
CHAPTER 39

PRESSURE CYLINDERS

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39.1 INTRODUCTION

The pressure vessels commonly used in industrial applications consist basically of a few closed shells of simple shape: spherical or cylindrical with hemispherical, conical, ellipsoidal, or flat ends. The shell components are joined together mostly by welding and riveting; sometimes they are bolted together using flanges.

Generally, the shell elements are axisymmetrical surfaces of revolution formed by rotation of a straight line or a plane curve known as a *meridian* or a *generator* about an axis of rotation. The plane containing the axis of rotation is called the *meridional plane*. The geometry of such simple shells is specified by the form of the midwall surface, usually two radii of curvature and the wall thickness at every point. The majority of pressure vessels are cylindrical.

In practice, the shell is considered thin if the wall thickness t is small in comparison with the circumferential radius of curvature R_θ and the longitudinal radius of curvature R_L . If the ratio $R_\theta/t > 10$, the shell is considered to be thin shell. This implies that the stresses developed in the shell wall by external loads can be considered to be uniformly distributed over the wall thickness. Many shells used in pressure-vessel construction are relatively thin ($10 < R_\theta/t < 500$), with the associated uniform distribution of stresses throughout the cylinder wall. Bending stresses in the walls of such membrane shells due to concentrated external loads are of higher intensity near the area of application of the load. The attenuation distance from the load where the stresses die out is short. The radial deformation of a shell subjected to internal pressure is assumed smaller than one-half the shell thickness. The shell thickness is designed to keep the maximum stresses below the yield strength of the material.

39.2 DESIGN PRINCIPLES OF PRESSURE CYLINDERS

In the design of a pressure vessel as a unit, a number of criteria should be considered. These are (1) selection of the material for construction of the vessel based on a working knowledge of the properties of the material, (2) determination of the magnitude of the induced stress in conformity with the requirements of standard codes, and (3) determination of the elastic stability. To simplify the design and keep the cost of fabrication low, the components of a vessel should be made in the form of simple geometric shapes, such as spherical, cylindrical, or conical. A spherical geometry provides minimum surface area per unit volume and requires minimum wall thickness for a given pressure. From the point of view of material savings and uniform distribution of induced stresses in the shell wall, a spherical shape is favorable. However, the fabrication of spherical vessels is more complicated and expensive than that of cylindrical ones. Spherical vessels are used commonly for storage of gas and liquids. For large-volume, low-pressure storage, spherical vessels are economical. But for higher-pressure storage, cylindrical vessels are more economical.

The most common types of vessels can be classified according to their

1. Functions: Storage vessels, reactors, boilers, mixers, and heat exchangers
2. Structural materials: Steel, cast iron, copper, plastics, etc.
3. Method of fabrication: Welded, cast, brazed, flanged, etc.
4. Geometry: Cylindrical, spherical, conical, combined
5. Scheme of loading: Working under internal or external pressure
6. Wall temperatures: Heated, unheated
7. Corrosion action: Moderate or high corrosion effects
8. Orientation in space: Vertical, horizontal, sloped
9. Method of assembly: Detachable, nondetachable
10. Wall thickness: Thin-walled ($d_o/d_i < 1.5$); thick-walled ($d_o/d_i \geq 1.5$)

Most vessels are designed as cylindrical shells fabricated of rolled sheet metal or as cylindrical shells that are cast. From the point of view of simplified structural design, the stressed state of the material of a thin-wall shell is considered biaxial. This is permissible because the magnitude of the radial stress in such a vessel wall is very small. The stressed state of the shell wall is generally the sum of the two basic components: (1) stressed state due to uniformly distributed forces on the surface as a result of fluid pressure, and (2) stressed state due to the action of the forces and moments distributed around the contour.

The stressed condition due to uniform pressure of fluid on the surface of the shell can be determined either by the membrane or moment theory. The *membrane theory* yields accurate enough results for most engineering applications and is widely used for structural designs. The *moment theory* is not usually applied for the determination of stresses due to uniformly distributed fluid forces on a surface. The equations resulting from the application of the moment theory are complex and the design process is quite involved.

End forces and moments are calculated in sections where a sudden change in load, wall thickness, or the properties of the shell material occurs. The stressed state induced by the applied end forces and moments are determined by application of the moment theory [39.1]. The induced stress and deformations due to end effects

influence mostly the zones where the end forces and moments are applied. In general, end stresses should be carefully evaluated, and design measures must be taken to keep them within the safe limit. High values of end stresses are to be avoided, especially for brittle materials and vessels operating under high alternating loads.

The structural reliability of the equipment parts is determined by two different approaches. The *theory of elasticity* requires that the strength be determined by the ultimate stress which the part can withstand without rupture, whereas the *theory of plasticity* suggests that the strength be determined by the ultimate load the part can withstand without residual deformation. The elastic theory is based on the assumption that the material of the component parts of the vessel is in an elastic state everywhere and nowhere does the state of stress exceed the yield point.

The parts of a pressure vessel are not generally bonded uniformly. When the structural design is based on the ultimate stress occurring at the most loaded region of the construction, the required material consumption could be excessive. In design works using the membrane theory, only average stresses are assumed, and no efforts are made to include the local stresses of significant magnitude. However, the designer should consider the probable adverse effects of very high local stress intensities and modify the design accordingly.

In heavily loaded parts of pressure vessels made of plastic materials, partial transition to the elastic-plastic state occurs. The plastic design method permits a realistic evaluation of the maximum load this vessel can withstand without failure.

39.3 DESIGN LOADS

The principal loads (i.e., forces) applied in actual operations to a vessel or its structural attachments to be considered in the design of such a vessel are

1. Design pressure (internal or external or both)
2. Dead loads
3. Wind loads
4. Thermal loads
5. Piping load
6. Impact load
7. Cyclic loads

Several loading combinations are often possible. The designer should consider them carefully for a particular situation and select the most probable combination of simultaneous loads for an economical and reliable design. Failure of a pressure vessel may be due to improper selection of materials, defects in materials due to inadequate quality control, incorrect design conditions, computational errors in design, improper or faulty fabrication procedures, or inadequate inspection and shop testing.

Design pressure is the pressure used to determine the minimum thickness of each of the vessel components. It is the difference between the internal and external pressures. A suitable margin above the operating pressure (usually 10 percent of operating pressure and a minimum of 10 psi) plus a static head of operating liquid must be included. The minimum design pressure for a “code” nonvacuum vessel is 15 psi. Vessels with negative gauge operating pressures are designed for full vacuum. In

determining the design load, the designer must consult the *ASME Boiler and Pressure Vessel Code*, Sec. VIII, Pressure Vessels, Division I. The maximum operating pressure is, according to code definition, the maximum gauge pressure permissible at the top of the completed vessel in its operating condition at the designated temperature. It is based on the nominal thickness, exclusive of the corrosion allowance, and the thickness required for loads other than fluid pressure.

The *required thickness* is the minimum thickness of the vessel wall, computed by code formula. The *design thickness* is the minimum required thickness plus an allowance for corrosion. The *nominal thickness* is the design thickness of the commercially available material actually used in making the vessel.

The vessel shell must be designed to withstand the combined effect of the pressure and temperature under the anticipated operating conditions. One has to use the pressure-vessel code and standard engineering analysis whenever applicable to determine the nominal stress at any part of the vessel. The stresses cannot in general exceed the allowable stress obtained by the code guidelines.

Design temperature is really more a design environmental condition than a load. *Thermal loads* originate from temperature changes combined with body restraints or existing temperature gradients. Reduction in structural strength due to rising temperature, increase in brittleness with rapidly falling temperature, and the associated changes in the vessel dimensions should be taken into careful consideration for the design. The code design temperature must not be less than the mean vessel-wall temperature under operating conditions. It can be determined by the standard heat-transfer methods. Generally, the standard vessel design temperature is equal to the maximum operating temperature of the fluid in the vessel plus 50°F for safety considerations. For low-temperature operation, the minimum fluid temperature is used for the design temperature. The designer should consult the *ASME Boiler and Pressure Vessel Codes* and the excellent books by Bednar [39.2] and Chuse [39.3].

Dead loads are the forces due to the weight of the vessel and the parts permanently connected with the vessel. The *pipng loads* acting on the vessel must be evaluated carefully. They consist of the weight of the pipe sections supported by the nozzles into the vessel shell, as well as operating forces due to thermal expansion of the pipes.

If the pressure vessel is exposed to the environment, the dynamic pressure forces due to the turbulent flow of air and associated gustiness must be evaluated for structural stability of the vessel support system. Determination of *wind load* is outlined in the SEI/ASCE 7-02 standard, and the designer must follow the guidelines to determine the effective dynamic load due to wind speed past the vessel and support-structure system. The same code provides guidelines for evaluation of seismic loads on flexible tall vessels.

In actual design situations, many combinations of loads are possible. Consequently, the designer should use discretion to determine the important ones and select only certain sets of design loads which can most probably occur simultaneously.

39.4 CYLINDRICAL SHELLS—STRESS ANALYSIS

Cylindrical shells are widely used in the manufacture of pressure vessels. They can be easily fabricated and have great structural strength. Depending on their function, they may be vertical or horizontal. Vertical pressure vessels are often preferred, especially for a thin-walled vessel operating under low internal pressure.

The design of a vertical cylindrical vessel becomes simple because the additional bending stresses due to weight of the vessel itself and of the fluid can be eliminated.

Cylindrical shells of malleable materials, such as steels, nonferrous metals, and their alloys, operating under internal pressures up to 10 MPa are fabricated mostly of rolled and welded sheets of corrosion-resistant material (see Ref. [39.1]). The minimum thickness of a shell rolled from low-carbon sheet metal and welded is 4.0 mm. The minimum thickness for an austenitic steel shell is 3.4 mm; for copper shells, 2.5 to 30 mm; and for cast shells, 20 to 25 mm.

39.4.1 Cylindrical Storage Vessel

Before the actual design calculation of a vertical cylindrical vessel is done, the relationship between the optimum height and the diameter should be determined. The volume of the sheet metal needed to make a vertical cylindrical storage tank (Fig. 39.1) is determined by the following formula:

$$\begin{aligned} V_s &= \pi d_o H t_1 + \frac{\pi d_o^2}{4} (t_k + t_a) \\ &= \pi d_o H t_1 + \frac{\pi d_o^2}{4} t_2 \end{aligned} \quad (39.1)$$

Vessel capacity is

$$V = \frac{\pi d_i^2}{4} H \quad (39.2)$$

The inside diameter of the vessel is

$$d_i = \sqrt{\frac{4V}{\pi H}} \quad (39.3)$$

Since thickness t_1 is very small in comparison with d_o and d_i , we may consider $d_i \approx d_o$. Substitution of Eq. (39.3) into Eq. (39.1) gives

$$V_s = 2t_1 \sqrt{\pi V H} + \frac{V t_2}{H} \quad (39.4)$$

The minimum volume of sheet metal is obtained by differentiating Eq. (39.4) with respect to H and equating the derivative to zero. Thus

$$\frac{dV_s}{dH} = t_1 \sqrt{\pi V/H} - \frac{V t_2}{H^2} = 0 \quad (39.5)$$

The optimum height and the optimum diameter of the tank are given by

$$H_{\text{opt}} = \left[\frac{V}{\pi} \left(\frac{t_2}{t_1} \right)^2 \right]^{1/3} \quad (39.6)$$

and

$$d_{\text{opt}} = 2 \left[\frac{V t_1}{\pi t_2} \right]^{1/3} \quad (39.7)$$

The shell thickness is given by

$$t_1 = \frac{H_{\text{opt}} \gamma d_{\text{opt}}}{2\sigma_a} \quad (39.8)$$

where γ = specific weight of the fluid in the tank, and σ_a = allowable stress. Substituting the optimum values of height and outside diameter in Eq. (39.8), the tank capacity V is recovered as

$$V = \pi t_1^2 \sqrt{\frac{\sigma_a^3}{\gamma^3 t_2}} \quad (39.9)$$

Considering a minimum shell thickness of $t_1 = 4$ mm and top and bottom thicknesses of 4 mm, we find $t_2 = 8$ mm. The specific weight of the liquid can be obtained from appropriate property tables. The allowable stress for low-carbon steel is 137 MPa. Using this information in Eq. (39.9), one can easily find the corresponding tank volume.

Pressure vessels subjected to internal and/or external pressures require application of the theoretical principles involved in shell analysis. The structural configurations of relatively simple geometric shapes such as cylinders and spheres have been studied extensively in the theories of plates and shells (see Refs. [39.2], [39.4], and [39.5]). In fact, no single chapter or even an entire textbook can cover all the advancements in the field, particularly where internal pressure, external pressure, and other modes of loading are present. Therefore, attempts will be made here only to cover the materials which are pertinent for design of simple pressure vessels and piping. The majority of piping and vessel components are designed for internal pressure and have been analyzed to a great degree of sophistication. Numerous cases involve application of external pressure as

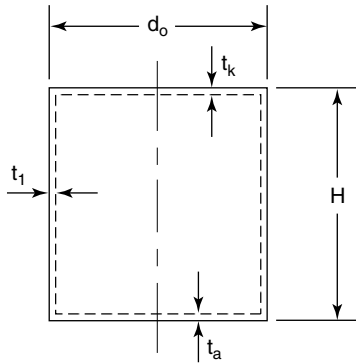


FIGURE 39.1 Cylindrical storage vessel.

well, where stresses, elastic stability, and possible structural failure must be analyzed and evaluated.

39.4.2 Membrane Theory

The common geometric shapes of pressure vessels used in industrial processes are spheres, cylinders, and ellipsoids. Conical and toroidal configurations are also used. Membrane theory can be applied to determine the stresses and deformations of such vessels when they have small thicknesses compared with other dimensions and have limited and small bending resistance normal to their surface. The stresses are considered to be average tension or compression over the thickness of the pipe or vessel wall acting tangential to the surface subjected to normal pressure. The imaginary surface passing through the middle of the wall thickness, however, extends in two directions and calls for rather complicated mathematical analysis, particularly when more than one expression for the curvature is necessary to describe the displacement of a point in the shell wall. In fact, in the more general sense it is neces-

sary to define a normal force, two transverse shear forces, two bending moments, and a torque in order to evaluate the state of stress at a point. Membrane theory simplifies this analysis to a great extent and permits one to ignore bending and twisting moments when shell thickness is small. In many practical cases, consideration of equilibrium of the forces allows us to develop necessary relations for stresses and displacements in terms of the shell parameters for adequate design.

39.4.3 Thin Cylindrical Shells under Internal Pressure

When a thin cylinder is subjected to an internal pressure, three mutually perpendicular principal stresses—hoop stress, longitudinal stress, and radial stress—are developed in the cylinder material. If the ratio of thickness t and the inside diameter of the cylinder d_i is less than 1:20, membrane theory may be applied and we may assume that the hoop and longitudinal stresses are approximately constant across the wall thickness. The magnitude of radial stress is negligibly small and can be ignored. It is to be understood that this simplified approximation is used extensively for the design of thin cylindrical pressure vessels. However, in reality, radial stress varies from zero at the outside surface to a value equal to the internal pressure at the inside surface. The ends of the cylinder are assumed closed. Hoop stress is set up in resisting the bursting effect of the applied pressure and is treated by taking the equilibrium of half of the cylindrical vessel, as shown in Fig. 39.2. Total force acting on the half cylinder is

$$F_h = p_i d_i L \quad (39.10)$$

where d_i = inside diameter of cylinder, and L = length of cylinder. The resisting force due to hoop stress σ_h acting on the cylinder wall, for equilibrium, must equal the force F_h . Thus

$$F_h = 2\sigma_h t L \quad (39.11)$$

Substituting for F_h from Eq. (39.10) into Eq. (39.11), one obtains the following relation:

$$\sigma_h = \frac{p_i d_i}{2t} \quad \text{or} \quad \sigma_h = \frac{p_i r_i}{t} \quad (39.12)$$

Despite its simplicity, Eq. (39.12) has wide practical applications involving boiler drums, accumulators, piping, casing chemical processing vessels, and nuclear pressure vessels. Equation (39.12) gives the maximum tangential stress in the vessel wall on the assumption that the end closures do not provide any support, as is the case with long cylinders and pipes. Hoop stress can also be expressed in terms of the radius of the circle passing through the midpoint of the thickness. Then we can write

$$\sigma_h = \frac{p_i (r_i + 0.5t)}{t} \quad (39.13)$$

The shell thickness is then expressed as

$$t = \frac{p_i r_i}{\sigma_h - 0.5p_i} \quad (39.14)$$

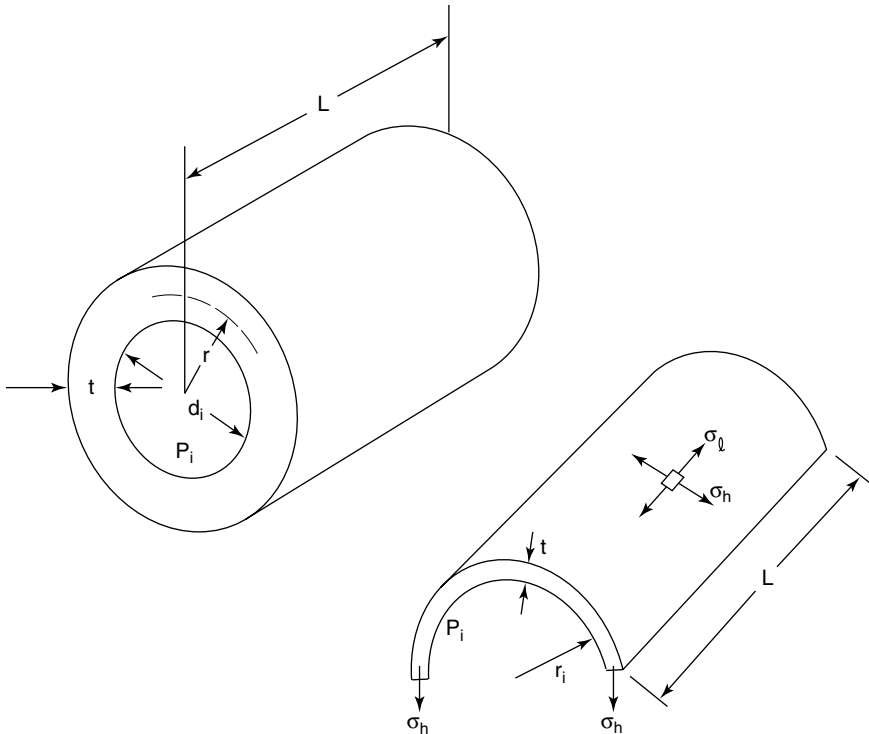


FIGURE 39.2 Thin cylindrical shell under internal pressure.

The code stress and shell thickness formulas based on inside radius approximate the more accurate thick-wall formula of Lamé, which is

$$t = \frac{p r_i}{S e - 0.6 p_i} \quad (39.15)$$

where e = code weld-joint efficiency, and S = allowable code stress.

Consideration of the equilibrium forces in the axial direction gives the longitudinal stress as

$$\sigma_\ell = \frac{p_i d_i}{4t}$$

or

$$\sigma_\ell = \frac{p_i r_i}{2t} \quad (39.16)$$

Equations (39.12) and Eqs. (39.16) reveal that the efficiency of the circumferential joint needs only be one-half that of the longitudinal joint. The preceding relations are good for elastic deformation only. The consequent changes in length, diameter, and intervolum of the cylindrical vessel subjected to inside fluid pressure can be determined easily.

The change in length is determined from the longitudinal strain ϵ_ℓ given by

$$\epsilon_\ell = \frac{1}{E} (\sigma_\ell - \nu \sigma_h) \quad (39.17)$$

where E = modulus of elasticity, and ν = Poisson's ratio. The change in length ΔL is then given by

$$\Delta L = \frac{L}{E} (\sigma_\ell - \nu \sigma_h)$$

$$\text{or} \quad \Delta L = \frac{p_i d_i}{4Et} (1 - 2\nu)L \quad (39.18)$$

Radial growth or dilatation under internal pressure is an important criterion in pipe and vessel analysis. For a long cylindrical vessel, the change in diameter is determined from consideration of the change in the circumference due to hoop stress. The change in circumference is obtained by multiplying hoop strain ϵ_h by the original circumference. The changed, or new, circumference is found to be equal $(\pi d_i + \pi d_i \epsilon_h)$. This is the circumference of the circle of diameter $d_i(1 + \epsilon_h)$. It can be shown easily that the diametral strain equals the hoop or circumferential strain. The change in diameter Δd_i is given by

$$\Delta d_i = \frac{d_i}{E} (\sigma_h - \nu \sigma_\ell)$$

$$\text{or} \quad \Delta d_i = \frac{p_i d_i^2}{4Et} (2 - \nu) \quad (39.19)$$

The change in the internal volume ΔV of the cylindrical vessel is obtained by multiplying the volume strain by the original volume of the vessel and is given by

$$\Delta V = \frac{p_i d_i}{4Et} (5 - 4\nu) V_o \quad (39.20)$$

where ΔV = change in volume, and V_o = original volume.

39.4.4 Thin Spherical Shell under Internal Pressure

A sphere is a symmetrical body. The internal pressure in a thin spherical shell will set up two mutually perpendicular hoop stresses of equal magnitude and a radial stress. When the thickness-to-diameter ratio is less than 1:20, membrane theory permits us to ignore the radial stress component. The stress system then reduces to one of equal biaxial hoop or circumferential stresses.

Considering the equilibrium of the half sphere, it can be seen that the force on the half sphere (Fig. 39.3) due to internal pressure p_i is

$$F = \frac{\pi}{4} d_i^2 p_i \quad (39.21)$$

The resisting force due to hoop stress is given by

$$F_h = \pi d_i t \sigma_h \quad (39.22)$$

For equilibrium, the resistive force must be equal to the force due to pressure. Therefore,

$$\frac{\pi}{4} d_i^2 p_i = \pi d_i t \sigma_h$$

$$\sigma_h = \frac{p_i d_i}{4t} \quad \text{or} \quad \sigma_h = \frac{p_i r_i}{2t} \quad (39.23)$$

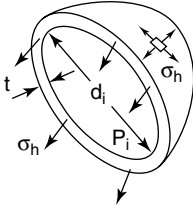


FIGURE 39.3 Spherical shell under internal pressure.

Equation (39.23) gives the relevant maximum stress in a spherical shell. The expression for hoop stress for a thin cylindrical shell and that for a thin spherical shell are similar. This simple deduction is of great importance in the design of pressure vessels because the thickness requirement for a spherical vessel of the same material strength and thickness-to-diameter ratio is only one-half that required for a cylindrical shell.

The change in internal volume ΔV of a spherical shell can be evaluated easily from consideration of volumetric strain and the original volume. Volumetric strain is equal to the sum of three equal and mutually perpendicular strains. The change in internal volume due to internal pressure is given by

$$\Delta V = \frac{3p_i d_i}{4Et} (1 - \nu) V_o \quad (39.24)$$

39.4.5 Vessels Subjected to Fluid Pressure

During the process of pressurization of a vessel, the fluid used as the medium changes in volume as the pressure is increased, and this must be taken into account when determining the amount of fluid which must be pumped into a cylinder in order to increase the pressure level in the vessel by a specified amount. The cylinder is considered initially full of fluid at atmospheric pressure. The necessary change in volume of the pressurizing fluid is given by

$$\Delta V_f = \frac{p V_o}{K} \quad (39.25)$$

where K = bulk modulus of fluid

p = pressure

V_o = original volume

The additional amount of fluid necessary to raise the pressure must take up the change in volume given by Eq. (39.25) together with the increase in internal volume of the cylinder. The amount of additional fluid V_a required to raise the cylinder pressure by p is given by

$$V_a = \frac{p d_i}{4Et} [5 - 4\nu] V + \frac{p V}{K} \quad (39.26)$$

It can be shown by similar analysis that the additional volume of fluid V_a required to pressurize a spherical vessel is given by

$$V_a = \frac{3pd_i}{4Et} (1 - \nu)V + \frac{pV}{K} \quad (39.27)$$

39.4.6 Cylindrical Vessel with Hemispherical Ends

One of the most commonly used configurations of pressure vessels is a cylindrical vessel with hemispherical ends, as shown in Fig. 39.4. The wall thickness of the cylindrical and hemispherical parts may be different. This is often necessary because the hoop stress in the cylinder is twice that in a sphere of the same inside diameter and wall thickness. The internal diameters of both parts are generally considered equal. In order that there should be no distortion or mismatch of hoop stress at the junction, the hoop stresses for the cylindrical part and the hemispherical part must be equal at the end junctions. Therefore,

$$\frac{pd_i}{4Et_c} [2 - \nu] = \frac{pd_i}{4Et_s} [1 - \nu] \quad (39.28)$$

where t_c = thickness of the cylinder, and t_s = thickness of the hemisphere. Simplification of Eq. (39.28) gives

$$\frac{t_s}{t_c} = \frac{(1 - \nu)}{(2 - \nu)} \quad (39.29)$$

39.4.7 Effects of Joints and End Plates in Vessel Fabrication

The preceding sections have assumed homogeneous materials and uniform material properties of the components. The effects of the end plates and joints will be the reduction of strength of the components due to characteristic fabrication techniques, such as riveted joints, welding, etc. To some extent, this reduction is taken into account by using a parameter, joint efficiency, in the equation of the stresses. For a thin cylindrical vessel as depicted in Fig. 39.5 the actual hoop and longitudinal stresses are given by the following equations:

$$\sigma_h = \frac{pd_i}{2t\eta_l} \quad (39.30)$$

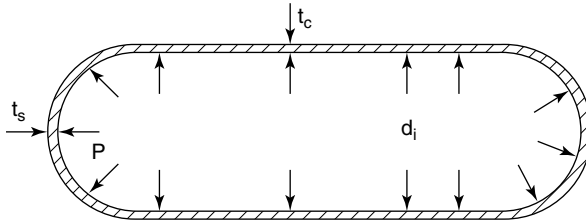


FIGURE 39.4 Thin cylindrical shell with hemispherical ends.

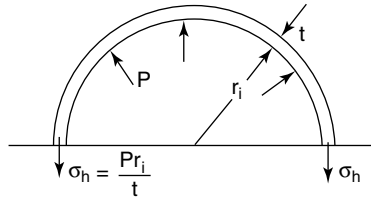


FIGURE 39.5 Thin cylindrical shell subjected to internal pressure.

$$\sigma_l = \frac{p d_l}{4 t \eta_c} \quad (39.31)$$

where η_l = efficiency of longitudinal joint, and η_c = efficiency of the circumferential joints. Similarly, for a thin sphere, hoop stress is given by

$$\sigma_h = \frac{p d_l}{4 t \eta} \quad (39.32)$$

where η = the joint efficiency.

39.5 THICK CYLINDRICAL SHELLS

The theoretical treatment of thin cylindrical shells assumes that hoop stress is constant across the thickness of the shell wall and also that there is no pressure gradient across the wall. The Lamé theory for determination of the stresses in the walls of thick cylindrical shells considers a mutually perpendicular, triaxial, principal-stress system consisting of the radial, hoop or tangential, and longitudinal stresses acting at any element in the wall. For the case of the shell depicted in Fig. 39.6, subjected to internal pressure only, radial stress σ_r , and hoop stress σ_h are given by

$$\sigma_r = \frac{p r_i^2}{(r_o^2 - r_i^2)} \left(\frac{r_o^2}{r^2} - 1 \right) \quad (39.33)$$

and

$$\sigma_h = \frac{p r_i^2}{(r_o^2 - r_i^2)} \left(\frac{r_o^2}{r^2} + 1 \right) \quad (39.34)$$

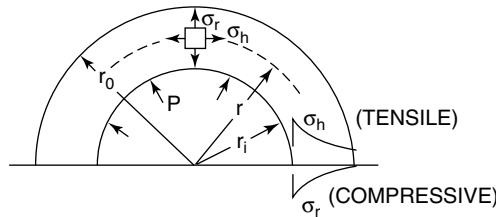


FIGURE 39.6 Thick cylindrical shell subjected to internal pressure.

where r_i and r_o = inside and outside radii of the shell, respectively, and r = any radius such that $r_i < r < r_o$.

In order to get an expression for the longitudinal stress σ_l , the shell is considered closed at both ends, as shown in Fig. 39.7. Simple consideration of the force equilibrium in the longitudinal direction yields

$$\sigma_l = \frac{pr_i^2}{r_o^2 - r_i^2} \quad (39.35)$$

The changes in the dimensions of the cylindrical shell can be determined by the following strain formulas:

Hoop strain:

$$E_h = \frac{1}{E} [\sigma_h - \nu\sigma_r - \nu\sigma_l] \quad (39.36)$$

Longitudinal strain:

$$E_l = \frac{1}{E} [\sigma_l - \nu\sigma_r - \nu\sigma_h] \quad (39.37)$$

It can be shown easily that diametral stress and the circumferential or hoop stress are equal. It is seen that the inside diameter-to-thickness ratio d_i/t is an important parameter in the stress formulas. For d_i/t values greater than 15, the error involved in using the thin-shell theory is within 5 percent. If, however, the mean diameter d_m is used for calculation of the thin-shell values instead of the inside diameter, the error reduces from 5 percent to approximately 0.25 percent at $d_m/t = 15.0$.

When the cylindrical shell is subjected to both the external pressure p_o and the internal pressure p_i , the radial and hoop stresses can be expressed by

$$\sigma_r = \frac{r_i^2 p_i - r_o^2 p_o}{r_o^2 - r_i^2} - \frac{(p_i - p_o) r_i^2 r_o^2}{r^2 (r_o^2 - r_i^2)} \quad (39.38)$$

and

$$\sigma_h = \frac{r_i^2 p_i - r_o^2 p_o}{r_o^2 - r_i^2} + \frac{(p_i - p_o) r_i^2 r_o^2}{r^2 (r_o^2 - r_i^2)} \quad (39.39)$$

It is observed from the preceding equations that the maximum value of σ_h occurs at the inner surface. The maximum radial stress σ_r is equal to the larger of the two pressures p_i and p_o . These equations are known as the *Lamé solution*. The sum of the two stresses remains constant, which indicates that the deformation of all elements in the

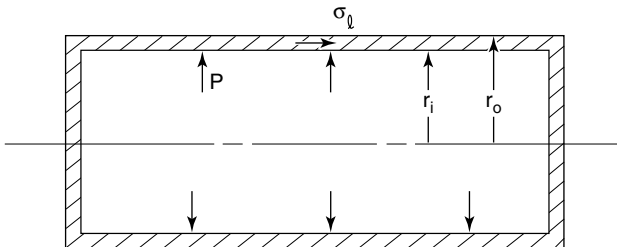


FIGURE 39.7 Thick cylindrical shell closed at both ends and subjected to internal pressure.

axial direction is the same, and the cross sections of the cylinder remain plane after deformation.

The maximum shearing stress at any point in the wall of a cylinder is equal to one-half the algebraic difference between the maximum and minimum principal stresses at that point. The axial or longitudinal stress is usually small compared with the radial and tangential stresses. The shear stress τ at any radial location in the wall is given by

$$\tau = \frac{\sigma_h - \sigma_r}{2}$$

or

$$\tau = \frac{(p_i - p_o)r_i^2 r_o^2}{(r_o^2 - r_i^2)r^2} \quad (39.40)$$

39.5.1 Compound Cylinders

Equation (39.34) indicates that there is a large variation in hoop stress across the wall of a cylindrical shell subjected to internal pressure p_i . In order to obtain a more uniform hoop-stress distribution, cylindrical shells are often fabricated by shrinking one onto the outside of another. When the outer cylinder contracts on cooling, the inner tube is brought into a state of compression. The outer cylinder will be in tension. If the compound cylinder is subjected to an internal pressure, the resultant hoop stresses will be the algebraic sum of those resulting from the internal pressure and those resulting from the shrinkage. The net result is a small total variation in hoop stress across the wall thickness. A similar effect is realized when a cylinder is wound with wire or steel tape under tension. Comings [39.4] gives a complete analysis for such systems. In order to obtain a favorable stress pattern, an analogous technique can be used by applying a sufficiently high internal pressure to produce plastic deformation in the inner part of the cylinder. On removal of this internal pressure, the residual stress persists, with the net effect of compression in the inner part and tension in the outer part of the cylinder. This process is called *autofrettage*. Harvey [39.5] discusses stress analysis for autofrettage in detail.

39.6 THERMAL STRESSES IN CYLINDRICAL SHELLS

Thermal stresses in the structure are caused by the temperature gradient in the wall and accompanying dimensional changes. In order to develop the thermal stresses, the structural member must be restrained in some way. The constraints in the thermal-stress problems in the design of vessels are usually divided into external and internal constraints. Whenever there is a significant temperature gradient across the vessel wall, thermal expansion takes place. Generally, the effects of the externally applied loads and the effects of thermal expansion or contraction are independently analyzed, and the final effects of the total combined stresses are considered additive when comparisons are made with maximum allowable stresses. It is usually assumed that thermal stresses are within the elastic range on the stress-strain curve of the vessel material. If the temperature is very high, the creep may become significant and must be considered. The induced thermal stresses often exceed the yield strength of the vessel material. Since thermal stresses are self-limiting, local plastic relaxation will tend to reduce the acting load.

The basic equations for thermal stresses can be developed by considering that a body is composed of unit cubes of uniform average temperature T . If the temperature of the unit cube is changed to T_1 such that $T_1 > T$ and is restricted, then there are three distinct cases according to the nature of restriction. If a cartesian coordinate system x, y, z is considered, then restrictions can be imagined in (1) the x direction, (2) in the x and y directions, and (3) in all three directions, i.e., in the x, y , and z directions. The corresponding thermal stresses are given by the following expressions:

$$\sigma_x = -\alpha E(T_1 - T) \quad (39.41)$$

$$\sigma_x = \sigma_y = \frac{-\alpha E}{(1 - \nu)} (T_1 - T) \quad (39.42)$$

$$\sigma_x = \sigma_y = \sigma_z = \frac{-\alpha E}{(1 - \nu)} (T_1 - T) \quad (39.43)$$

where α = coefficient of thermal expansion

ν = Poisson's ratio

E = modulus of elasticity

The equations are basic for direct thermal stresses under external or internal constraints and give the maximum thermal stress for the specific constraint.

The internal constraint is due to nonuniform temperature distribution in the body of the structure such that it does not allow free expansion of the individual body elements according to the local temperatures. In such a case, stresses are induced in the body in the absence of any external constraints. A thick cylindrical shell may be assumed to consist of thin, mutually connected, concentric cylindrical shells. Whenever a temperature gradient exists in the vessel wall due to heat transfer, the cylindrical elements will be at different average temperatures and will expand at different rates. The individual cylindrical elements will be constrained by each other, and consequently, thermal stresses will be induced in the otherwise non-restrained cylinder wall. Detailed treatments of the general analytical methods for solving thermal stresses caused by internal constraints are given by Gatewood [39.6], Bergen [39.7], and Goodier [39.8].

Most cylindrical shell and pressure-vessel problems can be reduced to two-dimensional stress problems. Generally, analytical solutions are possible for relatively simpler cases.

39.6.1 Thermal Stresses in Thin Cylindrical Vessels

Consider a hollow, thin cylindrical vessel (Fig. 39.8) subjected to a linear radial temperature gradient. The temperature at the inner wall is T_1 and is greater than temperature T_2 at the outer wall. If the vessel is long and restrained at the ends, then the longitudinal stress σ_z is given by

$$\sigma_z = \frac{E\alpha(T_1 - T_2)}{2(1 - \nu)} \quad (39.44)$$

where α = coefficient of thermal expansion, and ν = Poisson's ratio.

Estimation of thermal stresses in composite cylinders is quite involved. Faupel and Fisher [39.10] consider the thermal stresses in a multishell laminate of different materials with thermal gradients through each shell.

39.6.2 Thermal Stresses in Thick Cylindrical Vessels

When a thick-walled cylindrical vessel is subjected to a thermal gradient, nonuniform deformations are induced, and consequently, thermal stresses are developed.

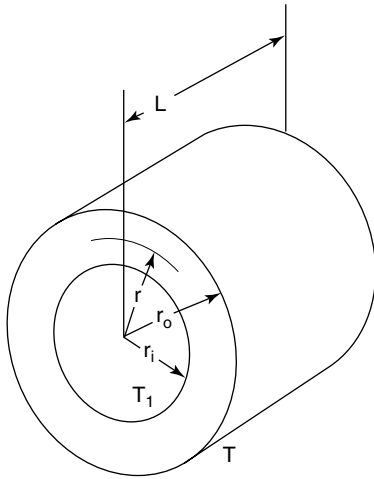


FIGURE 39.8 Hollow cylinder subjected to a temperature gradient.

Fluids under high pressure and temperature are generally transported through structures such as boilers, piping, heat exchangers, and other pressure vessels. Owing to the presence of a large temperature gradient between the inner and outer walls, thermal stresses are produced in these structures. Radial, hoop, and longitudinal stresses in thick, hollow cylinders with a thermal gradient across the wall may be estimated analytically, but generally the computations are lengthy, tedious, and time-consuming. Bhaduri [39.9] developed a set of dimensionless graphs from computer solutions, and they can be used to find the thermal-stress components with a few simple calculations.

In this technique, the temperature of the shell's inner surface at radius r_i and outer surface at radius r_o are considered to be T_i and T_o , respectively. The ends of the cylindrical shell are considered unrestrained. The longitudinal strain developed as a result of the thermal stresses is assumed to be uniform and constant. The temperature distribution in the shell wall is given by

$$T = T_i \left(\frac{\ln r_o/r}{\ln r_o/r_i} \right)$$

when the outer surface temperature $T_o = 0$, that is, when the temperature differences are measured relative to the outer surface temperature. The dimensionless hoop-stress function F_h , radial stress function F_r , and longitudinal stress function F_z are obtained analytically for cylindrical shells of various thickness ratios. The stress functions are given by the following equations:

$$\begin{aligned} F_h &= \frac{2(1-\nu)\sigma_h}{\alpha E T_i} = \frac{1}{\ln R_o} \left[1 - \ln \frac{R_o}{R} - \frac{(R_o/R)^2 + 1}{R_o^2 - 1} \ln R_o \right] \\ F_r &= \frac{2(1-\nu)\sigma_r}{\alpha E T_i} = \frac{1}{\ln R_o} \left[\ln \frac{R}{R_o} + \frac{(R_o/R)^2 - 1}{R_o^2 - 1} \ln R_o \right] \\ F_z &= \frac{2(1-\nu)\sigma_z}{\alpha E T_i} = \frac{1}{\ln R_o} \left(1 - 2 \ln \frac{R_o}{R} - \frac{2}{R_o^2 - 1} \ln R_o \right) \end{aligned} \quad (39.45)$$

where σ_h = hoop stress
 σ_r = radial stress
 σ_z = longitudinal stress
 ν = Poisson's ratio

$$\begin{aligned}\alpha &= \text{coefficient of thermal expansion} \\ E &= \text{Young's modulus} \\ R &= r/r_i \\ R_o &= r_o/r_i\end{aligned}$$

The stress functions are shown graphically for the radii ratios $r_o/r_i = 2.0, 2.5,$ and 3.00 , respectively, in Fig. 39.9. These curves are general enough to compute the hoop, radial, and shear stresses produced by temperature gradients encountered in most cylindrical-shell designs.

The procedure is simple. The value of R_o must be known to select the appropriate curve. For a particular value of R , the corresponding values of the stress functions can be read from the ordinate. The stresses at any radial location of the shell wall can be easily calculated from the known values of the stress function, property values of the shell material, and temperature of the inside surface of the shell.

39.7 FABRICATION METHODS AND MATERIALS

Welding is the most common method of fabrication of pressure-vessel shells. Structural parts such as stiffening rings, lifting lugs, support clips for piping, internal trays, and other parts are also attached to the vessel wall by welding. Welded joints are used for pipe-to-vessel connections to ensure optimum leak-proof design, particularly when the vessel contains hazardous fluid at a very high temperature. A structure whose parts are joined by welding is called a *weldment*. The most widely used industrial welding method is arc welding. It is any of several fusion welding processes wherein the heat of fusion is generated by electric arc. Residual stresses in a weld and in the adjoining areas cannot be avoided. They are quite complex. If the weld residual stress is superposed on the stress due to external loads and the resultant stress exceeds the yield point of the material, local plastic yielding will result in redistribution of the stress in ductile materials. A good weld requires a highly ductile material. In order to prevent loss of ductility in the welded region, low-carbon steels with less than 0.35 percent carbon content are used as construction materials. Carbon itself is a steel hardener. However, in the presence of a manganese content of 0.30 to 0.80 percent, carbon does not cause difficulties when present in steel up to 0.30 percent. Welding is a highly specialized manufacturing process. In pressure-vessel fabrication, the designer has to follow the code developed by the American Society of Mechanical Engineers. Chuse [39.3] provides simplified guidelines for designing welding joints for pressure vessels. For design purposes, welding is classified into three basic types: groove, fillet, and plug welds. Welding joints are described by the position of the pieces to be joined and are divided into five basic types: butt, tee, lap, corner, and edge. Bednar [39.2] describes various types of welds and outlines methods for stress calculation.

39.7.1 Construction Materials

A designer of a cylindrical pressure vessel should be familiar with commonly used construction materials to be able to specify them correctly in the design and in material specification. The selection of materials for code pressure vessels has to be made from the code-approved material specifications.

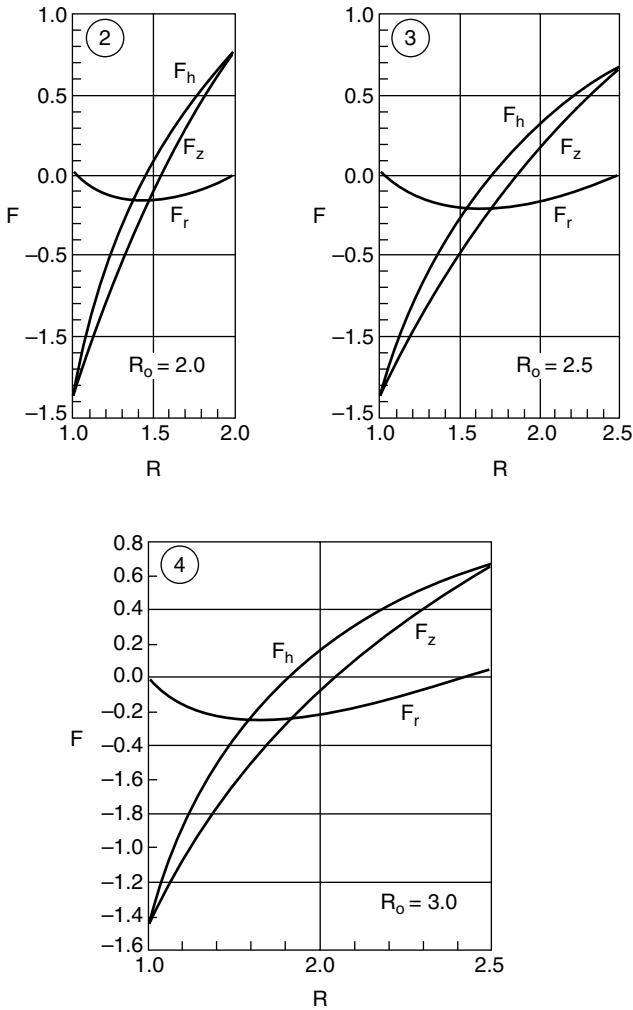


FIGURE 39.9 Thermal stress functions for thick cylindrical shell.
(From Ref. [39.9].)

39.8 DESIGN OF PRESSURE CYLINDERS

Cylindrical shells are commonly used in industrial applications for their adequate structural strength, ease of fabrication, and economical consumption of material. The vessels, depending on their function, can be vertical or horizontal. A vertical orientation is often preferred for thin-walled vessels operating under low internal pressure because of the additional bending stresses resulting from the weights of the vessel itself and the fluid in the vessel.

Cylindrical shells of such elastic materials as steels, nonferrous metals, and most alloys that operate under internal pressures up to 10 MPa are fabricated mostly of rolled and subsequently welded sheets. The joint connections of cylindrical shells made of copper or brass sheets are made by soldering with suitable solders. Cylindrical vessels of steel operating under pressures greater than 10 MPa are commonly fabricated of forged pieces that are heat-treated. Cylindrical shells of brittle materials for vessels operating under low internal pressures (approximately up to 0.8 MPa) are molded. Cast shells are usually fabricated with a bottom. In some cases, cast shells are made of elastic metals and their alloys.

The fabrication of the cylindrical vessels by the rolling of sheets is a very common method of manufacturing low- and medium-pressure (1.75 to 10.0 MPa) vessels.

Azbel and Cheremisinoff [39.1] give the following general guidelines for designing welded and soldered cylindrical shells:

1. The length of the seams should be minimized.
2. The minimum number of longitudinal seams should be provided.
3. Longitudinal and circumferential seams must be butt welded.
4. All joints should be accessible for inspection and repair.
5. It is not permissible to provide holes, access holes, or any opening on the seams, especially on longitudinal seams.

Shell thicknesses of vessels operating under very low pressures are not designed; they are selected on the basis of manufacturing considerations. Durability is estimated from the available information regarding corrosion resistance of the material. The minimum thicknesses of a shell rolled from sheet metals are given as follows: carbon and low-alloy steel, 4 mm; austenite steel, 3.0 to 4.0 mm; copper, 2.5 to 3.0 mm; and cast materials, 2.0 to 2.5 mm.

39.8.1 Design of Welded Cylindrical Shells

The structural design of cylindrical shells is based on membrane theory. In order to express the design relations in convenient form, it is necessary to (1) select the strength theory best reflecting the material behavior, (2) consider the weakening of the construction induced by welding and other connections, (3) consider wall thinning as a result of corrosion effects, and (4) establish a factor of safety and allowable stresses.

The distortion-energy theory (see Chap. 28) gives the basic design stress as

$$\sigma_d = [\sigma_r^2 + \sigma_\ell^2 + \sigma_r^2 - 2(\sigma_r\sigma_\ell + \sigma_\ell\sigma_r + \sigma_\ell\sigma_r)]^{1/2} \quad (39.46)$$

where σ_r , σ_ℓ , and σ_r = principal circumferential, longitudinal, and radial stresses, respectively. For a thin-walled cylindrical shell operating under an internal pressure p_i , the radial stress is assumed to be equal to zero, and the longitudinal stress induced in the shell is given by

$$\sigma_\ell = \frac{p_i d_i}{4t} \quad (39.16)$$

where d_i = inside diameter, and t = the thickness of the shell. The circumferential stress induced in the shell is given by

$$\sigma_i = \frac{p_i d_i}{2t} \quad (39.12)$$

When the deformation reaches plastic state, a definite amount of deformation energy is assumed (see Ref. [39.1]), and the value of the Poisson's coefficient ν is taken as equal to 0.5.

Substituting Eqs. (39.16) and (39.12) into Eq. (39.46) and assuming the value of Poisson's coefficient $\nu = 0.5$, the design stress can be expressed as

$$\sigma_d = 0.87 \frac{p_i r_o}{t} \quad (39.47)$$

where r_o = external radius of the shell. The external radius $r_o = d_o/2$, where d_o = external diameter of the shell.

It is observed that according to the distortion-energy theory, considering the combined effect of the longitudinal and tangential stresses, the design stress for plastic material is 13.0 percent less compared with the maximum value of the main stress. The allowable stress σ_a is therefore given by

$$\sigma_a = \frac{p_i d_o}{2.3t} \quad (39.48)$$

And the thickness of the shell t is given by

$$t = \frac{p d_o}{2.3\sigma_a} \quad (39.49)$$

Introducing the joint efficiency factor η_j and allowance factor t_c for corrosion, erosion, and negative tolerance of the shell thickness, the following relation for design thickness is obtained:

$$t_d = \frac{p d_o}{2.3\sigma_a \eta_j} + t_c \quad (39.50)$$

Azbel and Cheremisinoff [39.1], on the basis of similar analysis, give the following design formula for the determination of shell thickness:

$$t_d = \frac{p_i d_i}{2.3\eta_j \sigma_a - p_i} + t_c \quad (39.51)$$

where $d_o/d_i \leq 1.5$.

The proper selection of allowable stress to provide safe operation of a vessel is an important design consideration. The allowable stress value is influenced by a number of factors, such as (1) the strength and ductility of the material, (2) variations in load over time, (3) variations in temperature and their influence on ductility and strength of the material, and (4) the effects of local stress concentration, impact loading, fatigue, and corrosion.

In shell design, the criterion of determining the allowable stresses in an environmentally moderate temperature is the *ultimate tensile strength*. The pressure-vessel codes [39.3] use a safety factor of 4 based on the yield strength S_y for determining the allowable stresses for pressure vessels. The safety factor η_y is defined as

$$\eta_y = \frac{S_y}{\sigma_a} \quad (39.52)$$

It is known that for ductile metals, an increase in temperature results in an increase in ductility and a decrease in the yield-strength value. The allowable stress at higher temperatures can be estimated by using the following relation:

$$\sigma_a = \frac{S_y^T}{\eta_y} \quad (39.53)$$

where S_y^T = yield strength of the material at a given operating temperature.

Implied in the preceding analysis is an assumption that no creeping of the shell material is involved. This is generally a valid assumption for ferrous metals under load at temperatures less than 360°C. However, at higher operating temperatures, the material creeps under load, and an increase in stress takes place with time. The resulting stresses do not exceed the yield-strength values. It is to be noted that the yield point at high temperatures cannot generally be used as a criterion for allowance of the stressed state because creep of the material may induce failures as a result of the increased deformation over a long period of time. The basic design criterion for shells operating at moderate temperatures is the stability of size and dimensional integrity of the loaded elements. For a shell operating at higher temperatures, increase in size, which ensures that creep does not exceed a tolerable limit, should be considered. When a loaded material is under creep, there is an associated relaxation of stress. The stress decreases over time as a result of plastic deformation. The creep rate is dependent on temperature and state of stress in the metal. Azbel and Cheremisinoff [39.1] discuss the effects of creep and stress relaxation on loaded shells and give a formula for determining the allowable stresses and safety factors on the basis of fatigue stress.

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