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MACHINE ELEMENTS THAT ABSORB AND STORE ENERGY

CHAPTER 6

SPRINGS

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- 6.1 INTRODUCTION / 6.4**
- 6.2 GLOSSARY OF SPRING TERMINOLOGY / 6.4**
- 6.3 SELECTION OF SPRING MATERIALS / 6.6**
- 6.4 HELICAL COMPRESSION SPRINGS / 6.12**
- 6.5 HELICAL EXTENSION SPRINGS / 6.29**
- 6.6 HELICAL TORSION SPRINGS / 6.36**
- 6.7 BELLEVILLE SPRING WASHER / 6.40**
- 6.8 SPECIAL SPRING WASHERS / 6.51**
- 6.9 FLAT SPRINGS / 6.55**
- 6.10 CONSTANT-FORCE SPRINGS / 6.58**
- 6.11 TORSION BARS / 6.62**
- 6.12 POWER SPRINGS / 6.63**
- 6.13 HOT-WOUND SPRINGS / 6.66**
- REFERENCES / 6.69**

GENERAL NOMENCLATURE[†]

<i>A</i>	Area, mm ² (in ²)
<i>b</i>	Width, mm (in)
<i>C</i>	Spring index, D/d
<i>d</i>	Wire diameter, mm (in)
<i>D</i>	Mean diameter (OD minus wire diameter), mm (in)
<i>E</i>	Modulus of elasticity in tension or Young's modulus, MPa (psi)
<i>f</i>	Deflection, mm (in)
<i>g</i>	Gravitational constant, 9.807 m/s ² (386.4 in/s ²)
<i>G</i>	Shear modulus or modulus of rigidity, MPa (psi)
<i>I</i>	Moment of inertia, mm ⁴ (in ⁴)
<i>ID</i>	Inside diameter, mm (in)
<i>k</i>	Spring rate, N/mm (lb/in) or N·mm/r (lb·in/r)

[†] The symbols presented here are used extensively in the spring industry. They may differ from those used elsewhere in this Handbook.

K	Design constant
K_w	Stress correction factor for helical springs
L	Length, mm (in)
L_f	Free length, mm (in)
L_s	Length at solid, mm (in)
M	Moment or torque, N·mm (lb·in)
n	Frequency, Hz
N_a	Number of active coils or waves
N_t	Total number of coils
OD	Outside diameter, mm (in)
P	Load, N (lbf)
r	Radius, mm (in)
S	Stress, MPa (psi)
TS	Tensile strength, MPa (psi)
t	Thickness, mm (in)
YS	Yield strength, MPa (psi)
ρ	Density, g/cm ³ (lb/in ³)
θ	Angular deflection, expressed in number of revolutions
μ	Poisson's ratio

6.1 INTRODUCTION

Spring designing is a complex process. It is an interactive process which may require several iterations before the best design is achieved. Many simplifying assumptions have been made in the design equations, and yet they have proved reliable over the years. When more unusual or complex designs are required, designers should rely on the experience of a spring manufacturer.

The information in this chapter is offered for its theoretical value and should be used accordingly.

6.2 GLOSSARY OF SPRING TERMINOLOGY

active coils: those coils which are free to deflect under load.

baking: heating of electroplated springs to relieve hydrogen embrittlement.

buckling: bowing or lateral displacement of a compression spring; this effect is related to slenderness ratio L/D .

closed and ground ends: same as *closed ends*, except that the first and last coils are ground to provide a flat bearing surface.

closed ends: compression spring ends with coil pitch angle reduced so that they are square with the spring axis and touch the adjacent coils.

close-wound: wound so that adjacent coils are touching.

deflection: motion imparted to a spring by application or removal of an external load.

elastic limit: maximum stress to which a material may be subjected without permanent set.

endurance limit: maximum stress, at a given stress ratio, at which material will operate in a given environment for a stated number of cycles without failure.

free angle: angular relationship between arms of a helical torsion spring which is not under load.

free length: overall length of a spring which is not under load.

gradient: see *rate*.

heat setting: a process to prerelax a spring in order to improve stress-relaxation resistance in service.

helical springs: springs made of bar stock or wire coiled into a helical form; this category includes compression, extension, and torsion springs.

hooks: open loops or ends of extension springs.

hysteresis: mechanical energy loss occurring during loading and unloading of a spring within the elastic range. It is illustrated by the area between load-deflection curves.

initial tension: a force that tends to keep coils of a close-wound extension spring closed and which must be overcome before the coils start to open.

loops: formed ends with minimal gaps at the ends of extension springs.

mean diameter: in a helical spring, the outside diameter minus one wire diameter.

modulus in shear or torsion (modulus of rigidity G): coefficient of stiffness used for compression and extension springs.

modulus in tension or bending (Young's modulus E): coefficient of stiffness used for torsion or flat springs.

moment: a product of the distance from the spring axis to the point of load application and the force component normal to the distance line.

natural frequency: lowest inherent rate of free vibration of a spring vibrating between its own ends.

pitch: distance from center to center of wire in adjacent coils in an open-wound spring.

plain ends: end coils of a helical spring having a constant pitch and with the ends not squared.

plain ends, ground: same as *plain ends*, except that wire ends are ground square with the axis.

rate: spring gradient, or change in load per unit of deflection.

residual stress: stress mechanically induced by such means as set removal, shot peening, cold working, or forming; it may be beneficial or not, depending on the spring application.

set: permanent change of length, height, or position after a spring is stressed beyond material's elastic limit.

set point: stress at which some arbitrarily chosen amount of set (usually 2 percent) occurs; set percentage is the set divided by the deflection which produced it.

set removal: an operation which causes a permanent loss of length or height because of spring deflection.

solid height: length of a compression spring when deflected under load sufficient to bring all adjacent coils into contact.

spiral springs: springs formed from flat strip or wire wound in the form of a spiral, loaded by torque about an axis normal to the plane of the spiral.

spring index: ratio of mean diameter to wire diameter.

squared and ground ends: see *closed and ground ends*.

squared ends: see *closed ends*.

squareness: angular deviation between the axis of a compression spring in a free state and a line normal to the end planes.

stress range: difference in operating stresses at minimum and maximum loads.

stress ratio: minimum stress divided by maximum stress.

stress relief: a low-temperature heat treatment given springs to relieve residual stresses produced by prior cold forming.

torque: see *moment*.

total number of coils: the sum of the number of active and inactive coils in a spring body.

6.3 SELECTION OF SPRING MATERIALS

6.3.1 Chemical and Physical Characteristics

Springs are resilient structures designed to undergo large deflections within their elastic range. It follows that the materials used in springs must have an extensive elastic range.

Some materials are well known as spring materials. Although they are not specifically designed alloys, they do have the elastic range required. In steels, the medium- and high-carbon grades are suitable for springs. Beryllium copper and phosphor bronze are used when a copper-base alloy is required. The high-nickel alloys are used when high strength must be maintained in an elevated-temperature environment.

The selection of material is always a cost-benefit decision. Some factors to be considered are costs, availability, formability, fatigue strength, corrosion resistance, stress relaxation, and electric conductivity. The right selection is usually a compromise among these factors. Table 6.1 lists some of the more commonly used metal alloys and includes data which are useful in material selection.

Surface quality has a major influence on fatigue strength. This surface quality is a function of the control of the material manufacturing process. Materials with high surface integrity cost more than commercial grades but must be used for fatigue applications, particularly in the high cycle region.

6.3.2 Heat Treatment of Springs

Heat treatment is a term used in the spring industry to describe both low- and high-temperature heat treatments. Low-temperature heat treatment, from 350 to 950°F (175 to 510°C), is applied to springs after forming to reduce unfavorable residual stresses and to stabilize parts dimensionally.

When steel materials are worked in the spring manufacturing process, the yield point is lowered by the unfavorable residual stresses. A low-temperature heat treatment restores the yield point. Most heat treatment is done in air, and the minor oxide that is formed does not impair the performance of the springs.

When hardened high-carbon-steel parts are electroplated, a phenomenon known as *hydrogen embrittlement* occurs, in which hydrogen atoms diffuse into the metallic lattice, causing previously sound material to crack under sustained stress. Low-temperature baking in the range of 375 to 450°F (190 to 230°C) for times ranging from 0.5 to 3 h, depending on the type of plating and the degree of embrittlement, will reduce the concentration of hydrogen to acceptable levels.

High-temperature heat treatments are used to strengthen annealed material after spring forming. High-carbon steels are austenitized at 1480 to 1652°F (760 to 900°C), quenched to form martensite, and then tempered to final hardness. Some nickel-base alloys are strengthened by high-temperature aging. Oxidation will occur at these temperatures, and it is advisable to use a protective atmosphere in the furnace.

Heat treatments for many common materials are listed in Table 6.2. Unless otherwise noted, 20 to 30 min at the specified temperature is sufficient. Thin, flimsy cross-sectional springs can be distorted by the heat-treatment operation. Pretempered materials are available for use in such cases.

6.3.3 Relaxation

The primary concern in elevated-temperature applications is stress relaxation. *Stress relaxation* is the loss of load or spring length that occurs when a spring is held at load or cycled under load. Heat affects modulus and tensile strength. In addition to the factors of stress, time, and temperature which affect relaxation, other controllable factors are

1. Alloy type—the highly alloyed materials are generally more temperature-resistant.
2. Residual stresses—such stresses remaining from forming operations are detrimental to relaxation resistance. Use the highest practical stress-relief temperature.
3. Heat setting—procedures employed to expose springs under some load to stress and heat to prepare them for a subsequent exposure. The effect is to remove the first stage of relaxation.

6.3.4 Corrosion

The specific effect of a corrosive environment on spring performance is difficult to predict. In general, if the environment causes damage to the spring surface, the life and the load-carrying ability of the spring will be reduced.

The most common methods of combating corrosion are to use materials that are resistant or inert to the particular corrosive environment or to use coatings that slow

TABLE 6.1 Typical Properties of Common Spring Materials

Common Name	Young's Modulus E (1)		Modulus of Rigidity G (1)		Density (1) g/cm ³ (lb/in. ³)	Electrical Conductivity (1) % IACS	Sizes Normally Available (2)		Typical Surface Quality (3)	Maximum Service Temperature (4) °C °F
	MPa 10 ³	psi 10 ⁶	MPa 10 ³	psi 10 ⁶			Min. mm (in.)	Max. mm (in.)		
Carbon Steel Wires: Music (5) Hard Drawn (5) Oil Tempered Valve Spring	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.10 (0.004)	6.35 (0.250)	a	120 250
	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.13 (0.005)	16 (0.625)	c	150 250
	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.50 (0.020)	16 (0.625)	c	150 300
	207	(30)	79.3	(11.5)	7.86 (0.284)	7	1.3 (0.050)	6.35 (0.250)	a	150 300
Alloy Steel Wires: Chrome Vanadium Chrome Silicon	207	(30)	79.3	(11.5)	7.86 (0.284)	7	0.50 (0.020)	11 (0.435)	a,b	220 425
	207	(30)	79.3	(11.5)	7.86 (0.284)	5	0.50 (0.020)	9.5 (0.375)	a,b	245 475
Stainless Steel Wires: Austenitic Type 302 Precipitation Hardening 17-7 PH NiCr A286	193	(28)	69.0	(10.)	7.92 (0.286)	2	0.13 (0.005)	9.5 (0.375)	b	260 500
	203	(29.5)	75.8	(11)	7.81 (0.282)	2	0.08 (0.002)	12.5 (0.500)	b	315 600
	200	(29)	71.7	(10.4)	8.03 (0.290)	2	0.40 (0.016)	5 (0.200)	b	510 950
	200	(29)	71.7	(10.4)	8.03 (0.290)	2	0.40 (0.016)	5 (0.200)	b	510 950
Copper Base Alloy Wires: Phosphor Bronze (A) Silicon Bronze (A) Silicon Bronze (B) Beryllium Copper Spring Brass, CA260	103	(15)	43.4	(6.3)	8.86 (0.320)	15	0.10 (0.004)	12.5 (0.500)	b	95 200
	103	(15)	38.6	(5.6)	8.53 (0.308)	7	0.10 (0.004)	12.5 (0.500)	b	95 200
	117	(17)	44.1	(6.4)	8.75 (0.316)	12	0.10 (0.004)	12.5 (0.500)	b	95 200
	128	(18.5)	48.3	(7.0)	8.26 (0.298)	21	0.08 (0.003)	12.5 (0.500)	b	205 400
	110	(16)	42.0	(6.0)	8.53 (0.308)	17	0.10 (0.004)	12.5 (0.500)	b	95 200
	110	(16)	42.0	(6.0)	8.53 (0.308)	17	0.10 (0.004)	12.5 (0.500)	b	95 200
Nickel Base Alloys: Inconel® Alloy 600 Inconel Alloy X750 Ni-Span-C® Monel® Alloy 400 Monel Alloy K500	214	(31)	75.8	(11)	8.43 (0.304)	1.5	0.10 (0.004)	12.5 (0.500)	b	320 700
	214	(31)	79.3	(11.5)	8.25 (0.298)	1	0.10 (0.004)	12.5 (0.500)	b	595 1100
	186	(27)	62.9	(9.7)	8.14 (0.294)	1.6	0.10 (0.004)	12.5 (0.500)	b	95 200
	179	(26)	66.2	(9.6)	8.83 (0.319)	3.5	0.05 (0.002)	9.5 (0.375)	b	230 450
	179	(26)	66.2	(9.6)	8.46 (0.306)	3	0.05 (0.002)	9.5 (0.375)	b	260 500
	179	(26)	66.2	(9.6)	8.46 (0.306)	3	0.05 (0.002)	9.5 (0.375)	b	260 500

Carbon Steel Strip: AISI 1050 1065 1074, 1075 1095	207	(30)	79.3 (11.5)	7.86 (0.284)	7	0.25 (0.010) 0.08 (0.003)	3 (0.125)	b	95	200
	207	(30)	79.3 (11.5)	7.86 (0.284)	7	0.08 (0.003)	3 (0.125)	b	95	200
	207	(30)	79.3 (11.5)	7.86 (0.284)	7	0.08 (0.003)	3 (0.125)	b	120	250
	207	(30)	79.3 (11.5)	7.86 (0.284)	7	0.08 (0.003)	3 (0.125)	b	120	250
Stainless Steel Strip: Austenitic Types 301, 302 Precipitation Hardening 17-7 PH	193	(28)	69.0 (10)	7.92 (0.286)	2	0.08 (0.003)	1.5 (0.063)	b	315	600
	203	(29.5)	75.8 (11)	7.81 (0.282)	2	0.08 (0.003)	3 (0.125)	b	370	700
Copper Base Alloy Strip: Phosphor Bronze (A) Beryllium Copper	103	(15)	43 (6.3)	8.86 (0.320)	15	0.08 (0.003)	5 (0.188)	b	95	200
	128	(18.5)	48 (7.0)	8.26 (0.298)	21	0.08 (0.003)	9.5 (0.375)	b	205	400

(1) Elastic moduli, density and electrical conductivity can vary with cold work, heat treatment and operating stress. These variations are usually minor but should be considered if one or more of these properties is critical.

(2) Sizes normally available are diameters for wire; thicknesses for strip.

(3) Typical surface quality ratings. (For most materials, special processes can be specified to upgrade typical values.)

a. Maximum defect depth: 0 to 0.5% of d or t.

SOURCE: Associated Spring, Barnes Group Inc.

b. Maximum defect depth: 1.0% of d or t.

c. Defect depth: less than 3.5% of d or t.

(4) Maximum service temperatures are guidelines and may vary due to operating stress and allowable relaxation.

(5) Music and hard drawn are commercial terms for patented and cold-drawn carbon steel spring wire.

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TABLE 6.2 Typical Heat Treatments for Springs after Forming

Materials	Heat Treatment	
	°C	°F
Patented and Cold-Drawn Steel Wire	190–230	375–450
Tempered Steel Wire:		
Carbon	260–400	500–750
Alloy	315–425	600–800
Austenitic Stainless Steel Wire	230–510	450–950
Precipitation Hardening Stainless Wire (17–7 PH):		
Condition C	480/1 hour	900/1 hour
Condition A to TH 1050	760/1 hour cool to 15°C followed by 565/1 hour	1400/1 hour, cool to 60°F followed by 1050/1 hour
Monel:		
Alloy 400	300–315	575–600
Alloy K500, Spring Temper	525/4 hours	980/4 hours
Inconel:		
Alloy 600	400–510	750–950
Alloy X-750:		
#1 Temper	730/16 hours	1350/16 hours
Spring Temper	650/4 hours	1200/4 hours
Copper Base, Cold Worked (Brass, Phosphor Bronze, etc.)	175–205	350–400
Beryllium Copper:		
Pretempered (Mill Hardened)	205	400
Solution Annealed, Temper Rolled or Drawn	315/2–3 hours	600/2–3 hours
Annealed Steels:		
Carbon (AISI 1050 to 1095)	800–830*	1475–1525*
Alloy (AISI 5160H 6150, 9254)	830–885*	1525–1625*

*Time depends on heating equipment and section size. Parts are austenitized then quenched and tempered to the desired hardness.

SOURCE: Associated Spring, Barnes Group Inc.

down the rate of corrosion attack on the base metal. The latter approach is most often the most cost-effective method.

Spring Wire. The tensile strength of spring wire varies inversely with the wire diameter (Fig. 6.1).

Common spring wires with the highest strengths are ASTM A228 (music wire) and ASTM A401 (oil-tempered chrome silicon). Wires having slightly lower tensile strength and with surface quality suitable for fatigue applications are ASTM A313 type 302 (stainless steel), ASTM A230 (oil-tempered carbon valve-spring-quality steel), and ASTM A232 (oil-tempered chrome vanadium). For most static applica-

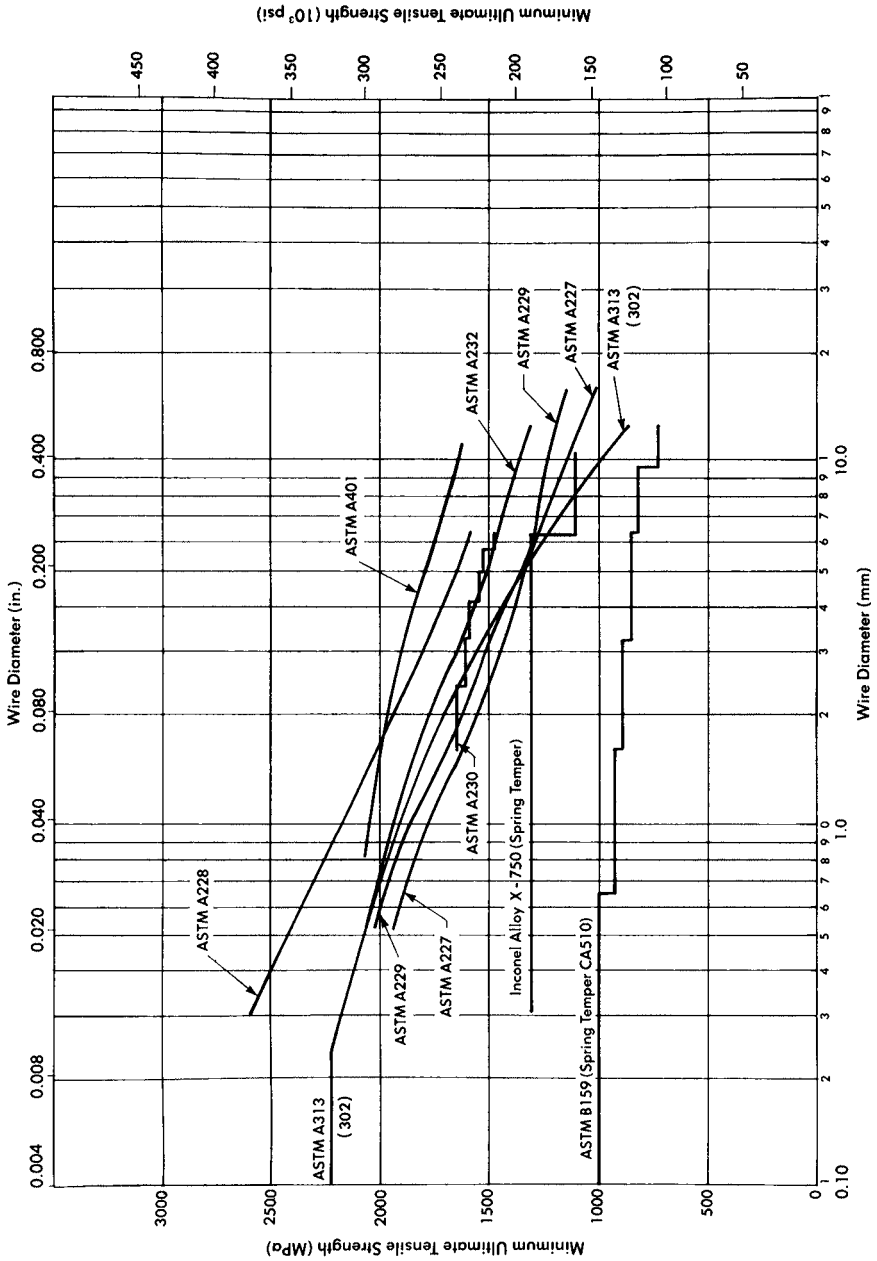


FIGURE 6.1 Minimum tensile strengths of spring wire. (Associated Spring, Barnes Group Inc.)

tions ASTM A227 (hard-drawn carbon steel) and ASTM A229 (oil-tempered carbon steel) are available at lower strength levels. Table 6.3 ranks the relative costs of common spring materials based on hard-drawn carbon steel as 1.0.

Spring Strip. Most “flat” springs are made from AISI grades 1050, 1065, 1074, and 1095 steel strip. Strength and formability characteristics are shown in Fig. 6.2, covering the range of carbon content from 1050 to 1095. Since all carbon levels can be obtained at all strength levels, the curves are not identified by composition. Figure 6.3 shows the tensile strength versus Rockwell hardness for tempered carbon-steel strip. Edge configurations for steel strip are shown in Fig. 6.4.

Formability of annealed spring steels is shown in Table 6.4, and typical properties of various spring-tempered alloy strip materials are shown in Table 6.5.

6.4 HELICAL COMPRESSION SPRINGS

6.4.1 General

A helical compression spring is an open-pitch spring which is used to resist applied compression forces or to store energy. It can be made in a variety of configurations and from different shapes of wire, depending on the application. Round, high-carbon-steel wire is the most common spring material, but other shapes and compositions may be required by space and environmental conditions.

Usually the spring has a uniform coil diameter for its entire length. Conical, barrel, and hourglass shapes are a few of the special shapes used to meet particular load-deflection requirements.

TABLE 6.3 Ranking of Relative Costs of Common Spring Wires

Wire	Specification	Relative Cost of 2 mm (0.079") Dia.	
		Mill Quantities	Ware- House Lots
Patented and Cold Drawn Oil Tempered	ASTM A227	1.0	1.0
	ASTM A229	1.3	1.3
Music Carbon Valve Spring	ASTM A228	2.6	1.4
	ASTM A230	3.1	1.9
Chrome Silicon Valve Stainless Steel (Type 302)	ASTM A401	4.0	3.9
	ASTM A313 (302)	7.6	4.7
Phosphor Bronze Stainless Steel (Type 631) (17-7 PH)	ASTM	8.0	6.7
	ASTM A 313 (631)	11	8.7
Beryllium Copper Inconel Alloy X-750	ASTM B197	27	17
		44	31

SOURCE: Associated Spring, Barnes Group Inc.

Helical compression springs are stressed in the torsional mode. The stresses, in the elastic range, are not uniform about the wire's cross section. The stress is greatest at the surface of the wire and, in particular, at the inside diameter (ID) of the spring.

In some circumstances, residual bending stresses are present as well. In such cases, the bending stresses become negligible after set is removed (or the elastic limit is exceeded) and the stresses are redistributed more uniformly about the cross section.

6.4.2 Compression Spring Terminology

The definitions that follow are for terms which have evolved and are commonly used in the spring industry. Figure 6.5 shows the relationships among the characteristics.

Wire Diameter d . Round wire is the most economical form. Rectangular wire is used in situations where space is limited, usually to reduce solid height.

Coil Diameter. The outside diameter (OD) is specified when a spring operates in a cavity. The inside diameter is specified when the spring is to operate over a rod. The mean diameter D is either OD minus the wire size or ID plus the wire size.

The coil diameter increases when a spring is compressed. The increase, though small, must be considered whenever clearances could be a problem. The diameter increase is a function of the spring pitch and follows the equation

$$OD_{\text{at solid}} = \sqrt{D^2 + \frac{p^2 - d^2}{\pi^2}} + d \quad (6.1)$$

where p = pitch and d = wire size.

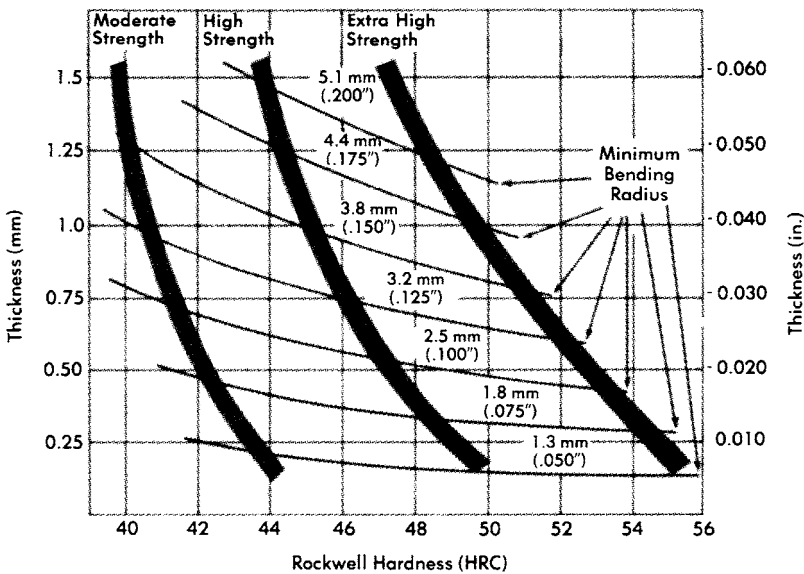


FIGURE 6.2 Minimum transverse bending radii for various tempers and thicknesses of tempered spring steel. (Associated Spring, Barnes Group Inc.)

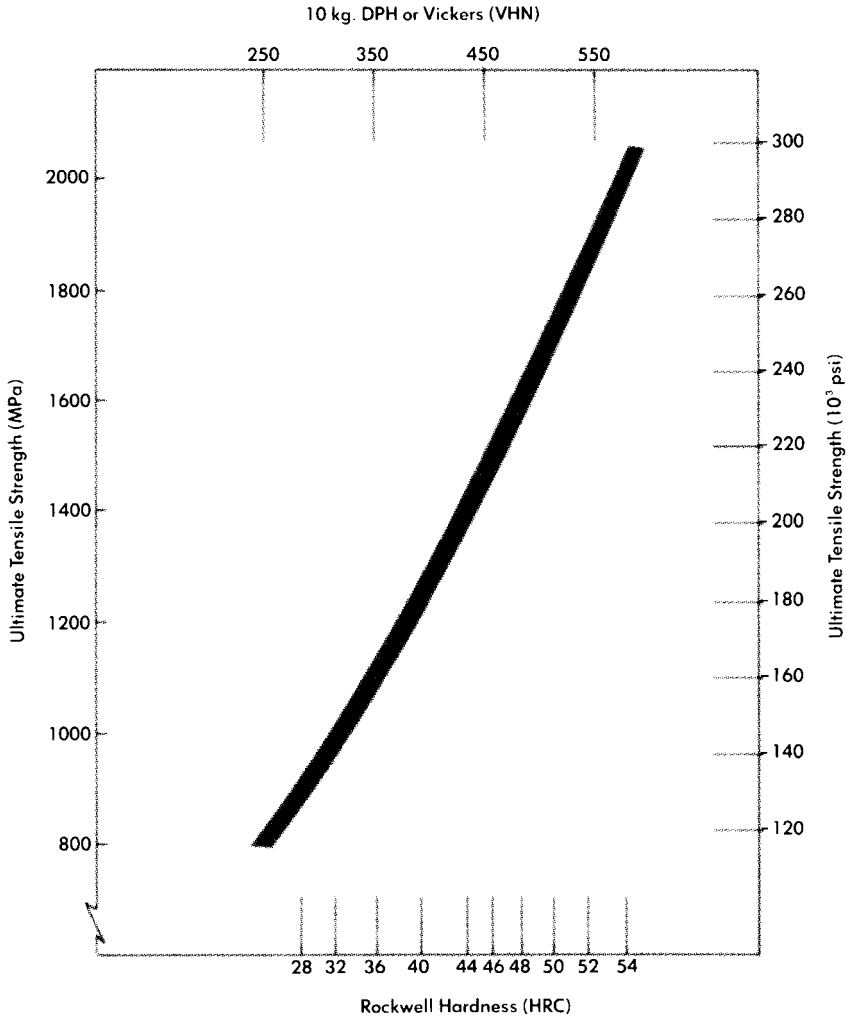


FIGURE 6.3 Tensile strength versus hardness of quenched and tempered spring steel. (*Associated Spring, Barnes Group Inc.*)

Spring Index. Spring index C is the ratio of the mean diameter to the wire diameter (or to the radial dimension if the wire is rectangular). The preferred range of index is 5 to 9, but ranges as low as 3 and as high as 15 are commercially feasible. The very low indices are hard to produce and require special setup techniques. High indices are difficult to control and can lead to spring tangling.

Free Length. Free length L_f is the overall length measured parallel to the axis when the spring is in a free, or unloaded, state. If loads are not given, the free length should be specified. If they are given, then free length should be a reference dimension which can be varied to meet the load requirements.

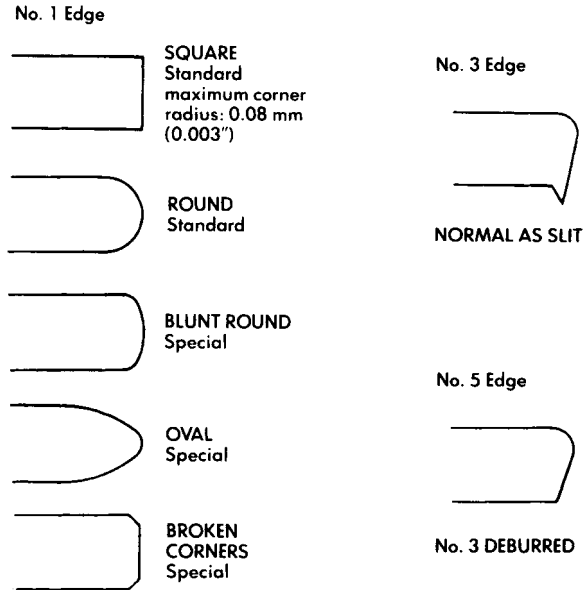


FIGURE 6.4 Edges available on steel strip. (*Associated Spring, Barnes Group Inc.*)

Types of Ends. Four basic types of ends are used: closed (squared) ends, closed (squared) ends ground, plain ends, and plain ends ground. Figure 6.6 illustrates the various end conditions. Closed and ground springs are normally supplied with a ground bearing surface of 270 to 330°.

Number of Coils. The number of coils is defined by either the total number of coils N_t or the number of active coils N_a . The difference between N_t and N_a equals the number of inactive coils, which are those end coils that do not deflect during service.

Solid Height. The solid height L_s is the length of the spring when it is loaded with enough force to close all the coils. For ground springs, $L_s = N_t d$. For unground springs, $L_s = (N_t + 1)d$.

Direction of the Helix. Springs can be made with the helix direction either right or left hand. Figure 6.7 illustrates how to define the direction. Springs that are nested one inside the other should have opposite helix directions. If a spring is to be assembled onto a screw thread, the direction of the helix must be opposite to that of the thread.

Spring Rate. Spring rate k is the change in load per unit deflection. It is expressed as

$$k = \frac{P}{f} = \frac{Gd^4}{8D^3N_a} \quad (6.2)$$

where G = shear modulus.

TABLE 6.4 Formability of Annealed Spring Steels

Thickness (t) mm (in.)	Direction of Bend	AISI 1050 N_t/t		AISI 1065 N_t/t		AISI 1074 N_t/t	AISI 1095 N_t/t
		Annealed (standard lowest max.)	WBS* Barco- Form®	Annealed (standard lowest max.)	WBS Barco- Form	Annealed (standard lowest max.)	Annealed (standard lowest max.)
1.9 mm (0.076)-over	⊥	2	0	2	0	2	3
		4	3	4	3	4	5
0.9-1.89 mm (0.036-0.075")	⊥	1	0	1	0	1	2
		2	1	2	1	2	3
0.37-0.89 mm (0.015-0.035")	⊥	0	0	0	0	1	1
		1	0	1 1/2	1	1 1/2	2
0.2-0.36 mm (0.008-0.014")	⊥	0	0	0	0	1	1
		0	0	0	0	1	1

Formability is determined by slowly bending a sample over 180° until its ends are parallel. The measured distance between the ends is N_t . For example, if $N_t = 4$ and $t = 2$, then $N_t/t = 2$.

*Wallace Barnes Steel.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.5 Typical Properties of Spring-Tempered Alloy Strip

Material	Tensile Strength MPa (10 ³ psi)	Rockwell Hardness	Elongation(1) Percent	Bend Factor (1) (2r/t trans. bends)	Modulus of Elasticity 10 ⁴ MPa (10 ⁶ psi)	Poisson's Ratio
Steel, spring temper	1700 (246)	C50	2	5	20.7 (30)	0.30
Stainless 301	1300 (189)	C40	8	3	19.3 (28)	0.31
Stainless 302	1300 (189)	C40	5	4	19.3 (28)	0.31
Monel 400	690 (100)	B95	2	5	17.9 (26)	0.32
Monel K500	1200 (174)	C34	40	5	17.9 (26)	0.29
Inconel 600	1040 (151)	C30	2	2	21.4 (31)	0.29
Inconel X-750	1050 (152)	C35	20	3	21.4 (31)	0.29
Copper-Beryllium	1300 (189)	C40	2	5	12.8 (18.5)	0.33
Ni-Span-C	1400 (203)	C42	6	2	18.6 (27)	—
Brass CA 260	620 (90)	B90	3	3	11 (16)	0.33
Phosphor Bronze	690 (100)	B90	3	2.5	10.3 (15)	0.20
17-7 PH RH950	1450 (210)	C44	6	flat	20.3 (29.5)	0.34
17-7 PH Condition C	1650 (239)	C46	1	2.5	20.3 (29.5)	0.34

(1) Before heat treatment.

SOURCE: Associated Spring, Barnes Group Inc.

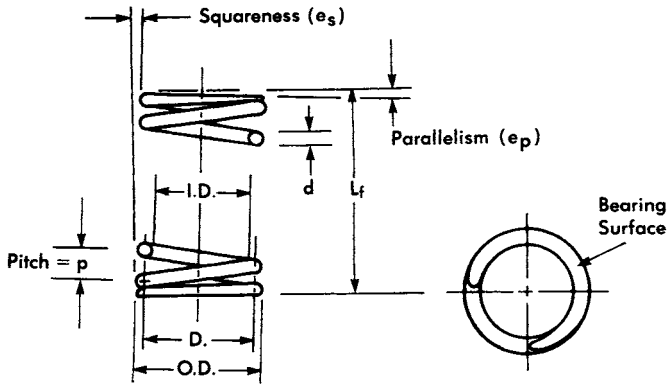


FIGURE 6.5 Dimensional terminology for helical compression springs. (*Associated Spring, Barnes Group Inc.*)

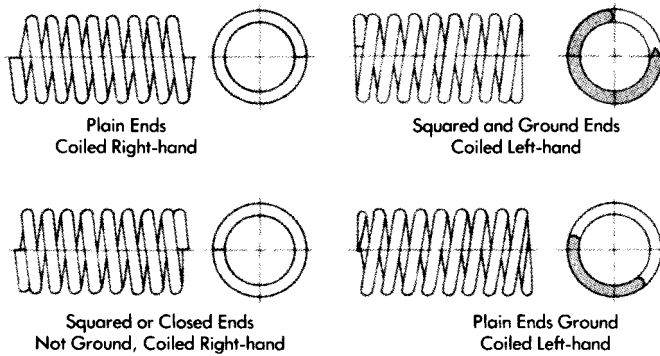


FIGURE 6.6 Types of ends for helical compression springs. (*Associated Spring, Barnes Group Inc.*)

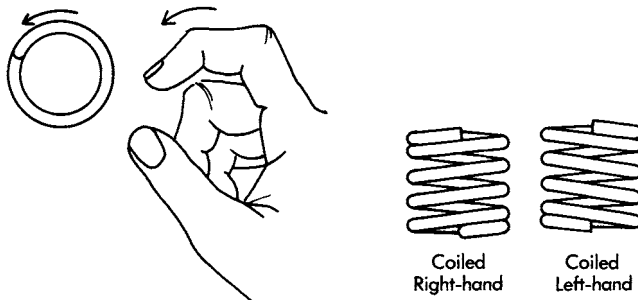


FIGURE 6.7 Direction of coiling of helical compression springs. (*Associated Spring, Barnes Group Inc.*)

The rate equation is accurate for a deflection range between 15 and 85 percent of the maximum available deflection. When compression springs are loaded in parallel, the combined rate of all the springs is the sum of the individual rates. When the springs are loaded in series, the combined rate is

$$k = \frac{1}{1/k_1 + 1/k_2 + 1/k_3 + \dots + 1/k_n} \quad (6.3)$$

This relationship can be used to design a spring with variable diameters. The design method is to divide the spring into many small increments and calculate the rate for each increment. The rate for the whole spring is calculated as in Eq. (6.3).

Stress. Torsional stress S is expressed as

$$S = \frac{8K_w PD}{\pi d^3} \quad (6.4)$$

Under elastic conditions, torsional stress is not uniform around the wire's cross section because of the coil curvature and direct shear loading.

The highest stress occurs at the surface in the inside diameter of the spring, and it is computed by using the stress factor K_w . In most cases, the correction factor is expressed as

$$K_{w_1} = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad (6.5)$$

The stress-concentration factor K_{w_1} becomes K_{w_2} after a spring has been set out because stresses become more uniformly distributed after subjecting the cross section to plastic flow during set-out:

$$K_{w_2} = 1 + \frac{0.5}{C} \quad (6.6)$$

The appropriate stress correction factor is discussed in Sec. 6.4.3.

Loads. If deflection is known, the load is found by multiplying deflection by the spring rate. When the stress is either known or assumed, loads can be obtained from the stress equation.

Loads should be specified at a test height so that the spring manufacturer can control variations by adjustments of the free length. The load-deflection curve is not usually linear at the start of deflection from free position or when the load is very close to solid height. It is advisable to specify loads at test heights between 15 and 85 percent of the load-deflection range.

Loads can be conveniently classified as static, cyclic, and dynamic. In static loading, the spring will operate between specified loads only a few times. In other instances, the spring may remain under load for a long time. In cyclic applications, the spring may typically be required to cycle between load points from 10^4 to more than 10^9 times. During dynamic loading, the rate of load application is high and causes a surge wave in the spring which usually induces stresses higher than calculated from the standard stress equation.

Buckling. Compression springs with a free length more than 4 times the mean coil diameter may buckle when compressed. Guiding the spring, either in a tube or over

a rod, can minimize the buckling but can result in additional friction which will affect loads, especially when the L_f/D ratio is high.

Buckling conditions are shown in Figs. 6.8 and 6.9 for springs loaded axially and with squared and ground ends. Buckling occurs at points above and to the right of the curves. Curve *A* is for the springs with one end on a fixed, flat surface and the other end free to tip. Curve *B* is for springs with both ends on fixed, flat surfaces. The tendency to buckle is clearly less for curve *B* springs.

6.4.3 Choice of Operating Stress

The choice of operating stress depends on whether the application is static or cyclic. For static applications, yield strength or stress-relaxation resistance of the material limits the load-carrying ability of the springs. The required cycles are few, if any, and the velocity of the end coils is so low as to preclude surging or impact conditions.

The maximum allowable torsional stresses for static applications are shown in Table 6.6 as percentages of tensile strengths for common spring materials. To calculate the stress before set removal, use the K_{w1} correction factor. If the calculated stress is greater than the indicated percentage of the tensile strength, then the spring will take a permanent set when deflected to solid. The amount of set is a function of the amount by which the calculated stress exceeds the tabular percentage.

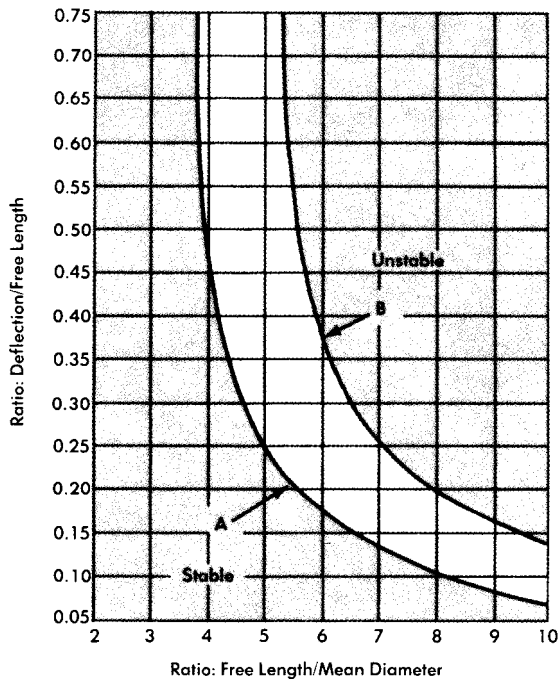


FIGURE 6.8 Critical buckling curves. (*Associated Spring, Barnes Group Inc.*)

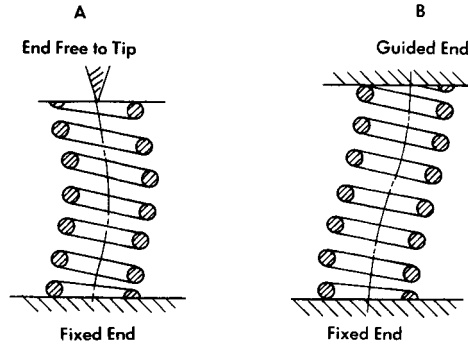


FIGURE 6.9 End conditions used to determine critical buckling. (Associated Spring, Barnes Group Inc.)

It is common practice, in static applications, to increase the load-carrying capability of a spring by making it longer than the desired free length and then compressing it to solid. The spring *sets* to its final desired length. This procedure is called *removing set*. It induces favorable residual stresses which allow for significantly higher stresses than in springs not having the set removed. The loss of the length should be at least 10 percent to be effective (see Fig. 6.10).

Note that set removal causes stresses to be more uniformly distributed about the cross section. Therefore, stress after set removal is calculated by using the K_{w_2} correction factor. If the stress calculated by using the K_{w_2} correction factor exceeds the percentage of tensile strength shown in Table 6.6, the spring cannot be made. It is then necessary either to lower the design stress or to select a higher-strength material.

For cyclic applications, the load-carrying ability of the spring is limited by the fatigue strength of the material. To select the optimum stress level, spring costs must be balanced against reliability. The designer should know the operating environ-

TABLE 6.6 Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Materials	Maximum % of Tensile Strength	
	Before Set Removed (K_{w_1})	After Set Removed (K_{w_2})
Patented and cold drawn carbon steel	45%	65–75%
Hardened and tempered carbon and low alloy steel	50%	
Austenitic stainless steels	35%	
Nonferrous alloys	35%	

SOURCE: Associated Spring, Barnes Group Inc.

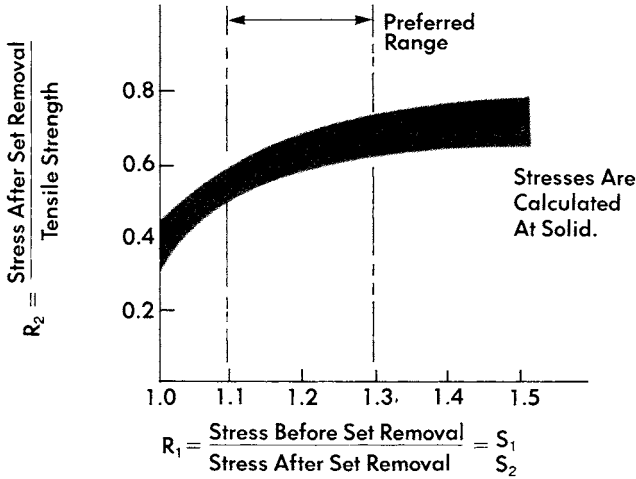


FIGURE 6.10 Spring load-carrying ability versus amount of set removed. (*Associated Spring, Barnes Group Inc.*)

ment, desired life, stress range, frequency of operation, speed of operation, and permissible levels of stress relaxation in order to make a cost-reliability decision.

Fatigue life can be severely reduced by pits, seams, or tool marks on the wire surface where stress is at a maximum. Shot peening improves fatigue life, in part, by minimizing the harmful effects of surface defects. It does not remove them. Additionally, shot peening imparts favorable compression stresses to the surface of the spring wire.

Maximum allowable stresses for fatigue applications should be calculated by using the K_{w1} stress correction factor. Table 6.7 shows the estimated fatigue life for common spring materials. Note the significant increase in fatigue strength from shot peening.

TABLE 6.7 Maximum Allowable Torsional Stress for Round-Wire Helical Compression Springs in Cyclic Applications

Fatigue Life (cycles)	Percent of Tensile Strength			
	ASTM A228, Austenitic Stainless Steel and Nonferrous		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened	Not Shot-Peened	Shot-Peened
10^5	36	42	42	49
10^6	33	39	40	47
10^7	30	36	38	46

This information is based on the following conditions: no surging, room temperature and noncorrosive environment.

Stress ratio in fatigue = $\frac{S_{\text{minimum}}}{S_{\text{maximum}}} = 0$

SOURCE: Associated Spring, Barnes Group Inc.

The fatigue life estimates in Table 6.7 are guideline values which should be used only where specific data are unavailable. The values are conservative, and most springs designed using them will exceed the anticipated lives.

6.4.4 Dynamic Loading under Impact

When a spring is loaded or unloaded, a surge wave is established which transmits torsional stress from the point of load along the spring's length to the point of restraint. The surge wave will travel at a velocity approximately one-tenth that of a normal, torsional-stress wave. The velocity of the torsional-stress wave V_T , in meters per second (m/s) [inches per second (in/s)], is given by

$$V_T = 10.1 \sqrt{\frac{Gg}{\rho}} \quad \text{m/s} \quad \text{or} \quad V_T = \sqrt{\frac{Gg}{\rho}} \quad \text{in/s} \quad (6.7)$$

The velocity of the surge wave V_s varies with material and design but is usually in the range of 50 to 500 m/s. The surge wave limits the rate at which a spring can absorb or release energy by limiting the impact velocity V . *Impact velocity* is defined as the spring velocity parallel to the spring axis and is a function of stress and material as shown:

$$V \approx 10.1S \sqrt{\frac{g}{2\rho G}} \quad \text{m/s} \quad \text{or} \quad V \approx S \sqrt{\frac{g}{2\rho G}} \quad \text{in/s} \quad (6.8)$$

For steel, this reduces to

$$V = \frac{S}{35.5} \quad \text{m/s} \quad \text{or} \quad V = \frac{S}{131} \quad \text{in/s} \quad (6.9)$$

If a spring is compressed to a given stress level and released instantaneously, the maximum spring velocity is the stress divided by 35.5. Similarly, if the spring is loaded at known velocity, the instantaneous stress can be calculated. At very high load velocities, the instantaneous stress will exceed the stress calculated by the conventional equation. This will limit design performance. Since the surge wave travels the length of the spring, springs loaded at high velocity often are subject to resonance.

6.4.5 Dynamic Loading—Resonance

A spring experiences resonance when the frequency of cyclic loading is near the natural frequency or a multiple of it. Resonance can cause an individual coil to deflect to stress levels above those predicted by static stress analysis. Resonance can also cause the spring to bounce, resulting in loads lower than calculated. To avoid these effects, the natural frequency should be a minimum of 13 times the operating frequency.

For a compression spring with both ends fixed and no damper, the natural frequency in International System (SI) units is

$$n = \frac{1.12(10^3)d}{D^2 N_a} \sqrt{\frac{Gg}{\rho}} \quad (6.10)$$

For steel, this equation becomes

$$n = \frac{3.5(10^5)d}{D^2 N_a} \quad (6.11)$$

where n = frequency in hertz (Hz). The corresponding equation in U.S. Customary System (USCS) units is

$$n = \frac{d}{9D^2 N_a} \sqrt{\frac{Gg}{\rho}} \quad (6.12)$$

and for steel we have

$$n = \frac{14(10^3)d}{D^2 N_a} \quad (6.13)$$

If the spring cannot be designed to have a natural frequency more than 13 times the operating frequency, energy dampers may be employed. They are generally friction devices which rub against the coils. Often, variable-pitch springs are used to minimize resonance effects.

6.4.6 Rectangular-Wire Springs

In applications where high loads and relatively low stresses are required but solid height is also restricted, rectangular wire can be used to increase the material volume while maintaining the maximum solid-height limitation.

Springs made of rectangular wire with the long side of the wire cross section perpendicular to the axis of the coils can store more energy in a smaller space than an equivalent, round-wire spring.

When rectangular wire is coiled, it changes from a rectangular to a keystone shape, as shown in Fig. 6.11. Similarly, if the wire is made to the keystone shape, it will become rectangular after coiling. The cross-sectional distortion can be approximated by

$$t_1 = t \frac{C + 0.5}{C} \quad (6.14)$$

where t_1 = wider end of keystone section and t = original, smaller dimension of rectangle.

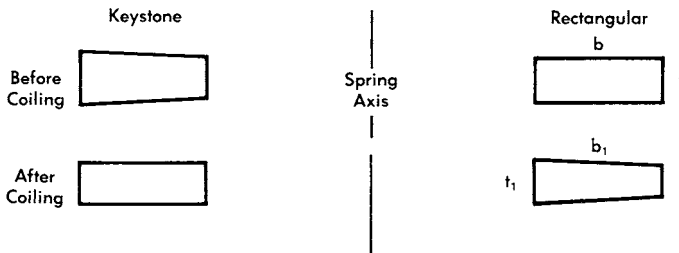


FIGURE 6.11 Wire cross section before and after coiling. (*Associated Spring, Barnes Group Inc.*)

The spring rate for a rectangular-wire spring is

$$k = \frac{P}{f} = \frac{K_2 G b t^3}{N_a D^3} \quad (6.15)$$

Since the wire is loaded in torsion, it makes no difference whether the wire is wound on the flat or on edge. See Fig. 6.12.

Stress is calculated by

$$S = \frac{K_E P D}{K_1 b t^2} \quad \text{or} \quad S = \frac{K_F P D}{K_1 b t^2} \quad (6.16)$$

Values for K_1 and K_2 are found in Fig. 6.13, and those for K_E and K_F are found in Figs. 6.15 and 6.14, respectively.

When a round wire cannot be used because the solid height exceeds the specification, the approximate equivalent rectangular dimensions are found from

$$t = \frac{2d}{1 + b/t} \quad (6.17)$$

where d = round-wire diameter.

6.4.7 Variable-Diameter Springs

Conical, hourglass, and barrel-shaped springs, shown in Fig. 6.16, are used in applications requiring a low solid height and an increased lateral stability or resistance to surging. Conical springs can be designed so that each coil nests wholly or partly within an adjacent coil. Solid height can be as low as one wire diameter. The rate for conical springs usually increases with deflection (see Fig. 6.17) because the number of active coils decreases progressively as the spring approaches solid. By varying the pitch, conical springs can be designed to have a uniform rate. The rate for conical springs is calculated by considering the spring as many springs in series. The rate for

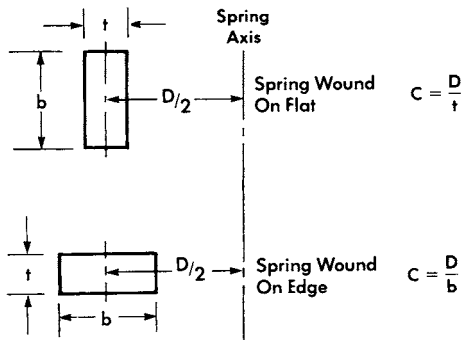


FIGURE 6.12 Rectangular-wire compression spring wound on flat or edge. (*Associated Spring, Barnes Group Inc.*)

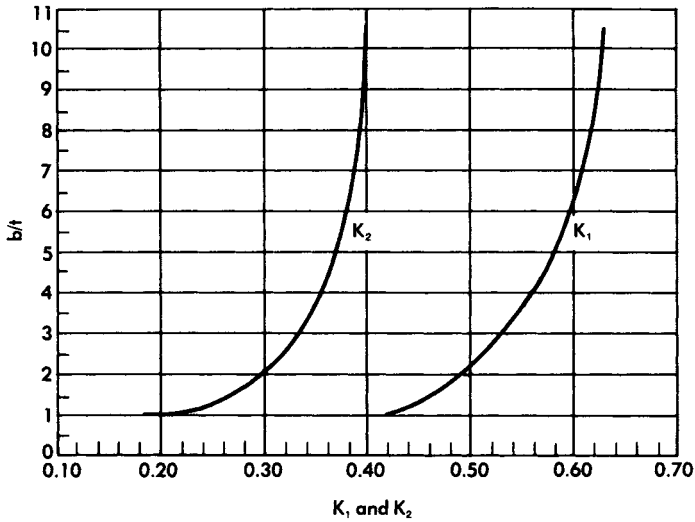


FIGURE 6.13 Constants for rectangular wire in torsion. (*Associated Spring, Barnes Group Inc.*)

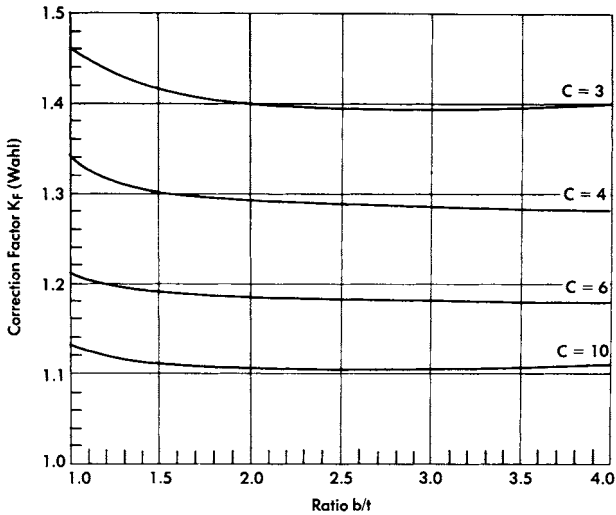


FIGURE 6.14 Stress correction factors for rectangular-wire compression springs wound on flat. (*Associated Spring, Barnes Group Inc.*)

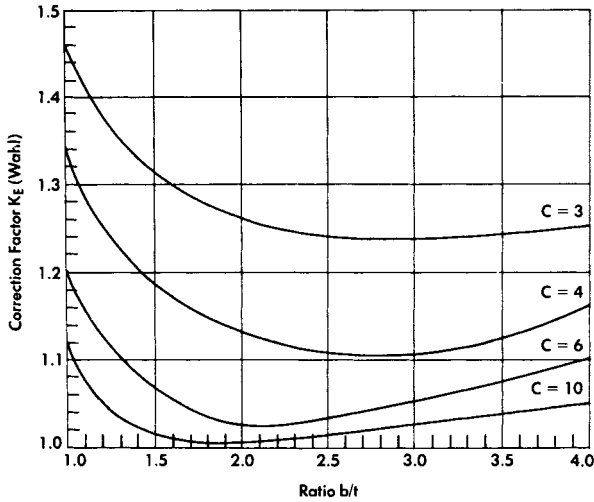


FIGURE 6.15 Stress correction factors for rectangular-wire compression springs wound on edge. (Associated Spring, Barnes Group Inc.)

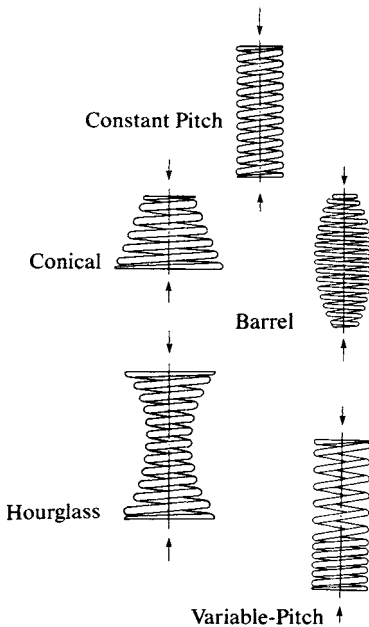


FIGURE 6.16 Various compression-spring body shapes. (Associated Spring, Barnes Group Inc.)

each turn or fraction of a turn is calculated by using the standard rate equation. The rate for a complete spring is then determined, given that the spring rate follows the series relationship in Eq. (6.3).

To calculate the highest stress at a given load, the mean diameter of the largest active coil at load is used. The solid height of a uniformly tapered, but not telescoping, spring with squared and ground ends made from round wire can be estimated from

$$L_s = N_a \sqrt{d^2 - u^2} + 2d \quad (6.18)$$

where u = OD of large end minus OD of small end, divided by $2N_a$.

Barrel- and hourglass-shaped springs are calculated as two conical springs in series.

6.4.8 Commercial Tolerances

Standard commercial tolerances are presented in Tables 6.8, 6.9, and 6.10 for free length, coil diameter, and load tolerances, respectively. These tolerances represent the best tradeoffs between manufacturing costs and performance.

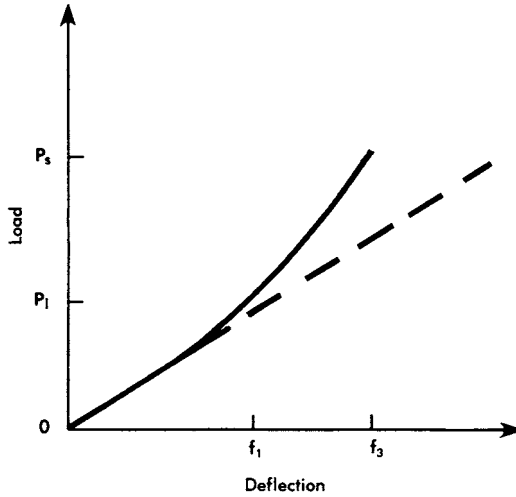


FIGURE 6.17 Typical load-deflection curve for variable-diameter springs (solid line). (*Associated Spring, Barnes Group Inc.*)

TABLE 6.8 Free-Length Tolerances of Squared and Ground Helical Compression Springs

Number of Active coils per mm(in.)	Tolerances: $\pm$ mm/mm (in./in.) of Free Length						
	Spring Index (D/d)						
	4	6	8	10	12	14	16
0.02 (0.5)	0.010	0.011	0.012	0.013	0.015	0.016	0.016
0.04 (1)	0.011	0.013	0.015	0.016	0.017	0.018	0.019
0.08 (2)	0.013	0.015	0.017	0.019	0.020	0.022	0.023
0.2 (4)	0.016	0.018	0.021	0.023	0.024	0.026	0.027
0.3 (8)	0.019	0.022	0.024	0.026	0.028	0.030	0.032
0.5 (12)	0.021	0.024	0.027	0.030	0.032	0.034	0.036
0.6 (16)	0.022	0.026	0.029	0.032	0.034	0.036	0.038
0.8 (20)	0.023	0.027	0.031	0.034	0.036	0.038	0.040

For springs less than 12.7 mm (0.500") long, use the tolerances for 12.7 mm (0.500"). For closed ends not ground, multiply above values by 1.7.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.9 Coil Diameter Tolerances of Helical Compression and Extension Springs

Wire Dia., mm(in.)	Tolerances: $\pm$ mm (in.)						
	Spring Index (D/d)						
	4	6	8	10	12	14	16
0.38 (0.015)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.13 (0.005)	0.15 (0.006)	0.18 (0.007)
0.58 (0.023)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.20 (0.008)	0.25 (0.010)
0.89 (0.035)	0.05 (0.002)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.23 (0.009)	0.28 (0.011)	0.33 (0.013)
1.30 (0.051)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.25 (0.010)	0.30 (0.012)	0.38 (0.015)	0.43 (0.017)
1.93 (0.076)	0.10 (0.004)	0.18 (0.007)	0.25 (0.010)	0.33 (0.013)	0.41 (0.016)	0.48 (0.019)	0.53 (0.021)
2.90 (0.114)	0.15 (0.006)	0.23 (0.009)	0.33 (0.013)	0.46 (0.018)	0.53 (0.021)	0.64 (0.025)	0.74 (0.029)
4.34 (0.171)	0.20 (0.008)	0.30 (0.012)	0.43 (0.017)	0.58 (0.023)	0.71 (0.028)	0.84 (0.033)	0.97 (0.038)
6.35 (0.250)	0.28 (0.011)	0.38 (0.015)	0.53 (0.021)	0.71 (0.028)	0.90 (0.035)	1.07 (0.042)	1.24 (0.049)
9.53 (0.375)	0.41 (0.016)	0.51 (0.020)	0.66 (0.026)	0.94 (0.037)	1.17 (0.046)	1.37 (0.054)	1.63 (0.064)
12.70 (0.500)	0.53 (0.021)	0.76 (0.030)	1.02 (0.040)	1.57 (0.062)	2.03 (0.080)	2.54 (0.100)	3.18 (0.125)

SOURCE: Associated Spring, Barnes Group Inc.

6.5 HELICAL EXTENSION SPRINGS

6.5.1 General

Helical extension springs store energy and exert a pulling force. They are usually made from round wire and are close-wound with initial tension. They have various types of end hooks or loops by which they are attached to the loads.

Like compression springs, extension springs are stressed in torsion in the body coils. The design procedures for the body coil are similar to those discussed in Sec. 6.4 except for the initial tension and the hook stresses.

Most extension springs are made with the body coils held tightly together by a force called *initial tension*. The measure of initial tension is the load required to overcome the internal force and start coil separation.

Extension springs, unlike compression springs, seldom have set removed. Furthermore, they have no solid stop to prevent overloading. For these reasons, the design stresses are normally held to lower values than those for compression springs.

The pulling force exerted by an extension spring is transmitted to the body coils through hooks or loops. Careful attention must be given to the stresses in the hooks. The hook ends must be free of damaging tool marks so that spring performance will not be limited by hook failure.

TABLE 6.10 Load Tolerances of Helical Compression Springs

Length Tolerance $\pm$ mm (in.)	Tolerances: $\pm$ % of Load. Start with Tolerance from Table 6-8 Multiplied by L_F .														
	Deflection from Free Length to Load, mm (in.)														
	1.27 (0.050)	2.54 (0.100)	3.81 (0.150)	5.08 (0.200)	6.35 (0.250)	7.62 (0.300)	10.2 (0.400)	12.7 (0.500)	19.1 (0.750)	25.4 (1.00)	38.1 (1.50)	50.8 (2.00)	76.2 (3.00)	102 (4.00)	152 (6.00)
0.13 (0.005)	12.	7.	6.	5.	—	—	—	—	—	—	—	—	—	—	—
0.25 (0.010)	—	12.	8.5	7.	6.5	5.5	5.	—	—	—	—	—	—	—	—
0.51 (0.020)	—	22.	15.5	12.	10.	8.5	7.	6.	5.	—	—	—	—	—	—
0.76 (0.030)	—	—	22.	17.	14.	12.	9.5	8.	6.	5.	—	—	—	—	—
1.0 (0.040)	—	—	—	22.	18.	15.5	12.	10.	7.5	6.	5.	—	—	—	—
1.3 (0.050)	—	—	—	—	22.	19.	14.5	12.	9.	7.	5.5	—	—	—	—
1.5 (0.060)	—	—	—	—	25.	22.	17.	14.	10.	8.	6.	5.	—	—	—
1.8 (0.070)	—	—	—	—	—	25.	19.5	16.	11.	9.	6.5	5.5	—	—	—
2.0 (0.080)	—	—	—	—	—	—	22.	18.	12.5	10.	7.5	6.	5.	—	—
2.3 (0.090)	—	—	—	—	—	—	25.	20.	14.	11.	8.	6.	5.	—	—
2.5 (0.100)	—	—	—	—	—	—	—	22.	15.5	12.	8.5	7.	5.5	—	—
5.1 (0.200)	—	—	—	—	—	—	—	—	—	22.	15.5	12.	8.5	7.	5.5
7.6 (0.300)	—	—	—	—	—	—	—	—	—	—	22.	17.	12.	9.5	7.
10.2 (0.400)	—	—	—	—	—	—	—	—	—	—	—	21.	15.	12.	8.5
12.7 (0.500)	—	—	—	—	—	—	—	—	—	—	—	25.	18.5	14.5	10.5

First load test at not less than 15% of available deflection.

Final load test at not more than 85% of available deflection.

SOURCE: Associated Spring, Barnes Group Inc.

6.5.2 Initial Tension

Initial tension is illustrated in Fig. 6.18. The point of intersection on the ordinate is initial tension P_I . The amount of initial tension is governed by the spring index, material, method of manufacture, and the post-stress-relief heat treatment temperature. Note that a high stress-relief temperature can reduce the initial tension. This is sometimes used as a means to control initial tension in low-stress, low-index springs. It follows that an extension spring requiring no initial tension can be made either by removing the initial tension with heat treatment or by keeping the coils open during coiling. The levels of initial tension obtainable are shown in Fig. 6.19.

6.5.3 Types of Ends

Extension springs require a means of attachment to the system which is to be loaded. A variety of end configurations have been developed over the years. The configurations most commonly used are shown in Fig. 6.20. Loops or hooks longer than recommended will require special setup and are more expensive. Specifying an angular relationship for the loops may also add to the cost. Allow a random relationship of loops whenever possible.

Stresses in the loops are often higher than those in the body coils. In such cases, the loops are the performance limiters, particularly in cyclic applications. Generous bend radii, elimination of tool marks, and a reduced diameter of end coils are methods used to reduce loop stresses. In a full-twist loop, stress reaches a maximum in bending at point A (Fig. 6.21) and a maximum in torsion at point B . The stresses at these locations are complex, but useful approximations are, for bending,

$$S_A = \frac{16K_1DP}{\pi d^3} + \frac{4P}{\pi d^2} \quad (6.19)$$

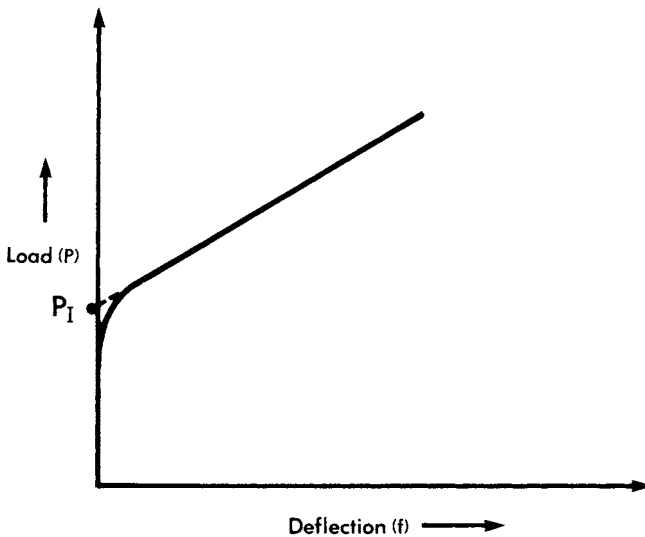


FIGURE 6.18 Load-deflection curve for a helical extension spring with initial tension. (*Associated Spring, Barnes Group Inc.*)

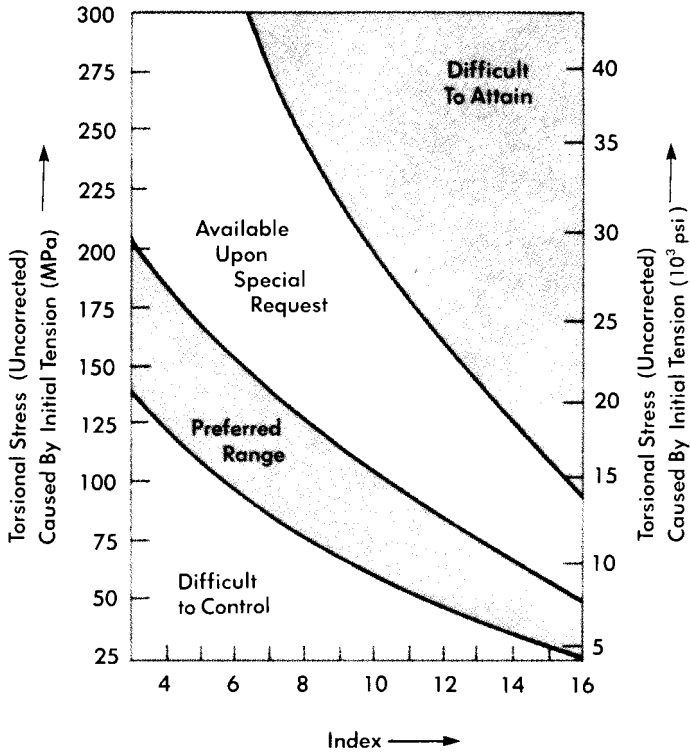


FIGURE 6.19 Torsional stress resulting from initial tension as a function of index in helical extension springs. (*Associated Spring, Barnes Group Inc.*)

where the constants are

$$K_1 = \frac{4C^2 - C_1 - 1}{4C_1(C_1 - 1)} \quad (6.20)$$

and

$$C_1 = \frac{2R_1}{d} \quad (6.21)$$

The torsional stresses are

$$S_B = \frac{8DP}{\pi d^3} \frac{4C_2 - 1}{4C_2 - 4} \quad (6.22)$$

where

$$C_2 = \frac{2R_2}{d} \quad (6.23)$$

General practice is to make C_2 greater than 4.

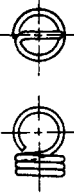
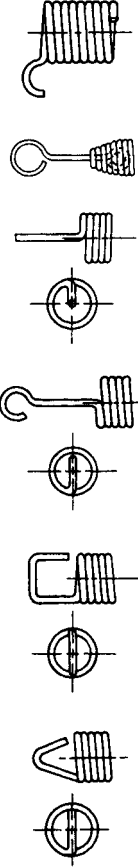
Type	Configurations	Recommended Length Min.-Max.
Twist Loop or Hook		0.5-1.7 I.D.
Cross Center Loop or Hook		I.D.
Side Loop or Hook		0.9-1.0 I.D.
Extended Hook		1.1 I.D. and up, as required by design
Special Ends		As required by design

FIGURE 6.20 Common end configurations for helical extension springs. Recommended length is distance from last body coil to inside of end. ID is inside diameter of adjacent coil in spring body. (*Associated Spring, Barnes Group Inc.*)

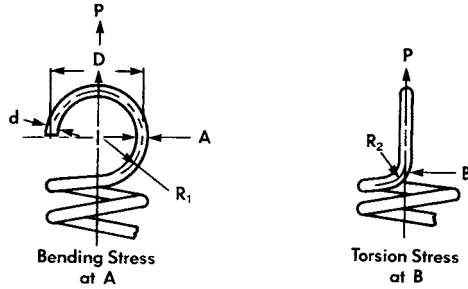


FIGURE 6.21 Location of maximum bending and torsional stresses in twist loops. (*Associated Spring, Barnes Group Inc.*)

6.5.4 Extension Spring Dimensioning

The dimensioning shown in Fig. 6.22 is generally accepted for extension springs. The free length is the distance between the inside surfaces of the loops. The body length is $L_B = d(N + 1)$. The loop opening, or gap, can be varied. The number of active coils is equal to the number of coils in the body of the spring. However, with special ends such as threaded plugs or swivel hooks, the number of active coils will be less than the number of body coils.

6.5.5 Design Equations

The design equations are similar to those for compression springs with the exception of initial tension and loop stresses. The rate is given by

$$k = \frac{P - P_I}{f} = \frac{Gd^4}{8D^3N_a} \quad (6.24)$$

where P_I is initial tension. Stress is given by

$$S = \frac{K_w 8PD}{\pi d^3} \quad (6.25)$$

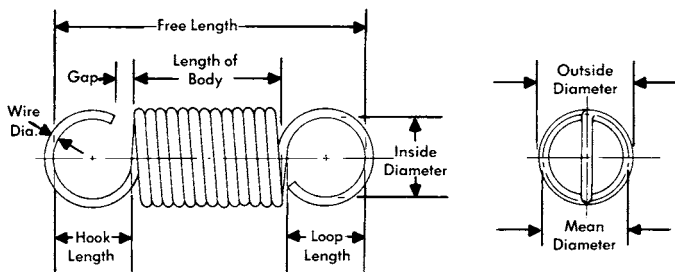


FIGURE 6.22 Typical extension-spring dimensions. (*Associated Spring, Barnes Group Inc.*)

Dynamic considerations discussed previously are generally applicable to extension springs. Natural frequency with one end fixed, in SI units, is

$$n = \frac{560d}{D^2 N_a} \sqrt{\frac{Gg}{\rho}} \quad (6.26)$$

For steel, this equation becomes

$$n = \frac{176\,000d}{N_a D^2} \quad (6.27)$$

where n = frequency in hertz. The corresponding equation in USCS units is

$$n = \frac{d}{18D^2 N_a} \sqrt{\frac{Gg}{\rho}} \quad (6.28)$$

And for steel we have

$$n = \frac{7000d}{N_a D^2} \quad (6.29)$$

6.5.6 Choice of Operating Stress—Static

The maximum stresses recommended for extension springs in static applications are given in Table 6.11. Note that extension springs are similar to compression springs without set removed. For body coil stresses in springs that cannot be adequately stress-relieved because of very high initial-tension requirements, use the maximum recommended stress in torsion, given for the end loops.

6.5.7 Choice of Operating Stress—Cyclic

Table 6.12 presents the maximum stresses for extension springs used in cyclic applications. The data are for stress-relieved springs with initial tension in the preferred range.

TABLE 6.11 Maximum Allowable Stresses (K_{w1} Corrected) for Helical Extension Springs in Static Applications

Materials	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
Patented, cold-drawn or hardened and tempered carbon and low alloy steels	45–50	40	75
Austenitic stainless steel and nonferrous alloys	35	30	55

This information is based on the following conditions: set not removed and low temperature heat treatment applied.

For springs that require high initial tension, use the same percent of tensile strength as for end.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.12 Maximum Allowable Stresses for ASTM A228 and Type 302 Stainless-Steel Helical Extension Springs in Cyclic Applications

Number of Cycles	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
10^5	36	34	51
10^6	33	30	47
10^7	30	28	45

This information is based on the following conditions: not shot-peened, no surging and ambient environment with a low temperature heat treatment applied. Stress ratio = 0.

SOURCE: Associated Spring, Barnes Group Inc.

6.5.8 Tolerances

Extension springs do not buckle or require guide pins when they are deflected, but they may vibrate laterally if loaded or unloaded suddenly. Clearance should be allowed in these cases to eliminate the potential for noise or premature failure. The load tolerances are the same as those given for compression springs. Tolerances for free length and for angular relationship of ends are given in Tables 6.13 and 6.14.

6.6 HELICAL TORSION SPRINGS

Helical springs that exert a torque or store rotational energy are known as *torsion springs*. The most frequently used configuration of a torsion spring is the single-body type (Fig. 6.23). Double-bodied springs, known as double-torsion springs, are sometimes used where dictated by restrictive torque, stress, and space requirements. It is often less costly to make a pair of single-torsion springs than a double-torsion type.

TABLE 6.13 Commercial Free-Length Tolerances for Helical Extension Springs with Initial Tension

Spring Free Length (inside hooks) mm (in.)	Tolerance ± mm (in.)
Up to 12.7 (0.500)	0.51 (0.020)
Over 12.7 to 25.4 (0.500 to 1.00)	0.76 (0.030)
Over 25.4 to 50.8 (1.00 to 2.00)	1.0 (0.040)
Over 50.8 to 102 (2.00 to 4.00)	1.5 (0.060)
Over 102 to 203 (4.00 to 8.00)	2.4 (0.093)
Over 203 to 406 (8.00 to 16.0)	4.0 (0.156)
Over 406 to 610 (16.0 to 24.0)	5.5 (0.218)

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.14 Tolerances on Angular Relationship of Extension Spring Ends

Angular Tolerance per Coil: $\pm$ Degrees									
Index									
4	5	6	7	8	9	10	12	14	16
0.75	0.9	1.1	1.3	1.5	1.7	1.9	2.3	2.6	3

For example, tolerance for a 10-coil spring with an index of 8 is $10 \times \pm 1.5 = \pm 15^\circ$.

If angular tolerance is greater than $\pm 45^\circ$, or if closer tolerances than indicated must be held, consult with Associated Spring.

SOURCE: Associated Spring, Barnes Group Inc.

Torsion springs are used in spring-loaded hinges, oven doors, clothespins, window shades, ratchets, counterbalances, cameras, door locks, door checks, and many other applications. Torsion springs are almost always mounted on a shaft or arbor with one end fixed. They can be wound either right or left hand.

In most cases the springs are not stress-relieved and are loaded in the direction that winds them up or causes a decrease in body diameter. The residual forming stresses which remain are favorable in that direction. Although it is possible to load a torsion spring in the direction to unwind and enlarge the body coils, ordinarily it is not good design practice and should be avoided. Residual stresses in the unwind direction are unfavorable. Torsion springs which are plated or painted and subsequently baked or are stress-relieved will have essentially no residual stresses and can be loaded in either direction, but at lower stress levels than springs which are not heat-treated.

Correlation of test results between manufacturer and user may be difficult because there are few, if any, standardized torsion-spring testing machines. The springs will have varying degrees of intercoil friction and friction between the mounting arbor and the body coils. Often, duplicate test fixtures must be made and test methods coordinated.

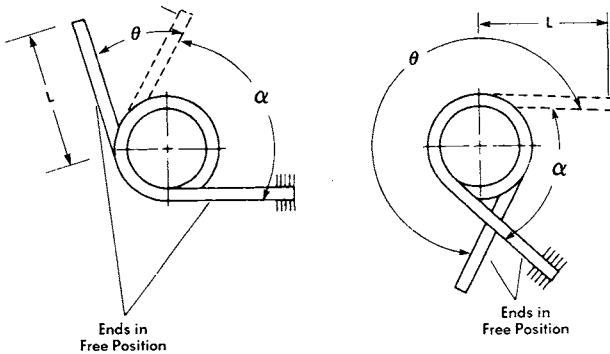


FIGURE 6.23 Specifying load and deflection requirements for torsion spring: α = angle at ends; P = load on ends at α ; L = moment arm; θ = angular deflection from free position. (Associated Spring, Barnes Group Inc.)

Spring ends most commonly used are shown in Fig. 6.24, although the possible variations are unlimited. In considering spring mounting, it must be recognized that for each turn of windup, the overall length L of the spring body will increase as

$$L_1 = d(N_a + 1 + \theta) \quad (6.30)$$

where θ = deflection in revolutions.

Also note that the body coil diameter will be reduced to

$$D = \frac{D_i N_a}{N_a + \theta} \quad (6.31)$$

where D_i = initial mean coil diameter. Experience indicates that the diameter of the arbor over which the spring operates should be approximately 90 percent of the smallest inside diameter to which the spring is reduced under maximum load. Too large an arbor will interfere with deflection, while too small an arbor will provide too little support. Both conditions lead to unexpectedly early failure. Coil diameter tolerances are given in Table 6.17.

6.6.1 Spring Rate

The spring rate, or moment per turn, is given by

$$k = \frac{M}{\theta} = \frac{Ed^4}{10.8DN_a} \quad (6.32)$$

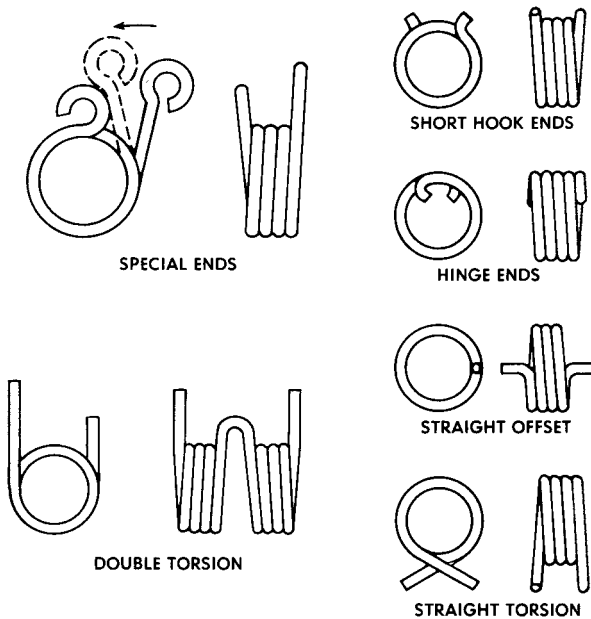


FIGURE 6.24 Common helical torsion-spring end configurations.
(Associated Spring, Barnes Group Inc.)

The number of coils is equal to the number of body coils plus a contribution from the ends. The effect is more pronounced when the ends are long. The number of equivalent coils in the ends is

$$N_e = \frac{L_1 + L_2}{3\pi D} \quad (6.33)$$

where L_1 and L_2 = lengths of ends, and so $N_a = N_b + N_e$, where N_b = number of body coils.

The load should be specified at a fixed angular relationship of the spring ends rather than at a specific angular deflection from free or load positions. Helical torsion springs are stressed in bending. Rectangular sections are more efficient than round sections, but round sections are normally used because there is usually a premium cost for rectangular wire.

6.6.2 Stresses

Stress in round-wire torsion springs is given by

$$S = \frac{32K_B M}{\pi d^3} \quad (6.34)$$

where K_B = a stress correction factor. Stress is higher on the inner surface of the coil. A useful approximation of this factor is

$$K_B = \frac{4C - 1}{4C - 4} \quad (6.35)$$

6.6.3 Rectangular-Wire Torsion Springs

When rectangular wire is formed into coils, it approaches a keystone according to the relation

$$b_I = b \frac{C + 0.5}{C} \quad (6.36)$$

where b_I = axial dimension b after keystoneing. The radial dimension is always t . The rate equation is

$$k = \frac{M}{\theta} = \frac{Ebt^3}{6.6DN_a} \quad (6.37)$$

Stress in rectangular-wire torsion springs is given by

$$S = \frac{6K_B M}{bt^2} \quad (6.38)$$

where $K_{B_{1D}} = 4C/(4C - 3)$ and b = axial dimension of rectangular cross section. Maximum recommended stresses are given in Table 6.15 for static applications and in Table 6.16 for cyclic applications.

TABLE 6.15 Maximum Recommended Bending Stresses for Helical Torsion Springs in Static Applications

Material	Percent of Tensile Strength	
	Stress-Relieved (1) (K_B Corrected)	With Favorable Residual Stress (2) (No Correction Factor)
Patented and Cold Drawn	80	100
Hardened and Tempered Carbon and Low Alloy Steels	85	100
Austenitic Stainless Steels and Non-Ferrous Alloys	60	80

(1) Also for springs without residual stresses.

(2) Springs that have not been stress-relieved and which have bodies and ends loaded in a direction that decreases the radius of curvature.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.16 Maximum Recommended Bending Stresses (K_B Corrected) for Helical Torsion Springs in Cyclic Applications

Fatigue Life (cycles)	ASTM A228 and Type 302 Stainless Steel		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened*	Not Shot-Peened	Shot-Peened*
10^5	53	62	55	64
10^6	50	60	53	62

This information is based on the following conditions: no surging, springs are in the "as-stress-relieved" condition

*Not always possible.

SOURCE: Associated Spring, Barnes Group Inc.

6.6.4 Tolerances

The tolerances for coil diameter and end position are given in Tables 6.17 and 6.18, respectively. Use them as guides.

6.7 BELLEVILLE SPRING WASHER

Belleville washers, also known as *coned-disk springs*, take their name from their inventor, Julian F. Belleville. They are essentially circular disks formed to a conical shape, as shown in Fig. 6.25. When load is applied, the disk tends to flatten. This elastic deformation constitutes the spring action.

TABLE 6.17 Commercial Tolerances for Torsion-Spring Coil Diameters

Wire Diameter mm (in.)	Tolerance: $\pm$ mm (in.)						
	Spring Index D/d						
	4	6	8	10	12	14	16
0.38 (0.015)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.08 (0.003)	0.10 (0.004)
0.58 (0.023)	0.05 (0.002)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.13 (0.005)	0.15 (0.006)
0.89 (0.035)	0.05 (0.002)	0.05 (0.002)	0.08 (0.003)	0.10 (0.004)	0.15 (0.006)	0.18 (0.007)	0.23 (0.009)
1.30 (0.051)	0.05 (0.002)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.20 (0.008)	0.25 (0.010)	0.31 (0.012)
1.93 (0.076)	0.08 (0.003)	0.13 (0.005)	0.18 (0.007)	0.23 (0.009)	0.31 (0.012)	0.38 (0.015)	0.46 (0.018)
2.90 (0.114)	0.10 (0.004)	0.18 (0.007)	0.25 (0.010)	0.33 (0.013)	0.46 (0.018)	0.56 (0.022)	0.71 (0.028)
4.37 (0.172)	0.15 (0.006)	0.25 (0.010)	0.33 (0.013)	0.51 (0.020)	0.69 (0.027)	0.86 (0.034)	1.07 (0.042)
6.35 (0.250)	0.20 (0.008)	0.36 (0.014)	0.56 (0.022)	0.76 (0.030)	1.02 (0.040)	1.27 (0.050)	1.52 (0.060)

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.18 End-Position Tolerances (for D/d Ratios up to and Including 16)

Total Coils	Tolerance: $\pm$ Degrees*
Up to 3	8
Over 3–10	10
Over 10–20	15
Over 20–30	20
Over 30	25

*Closer tolerances available

SOURCE: Associated Spring, Barnes Group Inc.

Belleville springs are used in two broad types of applications. First, they are used to provide very high loads with small deflections, as in stripper springs for punch-press dies, recoil mechanisms, and pressure-relief valves. Second, they are used for their special nonlinear load-deflection curves, particularly those with a constant-load portion. In loading a packing seal or a live center for a lathe, or in injection molding machines, Belleville washers can maintain a constant force throughout dimensional changes in the mechanical system resulting from wear, relaxation, or thermal change.

The two types of performance depend on the ratio of height to thickness. Typical load-deflection curves for various height-thickness ratios are shown in Fig. 6.26. Note that the curve for a small h/t ratio is nearly a straight line. At $h/t = 1.41$ the curve shows a nearly constant load for approximately the last 50 percent of deflection

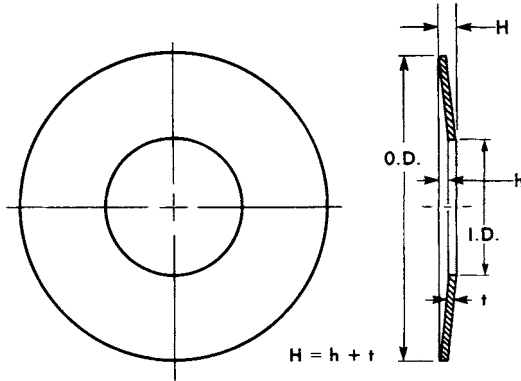


FIGURE 6.25 Belleville washer. (*Associated Spring, Barnes Group Inc.*)

before the flat position. Above $h/t = 1.41$ the load decreases after reaching a peak. When h/t is 2.83 or more, the load will go negative at some point beyond flat and will require some force to be restored to its free position. In other words, the washer will turn inside out.

The design equations given here are complex and may present a difficult challenge to the occasional designer. Use of charts and the equation transpositions presented here have proved helpful. Note that these equations are taken from the mathematical analysis by Almen and Laszlo [6.5]. The symbols used here are those originally used by the authors and may not necessarily agree with those used elsewhere in the text.

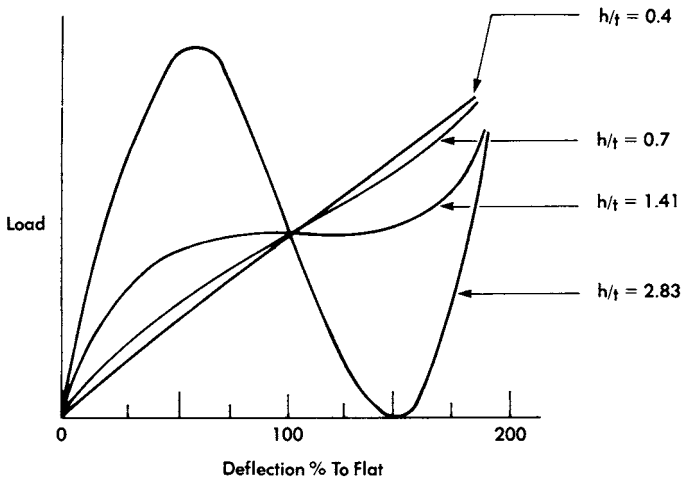


FIGURE 6.26 Load-deflection curves for Belleville washers with various h/t ratios. (*Associated Spring, Barnes Group Inc.*)

6.7.1 Nomenclature

a	OD/2, mm (in)
C_1	Compressive stress constant (see formula and Fig. 6.28)
C_2	Compressive stress constant (see formula and Fig. 6.28)
E	Modulus of elasticity (see Table 6.19), MPa (psi)
f	Deflection, mm (in)
h	Inside height, mm (in)
ID	Inside diameter, mm (in)
M	Constant
OD	Outside diameter, mm (in)
P	Load, N (lb)
P_f	Load at flat position, N (lb)
R	OD/ID
S_c	Compressive stress (Fig. 6.27), MPa (psi)
S_{T_1}	Tensile stress (Fig. 6.27), MPa (psi)
S_{T_2}	Tensile stress (Fig. 6.27), MPa (psi)
t	Thickness, mm (in)
T_1	Tensile stress constant (see formula and Fig. 6.29)
T_2	Tensile stress constant (see formula and Fig. 6.29)
μ	Poisson's ratio (Table 6.19)

6.7.2 Basic Equations

$$P = \frac{Ef}{(1 - \mu^2)Ma^2} \left[(h - f) \left(h - \frac{f}{2} \right) t + t^3 \right] \quad (6.39)$$

$$P_F = \frac{Eht^3}{(1 - \mu^2)Ma^2} \quad (6.40)$$

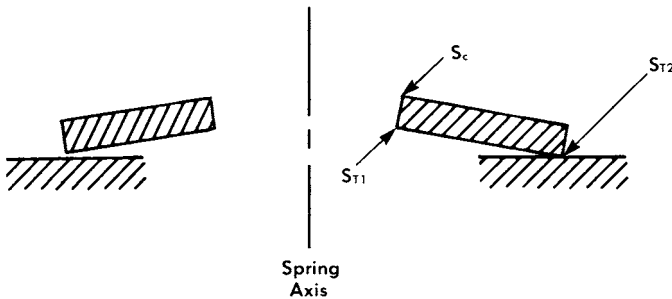


FIGURE 6.27 Highest-stressed regions in Belleville washers. (*Associated Spring, Barnes Group Inc.*)

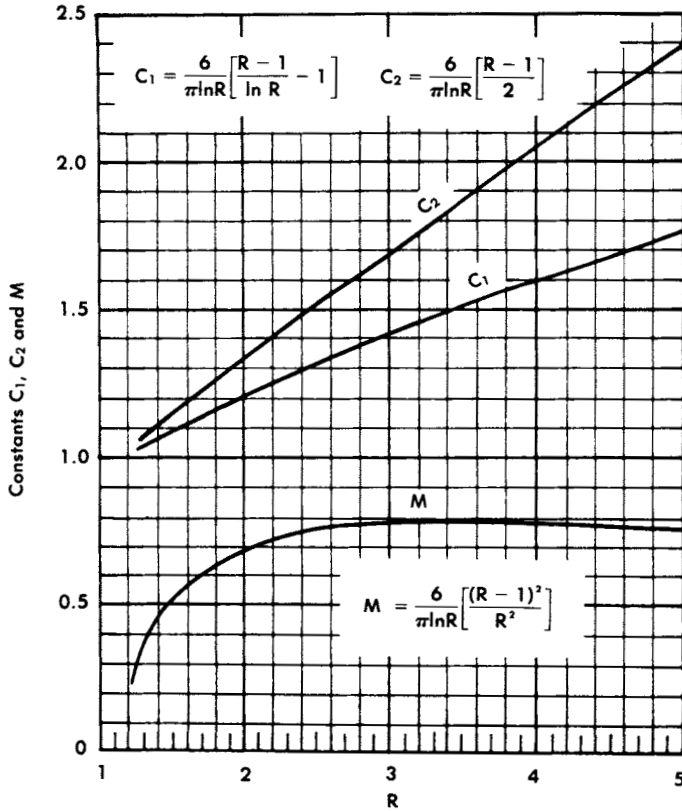


FIGURE 6.28 Compressive stress constants for Belleville washers. (*Associated Spring, Barnes Group Inc.*)

TABLE 6.19 Elastic Constants of Common Spring Materials

Material	Modulus of Elasticity E		Poisson's ratio μ
	Mpsi	GPa	
Steel	30	207	0.30
Phosphor bronze	15	103	0.20
17-7 PH stainless	29	200	0.34
302 stainless	28	193	0.30
Beryllium copper	18.5	128	0.33
Inconel	31	214	0.29
Inconel X	31	214	0.29

SOURCE: Associated Spring, Barnes Group Inc.

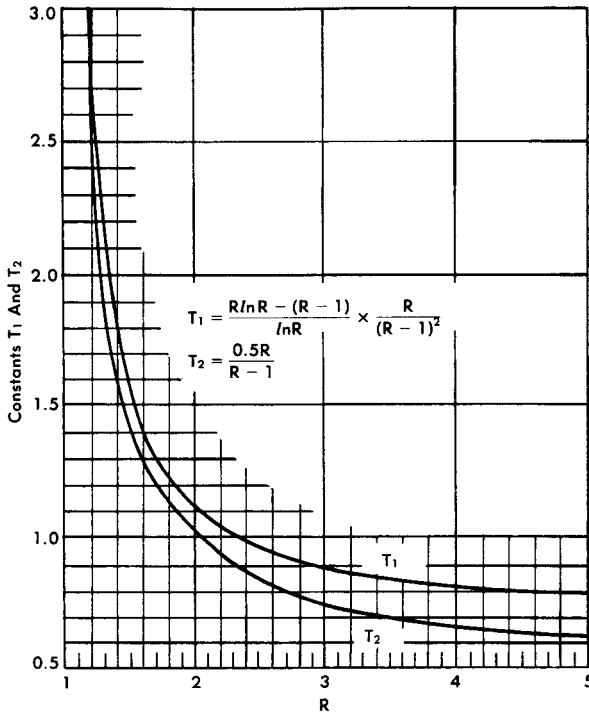


FIGURE 6.29 Tensile stress constants for Belleville washers.
(Associated Spring, Barnes Group Inc.)

$$S_c = \frac{Ef}{(1 - \mu^2)Ma^2} \left[C_1 \left(h - \frac{f}{2} \right) + C_2 t \right] \quad (6.41)$$

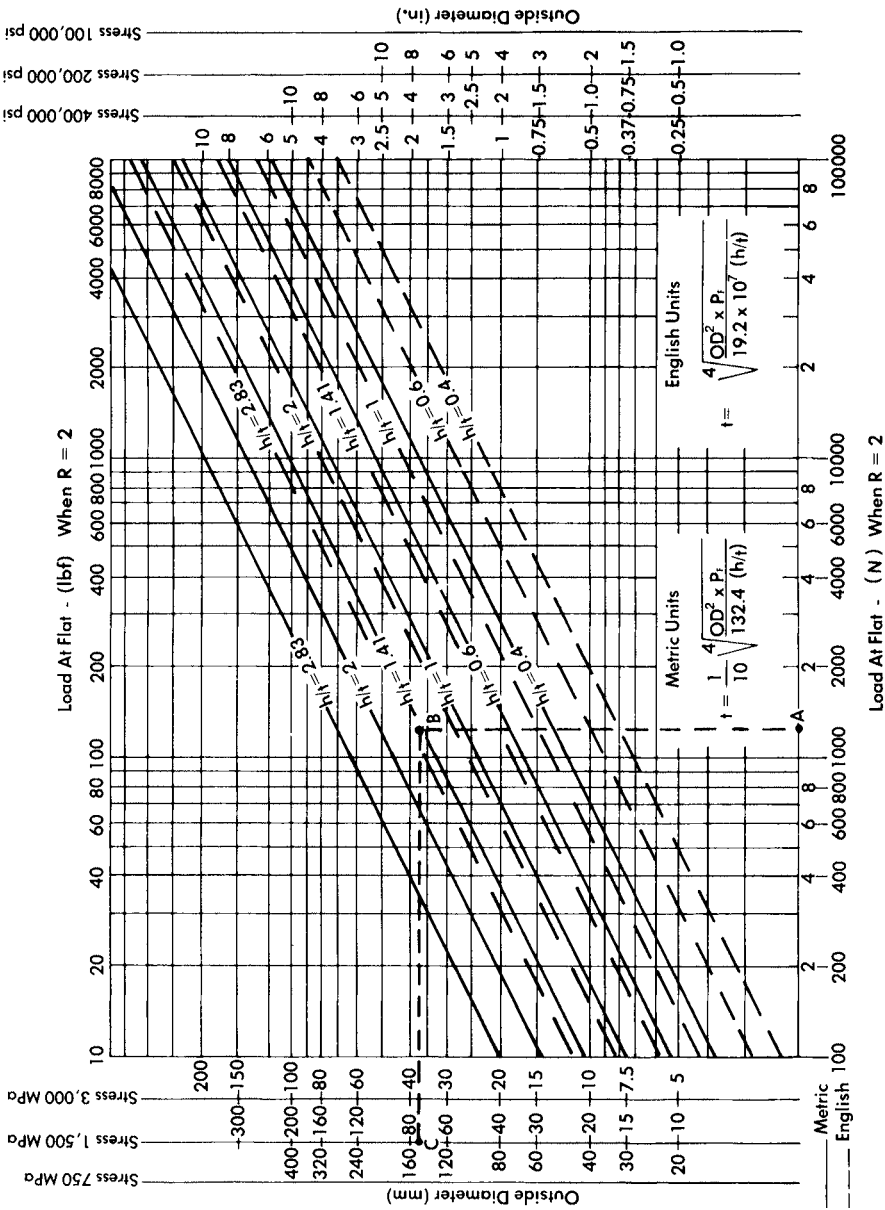
$$S_{T_1} = \frac{Ef}{(1 - \mu^2)Ma^2} \left[C_1 \left(h - \frac{f}{2} \right) - C_2 t \right] \quad (6.42)$$

$$S_{T_2} = \frac{Ef}{(1 - \mu^2)a^2} \left[T_1 \left(h - \frac{f}{2} \right) + T_2 t \right] \quad (6.43)$$

The design approach recommended here depends on first determining the loads and stresses at flat position, as shown in Fig. 6.30. Intermediate loads are determined from the curves in Fig. 6.31.

Figure 6.30 gives the values graphically for compressive stresses S_c at flat position. The stress at intermediate stages is approximately proportional to the deflection. For critical applications involving close tolerances or unusual proportions, stresses should be checked by using the equation before the design is finalized.

The stress level for static applications is evaluated in accordance with Eq. (6.41). This equation has been used most commonly for appraising the design of a Belleville spring because it gives the highest numerical value. It gives the compressive stress at the point shown in Fig. 6.27.



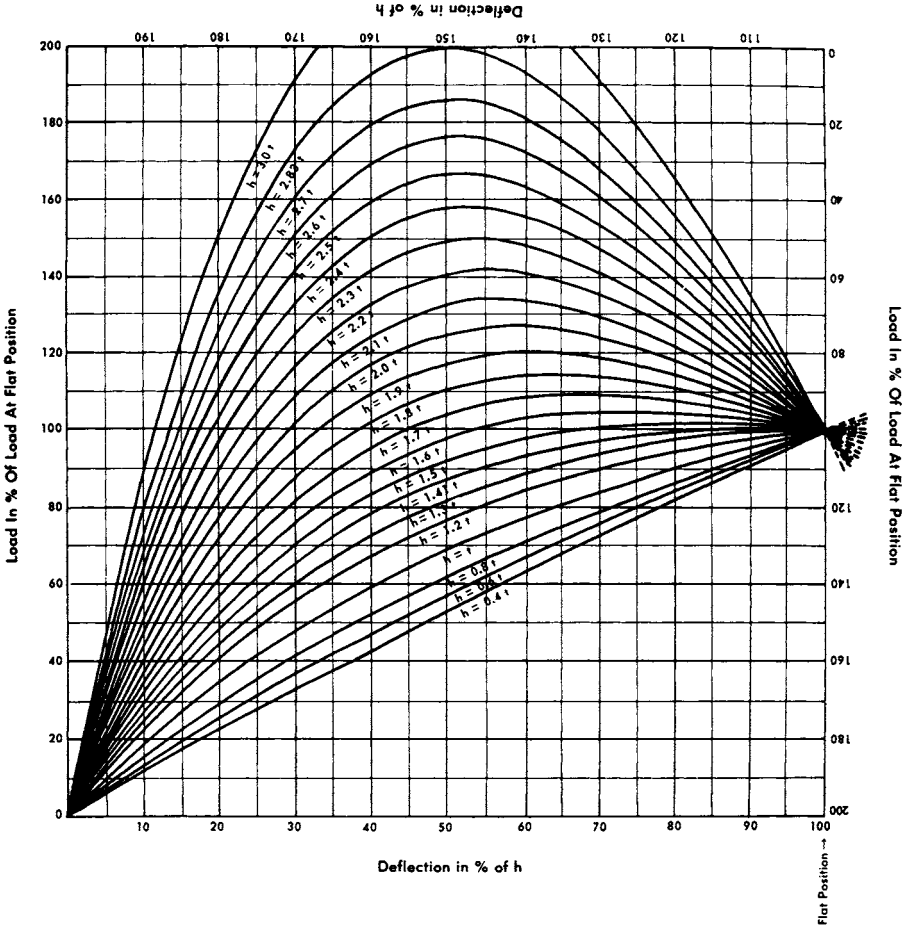


FIGURE 6.31 Load-deflection characteristics for Belleville washers. If a washer is supported and loaded at its edges so that it is deflected beyond the flat position, then the greatest possible deflection can be utilized. Since the load-deflection curve beyond the horizontal position is symmetric with the first part of the curve, this chart has been labeled at the right and top to be read upside down for deflection beyond horizontal. Dotted lines extending beyond the chart indicate continuation of curves beyond flat. (*Associated Spring, Barnes Group Inc.*)

A Belleville spring washer should be designed so that it can be compressed flat by accidental overloading, without setting. This can be accomplished either by using a stress so low that the spring will not set or by forming the spring higher than the design height and removing set by compressing flat or beyond flat (see Table 6.21). The table values should be reduced if the washers are plated or used at elevated temperatures.

For fatigue applications it is necessary to consider the tensile stresses at the points marked S_{T_1} and S_{T_2} in Fig. 6.27. The higher value of the two can occur at either

the ID or the OD, depending on the proportions of the spring. Therefore, it is necessary to compute both values.

Fatigue life depends on the stress range as well as the maximum stress value. Figure 6.32 predicts the endurance limits based on either S_{T1} or S_{T2} , whichever is higher. Fatigue life is adversely affected by surface imperfections and edge fractures and can be improved by shot peening.

Since the deflection in a single Belleville washer is relatively small, it is often necessary to combine a number of washers. Such a combination is called a *stack*.

The deflection of a series stack (Fig. 6.33) is equal to the number of washers times the deflection of one washer, and the load of the stack is equal to that of one washer. The load of a parallel stack is equal to the load of one washer times the number of washers, and the deflection of the stack is that of one washer.

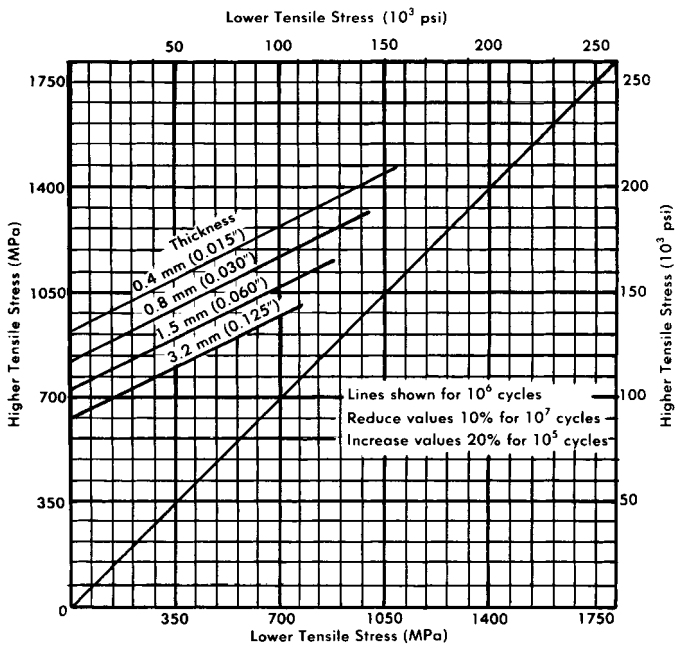


FIGURE 6.32 Modified Goodman diagram for Belleville washers; for carbon and alloy steels at 47 to 49 R_c with set removed, but not shot-peened. (Associated Spring, Barnes Group Inc.)

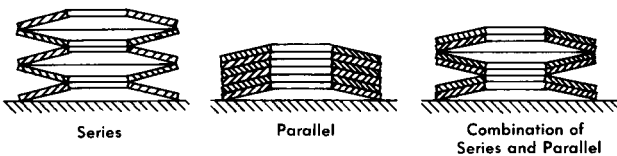


FIGURE 6.33 Stacks of Belleville washers. (Associated Spring, Barnes Group Inc.)

Because of production variations in washer parameters, both the foregoing statements carry cautionary notes. In the series stack, springs of the constant-load type ($h/t = 1.41$) may actually have a negative rate in some portion of their deflection range. When such a series stack is deflected, some washers will snap through, producing jumps in the load-deflection curve. To avoid this problem, the h/t ratio in a series stack design should not exceed 1.3.

In the parallel stack, friction between the washers causes a hysteresis loop in the load-deflection curve (Fig. 6.34). The width of the loop increases with each washer added to the stack but may be reduced by adding lubrication as the washers burnish each other during use.

Stacked washers normally require guide pins or sleeves to keep them in proper alignment. These guides should be hardened steel at HRC 48 minimum hardness. Clearance between the washer and the guide pin or sleeve should be about 1.5 percent of the appropriate diameter.

6.7.3 Tolerances

Load tolerances should be specified at test height. For carbon-steel washers with $h/t < 0.25$, use load tolerance of ± 15 percent. For washers with $h/t > 0.25$, use ± 10 percent. The recommended load tolerance for stainless steel and nonferrous washers is ± 15 percent. See Table 6.20 for outside- and inside-diameter tolerances.

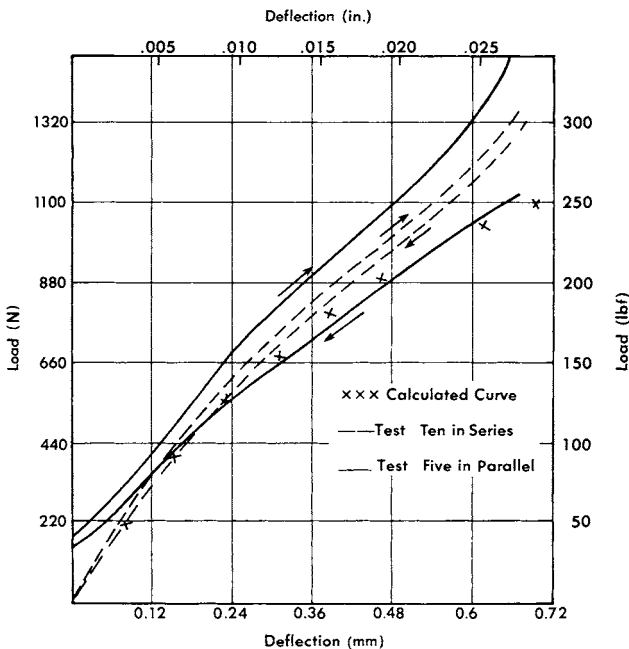


FIGURE 6.34 Hysteresis in stacked Belleville washers. (*Associated Spring, Barnes Group Inc.*)

TABLE 6.20 Belleville Washer Diameter Tolerances

Diameter, mm (in.)	O.D. mm (in.)	I.D. mm (in.)
	+0.00	-0.00
Up to 5 (0.197)	-0.20 (-0.008)	+0.20 (+0.008)
5-10 (0.197-0.394)	-0.25 (-0.010)	+0.25 (+0.010)
10-25 (0.394-0.984)	-0.30 (-0.012)	+0.30 (+0.012)
25-50 (0.984-1.969)	-0.40 (-0.016)	+0.40 (+0.016)
50-100 (1.969-3.937)	-0.50 (-0.020)	+0.50 (+0.020)

Based on $R = 2$, increased tolerances are required for lower R ratios.

SOURCE: Associated Spring, Barnes Group Inc.

Example. In a clutch, a minimum pressure of 202 lb (900 N) is required. This pressure must be held nearly constant as the clutch facing wears down 0.31 in (7.9 mm). The washer OD is 2.99 in (76 mm). The material washer OD is 2.99 in (76 mm). The material selected for the application is spring steel HRC 47-50.

Solution

1. Base the load on a value 10 percent above the minimum load, or $202 + 10$ percent = 223 lb (998 N). Assume $OD/ID = 2$. From Fig. 6.31, select a load-deflection curve which gives approximately constant load between 50 and 100 percent of deflection to flat. Choose the $h/t = 1.41$ curve.
2. From Fig. 6.31, the load at 50 percent of deflection to flat is 88 percent of the flat load.
3. Flat load is $P_F = 223/0.88 = 252$ lb (1125 N).
4. From Fig. 6.30 [follow line AB from 1125 N to $h/t = 1.41$ and line BC to approximately 76-mm (2.99-in) OD], the estimated stress is 1500 MPa [218 kilopounds per square inch (kpsi)].
5. From Table 6.21 maximum stress without set removed is 120 percent of tensile strength. From Fig. 6.3, the tensile strength at HRC 48 will be approximately 239 kpsi (1650 MPa). Yield point without residual stress will be $(239 \text{ kpsi})(1.20) = 287$ kpsi. Therefore 218 kpsi stress is less than the maximum stress of 287 kpsi.
6. Stock thickness is

$$t = \sqrt[4]{\frac{OD^2(P_F)}{19.2(10^7)(h/t)}} = 0.054 \text{ in (1.37 mm)}$$

TABLE 6.21 Maximum Recommended Stress Levels for Belleville Washers in Static Applications

Material	Percent of Tensile Strength	
	Set Not Removed	Set Removed
Carbon or Alloy Steel	120	275
Nonferrous and Austenitic Stainless Steel	95	160

SOURCE: Associated Spring, Barnes Group Inc.

7. $h = 1.41t = 1.41(0.054) = 0.076$ in
 $H = h + t = 0.076 + 0.054 = 0.130$ in
8. Refer to Fig. 6.31. The load of 202 lb will be reached at $f_1 = 50$ percent of maximum available deflection. And $f_1 = 0.50(0.076) = 0.038$ in deflection, or the load of 223 lb will be reached at $H_1 = H - f_1 = 0.130 - 0.038 = 0.092$ in height at load. To allow for wear, the spring should be preloaded at $H_2 = H_1 - f(\text{wear}) = 0.092 - 0.032 = 0.060$ in height. This preload corresponds to a deflection $f_2 = H - H_2 = 0.130 - 0.060 = 0.070$ in. Then $f_2/h = 0.070/0.076 = 0.92$, or 92 percent of h .
9. Because 92 percent of h exceeds the recommended 85 percent (the load-deflection curve is not reliable beyond 85 percent deflection when the washer is compressed between flat surfaces), increase the deflection range to 40 to 85 percent. From Fig. 6.31, the load at 40 percent deflection is 78.5 percent, and $P_F = 223/0.785 = 284$ lb. Repeat previous procedures 4, 5, 6, 7, and 8, and find that $100(f_2/h) = 81$ percent of h . The final design is as follows:
- Material: AISI 1074
 OD = 2.99 in (76 mm)
 ID = 1.50 in (38 mm)
 $t = 0.055$ in (1.40 mm) nominal
 $h = 0.078$ in (1.95 mm) nominal
 Tensile stress $S_{T_1} = -29.5$ kpsi (-203 MPa) at $f_2 = 85$ percent of h
 Tensile stress $S_{T_2} = 103$ kpsi (710 MPa) at $f_2 = 85$ percent of h

6.8 SPECIAL SPRING WASHERS

Spring washers are being used increasingly in applications where there is a requirement for miniaturization and compactness of design. They are used to absorb vibrations and both side and end play, to distribute loads, and to control end pressure.

Design equations have been developed for determining the spring characteristics of curved, wave, and Belleville washers. There are no special design equations for slotted and finger washers. They are approximated by using Belleville and cantilever equations and then are refined through sampling and testing.

6.8.1 Curved Washers

These springs (Fig. 6.35) exert relatively light thrust loads and are often used to absorb axial end play. The designer must provide space for diametral expansion which occurs as the washer is compressed during loading. Bearing surfaces should be hard, since the washer edges tend to dig in. The spring rate is approximately linear up to 80 percent of the available deflection. Beyond that the rate will be much higher than calculated. Load tolerance should not be specified closer than ± 20 percent.

Approximate equations are

$$P = \frac{4fEt^3}{OD^2(K)} \quad (6.44)$$

and

$$S = \frac{1.5KP}{t^2} \quad (6.45)$$

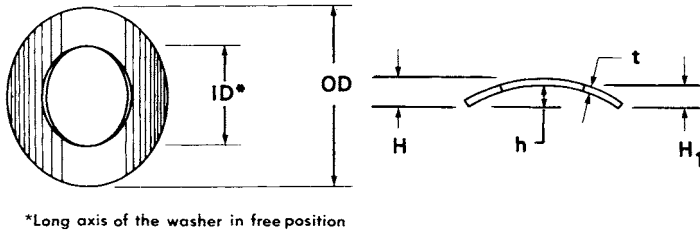


FIGURE 6.35 Curved washer. (Associated Spring, Barnes Group Inc.)

where K is given in Fig. 6.36 and f is 80 percent of h or less.

Maximum recommended stress levels for static operations are given in Table 6.22. Favorable residual stresses can be induced by shot peening and, to a lesser extent, by removing set. The maximum recommended stresses for cyclic applications are given in Table 6.23.

Tensile strengths for carbon steel are obtained from Fig. 6.3.

6.8.2 Wave Washer

These spring washers (Fig. 6.37) are regularly used in thrust loading applications, for small deflections, and for light to medium loads. The rate is linear between 20 and 80 percent of available deflection. Load tolerances should be no less than ± 20 percent. In the most commonly used range of sizes, these washers can have three, four, or six waves.

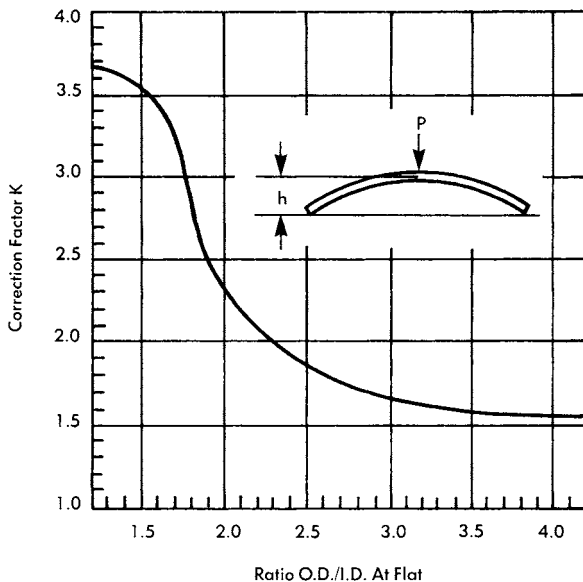


FIGURE 6.36 Empirical correction factor K for curved spring washers. (Associated Spring, Barnes Group Inc.)

TABLE 6.22 Maximum Recommended Operating Stress Levels for Special Spring Washers in Static Applications

Material	Percent of Tensile Strength	
	Stress-Relieved	With Favorable Residual Stresses
Steels, Alloy Steels	80	100
Nonferrous Alloys and Austenitic Steel	—	80

Finger washers are not generally supplied with favorable residual stresses.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.23 Maximum Recommended Operating Stress Levels for Steel Curved and Wave Washers in Cyclic Applications

Life (Cycles)	Percent of Tensile Strength
	Maximum Stress
10^4	80
10^5	53
10^6	50

This information is based on the following conditions: ambient environment, free from sharp bends, burrs, and other stress concentrations. AISI 1075

SOURCE: Associated Spring, Barnes Group Inc.

Design equations are

$$\frac{P}{f} = \frac{Ebt^3N^4(OD)}{2.4D^3(ID)} \quad (6.46)$$

and

$$S = \frac{3\pi PD}{4bt^2N^2} \quad (6.47)$$

where $D = OD - b$. The washer expands in diameter when compressed, according to the formula

$$D' = \sqrt{D^2 + 0.458h^2N^2} \quad (6.48)$$

Maximum recommended stress levels for static applications are given in Table 6.22. Favorable residual stresses are induced by shot peening or removing set. Table 6.23 gives the maximum recommended stress levels for cyclic applications. Figure 6.3 provides tensile strengths for carbon steel.

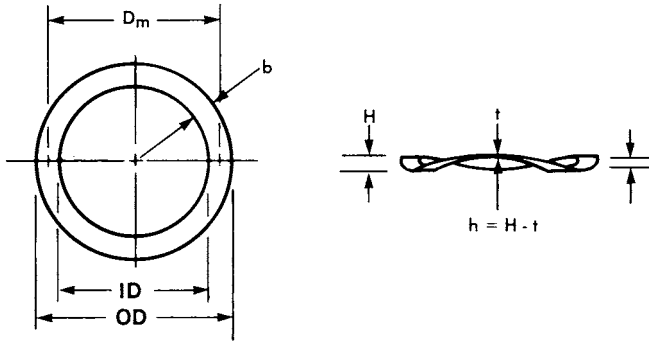


FIGURE 6.37 Typical wave spring washer. (*Associated Spring, Barnes Group Inc.*)

6.8.3 Finger Washers

Finger washers (Fig. 6.38) have both the flexibility of curved washers and the distributed points of loading of wave washers. They are calculated, approximately, as groups of cantilever springs; then samples are made and tested to prove the design. They are most frequently used in static applications such as applying axial load to ball-bearing races to reduce vibration and noise. These washers are not used in cyclic applications because of the shear cuts.

6.8.4 Slotted Washers

These are more flexible than plain dished washers but should be designed to maintain a constant pressure rather than to operate through a deflection range (see Fig. 6.39).

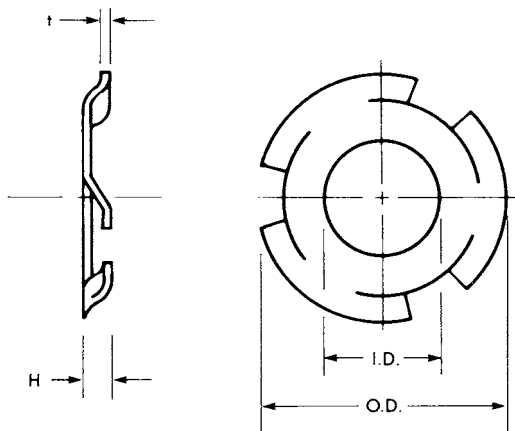


FIGURE 6.38 Finger washer. (*Associated Spring, Barnes Group Inc.*)

6.9 FLAT SPRINGS

6.9.1 Introduction

The classification *flat springs* applies to a wide range of springs made from sheet, strip, or plate material. Exceptions to this classification are power springs and washers. Flat springs may contain bends and forms. Thus the classification refers to the raw material and not to the spring itself.

Flat springs can perform functions beyond normal spring functions. A flat spring may conduct electricity, act as a latch, or hold a part in position. In some flat springs, only a portion of the part may have a spring function.



FIGURE 6.39 Slotted washers. (*Associated Spring, Barnes Group Inc.*)

Most flat springs are custom designs, and the tooling is often a major cost consideration. Flat springs can be cantilever or simple elliptical beams or combinations of both. These two elementary forms are discussed in this section. For a description of the methods used to compute complex flat-spring designs, see [6.6].

Load specification in flat springs is closely connected with the dimensioning of the form of the spring. From the equations it can be seen that the deflection and load vary in proportion to the third power of the material thickness. The important factors in load control are first, the material thickness and second, the deflection. Where close load control is required, the material may have to be selected to restricted thickness tolerance, and/or the free shape may be trued.

6.9.2 Cantilever Springs

The basic type of cantilever is a rectangular spring as shown in Fig. 6.40. The maximum bending stress occurs at the clamping point, and the stress is not uniform through the section. This stress is

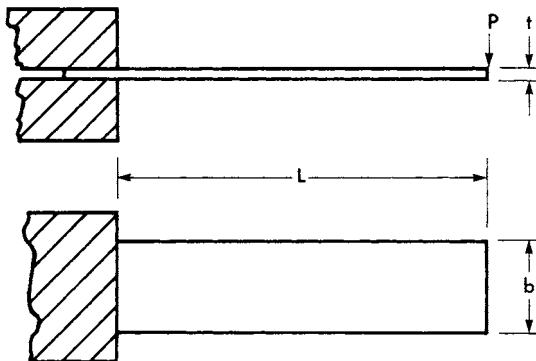


FIGURE 6.40 Rectangular cantilever spring. (*Associated Spring, Barnes Group Inc.*)

$$S = \frac{6PL}{bt^2} \quad (6.49)$$

The load is given by

$$P = \frac{fEbt^3}{4L^3} \quad (6.50)$$

These equations are satisfactory when the ratio of deflection to length f/L is less than 0.3. For larger deflections, use the method described in Fig. 6.41.

In cantilever springs with a trapezoidal or triangular configuration (Fig. 6.42), the stress is uniform throughout and is

$$S = \frac{6PL}{b_o t^2} \quad (6.51)$$

The corresponding load is

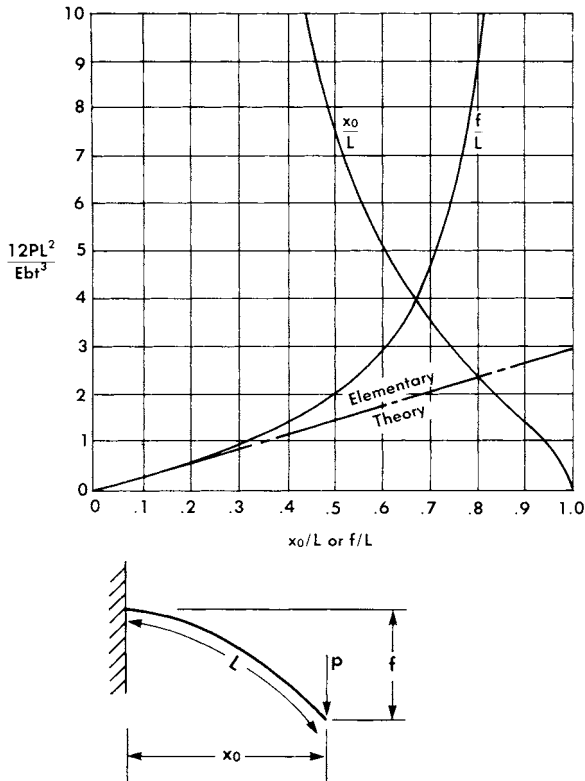


FIGURE 6.41 Calculating large deflection in cantilever beams [6.7]. To utilize this figure for any load P , first calculate the quantity $12PL^2/Ebt^3$. Using this value, from the curves find f/L and x_o/L , where x_o is the moment arm of the load P . Deflection then equals L multiplied by f/L . The maximum stress is reduced in the ratio x_o/L . (Associated Spring, Barnes Group Inc.)

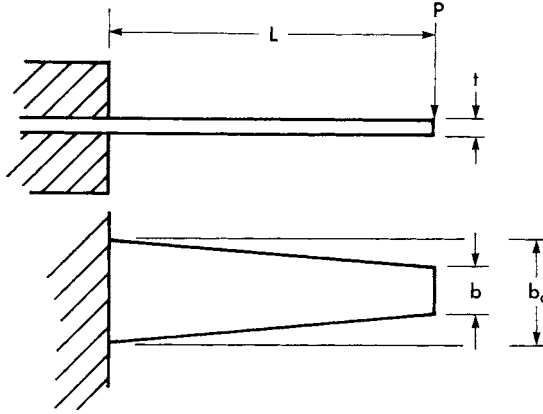


FIGURE 6.42 Trapezoidal cantilever spring. (*Associated Spring, Barnes Group Inc.*)

$$P = \frac{fEb_ot^3}{4L^3K} \quad (6.52)$$

where K = constant based on the ratio b/b_o (Fig. 6.43). These equations are valid for f/L ratios of less than 0.3.

6.9.3 Simple Beams or Elliptical Springs

Simple beams are usually rectangular and are formed into an arc as in Fig. 6.44. If holes are introduced for clamping purposes, stress will increase at the hole and at the clamping point owing to stress concentration.

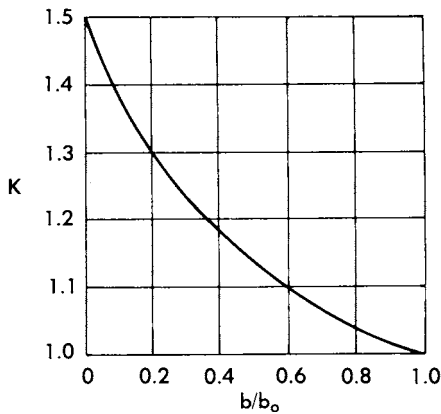


FIGURE 6.43 Correction factor for trapezoidal beam-load equation. (*Associated Spring, Barnes Group Inc.*)

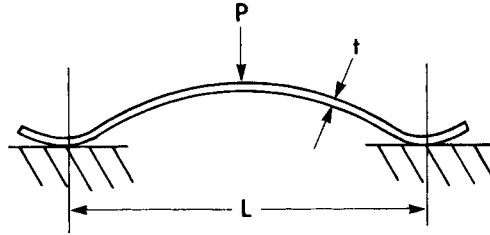


FIGURE 6.44 Simple beam spring. (*Associated Spring, Barnes Group Inc.*)

When ends are free to move laterally, the equation for load is

$$P = \frac{4fEbt^3}{L^3} \quad (6.53)$$

and stress is given by

$$S = \frac{1.5PL}{bt^2} \quad (6.54)$$

These equations apply when the ratio f/L is less than 0.15.

Stress Considerations. The maximum design stresses for cantilevers and simple beams are given in Table 6.24 for static applications and in Table 6.25 for cyclic applications. These recommendations do not apply when holes, sharp corners, notches, or abrupt changes in cross section are incorporated in the design, and should be used for guidance only.

6.10 CONSTANT-FORCE SPRINGS

A constant-force spring is a roll of prestressed material which exerts a nearly constant restraining force to resist uncoiling. Its unique characteristic is *force independent of deflection*. The force required to produce a unit deflection is the same for

TABLE 6.24 Maximum Design Stresses for Cantilever and Simple Beam Springs in Static Applications

Percent of Tensile Strength			
Ferrous Material		Nonferrous Material	
No Residual Stress	Maximum Residual Stress	No Residual Stress	Maximum Residual Stress
80	100	75	80

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.25 Maximum Design Stresses for Carbon-Steel Cantilever and Simple Beam Springs in Cyclic Applications

Number of Cycles	Percent of Tensile Strength	
	Not Shot-Peened	Shot-Peened*
10^5	53	62
10^6	50	60
10^7	48	58

*Shot peening is not recommended for thin materials and complex shapes. This information is based on an ambient environment. Stress ratio = 0.

SOURCE: Associated Spring, Barnes Group Inc.

each increment of coil because the radius of curvature of each increment is the same as any other.

Although these springs are not constant-load or constant-torque springs in the precise meaning of those terms, they produce a more nearly constant load over a greater deflection than any other spring design covered here. See Fig. 6.45. Constant-force springs are made of both type 301 stainless steel and ultra-high-strength high-carbon steels, with many of the applications using stainless steel because of its inherent resistance to corrosion.

One of the most severe limitations on the use of constant-force springs is their relatively short operating life. The most efficient use of material will produce a life of about 3000 cycles. Although life of hundreds of thousands of cycles is possible,

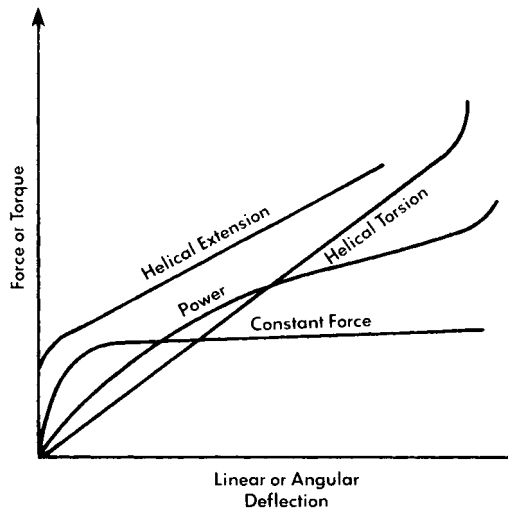


FIGURE 6.45 Load-deflection curves for various spring configurations. (Associated Spring, Barnes Group Inc.)

most applications fall into the range of 3000 to 30 000 cycles. Figure 6.46 shows the relationship between stress and fatigue life. These curves are derived from experimentally obtained data.

Some applications involving constant-force and constant-torque springs are simple extension springs, window sash counterbalances, camera motors, toys, machine carriage returns, constant-pressure electric-motor brush springs, space vehicle applications, and retraction devices.

6.10.1 Extension Type

This type of spring is a spiral spring made of strip material wound on the flat with an inherent curvature such that, in repose, each coil wraps tightly on its inner neighbor. In use the strip is extended with the free end loaded and the inner end supported on a drum or arbor. Very long deflections are possible, but the strip becomes unstable in long deflections and must be guided or supported to avoid kinking or snarling on the return stroke.

The rated load is not reached until after an initial deflection of 1.25 times the drum diameter, as shown in Fig. 6.47. Idler pulleys can be used but should be no smaller in diameter than the natural diameter of the coils and should never be used in a direction to cause backbending against the strip curvature.

6.10.2 Design Equations

$$P = \frac{Ebt^3}{6.5D_n^2} \quad \text{for } N \leq 10 \quad (6.55)$$

$$P = \frac{Ebt^3}{6.5D_1} \left(\frac{2}{D_n} - \frac{1}{D_1} \right) \quad \text{for } N > 10 \quad (6.56)$$

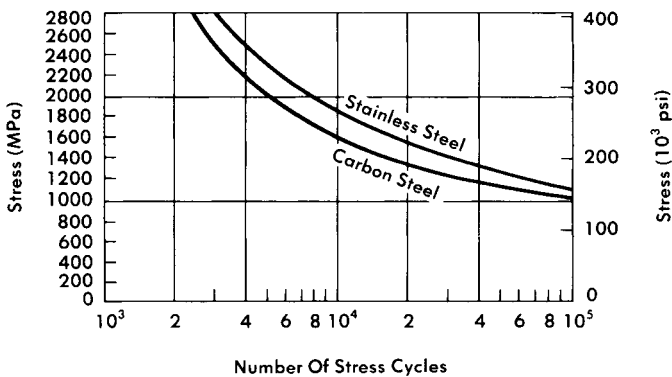


FIGURE 6.46 Maximum bending stress versus number of stress cycles for constant-force springs. These curves are based on no. 1 round-edge strip. (Associated Spring, Barnes Group Inc.)

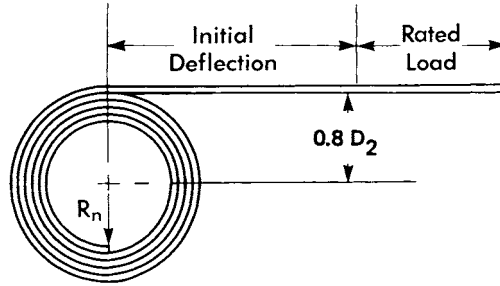


FIGURE 6.47 Typical constant-force extension spring (extension form). (*Associated Spring, Barnes Group Inc.*)

If unknown, let $b/t = 100/1$, $D_2 = 1.2 D_n$,

$$S = \frac{Et}{D_n} \quad (6.57)$$

and

$$L = 1.57N(D_1 + D_2) \quad \text{or} \quad L \approx f + 5D_2$$

where N = number of turns
 D_1 = outside coil diameter
 D_2 = drum (arbor) diameter
 D_n = natural diameter
 E = modulus of elasticity

6.10.3 Spring Motor Type

When a constant-force spring is mounted on two drums of different diameters and the spring is backbent onto the larger diameter, the result is a constant-torque spring motor. The strip is in repose on the smaller (storage) drum and is backbent onto the larger (output) drum. Torque is taken from the output drum shaft as shown in Fig. 6.48.

Note here that constant torque does not mean constant speed. Constant torque implies uniform acceleration, and the mechanism so driven will continue to speed up unless restrained by a governor mechanism. Load tolerances are normally held within ± 10 percent.

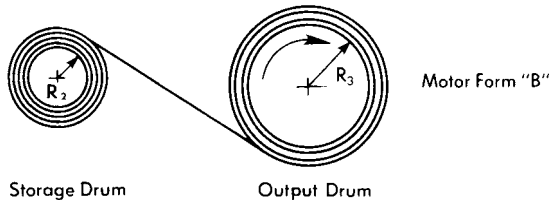


FIGURE 6.48 Typical constant-torque motor spring. (*Associated Spring, Barnes Group Inc.*)

6.10.4 Design Equations

$$M = \frac{Ebt^3D_3}{13} \left(\frac{1}{D_n} + \frac{1}{D_3} \right)^2 \quad (6.58)$$

$$S = Et \left(\frac{1}{D_n} + \frac{1}{D_3} \right) \quad (6.59)$$

$$L = \pi N(D_3 + Nt) + 10D_3 \quad (6.60)$$

$$R_c = R_n \sqrt{4 + \frac{4R_3}{R_n} + \frac{R_n}{R_3} + \left(\frac{R_3}{R_n} \right)^2} \quad (6.61)$$

Design Suggestions. Let

$$\frac{b}{t} = 100 \quad \frac{D_n}{t} = 250 \quad \frac{D_3}{D_n} = 2 \quad \frac{D_3}{D_2} = 1.6$$

where D_n = natural diameter

R_n = natural radius

D_2 = storage-drum diameter

D_3 = output-drum diameter

R_3 = output-drum radius

N = number of revolutions

R_c = minimum center-to-center distance of drums

6.11 TORSION BARS

Torsion bars used as springs are usually straight bars of spring material to which a twisting couple is applied. The stressing mode is torsional. This type of spring is very efficient in its use of material to store energy. The major disadvantage with the torsion bar is that unfavorable stress concentrations occur at the point where the ends are fastened.

Although both round and rectangular bar sections are used, the round section is used more often.

6.11.1 Design Equations: Round Sections

$$\phi = \frac{584ML}{d^4G} \quad (6.62)$$

$$S = \frac{16M}{\pi d^3} \quad (6.63)$$

where ϕ = rotation angle in degrees

S = shear stress

L = active length

6.11.2 Design Equations: Rectangular Sections

$$\phi = \frac{57.3ML}{K_1 b t^3 G} \quad (6.64)$$

$$S = \frac{M}{K_2 b t^2} \quad (6.65)$$

where factors K_1 and K_2 are taken from Table 6.26.

The assumptions used in deriving these equations are (1) the bar is straight, (2) the bar is solid, and (3) loading is in pure torsion.

Torsion-bar springs are often preset in the direction in which they are loaded by twisting the bar beyond the torsional elastic limit. Care must be taken in the use of a preset bar: It must be loaded in the same direction in which it was preset; otherwise, excessive set will occur.

6.12 POWER SPRINGS

Power springs, also known as clock, motor, or flat coil springs, are made of flat strip material which is wound on an arbor and confined in a case. Power springs store and release rotational energy through either the arbor or the case in which they are retained. They are unique among spring types in that they are almost always stored in a case or housing while unloaded. Figure 6.49 shows typical retainers, a case, and various ends.

6.12.1 Design Considerations

Power springs are stressed in bending, and stress is related to torque by

$$S = \frac{6M}{b t^2} \quad (6.66)$$

Load-deflection curves for power springs are difficult to predict. As a spring is wound up, material is wound onto the arbor. This material is drawn from that which

TABLE 6.26 Factors for Computing Rectangular Bars in Torsion

b/t	K_1	K_2
1.0	0.140	0.208
1.5	0.196	0.231
2.0	0.229	0.246
2.5	0.249	0.258
3.0	0.263	0.267
5.0	0.291	0.291

SOURCE: A. M. Wahl, *Mechanical Springs*, 2d ed., McGraw-Hill Book Company, New York, 1963.

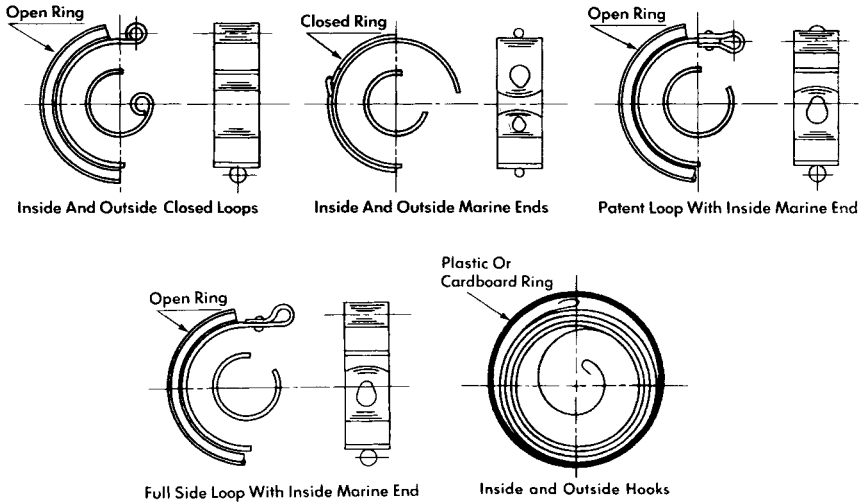


FIGURE 6.49 Typical power spring retainers and ends. (*Associated Spring, Barnes Group Inc.*)

was at rest against the case. Thus, the length of active material is constantly changing, which makes it difficult to develop a workable expression for the spring rate. For these reasons, ratios, tables, and graphical presentations are used to develop the design criteria.

The ratio of arbor diameter to thickness D_a/t is sometimes called the *life factor*. If it is too small, fatigue life will suffer. The life factor is usually maintained from 15 to 25. The ratio of active strip length to thickness L/t determines the flatness of the spring-gradient (torque-revolution) curve. The curve is flatter when L is longer. The usual range of the L/t ratio is from 5000 to 10 000. The ratio of the inside diameter of cup (case or housing) to thickness D_c/t is the *turns factor*. This determines the motion capability of the spring or indicates how much space is available between the arbor and the material lying against the inside of the case.

6.12.2 Design Procedure

In order to design a power spring that will deliver a given torque and number of turns, first determine its maximum torque in the fully wound condition. If a spring is required to deliver a minimum torque of 0.5 N·m for 10 revolutions (r) of windup and 10 r equals 80 percent unwound from solid, then from Fig. 6.50 we see that the torque at that point is 50 percent of the fully wound. Thus the fully wound torque is 1.0 N·m. Table 6.27 shows that a strip of steel 0.58 mm thick and 10 mm wide will provide 1.0 N·m of torque at the fully wound position per 10 mm of strip width.

Figure 6.51 shows that the average maximum solid stress for 0.58-mm-thick stock is about 1820 MPa. At the hardness normally supplied in steel strip for power springs, this is about 95 percent of tensile strength.

In Fig. 6.52, 10 turns relate to a length-to-thickness L/t ratio of 4300. With $t = 0.58$, L equals 2494 mm. Similarly, 4300 L/t relates to a D_c/t ratio of 107. Then $D_c = 62.06$ mm. If

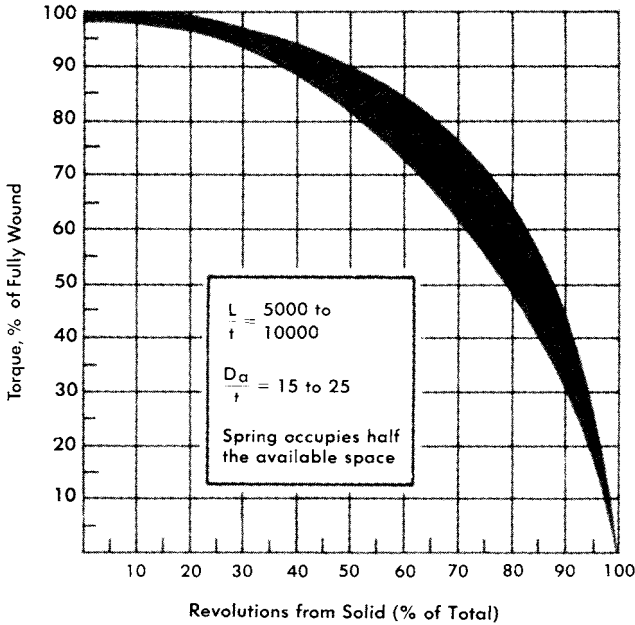


FIGURE 6.50 Typical normalized torque-revolution curve for power springs. (Associated Spring, Barnes Group Inc.)

$$L = \frac{D_c^2 - D_a^2}{2.55t} \quad (6.67)$$

then $D_a = \sqrt{D_c^2 - 2.55Lt} = 12.72 \text{ mm}$ and $D_a/t = 22$.

The equation for the number of turns a power spring will deliver, when it occupies half the space between arbor and case, is

$$\theta = \frac{\sqrt{2(D_c^2 + D_a^2)} - (D_c + D_a)}{2.55t} \quad (6.68)$$

In this example $\theta = 10 \text{ r}$.

Experience shows that highly stressed power springs, made from pretempered AISI 1095 steel with a hardness of HRC 50 to 52 and stressed to 100 percent of tensile strength, could be expected to provide approximately 10 000 full-stroke life cycles. If the maximum stress were 50 percent of tensile strength at full stroke, then a life of about 100 000 cycles could be expected.

The final design is as follows:

$$t = 0.023 \text{ in (0.58 mm)}$$

1095 carbon steel, HRC 51, no. 1 round edge

$$b = 0.394 \text{ in (10 mm)}$$

$$L = 98.188 \text{ in (2494 mm)}$$

$$D_a = 0.501 \text{ in (12.72 mm)}$$

$$D_c = 2.443 \text{ in (62.06 mm)}$$

TABLE 6.27 Torque per Unit of Width at Maximum Allowable Stress for Steel; L/t Range Is 5000 to 10 000

Thickness t		Unit Torque M		Thickness t		Unit Torque M	
mm	in	N · m/10 mm of width	lb · in/in of width	mm	in	N · m/10 mm of width	lb · in/in of width
0.127	0.005	0.0587	1.32	1.30	0.051	4.132	92.90
0.152	0.006	0.0841	1.89	1.37	0.054	4.541	102.1
0.178	0.007	0.1094	2.46	1.45	0.057	4.991	112.2
0.203	0.008	0.1419	3.19	1.60	0.063	5.947	133.7
0.229	0.009	0.1775	3.99	1.70	0.067	6.619	148.8
0.254	0.010	0.2171	4.88	1.83	0.072	7.504	168.7
0.279	0.011	0.2620	5.89	1.93	0.076	8.282	186.2
0.305	0.012	0.3074	6.91	2.03	0.080	8.981	201.9
0.330	0.013	0.3567	8.02	2.18	0.086	10.37	233.2
0.356	0.014	0.4101	9.22	2.34	0.092	11.74	264.0
0.381	0.015	0.4679	10.52	2.49	0.098	13.12	295
0.406	0.016	0.5271	11.85	2.67	0.105	14.86	334
0.432	0.017	0.5876	13.21	2.84	0.112	16.59	373
0.457	0.018	0.6530	14.68	3.05	0.120	18.82	423
0.483	0.019	0.7215	16.22	3.18	0.125	20.24	455
0.508	0.020	0.7953	17.88	3.43	0.135	23.35	525
0.584	0.023	1.025	23.05	3.58	0.141	25.35	570
0.635	0.025	1.189	26.72	3.76	0.148	27.76	624
0.711	0.028	1.452	32.65	3.96	0.156	30.69	690
0.813	0.032	1.841	41.40	4.11	0.162	33.00	742
0.889	0.035	2.144	48.20	4.50	0.177	39.23	882
1.041	0.041	2.824	63.50	4.75	0.187	43.81	985
1.19	0.047	3.585	80.60				

SOURCE: Associated Spring, Barnes Group Inc.

6.13 HOT-WOUND SPRINGS

6.13.1 Introduction

Springs are usually cold-formed when bar or wire diameters are less than 10 mm (approximately $\frac{3}{8}$ in). When the bar diameter exceeds 16 mm (approximately $\frac{5}{8}$ in), cold forming becomes impractical and springs are hot-wound.

Hot winding involves heating the steel into the austenitic range, winding hot, quenching to form martensite, and then tempering to the required properties. Although the most common types of hot-wound springs are compression springs for highway, off-highway, and railroad-vehicle suspension applications, torsion and extension springs can also be hot-wound.

6.13.2 Special Design Considerations

Design equations for hot-wound springs are the same as those for cold-formed springs except for the use of an empirical factor K_H which adjusts for effects related to hot-winding springs. Multiply the spring rate by K_H .

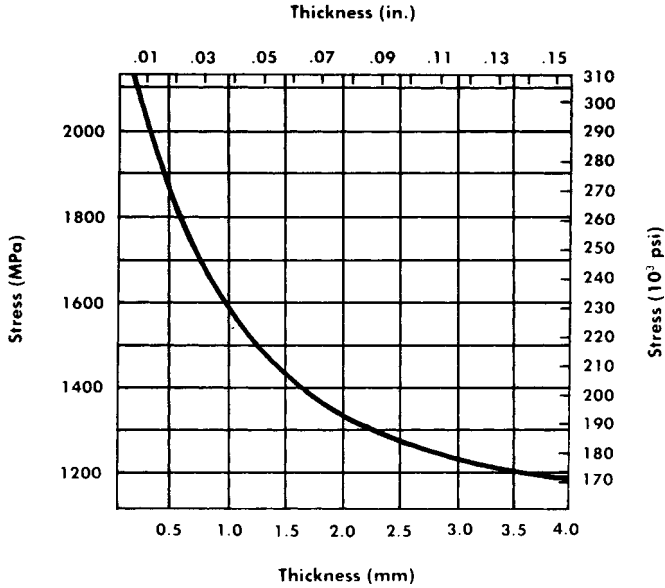


FIGURE 6.51 Average maximum solid stress in carbon-steel power springs. (*Associated Spring, Barnes Group Inc.*)

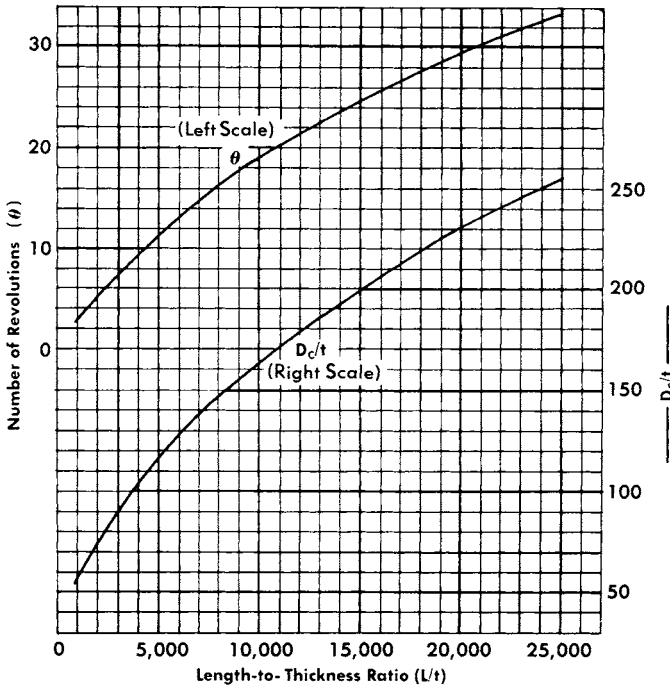


FIGURE 6.52 Relationships among number of revolutions, case diameter, strip length, and thickness for power springs. (*Associated Spring, Barnes Group Inc.*)

The values for factor K_H are 0.91 for springs made from hot-rolled carbon or low-alloy steel, *not* centerless ground; 0.96 for springs made from hot-rolled carbon or low-alloy steel, centerless ground; and 0.95 for torsion springs made from carbon or low-alloy steel.

The ends of hot-wound springs can be open or squared or either ground or not ground. Solid height is calculated in the same way as for cold-wound springs; but when space is limited, L_s can be reduced to $(N_t - 0.5)d$ by using a heavy grind.

The end configurations of extension or torsion springs must be formed hot at the same time as the spring is wound. If the configuration is complex, they may become cool in the process, and the whole spring may have to be reheated into the austenitic range. Note that hot-wound extension springs cannot have initial tension.

6.13.3 Materials

The common hot-wound alloys are AISI 5160, 5160H, and 1095 steels. The normal range of hardness is from HRC 44 to 48. Corresponding tensile strengths are 1430 to 1635 MPa.

The hot-rolled wire used in hot-wound springs is produced in standard sizes. Bar diameter variation and bar out-of-roundness tolerances are approximated in Table 6.28.

6.13.4 Choice of Operating Stress

Static Applications. The stress is calculated as in cold-wound springs. Use Table 6.29 for set-point information.

Cyclic Applications. Hot-wound springs made from hot-rolled wire are used in cyclic applications because rolled bars are subject to a variety of characteristic material defects mostly related to the bar surface condition. Therefore Table 6.30 can be

TABLE 6.28 Diameter and Out-of-Roundness Tolerances
for Hot-Rolled Carbon-Steel Bars

Diameter mm(in.)		Tolerance ± mm(in.)	Out-of-Roundness mm(in.)
Over	Through		
	8 (0.315)	0.13 (0.005)	0.20 (0.008)
8 (0.315)	10 (0.394)	0.15 (0.006)	0.22 (0.009)
10 (0.394)	15 (0.591)	0.18 (0.007)	0.27 (0.011)
15 (0.591)	20 (0.787)	0.20 (0.008)	0.30 (0.012)
20 (0.787)	25 (0.984)	0.23 (0.009)	0.34 (0.013)
25 (0.984)	30 (1.181)	0.25 (0.010)	0.38 (0.015)
30 (1.181)	35 (1.378)	0.30 (0.012)	0.45 (0.018)
35 (1.378)	40 (1.575)	0.35 (0.014)	0.52 (0.020)
40 (1.575)	60 (2.362)	0.40 (0.016)	0.60 (0.024)
60 (2.362)	80 (3.150)	0.60 (0.024)	0.90 (0.035)

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.29 Maximum Allowable Torsional Stress for Hot-Wound Helical Compression Springs in Static Applications

Before Set Removal	After Set Removal
50% of TS	65–75% of TS

Torsional stress after set removal depends on material size and amount of set removed.

SOURCE: Associated Spring, Barnes Group Inc.

TABLE 6.30 Maximum Allowable Torsional Stress for Hot-Wound Helical Compression Springs in Cyclic Applications

Fatigue Life (Cycles)	Percent of Tensile Strength	
	Not Shot-Peened	Shot-Peened
10^5	40	48
10^6	38	46
10^7	35	43

This information is based on **centerless ground** AISI 5160, 5160H and 1095, HRC 44 to 48, 25 mm (1") diameter. Set has not been removed. Conditions are: no surging, room temperature and non-corrosive environment.

$$\text{Stress ratio in fatigue} = \frac{S_{\text{Minimum}}}{S_{\text{Maximum}}} = 0.$$

SOURCE: Associated Spring, Barnes Group Inc.

used only for centerless ground alloy bars. Practical manufacturing tolerances for hot-wound springs can be found in ASTM A125.

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