

CHAPTER 2

Basic Thermodynamics, Fluid Mechanics: Definitions of Efficiency

Take your choice of those that can best aid your action. (SHAKESPEARE, Coriolanus.)

Introduction

THIS chapter summarises the basic physical laws of fluid mechanics and thermodynamics, developing them into a form suitable for the study of turbomachines. Following this, some of the more important and commonly used expressions for the efficiency of compression and expansion flow processes are given.

The laws discussed are:

- (1) the *continuity of flow equation*;
- (2) the *first law of thermodynamics* and the *steady flow energy equation*;
- (3) the *momentum equation*;
- (4) the *second law of thermodynamics*.

All of these laws are usually covered in first-year university engineering and technology courses, so only the briefest discussion and analysis is given here. Some fairly recent textbooks dealing comprehensively with these laws are those written by Cengel and Boles (1994), Douglas, Gasiorek and Swaffield (1995), Rogers and Mayhew (1992) and Reynolds and Perkins (1977). It is worth remembering that these laws are completely general; they are independent of the nature of the fluid or whether the fluid is compressible or incompressible.

The equation of continuity

Consider the flow of a fluid with density ρ , through the element of area dA , during the time interval dt . Referring to Figure 2.1, if c is the stream velocity the elementary mass is $dm = \rho c dt dA \cos \theta$, where θ is the angle subtended by the normal of the area element to the stream direction. The velocity component perpendicular to the area dA is $c_n = c \cos \theta$ and so $dm = \rho c_n dA dt$. The elementary rate of mass flow is therefore

$$\dot{m} = \frac{dm}{dt} = \rho c_n dA. \quad (2.1)$$

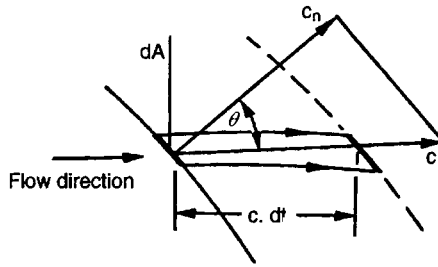


FIG. 2.1. Flow across an element of area.

Most analyses in this book are limited to one-dimensional steady flows where the velocity and density are regarded as constant across each section of a duct or passage. If A_1 and A_2 are the flow areas at stations 1 and 2 along a passage respectively, then

$$\dot{m} = \rho_1 c_{n1} A_1 = \rho_2 c_{n2} A_2 = \rho c_n A, \quad (2.2)$$

since there is no accumulation of fluid within the control volume.

The first law of thermodynamics – internal energy

The *first law of thermodynamics* states that if a system is taken through a complete cycle during which heat is supplied and work is done, then

$$\oint (dQ - dW) = 0, \quad (2.3)$$

where $\oint dQ$ represents the heat supplied to the system during the cycle and $\oint dW$ the work done by the system during the cycle. The units of heat and work in eqn. (2.3) are taken to be the same.

During a change of state from 1 to 2, there is a change in the property internal energy,

$$E_2 - E_1 = \int_1^2 (dQ - dW). \quad (2.4)$$

For an infinitesimal change of state

$$dE = dQ - dW. \quad (2.4a)$$

The steady flow energy equation

Many textbooks, e.g. Çengel and Boles (1994), demonstrate how the first law of thermodynamics is applied to the steady flow of fluid through a control volume so that the steady flow energy equation is obtained. It is unprofitable to reproduce this proof here and only the final result is quoted. Figure 2.2 shows a control volume representing a turbomachine, through which fluid passes at a steady rate of mass

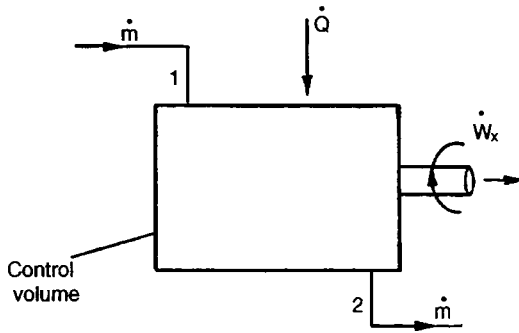


FIG. 2.2. Control volume showing sign convention for heat and work transfers.

flow $\dot{m}$, entering at position 1 and leaving at position 2. Energy is transferred from the fluid to the blades of the turbomachine, positive work being done (via the shaft) at the rate $\dot{W}_x$. In the general case positive heat transfer takes place at the rate $\dot{Q}$, from the surroundings to the control volume. Thus, with this sign convention the steady flow energy equation is

$$\dot{Q} - \dot{W}_x = \dot{m}[(h_2 - h_1) + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)], \quad (2.5)$$

where h is the specific enthalpy, $\frac{1}{2}c^2$ the kinetic energy per unit mass and gz the potential energy per unit mass.

Apart from hydraulic machines, the contribution of the last term in eqn. (2.5) is small and usually ignored. Defining stagnation enthalpy by $h_0 = h + \frac{1}{2}c^2$ and assuming $g(z_2 - z_1)$ is negligible, eqn. (2.5) becomes

$$\dot{Q} - \dot{W}_x = \dot{m}(h_{02} - h_{01}). \quad (2.6)$$

Most turbomachinery flow processes are adiabatic (or very nearly so) and it is permissible to write $\dot{Q} = 0$. For work producing machines (turbines) $\dot{W}_x > 0$, so that

$$\dot{W}_x = \dot{W}_t = \dot{m}(h_{01} - h_{02}). \quad (2.7)$$

For work absorbing machines (compressors) $\dot{W}_x < 0$, so that it is more convenient to write

$$\dot{W}_c = -\dot{W}_x = \dot{m}(h_{02} - h_{01}). \quad (2.8)$$

The momentum equation – Newton's second law of motion

One of the most fundamental and valuable principles in mechanics is *Newton's second law of motion*. The momentum equation relates the sum of the external forces acting on a fluid element to its acceleration, or to the rate of change of momentum in the direction of the resultant external force. In the study of turbomachines many applications of the momentum equation can be found, e.g. the force exerted upon

a blade in a compressor or turbine cascade caused by the deflection or acceleration of fluid passing the blades.

Considering a system of mass m , the sum of all the body and surface forces acting on m along some arbitrary direction x is equal to the time rate of change of the total x -momentum of the system, i.e.

$$\Sigma F_x = \frac{d}{dt}(mc_x). \quad (2.9)$$

For a control volume where fluid enters steadily at a uniform velocity c_{x1} and leaves steadily with a uniform velocity c_{x2} , then

$$\Sigma F_x = \dot{m}(c_{x2} - c_{x1}) \quad (2.9a)$$

Equation (2.9a) is the one-dimensional form of the steady flow momentum equation.

Euler's equation of motion

It can be shown for the steady flow of fluid through an elementary control volume that, in the absence of all shear forces, the relation

$$\frac{1}{\rho} dp + c dc + g dz = 0 \quad (2.10)$$

is obtained. This is Euler's equation of motion for one-dimensional flow and is derived from Newton's second law. By shear forces being absent we mean there is neither friction nor shaft work. However, it is not necessary that heat transfer should also be absent.

Bernoulli's equation

The one-dimensional form of Euler's equation applies to a control volume whose thickness is infinitesimal in the stream direction (Figure 2.3). Integrating this equation in the stream direction we obtain

$$\int_1^2 \frac{1}{\rho} dp + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1) = 0 \quad (2.10a)$$

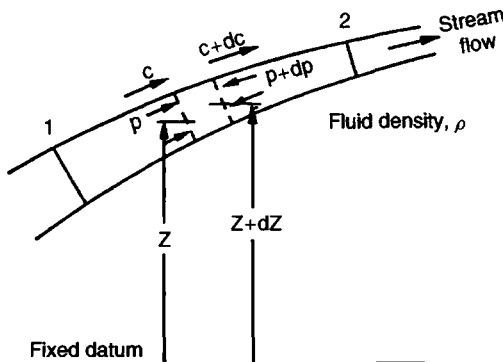


FIG. 2.3. Control volume in a streaming fluid.

which is Bernoulli's equation. For an incompressible fluid, ρ is constant and eqn. (2.10a) becomes

$$\frac{1}{\rho}(p_{02} - p_{01}) + g(z_2 - z_1) = 0, \quad (2.10b)$$

where stagnation pressure is $p_0 = p + \frac{1}{2}\rho c^2$.

When dealing with hydraulic turbomachines, the term *head* H occurs frequently and describes the quantity $z + p_0/(\rho g)$. Thus eqn. (2.10b) becomes

$$H_2 - H_1 = 0. \quad (2.10c)$$

If the fluid is a gas or vapour, the change in gravitational potential is generally negligible and eqn. (2.10a) is then

$$\int_1^2 \frac{1}{\rho} dp + \frac{1}{2}(c_2^2 - c_1^2) = 0. \quad (2.10d)$$

Now, if the gas or vapour is subject to only a small pressure change the fluid density is sensibly constant and

$$p_{02} = p_{01} = p_0, \quad (2.10e)$$

i.e. the stagnation pressure is constant (this is also true for a *compressible isentropic process*).

Moment of momentum

In dynamics much useful information is obtained by employing Newton's second law in the form where it applies to the moments of forces. This form is of central importance in the analysis of the energy transfer process in turbomachines.

For a system of mass m , the vector sum of the moments of all external forces acting on the system about some arbitrary axis A — A fixed in space is equal to the time rate of change of angular momentum of the system about that axis, i.e.

$$\tau_A = m \frac{d}{dt}(rc_\theta), \quad (2.11)$$

where r is distance of the mass centre from the axis of rotation measured along the normal to the axis and c_θ the velocity component mutually perpendicular to both the axis and radius vector r .

For a control volume the *law of moment of momentum* can be obtained. Figure 2.4 shows the control volume enclosing the rotor of a generalised turbomachine. Swirling fluid enters the control volume at radius r_1 with tangential velocity $c_{\theta 1}$ and leaves at radius r_2 with tangential velocity $c_{\theta 2}$. For one-dimensional steady flow

$$\tau_A = \dot{m}(r_2 c_{\theta 2} - r_1 c_{\theta 1}) \quad (2.11a)$$

which states that, the sum of the moments of the external forces acting on fluid temporarily occupying the control volume is equal to the net time rate of efflux of angular momentum from the control volume.

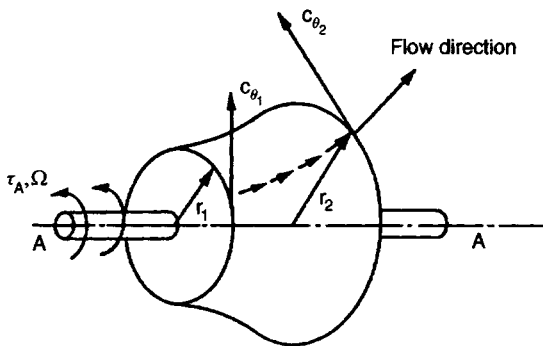


FIG. 2.4. Control volume for a generalised turbomachine.

Euler's pump and turbine equations

For a pump or compressor rotor running at angular velocity Ω , the rate at which the rotor does work on the fluid is

$$\tau_A \Omega = \dot{m}(U_2 c_{\theta 2} - U_1 c_{\theta 1}), \quad (2.12)$$

where the blade speed $U = \Omega r$.

Thus the work done on the fluid per unit mass or specific work, is

$$\Delta W_c = \frac{\dot{W}_c}{\dot{m}} = \frac{\tau_A \Omega}{\dot{m}} = U_2 c_{\theta 2} - U_1 c_{\theta 1} > 0. \quad (2.12a)$$

This equation is referred to as *Euler's pump equation*.

For a turbine the fluid does work *on* the rotor and the sign for work is then reversed. Thus, the specific work is

$$\Delta W_t = \frac{\dot{W}_t}{\dot{m}} = U_1 c_{\theta 1} - U_2 c_{\theta 2} > 0. \quad (2.12b)$$

Equation (2.12b) will be referred to as *Euler's turbine equation*.

Defining rothalpy

In a compressor or pump the specific work done on the fluid equals the rise in stagnation enthalpy. Thus, combining eqns. (2.8) and (2.12a),

$$\Delta W_c = \dot{W}_c / \dot{m} = U_2 c_{\theta 2} - U_1 c_{\theta 1} = h_{02} - h_{01}. \quad (2.12c)$$

This relationship is true for steady, adiabatic and irreversible flow in compressor or in pump impellers. After some rearranging of eqn. (2.12c) and writing $h_0 = h + \frac{1}{2}c^2$, then

$$h_1 + \frac{1}{2}c_1^2 - U_1 c_{\theta 1} = h_2 + \frac{1}{2}c_2^2 - U_2 c_{\theta 2} = I. \quad (2.12d)$$

According to the above reasoning a new function I has been defined having the same value at exit from the impeller as at entry. The function I has acquired the

widely used name *rothalpy*, a contraction of rotational stagnation enthalpy, and is a fluid mechanical property of some importance in the study of relative flows in rotating systems. As the value of rothalpy is apparently* unchanged between entry and exit of the impeller it is deduced that it must be constant along the flow lines between these two stations. Thus, the rothalpy can be written generally as

$$I = h + \frac{1}{2}c^2 - U c_{\theta}. \quad (2.12e)$$

The same reasoning can be applied to the thermomechanical flow through a turbine with the same result.

The second law of thermodynamics – entropy

The *second law of thermodynamics*, developed rigorously in many modern thermodynamic textbooks, e.g. Çengel and Boles (1994), Reynolds and Perkins (1977), Rogers and Mayhew (1992), enables the concept of entropy to be introduced and ideal thermodynamic processes to be defined.

An important and useful corollary of the second law of thermodynamics, known as the *Inequality of Clausius*, states that for a system passing through a cycle involving heat exchanges,

$$\oint \frac{dQ}{T} \leq 0, \quad (2.13)$$

where dQ is an element of heat transferred to the system at an absolute temperature T . If all the processes in the cycle are reversible then $dQ = dQ_R$ and the equality in eqn. (2.13) holds true, i.e.

$$\oint \frac{dQ_R}{T} = 0. \quad (2.13a)$$

The property called entropy, for a finite change of state, is then defined as

$$S_2 - S_1 = \int_1^2 \frac{dQ_R}{T}. \quad (2.14)$$

For an incremental change of state

$$dS = mds = \frac{dQ_R}{T}, \quad (2.14a)$$

where m is the mass of the system.

With steady one-dimensional flow through a control volume in which the fluid experiences a change of state from condition 1 at entry to 2 at exit,

$$\int_1^2 \frac{d\dot{Q}}{T} \leq \dot{m}(s_2 - s_1). \quad (2.15)$$

* A discussion of recent investigations into the conditions required for the conservation of rothalpy is deferred until Chapter 7.

If the process is adiabatic, $d\dot{Q} = 0$, then

$$s_2 \geq s_1. \quad (2.16)$$

If the process is *reversible* as well, then

$$s_2 = s_1. \quad (2.16a)$$

Thus, for a flow which is adiabatic, the ideal process will be one in which the entropy remains unchanged during the process (the condition of *isentropy*).

Several important expressions can be obtained using the above definition of entropy. For a system of mass m undergoing a reversible process $dQ = dQ_R = mTds$ and $dW = dW_R = mpdv$. In the absence of motion, gravity and other effects the first law of thermodynamics, eqn. (2.4a) becomes

$$Tds = du + pdv. \quad (2.17)$$

With $h = u + pv$ then $dh = du + pdv + vdp$ and eqn. (2.17) then gives

$$Tds = dh - vdp. \quad (2.18)$$

Definitions of efficiency

A large number of efficiency definitions are included in the literature of turbomachines and most workers in this field would agree there are too many. In this book only those considered to be important and useful are included.

Efficiency of turbines

Turbines are designed to convert the available energy in a flowing fluid into useful mechanical work delivered at the coupling of the output shaft. The efficiency of this process, the *overall efficiency* η_0 , is a performance factor of considerable interest to both designer and user of the turbine. Thus,

$$\eta_0 = \frac{\text{mechanical energy available at coupling of output shaft in unit time}}{\text{maximum energy difference possible for the fluid in unit time}}.$$

Mechanical energy losses occur between the turbine rotor and the output shaft coupling as a result of the work done against friction at the bearings, glands, etc. The magnitude of this loss as a fraction of the total energy transferred to the rotor is difficult to estimate as it varies with the size and individual design of turbomachine. For small machines (several kilowatts) it may amount to 5% or more, but for medium and large machines this loss ratio may become as little as 1%. A detailed consideration of the mechanical losses in turbomachines is beyond the scope of this book and is not pursued further.

The *isentropic efficiency* η_i or *hydraulic efficiency* η_h for a turbine is, in broad terms,

$$\eta_i (\text{or } \eta_h) = \frac{\text{mechanical energy supplied to the rotor in unit time}}{\text{maximum energy difference possible for the fluid in unit time}}.$$

Comparing the above definitions it is easily deduced that the *mechanical efficiency* η_m , which is simply the ratio of shaft power to rotor power, is

$$\eta_m = \eta_0 / \eta_t \text{ (or } \eta_0 / \eta_h \text{)}.$$

In the following paragraphs the various definitions of hydraulic and adiabatic efficiency are discussed in more detail.

For an incremental change of state through a turbomachine the steady flow energy equation, eqn. (2.5), can be written

$$d\dot{Q} - d\dot{W}_x = \dot{m}[dh + \frac{1}{2}d(c^2) + g dz].$$

From the second law of thermodynamics

$$d\dot{Q} \leq \dot{m}Tds = \dot{m} \left(dh - \frac{1}{\rho} dp \right).$$

Eliminating $d\dot{Q}$ between these two equations and rearranging

$$d\dot{W}_x \leq -\dot{m} \left[\frac{1}{\rho} dp + \frac{1}{2}d(c^2) + g dz \right]. \quad (2.19)$$

For a turbine expansion, noting $\dot{W}_x = \dot{W}_t > 0$, integrate eqn. (2.19) from the initial state 1 to the final state 2,

$$\dot{W}_x \leq \dot{m} \left[\int_2^1 \frac{1}{\rho} dp + \frac{1}{2}(c_1^2 - c_2^2) + g(z_1 - z_2) \right]. \quad (2.20)$$

For a reversible adiabatic process, $Tds = 0 = dh - dp/\rho$. The incremental maximum work output is then

$$d\dot{W}_{x_{\max}} = -\dot{m}[dh + \frac{1}{2}d(c^2) + g dz]$$

Hence, the overall maximum work output between initial state 1 and final state 2 is

$$\begin{aligned} \dot{W}_{x_{\max}} &= \dot{m} \int_2^1 \left[dh + \frac{1}{2}d(c^2) + g dz \right] \\ &= \dot{m}[(h_{01} - h_{02s}) + g(z_1 - z_2)] \end{aligned} \quad (2.20a)$$

where the subscript s in eqn. (2.20a) denotes that the change of state between 1 and 2 is isentropic.

For an incompressible fluid, in the absence of friction, the maximum work output from the turbine (ignoring frictional losses) is

$$\dot{W}_{x_{\max}} = \dot{m}g[H_1 - H_2], \quad (2.20b)$$

where $gH = p/\rho + \frac{1}{2}c^2 + gz$.

Steam and gas turbines

Figure 2.5a shows a Mollier diagram representing the expansion process through an adiabatic turbine. Line 1–2 represents the actual expansion and line 1–2 $_s$ the ideal or reversible expansion. The fluid velocities at entry to and at exit from a turbine

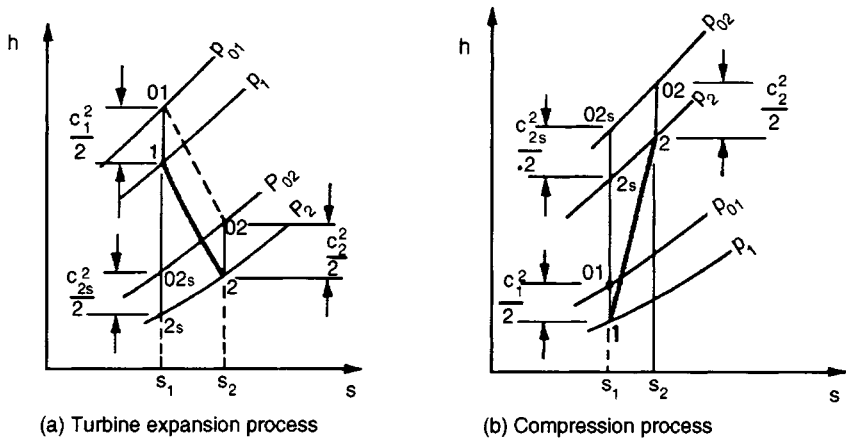


FIG. 2.5. Enthalpy-entropy diagrams for turbines and compressors.

may be quite high and the corresponding kinetic energies may be significant. On the other hand, for a compressible fluid the potential energy terms are usually negligible. Hence the *actual* turbine rotor *specific work*

$$\Delta W_x = \dot{W}_x / \dot{m} = h_{01} - h_{02} = (h_1 - h_2) + \frac{1}{2}(c_1^2 - c_2^2)$$

Similarly, the *ideal* turbine rotor specific work between the same two pressures is

$$\Delta W_{\max} = \dot{W}_{x_{\max}} / \dot{m} = h_{01} - h_{02s} = (h_1 - h_{2s}) + \frac{1}{2}(c_1^2 - c_{2s}^2).$$

In Figure 2.5a the actual turbine work/unit mass of fluid is the stagnation enthalpy change between state points 01 and 02 which lie on the stagnation pressure lines p_{01} and p_{02} respectively. The ideal turbine work per unit mass of fluid is the stagnation enthalpy change during the *isentropic process* between the same two pressures. The kinetic energy of the fluid at the end of the ideal process $\frac{1}{2}c_{2s}^2$ is not, however, the same as that at the end of the actual process $\frac{1}{2}c_2^2$. This may be adduced as follows. Taking for simplicity a perfect gas, then $h = C_p T$ and $p/\rho = RT$. Consider the constant pressure line p_2 (Figure 2.5a); as $T_2 > T_{2s}$ then $\rho_{2s} > \rho_2$. From continuity $\dot{m}/A = \rho c$ and since we are dealing with the same area, $c_2 > c_{2s}$, and the kinetic energy terms are not equal. The difference in practice is usually negligible and often ignored.

There are several ways of expressing efficiency, the choice of definition depending largely upon whether the *exit kinetic energy* is usefully employed or is wasted. An example where the exhaust kinetic energy is not wasted is from the last stage of an aircraft gas turbine where it contributes to the jet propulsive thrust. Likewise, the exit kinetic energy from one stage of a multistage turbine where it is used in the next stage, provides another example. For these two cases the turbine and stage adiabatic efficiency η , is the *total-to-total efficiency* and is defined as

$$\eta_{tt} = \Delta W_x / \Delta W_{x_{\max}} = (h_{01} - h_{02}) / (h_{01} - h_{02s}). \quad (2.21)$$

If the difference between the inlet and outlet kinetic energies is small, i.e. $\frac{1}{2}c_1^2 \div \frac{1}{2}c_2^2$, then

$$\eta_{it} = (h_1 - h_2)/(h_1 - h_{2s}) \quad (2.21a)$$

When the exhaust kinetic energy is not usefully employed and entirely wasted, the relevant adiabatic efficiency is the *total-to-static efficiency* η_{ts} . In this case the ideal turbine work is that obtained between state points 01 and 2s. Thus

$$\begin{aligned} \eta_{ts} &= (h_{01} - h_{02})/(h_{01} - h_{02s} + \frac{1}{2}c_{2s}^2) \\ &= (h_{01} - h_{02})/(h_{01} - h_{2s}). \end{aligned} \quad (2.22)$$

If the difference between inlet and outlet kinetic energies is small, eqn. (2.22) becomes

$$\eta_{ts} = (h_1 - h_2)/(h_1 - h_{2s} + \frac{1}{2}c_1^2). \quad (2.22a)$$

A situation where the outlet kinetic energy is wasted is a turbine exhausting directly to the surroundings rather than through a diffuser. For example, auxiliary turbines used in rockets often do not have exhaust diffusers because the disadvantages of increased mass and space utilisation are greater than the extra propellant required as a result of reduced turbine efficiency.

Hydraulic turbines

When the working fluid is a liquid, the turbine hydraulic efficiency η_h , is defined as the work supplied by the rotor in unit time divided by the hydrodynamic energy difference of the fluid per unit time, i.e.

$$\eta_h = \frac{\Delta W_x}{\Delta W_{x_{\max}}} = \frac{\Delta W_x}{g(H_1 - H_2)}. \quad (2.23)$$

Efficiency of compressors and pumps

The isentropic efficiency η_c of a compressor or the *hydraulic efficiency* of a pump η_h is broadly defined as,

$$\eta_c(\text{or } \eta_h) = \frac{\text{useful (hydrodynamic) energy input to fluid in unit time}}{\text{power input to rotor}}.$$

The power input to the rotor (or impeller) is always less than the power supplied at the coupling because of external energy losses in the bearings and glands, etc. Thus, the overall efficiency of the compressor or pump is

$$\eta_o = \frac{\text{useful (hydrodynamic) energy input to fluid in unit time}}{\text{power input to coupling of shaft}}$$

Hence the mechanical efficiency is

$$\eta_m = \eta_o/\eta_c(\text{or } \eta_o/\eta_h).$$

In eqn. (2.19), for a compressor or pump process, replace $-d\dot{W}_x$ with $d\dot{W}_c$ and rearrange the inequality to give the incremental work input

$$d\dot{W}_c \geq \dot{m} \left[\frac{1}{\rho} dp + \frac{1}{2} d(c^2) + g dz \right]. \quad (2.24)$$

The student should carefully check the fact that the rhs of this inequality is *positive*, working from eqn. (2.19)

For a complete adiabatic compression process going from state 1 to state 2, the overall work input rate is

$$\dot{W}_c \geq \dot{m} \left[\int_1^2 \frac{dp}{\rho} + \frac{1}{2} (c_2^2 - c_1^2) + g(z_2 - z_1) \right]. \quad (2.25)$$

For the corresponding *reversible* adiabatic compression process, noting that $T ds = 0 = dh - dp/\rho$, the minimum work input rate is

$$\dot{W}_{c \min} = \dot{m} \int_1^{2s} \left[dh + \frac{1}{2} dc^2 + g dz \right] = \dot{m} [(h_{02s} - h_{01}) + g(z_2 - z_1)]. \quad (2.26)$$

From the steady flow energy equation, for an adiabatic process in a compressor

$$\dot{W}_c = \dot{m}(h_{02} - h_{01}). \quad (2.27)$$

Figure 2.5b shows a Mollier diagram on which the actual compression process is represented by the state change 1–2 and the corresponding ideal process by 1–2s. For an adiabatic compressor the only meaningful efficiency is the total-to-total efficiency which is

$$\begin{aligned} \eta_c &= \frac{\text{minimum adiabatic work input per unit time}}{\text{actual adiabatic work input to rotor per unit time}} \\ &= \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}. \end{aligned} \quad (2.28)$$

If the difference between inlet and outlet kinetic energies is small, $\frac{1}{2}c_1^2 \div \frac{1}{2}c_2^2$ and

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}. \quad (2.28a)$$

For *incompressible* flow, eqn. (2.25) gives

$$\Delta W_p = \dot{W}_p / \dot{m} \geq [(p_2 - p_1)/\rho + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)] \geq g[H_2 - H_1].$$

For the ideal case with no fluid friction

$$\Delta W_{p \min} = g[H_2 - H_1]. \quad (2.29)$$

For a pump the hydraulic efficiency is defined as

$$\eta_h = \frac{\Delta W_{p \min}}{\Delta W_p} = \frac{g[H_2 - H_1]}{\Delta W_p}. \quad (2.30)$$

From the relation $Tds = dh - vdp$, for a constant pressure process $(\partial h/\partial s)_{p_1} = T$. This means that the higher the fluid temperature the *greater* is the slope of the constant pressure lines on the Mollier diagram. For a gas where h is a function of T , constant pressure lines diverge and the slope of the line p_2 is greater than the slope of line p_1 at the same value of entropy. At equal values of T , constant pressure lines are of equal slope as indicated in Figure 2.6. For the special case of a *perfect gas* (where C_p is constant), $C_p(dT/ds) = T$ for a constant pressure process. Integrating this expression results in the equation for a constant pressure line, $s = C_p \log T + \text{constant}$.

Returning now to the more general case, since

$$\Sigma \delta W = \{(h_x - h_1) + (h_y - h_x) + \dots\} = (h_2 - h_1),$$

then

$$\eta_p = [(h_{xs} - h_1) + (h_{ys} - h_x) + \dots]/(h_2 - h_1).$$

The adiabatic efficiency of the *whole* compression process is

$$\eta_c = (h_{2s} - h_1)/(h_2 - h_1).$$

Because of the divergence of the constant pressure lines

$$\{(h_{xs} - h_1) + (h_{ys} - h_x) + \dots\} > (h_{2s} - h_1),$$

i.e.

$$\Sigma \delta W_{\min} > W_{\min}.$$

Therefore,

$$\eta_p > \eta_c.$$

Thus, for a compression process the isentropic efficiency of the machine is *less* than the small stage efficiency, the difference being dependent upon the divergence of the constant pressure lines. Although the foregoing discussion has been in terms of static states it can be regarded as applying to stagnation states if the inlet and outlet kinetic energies from each stage are equal.

Small stage efficiency for a perfect gas

An explicit relation can be readily derived for a perfect gas (C_p is constant) between small stage efficiency, the overall isentropic efficiency and pressure ratio. The analysis is for the limiting case of an infinitesimal compressor stage in which the incremental change in pressure is dp as indicated in Figure 2.7. For the actual process the incremental enthalpy rise is dh and the corresponding ideal enthalpy rise is dh_{is} .

The polytropic efficiency for the small stage is

$$\eta_p = \frac{dh_{is}}{dh} = \frac{vdp}{C_p dT}, \quad (2.31)$$

since for an isentropic process $Tds = 0 = dh_{is} - vdp$.

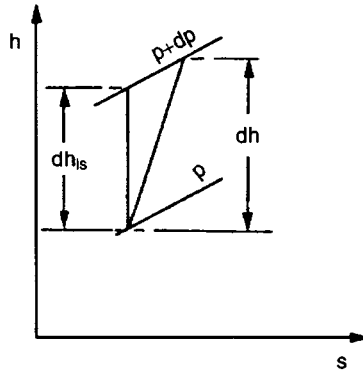


FIG. 2.7. Incremental change of state in a compression process.

Substituting $v = RT/p$ in eqn. (2.31), then

$$\eta_p = \frac{R}{C_p} \frac{T}{p} \frac{dp}{dT}$$

and hence

$$\frac{dT}{T} = \frac{(\gamma - 1)}{\gamma \eta_p} \frac{dp}{p} \quad (2.32)$$

as $C_p = \gamma R/(\gamma - 1)$.

Integrating eqn. (2.32) across the whole compressor and taking equal efficiency for each infinitesimal stage gives,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\eta_p \gamma} \quad (2.33)$$

Now the isentropic efficiency for the whole compression process is

$$\eta_c = (T_{2s} - T_1)/(T_2 - T_1) \quad (2.34)$$

if it is assumed that the velocities at inlet and outlet are equal.

For the *ideal* compression process put $\eta_p = 1$ in eqn. (2.32) and so obtain

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \quad (2.35)$$

which is also obtainable from $pv^\gamma = \text{constant}$ and $pv = RT$. Substituting eqns. (2.33) and (2.35) into eqn. (2.34) results in the expression

$$\eta_c = \left[\left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} - 1 \right] / \left[\left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\eta_p \gamma} - 1 \right] \quad (2.36)$$

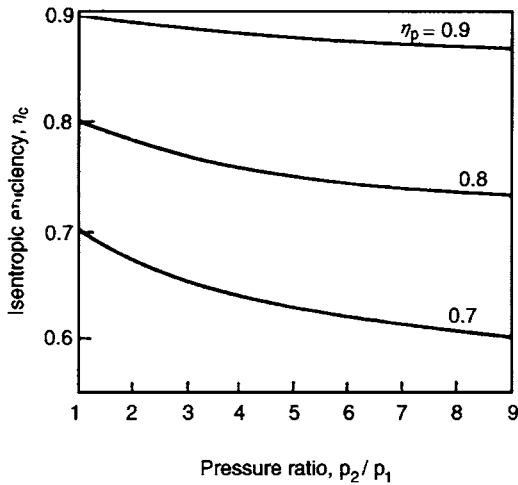


FIG. 2.8. Relationship between isentropic (overall) efficiency, pressure ratio and small stage (polytropic) efficiency for a compressor ($\gamma = 1.4$).

Values of “overall” isentropic efficiency have been calculated using eqn. (2.36) for a range of pressure ratio and different values of η_p , and are plotted in Figure 2.8. This figure amplifies the observation made earlier that the isentropic efficiency of a finite compression process is *less* than the efficiency of the small stages. Comparison of the isentropic efficiency of two machines of different pressure ratios is not a valid procedure since, for equal polytropic efficiency, the compressor with the highest pressure ratio is penalised by the *hidden* thermodynamic effect.

The term *polytropic* used above arises in the context of a reversible compressor compressing a gas from the same initial state to the same final state as the irreversible adiabatic compressor but obeying the relation $pv^n = \text{constant}$. The index n is called the *polytropic index*. Since an increase in entropy occurs for the change of state in both compressors, for the reversible compressor this is only possible if there is a reversible heat transfer $dQ_R = Tds$. Proceeding farther, it follows that the value of the index n must always exceed that of γ . This is clear from the following argument. For the polytropic process,

$$\begin{aligned} dQ_R &= du + pdv. \\ &= \frac{C_v}{R}d(pv) + pdv. \end{aligned}$$

Using $pv^n = \text{constant}$ and $C_v = R/(\gamma - 1)$, after some manipulation the expression $dQ_R = (\gamma - n)/(\gamma - 1)pdv$ is derived. For a compression process $dv < 0$ and $dQ_R > 0$ then $n > \gamma$. For an expansion process $dv > 0$, $dQ_R < 0$ and again $n > \gamma$.

EXAMPLE 2.1. An axial flow air compressor is designed to provide an overall total-to-total pressure ratio of 8 to 1. At inlet and outlet the stagnation temperatures are 300 K and 586.4 K, respectively.

Determine the overall total-to-total efficiency and the polytropic efficiency for the compressor. Assume that γ for air is 1.4.

Solution. From eqn. (2.28), substituting $h = C_p T$, the efficiency can be written as,

$$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1}{T_{02}/T_{01} - 1} = \frac{8^{1/3.5} - 1}{586 \cdot 4/300 - 1} = 0.85.$$

From eqn. (2.33), taking logs of both sides and re-arranging, we get,

$$\eta_p = \frac{\gamma - 1}{\gamma} \frac{\ln(p_{02}/p_{01})}{\ln(T_{02}/T_{01})} = \frac{1}{3.5} \times \frac{\ln 8}{\ln 1.9547} = 0.8865$$

Turbine polytropic efficiency

A similar analysis to the compression process can be applied to a perfect gas expanding through an adiabatic turbine. For the turbine the appropriate expressions for an expansion, from a state 1 to a state 2, are

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\eta_p(\gamma-1)/\gamma} \quad (2.37)$$

$$\eta_t = \left[1 - \left(\frac{p_2}{p_1}\right)^{\eta_p(\gamma-1)/\gamma}\right] / \left[1 - \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}\right]. \quad (2.38)$$

The derivation of these expressions is left as an exercise for the student. “Overall” isentropic efficiencies have been calculated for a range of pressure ratio and different polytropic efficiencies and are shown in Figure 2.9. The most notable feature of these results is that, in contrast with a compression process, for an expansion, isentropic efficiency *exceeds* small stage efficiency.

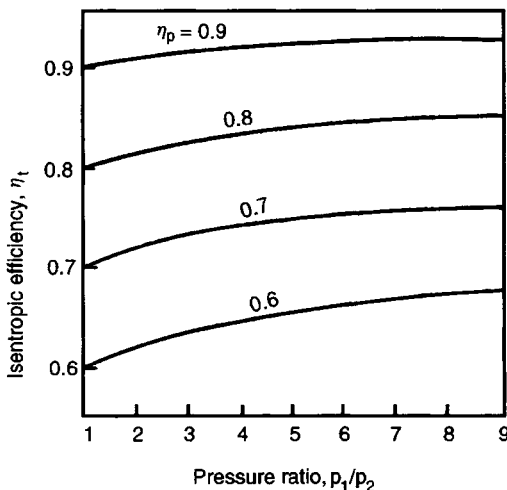


FIG. 2.9. Turbine isentropic efficiency against pressure ratio for various polytropic efficiencies ($\gamma = 1.4$).

Reheat factor

The foregoing relations obviously cannot be applied to steam turbines as vapours do not in general obey the gas laws. It is customary in steam turbine practice to use a *reheat factor* R_H as a measure of the inefficiency of the complete expansion. Referring to Figure 2.10, the expansion process through an adiabatic turbine from state 1 to state 2 is shown on a Mollier diagram, split into a number of small stages. The reheat factor is defined as

$$R_H = [(h_1 - h_{x_s}) + (h_x - h_{y_s}) + \dots]/(h_1 - h_{2_s}) = (\Sigma \Delta h_{is})/(h_1 - h_{2_s}).$$

Due to the gradual divergence of the constant pressure lines on a Mollier chart, R_H is always greater than unity. The actual value of R_H for a large number of stages will depend upon the position of the expansion line on the Mollier chart and the overall pressure ratio of the expansion. In normal steam turbine practice the value of R_H is usually between 1.03 and 1.08. For an isentropic expansion in the superheated region with $pv^n = \text{constant}$, the tables of Rogers and Mayhew (1995) give a value for $n = 1.3$. Assuming this value for n is valid, the relationship between reheat factor and pressure ratio for various fixed values of the polytropic efficiency has been calculated and is shown in Figure 2.11.

Now since the isentropic efficiency of the turbine is

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2_s}} = \frac{h_1 - h_2}{\Sigma \Delta h_{is}} \cdot \frac{\Sigma \Delta h_{is}}{h_1 - h_{2_s}}$$

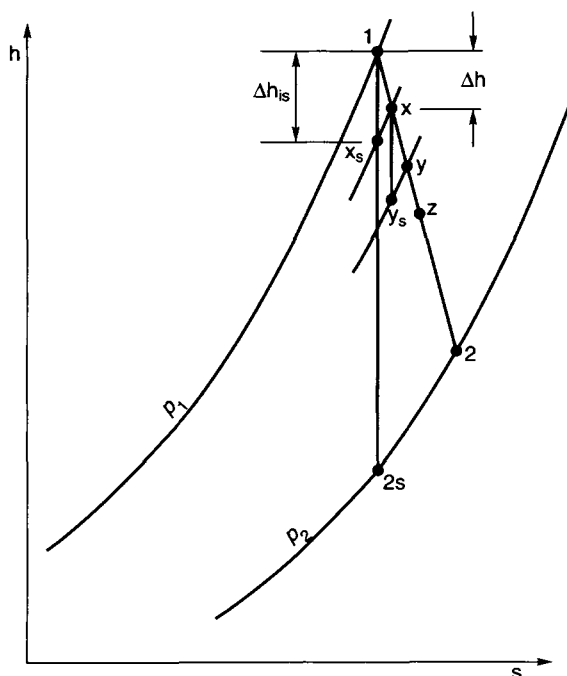


FIG. 2.10. Mollier diagram showing expansion process through a turbine split up into a number of small stages.

then

$$\eta_t = \eta_p R_H \quad (2.39)$$

which establishes the connection between polytropic efficiency, reheat factor and turbine isentropic efficiency.

Nozzle efficiency

In a large number of turbomachinery components the flow process can be regarded as a purely nozzle flow in which the fluid receives an acceleration as a result of a drop in pressure. Such a nozzle flow occurs at entry to all turbomachines and in the stationary blade rows in turbines. In axial machines the expansion at entry is assisted by a row of stationary blades (called *guide vanes* in compressors and *nozzles* in turbines) which direct the fluid on to the rotor with a large swirl angle. Centrifugal compressors and pumps, on the other hand, often have no such provision for flow guidance but there is still a velocity increase obtained from a contraction in entry flow area.

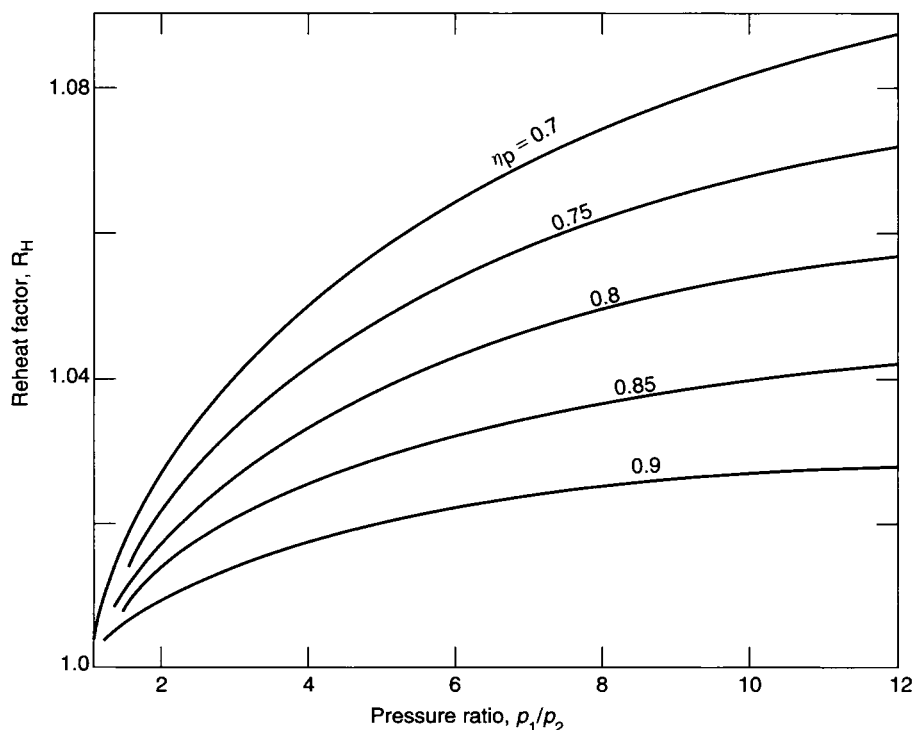


FIG. 2.11. Relationship between reheat factor, pressure ratio and polytropic efficiency ($n = 1.3$).

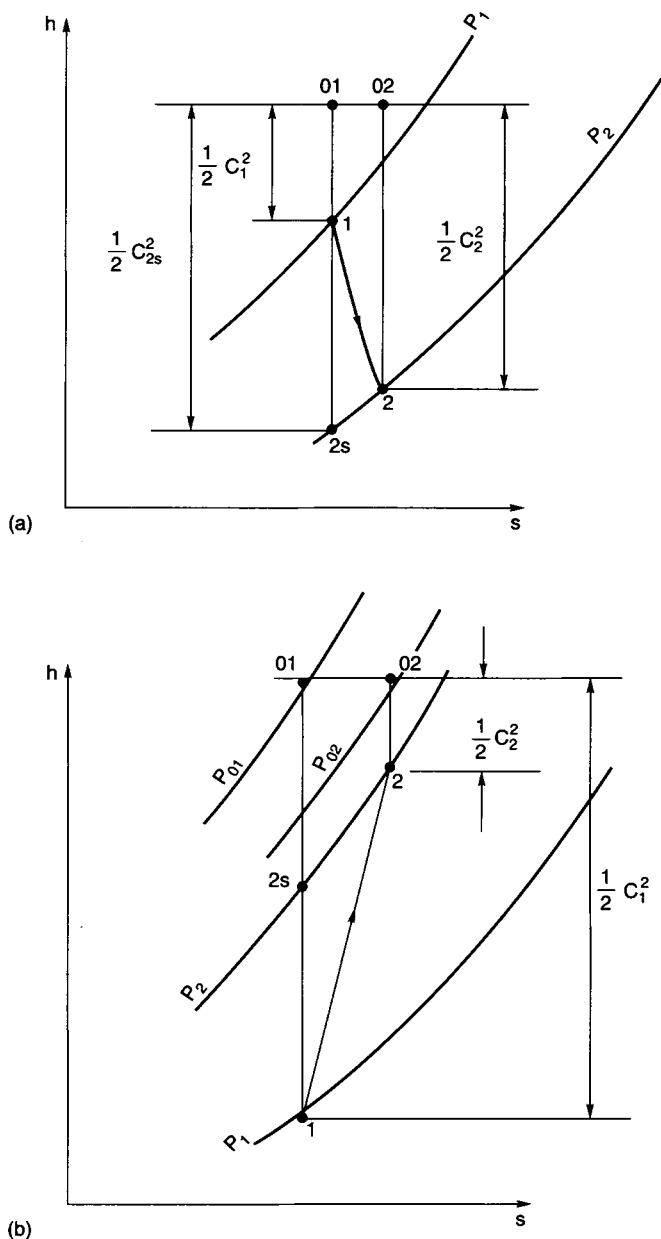


FIG. 2.12. Mollier diagrams for the flow processes through a nozzle and a diffuser: (a) nozzle; (b) diffuser.

Figure 2.12a shows the process on a Mollier diagram, the expansion proceeding from state 1 to state 2. It is assumed that the process is steady and adiabatic such that $h_{01} = h_{02}$.

According to Horlock (1966), the most frequently used definition of nozzle efficiency, η_N is, the ratio of the final kinetic energy per unit mass to the maximum

theoretical kinetic energy per unit mass obtained by an isentropic expansion to the same back pressure, i.e.

$$\eta_N = (\frac{1}{2}c_2^2)/(\frac{1}{2}c_{2s}^2) = (h_{01} - h_2)/(h_{01} - h_{2s}). \quad (2.40)$$

Nozzle efficiency is sometimes expressed in terms of various loss or other coefficients. An enthalpy loss coefficient for the nozzle can be defined as

$$\zeta_N = (h_2 - h_{2s})/(\frac{1}{2}c_2^2), \quad (2.41)$$

and, also, a velocity coefficient for the nozzle,

$$K_N = c_2/c_{2s}. \quad (2.42)$$

It is easy to show that these definitions are related to one another by

$$\eta_N = 1/(1 + \zeta_N) = K_N^2. \quad (2.43)$$

EXAMPLE 2.2. Grs enters the nozzles of a turbine stage at a stagnation pressure and temperature of 4.0 bar and 1200 K and leaves with a velocity of 572 m/s and at a static pressure of 2.36 bar. Determine the nozzle efficiency assuming the gas has the average properties over the temperature range of the expansion of $C_p = 1.160 \text{ kJ/kg K}$ and $\gamma = 1.33$.

Solution. From eqns. (2.40) and (2.35) the nozzle efficiency becomes

$$\eta_N = \frac{1 - T_2/T_{01}}{1 - T_{2s}/T_{01}} = \frac{1 - T_2/T_{01}}{1 - (p_2/p_{01})^{(\gamma-1)/\gamma}}.$$

Assuming adiabatic flow ($T_{02} = T_{01}$):

$$T_2 = T_{02} - \frac{1}{2}c_2^2/C_p = 1200 - \frac{1}{2} \times 572^2/1160 = 1059 \text{ K},$$

and thus

$$\eta_N = \frac{1 - 1059/1200}{1 - (2.36/4)^{0.33/1.33}} = \frac{0.1175}{0.12271} = \underline{0.9576}.$$

Diffusers

A diffuser is a component of a fluid flow system designed to reduce the flow velocity and thereby increase the fluid pressure. All turbomachines and many other flow systems incorporate a diffuser (e.g. closed circuit wind tunnels, the duct between the compressor and burner of a gas turbine engine, the duct at exit from a gas turbine connected to the jet pipe, the duct following the impeller of a centrifugal compressor, etc.). Turbomachinery flows are, in general, subsonic ($M < 1$) and the diffuser can be represented as a channel *diverging* in the direction of flow (see Figure 2.13).

The basic diffuser is a geometrically simple device with a rather long history of investigation by many researchers. The long timespan of the research is an indicator that the fluid mechanical processes within it are complex, the research rather more

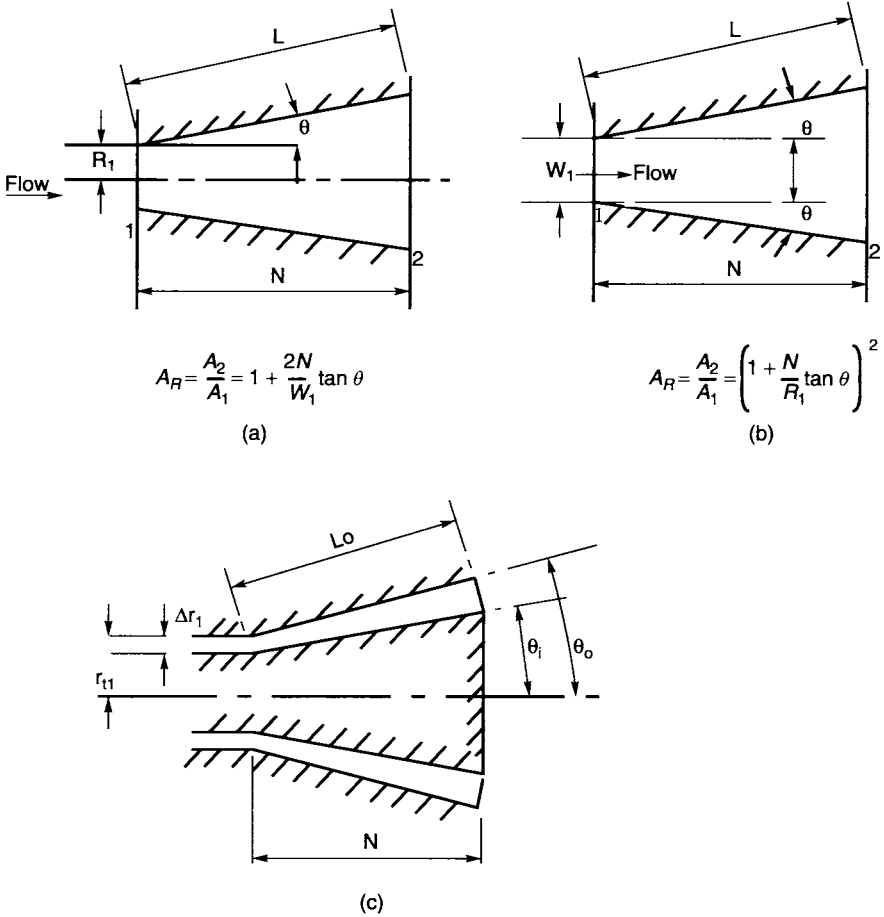


FIG. 2.13. Some subsonic diffuser geometries and their parameters: (a) two-dimensional; (b) conical; (c) annular.

difficult than might be anticipated, and some aspects of the flow processes are still not fully understood. There is now a vast literature about the flow in diffusers and their performance. Only a few of the more prominent investigations are referenced here. A noteworthy and recommended reference, however, which reviews many diverse and recondite aspects of diffuser design and flow phenomena is that of Kline and Johnson (1986).

The primary fluid mechanical problem of the diffusion process is caused by the tendency of the boundary layers to separate from the diffuser walls if the rate of diffusion is too rapid. The result of too rapid diffusion is always large losses in stagnation pressure. On the other hand, if the rate of diffusion is too low, the fluid is exposed to an excessive length of wall and fluid friction losses become predominant. Clearly, there must be an *optimum rate of diffusion* between these two extremes for which the losses are minimised. Test results from many sources indicate that an included angle of about $2\theta = 7$ degrees gives the optimum recovery for both two-dimensional and conical diffusers.

Diffuser performance parameters

The diffusion process can be represented on a Mollier diagram, Figure 2.12b, by the change of state from point 1 to point 2, and the corresponding changes in pressure and velocity from p_1 and c_1 to p_2 and c_2 . The actual performance of a diffuser can be expressed in several different ways:

- (1) as the ratio of the actual enthalpy change to the isentropic enthalpy change;
- (2) as the ratio of an actual pressure rise coefficient to an ideal pressure rise coefficient.

For steady and adiabatic flow in stationary passages, $h_{01} = h_{02}$, so that

$$h_2 - h_1 = \frac{1}{2}(c_1^2 - c_2^2). \quad (2.44a)$$

For the equivalent reversible adiabatic process from state point 1 to state point 2s,

$$(h_{2s} - h_1 = \frac{1}{2}(c_1^2 - c_{2s}^2). \quad (2.44b)$$

A *diffuser efficiency*, η_D , also called the *diffuser effectiveness*, can be defined as

$$\eta_D = (h_{2s} - h_1)/(h_2 - h_1) = (c_1^2 - c_{2s}^2)/(c_1^2 - c_2^2). \quad (2.45a)$$

In a low speed flow or a flow in which the density ρ can be considered nearly constant,

$$h_{2s} - h_1 = (p_2 - p_1)/\rho$$

so that the diffuser efficiency can be written

$$\eta_D = 2(p_2 - p_1)/\{ \rho(c_1^2 - c_2^2) \}. \quad (2.45b)$$

Equation (2.45a) can be expressed entirely in terms of pressure differences, by writing

$$\begin{aligned} h_2 - h_{2s} &= (h_2 - h_1) - (h_{2s} - h_1) \\ &= \frac{1}{2}(c_1^2 - c_2^2) - (p_2 - p_1)/\rho = (p_{01} - p_{02})/\rho, \end{aligned}$$

then, with eqn. (2.45a),

$$\begin{aligned} \eta_D &= \frac{(h_{2s} - h_1)}{(h_{2s} - h_1) - (h_2 - h_{2s})} = \frac{1}{1 - (h_{2s} - h_2)/(h_{2s} - h_1)} \\ &= \frac{1}{1 + (p_{01} - p_{02})/(p_2 - p_1)}. \end{aligned} \quad (2.46)$$

Alternative expressions for diffuser performance

- (1) A *pressure rise coefficient* C_p can be defined:

$$C_p = (p_2 - p_1)/q_1, \quad (2.47a)$$

where $q_1 = \frac{1}{2}\rho c_1^2$.

For an *incompressible* flow through the diffuser the energy equation can be written as

$$p_1/\rho + \frac{1}{2}c_1^2 = p_2/\rho + \frac{1}{2}c_2^2 + \Delta p_0/\rho, \quad (2.48)$$

where the loss in total pressure, $\Delta p_0 = p_{01} - p_{02}$. Also, using the continuity equation across the diffuser, $c_1 A_1 = c_2 A_2$, we obtain

$$c_1/c_2 = A_2/A_1 = A_R, \quad (2.49)$$

where A_R is the area ratio of the diffuser.

From eqn. (2.48), by setting Δp_0 to zero and with eqn. (2.49), it is easy to show that the *ideal pressure rise coefficient* is

$$C_{pi} = 1 - (c_2/c_1)^2 = 1 - \left[\frac{1}{A_R^2} \right] \quad (2.47b)$$

Thus, eqn. (2.48) can be rewritten as

$$C_p = C_{pi} - \Delta p_0/q_1. \quad (2.50)$$

Using the definition given in eqn. (2.46), then the diffuser efficiency (referred to as the *diffuser effectiveness* by Sovran and Klomp (1967)), is

$$\eta_D = C_p/C_{pi}. \quad (2.51)$$

(2) A *total pressure recovery factor*, p_{02}/p_{01} , is sometimes used as an indicator of the performance of diffusers. From eqn. (2.45a), the diffuser efficiency can be written

$$\eta_D = (T_{2s}/T_1 - 1)/(T_2/T_1 - 1). \quad (2.52)$$

For the isentropic process 1–2s:

$$\frac{T_{2s}}{T_1} = \left[\frac{p_2}{p_1} \right]^{(\gamma-1)/\gamma}.$$

For the constant temperature process 01–02, $Tds = -dp/\rho$ which, when combined with the gas law, $p/\rho = RT$, gives $ds = -Rdp/p$:

$$\therefore \Delta s = R \ln \left(\frac{p_{01}}{p_{02}} \right).$$

For the constant pressure process 2s–2, $Tds = dh = C_p dT$,

$$\therefore \Delta s = C_p \ln \left(\frac{T_2}{T_{2s}} \right).$$

Equating these expressions for the entropy increase and using $R/C_p = (\gamma - 1)/\gamma$, then

$$\begin{aligned} \frac{T_2}{T_{2s}} &= \left(\frac{p_{01}}{p_{02}} \right)^{(\gamma-1)/\gamma}, \\ \therefore \frac{T_2}{T_1} &= \left(\frac{T_2}{T_{2s}} \right) \left(\frac{T_{2s}}{T_1} \right) = \left[\left(\frac{p_{01}}{p_{02}} \right) \left(\frac{p_2}{p_1} \right) \right]^{(\gamma-1)/\gamma}. \end{aligned}$$

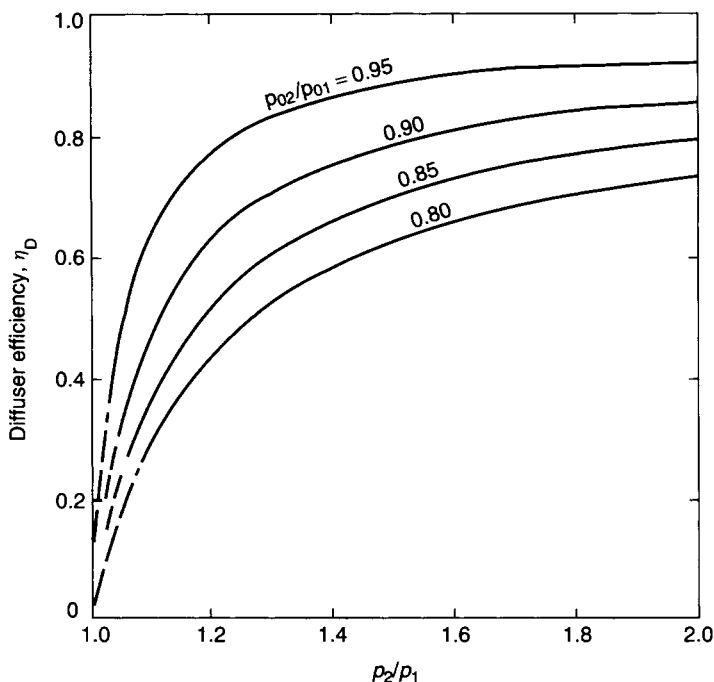


FIG. 2.14. Variation of diffuser efficiency with static pressure ratio for constant values of total pressure recovery factor ($\gamma = 1.4$).

Substituting these two expressions into eqn. (2.52):

$$\eta_D = \frac{(p_2/p_1)^{(\gamma-1)/\gamma} - 1}{[(p_{01}/p_{02})(p_2/p_1)]^{(\gamma-1)/\gamma} - 1}. \quad (2.53)$$

The variation of η_D as a function of the static pressure ratio, p_2/p_1 , for specific values of the total pressure recovery factor, p_{02}/p_{01} , is shown in Figure 2.14.

Some remarks on diffuser performance

It was pointed out by Sovran and Klomp (1967) that the uniformity or steadiness of the flow at the diffuser exit is as important as the reduction in flow velocity (or the static pressure rise) produced. This is particularly so in the case of a compressor located at the diffuser exit since the compressor performance is sensitive to non-uniformities in velocity in its inlet flow. Figure 2.15, from Sovran and Klomp (1967), shows the occurrence of flow unsteadiness and/or non-uniform flow at the exit from two-dimensional diffusers (correlated originally by Kline, Abbott and Fox 1959). Four different flow regimes exist, three of which have steady or reasonably steady flow. The region of “no appreciable stall” is steady and uniform. The region marked “large transitory stall” is unsteady and non-uniform, while the “fully-developed” and “jet flow” regions are reasonably steady but very non-uniform.

The line marked a—a will be of interest in turbomachinery applications. However, a sharply marked transition does not exist and the definition of an appropriate line

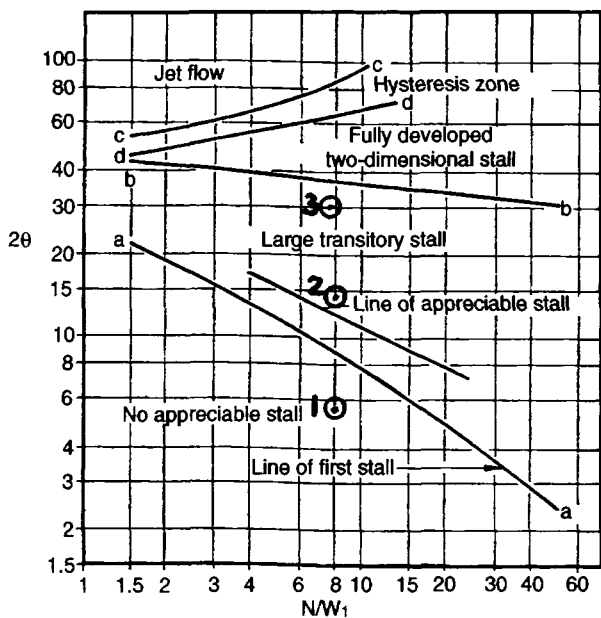


FIG. 2.15. Flow regime chart for two-dimensional diffusers (adapted from Sovran and Klomp 1967).

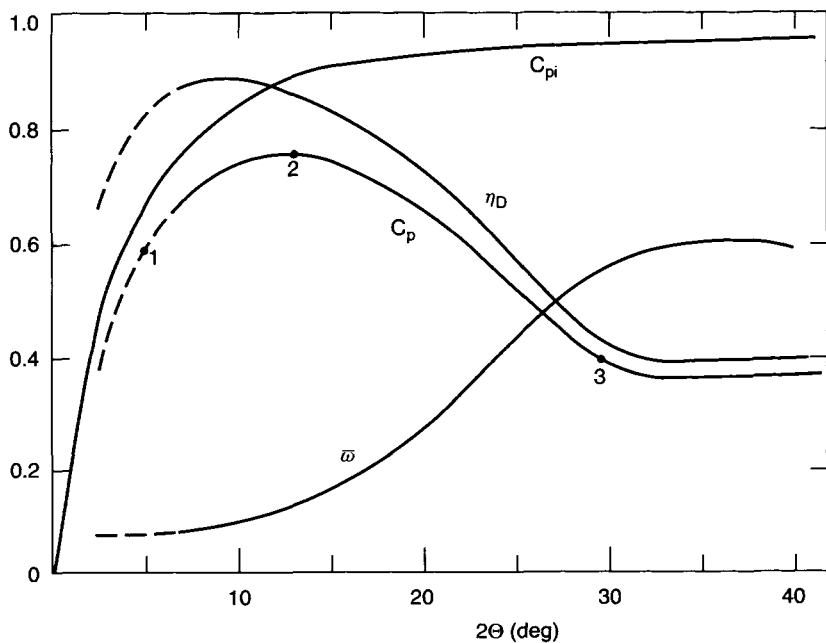


FIG. 2.16. Typical diffuser performance curves for a two-dimensional diffuser, with $L/W_1 = 8.0$ (adapted from Kline *et al.* 1959).

involves a certain degree of arbitrariness and subjectivity on the occurrence of “first stall”.

Figure 2.16 shows typical performance curves for a rectangular diffuser with a fixed sidewall to length ratio, $L/W_1 = 8.0$, given in Kline *et al.* (1959). On the line labelled C_p , points numbered 1, 2 and 3 are shown. These same numbered points are redrawn onto Figure 2.15 to show where they lie in relation to the various flow regimes. Inspection of the location of point 2 shows that optimum recovery at constant length occurs slightly above the line marked “no appreciable stall”. The performance of the diffuser between points 2 and 3 in Figure 2.16 is shows a very significant deterioration and is in the regime of large amplitude, very unsteady flow.

Maximum pressure recovery

From an inspection of eqn. (2.46) it will be observed that when diffuser efficiency η_D is a maximum, the total pressure loss is a minimum for a given rise in static pressure. Another optimum problem is the requirement of *maximum pressure recovery* for a given diffuser length in the flow direction *regardless of the area ratio* $A_r = A_2/A_1$. This may seem surprising but, in general, this optimum condition produces a different diffuser geometry from that needed for optimum efficiency. This can be demonstrated by means of the following considerations.

From eqn. (2.51), taking logs of both sides and differentiating, we get:

$$\frac{\partial}{\partial \theta}(\ln \eta_D) = \frac{\partial}{\partial \theta}(\ln C_p) - \frac{\partial}{\partial \theta}(\ln C_{pi}).$$

Setting the L.H.S to zero for the condition of maximum η_D , then

$$\frac{1}{C_p} \frac{\partial C_p}{\partial \theta} = \frac{1}{C_{pi}} \frac{\partial C_{pi}}{\partial \theta}. \quad (2.54)$$

Thus, at the maximum efficiency the fractional rate of increase of C_p with a change in θ is equal to the fractional rate of increase of C_{pi} with a change in θ . At this point C_p is positive and, by definition, both C_{pi} and $\partial C_p / \partial \theta$ are also positive. Equation (2.54) shows that $\partial C_p / \partial \theta > 0$ at the maximum efficiency point. Clearly, C_p *cannot* be at its maximum when η_D is at its peak value! What happens is that C_p *continues to increase* until $\partial C_p / \partial \theta = 0$, as can be seen from the curves in Figure 2.16.

Now, upon differentiating eqn. (2.50) with respect to θ and setting the lhs to zero, the condition for maximum C_p is obtained, namely

$$\frac{\partial C_{pi}}{\partial \theta} = \frac{\partial}{\partial \theta}(\Delta p_0/q_1).$$

Thus, as the diffuser angle is increased beyond the divergence which gave maximum efficiency, the actual pressure rise will continue to rise until the additional losses in total pressure balance the theoretical gain in pressure recovery produced by the increased area ratio.

Diffuser design calculation

The performance of a conical diffuser has been chosen for this purpose using data presented by Sovran and Klomp (1967). This is shown in Figure 2.17 as contour

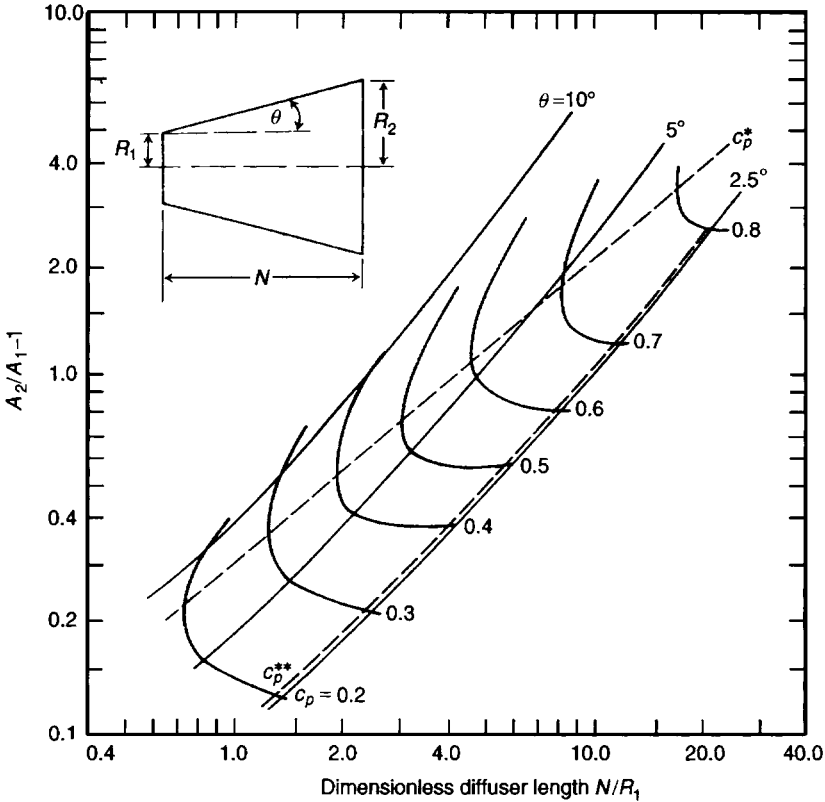


FIG. 2.17. Performance chart for conical diffusers, $B_1 \cong 0.02$. (adapted from Sovran and Klomp 1967).

plots of C_p in terms of the geometry of the diffuser, L/R_1 and the area ratio A_R . Two optimum diffuser lines, useful for design purposes, were added by the authors. The first is the line C_p^* , the locus of points which defines the diffuser area ratio A_R , producing the maximum pressure recovery for a prescribed non-dimensional length, L/R_1 . The second is the line C_p^{**} , the locus of points defining the diffuser non-dimensional length, producing the maximum pressure recovery at a prescribed area ratio.

EXAMPLE 2.3. Design a conical diffuser to give maximum pressure recovery in a non-dimensional length $N/R_1 = 4.66$ using the data given in Figure 2.17.

Solution. From the graph, using log-linear scaling, the appropriate value of C_p is 0.6 and the corresponding value of A_R is 2.13. From eqn. (2.47b), $C_{pi} = 1 - (1/2 \cdot 13^2) = 0.78$. Hence, $\eta_D = 0.6/0.78 = 0.77$.

Transposing the expression given in Figure 2.13b, the included cone angle can be found:

$$2\theta = 2 \tan^{-1}\{(A_R^{0.5} - 1)/(L/R_1)\} = \underline{11.26 \text{ deg.}}$$

EXAMPLE 2.4. Design a conical diffuser to give maximum pressure recovery at a prescribed area ratio $A_R = 1.8$ using the data given in Figure 2.17.

Solution. From the graph, $C_p = 0.6$ and $N/R_1 = 7.85$ (using log-linear scaling). Thus,

$$2\theta = 2 \tan^{-1}\{(1.8^{0.5} - 1)/7.85\} = 5 \text{ deg.}$$

$$C_{pi} = 1 - (1/1.8^2) = 0.69 \text{ and } \eta_D = 0.6/0.69 = \underline{0.87}.$$

Analysis of a non-uniform diffuser flow

The actual pressure recovery produced by a diffuser of optimum geometry is known to be strongly affected by the shape of the velocity profile at inlet. A large reduction in the pressure rise which might be expected from a diffuser can result from inlet flow non-uniformities (e.g. wall boundary layers and, possibly, wakes from a preceding row of blades). Sovran and Klop (1967) presented an incompressible flow analysis which helps to explain how this deterioration in performance occurs and some of the main details of their analysis are included in the following account.

The mass-averaged total pressure $\bar{p}_0$ at any cross-section of a diffuser can be obtained by integrating over the section area. For symmetrical ducts with straight centre lines the static pressure can be considered constant, as it is normally. Thus,

$$\begin{aligned} \bar{p}_0 &= \int_A c(p + \tfrac{1}{2}\rho c^2) dA \bigg/ \int_A c dA, \\ &= p + \tfrac{1}{2}\rho \int_A c^3 dA \bigg/ \int_A c dA. \end{aligned} \quad (2.55)$$

The average axial velocity U and the average dynamic pressure q at a section are

$$U = \frac{1}{A} \int_A c dA \text{ and } q = \frac{1}{2} \rho U^2.$$

Substituting into eqn. (2.55),

$$\begin{aligned} \bar{p}_0 &= p + \tfrac{1}{2}\rho U^3 \int_A \left(\frac{c}{U}\right)^3 dA / UA \\ &= p + \frac{q}{A} \int_A \left(\frac{c}{U}\right)^3 dA = p + \alpha q, \end{aligned} \quad (2.56)$$

where α is the kinetic energy flux coefficient of the velocity profile, i.e.

$$\alpha = \frac{1}{A} \int_A \left(\frac{c}{U}\right)^3 dA = \frac{1}{Q} \int_A \left(\frac{c}{U}\right)^2 dQ = \overline{c^2}/U^2, \quad (2.57)$$

where $\overline{c^2}$ is the mean square of the velocity in the cross-section and $Q = AU$, i.e.

$$\overline{c^2} = (1/Q) \int_A c^2 dQ.$$

From eqn. (2.56) the change in static pressure is found as

$$p_2 - p_1 = (\alpha_1 q_1 - \alpha_2 q_2) - (\bar{p}_{01} - \bar{p}_{02}). \quad (2.58)$$

From eqn. (2.51), with eqns. (2.47a) and (2.47b), the diffuser efficiency (or diffuser effectiveness) can now be written:

$$\eta_D = C_p / C_{pi} = \frac{(p_2 - p_1)/q_1}{1 - 1/A_R^2}.$$

Substituting eqn. (2.58) into the above expression,

$$\eta_D = \frac{\alpha_1 [1 - (\alpha_2/\alpha_1)/A_R^2]}{(1 - 1/A_R^2)} - \frac{\varpi}{(1 - 1/A_R^2)} \quad (2.59)$$

where ϖ is the total pressure loss coefficient for the whole diffuser, i.e.

$$\varpi = (\bar{p}_{01} - \bar{p}_{02})/q_1. \quad (2.60)$$

Equation (2.59) is particularly useful as it enables the separate effects due the changes in the velocity profile and total pressure losses on the diffuser effectiveness to be found. The first term in the equation gives the reduction in η_D caused by *insufficient flow diffusion*. The second term gives the reduction in η_D produced by viscous effects and represents *inefficient flow diffusion*. An assessment of the relative proportion of these effects on the effectiveness requires the accurate measurement of both the inlet and exit velocity profiles as well as the static pressure rise. Such complete data is seldom derived by experiments. However, Sovran and Klomp (1967) made the observation that there is a widely held belief that fluid mechanical losses are the primary cause of poor performance in diffusers. One of the important conclusions they drew from their work was that it is the thickening of the inlet boundary layer which is primarily responsible for the reduction in η_D . Thus, it is *insufficient* flow diffusion rather than *inefficient* flow diffusion which is often the cause of poor performance.

Some of the most comprehensive tests made of diffuser performance were those of Stevens and Williams (1980) who included traverses of the flow at inlet and at exit as well as careful measurements of the static pressure increase and total pressure loss in low speed tests on annular diffusers. In the following worked example, to illustrate the preceding theoretical analysis, data from this source has been used.

EXAMPLE 2.5. An annular diffuser with an area ratio, $A_R = 2.0$ is tested at low speed and the results obtained give the following data:

at entry, $\alpha_1 = 1.059$, $B_1 = 0.109$

at exit, $\alpha_2 = 1.543$, $B_2 = 0.364$, $C_p = 0.577$

Determine the diffuser efficiency.

NB B_1 and B_2 are the fractions of the area blocked by the wall boundary layers at inlet and exit (displacement thicknesses) and are included only to illustrate the profound effect the diffusion process has on boundary layer thickening.

Solution. From eqns. (2.47a) and (2.58):

$$C_p = \frac{p_2 - p_1}{q_1} = (\alpha_1 - \alpha_2/A_R^2) - w$$

$$= 1.059 - 1.543/4 - 0.09 = \underline{0.583}.$$

Using eqn. (2.59) directly,

$$\eta_D = C_p/C_{pi} = C_p/(1 - 1/A_R^2) = 0.583/0.75$$

$$\therefore \eta_D = \underline{0.7777}.$$

Stevens and Williams observed that an incipient transitory stall was in evidence on the diffuser outer wall which affected the accuracy of the results. So, it is not surprising that a slight mismatch is evident between the above calculated result and the measured result.

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Problems

1. For the adiabatic expansion of a perfect gas through a turbine, show that the overall efficiency η_t and small stage efficiency η_p are related by

$$\eta_t = (1 - \epsilon^{\eta_p})/(1 - \epsilon),$$

where $\epsilon = r^{(1-\gamma)/\gamma}$, and r is the expansion pressure ratio, γ is the ratio of specific heats.

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An axial flow turbine has a small stage efficiency of 86%, an overall pressure ratio of 4.5 to 1 and a mean value of γ equal to 1.333. Calculate the overall turbine efficiency.

2. Air is expanded in a multi-stage axial flow turbine, the pressure drop across each stage being very small. Assuming that air behaves as a perfect gas with ratio of specific heats γ , derive pressure-temperature relationships for the following processes:

- (i) reversible adiabatic expansion;
 - (ii) irreversible adiabatic expansion, with small stage efficiency η_p ;
 - (iii) reversible expansion in which the heat loss in each stage is a constant fraction k of the enthalpy drop in that stage;
 - (iv) reversible expansion in which the heat loss is proportional to the absolute temperature T .
- Sketch the first three processes on a T, s diagram.

If the entry temperature is 1100 K, and the pressure ratio across the turbine is 6 to 1, calculate the exhaust temperatures in each of these three cases. Assume that γ is 1.333, that $\eta_p = 0.85$, and that $k = 0.1$.

3. A multi-stage high-pressure steam turbine is supplied with steam at a stagnation pressure of 7 MPa, and a stagnation temperature of 500°C. The corresponding specific enthalpy is 3410 kJ/kg. The steam exhausts from the turbine at a stagnation pressure of 0.7 MPa, the steam having been in a superheated condition throughout the expansion. It can be assumed that the steam behaves like a perfect gas over the range of the expansion and that $\gamma = 1.3$. Given that the turbine flow process has a small-stage efficiency of 0.82, determine.

- (i) the temperature and specific volume at the end of the expansion;
- (ii) the reheat factor.

The specific volume of superheated steam is represented by $pv = 0.231(h - 1943)$, where p is in kPa, v is in m^3/kg and h is in kJ/kg.

4. A 20 MW back-pressure turbine receives steam at 4 MPa and 300°C, exhausting from the last stage at 0.35 MPa. The stage efficiency is 0.85, reheat factor 1.04 and external losses 2% of the actual isentropic enthalpy drop. Determine the rate of steam flow.

At the exit from the first stage nozzles the steam velocity is 244 m/s, specific volume $68.6 \text{ dm}^3/\text{kg}$, mean diameter 762 mm and steam exit angle 76 deg measured from the axial direction. Determine the nozzle exit height of this stage.

5. Steam is supplied to the first stage of a five stage pressure-compounded steam turbine at a stagnation pressure of 1.5 MPa and a stagnation temperature of 350°C. The steam leaves the last stage at a stagnation pressure of 7.0 kPa with a corresponding dryness fraction of 0.95. By using a Mollier chart for steam and assuming that the stagnation state point locus is a straight line joining the initial and final states, determine

- (i) the stagnation conditions between each stage assuming that each stage does the same amount of work;
- (ii) the total-to-total efficiency of each stage;
- (iii) the overall total-to-total efficiency and total-to-static efficiency assuming the steam enters the condenser with a velocity of 200 m/s;
- (iv) the reheat factor based upon stagnation conditions.