12.17), we find the input horsepower to be

$$\text{hp(in)} = \frac{W_{G_t} D n_W}{126\,000 m_G} + \frac{W_f V_S}{396\,000}$$

$$= \frac{779(6)(1800)}{126\,000(24)} + \frac{18.6(17\,024)}{396\,000}$$

$$= 2.78 + 0.80 = 3.58$$

12.6 HEAT DISSIPATION

In the last section we noted that the input and output horsepowers differ by the amount of power resulting from friction between the gear teeth. This difference represents energy input to the gear set unit, which will result in a temperature rise. The capacity of the gear reducer will thus be limited by its heat-dissipating capacity.

The cooling rate for rectangular housings can be estimated from

$$C_1 = \begin{cases} \dfrac{n}{84\,200} + 0.01 & \text{without fan} \\[2ex] \dfrac{n}{51\,600} + 0.01 & \text{with fan} \end{cases} \tag{12.19}$$

where C_1 is the heat dissipated in Btu/(h)(in^2)(°F), British thermal units per hour–inch squared–degrees Fahrenheit, and n is the speed of the worm shaft in rotations per minute. Note that the rates depend on whether there is a fan on the worm shaft. The rates are based on the area of the casing surface, which can be estimated from

$$A_c = 43.2C^{1.7} \tag{12.20}$$

where A_c is in square inches.

The temperature rise can be computed by equating the friction horsepower to the heat-dissipation rate. Thus

$$\text{hp(friction)} = \frac{778 C_1 A_c \Delta T}{60(33\,000)} \tag{12.21}$$

or

$$\Delta T(°\text{F}) = \frac{\text{hp(friction)}(60)(33\,000)}{778 C_1 A_c} \tag{12.22}$$

The oil temperature should not exceed 180°F. Clearly the horsepower rating of a gear set may be limited by temperature rather than by gear strength. Both must be checked. Of course, means other than natural radiation and convection can be employed to solve the heat problem.

12.7 DESIGN STANDARDS

The American Gear Manufacturer's Association† has issued certain standards relating to worm-gear design. The purpose of these publications, which are the work of broad committees, is to share the experience of the industry and thus to arrive at good standard design practice. The following relate to industrial worm-gear design and are extracted from [12.1] with the permission of the publisher.

Gear sets with axial pitches of 3/16 in and larger are termed *coarse-pitch*. Another standard deals with fine-pitch worm gearing, but we do not include these details here. It is not recommended that gear and worm be obtained from separate sources. Utilizing a worm design for which a comparable hob exists will reduce tooling costs.

12.7.1 Number of Teeth of Gear

Center distance influences to a large extent the minimum number of teeth for the gear. Recommended minimums are shown in Table 12.5. The maximum number of teeth selected is governed by high ratios of reduction and considerations of strength and load-carrying capacity.

12.7.2 Number of Threads in Worm

The minimum number of teeth in the gear and the reduction ratio determine the number of threads for the worm. Generally, 1 to 10 threads are used. In special cases, a larger number may be required.

12.7.3 Gear Ratio

Either prime or even gear ratios may be used. However, if the gear teeth are to be generated by a single-tooth "fly cutter," the use of a prime ratio will eliminate the need for indexing the cutter.

† American Gear Manufacturer's Association (AGMA), Alexandria, Virginia.

TABLE 12.5 Recommended Minimum Number of Gear Teeth

Center distance, in	Minimum number of teeth†
2	20
3	25
5	25
10	29
15	35
20	40
24	45

†Lower numbers are permissible for specific applications.

12.7.4 Pitch

It is recommended that pitch be specified in the axial plane of the worm and that it be a simple fraction, to permit accurate factoring for change-gear ratios.

12.7.5 Worm Pitch Diameter

The pitch diameter of the worm for calculation purposes is assumed to be at the mean of the working depth. A worm does not have a true pitch diameter until it is mated with a gear at a specified center distance. If the actual addendum and dedendum of the worm are equal, respectively, to the addendum and dedendum of the gear, then the nominal and actual pitch diameters of the worm are the same. However, it is not essential that this condition exist for satisfactory operation of the gearing.

Although a relatively large variation in worm pitch diameter is permissible, it should be held within certain limits if the power capacity is not to be adversely affected. Therefore, when a worm pitch diameter is selected, the following factors should be considered:

1. Smaller pitch diameters provide higher efficiency and reduce the magnitude of tooth loading.
2. The root diameter which results from selection of a pitch diameter must be sufficiently large to prevent undue deflection and stress under load.
3. Larger worm pitch diameters permit utilization of larger gear face widths, providing higher strength for the gear set.
4. For low ratios, the minimum pitch diameter is governed, to some degree, by the desirability of avoiding too high a lead angle. Generally, the lead is limited to a maximum of 45°. However, lead angles up to 50° are practical.

12.7.6 Gear Pitch Diameter

The selection of an approximate worm pitch diameter permits the determination of a corresponding approximate gear pitch diameter. In the normal case where the addendum and dedendum of the worm are to be equal, respectively, to the addendum and dedendum of the gear, a trial value of gear pitch diameter may be found by subtracting the approximate worm pitch diameter from twice the center distance of the worm and gear. Once the number of teeth for the gear has been selected, it is desirable to arrive at an exact gear pitch diameter by selecting for the gear circular pitch a fraction, which can be conveniently factored into a gear train for processing purposes, and calculating gear pitch diameter from the formula in Table 12.6. Should the actual value of gear pitch diameter differ from the trial value, the worm pitch diameter must be adjusted accordingly through the use of the formula in Table 12.7.

It is not essential that the pitch circle of the gear be at the mean of the working depth. Where there are sufficient teeth in the gear and the pressure angle is high enough to prevent undercutting, the pitch line can be anywhere between the mean of the depth and the throat diameter of the gear, or even outside the throat. This results in a short addendum for the gear teeth and lengthens the angle of recess. It is

TABLE 12.6 Dimensions of the Gear

Quantity	Symbol	Formula
Pitch diameter	D	$\frac{N_G p_x}{\pi}$
Throat diameter	D_t	$D + 2a$
Effective face width	F_e	$\sqrt{(d + h_k)^2 - d^2}$

also practical for the gear pitch diameter to be located somewhat below the mean of the working depth.

12.7.7 Worm Thread and Gear-Tooth Proportions

Pressure Angle. Several factors deserve consideration in the selection of the pressure angle. Smaller values of pressure angle decrease the separating forces, extend the line of action, and result in less backlash change with change in center distance. Larger values of pressure angle provide stronger teeth and assist in preventing undercutting of the teeth where lead angles are larger. The recommended pressure angles are listed in Table 12.8. These, used with the system for stubbing teeth (Table 12.9), will avoid undercutting.

TABLE 12.7 Dimensions of the Worm

Quantity	Symbol	Formula
Lead	l	$N_W P_x$
Pitch diameter†	d	$2C - D$
Outside diameter	d_o	$d + 2a$
Minimum face width	f	$2\sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D}{2} - a\right)^2}$
Lead angle	λ	$\tan^{-1} \frac{l}{\pi d}$
Normal pitch	p_n	$p_x \cos \lambda$
Normal pressure angle	ϕ_n	See Table 12.8

†Use only where addenda and dedenda of worm and gear are equal.

TABLE 12.8 Recommended Values for the Normal Pressure Angle

Normal pressure angle ϕ_n, deg	Lead angle λ, deg
20	Less than 30
25	30–45

Although its use is discouraged, a 14° normal pressure angle may be used for lead angles up to 17°. A detailed study of gear-tooth action is employed by some designers to utilize pressure angles less than 25° where worm lead angles are above 30°.

Tooth Depth Proportions. The choice of tooth depth proportions is governed, to a great extent, by the need to avoid undercutting of the gear teeth. Commonly used tooth depth proportions for lead angles to, but not including, 30° are listed in Table 12.10. However, other acceptable practices are used by several manufacturers.

TABLE 12.9 System for Stubbing Teeth†

Depth, percent	Lead angle λ, deg
90	30–34.99
80	35–39.99
70	40–45

†Other systems for stubbing gear teeth such as reducing the depth by 2 percent per degree of lead angle over 30° are also in common use.

TABLE 12.10 Dimensions Common to Both Worm and Gear†

Quantity	Symbol	Formula
Addendum	a	$0.3183p_x$
Whole depth	h_t	$0.6866p_x$
Working depth	h_k	$0.6366p_x$
Center distance‡	C	$\frac{D+d}{2}$

†Recommended for lead angles less than 30°. See Table 12.9 for others.

‡Nominal, where addenda and dedenda of worm and gear are equal.

Table 12.9 presents a system for stubbing teeth to be used in conjunction with the pressure angles in Table 12.8 for lead angles 30° and above.

Tooth Thickness. The gear-tooth normal thickness preferably should be not less than half the normal pitch at the mean of the working depth. In view of the lower-strength material normally used for the gear, it is the practice of some manufacturers to make the gear tooth appreciably thicker than the worm thread. The extent to which this procedure can be followed is limited by the necessity for providing adequate land thickness at the thread peaks.

Tooth or Thread Forms. The most important detail of the worm thread form is that it must be conjugate to that of the gear tooth. The thread form varies with individual manufacturers' practices and may be anything between the extremes of a straight side and the normal section of an involute helicoid.

12.7.8 Gear Blank Dimensions

Face Width. The effective face width of a worm gear varies with the nominal pitch diameter of the worm and the depth of the thread. The formula for gear face width given in Table 12.6 is based on the maximum effective face width of a worm gear (the length of a tangent to the mean worm diameter) between the points where it is intersected by the outside diameter of the worm. Any additional face width is of very little value and is wasteful of material.

Diameter Increment. This is the amount that is added to the throat diameter of the gear to obtain the outside diameter. The magnitude of this increment is not critical and may vary with manufacturers' practice. Normal practice is to use approximately one addendum. It is general practice to round the outside diameter to the nearest fraction of an inch.

The sharp corners at the point where gear face and outside diameter intersect should be removed by the use of either a chamfer or a radius, as shown in Fig. 12.7.

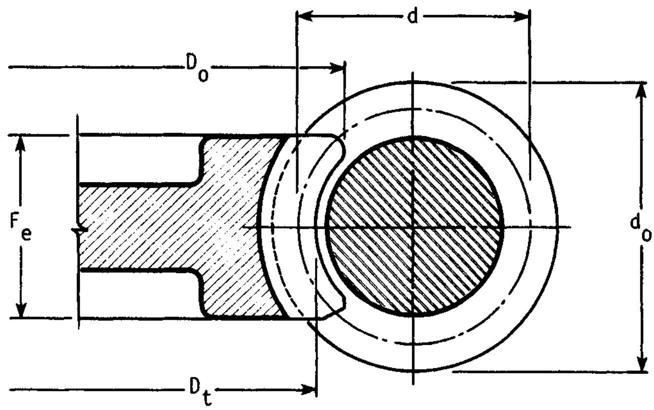

FIGURE 12.7 Section of worm and gear. Note that corners of gear teeth are usually rounded, as shown above the gear centerline; they may, however, be chamfered, as shown below.

Rim thicknesses are generally equal to or slightly greater than the whole depth of the teeth.

12.7.9 Worm Face

The face or length of the worm should be such that it extends beyond the point where contact between its threads and the teeth of the gear begins. Unlike with spur and helical gears, the pressure angle of a worm gear varies along the length of the tooth and becomes quite low on the leaving, or recess, side. This causes contact to occur on the worm almost to the point where the outside diameter of the worm intersects the throat diameter of the gear.

The formula in Table 12.7 provides a conservative value of the worm face width and is based on intersection of worm outside diameter with gear throat diameter.

More exact worm face widths may be determined by detailed calculations or layouts which take into consideration the face width of the gear and fix more definitely the extent of contact along the worm threads.

Good practice includes the breaking or rounding of the sharp edge of the worm threads at the end of the worm face. This procedure is particularly important where the worm face is less than provided for in the formula in Table 12.7.

12.7.10 Bored Worm Blanks

Where it is necessary to use a bored worm, the blank is normally designed with a key seat for driving purposes. The thickness of material between the worm root and the key seat should be at least $0.5h_t$. This is a general recommendation which is governed to some extent by whether the blank is hardened or unhardened. An increase in this amount may be necessary if the blank is hardened, particularly if a case-hardening process is used.

12.8 DOUBLE-ENVELOPING GEAR SETS†

12.8.1 Number of Teeth in Gear

The number of teeth for the gear is influenced to a large extent by center distance. The recommended number of teeth for various center distances is listed in Table 12.11. Should special considerations indicate a requirement for fewer teeth, it is advisable to consult a manufacturer of this type of gearing before you complete the design. For multiple-thread worms, the number of teeth in the gear should be within the limits listed in Table 12.11. The maximum number of teeth for single-threaded worms is limited only by the machines available for cutting gear sets and manufacturing tooling.

12.8.2 Number of Threads in Worm

The minimum number of teeth in the gear and the ratio determine the number of threads for the worm. Generally, one to nine threads are used. In special cases, a larger number of threads may be required.

† See Ref. [12.2].

TABLE 12.11 Range of Recommended Gear-Tooth Numbers

Center distance, in	No. teeth
2	24–40
3	24–50
4	30–50
8	40–60
15	50–60
20	50–70
24	60–80

12.8.3 Gear Ratio

The gear ratio is the quotient of the number of teeth in the gear and the number of threads in the worm. Either prime or even ratios may be used; however, hob life is improved with even ratios.

12.8.4 Pitch

It is recommended that pitch be specified in the axial section. Pitch is the result of design proportions.

12.8.5 Worm Root Diameter

The recommended root diameter for the worm is

$$d_R = \frac{C^{0.875}}{3} \tag{12.23}$$

It is desirable that the root diameter be not less than that indicated by this formula, even where the worm threads are cut integral with the shaft. For ratios less than 8/1, the worm root diameter may be increased. This increase may vary from zero for an 8/1 ratio to plus 15 percent for a 3/1 ratio.

12.8.6 Worm Pitch Diameter

The pitch diameter of the worm is assumed to be at the mean of the working depth at the center of the worm and is so considered for all calculations. The approximate worm pitch diameter is

$$d = \frac{C^{0.875}}{2.2}$$

and the corresponding root diameter is

$$d_R = d - 2b_G$$

where b_G is the dedendum of gear teeth in inches. Compare this root diameter with that given by Eq. (12.23). If it does not agree, alter the pitch diameter until the root diameter is within the desired limits.

Where horsepower rating is not a factor, there is no limitation regarding pitch diameter of the worm. Where efficiency is not as important as strength or load-carrying capacity, increasing the worm root diameter and gear face width will result in greater capacity.

12.8.7 Base Circle

The base circle may be secured from a layout in the following way. The normal pressure angle is always 20°. The axial pressure angle may be obtained from

$$\phi_x = \tan^{-1} \frac{\tan \phi_n}{\cos \lambda} \tag{12.24}$$

Once the centerline of the worm and gear, the vertical centerline, and the gear pitch circle are laid out, measure along the common worm and gear pitch circle to the right or left of the vertical centerline an amount equal to one-fourth the axial circular pitch p_x. Through the point thus established and at an angle to the vertical centerline equal to the axial pressure angle ϕ_x, extend a line upward. A circle tangent to this line and concentric to the gear axis is the base circle. Adjust this diameter to the nearest 0.01 in. The formula for figuring the base circle diameter is

$$D_b = D \sin \left(\phi_x + \frac{90°}{N_G} \right) \tag{12.25}$$

12.8.8 Tooth Depth Proportions

Formulas for figuring the whole depth, working depth, and dedendum of gear teeth are found in Table 12.12. Note that the working depth is based on the normal circular pitch and so varies for a given axial pitch.

It is common practice in double-enveloping worm gears to proportion the gear tooth and worm thread thickness as follows: The gear tooth thickness is 55 percent of the circular pitch, and the worm thread thickness is 45 percent of the circular pitch. The backlash in the gear set is subtracted from the worm thread thickness. This practice has been followed to secure greater tooth strength in the gear, which is the weaker member.

12.8.9 Tooth or Thread Forms

The thread form is usually straight in the axial section, but any other form may be used. Since there is no rolling action up or down the flanks, the form is unimportant, except that it must be the same on the worm and the hob. The straight-sided tooth in the axial section provides the greatest ease of manufacture and checking of both the gear sets and the cutting tools.

TABLE 12.12 Recommended Worm Tooth Dimensions

Quantity	Formula
Length of flat on outside diameter of worm, in	$f = \frac{p_x}{5.5}$
Whole depth of tooth	$h_t = \frac{p_n}{2}$
Working depth of tooth	$h_k = 0.9h_t$
Dedendum	$b_G = 0.611h_k$
Normal pressure angle	$\phi_n = 20°$
Axial pressure angle at center of worm	$\phi_x = \tan^{-1} \frac{\tan \phi_n}{\cos \lambda_c}$
Lead angle at center of worm	$\lambda_c = \tan^{-1} \frac{D}{m_G d}$

12.8.10 Worm Length

The effective length of the worm thread should be the base circle diameter minus $0.10C$ for lead angles up to and including 20° and minus $0.20C$ to $0.30C$ for lead angles from 20 to 45°. The principal reason for altering this length is to secure the proper amount of worm thread overlap. The overlap should be a distance along the worm thread greater than the face width of the gear. The worm thread extending beyond the effective length must be relieved to prevent interference.

The outside diameter of the worm equals the diameter at the tip of the worm thread at the effective length.

A formula for computing the flat on the outside diameter of the worm at the effective length is given in Table 12.12; the worm face equals the effective length plus twice the flat. The worm face angle is generally 45°.

12.8.11 Gear Blank Dimensions

The face width of the gear should be equal to the root diameter of the worm. Additional face width will not add proportional capacity and is wasteful of material. Where gear sets are to be used at less than their rated horsepower, the face width may be reduced in proportion.

The gear outside diameter may be the point at which the gear face angle intersects the gear throat radius or any desired amount less, except not less than the throat diameter. The gear throat diameter equals the gear pitch diameter plus one working depth.

There are generally three types of gear blanks in use: those having the hub integral, those flanged and counterbored for a bolted spider, and those having a through bore and fastened by setscrews (or bolts) inserted in drilled and tapped holes located half in the joint between the blank and spider. In all designs, the thickness of metal beneath the teeth should be 1¼ to 1½ times the whole depth of the tooth.

12.8.12 Materials†

Most of the rating standards are based on the use of worms made from a through-hardened, high-carbon steel heat-treated to 32 to 38 R_C. Where case-hardened worms are employed, somewhat higher ratings may be used.

Many high-strength gear materials (such as aluminum, heat-treated aluminum, and nickel bronzes) are used for slow speeds and heavy loads at higher ratings than shown in [12.2].

REFERENCES

12.1 ANSI/AGMA 6022-C93 (R2000), "Design Manual for Cylindrical Wormgearing."

12.2 ANSI/AGMA 6035-A02, "Design, Rating and Application of Industrial Globoidal Wormgearing."

ADDITIONAL REFERENCE

ANSI/AGMA 6034-B92 (R1999), "Practice for Enclosed Cylindrical Wormgear Speed Reducers and Gearmotors."

† See Ref. [12.2].