CHAPTER 33

THE STRENGTH OF COLD-WORKED AND HEAT-TREATED STEELS

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GLOSSARY

AR	Fractional area reduction
A	Area
B	Critical hardness for carbon content and tempering temperature, Rockwell C scale
d	Diameter
D	Tempering decrement, Rockwell C scale; carbon ideal diameter, in
D_I	Ideal critical diameter, in
DH	Distant hardness, Rockwell C scale
EJD	Equivalent Jominy distance, sixteenths of inch
f	Tempering factor for carbon content and tempering temperature
F	Load, temperature, degrees Fahrenheit
H	Quench severity, in^{-1}
IH	Initial hardness, Rockwell C scale

m	Strain-strengthening exponent
n	Design factor
r	Radius
R_{max}	Maximum hardness attainable, Rockwell C scale
R_Q	As-quenched Jominy test hardness, Rockwell C scale
R_T	Tempered hardness, Rockwell C scale
S_e'	Engineering endurance limit
S_u	Engineering ultimate strength in tension
S_y	Engineering yield strength, 0.2 percent offset
t	Time
ε	True strain
η	Factor of safety
$\bar{\sigma}_0$	Strain-strengthening coefficient
σ	Normal stress
ΣA	Sum of alloy increments, Rockwell C scale
τ_o	Octahedral shear stress
τ	Shearing stress

Subscripts

a	Axial
B	Long traverse
c	Compression
C	Circumferential
D	Short traverse
e	Endurance
f	Fracture
L	Longitudinal
R	Radial
s	Shear
t	Tension
u	Ultimate
y	Yield
0	No prior strain

33.1 INTRODUCTION

The mechanical designer needs to know the yield strength of a material so that a suitable margin against permanent distortion can be provided. The yield strength provided by a standardized tensile test is often not helpful because the manufacturing process has altered this property. Hot or cold forming and heat treatment (quenching and tempering) change the yield strength. The designer needs to know the yield strength of the material at the critical location in the geometry and at condition of use.

The designer also needs knowledge of the ultimate strength, principally as an estimator of fatigue strength, so that a suitable margin against fracture or fatigue can be provided. Hot and cold forming and various thermomechanical treatments during manufacture have altered these properties too. These changes vary within the part and can be directional. Again, the designer needs strength information for the material at the critical location in the geometry and at condition of use.

This chapter addresses the effect of plastic strain or a sequence of plastic strains on changes in yield and ultimate strengths (and associated endurance limits) and gives quantitative methods for the estimation of these properties. It also examines the changes in ultimate strength in heat-treated plain carbon and low-alloy steels.

33.2 STRENGTH OF PLASTICALLY DEFORMED MATERIALS

Methods for strength estimation include the conventional uniaxial tension test, which routinely measures true and engineering yield and ultimate strengths, percentage elongation and reduction in area, true ultimate and fracture strains, strain-strengthening exponent, strain-strengthening coefficient, and Young's modulus. These results are for the material in specimen form. Machine parts are of different shape, size, texture, material treatment, and manufacturing history and resist loading differently. Hardness tests can be made on a prototype part, and from correlations of strength with hardness and indentor size ([33.1], p. 5–35) and surface, ultimate strength can be assessed. Such information can be found in corporate manuals and catalogs or scattered in the literature. Often these are not helpful.

In the case of a single plastic deformation in the manufacturing process, one can use the true stress-strain curve of the material in the condition prior to straining provided the plastic strain can be determined. The results are good. For a sequence of successive strains, an empirical method is available which approximates what happens but is sometimes at variance with test results.

Cold work or *strain strengthening* is a common result of a cold-forming process. The process changes the properties, and such changes must be incorporated into the application of a theory of failure. The important strength is that of the part in the critical location in the geometry and at condition of use.

33.2.1 Datsko's Notation

In any discussion of strength it is necessary to identify

1. The kind of strength: ultimate, *u;* yield, *y;* fracture, *f;* endurance, *e.*
2. The sense of the strength: tensile, *t;* compressive, *c;* shear, *s.*
3. The direction or orientation of the strength: longitudinal, *L;* long transverse, *B;* short transverse, *D;* axial, *a;* radial, *R;* circumferential, *C.*
4. The sense of the most recent prior strain in the axial direction of the envisioned test specimen: tension, *t;* compression, *c.* If there is no prior strain, the subscript 0 is used.

33.2.2 Datsko's Rules

Datsko [33.1] suggests a notation $(S_1)_{234}$, where the subscripts correspond to 1, 2, 3, and 4 above. In Fig. 33.1 an axially deformed round and a rolled plate are depicted. A strength $(S_u)_{tLc}$ would be read as the engineering ultimate strength S_u, in tension $(S_u)_t$, in the longitudinal direction $(S_u)_{tL}$, after a last prior strain in the specimen direction that was compressive $(S_u)_{tLc}$. Datsko [33.1] has articulated rules for strain strengthening that are in approximate agreement with data he has collected. Briefly,

Rule 1. Strain strengthening is a bulk mechanism, exhibiting changes in strength in directions free of strain.

Rule 2. The maximum strain that can be imposed lies between the true strain at ultimate load ε_u and the true fracture strain ε_f. In upsetting procedures devoid of flexure, the limit is ε_f, as determined in the tension test.

Rule 3. The significant strain in a deformation cycle is the largest absolute strain, denoted ε_w. In a round $\varepsilon_w = \max(|\varepsilon_r|, |\varepsilon_\theta|, |\varepsilon_x|)$. The largest absolute strain ε_w is used in calculating the equivalent plastic strain ε_q, which is defined for two categories of strength, ultimate and yield, and in four groups of strength in Table 33.1.

Rule 4. In the case of several strains applied sequentially (say, cold rolling then upsetting), in determining ε_{qu}, the significant strains in each cycle ε_{wi} are added in decreasing order of magnitude rather than in chronological order.

Rule 5. If the plastic strain is imposed below the material's recrystallization temperature, the ultimate tensile strength is given by

$$S_u = (S_u)_o \exp \varepsilon_{qu} \qquad \varepsilon_{qu} < m$$
$$= \overline{\sigma}_0(\varepsilon_{qu})^m \qquad \varepsilon_{qu} > m$$

Rule 6. The yield strength of a material whose recrystallization temperature was not exceeded is given by

$$S_y = \overline{\sigma}_0(\varepsilon_{qy})^m$$

Table 33.1 summarizes the strength relations for plastically deformed metals.

33.3 ESTIMATING ULTIMATE STRENGTH AFTER PLASTIC STRAINS

This topic is best illuminated by example, applying ideas expressed in Secs. 33.2.1 and 33.2.2.

Example 1. A 1045HR bar has the following properties from tension tests:

$$S_y = 60 \text{ kpsi} \qquad S_u = 92.5 \text{ kpsi}$$
$$\text{AR} = 0.44 \qquad m = 0.14$$

The material is to be used to form an integral pinion on a shaft by cold working from 2¼ in to 2 in diameter and then upsetting to 2½ in to form a pinion blank, as depicted in Fig. 33.2. Find, using Datsko's rules, an estimate of the ultimate strength in a direction resisting tooth bending at the root of the gear tooth to be cut in the blank.

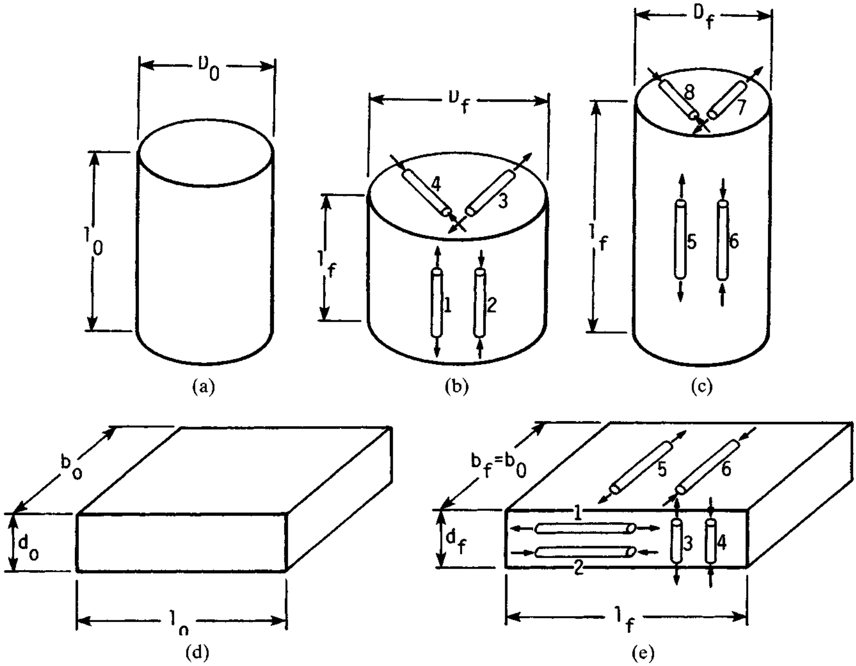


FIGURE 33.1 Sense of strengths in bar and plate. *(Adapted from [33.1], p. 7-7 with permission.)*
(*a*) Original bar before axial deformation.

	Specimen	Sense of strength	Direction in the bar	Prior strain	Designation
(*b*)	1	*t*	*L*	*c*	$(S)_{tLc}$
	2	*c*	*L*	*c*	$(S)_{cLc}$
	3	*t*	*T*	*t*	$(S)_{tTt}$
	4	*c*	*T*	*t*	$(S)_{cTt}$
(*c*)	5	*t*	*L*	*t*	$(S)_{tLt}$
	6	*c*	*L*	*t*	$(S)_{cLt}$
	7	*t*	*T*	*c*	$(S)_{tTc}$
	8	*c*	*T*	*c*	$(S)_{cTc}$

(*d*) Plate prior to rolling.

	Specimen	Sense of strength	Direction in the bar	Prior strain	Designation
(*e*)	1	*t*	*L*	*t*	$(S)_{tLt}$
	2	*c*	*L*	*t*	$(S)_{cLt}$
	3	*t*	*D*	*c*	$(S)_{tDc}$
	4	*c*	*D*	*c*	$(S)_{cDc}$
	5	*t*	*B*	0	$(S)_{tB0}$
	6	*c*	*B*	0	$(S)_{cB0}$

TABLE 33.1 Strength Relations for Plastically Deformed Metals[†]

$$(s_y)_w = \bar{\sigma}_0(\varepsilon_{qy})^m \qquad (S_u)_w = \begin{cases} (S_u)_0 \exp \varepsilon_{qu} & \varepsilon_{qu} < m \\ \bar{\sigma}_w & \varepsilon_{qu} > m \end{cases}$$

Group	Strength designation	ε_{qu}	ε_{qy}
1	$(S\ )_{cLc}$ $(S\ )_{tLt}$ $(S\ )_{tB0}$ $(S\ )_{cB0}$ $(S\ )_{cDc}$	$\varepsilon_{qus} = \sum_{i=1}^{n} \frac{\varepsilon_{wi}}{i}$	$\varepsilon_{qys} = \frac{\varepsilon_{qus}}{1 + 0.2\varepsilon_{qus}}$
2	$(S\ )_{tTt}$ $(S\ )_{cTc}$	$\varepsilon_{qus} = \sum_{i=1}^{n} \frac{\varepsilon_{wi}}{i}$	$\varepsilon_{qys} = \frac{\varepsilon_{qus}}{1 + 0.5\varepsilon_{qus}}$
3	$(S\ )_{cLt}$ $(S\ )_{tLc}$ $(S\ )_{tDc}$	$\varepsilon_{qu0} = \sum_{i=1}^{n} \frac{\varepsilon_{wi}}{i+1}$	$\varepsilon_{qy0} = \frac{\varepsilon_{qu0}}{1 + 2\varepsilon_{qu0}}$
4	$(S\ )_{tTc}$ $(S\ )_{cTt}$	$\varepsilon_{qu0} = \sum_{i=1}^{n} \frac{\varepsilon_{wi}}{i+1}$	‡

† Plastic deformation below material's recrystallization temperature.
‡ $(S_y)_{tTc} = (S_y)_{cTt} = 0.95(S_y)_{tTt}$ or $0.95(S_y)_{cTc}$
ε_{qus} = equivalent strain when prestrain sense is same as sense of strength
ε_{qu0} = equivalent strain when prestrain sense is opposite to sense of strength
SOURCE: From Datsko [33.1] and Hertzberg [33.2].

The strain-strengthening coefficient $\bar{\sigma}_0$ is, after [33.3],

$$\bar{\sigma}_0 = S_u \exp (m)m^{-m} = 92.5 \exp (0.14)0.14^{-0.14} = 140.1 \text{ kpsi}$$

The fracture strain (true) of the hot-rolled material from the tension test is

$$\varepsilon_f = \ln \frac{1}{1 - \text{AR}} = \ln \frac{1}{1 - 0.44} = 0.58$$

which represents limiting strain in deformation free of bending (rule 2). In the first step (cold rolling), the largest strain is axial, and it has a magnitude of (rule 3)

$$\varepsilon_1 = \left| \ln \left(\frac{D_0}{D_1} \right)^2 \right| = \left| \ln \left(\frac{2.25}{2} \right)^2 \right| = 0.236$$

In the second step (upsetting), the largest strain is axial, and it has a magnitude (rule 3) of

$$\varepsilon_2 = \left| \ln \left(\frac{D_1}{D_2} \right)^2 \right| = \left| \ln \left(\frac{2}{2.5} \right)^2 \right| = |-0.446| = 0.446$$

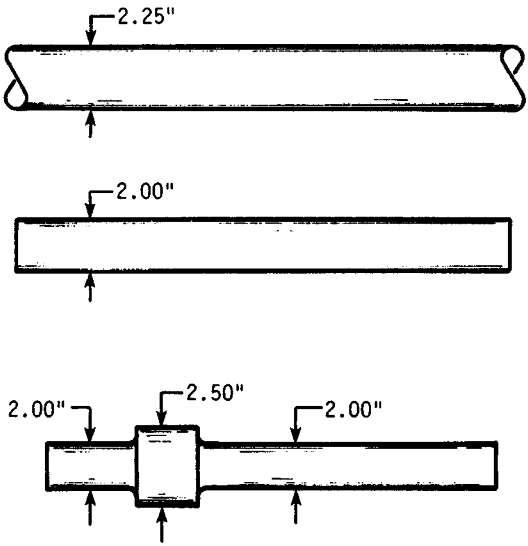


FIGURE 33.2 Cold working bar stock in two steps to form integral pinion blank on spindle.

The significant strains ε_{w1} and ε_{w2} are (rule 4) $\varepsilon_{w1} = 0.446$ and $\varepsilon_{w2} = 0.236$. Strengths will be carried with four computational digits until numerical work is done. For group 1 strengths,

$$\varepsilon_{qu} = \sum \frac{\varepsilon_{wi}}{i} = \frac{0.446}{1} + \frac{0.236}{2} = 0.564$$

$$S_u = \overline{\sigma}_0(\varepsilon_{qu})^m = 140.1(0.564)^{0.14} = 129.3 \text{ kpsi}$$

According to rule 5, $\varepsilon_{qu} > m$.

For group 2 strengths,

$$\varepsilon_{qu} = \sum \frac{\varepsilon_{wi}}{i} = \frac{0.446}{1} + \frac{0.236}{2} = 0.564$$

$$S_u = \overline{\sigma}_0(\varepsilon_{qu})^m = 140.1(0.564)^{0.14} = 129.3 \text{ kpsi}$$

For group 3 strengths,

$$\varepsilon_{qu} = \sum \frac{\varepsilon_{wi}}{1+i} = \frac{0.446}{2} + \frac{0.236}{3} = 0.302$$

$$S_u = \overline{\sigma}_0(\varepsilon_{qu})^m = 140.1(0.302)^{0.14} = 118.5 \text{ kpsi}$$

For group 4 strengths,

$$\varepsilon_{qu} = \sum \frac{\varepsilon_{wi}}{1+i} = \frac{0.446}{2} + \frac{0.236}{3} = 0.302$$

$$S_u = \overline{\sigma}_0(\varepsilon_{qu})^m = 140.1(0.302)^{0.14} = 118.5 \text{ kpsi}$$

The endurance limit and the ultimate strength resisting tensile bending stresses are $(S_e')_{tTt}$ and $(S_u)_{tTt}$, namely, 129.3/2 = 64.7 kpsi and 129.3 kpsi, respectively (group 2 strengths). The endurance limit and the ultimate strength resisting compressive bending stresses are $(S_e')_{cTt}$ and $(S_u)_{cTt}$, namely, 118.5/2 = 59.3 kpsi and 118.5 kpsi, respectively (group 4 strengths). In fatigue the strength resisting tensile stresses is the significant one, namely, 64.7 kpsi. A summary of this information concerning the four group ultimate strengths forms part of Table 33.2. Note that these two successive plastic strains have improved the ultimate tensile strength (which has become directional). The pertinent endurance limit has risen from 92.5/2 = 46.3 kpsi to 59.3 kpsi.

33.4 ESTIMATING YIELD STRENGTH AFTER PLASTIC STRAINS

This topic is best presented by extending the conditions of Example 1 to include the estimation of yield strengths.

Example 2. The same material as in Example 1 is doubly cold-worked as previously described. The strain-strengthening coefficient $\bar{\sigma}_0$ is still 140.1 kpsi, true fracture strain ε_f is 0.58, and $\varepsilon_1 = 0.236$, $\varepsilon_2 = 0.446$, $\varepsilon_{w1} = 0.446$, and $\varepsilon_{w2} = 0.236$ as before. For group 1 strengths,

$$\varepsilon_{qy} = \frac{\varepsilon_{qu}}{1 + 0.2\varepsilon_{qu}} = \frac{0.564}{1 + 0.2(0.564)} = 0.507$$

$$S_y = \bar{\sigma}_0(\varepsilon_{qy})^m = 140.1(0.507)^{0.14} = 127.4 \text{ kpsi} \qquad \text{(rule 6)}$$

For group 2 strengths,

$$\varepsilon_{qy} = \frac{\varepsilon_{qu}}{1 + 0.5\varepsilon_{qu}} = \frac{0.564}{1 + 0.5(0.564)} = 0.440$$

$$S_y = \bar{\sigma}_0(\varepsilon_{qy})^m = 140.1(0.440)^{0.14} = 124.9 \text{ kpsi}$$

For group 3 strengths,

$$\varepsilon_{qy} = \frac{\varepsilon_{qu}}{1 + 2\varepsilon_{qu}} = \frac{0.302}{1 + 2(0.302)} = 0.188$$

$$S_y = \bar{\sigma}_0(\varepsilon_{qy})^m = 140.1(0.188)^{0.14} = 110.9 \text{ kpsi}$$

TABLE 33.2 Summary of Ultimate and Yield Strengths for Groups 1 to 4 for Upset Pinion Blank

Group	ε_{qu}	S_u, kpsi	ε_{qy}	S_y, kpsi
1	0.564	129.3	0.507	127.4
2	0.564	129.3	0.440	124.9
3	0.302	118.5	0.188	110.9
4	0.302	118.5	. . .	118.7

Group 4 yield strengths are 0.95 of group 2:

$$S_y = 0.95(S_y)_2 = 0.95(124.9) = 118.7 \text{ kpsi}$$

Table 33.2 summarizes the four group strengths.

The yield strength resisting tensile bending stresses is $(S_y)_{tTt}$, a group 2 strength equaling 124.9 kpsi. The yield strength resisting compressive bending stresses is $(S_y)_{cTt}$, a group 4 strength equaling 118.7 kpsi. Yielding will commence at the weaker of the two strengths. If the bending stress level is 60 kpsi, the factor of safety against yielding is

$$\eta_y = \frac{(S_y)_{cTt}}{\sigma} = \frac{118.7}{60} = 1.98$$

If the estimate were to be based on the original material,

$$\eta_y = \frac{(S_y)_0}{\sigma} = \frac{60}{60} = 1$$

Datsko reports that predictions of properties after up to five plastic strains are reasonably accurate. For a longer sequence of different strains, Datsko's rules are approximate. They give the sense (improved or impaired) of the strength change and a prediction of variable accuracy. This is the only method of estimation we have, and if it is used cautiously, it has usefulness in preliminary design and should be checked by tests later in the design process.

33.5 ESTIMATING ULTIMATE STRENGTH OF HEAT-TREATED PLAIN CARBON STEELS

For a plain carbon steel the prediction of heat-treated properties requires that Jominy tests be carried out on the material. The addition method of Crafts and Lamont [33.4] can be used to estimate tempered-part strengths. Although the method was devised over 30 years ago, it is still the best approximation available, in either graphic or tabular form. The method uses the Jominy test, the ladle analysis, and the tempering time and temperature.

A 1040 steel has a ladle analysis as shown in Table 33.3 and a Jominy test as shown in Table 33.4. The symbol R_Q is the Jominy-test Rockwell C-scale hardness. The Jominy distance numbers are sixteenths of an inch from the end of the standard Jominy specimen. The tempered hardness after 2 hours (at 1000°F, for example) may be predicted from

$$R_T = (R_Q - D - B)f + B + \Sigma A \qquad R_T < R_Q - D \tag{33.1}$$

$$R_T = R_Q - D \qquad R_T > R_Q - D \tag{33.2}$$

TABLE 33.3 Ladle Analysis of a 1040 Steel

Element	C	Mn	P	S	Si
Percent	0.39	0.71	0.019	0.036	0.15

TABLE 33.4 Jominy Test of a 1040 Steel

Station	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	20	24	28	32
R_Q	55	49	29	25	25	24	23	22	21	20	19	18	17	17	16	16	14	12	11	9

where R_T = tempered hardness, Rockwell C scale
R_Q = as-quenched hardness, Rockwell C scale
D = tempering decrement, Rockwell C scale
B = critical hardness for carbon content and tempering temperature, Rockwell C scale
f = tempering factor of carbon content and tempering temperature
ΣA = sum of alloy increments, Rockwell C scale

From the appropriate figures for tempering for 2 hours at 1000°F, we have

$$D = 5.4 \quad \text{(Fig. 33.3)} \qquad A_{\text{Mn}} = 1.9 \quad \text{(Fig. 33.6)}$$

$$B = 10 \quad \text{(Fig. 33.4)} \qquad A_{\text{Si}} = 0.7 \quad \text{(Fig. 33.7)}$$

$$f = 0.34 \quad \text{(Fig. 33.5)} \qquad \Sigma A = 2.6$$

The transition from Eq. (33.1) to Eq. (33.2) occurs at a Rockwell hardness determined by equating these two expressions:

$$(R_Q - 5.4 - 10)0.34 + 10 + 2.6 = R_Q - 5.4$$

from which R_Q = 19.3, Rockwell C scale. The softening at each station and corresponding ultimate tensile strength can be found using Eq. (33.1) or Eq. (33.2) as appropriate and converting R_T to Brinell hardness and then to tensile strength or converting directly from R_T to tensile strength. Table 33.5 displays the sequence of steps in estimating the softening due to tempering at each Jominy distance of interest.

A shaft made from this material, quenched in oil ($H = 0.35$)† and tempered for 2 hours at 1000°F would have surface properties that are a function of the shaft's diameter. Figures 33.8 through 33.11 express graphically and Tables 33.6 through 33.9 express numerically the equivalent Jominy distance for the surface and interior of rounds for various severities of quench. A 1-in-diameter round has a rate of cooling at the surface that is the same as at Jominy distance 5.1 (see Table 33.6). This means an as-quenched hardness of about 15.9 and a surface ultimate strength of about 105.7 kpsi. Similar determinations for other diameters in the range 0.1 to 4 in lead to the display that is Table 33.10. A table such as this is valuable to the designer and can be routinely produced by computer [33.5]. A plot of the surface ultimate strength versus diameter from this table provides the 1000°F contour shown in Fig. 33.12. An estimate of 0.2 percent yield strength at the surface can be made (after Ref. [33.4], p. 191):

$$S_y = [0.92 - 0.006(R_{\max} - R_Q)]S_u \tag{33.3}$$

† The quench severity H is the ratio of the film coefficient of convective heat transfer h [Btu/(h·in²·°F)] to the thermal conductivity of the metal k [Btu/(h·in·°F)], making the units of H in $^{-1}$.

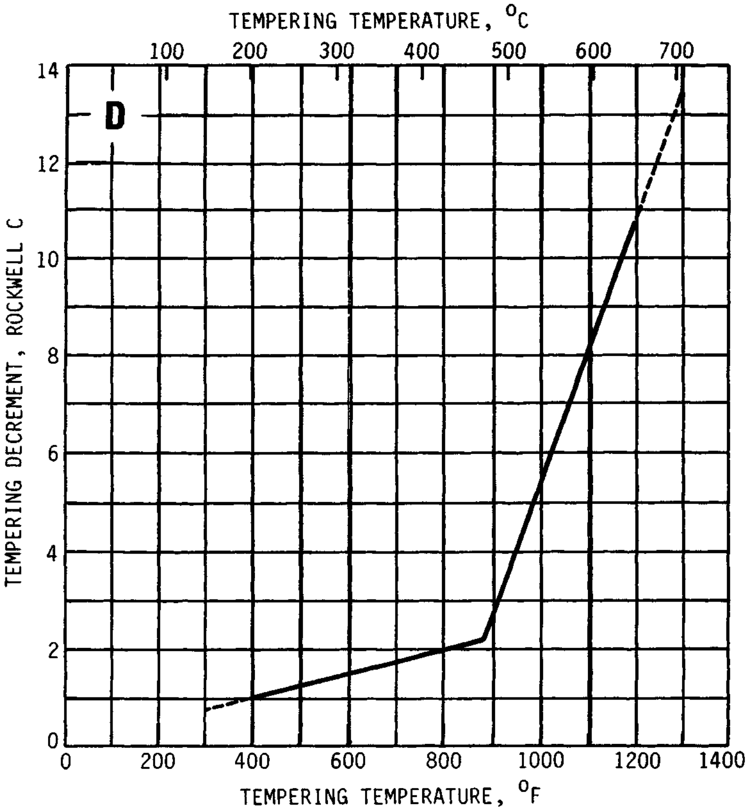


FIGURE 33.3 Hardness decrement D caused by tempering for "unhardened" steel. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

where R_{max} = maximum Rockwell C-scale hardness attainable for this steel, 32 + 60(%C), and R_Q = as-quenched hardness. An estimate of yield strength at the surface of a 1-in round of this material is as follows (equivalent Jominy distance is 5.1):

$$S_y = [0.92 - 0.006(55 - 25)]105.7 = 78.2 \text{ kpsi}$$

Different properties exist at different radii. For example, at the center of a 1-in round the properties are the same as at Jominy distance 6.6, namely, a predicted ultimate strength of 104.5 kpsi and a yield strength of 76.3 kpsi, which are not very different from surface conditions. This is not always the case.

33.6 ESTIMATING ULTIMATE STRENGTH OF HEAT-TREATED LOW-ALLOY STEELS

For heat-treated low-alloy steels, the addition method of Crafts and Lamont changes only in that additional constituents are present in the ΣA term if a Jominy test is

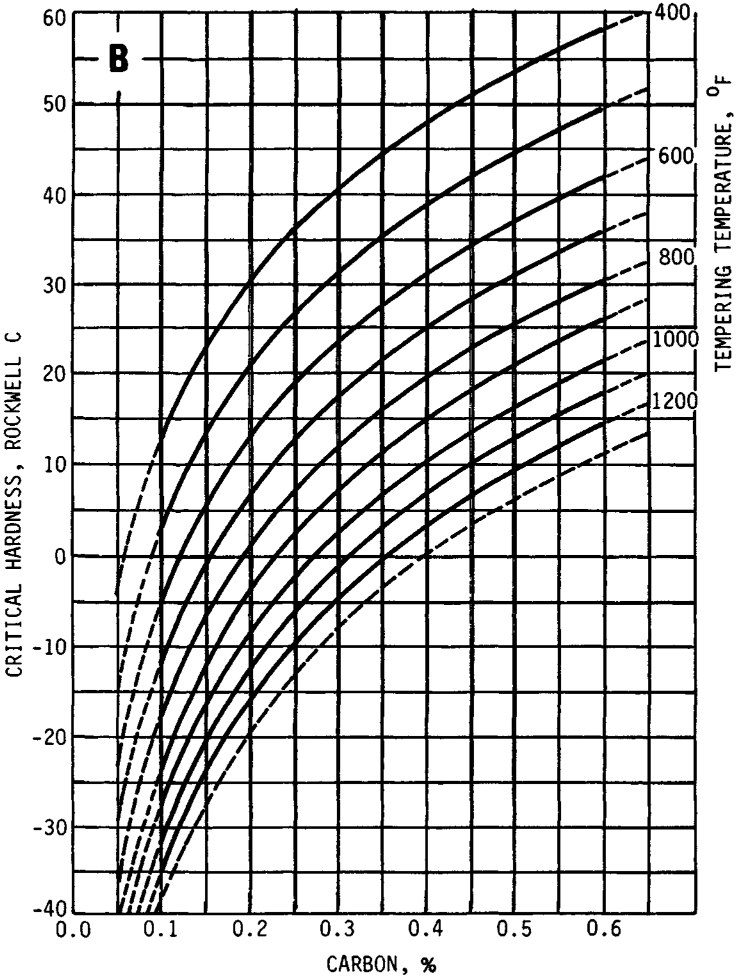


FIGURE 33.4 Critical hardness B for alloy-free steel as affected by carbon content and tempering temperature. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

available. However, for heat-treated low-alloy steels, the Jominy test may be replaced by an estimate based on the multiplication method of Grossmann and Fields coupled with knowledge of grain size and ladle analysis. Again, although the method was devised over 30 years ago, it is still the best approach available, in either graphic or tabular form. The multiplying factors for sulfur and phosphorus in this method are close to unity in the trace amounts of these two elements. The basic equation is

$$\begin{matrix}\text{Ideal}\\ \text{critical}\\ \text{diameter } D_I\end{matrix} = \begin{pmatrix}\text{carbon}\\ \text{ideal}\\ \text{diameter } D\end{pmatrix}\begin{pmatrix}\text{Mn}\\ \text{multiplying}\\ \text{factor}\end{pmatrix}\begin{pmatrix}\text{Cr}\\ \text{multiplying}\\ \text{factor}\end{pmatrix}\cdots$$

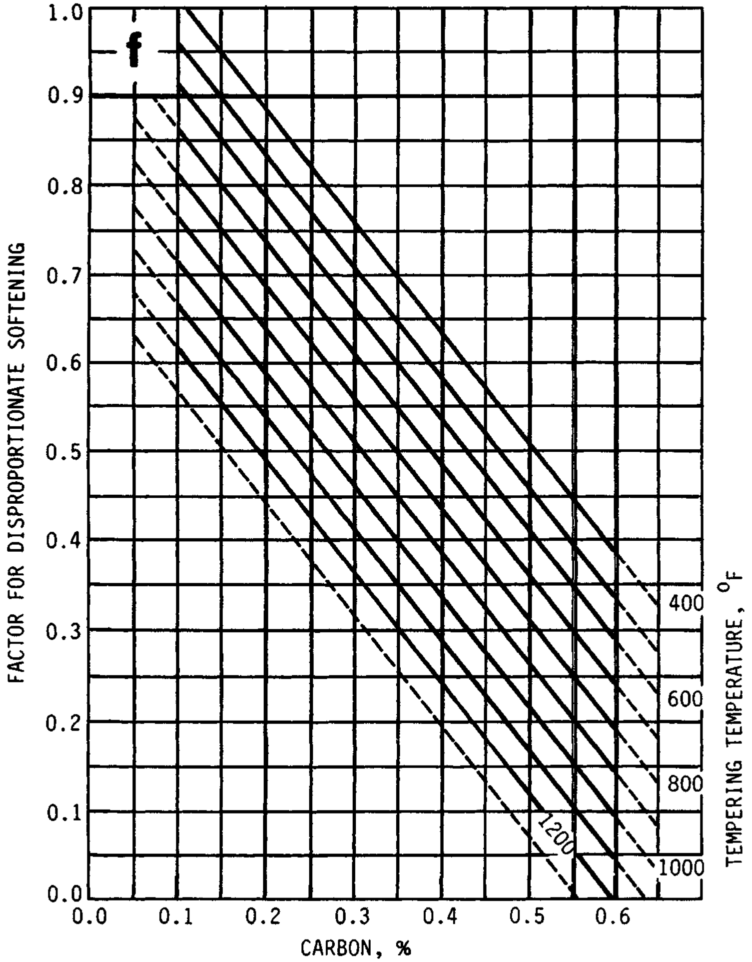


FIGURE 33.5 Factor *f* for disproportionate softening in "hardened" steel as affected by carbon content and tempering temperature. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

The multiplying factors for the elements Mn, Si, Cr, Ni, Mo, and Cu are presented in Fig. 33.13. The carbon ideal diameter D is available from Fig. 33.14 as a function of percent carbon and grain size of the steel.

Example 3. Determine the surface properties of an 8640 steel with average grain size 8 that was oil-quenched ($H = 0.35$) and tempered 2 hours at 1000°F. The ladle analysis and the multiplying factors are shown in Table 33.11. The multiplying factors are determined from Figs. 33.13 and 33.14. If boron were present, the multiplying factor would be

$$B = 17.23(\text{percent boron})^{-0.268}$$

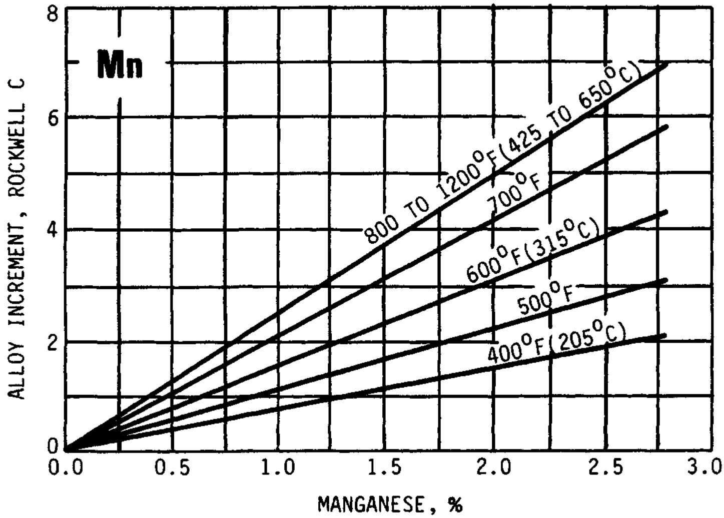


FIGURE 33.6 Effect of manganese on resistance to softening at various temperatures. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

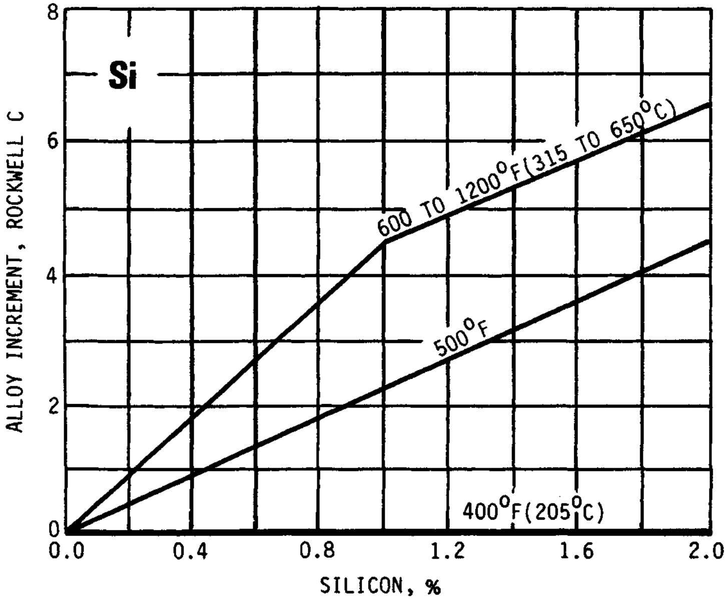


FIGURE 33.7 Effect of silicon on resistance to softening at various tempering temperatures. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

TABLE 33.5 Softening of 1040 Round Due to Tempering at 1000°F for 2 Hours

Jominy distance	R_Q	R_T	H_B	S_u, kpsi
1	55	26.1	258.6	129.3
2	49	24.0	247.0	123.5
3	29	17.2	216.2	108.1
4	25	15.9	211.6	105.8
5	25	15.9	211.6	105.8
6	24	15.3	209.8	104.9
7	23	15.2	208.4	104.2
8	22	14.8	206.6	103.3
9	21	14.5	205.3	102.6
10	20	14.2	203.9	102.0
				←Transition
11	19	13.6	201.2	100.6
12	18	12.6	196.7	98.4

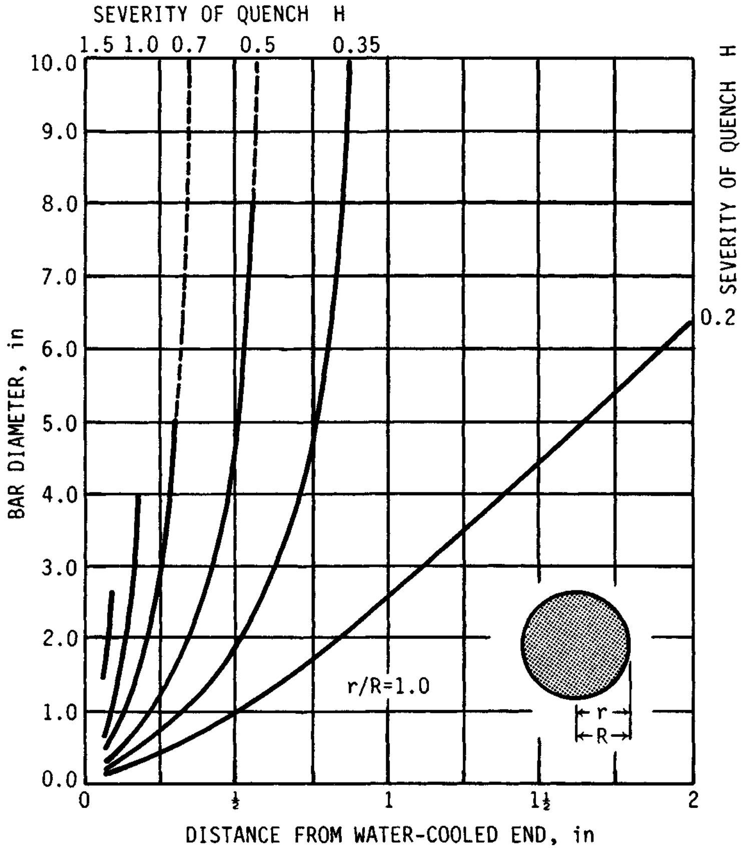


FIGURE 33.8 Location on end-quenched Jominy hardenability specimen corresponding to the surface of round bars. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

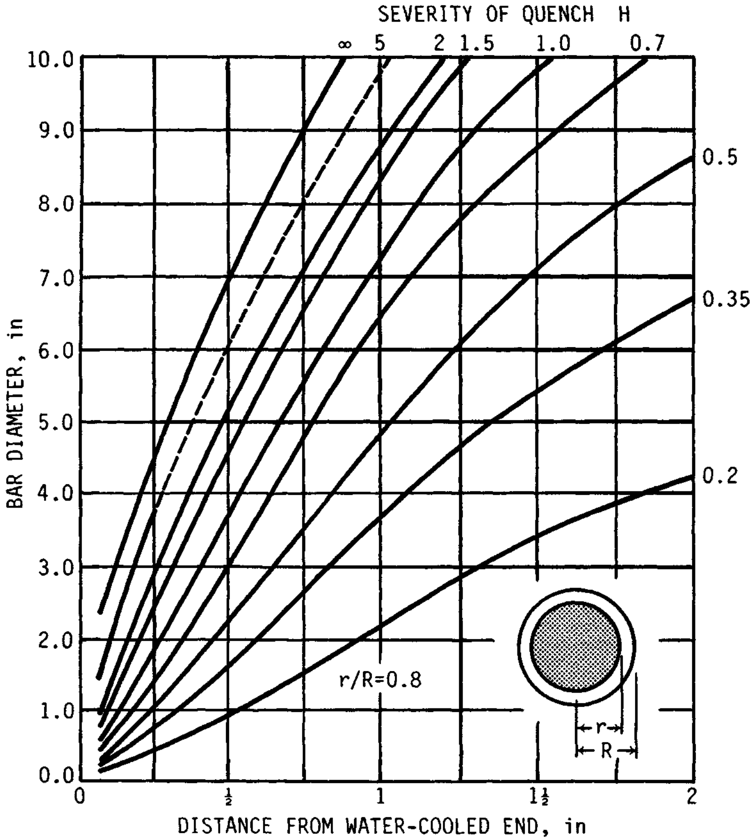


FIGURE 33.9 Location on end-quenched Jominy hardenability specimen corresponding to 80 percent from center of round. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

where percent boron is less than about 0.002. The calculation for ideal critical diameter D_I is

$$D_I = 0.197(3.98)(1.18)(2.08)(1.20)(1.60)(1.00) = 3.70 \text{ in}$$

The meaning of D_I is that it describes the largest diameter of a round that has at least 50 percent martensite structure everywhere in the cross section and exactly 50 percent at the center. The surface hardness of quenched steels is independent of alloy content and a function of carbon content alone. The Rockwell C-scale hardness is approximated by 32 + 60(%C), although it is not a strictly linear relationship ([33.4], p. 88. For the 8640 steel, the hardness at Jominy distance 1 is estimated to be 32 + 60(0.40) or 56 Rockwell C scale.

The ratio of initial hardness (distance 1), denoted IH, to distant hardness (at any other Jominy distance), denoted DH, is available as a function of the ideal critical

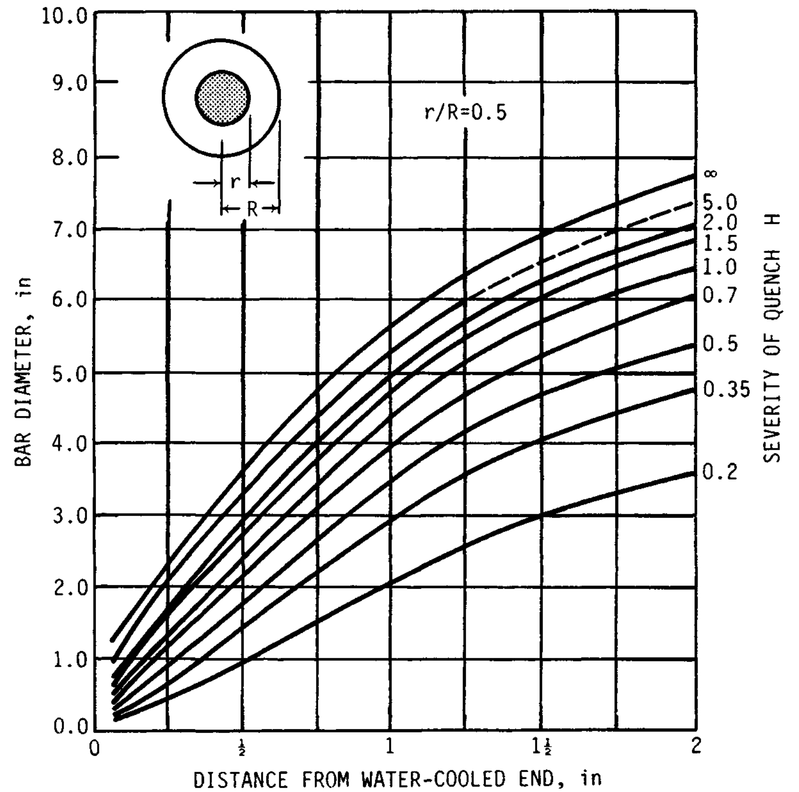


FIGURE 33.10 Location on end-quenched Jominy hardenability specimen corresponding to 50 percent from the center of round bars. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

diameter and the Jominy distance (Fig. 33.15). For the 8640 steel the Jominy hardnesses are estimated as displayed in Table 33.12. The Rockwell C-scale hardness is plotted against Jominy distance in Fig. 33.16, upper contour. The softening due to 2 hours of tempering at 1000°F can be estimated as before using the addition method of Crafts and Lamont. The ΣA term is evaluated as follows:

$D = 5.31$	(Fig. 33.3)	$A_{Mn} = 2.25$	(Fig. 33.6)
$B = 9.90$	(Fig. 33.4)	$A_{Si} = 1.13$	(Fig. 33.7)
$f = 0.34$	(Fig. 33.5)	$A_{Cr} = 2.59$	(Fig. 33.17)
		$A_{Ni} = 0.11$	(Fig. 33.18)
		$A_{Mo} = 3.60$	(Fig. 33.19)
		$\Sigma A = 9.67$	

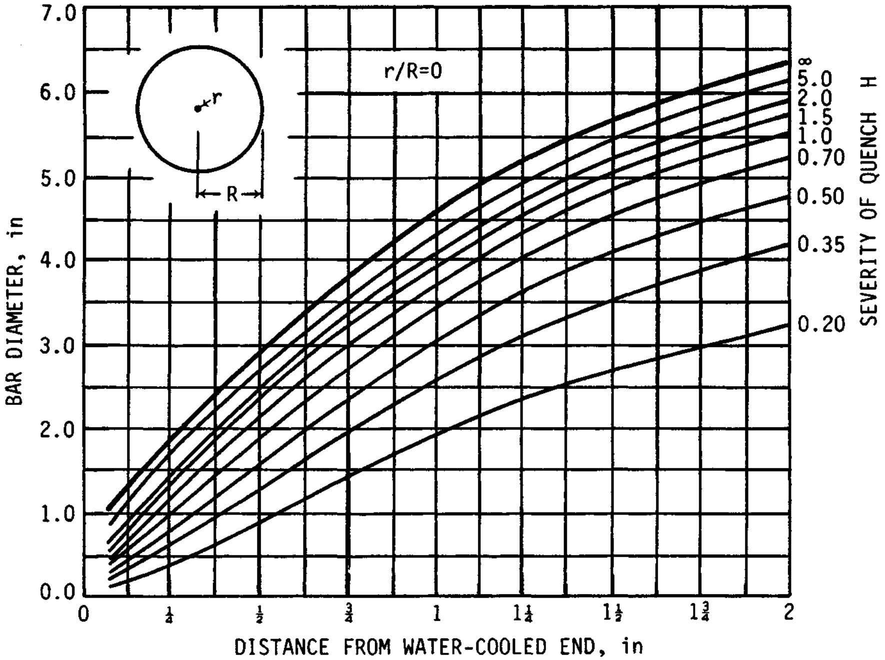


FIGURE 33.11 Location on end-quenched Jominy hardenability specimen corresponding to the center of round bars. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

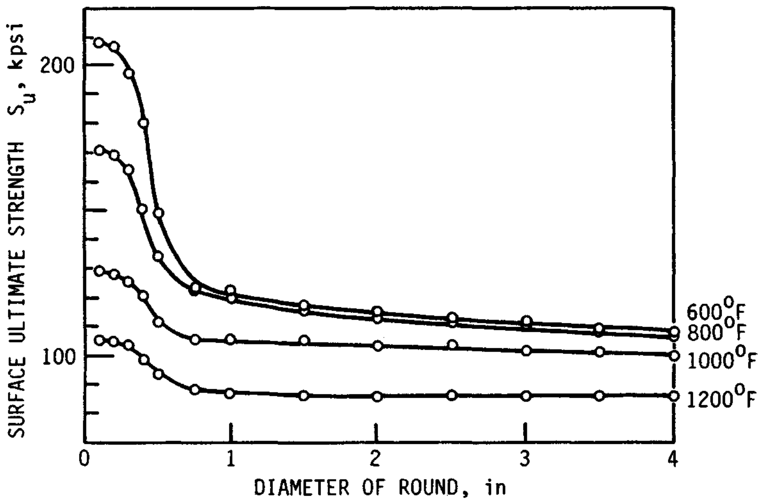


FIGURE 33.12 Variation of surface ultimate strength with diameter for a 1040 steel oil-quenched ($H = 0.35$) from 1575°F and tempered 2 hours at 1000°F.

TABLE 33.6 Equivalent Jominy Distances for Quenched Rounds at $r/R = 1$

	Severity of quench H, in^{-1}						
Diameter, in	0.20	0.30	0.35	0.40	0.50	0.60	0.70
0.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.2	1.8	1.3	1.1	1.0	1.0	1.0	1.0
0.3	2.7	1.9	1.6	1.4	1.2	1.0	1.0
0.4	3.6	2.5	2.2	1.9	1.5	1.2	1.0
0.5	4.5	3.2	2.7	2.3	1.9	1.5	1.2
0.75	6.7	4.8	4.0	3.5	2.9	2.2	1.7
1.0	8.3	6.0	5.1	4.4	3.5	2.7	2.2
1.5	10.7	8.0	6.9	6.0	4.6	3.5	2.8
2.0	13.2	9.6	8.2	7.1	5.4	4.2	3.3
2.5	15.4	11.0	9.2	7.8	6.1	4.6	3.7
3.0	17.6	12.1	10.0	8.4	6.6	5.0	4.0
3.5	19.8	13.1	10.7	8.9	7.0	5.4	4.3
4.0	22.1	14.2	11.4	9.4	7.6	5.7	4.5

The tempered hardness equations become either

$$R_T = (R_Q - 5.31 - 9.90)0.34 + 9.90 + 9.67$$

$$= 0.34R_Q + 14.4$$

or

$$R_T = R_Q - 5.31$$

TABLE 33.7 Equivalent Jominy Distances for Quenched Rounds at $r/R = 0.8$

	Severity of quench H, in^{-1}						
Diameter, in	0.20	0.30	0.35	0.40	0.50	0.60	0.70
0.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.2	1.8	1.3	1.1	1.0	1.0	1.0	1.0
0.3	2.7	1.9	1.7	1.5	1.3	1.1	1.0
0.4	3.6	2.6	2.2	2.0	1.7	1.4	1.2
0.5	4.5	3.2	2.8	2.5	2.1	1.8	1.5
0.75	6.7	4.9	4.2	3.7	3.2	2.6	2.2
1.0	8.3	6.2	5.4	4.8	4.0	3.4	3.0
1.5	11.5	8.7	7.6	6.7	5.6	4.8	4.4
2.0	14.6	10.9	9.6	8.5	7.3	6.3	5.7
2.5	17.7	13.1	11.4	10.2	8.9	7.7	7.0
3.0	21.0	15.4	13.4	11.9	10.4	9.0	8.1
3.5	24.9	18.0	15.5	13.7	12.0	10.3	9.3
4.0	29.4	21.1	18.0	15.9	13.4	11.5	10.3

TABLE 33.8 Equivalent Jominy Distances for Quenched Rounds at $r/R = 0.5$

	Severity of quench H, in^{-1}						
Diameter, in	0.20	0.30	0.35	0.40	0.50	0.60	0.70
0.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.2	1.8	1.3	1.2	1.1	1.0	1.0	1.0
0.3	2.7	2.0	1.8	1.6	1.4	1.2	1.0
0.4	3.6	2.7	2.4	2.1	1.8	1.6	1.4
0.5	4.5	3.4	3.0	2.6	2.3	2.0	1.1
0.75	6.7	5.0	4.4	4.0	3.4	2.9	2.6
1.0	8.3	6.4	5.7	5.2	4.5	4.0	3.5
1.5	11.9	9.2	8.3	7.5	6.7	5.9	5.4
2.0	15.4	12.0	10.8	9.8	8.9	8.0	7.3
2.5	19.3	15.0	13.4	12.2	11.1	10.1	9.3
3.0	24.2	18.4	16.3	14.8	13.6	12.3	11.5
3.5	30.3	22.4	19.6	17.7	16.2	14.7	13.8
4.0	32.0	25.9	23.5	21.6	19.1	17.3	16.4

The transition hardness obtained by equating the preceding pair of equations is $R_Q = 29.9$. The Jominy curve may be corrected for tempering. Table 33.13 shows the tempered hardness and ultimate strength corresponding to the Jominy distances. The column R_T is plotted against Jominy distance as the lower curve in Fig. 33.16. The surface ultimate strength can be estimated for diameters 0.5, 1, 2, 3, and 4 in. At a diameter of 2 in, the equivalent Jominy distance is 8.2 from Table 33.6. The surface ultimate strength as a function of diameter of round is displayed in Table 33.14. The ultimate tensile strength is found by interpolation in the prior display, entering with equivalent Jominy distance. The tensile ultimate strength at the surface versus diam-

TABLE 33.9 Equivalent Jominy Distances for Quenched Rounds at $r/R = 0$

	Severity of quench H, in^{-1}						
Diameter, in	0.20	0.30	0.35	0.40	0.50	0.60	0.70
0.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.2	1.8	1.5	1.4	1.2	1.0	1.0	1.0
0.3	2.7	2.2	2.0	1.9	1.5	1.3	1.2
0.4	3.6	3.0	2.7	2.5	2.0	1.8	1.6
0.5	4.5	3.7	3.4	3.1	2.6	2.2	2.0
0.75	6.7	5.6	5.1	4.6	3.8	3.3	3.0
1.0	8.3	7.1	6.6	6.1	5.1	4.5	4.1
1.5	12.4	10.3	9.5	8.7	7.7	6.9	6.4
2.0	16.7	13.5	12.3	11.4	10.2	9.2	8.6
2.5	21.8	17.2	15.5	14.2	12.9	11.7	11.0
3.0	28.1	21.6	19.3	17.5	15.8	14.4	13.6
3.5	32.0	26.2	23.9	21.9	19.2	17.3	16.5
4.0	32.0	30.9	29.7	27.9	23.0	20.6	19.9

TABLE 33.10 Surface Ultimate Strength of a 1040 Steel Heat-Treated Round as a Function of Diameter†

Diameter, in	Equivalent Jominy distance, $\frac{1}{16}$ in	Surface ultimate strength S_u, kpsi
0.1	1.0	129.3
0.2	1.1	128.7
0.3	1.6	125.8
0.4	2.2	120.4
0.5	2.7	112.7
1.0	5.1	105.7
1.5	6.9	104.3
2.0	8.2	103.2
3.0	10.0	102.0
4.0	11.4	99.7

† Round quenched from 1575°F in still oil (H = 0.35) tempered for 2 hours at 1000°F. Predictions by the addition method of Crafts and Lamont.

eter of round is plotted in Fig. 33.20. Note the greater hardening ability of the 8640 compared to the 1040 steel of the previous section. Local interior properties are available using Figs. 33.9, 33.10, and 33.11. An estimate of the variation of properties across the section of a round 4 in in diameter will be made. The equivalent Jominy distances are 11.2 at $r = 2$ in, 18.0 at $r = 1.6$ in, 23.5 at $r = 1$ in, and 29.7 at $r = 0$. Thus Table 33.15 may be formed. The values of S_u are obtained by interpolation; the values of S_y are estimated using Eq. (33.3). A plot is shown in Fig. 33.21.

A common source for properties of steels is *Modern Steels and Their Properties* [33.5]. It is well to note that hardness was taken in this reference at the surface of a

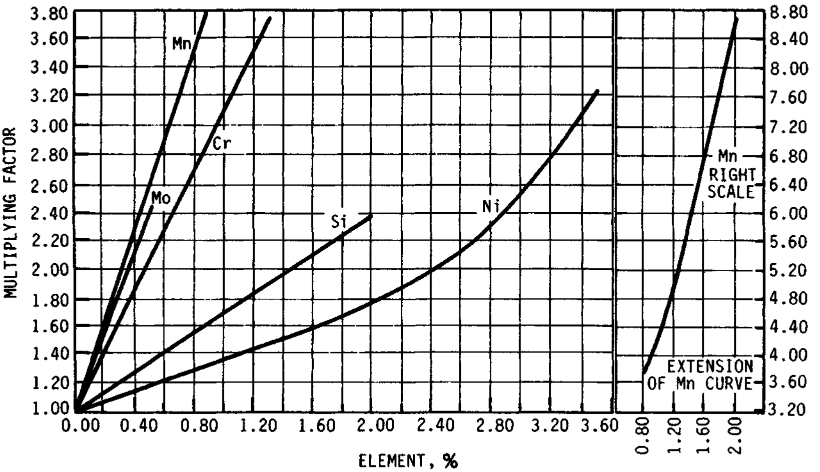


FIGURE 33.13 Multiplying factors for five common alloying elements (for trace copper, use nickel curve). *(From [33.4] with permission of Pitman Publishing Ltd., London).*

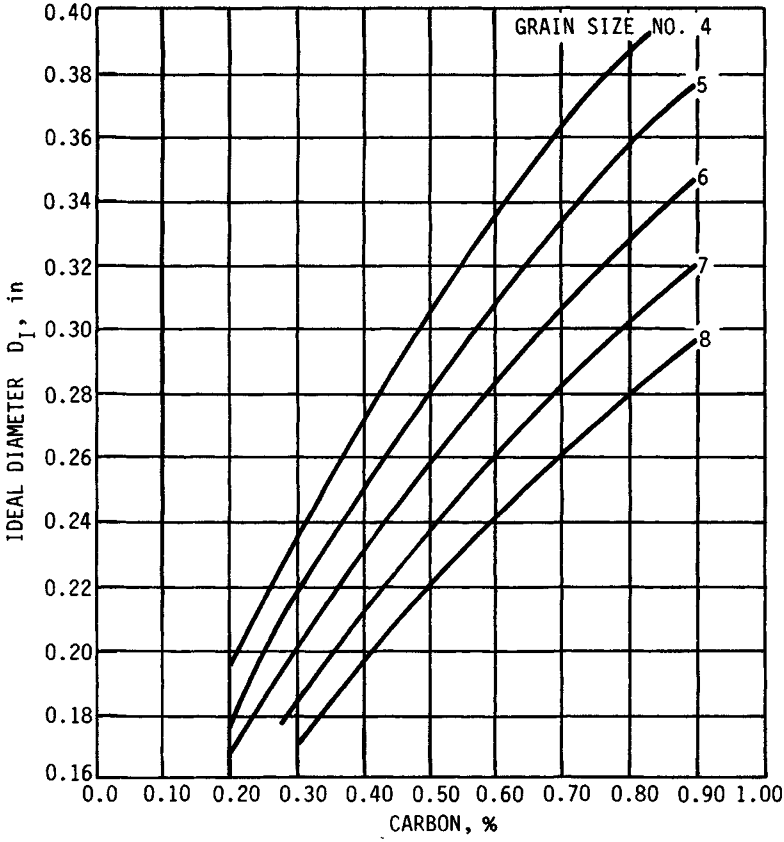


FIGURE 33.14 Relationship between ideal diameter D_I, carbon content, and grain size. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

TABLE 33.11 Ladle Analysis and Multiplying Factors for 8640 Steel, Grain Size 8

Element	C	Mn	Si	Cr	Ni	Mo	Cu
Percent	0.40	0.90	0.25	0.50	0.55	0.20	0.00
Factor	0.197	3.98	1.18	2.08	1.20	1.60	1.00

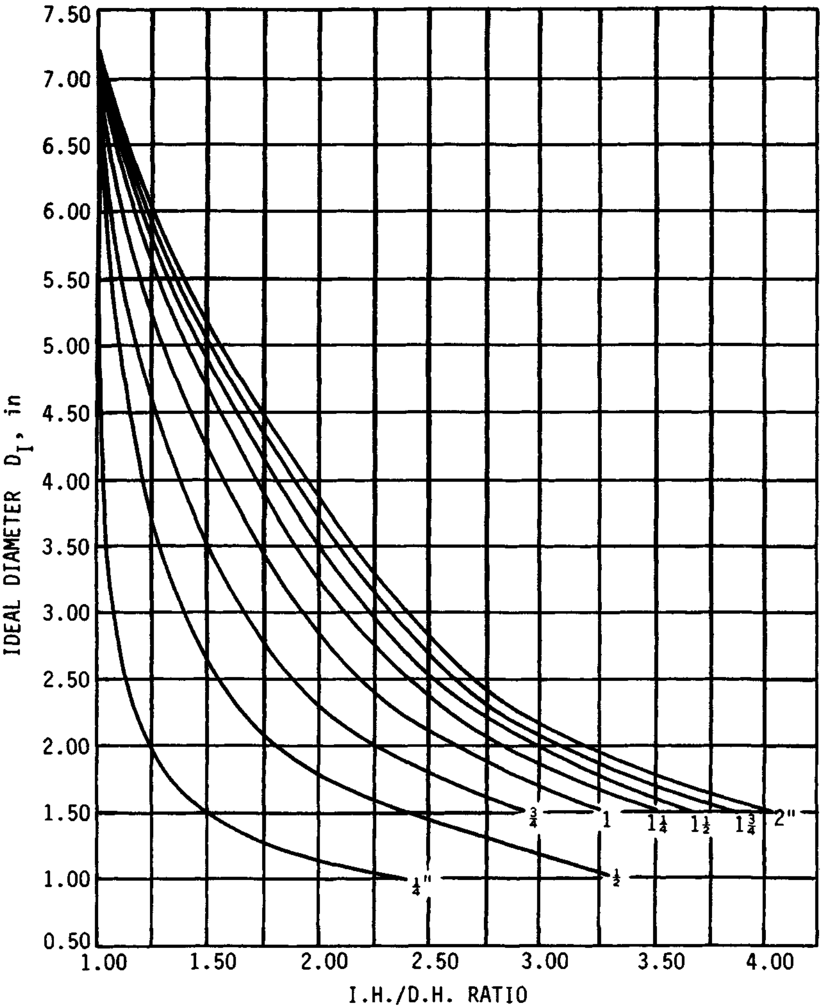


FIGURE 33.15 Relation between ideal critical diameter and the ratio of initial hardness IH to distant hardness DH. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

1-in-diameter quenched and tempered bar, and that the tensile specimen was taken from the center of that bar for plain carbon steels. Alloy-steel quenched and tempered bars were 0.532 in in diameter machined to a standard 0.505-in-diameter specimen. From the traverse of strengths in the previous array, it is clear that central and surface properties differ. In addition, the designer needs to know the properties of the critical location in the geometry and at condition of use. Methods of estimation such as the Crafts and Lamont addition method and the Grossmann and Fields multiplication method are useful prior to or in the absence of tests on the machine part.

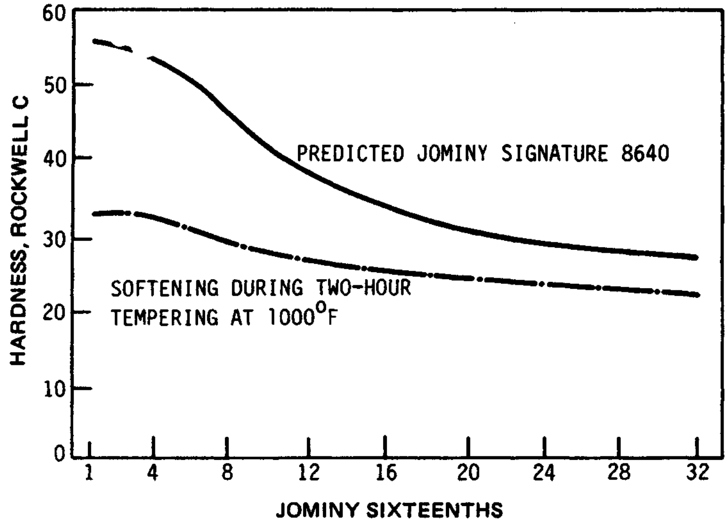


FIGURE 33.16 Predicted Jominy signature for a 8640 steel with softening produced by 2-hour tempering at 1000°F.

These methods have produced for a 4-in round of 8640, quenched in oil ($H = 0.35$) from 1575°F, and tempered for 2 hours at 1000°F, the property estimates displayed as Table 33.16. Reference [33.6] is a circular slide rule implementation of the multiplication method of Grossmann and Fields.

Current efforts are directed toward refining the information rather than displacing the ideas upon which Secs. 33.5 and 33.6 are based ([33.7], [33.8]). Probabilistic elements of the predicted Jominy curve are addressed in Ho [33.9].

TABLE 33.12 Prediction of Jominy Curve for 8640 Steel by Multiplication Method of Grossmann and Fields

Jominy distance	$\frac{IH}{DH}$	$R_Q = \frac{IH}{(IH/DH)}$
1	1.00	56.0
4	1.03	54.3
8	1.24	45.0
12	1.46	38.4
16	1.67	33.6
20	1.82	30.7
24	1.92	29.2
28	2.00	28.0
32	2.04	24.7

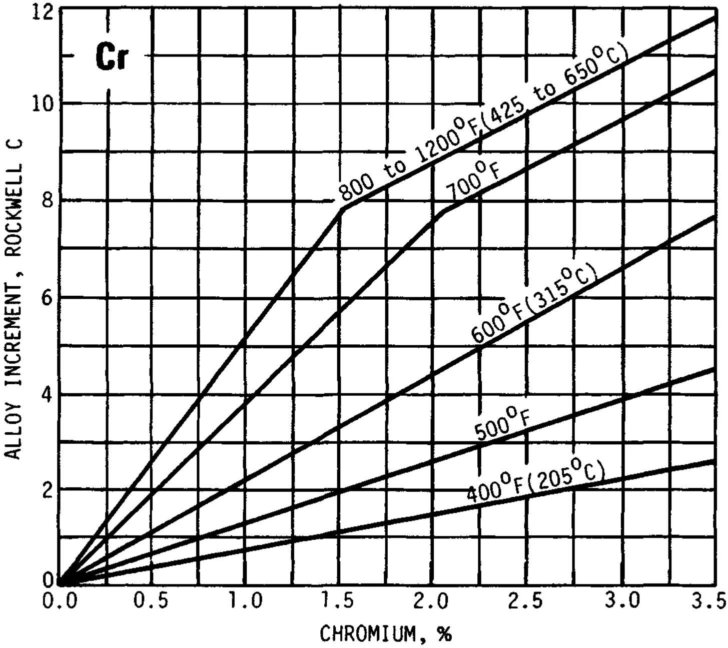


FIGURE 33.17 Effect of chromium on resistance to softening at various tempering temperatures. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

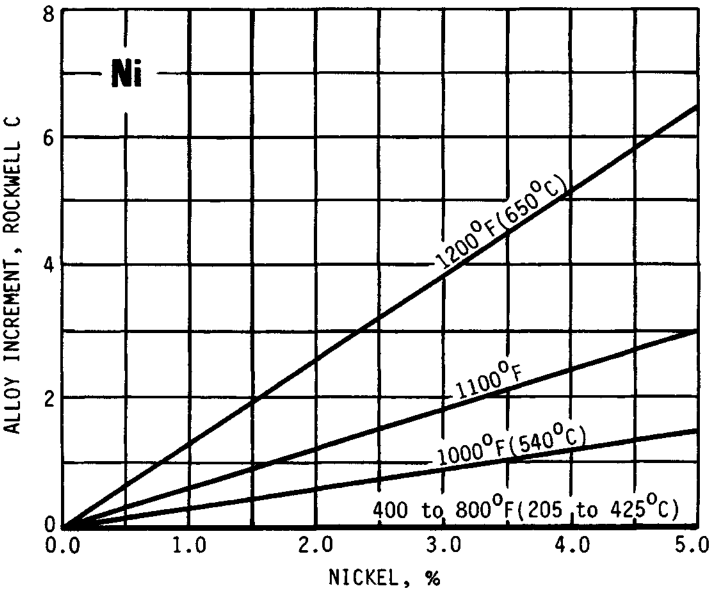


FIGURE 33.18 Effect of nickel on resistance to softening at various tempering temperatures. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

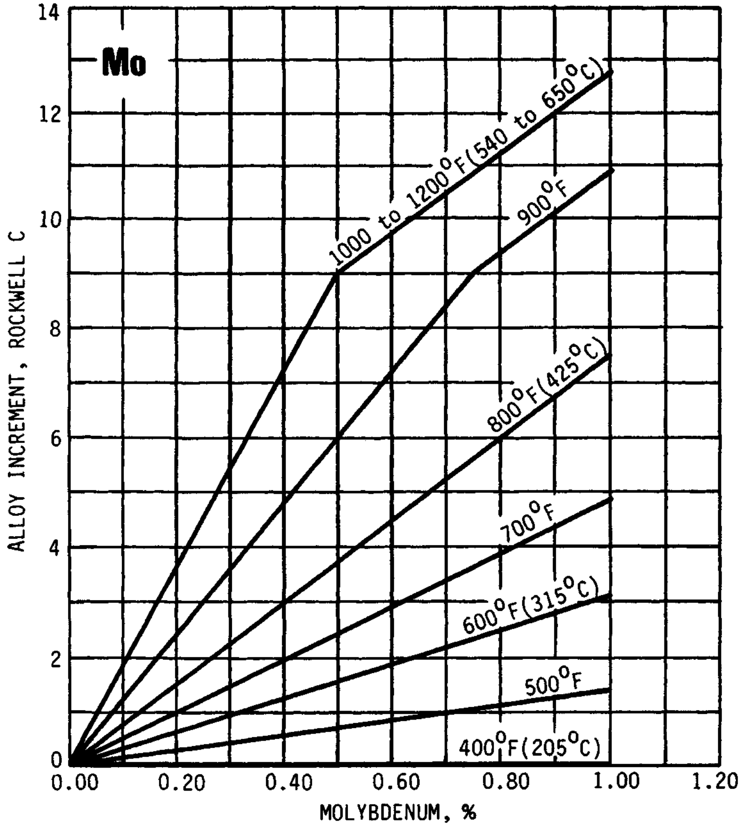


FIGURE 33.19 Effect of molybdenum on resistance to softening at various tempering temperatures. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

TABLE 33.13 Tempered Hardness and Ultimate Strength at Jominy Distances Due to Softening after Tempering 8640 Steel 2 Hours at 1000°F

Distance	R_Q	R_T	H_B	S_u, kpsi
1	56.0	33.4	314.2	157.1
4	54.3	32.9	310.2	155.1
8	45.0	29.7	283.9	142.0
12	38.4	27.5	267.5	133.8
16	33.6	25.8	257.0	128.5
20	30.7	24.8	252.4	126.2
				←Transition
24	29.2	23.9	246.6	123.3
28	28.0	22.7	241.2	120.6
32	27.4	22.1	237.6	118.8

TABLE 33.14 Surface Ultimate Strength of 8640 Steel Tempered for 2 Hours at 1000°F as a Function of Diameter of Round

Diameter, in	Equivalent Jominy distance, $\frac{1}{16}$ in	S_u, kpsi
0.5	2.7	156.0
1	5.1	151.5
2	8.2	141.6
3	10.0	137.3
4	11.4	135.0

TABLE 33.15 Ultimate and Yield Strength Traverse of a 4-in-Diameter Round of 8640 Steel Tempered 2 Hours at 1000°

Location r, in	Equivalent Jominy distance, $\frac{1}{16}$ in	S_u, kpsi	S_y, kpsi
2	11.4	135.0	110.8
1.6	18.0	127.4	99.0
1	23.5	123.7	94.1
0	29.7	119.8	89.9

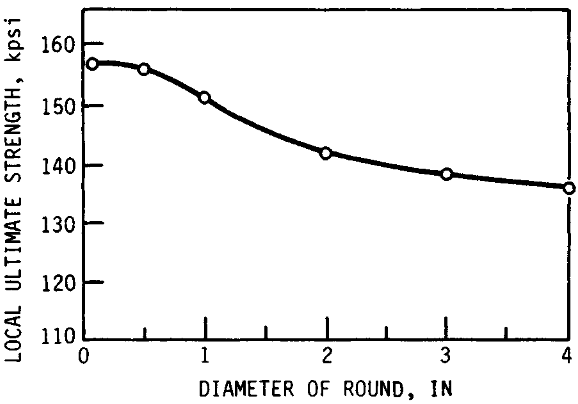


FIGURE 33.20 Variation on surface ultimate strength for 8640 steel oil-quenched (H = 0.35) from 1575°F and tempered for 2 hours at 1000°F as a function of diameter of round.

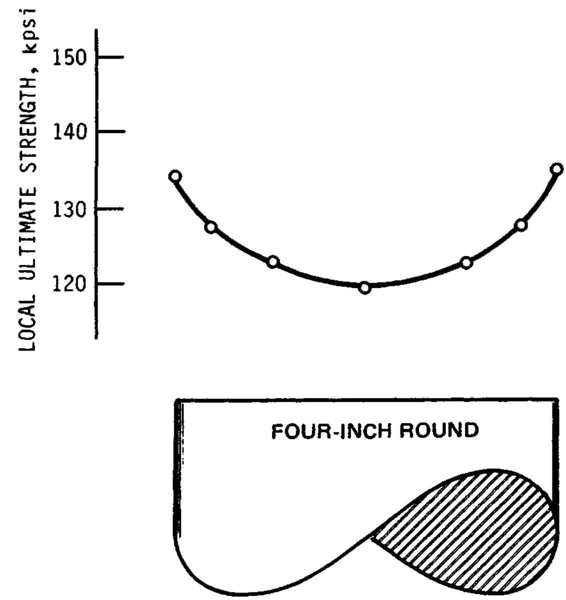


FIGURE 33.21 Variation in surface ultimate strength across a section of a 4-in round of 8640 steel oil-quenched (H = 0.35) from 1575°F and tempered for 2 hours at 1000°F as a function of radial position.

TABLE 33.16 Summary of Strength and Hardness Estimates for a 4-in Round of 8640 Steel Quenched in Oil (H = 0.35) from 1575°F and Tempered 2 Hours at 1000°F

Property	Estimate
Surface hardness	270 Brinell
Surface ultimate strength	135 kpsi
Surface yield strength	110.8 kpsi
Surface R. R. Moore endurance limit	67.5 kpsi
Contact endurance strength ($0.4H_B - 10$)	98 kpsi†
Central hardness	239.6 Brinell
Central ultimate strength	119.8 kpsi
Central yield strength	89.9 kpsi

† 10^8 cycles.

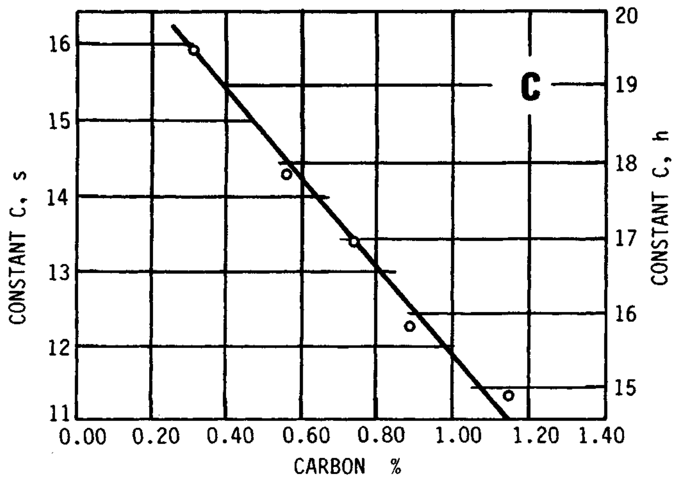


FIGURE 33.22 Variation with carbon content of constant C in time-temperature tradeoff equation for tempered, fully quenched plain carbon steels. *(From [33.4] with permission of Pitman Publishing Ltd., London.)*

33.7 TEMPERING TIME AND TEMPERATURE TRADEOFF RELATION

The tempering-temperature/time tradeoff equation is

$$(459 + F_1)(C + \log_{10} t_1) = (459 + F_2)(C + \log_{10} t_2) \tag{33.4}$$

where C is a function of carbon content determinable from Fig. 33.22. For 8640 steel, the value of C is 18.85 when the time is measured in hours. For a tempering temperature of 975°F, the tempering time is

$$(459 + 1000)(18.85 + \log_{10} 2) = (459 + 975)(18.85 + \log_{10} t_2)$$

from which $t_2 = 4.3$ h.

Since steel is bought in quantities for manufacturing purposes and the heat from which it came is identified as well as the ladle analysis, once such an estimation of properties procedure is carried out, the results are applicable for as long as the material is used. It is useful to employ a worksheet and display the results. Such a sheet is depicted in Fig. 33.23.

Heat treated steel worksheet for shafts
alloy # ________, grain size # __________

	C	Mn	Si	Ni	Cr	Mo			
%								D_I	
Mult.									ΣA
A									

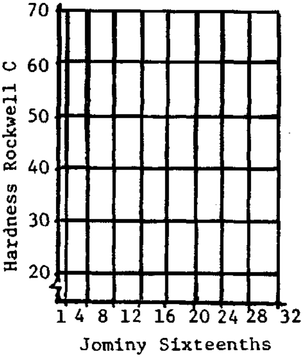
Change in tempering time from two hours?

$T(C+\log_{10}t) = \text{const.}$

predicted
tested

$IH = 32 + 60(\%C)$

Jominy Distance Sixteenths	$\frac{IH}{DH}$	$R_C = \frac{IH}{IH/DH}$
1	1	
4		
8		
12		
16		
20		
24		
28		
32		



heat treatment: quenched from ______ C/F in _____, quench severity, H = ______

$R_T = (R_Q - D - B)f + B + \Sigma A$ $\quad R_T < (R_Q - D)$ Tempered at ______ C/F for ______ hours

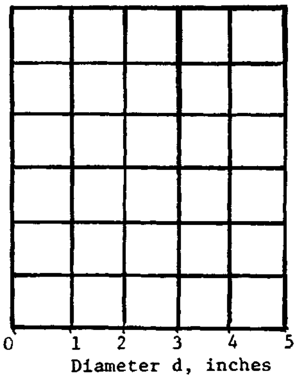
$R_T =$

or $R_T = (R_Q - D)$ $\quad R_T > (R_Q - D)$

$R_T =$

D	B	f

Surface of Round of Diameter d, inches	Equivalent Jominy Distance Sixteenths	Hardness Rockwell C	Hardness Rockwell C Tempered, R_T	Brinell Hardness BHN	Local S_u kpsi
0.1					
0.2					
0.3					
0.4					
0.5					
1.0					
1.5					
2.0					
3.0					
4.0					



Remarks:

FIGURE 33.23 Heat-treated-steel worksheet for shafts.

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