

CHAPTER 4

Axial-flow Turbines: Two-dimensional Theory

Power is more certainly retained by wary measures than by daring counsels.
(TACITUS, *Annals*.)

Introduction

The simplest approach to the study of axial-flow turbines (and also axial-flow compressors) is to assume that the flow conditions prevailing at the mean radius fully represent the flow at all other radii. This two-dimensional analysis at the *pitch-line* can provide a reasonable approximation to the actual flow, if the ratio of blade height to mean radius is small. When this ratio is large, however, as in the final stages of a steam turbine or, in the first stages of an axial compressor, a three-dimensional analysis is required. Some important aspects of three-dimensional flows in axial turbomachines are discussed in Chapter 6. Two further assumptions are, that radial velocities are zero, and that the flow is invariant along the circumferential direction (i.e. there are no “blade-to-blade” flow variations).

In this chapter the presentation of the analysis has been devised with compressible flow effects in mind. This approach is then applicable to both steam and gas turbines provided that, in the former case, the steam condition remains wholly within the *vapour* phase (i.e. superheat region). Much early work concerning flows in steam turbine nozzles and blade rows are reported in Stodola (1945), Kearton (1958) and Horlock (1960).

Velocity diagrams of the axial turbine stage

The axial turbine stage comprises a row of fixed guide vanes or nozzles (often called a *stator* row) and a row of moving blades or buckets (a *rotor* row). Fluid enters the stator with absolute velocity c_1 at angle α_1 and accelerates to an absolute velocity c_2 at angle α_2 (Figure 4.1). All angles are measured from the axial (x) direction. The *sign convention* is such that angles and velocities as drawn in Figure 4.1 will be taken as positive throughout this chapter. From the velocity diagram, the rotor inlet *relative* velocity w_2 , at an angle β_2 , is found by subtracting, vectorially, the blade speed U from the absolute velocity c_2 . The relative flow within the rotor accelerates to velocity w_3 at an angle β_3 at rotor outlet; the corresponding absolute flow (c_3, α_3) is obtained by adding, vectorially, the blade speed U to the relative velocity w_3 .

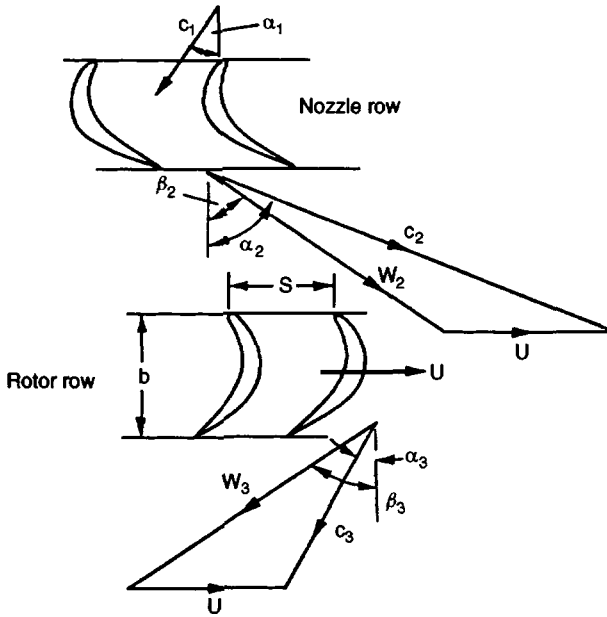


FIG. 4.1. Turbine stage velocity diagrams.

The continuity equation for uniform, steady flow is,

$$\rho_1 A_1 c_{x1} = \rho_2 A_2 c_{x2} = \rho_3 A_3 c_{x3}.$$

In two-dimensional theory of turbomachines it is usually assumed, for simplicity, that the axial velocity remains constant i.e. $c_{x1} = c_{x2} = c_{x3} = c_x$.

This must imply that,

$$\rho_1 A_1 = \rho_2 A_2 = \rho_3 A_3 = \text{constant.} \quad (4.1)$$

Thermodynamics of the axial turbine stage

The work done on the rotor by unit mass of fluid, the specific work, equals the stagnation enthalpy drop incurred by the fluid passing through the stage (assuming adiabatic flow), or,

$$\Delta W = \dot{W}/\dot{m} = h_{01} - h_{03} = U(c_{y2} + c_{y3}). \quad (4.2)$$

In eqn. (4.2) the absolute tangential velocity components (c_y) are *added*, so as to adhere to the agreed sign convention of Figure 4.1. As no work is done in the nozzle row, the stagnation enthalpy across it remains constant and

$$h_{01} = h_{02}. \quad (4.3)$$

Writing $h_0 = h + \frac{1}{2}(c_x^2 + c_y^2)$ and using eqn. (4.3) in eqn. (4.2) we obtain,

$$h_{02} - h_{03} = (h_2 - h_3) + \frac{1}{2}(c_{y2}^2 - c_{y3}^2) = U(c_{y2} + c_{y3}),$$

hence,

$$(h_2 - h_3) + \frac{1}{2}(c_{y2} + c_{y3})[(c_{y2} - U) - (c_{y3} + U)] = 0.$$

It is observed from the velocity triangles of Figure 4.1 that $c_{y2} - U = w_{y2}$, $c_{y3} + U = w_{y3}$ and $c_{y2} + c_{y3} = w_{y2} + w_{y3}$. Thus,

$$(h_2 - h_3) + \frac{1}{2}(w_{y2}^2 - w_{y3}^2) = 0.$$

Add and subtract $\frac{1}{2}c_x^2$ to the above equation

$$h_2 + \frac{1}{2}w_2^2 = h_3 + \frac{1}{2}w_3^2 \text{ or } h_{02\text{rel}} = h_{03\text{rel}}. \quad (4.4)$$

Thus, we have proved that the *relative* stagnation enthalpy, $h_{0\text{rel}} = h + \frac{1}{2}w^2$, remains unchanged through the rotor of an axial turbomachine. It is implicitly assumed that no radial shift of the streamlines occurs in this flow. In a *radial flow* machine a more general analysis is necessary (see Chapter 7) which takes account of the blade speed change between rotor inlet and outlet.

A Mollier diagram showing the change of state through a complete turbine stage, including the effects of irreversibility, is given in Figure 4.2.

Through the nozzles, the state point moves from 1 to 2 and the static pressure decreases from p_1 to p_2 . In the rotor row, the absolute static pressure reduces (in general) from p_2 to p_3 . It is important to note that the conditions contained in eqns. (4.2)–(4.4) are all satisfied in the figure.

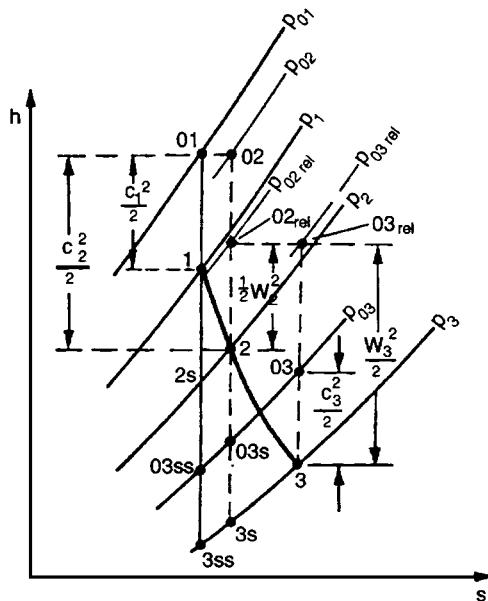


FIG. 4.2. Mollier diagram for a turbine stage.

Stage losses and efficiency

In Chapter 2 various definitions of efficiency for complete turbomachines were given. For a *turbine stage* the total-to-total efficiency is,

$$\eta_{tt} = \frac{\text{actual work output}}{\text{ideal work output when operating to same back pressure}} \\ = (h_{01} - h_{03}) / (h_{01} - h_{03ss}).$$

At the entry and exit of a *normal* stage the flow conditions (absolute velocity and flow angle) are identical, i.e. $c_1 = c_3$ and $a_1 = a_3$. If it is assumed that $c_{3ss} = c_3$, which is a reasonable approximation, the total-to-total efficiency becomes.

$$\eta_{tt} = (h_1 - h_3) / (h_1 - h_{3ss}) \\ = (h_1 - h_3) / \{(h_1 - h_3) + (h_3 - h_{3s}) + (h_{3s} - h_{3ss})\}. \quad (4.5)$$

Now the slope of a constant pressure line on a Mollier diagram is $(\partial h / \partial s)_p = T$, obtained from eqn. (2.18). Thus, for a finite change of enthalpy in a constant pressure process, $\Delta h \doteq T \Delta s$ and, therefore,

$$h_{3s} - h_{3ss} \doteq T_3(s_{3s} - s_{3ss}), \quad (4.6a)$$

$$h_2 - h_{2s} \doteq T_2(s_2 - s_{2s}). \quad (4.6b)$$

Noting, from Figure 4.2, that $s_{3s} - s_{3ss} = s_2 - s_{2s}$, the last two equations can be combined to give

$$h_{3s} - h_{3ss} = (T_3/T_2)(h_2 - h_{2s}). \quad (4.7)$$

The effects of irreversibility through the stator and rotor are expressed by the differences in static enthalpies, $(h_2 - h_{2s})$ and $(h_3 - h_{3s})$ respectively. Non-dimensional enthalpy "loss" coefficients can be defined in terms of the exit kinetic energy from each blade row. Thus, for the nozzle row,

$$h_2 - h_{2s} = \frac{1}{2} c_2^2 \zeta_N. \quad (4.8a)$$

For the rotor row,

$$h_3 - h_{3s} = \frac{1}{2} w_3^2 \zeta_R. \quad (4.8b)$$

Combining eqns. (4.7) and (4.8) with eqn. (4.5) gives

$$\eta_{tt} = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2 T_3 / T_2}{2(h_1 - h_3)} \right]^{-1}. \quad (4.9)$$

When the exit velocity is not recovered (in Chapter 2, examples of such cases are quoted) a total-to-static efficiency for the stage is used.

$$\eta_{ts} = (h_{01} - h_{03}) / (h_{01} - h_{3ss}) \\ = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2 T_3 / T_2 + c_1^2}{2(h_1 - h_3)} \right]^{-1}, \quad (4.10)$$

where, as before, it is assumed that $c_1 = c_3$.

In initial calculations or, in cases where the static temperature drop through the rotor is not large, the temperature ratio T_3/T_2 is set equal to unity, resulting in the more convenient approximations,

$$\eta_{it} = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2}{2(h_1 - h_3)} \right]^{-1}, \quad (4.9a)$$

$$\eta_{is} = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2 + c_1^2}{2(h_1 - h_3)} \right]^{-1}. \quad (4.10a)$$

So that estimates can be made of the efficiency of a proposed turbine stage as part of the preliminary design process, some means of determining the loss coefficients is required. Several methods for doing this are available with varying degrees of complexity. The blade row method proposed by Soderberg (1949) and reported by Horlock (1966), although old, is still remarkably valid despite its simplicity. Ainley and Mathieson (1952) correlated the profile loss coefficients for nozzle blades (which give 100% expansion) and impulse blades (which give 0% expansion) against flow deflection and pitch/chord ratio for stated values of Reynolds number and Mach number. Details of their method are given in Chapter 3. For blading between impulse and reaction the profile loss is derived from a combination of the impulse and reaction profile losses (see eqn. (3.42)). Horlock (1966) has given a detailed comparison between these two methods of loss prediction. A refinement of the Ainley and Mathieson prediction method was later published by Dunham and Came (1970).

Various other methods of predicting the efficiency of axial flow turbines have been devised such as those of Craig and Cox (1971), Kacker and Okapuu (1982) and Wilson (1987). It was Wilson who, tellingly, remarked that despite the emergence of “computer programs of great power and sophistication”, and “generally incorporating computational fluid dynamics”, that these have not yet replaced the preliminary design methods mentioned above. It is, clearly, essential for a design to converge as closely as possible to an optimum configuration using preliminary design methods before carrying out the final design refinements using computational fluid dynamics.

Soderberg's correlation

One method of obtaining design data on turbine blade losses is to assemble information on the overall efficiencies of a wide variety of turbines, and from this calculate the individual blade row losses. This system was developed by Soderberg (1949) from a large number of tests performed on steam turbines and on cascades, and extended to fit data obtained from small turbines with very low aspect ratio blading (small height–chord). Soderberg's method was intended only for turbines conforming to the standards of “good design”, as discussed below. The method was used by Stenning (1953) to whom reference can be made.

A paper by Horlock (1960) has critically reviewed several different and widely used methods of obtaining design data for turbines. His paper confirms the claim

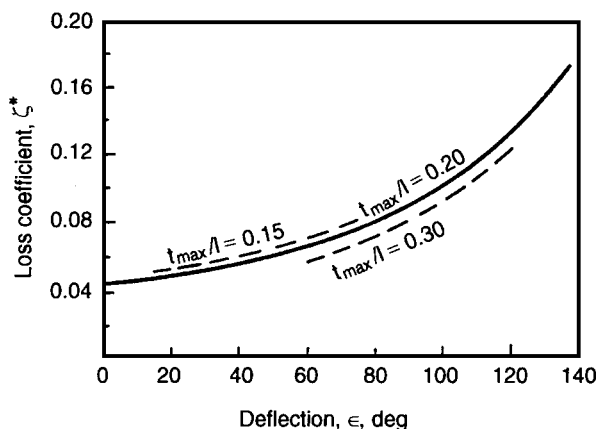


FIG. 4.3. Soderberg's correlation of turbine blade loss coefficient with fluid deflection (adapted from Horlock (1960)).

made for Soderberg's correlation that, although based on relatively few parameters, it is of comparable accuracy with the best of the other methods.

Soderberg found that for the *optimum* space–chord ratio, turbine blade losses (with “full admission” to the complete annulus) could be correlated with space–chord ratio, blade aspect ratio, blade thickness–chord ratio and Reynolds number. Soderberg used *Zweifel's criterion* (see Chapter 3) to obtain the optimum space–chord ratio of turbine cascades based upon the cascade geometry. Zweifel suggested that the aerodynamic load coefficient ψ_T should be approximately 0.8. Following the notation of Figure 4.1

$$\psi_T = 0.8 = 2(s/b)(\tan \alpha_1 + \tan \alpha_2) \cos^2 \alpha_2. \quad (4.11)$$

The optimum space–chord ratio may be obtained from eqn. (4.11) for specified values of α_1 and α_2 .

For turbine blade rows operating at this load coefficient, with a Reynolds number of 10^5 and aspect ratio H/b = blade height/axial chord) of 3, the “nominal” loss coefficient ζ^* is a simple function of the fluid deflection angle $\epsilon = \alpha_1 + \alpha_2$, for a given thickness–chord ratio ($t_{\max}/l$). Values of ζ^* are drawn in Figure 4.3 as a function of deflection ϵ , for several ratios of $t_{\max}/l$. A frequently used analytical simplification of this correlation (for $t_{\max}/l = 0.2$), which is useful in initial performance calculations, is

$$\zeta^* = 0.04 + 0.06 \left(\frac{\epsilon}{100} \right)^2. \quad (4.12)$$

This expression fits Soderberg's curve (for $t_{\max}/l = 0.2$) quite well for $\epsilon \leq 120^\circ$, but is less accurate at higher deflections. For turbine rows operating at zero incidence, which is the basis of Soderberg's correlation, the fluid deflection is little different from the blading deflection since, for *turbine cascades*, deviations are

usually small. Thus, for a nozzle row, $\epsilon = \epsilon_N = \alpha'_2 + \alpha'_1$ and for a rotor row, $\epsilon = \epsilon_R = \beta'_2 + \beta'_3$ can be used (the prime referring to the actual blade angles).

If the aspect ratio H/b is other than 3, a correction to the nominal loss coefficient ζ^* is made as follows:

for nozzles,

$$1 + \zeta_1 = (1 + \zeta^*)(0.993 + 0.021b/H), \quad (4.13a)$$

for rotors,

$$1 + \zeta_1 = (1 + \zeta^*)(0.975 + 0.075b/H), \quad (4.13b)$$

where ζ_1 is the loss coefficient at a Reynolds number of 10^5 .

A further correction can be made if the Reynolds number is different from 10^5 . As used in this section, Reynolds number is based upon exit velocity c_2 and the hydraulic mean diameter D_h at the throat section.

$$Re = \rho_2 c_2 D_h / \mu, \quad (4.14)$$

where

$$D_h = 2sH \cos \alpha_2 / (s \cos \alpha_2 + H).$$

(N.B. Hydraulic mean diameter = $4 \times$ flow area $\div$ wetted perimeter.)

The Reynolds number correction is

$$\zeta_2 = \left(\frac{10^5}{Re} \right)^{1/4} \zeta_1. \quad (4.15)$$

Soderberg's method of loss prediction gives turbine efficiencies with an error of less than 3% over a wide range of Reynolds number and aspect ratio when additional corrections are included to allow for tip leakage and disc friction. An approximate correction for tip clearance may be incorporated by the simple expedient of multiplying the final calculated stage efficiency by the ratio of "blade" area to *total* area (i.e. "blade" area + clearance area).

Types of axial turbine design

The process of choosing the best turbine design for a given application usually involves juggling several parameters which may be of equal importance, for instance, rotor angular velocity, weight, outside diameter, efficiency, so that the final design lies within acceptable limits for each parameter. In consequence, a simple presentation can hardly do justice to the real problem. However, a consideration of the factors affecting turbine efficiency for a simplified case can provide a useful guide to the designer.

Consider the problem of selecting an axial turbine design for which the mean blade speed U , the specific work ΔW , and the axial velocity c_x , have already been selected. The upper limit of blade speed is limited by stress; the limit on blade tip speed is roughly 450 m/s although some experimental turbines have been operated

at higher speeds. The axial velocity is limited by flow area considerations. It is assumed that the blades are sufficiently short to treat the flow as two-dimensional.

The specific work done is

$$\Delta W = U(c_{y2} + c_{y3}).$$

With ΔW , U and c_x fixed the only remaining parameter required to completely define the velocity triangles is c_{y2} , since

$$c_{y3} = \Delta W/U - c_{y2}. \quad (4.16)$$

For different values of c_{y2} the velocity triangles can be constructed, the loss coefficients determined and η_{tt} , η_{ts} calculated. In Shapiro *et al.* (1957) Stenning considered a family of turbines each having a flow coefficient $c_x/U = 0.4$, blade aspect ratio $H/b = 3$ and Reynolds number $Re = 10^5$, and calculated η_{tt} , η_{ts} for stage loading factors $\Delta W/U^2$ of 1, 2 and 3 using Soderberg's correlation. The results of this calculation are shown in Figure 4.4. It will be noted that these results relate to blading efficiency and make no allowance for losses due to tip clearance and disc friction.

EXAMPLE 4.1. Verify the peak value of the total to static efficiency η_{ts} shown in Figure 4.4 for the curve marked $\Delta W/U^2 = 1$, using Soderberg's correlation and the same data used by Stenning in Shapiro *et al.* (1957).

Solution. From eqn. (4.10a):

$$\frac{1}{\eta_{ts}} = 1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2 + c_1^2}{2\Delta W}.$$

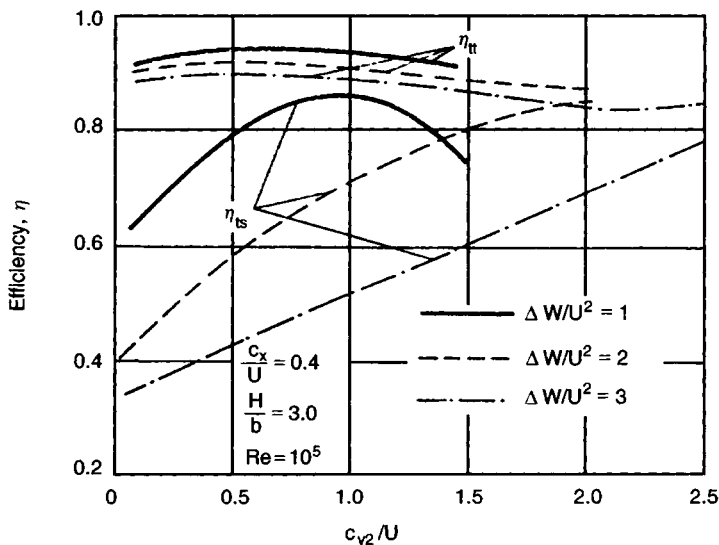


FIG. 4.4. Variation of efficiency with (c_{y2}/U) for several values of stage loading factor $\Delta W/U^2$ (adapted from Shapiro *et al.* 1957).

As $\Delta W = U^2 = U(c_{y2} + C_{y3})$ then as $c_{y2} = U, c_{y3} = 0$,

$$\phi = c_x/U = \cot \alpha_2 = 0.4, \text{ hence } \alpha_2 = 68.2^\circ.$$

The velocity triangles are symmetrical, so that $\alpha_2 = \beta_3$. Also, $\epsilon_R = \epsilon_N = \alpha_2 = 68.2^\circ$,

$$\therefore \zeta = 0.04 \times (1 + 1.5 \times 0.682^2) = 0.0679,$$

$$\frac{1}{\eta_{ts}} = 1 + \frac{2\zeta w_3^2 + c_x^2}{2U^2} = 1 + \zeta \phi^2 \sec^2 \beta_3 + \frac{1}{2} \phi^2$$

$$= 1 + \phi^2 (\zeta \sec^2 \beta_3 + 0.5)$$

$$= 1 + 0.4^2 \times (0.0679 \times 2.6928^2 + 0.5)$$

$$= 1 + 0.16 \times (0.49235 + 0.5),$$

$$\therefore \eta_{ts} = 0.863.$$

This value appears to be the same as the peak value of the efficiency curve $\Delta W/U^2 = 1.0$, in Figure 4.4.

Stage reaction

The classification of different types of axial turbine is more conveniently described by the *degree of reaction* or *reaction ratio* R , of each stage rather than by the ratio c_{y2}/U . As a means of description the term reaction has certain inherent advantages which become apparent later. Several definitions of reaction are available; the classical definition is given as the ratio of the static pressure drop in the rotor to the static pressure drop in the stage. However, it is more useful to define the reaction ratio as the static *enthalpy* drop in the rotor to the static *enthalpy* drop in the stage because it then becomes, in effect, a statement of the stage *flow geometry*. Thus,

$$R = (h_2 - h_3)/(h_1 - h_3). \quad (4.17)$$

If the stage is normal (i.e. $c_1 = c_3$) then,

$$R = (h_2 - h_3)/(h_{01} - h_{03}). \quad (4.18)$$

Using eqn. (4.4), $h_2 - h_3 = \frac{1}{2}(w_3^2 - w_2^2)$ and eqn. (4.18) becomes,

$$R = \frac{w_3^2 - w_2^2}{2U(c_{y2} + c_{y3})}. \quad (4.19)$$

Assuming constant axial velocity through the stage

$$R = \frac{(w_{y3} - w_{y2})(w_{y3} + w_{y2})}{2U(c_{y2} + c_{y3})} = \frac{w_{y3} - w_{y2}}{2U}, \quad (4.20)$$

since, upon referring to Figure 4.1, it is seen that

$$c_{y2} = w_{y2} + U \text{ and } c_{y3} = w_{y3} - U. \quad (4.21)$$

Thus,

$$R = \frac{c_x}{2U} (\tan \beta_3 - \tan \beta_2) \quad (4.22a)$$

or

$$R = \frac{1}{2} + \frac{c_x}{2U} (\tan \beta_3 - \tan \alpha_2), \quad (4.22b)$$

after using eqn. (4.21).

If $\beta_3 = \beta_2$, the reaction is zero; if $\beta_3 = \alpha_2$ the reaction is 50%. These two special cases are discussed below in more detail.

Zero reaction stage

From the definition of reaction, when $R = 0$, eqn. (4.18) indicates that $h_2 = h_3$ and eqn. (4.22a) that $\beta_2 = \beta_3$. The Mollier diagram and velocity triangles corresponding to these conditions are sketched in Figure 4.5. Now as $h_{02rel} = h_{03rel}$ and $h_2 = h_3$ for $R = 0$ it must follow, therefore, that $w_2 = w_3$. It will be observed from Figure 4.5 that, because of irreversibility, there is a *pressure drop* through the rotor row. The zero reaction stage is *not* the same thing as an *impulse* stage; in the latter case there is, by definition, no pressure drop through the rotor. The Mollier diagram for an impulse stage is shown in Figure 4.6 where it is seen that the enthalpy *increases*

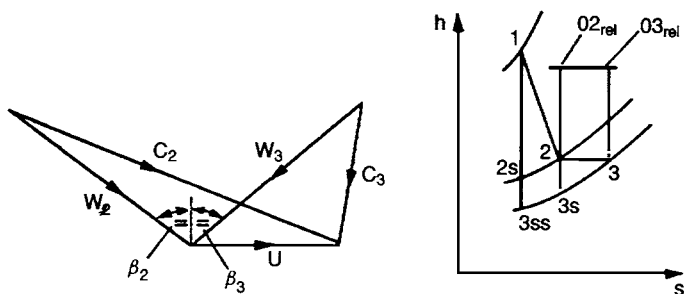


FIG. 4.5. Velocity diagram and Mollier diagram for a zero reaction turbine stage.

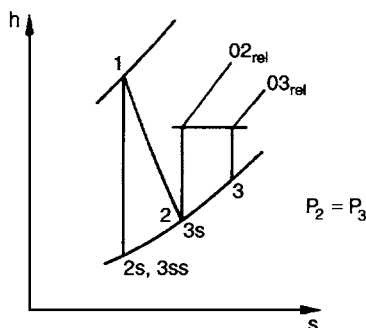


FIG. 4.6. Mollier diagram for an impulse turbine stage.

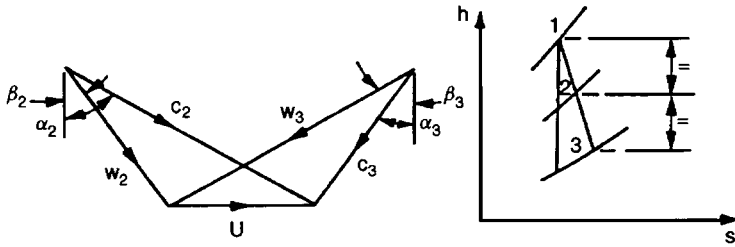


FIG. 4.7. Velocity diagram and Mollier diagram for a 50% reaction turbine stage.

through the rotor. The implication is clear from eqn. (4.18); the reaction is negative for the impulse turbine stage when account is taken of the irreversibility.

50 per cent reaction stage

The combined velocity diagram for this case is symmetrical as can be seen from Figure 4.7, since $\beta_3 = \alpha_2$. Because of the symmetry it is at once obvious that $\beta_2 = \alpha_3$, also. Now with $R = \frac{1}{2}$, eqn. (4.18) implies that the enthalpy drop in the nozzle row equals the enthalpy drop in the rotor, or

$$h_1 - h_2 = h_2 - h_3. \quad (4.23)$$

Figure 4.7 has been drawn with the same values of c_x , U and ΔW , as in Figure 4.5 (zero reaction case), to emphasise the difference in flow geometry between the 50% reaction and zero reaction stages.

Diffusion within blade rows

Any diffusion of the flow through turbine blade rows is particularly undesirable and must, at the design stage, be avoided at all costs. This is because the adverse pressure gradient (arising from the flow diffusion) coupled with large amounts of fluid deflection (usual in turbine blade rows), makes boundary-layer separation more than merely possible, with the result that large scale losses arise. A compressor blade row, on the other hand, is designed to cause the fluid pressure to rise in the direction of flow, i.e. an *adverse* pressure gradient. The magnitude of this gradient is strictly controlled in a compressor, mainly by having a fairly limited amount of fluid deflection in each blade row.

The comparison of the profile losses given in Figure 3.14 is illustrative of the undesirable result of negative “reaction” in a turbine blade row. The use of the term reaction here needs qualifying as it was only defined with respect to a complete stage. From eqn. (4.22a) the ratio R/ϕ can be expressed for a single row of blades if the flow angles are known. The original data provided with Figure 3.14 gives the blade inlet angles for impulse and reaction blades as 45.5 and 18.9 deg respectively. Thus, the flow angles can be found from Figure 3.14 for the range of incidence given, and R/ϕ can be calculated. For the reaction blades R/ϕ decreases as incidence increases going from 0.36 to 0.25 as i changes from 0 to 10 deg. The impulse blades, which it

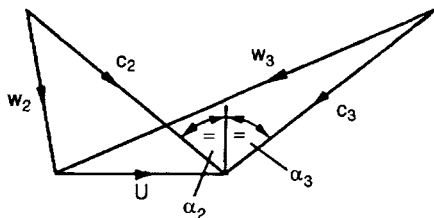


FIG. 4.8. Velocity diagram for 100% reaction turbine stage.

will be observed have a dramatic increase in blade profile loss, has R/ϕ decreasing from zero to -0.25 in the same range of incidence.

It was shown above that negative values of reaction indicated diffusion of the rotor relative velocity (i.e. for $R < 0$, $w_3 < w_2$). A similar condition which holds for diffusion of the nozzle absolute velocity is, that if $R > 1$, $c_2 < c_1$.

Substituting $\tan \beta_3 = \tan \alpha_3 + U/c_x$ into eqn. (4.22b) gives

$$R = 1 + \frac{c_x}{2U}(\tan \alpha_3 - \tan \alpha_2). \quad (4.22c)$$

Thus, when $\alpha_3 = \alpha_2$, the reaction is unity (also $c_2 = c_3$). The velocity diagram for $R = 1$ is shown in Figure 4.8 with the same values of c_x , U and ΔW used for $R = 0$ and $R = \frac{1}{2}$. It will be apparent that if R exceeds unity, then $c_2 < c_1$ (i.e. nozzle flow diffusion).

EXAMPLE 4.2. A single-stage gas turbine operates at its design condition with an axial absolute flow at entry and exit from the stage. The absolute flow angle at nozzle exit is 70° . At stage entry the total pressure and temperature are 311 kPa and 850°C respectively. The exhaust static pressure is 100 kPa , the total-to-static efficiency is 0.87 and the mean blade speed is 500 m/s .

Assuming constant axial velocity through the stage, determine

- (i) the specific work done;
- (ii) the Mach number leaving the nozzle;
- (iii) the axial velocity;
- (iv) the total-to-total efficiency;
- (v) the stage reaction.

Take $C_p = 1.148 \text{ kJ/(kg}^\circ\text{C)}$ and $\gamma = 1.33$ for the gas.

Solution. (i) From eqn. (4.10), total-to-static efficiency is

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} = \frac{\Delta W}{h_{01} \{1 - (p_3/p_{01})^{(\gamma-1)/\gamma}\}}.$$

Thus, the specific work is

$$\begin{aligned} \Delta W &= \eta_{ts} C_p T_{01} \{1 - (p_3/p_{01})^{(\gamma-1)/\gamma}\} \\ &= 0.87 \times 1148 \times 1123 \times \{1 - (1/3.11)^{0.248}\} \\ &= 276 \text{ kJ/kg.} \end{aligned}$$

(ii) At nozzle exit the Mach number is

$$M_2 = c_2 / (\gamma R T_2)^{1/2}$$

and it is necessary to solve the velocity diagram to find c_2 and hence to determine T_2 .

As $c_{y3} = 0$, $\Delta W = U c_{y2}$

$$c_{y2} = \frac{\Delta W}{U} = \frac{276 \times 10^3}{500} = 552 \text{ m/s}$$

$$c_2 = c_{y2} / \sin \alpha_2 = 588 \text{ m/s.}$$

Referring to Figure 4.2, across the nozzle $h_{01} = h_{02} = h_2 + \frac{1}{2} c_2^2$, thus

$$T_2 = T_{01} - \frac{1}{2} c_2^2 / C_p = 973 \text{ K.}$$

Hence, $M_2 = 0.97$ with $\gamma R = (\gamma - 1) C_p$.

(iii) The axial velocity, $c_x = c_2 \cos \alpha_2 = 200 \text{ m/s}$.

(iv) $\eta_{tt} = \Delta W / (h_{01} - h_{3ss} - \frac{1}{2} c_3^2)$.

After some rearrangement,

$$\frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{c_3^2}{2 \Delta W} = \frac{1}{0.87} - \frac{200^2}{2 \times 276 \times 10^3} = 1.0775.$$

Therefore $\eta_{tt} = 0.93$.

(v) Using eqn. (4.22a), the reaction is

$$R = \frac{1}{2} (c_x / U) (\tan \beta_3 - \tan \beta_2).$$

From the velocity diagram, $\tan \beta_3 = U / c_x$ and $\tan \beta_2 = \tan \alpha_2 - U / c_x$

$$R = 1 - \frac{1}{2} (c_x / U) \tan \alpha_2 = 1 - 200 \times 0.2745 / 1000 \\ = 0.451.$$

EXAMPLE 4.3. Verify the assumed value of total-to-static efficiency in the above example using Soderberg's correlation method. The average blade aspect ratio for the stage $H/b = 5.0$, the maximum blade thickness-chord ratio is 0.2 and the average Reynolds number, defined by eqn. (4.14), is 10^5 .

Solution. The approximation for total-to-static efficiency, eqn. (4.10a), is used and can be rewritten as

$$\frac{1}{\eta_{ts}} = 1 + \frac{\zeta_R (w_3 / U)^2 + \zeta_N (c_2 / U)^2 + (c_x / U)^2}{2 \Delta W / U^2}.$$

The loss coefficients ζ_R and ζ_N , uncorrected for the effects of blade aspect ratio, are determined using eqn. (4.12) which requires a knowledge of flow turning angle ϵ for each blade row.

For the nozzles, $\alpha_1 = 0$ and $\alpha_2 = 70^\circ$, thus $\epsilon_N = 70^\circ$.

$$\zeta_N^* = 0.04(1 + 1.5 \times 0.7^2) = 0.0694.$$

Correcting for aspect ratio with eqn. (4.13a),

$$\zeta_{N1} = 1.0694(0.993 + 0.021/5) - 1 = 0.0666.$$

For the rotor, $\tan \beta_2 = (c_{y2} - U)/c_x = (552 - 500)/200 = 0.26$,

$$\therefore \beta_2 = 14.55 \text{ deg.}$$

Therefore,

$$\tan \beta_3 = U/c_x = 2.5,$$

and $\beta_3 = 68.2 \text{ deg.}$

Therefore $\varepsilon_R = \beta_2 + \beta_3 = 82.75 \text{ deg.}$

$$\zeta_R^* = 0.04(1 + 1.5 \times 0.8275^2) = 0.0812.$$

Correcting for aspect ratio with eqn. (4.13b)

$$\zeta_{R1} = 1.0812(0.975 + 0.075/5) - 1 = 0.0712.$$

The velocity ratios are:

$$\left(\frac{w_3}{U}\right)^2 = 1 + \left(\frac{c_x}{U}\right)^2 = 1.16$$

$$\left(\frac{c_2}{U}\right)^2 = \left(\frac{588}{500}\right)^2 = 1.382; \left(\frac{c_x}{U}\right)^2 = 0.16$$

and the stage loading factor is,

$$\frac{\Delta W}{U^2} = \frac{c_{y2}}{U} = \frac{552}{500} = 1.104$$

Therefore

$$\frac{1}{\eta_{ts}} = 1 + \frac{0.0712 \times 1.16 + 0.0666 \times 1.382 + 0.16}{2 \times 1.104}$$

$$= 1 + 0.1515$$

$$\therefore \eta_{ts} = 0.869.$$

This result is very close to the value assumed in the first example.

It is not too difficult to include the temperature ratio T_3/T_2 implicit in the more exact eqn. (4.10) in order to see how little effect the correction will have. To calculate T_3

$$T_3 = T_{01} - \frac{\Delta W + \frac{1}{2}c_3^2}{C_p} = 1123 - \frac{276,000 + 20,000}{1148}$$

$$= 865 \text{ K.}$$

$$T_3/T_2 = 865/973 = 0.89.$$

Therefore

$$\begin{aligned}\frac{1}{\eta_{ts}} &= 1 + \frac{0.0712 \times 1.16 + 0.89 \times 0.0666 \times 1.382 + 0.16}{2 \times 1.104} \\ &= 1 + 0.1468.\end{aligned}$$

$$\therefore \eta_{ts} = 0.872.$$

Choice of reaction and effect on efficiency

In Figure 4.4 the total-to-total and total-to-static efficiencies are shown plotted against c_{y2}/U for several values of stage loading factor $\Delta W/U^2$. These curves can now easily be replotted against the degree of reaction R instead of c_{y2}/U . Equation (4.22c) can be rewritten as $R = 1 + (c_{y3} - c_{y2})/(2U)$ and c_{y3} eliminated using eqn. (4.16) to give

$$R = 1 + \frac{\Delta W}{2U^2} - \frac{c_{y2}}{U}. \quad (4.24)$$

The replotted curves are shown in Figure 4.9 as presented by Shapiro *et al.* (1957). In the case of total-to-static efficiency, it is at once apparent that this is optimised, at a given blade loading, by a suitable choice of reaction. When $\Delta W/U^2 = 2$, the maximum value of η_{ts} occurs with approximately zero reaction. With lighter blade loading, the optimum η_{ts} is obtained with higher reaction ratios. When $\Delta W/U^2 > 2$, the highest value of η_{ts} attainable without rotor *relative* flow diffusion occurring, is obtained with $R = 0$.

From Figure 4.4, for a fixed value of $\Delta W/U^2$, there is evidently only a relatively small changes in total-to-total efficiency (compared with η_{ts}), for a wide range of

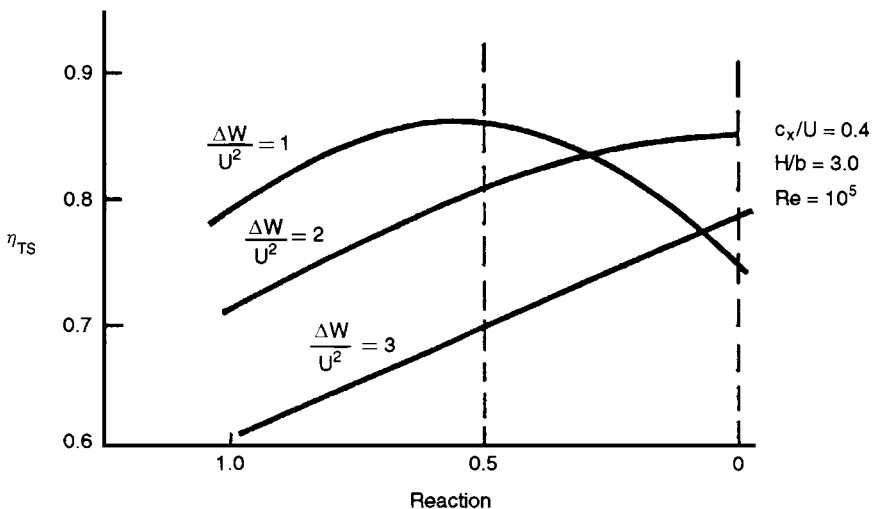


FIG. 4.9. Influence of reaction on total-to-static efficiency with fixed values of stage loading factor.

possible designs. Thus η_{tt} is not greatly affected by the choice of reaction. However, the maximum value of η_{tt} decreases as the stage loading factor increases. To obtain high total-to-total efficiency, it is therefore necessary to use the highest possible value of blade speed consistent with blade stress limitations (i.e. to reduce $\Delta W/U^2$).

Design point efficiency of a turbine stage

The performance of a turbine stage in terms of its efficiency is calculated for several types of design, i.e. 50 per cent reaction, zero reaction and zero exit flow angle, using the loss correlation method of Soderberg described earlier. These are most usefully presented in the form of carpet plots of stage loading coefficient, ψ , and flow coefficient, ϕ .

(1) Total-to-total efficiency of 50 per cent reaction stage

In a multistage turbine the total-to-total efficiency is the relevant performance criterion, the kinetic energy at stage exit being recovered in the next stage. After the last stage of a multistage turbine, or a single-stage turbine, the kinetic energy in the exit flow would be recovered in a diffuser or used for another purpose (e.g. as a contribution to the propulsive thrust).

From eqn. (4.9a), where it has already been assumed that $c_1 = c_3$ and $T_3 = T_2$, we have:

$$\frac{1}{\eta_{tt}} = 1 + \frac{(\zeta_R w_3^2 + \zeta_N c_2^2)}{2\Delta W},$$

where $\Delta W = \psi U^2$ and, for 50 per cent reaction, $w_3 = c_2$ and $\zeta_R = \zeta_N = \zeta$

$$w_3^2 = c_x^2 \sec^2 \beta_3 = c_x^2 (1 + \tan^2 \beta_3)$$

$$\therefore \frac{1}{\eta_{tt}} = 1 + \frac{\zeta \phi^2}{\psi} (1 + \tan^2 \beta_3) = 1 + \frac{\zeta \phi^2}{\psi} \left[1 + \left(\frac{1 + \psi}{2\phi} \right)^2 \right]$$

as $\tan \beta_3 = (\psi + 1)/(2\phi)$ and $\tan \beta_2 = (\psi - 1)/(2\phi)$.

From the above expressions the performance chart, shown in Figure 4.10, was derived for specified values of ψ and ϕ . From this chart it can be seen that the peak total-to-total efficiency, η_{tt} , is obtained at very low values of ϕ and ψ . As indicated in a survey by Kacker and Okapuu (1982), most aircraft gas turbine designs will operate with flow coefficients in the range, $0.5 \leq \psi \leq 1.5$, and values of stage loading coefficient in the range, $0.8 \leq \psi \leq 2.8$.

(2) Total-to-total efficiency of a zero reaction stage

The degree of reaction will normally vary along the length of the blade depending upon the type of design that is specified. The performance for $R = 0$ represents a limit, lower values of reaction are possible but undesirable as they would give rise to large losses in efficiency. For $R < 0$, $w_3 < w_2$, which means the relative flow decelerates across the rotor.

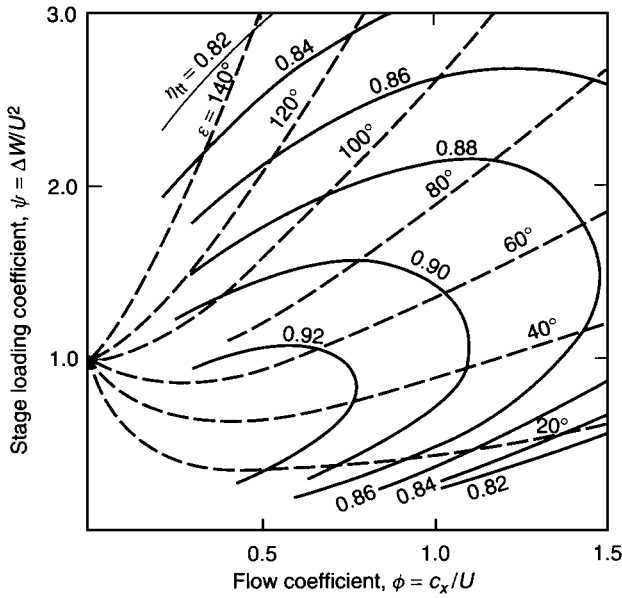


FIG. 4.10. Design point total-to-total efficiency and deflection angle contours for a turbine stage of 50 per cent reaction.

Referring to Figure 4.5, for zero reaction $\beta_2 = \beta_3$, and from eqn. (4.21)

$$\tan \alpha_2 = 1/\phi + \tan \beta_2 \text{ and } \tan \alpha_3 = \tan \beta_3 - 1/\phi.$$

$$\text{Also, } \psi = \Delta W/U^2 = \phi(\tan \alpha_2 + \tan \alpha_3) = \phi(\tan \beta_2 + \tan \beta_3) = 2\phi \tan \beta_2,$$

$$\therefore \tan \beta_2 = \frac{\psi}{2\phi}.$$

Thus, using the above expressions:

$$\tan \alpha_2 = (\psi/2 + 1)/\phi \text{ and } \tan \alpha_3 = (\psi/2 - 1)/\phi.$$

From these expressions the flow angles can be calculated if values for ψ and ϕ are specified. From an inspection of the velocity diagram,

$$c_2 = c_x \sec \alpha_2, \text{ hence } c_2^2 = c_x^2(1 + \tan^2 \alpha_2) = c_x^2[1 + (\psi/2 + 1)^2/\phi^2],$$

$$w_3 = c_x \sec \beta_3, \text{ hence } w_3^2 = c_x^2(1 + \tan^2 \beta_3) = c_x^2[1 + (\psi/2\phi)^2].$$

Substituting the above expressions into eqn. (4.9a):

$$\frac{1}{\eta_{tt}} = 1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2}{2\psi U^2},$$

$$\frac{1}{\eta_{tt}} = 1 + \frac{1}{2\psi} \left\{ \zeta_R \left[\phi^2 + \left(\frac{\psi}{2} \right)^2 \right] + \zeta_N \left[\phi^2 + \left(1 + \frac{\psi}{2} \right)^2 \right] \right\}.$$

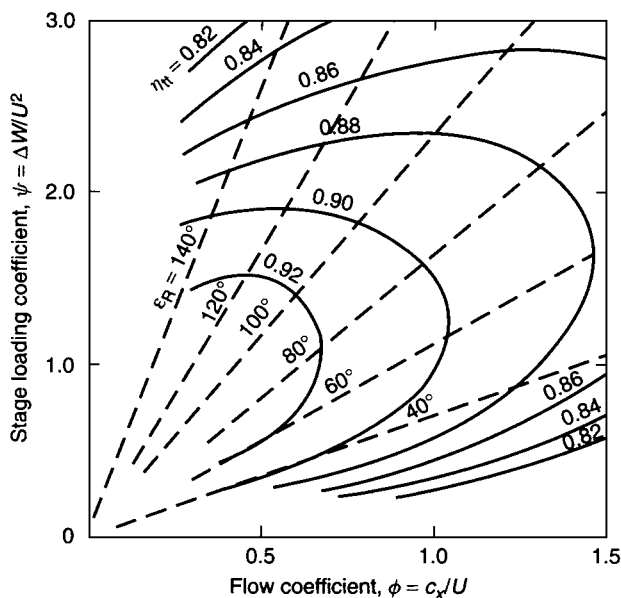


FIG. 4.11. Design point total-to-total efficiency and rotor flow deflection angle for a zero reaction turbine stage.

The performance chart shown in Figure 4.11 has been derived using the above expressions. This is similar in its general form to Figure 4.10 for 50 per cent reaction, with the highest efficiencies being obtained at the lowest values of ϕ and ψ , except that higher efficiencies are obtained at higher values of the stage loading but at reduced values of the flow coefficient.

(3) Total-to-static efficiency of stage with axial velocity at exit

A single-stage axial turbine will have axial flow at exit and the most appropriate efficiency is usually total-to-static. To calculate the performance, eqn. (4.10a) is used:

$$\begin{aligned}\frac{1}{\eta_{ts}} &= 1 + \frac{(\zeta_R w_3^2 + \zeta_N c_2^2 + c_1^2)}{2\Delta W}, \\ &= 1 + \frac{\phi^2}{2\psi} (\zeta_R \sec^2 \beta_3 + \zeta_N \sec^2 \alpha_2 + 1).\end{aligned}$$

With axial flow at exit, $c_1 = c_3 = c_x$, and from the velocity diagram, Figure 4.12,

$$\begin{aligned}\tan \beta_3 &= U/c_x, \quad \tan \beta_2 = \tan \alpha_2 - \tan \beta_3, \\ \sec^2 \beta_3 &= 1 + \tan^2 \beta_3 = 1 + 1/\phi^2, \\ \sec^2 \alpha_2 &= 1 + \tan^2 \alpha_2 = 1 + (\psi/\phi)^2, \\ \therefore \frac{1}{\eta_{ts}} &= 1 + \frac{1}{2\phi} [\zeta_R (1 + \phi^2) + \zeta_N (\psi^2 + \phi^2) + \phi^2].\end{aligned}$$

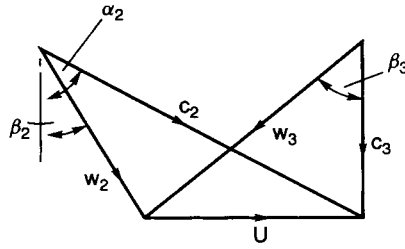


FIG. 4.12. Velocity diagram for turbine stage with axial exit flow.

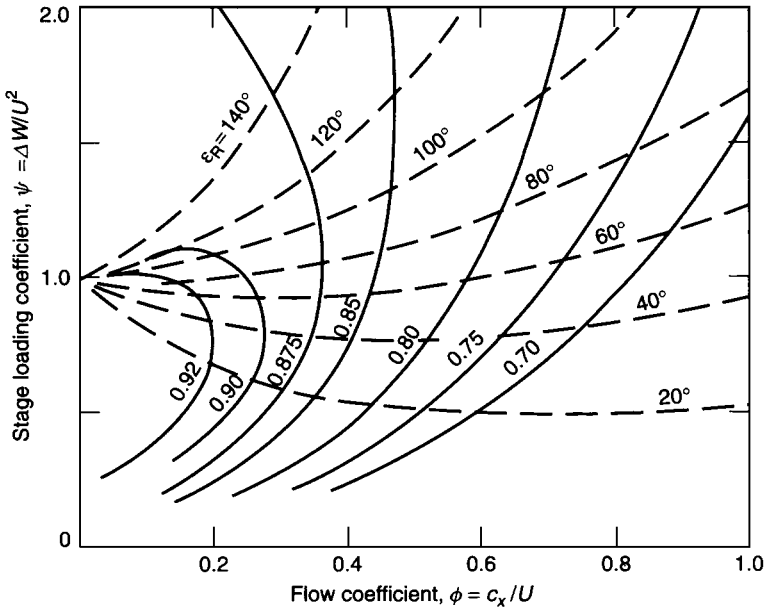


FIG. 4.13. Total-to-total efficiency contours for a stage with axial flow at exit.

Specifying ϕ , ψ , the unknown values of the loss coefficients, ζ_R and ζ_N , can be derived using Soderberg's correlation, eqn. (4.12), in which

$$\varepsilon_N = \alpha_2 = \tan^{-1}(\psi/\phi) \text{ and } \varepsilon_R = \beta_2 + \beta_3 = \tan^{-1}(1/\phi) + \tan^{-1}[(\psi - 1)/\phi].$$

From the above expressions the performance chart, Figure 4.13, has been derived.

An additional limitation is imposed on the performance chart because of the reaction which must remain greater than or, in the limit, equal to zero. From eqn. (4.22a),

$$R = \frac{\phi}{2}(\tan \beta_3 - \tan \beta_2) = 1 - \frac{\psi}{2}.$$

Thus at the limit, $R = 0$, the stage loading coefficient, $\psi = 2$.

Maximum total-to-static efficiency of a reversible turbine stage

When blade losses and exit kinetic energy loss are included in the definition of efficiency, we have shown, eqn. (4.10a), that the efficiency is

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} = \left[1 + \frac{w_3^2 \zeta_r + c_2^2 \zeta_n + c_3^2}{2(h_1 - h_3)} \right]^{-1}.$$

In the case of the *ideal* (or reversible) turbine stage the only loss is due to the exhaust kinetic energy and then the total-to-static efficiency is

$$\eta_{ts} = \frac{h_{01} - h_{03ss}}{h_{01} - h_{3ss}} = \left[1 + \frac{c_3^2}{2U(c_{y2} + c_{y3})} \right]^{-1} \quad (4.25a)$$

since $\Delta W = h_{01} - h_{03ss} = U(c_{y2} + c_{y3})$ and $h_{03ss} - h_{3ss} = \frac{1}{2}c_3^2$.

The maximum value of η_{ts} is obtained when the exit velocity c_3 is *nearly* a minimum for given turbine stage operating conditions (R , ϕ and α_2). On first thought it may appear obvious that maximum η_{ts} will be obtained when c_3 is absolutely axial (i.e. $\alpha_3 = 0^\circ$) but this is incorrect. By allowing the exit flow to have some counterclockwise (i.e. $\alpha_3 > 0^\circ$) the work done is increased for only a relatively small increase in the exit kinetic energy loss. Two analyses are now given to show how the total-to-static efficiency of the ideal turbine stage can be optimised for specified conditions.

Substituting $c_{y2} = c_x \tan \alpha_2$, $c_{y3} = c_x \tan \alpha_3$, $c_3 = c_x / \cos \alpha_3$ and $\phi = c_x / U$ into eqn. (4.25), leads to

$$\eta_{ts} = \left[1 + \frac{\phi(1 + \tan^2 \alpha_3)}{2(\tan \alpha_2 + \tan \alpha_3)} \right]^{-1} \quad (4.25b)$$

i.e. $\eta_{ts} = \text{fn}(\phi, \alpha_2, \alpha_3)$.

(i) To find the optimum η_{ts} when R and ϕ are specified

From eqn. (4.22c) the nozzle flow outlet angle α_2 can be expressed in terms of R , ϕ and α_3 as

$$\tan \alpha_2 = \tan \alpha_3 + 2(1 - R)/\phi. \quad (4.26)$$

Substituting into eqn. (4.25b)

$$\eta_{ts} = \left[1 + \frac{\phi^2(1 + \tan^2 \alpha_3)}{4(\phi \tan \alpha_3 + 1 - R)} \right]^{-1}.$$

Differentiating this expression with respect to $\tan \alpha_3$, and equating the result to zero,

$$\tan^2 \alpha_3 + 2k \tan \alpha_3 - 1 = 0$$

where $k = (1 - R)/\phi$. This quadratic equation has the solution

$$\tan \alpha_3 = -k + \sqrt{k^2 + 1} \quad (4.27)$$

the value of α_3 being the optimum flow outlet angle from the stage when R and ϕ are specified. From eqn. (4.26), $k = (\tan \alpha_2 - \tan \alpha_3)/2$ which when substituted into eqn. (4.27) and simplified gives

$$\tan \alpha_3 = \cot \alpha_2 = \tan(\pi/2 - \alpha_2).$$

Hence, the exact result that

$$\alpha_3 = \pi/2 - \alpha_2.$$

The corresponding idealised $\eta_{ts \max}$ and R are

$$\eta_{ts \max} = [1 + (\phi/2) \cot \alpha_2]^{-1} \quad (4.28a)$$

$$R = 1 - \phi(\tan \alpha_2 - \cot \alpha_2)/2.$$

(ii) To find the optimum η_{ts} when α_2 and ϕ are specified

Differentiating eqn. (4.25b) with respect to $\tan \alpha_3$ and equating the result to zero,

$$\tan^2 \alpha_3 + 2 \tan \alpha_2 \tan \alpha_3 - 1 = 0.$$

Solving this quadratic, the relevant root is

$$\tan \alpha_3 = \sec \alpha_2 - \tan \alpha_2.$$

Using simple trigonometric relations this simplifies still further to

$$\alpha_3 = (\pi/2 - \alpha_2)/2.$$

Substituting this expression for α_3 into eqn. (4.25b) the idealised maximum η_{ts} is obtained

$$\eta_{ts \max} = [1 + \phi(\sec \alpha_2 - \tan \alpha_2)]^{-1}. \quad (4.28b)$$

The corresponding expressions for the degree of reaction R and stage loading coefficient $\Delta W/U^2$ are

$$R = 1 - \phi(\tan \alpha_2 - \frac{1}{2} \sec \alpha_2)$$

$$\frac{\Delta W}{U^2} = \phi \sec \alpha_2 = \frac{c_2}{U}. \quad (4.29)$$

It is interesting that in this analysis the exit swirl angle α_3 is only half that of the constant reaction case. The difference is merely the outcome of the two different sets of constraints used for the two analyses.

For both analyses, as the flow coefficient is reduced towards zero, α_2 approaches $\pi/2$ and α_3 approaches zero. Thus, for such high nozzle exit angle turbine stages, the appropriate blade loading factor for maximum η_{ts} can be specified if the reaction is known (and conversely). For a turbine stage of 50% reaction (and with $\alpha_3 \rightarrow 0$ deg) the appropriate velocity diagram shows that $\Delta W/U^2 \doteq 1$ for maximum η_{ts} . Similarly, a turbine stage of zero reaction (which is an *impulse* stage for ideal, reversible flow) has a blade loading factor $\Delta W/U^2 \doteq 2$ for maximum η_{ts} .

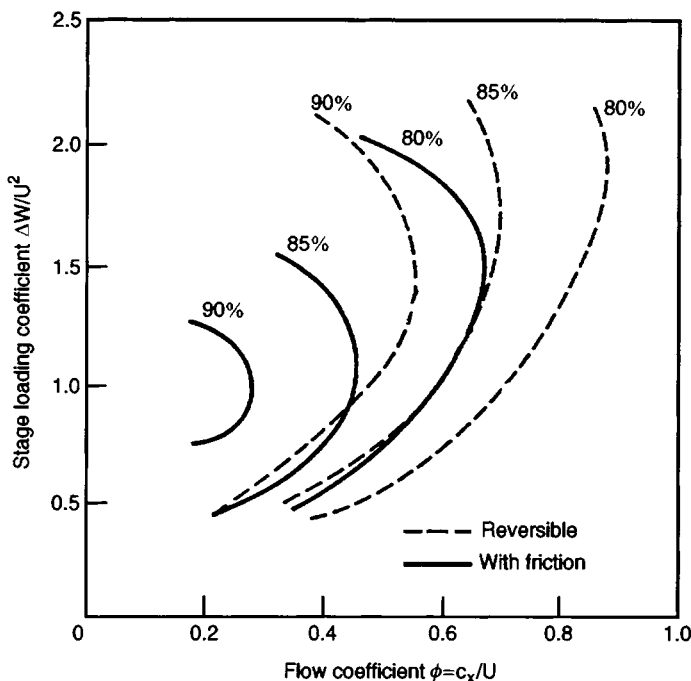


FIG. 4.14. Total-to-static efficiency of a 50% reaction axial flow turbine stage (adapted from Horlock 1966).

Calculations of turbine stage performance have been made by Horlock (1966) both for the reversible and irreversible cases with $R = 0$ and 50%. Figure 4.14 shows the effect of blade losses, determined with Soderberg's correlation, on the total-to-static efficiency of the turbine stage for the constant reaction of 50%. It is evident that exit losses become increasingly dominant as the flow coefficient is increased.

Stresses in turbine rotor blades

Although this chapter is primarily concerned with the fluid mechanics and thermodynamics of turbines, some consideration of stresses in rotor blades is needed as these can place restrictions on the allowable blade height and annulus flow area, particularly in high temperature, high stress situations. Only a very brief outline is attempted here of a very large subject which is treated at much greater length by Horlock (1966), in texts dealing with the mechanics of solids, e.g. Den Hartog (1952), Timoshenko (1957), and in specialised discourses, e.g. Japiske (1986) and Smith (1986). The stresses in turbine blades arise from centrifugal loads, from gas bending loads and from vibrational effects caused by non-constant gas loads. Although the centrifugal stress produces the biggest contribution to the total stress, the vibrational stress is very significant and thought to be responsible for fairly common vibratory fatigue failures (Smith 1986). The direct and simple approach to blade vibration is to "tune" the blades so that resonance does not occur in the

operating range of the turbine. This means obtaining a blade design in which none of its natural frequencies coincides with any excitation frequency. The subject is complex, interesting but outside of the scope of the present text.

Centrifugal stresses

Consider a blade rotating about an axis O as shown in Figure 4.15. For an element of the blade of length dr at radius r , at a rotational speed Ω the elementary centrifugal load dF_c is given by,

$$dF_c = -\Omega^2 r dm,$$

where $dm = \rho_m A dr$ and the negative sign accounts for the direction of the stress gradient (i.e. zero stress at the blade tip to a maximum at the blade root).

$$\frac{d\sigma_c}{\rho_m} = \frac{dF_c}{\rho_m A} = -\Omega^2 r dr.$$

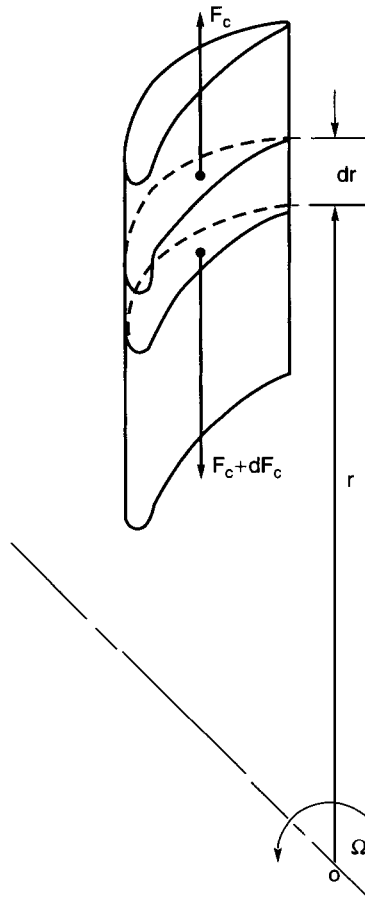


FIG. 4.15. Centrifugal forces acting on rotor blade element.

For blades with a constant cross-sectional area, we get

$$\frac{\sigma_c}{\rho_m} = \Omega^2 \int_{r_h}^{r_t} r \, dr = \frac{U_t^2}{2} \left[1 - \left(\frac{r_h}{r_t} \right)^2 \right]. \quad (4.30a)$$

A rotor blade is usually tapered both in chord and in thickness from root to tip such that the area ratio A_t/A_h is between 1/3 and 1/4. For such a blade taper it is often assumed that the blade stress is reduced to 2/3 of the value obtained for an untapered blade. A blade stress taper factor can be defined as:

$$K = \frac{\text{stress at root of tapered blade}}{\text{stress at root of untapered blade}}$$

Thus, for tapered blades

$$\frac{\sigma_c}{\rho_m} = \frac{K U_t^2}{2} \left[1 - \left(\frac{r_h}{r_t} \right)^2 \right]. \quad (4.30b)$$

Values of the taper factor K quoted by Emmert (1950), are shown in Figure 4.16 for various taper geometries.

Typical data for the allowable stresses of commonly used alloys are shown in Figure 4.17 for the “1000 hr rupture life” limit with maximum stress allowed plotted as a function of blade temperature. It can be seen that in the temperature range 900–1100 K, nickel or cobalt alloys are likely to be suitable and for temperatures up to about 1300 K molybdenum alloys would be needed.

By means of blade cooling techniques it is possible to operate with turbine entry temperatures up to 1650–1700 K, according to Le Grivès (1986). Further detailed information on one of the many alloys used for gas turbines blades is shown in Figure 4.18. This material is *Inconel*, a nickel-based alloy containing 13% chromium, 6% iron, with a little manganese, silicon and copper. Figure 4.18 shows the influence of the “rupture life” and also the “percentage creep”, which is the

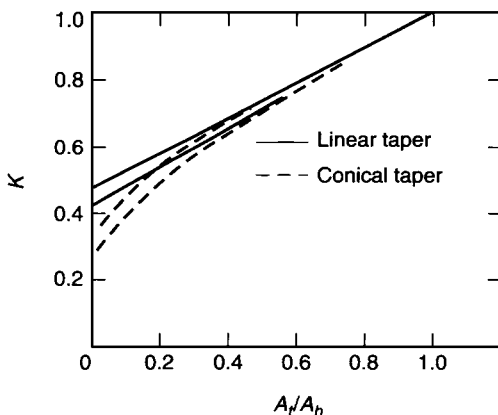


FIG. 4.16. Effect of tapering on centrifugal stress at blade root (adapted from Emmert 1950).

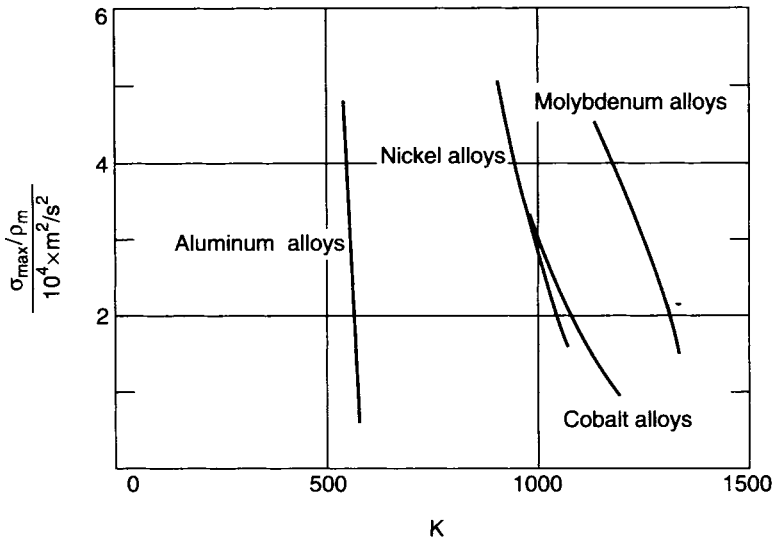


FIG. 4.17. Maximum allowable stress for various alloys (1000 hr rupture life) (adapted from Freeman 1955).

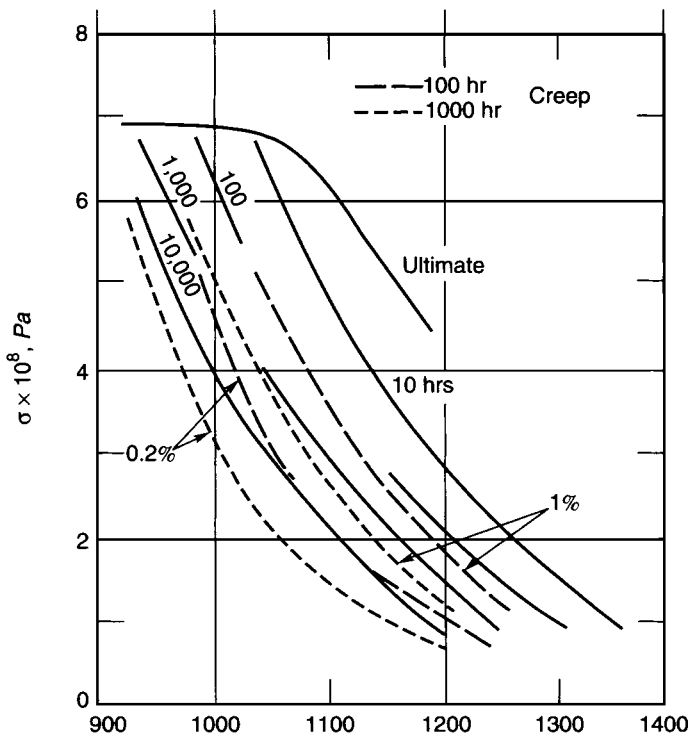


FIG. 4.18. Properties of Inconel 713 Cast (adapted from Balje 1981).

elongation strain at the allowable stress and temperature of the blade. To enable operation at high temperatures and for long life of the blades, the creep strength criterion is the one usually applied by designers.

An estimate of the average rotor blade temperature T_b can be made using the approximation,

$$T_b = T_2 + 0.85w_2^2/(2C_p), \quad (4.31)$$

i.e. 85% temperature recovery of the inlet *relative kinetic energy*.

EXAMPLE 4.4. Combustion gases enter the first stage of a gas turbine at a stagnation temperature and pressure of 1200 K and 4.0 bar. The rotor blade tip diameter is 0.75 m, the blade height is 0.12 m and the shaft speed is 10 500 rev/min. At the mean radius the stage operates with a reaction of 50 per cent, a flow coefficient of 0.7 and a stage loading coefficient of 2.5.

Determine:

- (1) the relative and absolute flow angles for the stage;
- (2) the velocity at nozzle exit;
- (3) the static temperature and pressure at nozzle exit assuming a nozzle efficiency of 0.96 and the mass flow;
- (4) the rotor blade root stress assuming the blade is tapered with a stress taper factor K of 2/3 and the blade material density is 8000 kg/m²;
- (5) the approximate mean blade temperature;
- (6) taking only the centrifugal stress into account suggest a suitable alloy from the information provided which could be used to withstand 1000 hr of operation.

Solution. (1) The stage loading is

$$\psi = \Delta h_0/U^2 = (w_{y3} + w_{y2})/U = \phi(\tan \beta_3 + \tan \beta_2).$$

From eqn. (4.20) the reaction is

$$R = \phi(\tan \beta_3 - \tan \beta_2)/2.$$

Adding and subtracting these two expressions, we get

$$\tan \beta_3 = (\psi/2 + R)/\phi \text{ and } \tan \beta_2 = (\psi/2 - R)/\phi.$$

Substituting values of ψ , ϕ and R into the preceding equations we obtain

$$\beta_3 = 68.2^\circ, \beta_2 = 46.98^\circ$$

and for similar triangles (i.e. 50% reaction)

$$\alpha_2 = \beta_3 \text{ and } \alpha_3 = \beta_2$$

(2) At the mean radius, $r_m = (0.75 - 0.12)/2 = 0.315$ m, the blade speed is $U_m = \Omega r_m = (10500/30) \times \pi \times 0.315 = 1099.6 \times 0.315 = 346.36$ m/s. The axial velocity $c_x = \phi U_m = 0.5 \times 346.36 = 242.45$ m/s and the velocity of the gas at nozzle exit is, $c_2 = c_x / \cos \alpha_2 = 242.45 / \cos 68.2 = 652.86$ m/s.

(3) To determine the conditions at nozzle exit, we have

$$T_2 = T_{02} - \frac{1}{2} c_2^2 / C_p = 1200 - 652.86^2 / (2 \times 1160) = 1016.3 \text{ K}$$

From eqn. (2.40), the nozzle efficiency is

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}} = \frac{1 - T_2/T_{01}}{1 - (p_2/p_{01})^{(\gamma-1)/\gamma}}$$

$$\therefore \left(\frac{p_2}{p_{01}} \right)^{(\gamma-1)/\gamma} = 1 - \frac{1 - T_2/T_{01}}{\eta_N} = 1 - (1 - 1016.3/1200)/0.96 = 0.84052$$

$$\therefore p_2 = 4 \times 0.840052^{4.0303} = 1.986 \text{ bar}$$

The mass flow is found from the continuity equation:

$$\dot{m} = \rho_2 A_2 c_{x2} = \left(\frac{p_2}{RT_2} \right) A_2 c_{x2}$$

$$\therefore \dot{m} = \left(\frac{1.986 \times 10^5}{287.8 \times 1016.3} \right) \times 0.2375 \times 242.45 = 39.1 \text{ kg/s}$$

(4) For a tapered blade, eqn. (4.30b) gives

$$\frac{\sigma_c}{\rho_m} = \frac{2}{3} \times \frac{412.3^2}{2} \left[1 - \left(\frac{0.51}{0.75} \right)^2 \right] = 30463.5 \text{ m}^2/\text{s}^2$$

where $U_t = 1099.6 \times 0.375 = 412.3 \text{ m/s}$.

The density of the blade material is taken to be 8000 kg/m^3 and so the root stress is

$$\sigma_c = 8000 \times 30463.5 = 2.437 \times 10^8 \text{ N/m}^2 = 243.7 \text{ MPa}$$

(5) The approximate average mean blade temperature is

$$T_b = 1016.3 + 0.85 \times (242.45 / \cos 46.975)^2 / (2 \times 1160)$$

$$= 1016.3 + 46.26 = 1062.6 \text{ K}$$

(6) The data in Figure 4.17 suggests that for this moderate root stress, cobalt or nickel alloys would not withstand a lifespan of 1000 hr to rupture and the use of molybdenum would be necessary. However, it would be necessary to take account of bending and vibratory stresses and the decision about the choice of a suitable blade material would be decided on the outcome of these calculations.

Inspection of the data for Inconel 713 cast alloy, Figure 4.18, suggests that it might be a better choice of blade material as the temperature–stress point of the above calculation is to the left of the line marked creep strain of 0.2% in 1000 hr. Again, account must be taken of the additional stresses due to bending and vibration.

Design is a process of trial and error; changes in the values of some of the parameters can lead to a more viable solution. In the above case (with bending and vibrational stresses included) it might be necessary to reduce one or more of the values chosen, e.g.

- (1) the rotational speed,
- (2) the inlet stagnation temperature,
- (3) the flow area.

NB. The combination of values for ψ and ϕ at $R = 0.5$ used in this example were selected from data given by Wilson (1987) and correspond to an optimum total-to-total efficiency of 91.9%.

Turbine blade cooling

In the gas turbine industry there has been a continuing trend towards higher turbine inlet temperatures, either to give increased specific thrust (thrust per unit air mass flow) or to reduce the specific fuel consumption. The highest allowable gas temperature at entry to a turbine with uncooled blades is 1250–1300 K while, with blade cooling systems, a range of gas temperatures up to 1800 K or so may be employed, depending on the nature of the cooling system.

Various types of cooling system for gas turbines have been considered in the past and a number of these are now in use. Wilde (1977) reviewed the progress in blade cooling techniques. He also considered the broader issues involving the various technical and design factors influencing the best choice of turbine inlet temperature for future turbofan engines. Le Grivès (1986) reviewed types of cooling system, outlining their respective advantages and drawbacks, and summarising important analytical considerations concerning their aerodynamics and heat transfer.

The system of blade cooling most commonly employed in aircraft gas turbines is where some cooling air is bled off from the exit stage of the high-pressure compressor and carried by ducts to the guide vanes and rotor of the high-pressure turbine. It was observed by Le Grivès that the cooling air leaving the compressor might be at a temperature of only 400 to 450 K less than the maximum allowable blade temperature of the turbine. Figure 4.19 illustrates a high-pressure turbine rotor blade, cut away to show the intricate labyrinth of passages through which the cooling air passes before it is vented to the blade surface via the rows of tiny holes along and around the leading edge of the blade. Ideally, the air emerges with little velocity and form a film of cool air around the blade surface (hence the term “film cooling”), insulating it from the hot gases. This type of cooling system enables turbine entry temperatures up to 1800 K to be used.

There is a rising thermodynamic penalty incurred with blade cooling systems as the turbine entry temperature rises, e.g. energy must be supplied to pressurise the air bled off from the compressor. Figure 4.21 is taken from Wilde (1977) showing how the net turbine efficiency decreases with increasing turbine entry temperature. Several in-service gas turbine engines are included in the graph. Wilde did question whether turbine entry temperatures greater than 1600 K could really be justified in turbofan engines because of the effect on the internal aerodynamic efficiency and specific fuel consumption.

Turbine flow characteristics

An accurate knowledge of the flow characteristics of a turbine is of considerable practical importance as, for instance, in the matching of flows between a compressor

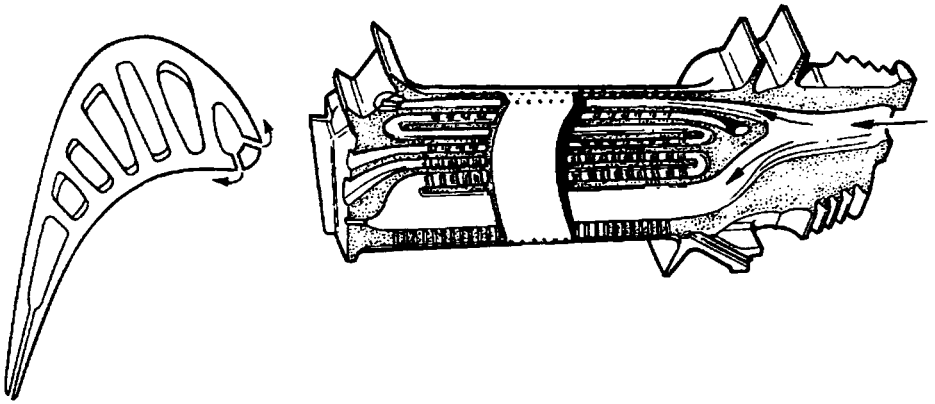


FIG. 4.19. Cooled HP turbine rotor blade showing the cooling passages (courtesy of Rolls-Royce plc).

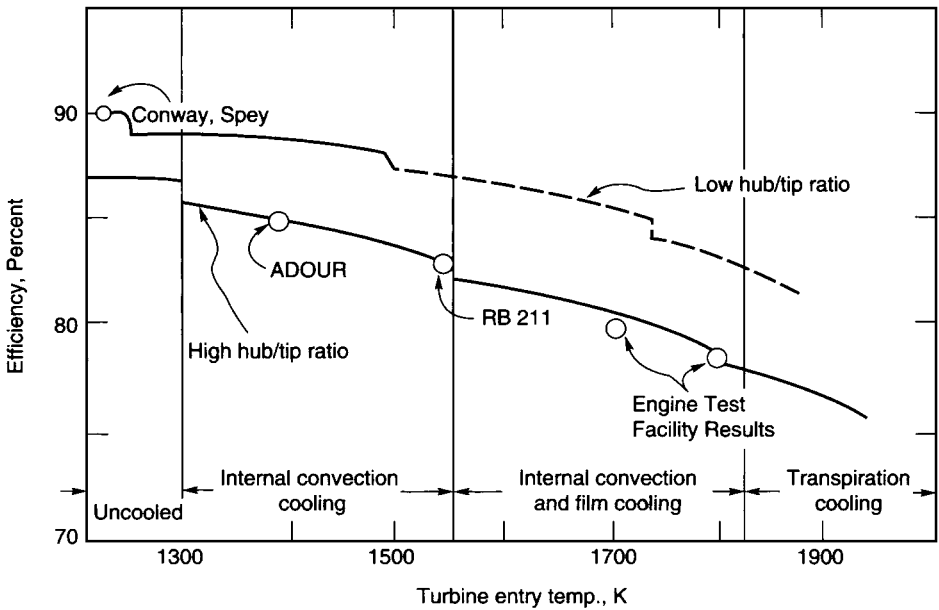


FIG. 4.20. Turbine thermal efficiency vs inlet gas temperature (adapted from le Grivès 1986).

and turbine of a jet engine. When a turbine can be expected to operate close to its design incidence (i.e. in the low loss region) the turbine characteristics can be reduced to a single curve. Figure 4.21, due to Mallinson and Lewis (1948), shows a comparison of typical characteristics for one, two and three stages plotted as turbine overall pressure ratio p_{0II}/p_{0I} against a mass flow coefficient $\dot{m}(\sqrt{T_{0I}})/p_{0I}$. There is a noticeable tendency for the characteristic to become more ellipsoidal as the number of stages is increased. At a given pressure ratio the mass flow coefficient, or “swallowing capacity” tends to decrease with the addition of further stages to

the turbine. One of the earliest attempts to assess the flow variation of a multistage turbine is credited to Stodola (1945), who formulated the much used “ellipse law”. The curve labelled “multistage” in Figure 4.21 is in agreement with the “ellipse law” expression

$$m(\sqrt{T_{01}})/p_{01} = k[1 - (p_{0II}/p_{0I})^2]^{1/2}, \quad (4.32)$$

where k is a constant.

This expression has been used for many years in steam turbine practice, but an accurate estimate of the variation in swallowing capacity with pressure ratio is of even greater importance in gas turbine technology. Whereas the average condensing steam turbine, even at part-load, operates at very high pressure ratios, some gas turbines may work at rather low pressure ratios, making flow matching with a compressor a difficult problem. The constant value of swallowing capacity, reached by the single-stage turbine at a pressure ratio a little above 2, and the other turbines at progressively higher pressure ratios, is associated with choking (sonic) conditions in the turbine stator blades.

Flow characteristics of a multistage turbine

Several derivations of the ellipse law are available in the literature. The derivation given below is a slightly amplified version of the proof given by Horlock (1958). A more general method has been given by Egli (1936) which takes into consideration the effects when operating outside the normal low loss region of the blade rows.

Consider a turbine comprising a large number of normal stages, each of 50% reaction; then, referring to the velocity diagram of Figure 4.22a, $c_1 = c_3 = w_2$ and $c_2 = w_3$. If the blade speed is maintained constant and the mass flow is reduced, the

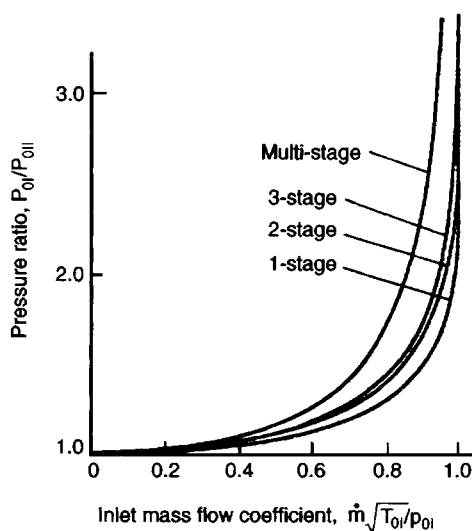


FIG. 4.21. Turbine flow characteristics (after Mallinson and Lewis 1948).

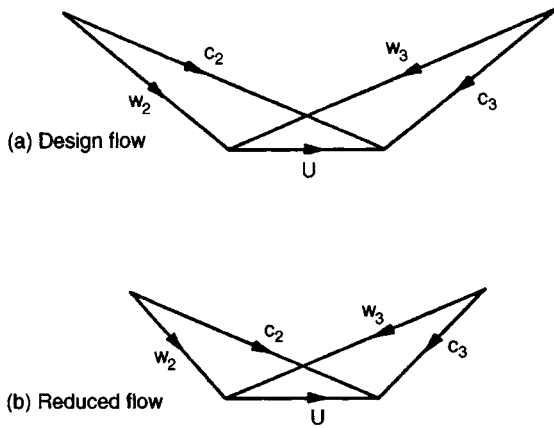


FIG. 4.22. Change in turbine stage velocity diagram with mass flow at constant blade speed.

fluid angles at exit from the rotor (β_3) and nozzles (α_2) will remain constant and the velocity diagram then assumes the form shown in Figure 4.22b. The turbine, if operated in this manner, will be of low efficiency, as the fluid direction at inlet to each blade row is likely to produce a negative incidence stall. To maintain high efficiency the fluid inlet angles must remain fairly close to the design values. It is therefore assumed that the turbine operates at its highest efficiency at *all off-design conditions* and, by implication, the blade speed is changed in direct proportion to the axial velocity. The velocity triangles are similar at off-design flows but of different scale.

Now the work done by unit mass of fluid through one stage is $U(c_{y2} + c_{y3})$ so that, assuming a perfect gas,

$$C_p \Delta T_0 = C_p \Delta T = U c_x (\tan \alpha_2 + \tan \alpha_3)$$

and, therefore,

$$\Delta T \propto c_x^2.$$

Denoting design conditions by subscript d , then

$$\frac{\Delta T}{\Delta T_d} = \left(\frac{c_x}{c_{xd}} \right)^2 \quad (4.33)$$

for equal values of c_x/U .

From the continuity equation, at off-design, $\dot{m} = \rho A c_x = \rho_I A_I c_{xI}$, and at design, $\dot{m}_d = \rho_d A c_{xd} = \rho_I A_I c_{xId}$, hence

$$\frac{c_x}{c_{xd}} = \frac{\rho_d}{\rho} \frac{c_{xId}}{c_{xI}} = \frac{\rho_d}{\rho} \frac{\dot{m}}{\dot{m}_d}. \quad (4.34)$$

Consistent with the assumed mode of turbine operation, the polytropic efficiency is taken to be constant at off-design conditions and, from eqn. (2.37), the relationship

between temperature and pressure is therefore,

$$T/p^{\eta_p(\gamma-1)/\gamma} = \text{constant}.$$

Combined with $p/\rho = RT$ the above expression gives, on eliminating p , $\rho/T^n = \text{constant}$, hence

$$\frac{\rho}{\rho_d} = \left(\frac{T}{T_d} \right)^n, \quad (4.35)$$

where $n = \gamma/[\eta_p(\gamma - 1)] - 1$.

For an infinitesimal temperature drop eqn. (4.33) combined with eqns. (4.34) and (4.35) gives, with little error,

$$\frac{dT}{dT_d} = \left(\frac{c_x}{c_{xd}} \right)^2 = \left(\frac{T_d}{T} \right)^{2n} \left(\frac{\dot{m}}{\dot{m}_d} \right)^2. \quad (4.36)$$

Integrating eqn. (4.36),

$$T^{2n+1} = \left(\frac{\dot{m}}{\dot{m}_d} \right)^2 T_d^{2n+1} + K,$$

where K is an arbitrary constant.

To establish a value for K it is noted that if the turbine entry temperature is constant $T_d = T_1$ and $T = T_1$ also.

Thus, $K = [1 - (\dot{m}/\dot{m}_d)^2] T_1^{2n+1}$ and

$$\left(\frac{T}{T_1} \right)^{2n+1} - 1 = \left(\frac{\dot{m}}{\dot{m}_d} \right)^2 \left[\left(\frac{T_d}{T_1} \right)^{2n+1} - 1 \right]. \quad (4.37)$$

Equation (4.37) can be rewritten in terms of pressure ratio since $T/T_1 = (p/p_1)^{\eta_p(\gamma-1)/\gamma}$. As $2n + 1 = 2\gamma/[\eta_p(\gamma - 1)] - 1$ then,

$$\frac{\dot{m}}{\dot{m}_d} = \left\{ \frac{1 - (p/p_1)^{2-\eta_p(\gamma-1)/\gamma}}{1 - (p_d/p_1)^{2-\eta_p(\gamma-1)/\gamma}} \right\}^{1/2} \quad (4.38a)$$

With $\eta_p = 0.9$ and $\gamma = 1.3$ the pressure ratio index is about 1.8; thus the approximation is often used

$$\frac{\dot{m}}{\dot{m}_d} = \left\{ \frac{1 - (p/p_1)^2}{1 - (p_d/p_1)^2} \right\}^{1/2}, \quad (4.38b)$$

which is ellipse law of a multistage turbine.

The Wells turbine

Introduction

Numerous methods for extracting energy from the motion of sea-waves have been proposed and investigated since the late 1970s. The problem is in finding an

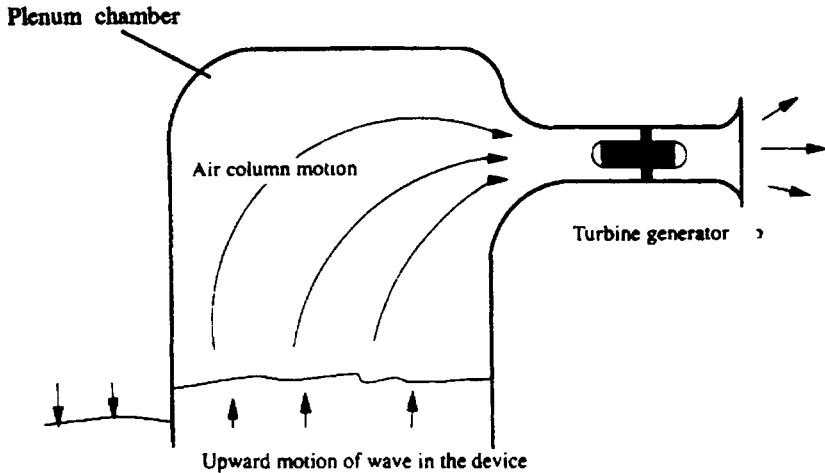


FIG. 4.23. Arrangement of Wells turbine and oscillating water column (adapted from Raghunathan *et al.* 1995).

efficient and economical means of converting an oscillating flow of energy into a unidirectional rotary motion for driving an electrical generator. A novel solution of this problem is the Wells turbine (Wells 1976), a version of the axial-flow turbine. For countries surrounded by the sea, such as the British Isles and Japan to mention just two, or with extensive shorelines, wave energy conversion is an attractive proposition. Energy conversion systems based on the oscillating water column and the Wells turbine have been installed at several locations (Islay in Scotland and at Trivandrum in India). Figure 4.23 shows the arrangement of a turbine and generator together with the oscillating column of sea-water. The cross-sectional area of the plenum chamber is made very large compared to the flow area of the turbine so that a substantial air velocity through the turbine is attained.

One version of the Wells turbine consists of a rotor with about eight *uncambered* aerofoil section blades set at a stagger angle of ninety degrees (i.e. with their chord lines lying in the plane of rotation). A schematic diagram of such a Wells turbine is shown in Figure 4.24. At first sight the arrangement might seem to be a highly improbable means of energy conversion. However, once the blades have attained design speed the turbine is capable of producing a time-averaged positive power output from the cyclically reversing airflow with a fairly high efficiency. According to Raghunathan *et al.* (1995) *peak* efficiencies of 65% have been measured at the experimental wave power station on Islay. The results obtained from a theoretical analysis by Gato and Falcão (1984) showed that fairly high values of the mean efficiency, of the order 70–80%, may be attained in an oscillating flow “with properly designed Wells turbines”.

Principle of operation

Figure 4.25(a) shows a blade in motion at the design speed U in a flow with an upward, absolute axial velocity c_1 . It can be seen that the *relative velocity* w_1 is inclined to the chordline of the blade at an angle α . According to classical aerofoil

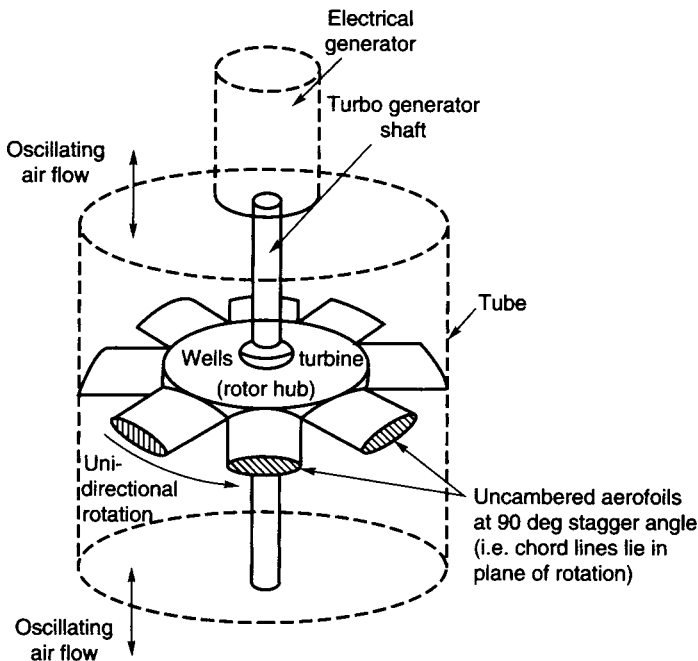


FIG. 4.24. Schematic of a Wells turbine (adapted from Raghunathan *et al.* 1995).

theory, an *isolated* aerofoil at an angle of incidence α to a free stream will generate a lift force L normal to the direction of the free stream. In a viscous fluid the aerofoil will also experience a drag force D in the direction of the free stream. These lift and drag forces can be resolved into the components of force X and Y as indicated in Figure 4.25a, i.e.

$$X = L \cos \alpha + D \sin \alpha, \quad (4.39)$$

$$Y = L \sin \alpha - D \cos \alpha. \quad (4.40)$$

The student should note, in particular, that the force Y acts in the direction of blade motion, giving positive work production.

For a symmetrical aerofoil, the direction of the tangential force Y is the same for both positive and negative values of α , as indicated in Figure 4.25b. If the aerofoils are secured to a rotor drum to form a turbine row, as in Figure 4.24, they will *always* rotate in the direction of the positive tangential force regardless of whether the air is approaching from above or below. With a time-varying, bi-directional air flow the torque produced will fluctuate cyclically but can be smoothed to a large extent by means of a high inertia rotor/generator.

It will be observed from the velocity diagrams that a residual swirl velocity is present for both directions of flow. It was suggested by Raghunathan *et al.* (1995) that the swirl losses at turbine exit can be reduced by the use of guide vanes.

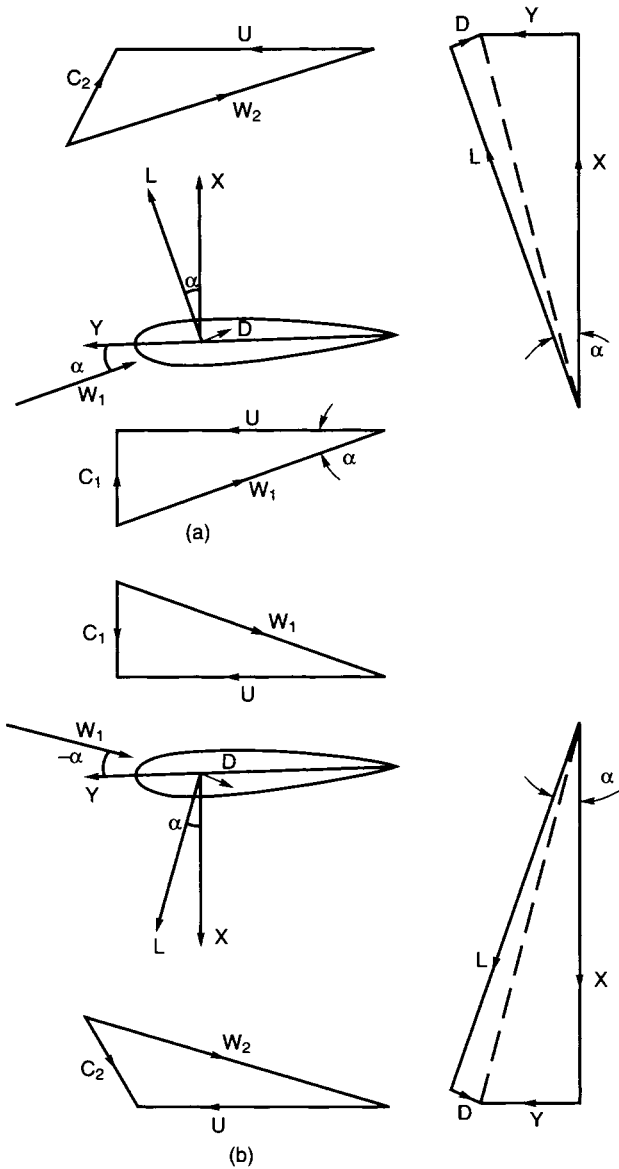


FIG. 4.25. Velocity and force vectors acting on a blade of a Wells turbine in motion: (a) upward absolute flow onto blade moving at speed U ; (b) downward absolute flow onto blade moving at speed U .

Two-dimensional flow analysis

The performance of the Wells turbine can be predicted by means of blade element theory. In this analysis the turbine annulus is considered to be made up of a series of concentric elementary rings, each ring being treated separately as a two-dimensional cascade.

The power output from an elementary ring of area $2\pi r dr$ is given by

$$dW = ZU dy,$$

where Z is the number of blades and the tangential force on each blade element is

$$dY = C_y(\frac{1}{2}\rho w_1^2 l)dr.$$

The axial force acting on the blade elements at radius r is $Z dX$, where

$$dX = C_x(\frac{1}{2}\rho w_1^2 l)dr,$$

and where C_x , C_y are the axial and tangential force coefficients. Now the axial force on all the blade elements at radius r can be equated to the pressure force acting on the elementary ring:

$$2\pi r(p_1 - p_2)dr = ZC_x(\frac{1}{2}\rho w_1^2 l)dr,$$

$$\therefore \frac{(p_1 - p_2)}{\frac{1}{2}\rho c_x^2} = \frac{ZC_x l}{2\pi r \sin^2 \alpha_1},$$

where $w_1 = c_x / \sin \alpha_1$.

An expression for the efficiency can now be derived from a consideration of all the power losses and the power output. The power lost due to the drag forces is $dW_f = w_1 dD$, where

$$dD = ZC_D(\frac{1}{2}\rho w_1^2 l)dr$$

and the power lost due to exit kinetic energy is given by

$$dW_k = (\frac{1}{2}c_2^2)d\dot{m},$$

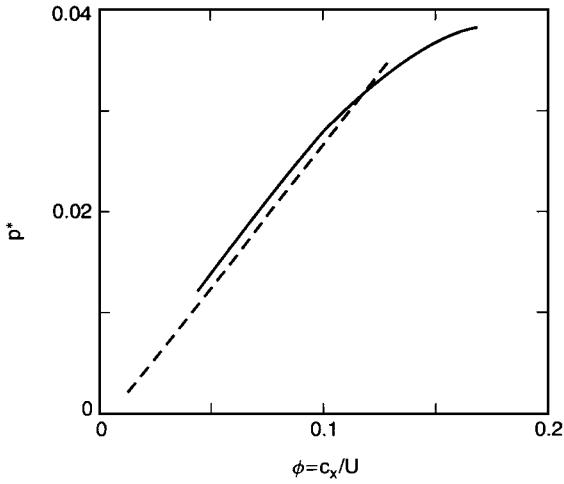
where $d\dot{m} = 2\pi r \rho c_x dr$ and c_2 is the absolute velocity at exit. Thus, the aerodynamic efficiency, defined as power output/power input, can now be written as

$$\eta = \frac{\int_h^t dW}{\int_h^t (dW + dW_f + dW_k)}.$$

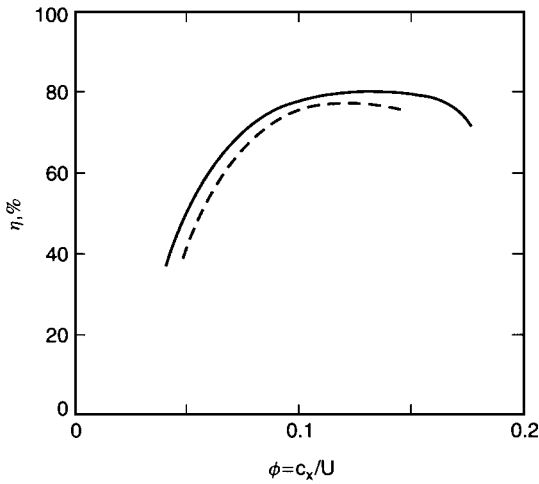
The predictions for non-dimensional pressure drop p^* and aerodynamic efficiency η determined by Raghunathan *et al.* (1995) are shown in Figure 4.26a and b, respectively, together with experimental results for comparison.

Design and performance variables

The primary input for the design of a Wells turbine is the air power based upon the pressure amplitude $(p_1 - p_2)$ and the volume flow rate Q at turbine inlet. The performance indicators are the pressure drop, power and efficiency and their variation with the flow rate. The aerodynamic design and consequent performance is a function of several variables which have been listed by Raghunathan. In non-dimensional form



(a)



(b)

FIG. 4.26. Comparison of theory with experiment for the Wells turbine: — theory
 - - - - experiment (adapted from Raghunathan 1995). (a) Non-dimensional pressure drop vs flow coefficient; (b) Efficiency vs flow coefficient.

these are:

flow coefficient

$$\phi = c_x/U$$

solidity at mean radius

$$\sigma = \frac{2lZ}{\pi D_t(1 + v)}$$

hub/tip ratio

$$v = D_h/D_t$$

blade aspect ratio

$$AR = \text{blade length/chord}$$

blade tip clearance ratio

$$= t_c/D_t$$

and also blade thickness ratio, turbulence level at inlet to turbine, frequency of waves and the relative Mach number. It was observed by Raghunathan *et al.* (1987) that the Wells turbine has a characteristic feature which makes it significantly different from most turbomachines: the absolute velocity of the flow is only a (small) fraction of the relative velocity. It is theoretically possible for transonic flow conditions to occur in the relative flow resulting in additional losses due to shock waves and an interaction with the boundary layers leading to flow separation. The effects of the variables listed above on the performance of the Wells turbine have been considered by Raghunathan (1995) and a summary of some of the main findings is given below.

Effect of flow coefficient

The flow coefficient ϕ is a measure of the angle of incidence of the flow and the aerodynamic forces developed are critically dependent upon this parameter. Typical results based on predictions and experiments of the non-dimensional pressure drop $p^* = \Delta p / (\rho \omega^2 D_t^2)$ and efficiency are shown in Figure 4.26. For a Wells turbine a linear relationship exists between pressure drop and the flowrate (Figure 4.26a) and this fact can be employed when making a match between a turbine and an oscillating water column which also has a similar characteristic.

The aerodynamic efficiency η (Figure 4.26b) is shown to increase up to a certain value, after which it decreases, because of boundary layer separation.

Effect of blade solidity

The solidity is a measure of the blockage offered by the blades to the flow of air and is an important design variable. The pressure drop across the turbine is, clearly, proportional to the axial force acting on the blades. An increase of solidity increases the axial force and likewise the pressure drop. Figure 4.27 shows how the variations of peak efficiency and pressure drop are related to the amount of the solidity.

Raghunathan gives correlations between pressure drop and efficiency with solidity:

$$p^*/p_0^* = 1 - \sigma^2 \text{ and } \eta/\eta_0 = \frac{1}{2}(1 - \sigma^2),$$

where the subscript 0 refers to values for a two-dimensional isolated aerofoil ($\sigma = 0$). A correlation between pressure drop and solidity (for $\sigma > 0$) was also expressed as

$$p^* = A\sigma^{1.6}$$

where A is a constant.

Effect of hub to tip ratio

The hub/tip ratio ν is an important parameter as it controls the volume flow rate through the turbine but also influences the stall conditions, the tip leakage and, most importantly, the ability of the turbine to run up to operating speed. Values of $\nu < 0.6$ are recommended for design.

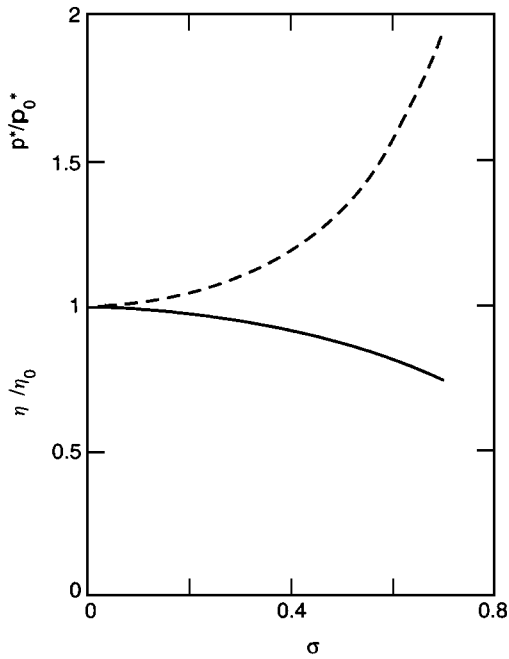


FIG. 4.27. Variation of peak efficiency and non-dimensional pressure drop (in comparison to the values for an isolated aerofoil) vs solidity: - - - pressure _____ efficiency (adapted from Raghunathan *et al.* 1995).

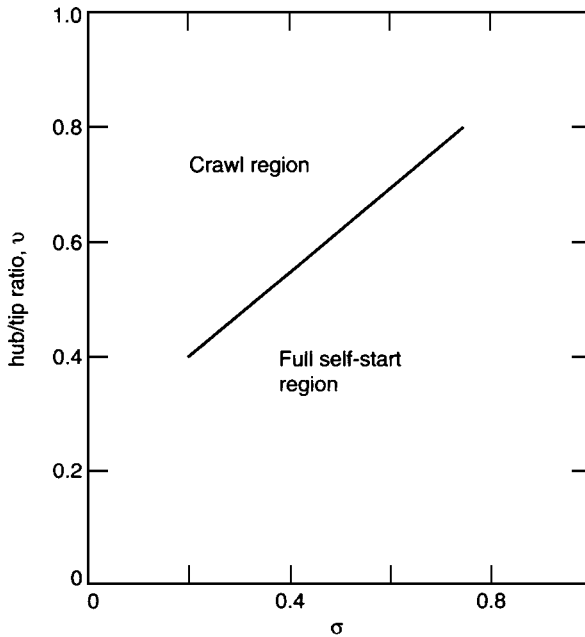


FIG. 4.28. Self-starting capability of the Wells turbine (adapted from Raghunathan *et al.* 1995).

The starting behaviour of the Wells turbine

When a Wells turbine is started from rest the incoming relative flow will be at 90 degrees to the rotor blades. According to the choice of the design parameters the blades could be severely stalled and, consequentially, the tangential force Y will be small and the acceleration negligible. In fact, if and when this situation occurs the turbine may only accelerate up to a speed much lower than the design operational speed, a phenomenon called *crawling*. The problem can be avoided either by choosing a suitable combination of hub/tip ratio and solidity values at the design stage or, by some other means such as incorporating a starter drive. Values of hub/tip ratio and solidity which have been found to allow self-starting of the Wells turbine are indicated in Figure 4.28.

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Problems

1. Show, for an axial flow turbine stage, that the *relative* stagnation enthalpy across the rotor row does not change. Draw an enthalpy–entropy diagram for the stage labelling all salient points.

Stage reaction for a turbine is defined as the ratio of the static enthalpy drop in the rotor to that in the stage. Derive expressions for the reaction in terms of the flow angles and draw velocity triangles for reactions of zero, 0.5 and 1.0.

2. (a) An axial flow turbine operating with an overall stagnation pressure of 8 to 1 has a polytropic efficiency of 0.85. Determine the total-to-total efficiency of the turbine.

(b) If the exhaust Mach number of the turbine is 0.3, determine the total-to-static efficiency.

(c) If, in addition, the exhaust velocity of the turbine is 160 m/s, determine the inlet total temperature.

Assume for the gas that $C_p = 1.175 \text{ kJ/(kg K)}$ and $R = 0.287 \text{ kJ/(kg K)}$.

3. The mean blade radii of the rotor of a mixed flow turbine are 0.3 m at inlet and 0.1 m at outlet. The rotor rotates at 20,000 rev/min and the turbine is required to produce 430 kW. The flow velocity at nozzle exit is 700 m/s and the flow direction is at 70° to the meridional plane.

Determine the absolute and relative flow angles and the absolute exit velocity if the gas flow is 1 kg/s and the velocity of the through-flow is constant through the rotor.

4. In a Parson's reaction turbine the rotor blades are similar to the stator blades but with the angles measured in the opposite direction. The efflux angle relative to each row of blades is 70 deg from the axial direction, the exit velocity of steam from the stator blades is 160 m/s, the blade speed is 152.5 m/s and the axial velocity is constant. Determine the specific work done by the steam per stage.

A turbine of 80% internal efficiency consists of ten such stages as described above and receives steam from the stop valve at 1.5 MPa and 300°C. Determine, with the aid of a Mollier chart, the condition of the steam at outlet from the last stage.

5. Values of pressure (kPa) measured at various stations of a zero-reaction gas turbine stage, all at the mean blade height, are shown in the table given below.

Stagnation pressure	Static pressure
Nozzle entry 414	Nozzle exit 207
Nozzle exit 400	Rotor exit 200

The mean blade speed is 291 m/s, inlet stagnation temperature 1100 K, and the flow angle at nozzle exit is 70 deg measured from the axial direction. Assuming the magnitude and direction of the velocities at entry and exit of the stage are the same, determine the total-to-total efficiency of the stage. Assume a perfect gas with $C_p = 1.148 \text{ kJ/(kg}^\circ\text{C)}$ and $\gamma = 1.333$.

6. In a certain axial flow turbine stage the axial velocity c_x is constant. The absolute velocities entering and leaving the stage are in the axial direction. If the flow coefficient c_x/U is 0.6 and the gas leaves the stator blades at 68.2 deg from the axial direction, calculate:

- the stage loading factor, $\Delta W/U^2$;
- the flow angles relative to the rotor blades;
- the degree of reaction;
- the total-to-total and total-to-static efficiencies.

The Soderberg loss correlation, eqn. (4.12) should be used.

7. An axial flow gas turbine stage develops 3.36 MW at a mass flow rate of 27.2 kg/s. At the stage entry the stagnation pressure and temperature are 772 kPa and 727°C, respectively. The static pressure at exit from the nozzle is 482 kPa and the corresponding absolute flow direction is 72° to the axial direction. Assuming the axial velocity is constant across the stage and the gas enters and leaves the stage without any absolute swirl velocity, determine:

- the nozzle exit velocity;
- the blade speed;
- the total-to-static efficiency;
- the stage reaction.

The Soderberg correlation for estimating blade row losses should be used. For the gas assume that $C_p = 1.148 \text{ kJ/(kg K)}$ and $R = 0.287 \text{ kJ/(kg K)}$.

8. Derive an approximate expression for the total-to-total efficiency of a turbine stage in terms of the enthalpy loss coefficients for the stator and rotor when the absolute velocities at inlet and outlet are **not** equal.

A steam turbine stage of high hub/tip ratio is to receive steam at a stagnation pressure and temperature of 1.5 MPa and 325°C respectively. It is designed for a blade speed of 200 m/s

and the following *blade* geometry was selected:

	Nozzles	Rotor
Inlet angle, deg	0	48
Outlet angle, deg	70.0	56.25
Space/chord ratio, s/l	0.42	—
Blade length/axial chord ratio, H/b	2.0	2.1
Max. thickness/axial chord	0.2	0.2

The deviation angle of the flow from the rotor row is known to be 3 deg on the evidence of cascade tests at the design condition. In the absence of cascade data for the nozzle row, the designer estimated the deviation angle from the approximation $0.19\theta s/l$ where θ is the blade camber in degrees. Assuming the incidence onto the nozzles is zero, the incidence onto the rotor 1.04 deg and the axial velocity across the stage is constant, determine:

- the axial velocity;
- the stage reaction and loading factor;
- the approximate total-to-total stage efficiency on the basic of Soderberg's loss correlation, assuming Reynolds number effects can be ignored;
- by means of a large steam chart (Mollier diagram) the stagnation temperature and pressure at stage exit.

9. (a) A single-stage axial flow turbine is to be designed for zero reaction without any absolute swirl at rotor exit. At nozzle inlet the stagnation pressure and temperature of the gas are 424 kPa and 1100 K. The static pressure at the mean radius between the nozzle row and rotor entry is 217 kPa and the nozzle exit flow angle is 70°.

Sketch an appropriate Mollier diagram (or a $T-s$ diagram) for *this* stage allowing for the effects of losses and sketch the corresponding velocity diagram. Hence, using Soderberg's correlation to calculate blade row losses, determine for the mean radius,

- the nozzle exit velocity,
- the blade speed,
- the total-to-static efficiency.

(b) Verify for this turbine stage that the total-to-total efficiency is given by

$$\frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \left(\frac{\phi}{2}\right)^2$$

where $\phi = c_x/U$. Hence, determine the value of the total-to-total efficiency.

Assume for the gas that $C_p = 1.15 \text{ kJ/(kg K)}$ and $\gamma = 1.333$.

10. (a) Prove that the centrifugal stress at the root of an untapered blade attached to the drum of an axial flow turbomachine is given by

$$\sigma_c = \pi \rho_m N^2 A_n / 1800,$$

where ρ_m = density of blade material, N = rotational speed of drum and A_n = area of the flow annulus.

(b) The preliminary design of an axial-flow gas turbine stage with stagnation conditions at stage entry of $p_{01} = 400 \text{ kPa}$, $T_{01} = 850 \text{ K}$, is to be based upon the following data *applicable to the mean radius*:

Flow angle at nozzle exit, $\alpha_2 = 63.8 \text{ deg}$

Reaction, $R = 0.5$

Flow coefficient, $c_x/U_m = 0.6$

Static pressure at stage exit, $p_3 = 200 \text{ kPa}$

Estimated total-to-static efficiency, $\eta_{ts} = 0.85$.

Assuming that the axial velocity is unchanged across the stage, determine:

- (1) the specific work done by the gas;
- (2) the blade speed;
- (3) the static temperature at stage exit.

(c) The blade material has a density of 7850 kg/m^3 and the maximum allowable stress in the rotor blade is 120 MPa . Taking into account only the centrifugal stress, assuming untapered blades and constant axial velocity at all radii, determine for a mean flow rate of 15 kg/s :

- (1) the rotor speed (rev/min);
- (2) the mean diameter;
- (3) the hub/tip radius ratio.

For the gas assume that $C_p = 1050 \text{ J/(kg K)}$ and $R = 287 \text{ J/(kg K)}$.

11. The design of a single-stage axial-flow turbine is to be based on constant axial velocity with axial discharge from the rotor blades directly to the atmosphere.

The following design values have been specified:

Mass flow rate	16.0 kg/s
Initial stagnation temperature, T_{01}	1100 K
Initial stagnation pressure, p_{01}	230 kN/m ²
Density of blading material, ρ_m	7850 kg/m ³
Maximum allowable centrifugal stress at blade root,	$1.7 \times 10^8 \text{ N/m}^2$
Nozzle profile loss coefficient, $Y_P = (p_{01} - p_{02})/(p_{02} - p_2)$	0.06
Taper factor for blade stressing, K	0.75

In addition the following may be assumed:

Atmospheric pressure, p_3	102 kPa
Ratio of specific heats, γ	1.333
Specific heat at constant pressure, C_p	1150 J/(kg K)

In the design calculations values of the parameters at the mean radius are as follows:

Stage loading coefficient, $\psi = \Delta W/U^2$	1.2
Flow coefficient, $\phi = c_x/U$	0.35
Isentropic velocity ratio, U/c_0	0.61
where $c_0 = \sqrt{2(h_{01} - h_{3SS})}$	

Determine:

- (1) the velocity triangles at the mean radius;
- (2) the required annulus area (based on the density at the mean radius);
- (3) the maximum allowable rotational speed;
- (4) the blade tip speed and the hub/tip radius ratio.