

CHAPTER 5

Axial-flow Compressors and Fans

A solemn, strange and mingled air, 't was sad by fits, by starts was wild.
(W. COLLINS, *The Passions*.)

Introduction

The idea of using a form of *reversed turbine* as an axial compressor is as old as the reaction turbine itself. It is recorded by Stoney (1937) that Sir Charles Parsons obtained a patent for such an arrangement as early as 1884. However, simply reversing a turbine for use as a compressor gives efficiencies which are, according to Howell (1945), less than 40% for machines of high pressure ratio. Parsons actually built a number of these machines (*circa* 1900), with blading based upon improved propeller sections. The machines were used for blast furnace work, operating with delivery pressures between 10 and 100 kPa. The efficiency attained by these early, low pressure compressors was about 55%; the reason for this low efficiency is now attributed to blade stall. A high pressure ratio compressor (550 kPa delivery pressure) was also built by Parsons but is reported by Stoney to have “run into difficulties”. The design, comprising two axial compressors in series, was abandoned after many trials, the flow having proved to be unstable (presumably due to *compressor surge*). As a result of low efficiency, axial compressors were generally abandoned in favour of multistage centrifugal compressors with their higher efficiency of 70–80%.

It was not until 1926 that any further development on axial compressors was undertaken when A. A. Griffith outlined the basic principles of his aerofoil theory of compressor and turbine design. The subsequent history of the axial compressor is closely linked with that of the aircraft gas turbine and has been recorded by Cox (1946) and Constant (1945). The work of the team under Griffith at the Royal Aircraft Establishment, Farnborough, led to the conclusion (confirmed later by rig tests) that efficiencies of at least 90% could be achieved for ‘small’ stages, i.e. low pressure ratio stages.

The early difficulties associated with the development of axial-flow compressors stemmed mainly from the fundamentally different nature of the flow process compared with that in axial-flow turbines. Whereas in the axial turbine the flow relative to each blade row is *accelerated*, in axial compressors it is *decelerated*. It is now widely known that although a fluid can be rapidly accelerated through a passage and sustain a small or moderate loss in total pressure the same is not true for a rapid deceleration. In the latter case large losses would arise as a result of severe

stall caused by a large adverse pressure gradient. So as to limit the total pressure losses during flow diffusion it is necessary for the rate of deceleration (and turning) in the blade passages to be severely restricted. (Details of these restrictions are outlined in Chapter 3 in connection with the correlations of Lieblein and Howell.) It is mainly because of these restrictions that axial compressors need to have many stages for a given pressure ratio compared with an axial turbine which needs only a few. Thus, the reversed turbine experiment tried by Parsons was doomed to a low operating efficiency.

The performance of axial compressors depends upon their usage category. Carchedi and Wood (1982) described the design and development of a single-shaft 15-stage axial-flow compressor which provided a 12 to 1 pressure ratio at a mass flow of 27.3 kg/s for a 6 MW industrial gas turbine. The design was based on subsonic flow and the compressor was fitted with variable stagger stator blades to control the position of the low-speed surge line. In the field of aircraft gas turbines, however, the engine designer is more concerned with *maximising* the work done per stage while retaining an acceptable level of overall performance. Increased stage loading almost inevitably leads to some aerodynamic constraint. This constraint will be increased Mach number, possibly giving rise to shock-induced boundary layer separation or increased losses arising from poor diffusion of the flow. Wennerstrom (1990) has outlined the history of highly loaded axial-flow compressors with special emphasis on the importance of reducing the number of stages and the ways that improved performance can be achieved. Since about 1970 a significant and special change occurred with respect to one design feature of the axial compressor and that was the introduction of low aspect ratio blading. It was not at all obvious why blading of large chord would produce any performance advantage, especially as the trend was to try to make engines more compact and lighter by using high aspect ratio blading. Wennerstrom (1989) has reviewed the increased usage of low aspect ratio blading in aircraft axial-flow compressors and reported on the high loading capability, high efficiency and good range obtained with this type of blading. One early application was an axial-flow compressor that achieved a pressure ratio of 12.1 in only five stages, with an isentropic efficiency of 81.9% and an 11% stall margin. The blade tip speed was 457 m/s and the flow rate per unit frontal area was 192.5 kg/s/m². It was reported that the mean aspect ratio ranged from a "high" of 1.2 in the first stage to less than 1.0 in the last three stages. A related later development pursued by the US Air Force was an alternative inlet stage with a rotor mean aspect ratio of 1.32 which produced, at design, a pressure ratio of 1.912 with an isentropic efficiency of 85.4% and an 11% stall margin. A maximum efficiency of 90.9% was obtained at a pressure ratio of 1.804 and lower rotational speed.

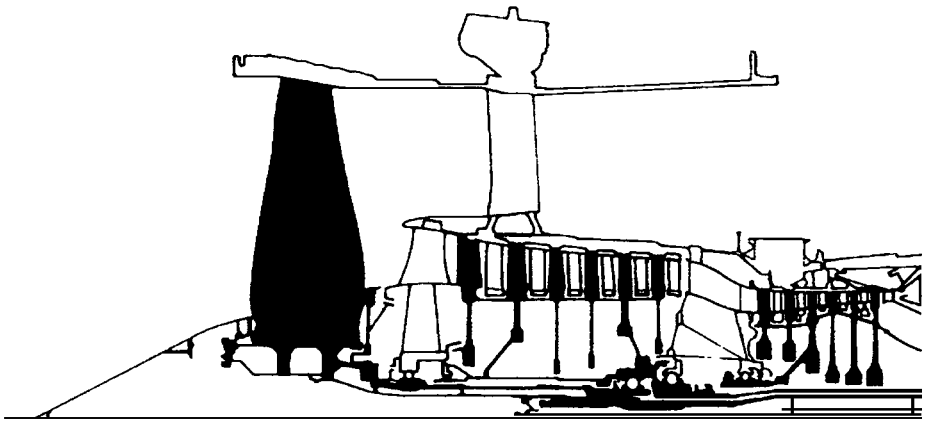
The flow within an axial-flow compressor is exceedingly complex which is one reason why research and development on compressors has proliferated over the years. In the following pages a very simplified and basic study is made of this compressor so that the student can grasp the essentials.

Two-dimensional analysis of the compressor stage

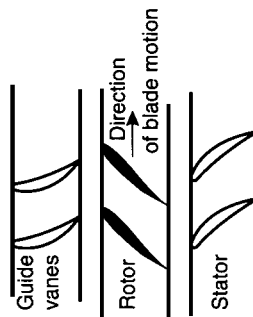
The analysis in this chapter is simplified (as it was for axial turbines) by assuming the flow is two-dimensional. This approach can be justified if the blade height is

small compared with the mean radius. Again, as for axial turbines, the flow is assumed to be invariant in the circumferential direction and that no spanwise (radial) velocities occur. Some of the three-dimensional effects of axial turbomachines are considered in Chapter 6.

To illustrate the layout of an axial compressor, Figure 5.1(a) shows a sectional drawing of the three-shaft compressor system of the Rolls-Royce RB211 gas-turbine engine. The very large blade on the left is part of the fan rotor which is on one shaft; this is followed by two, six-stage compressors of the “core” engine, each on its own shaft. A *compressor stage* is defined as a rotor blade row followed by a stator blade row. Figure 5.1b shows some of the blades of the first stage of the low-pressure compressor opened out into a plane array. The rotor blades (black) are fixed to the



(a)



(b)

FIG. 5.1. Axial-flow compressor and blading arrays. (a) Section of the compression system of the RB211-535E4 gas-turbine engine (courtesy of Rolls-Royce plc).

(b) Development of the first stage-blade rows and inlet guide vanes.

rotor drum and the stator blades are fixed to the outer casing. The blades upstream of the first rotor row are inlet guide vanes. These are not considered to be a part of the first stage and are treated separately. Their function is quite different from the other blade rows since, by directing the flow away from the axial direction, they act to *accelerate* the flow rather than diffuse it. Functionally, inlet guide vanes are the same as turbine nozzles; they increase the kinetic energy of the flow at the expense of the pressure energy.

Velocity diagrams of the compressor stage

The velocity diagrams for the stage are given in Figure 5.2 and the convention is adopted throughout this chapter of accepting all angles and swirl velocities in this figure as positive. As for axial turbine stages, a *normal* compressor stage is one where the absolute velocities and flow directions at stage outlet are the same as at stage inlet. The flow from a previous stage (or from the guide vanes) has a velocity c_1 and direction α_1 ; subtracting vectorially the blades speed U gives the inlet relative velocity w_1 at angle β_1 (the axial direction is the datum for all angles). Relative to the blades of the rotor, the flow is turned to the direction β_2 at outlet with a relative velocity w_2 . Clearly, by adding vectorially the blade speed U on to w_2 gives the absolute velocity from the rotor, c_2 at angle α_2 . The stator blades deflect the flow towards the axis and the exit velocity is c_3 at angle α_3 . For the normal stage $c_3 = c_1$ and $\alpha_3 = \alpha_1$. It will be noticed that as drawn in Figure 5.2, both the

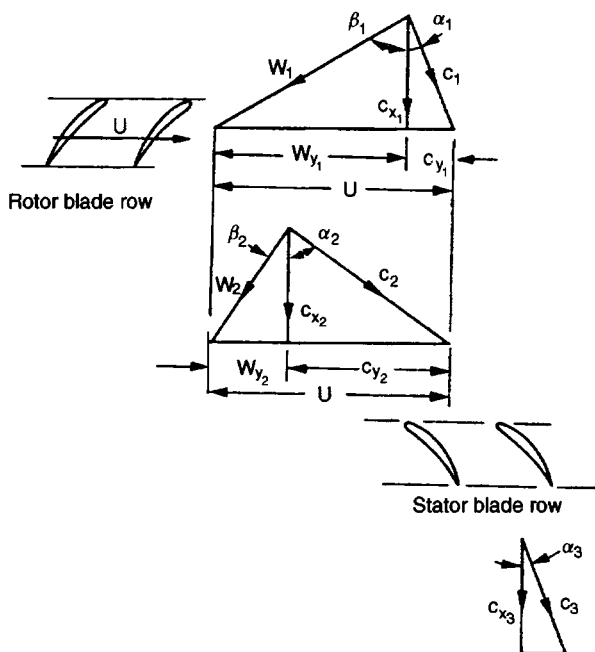


FIG. 5.2. Velocity diagrams for a compressor stage.

relative velocity in the rotor and the absolute velocity in the stator are diffused. It will be shown later in this chapter, that the relative amount of diffusion of kinetic energy in the rotor and stator rows, significantly influences the stage efficiency.

Thermodynamics of the compressor stage

The specific work done by the rotor on the fluid, from the steady flow energy equation (assuming adiabatic flow) and momentum equation is

$$\Delta W = \dot{W}_p / \dot{m} = h_{02} - h_{01} = U(c_{y2} - c_{y1}). \quad (5.1)$$

In Chapter 4 it was proved for all axial turbomachines that $h_{0rel} = h + \frac{1}{2}w^2$ is constant in the rotor. Thus,

$$h_1 + \frac{1}{2}w_1^2 = h_2 + \frac{1}{2}w_2^2. \quad (5.2)$$

This is a valid result as long as there is no radial shift of the streamlines across the rotor (i.e. $U_1 = U_2$).

Across the stator, h_0 is constant, and

$$h_2 + \frac{1}{2}c_2^2 = h_3 + \frac{1}{2}c_3^2. \quad (5.3)$$

The compression process for the complete stage is represented on a Mollier diagram in Figure 5.3, which is generalised to include the effects of irreversibility.

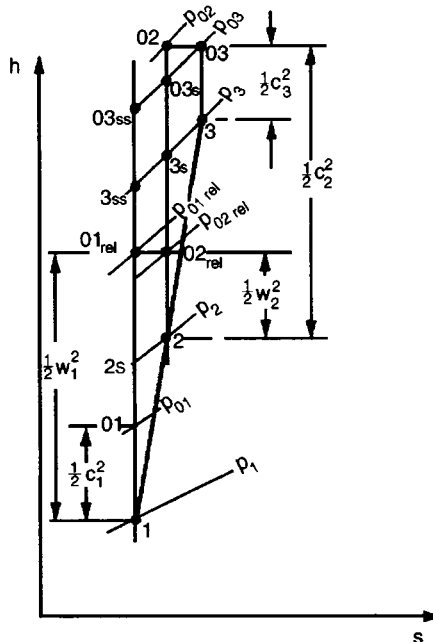


FIG. 5.3. Mollier diagram for an axial compressor stage.

Stage loss relationships and efficiency

From eqns. (5.1) and (5.3) the actual work performed by the rotor on unit mass of fluid is $\Delta W = h_{03} - h_{01}$. The reversible or *minimum* work required to attain the same final stagnation pressure as the real process is,

$$\begin{aligned}\Delta W_{\min} &= h_{03ss} - h_{01} \\ &= (h_{03} - h_{01}) - (h_{03s} - h_{03ss}) - (h_{03} - h_{03s}) \\ &\div \Delta W - (T_{03}/T_2)(h_2 - h_{2s}) - (T_{03}/T_3)(h_3 - h_{3s}),\end{aligned}$$

using the approximation that $\Delta h = T \Delta s$.

The temperature rise in a compressor stage is only a small fraction of the absolute temperature level and therefore, to a *close* approximation.

$$\Delta W_{\min} = \Delta W - (h_2 - h_{2s}) - (h_3 - h_{3s}). \quad (5.4)$$

Again, because of the small stage temperature rise, the density change is also small and it is reasonable to assume incompressibility for the fluid. This approximation is applied *only* to the stage and a *mean* stage density is implied; across a multistage compressor an appreciable density change can be expected.

The enthalpy losses in eqn. (5.4) can be expressed as stagnation pressure losses as follows. As $h_{02} = h_{03}$ then,

$$\begin{aligned}h_3 - h_2 &= \frac{1}{2}(c_2^2 - c_3^2) \\ &= [(p_{02} - p_2) - (p_{03} - p_3)]/\rho,\end{aligned} \quad (5.5)$$

since $p_0 - p = \frac{1}{2}\rho c^2$ for an incompressible fluid.

Along the isentrope 2 – 3_s in Figure 5.3, $Tds = 0 = dh - (1/\rho)dp$, and so,

$$h_{3s} - h_2 = (p_3 - p_2)/\rho. \quad (5.6)$$

Thus, subtracting eqn. (5.6) from eqn. (5.5)

$$h_3 - h_{3s} = (p_{02} - p_{03})/\rho = (1/\rho)\Delta p_{0\text{stator}}. \quad (5.7)$$

Similarly, for the rotor,

$$h_2 - h_{2s} = (p_{01\text{rel}} - p_{02\text{rel}})/\rho = (1/\rho)\Delta p_{0\text{rotor}}. \quad (5.8)$$

The total-to-total stage efficiency is,

$$\begin{aligned}\eta_{tt} &= \frac{\dot{W}_{p\min}}{\dot{W}_p} \div 1 - \frac{(h_2 - h_{2s}) + (h_3 - h_{3s})}{(h_{03} - h_{01})} \\ &\div 1 - \frac{\Delta p_{0\text{stator}} + \Delta p_{0\text{rotor}}}{\rho(h_{03} - h_{01})}\end{aligned} \quad (5.9)$$

It is to be observed that eqn. (5.9) also has direct application to pumps and fans.

Reaction ratio

For the case of *incompressible and reversible* flow it is permissible to define the reaction R , as the ratio of static pressure rise in the rotor to the static pressure rise in the stage

$$R = (p_2 - p_1)/(p_3 - p_1). \quad (5.10a)$$

If the flow is both *compressible and irreversible* a more general definition of R is the ratio of the rotor static enthalpy rise to the stage static enthalpy rise,

$$R = (h_2 - h_1)/(h_3 - h_1). \quad (5.10b)$$

From eqn. (5.2), $h_2 - h_1 = \frac{1}{2}(w_1^2 - w_2^2)$. For normal stages ($c_1 = c_3$), $h_3 - h_1 = h_{03} - h_{01} = U(c_{y2} - c_{y1})$. Substituting into eqn. (5.10b)

$$\begin{aligned} R &= \frac{w_1^2 - w_2^2}{2U(c_{y2} - c_{y1})} \\ &= \frac{(w_{y1} + w_{y2})(w_{y1} - w_{y2})}{2U(c_{y2} - c_{y1})}, \end{aligned} \quad (5.10c)$$

where it is assumed that c_x is constant across the stage. From Figure 5.2, $c_{y2} = U - w_{y2}$ and $c_{y1} = U - w_{y1}$ so that $c_{y2} - c_{y1} = w_{y1} - w_{y2}$. Thus,

$$R = (w_{y1} + w_{y2})/(2U) = (c_x/U) \tan \beta_m, \quad (5.11)$$

where

$$\tan \beta_m = \frac{1}{2}(\tan \beta_1 + \tan \beta_2). \quad (5.12)$$

An alternative useful expression for reaction can be found in terms of the fluid outlet angles from each blade row in a stage. With $w_{y1} = U - c_{y1}$, eqn. (5.11) gives,

$$R = \frac{1}{2} + (\tan \beta_2 - \tan \alpha_1)c_x/(2U). \quad (5.13)$$

Both expressions for reaction given above may be derived on a basis of incompressible, reversible flow, together with the definition of reaction in eqn. (5.10a).

Choice of reaction

The reaction ratio is a design parameter which has an important influence on stage efficiency. Stages having 50% reaction are widely used as the adverse (retarding) pressure gradient through the rotor and stator rows is equally shared. This choice of reaction minimises the tendency of the blade boundary layers to separate from the solid surfaces, thus avoiding large stagnation pressure losses.

If $R = 0.5$, then $\alpha_1 = \beta_2$ from eqn. (5.13), and the velocity diagram is symmetrical. The stage enthalpy rise is equally distributed between the rotor and stator rows.

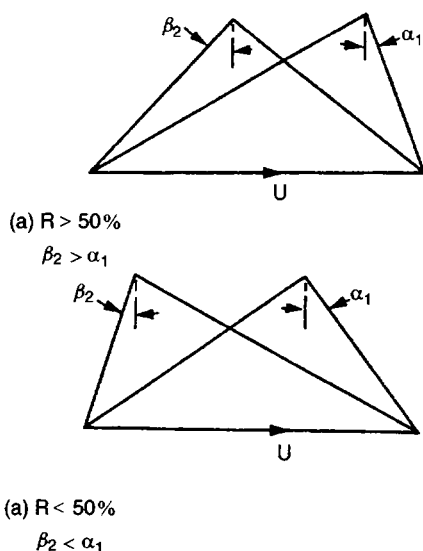


FIG. 5.4. Asymmetry of velocity diagrams for reactions greater or less than 50%.

If $R > 0.5$ then $\beta_2 > \alpha_1$ and the velocity diagram is skewed to the *right* as shown in Figure 5.4a. The static enthalpy rise in the rotor exceeds that in the stator (this is also true for the static pressure rise).

If $R < 0.5$ then $\beta_2 < \alpha_1$ and the velocity diagram is skewed to the *left* as indicated in Figure 5.4b. Clearly, the stator enthalpy (and pressure) rise exceeds that in the rotor.

In axial turbines the limitation on stage work output is imposed by rotor blade stresses but, in axial compressors, stage performance is limited by Mach number considerations. If Mach number effects could be ignored, the permissible temperature rise, based on incompressible flow cascade limits, increases with the amount of reaction. With a limit of 0.7 on the allowable Mach number, the temperature rise and efficiency are at a maximum with a reaction of 50%, according to Horlock (1958).

Stage loading

The stage loading factor ψ is another important design parameter of a compressor stage and is one which strongly affects the off-design performance characteristics. It is defined by

$$\psi = \frac{h_{03} - h_{01}}{U^2} = \frac{c_{y2} - c_{y1}}{U}. \quad (5.14a)$$

With $c_{y2} = U - w_{y2}$ this becomes,

$$\psi = 1 - \phi(\tan \alpha_1 + \tan \beta_2), \quad (5.14b)$$

where $\phi = c_x/U$ is called the *flow coefficient*.

The stage loading factor may also be expressed in terms of the lift and drag coefficients for the *rotor*. From Figure 3.5, replacing α_m with β_m , the tangential blade force on the *moving* blades per unit span is,

$$\begin{aligned} Y &= L \cos \beta_m + D \sin \beta_m \\ &= L \cos \beta_m \left(1 + \frac{C_D}{C_L} \tan \beta_m \right), \end{aligned}$$

where $\tan \beta_m = \frac{1}{2}(\tan \beta_1 + \tan \beta_2)$.

Now $C_L = L/(\frac{1}{2}\rho w_m^2 l)$ hence substituting for L above,

$$Y = \frac{1}{2}\rho c_x^2 l C_L \sec \beta_m (1 + \tan \beta_m C_D/C_L). \quad (5.15)$$

The work done by *each* moving blade per second is YU and is transferred to the fluid through *one* blade passage during that period. Thus, $YU = \rho s c_x (h_{03} - h_{01})$.

Therefore, the stage loading factor may now be written

$$\psi = \frac{h_{03} - h_{01}}{U^2} = \frac{Y}{\rho s c_x U}. \quad (5.16)$$

Substituting eqn. (5.15) in eqn. (5.16) the final result is

$$\psi = (\phi/2) \sec \beta_m (l/s)(C_L + C_D \tan \beta_m). \quad (5.17)$$

In Chapter 3, the approximate analysis indicated that maximum efficiency is obtained when the mean flow angle is 45 deg. The corresponding optimum stage loading factor at $\beta_m = 45$ deg is,

$$\psi_{\text{opt}} = (\phi/\sqrt{2})(l/s)(C_L + C_D). \quad (5.18)$$

Since $C_D \ll C_L$ in the normal low loss operating range, it is permissible to drop C_D from eqn. (5.18).

Simplified off-design performance

Horlock (1958) has considered how the stage loading behaves with varying flow coefficient, ϕ and how this off-design performance is influenced by the choice of design conditions. Now cascade data suggests that fluid *outlet angles* β_2 (for the rotor) and $\alpha_1 (= \alpha_3)$ for the stator, *do not change appreciably* for a range of incidence up to the stall point. The simplification may therefore be made that, for a given stage,

$$\tan \alpha_1 + \tan \beta_2 = t = \text{constant}. \quad (5.19)$$

Inserting this expression into eqn. (5.14b) gives

$$\psi = 1 - \phi t. \quad (5.20a)$$

An inspection of eqns. (5.20a) and (5.14a) indicates that the stagnation enthalpy rise of the stage increases as the mass flow is reduced, when running at constant

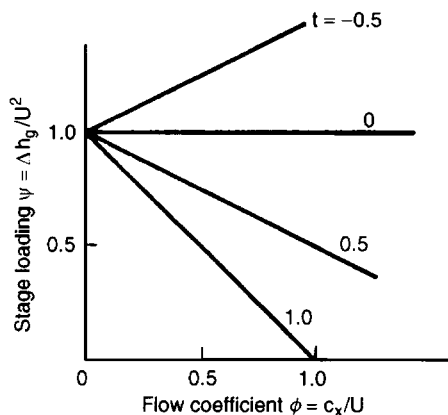


FIG. 5.5. Simplified off-design performance of a compressor stage (adapted from Horlock 1958).

rotational speed, provided t is positive. The effect is shown in Figure 5.5, where ψ is plotted against ϕ for several values of t .

Writing $\psi = \psi_d$ and $\phi = \phi_d$ for conditions at the design point, then

$$\psi_d = 1 - \phi_d t. \quad (5.20b)$$

The values of ψ_d and ϕ_d chosen for a particular stage design, determines the value of t . Thus t is fixed without regard to the degree of reaction and, therefore, the variation of stage loading at off-design conditions is not dependent on the choice of design reaction. However, from eqn. (5.13) it is apparent that, except for the case of 50% reaction when $\alpha_1 = \beta_2$, the reaction *does* change away from the design point. For design reactions exceeding 50% ($\beta_2 > \alpha_1$), the reaction decreases towards 50% as ϕ decreases; conversely, for design reactions less than 50% the reaction approaches 50% with diminishing flow coefficient.

If t is eliminated between eqns. (5.20a) and (5.20b) the following expression results,

$$\frac{\psi}{\psi_d} = \frac{1}{\psi_d} - \frac{\phi}{\phi_d} \left(\frac{1 - \psi_d}{\psi_d} \right). \quad (5.21)$$

This equation shows that, for a given design stage loading ψ_d , the fractional change in stage loading corresponding to a fractional change in flow coefficient is always the same, independent of the stage reaction. In Figure 5.6 it is seen that heavily loaded stages ($\psi_d \rightarrow 1$) are the most flexible producing little variation of ψ with change of ϕ . Lightly loaded stages ($\psi_d \rightarrow 0$) produce large changes in ψ with changing ϕ . Data from cascade tests show that ψ_d is limited to the range 0.3 to 0.4 for the most efficient operation and so substantial variations of ψ can be expected away from the design point.

In order to calculate the pressure rise at off-design conditions the variation of stage efficiency with flow coefficient is required. For an ideal stage (no losses) the pressure rise in incompressible flow is given by

$$\psi = \frac{\Delta h}{U^2} = \frac{\Delta p}{\rho U^2}. \quad (5.22)$$

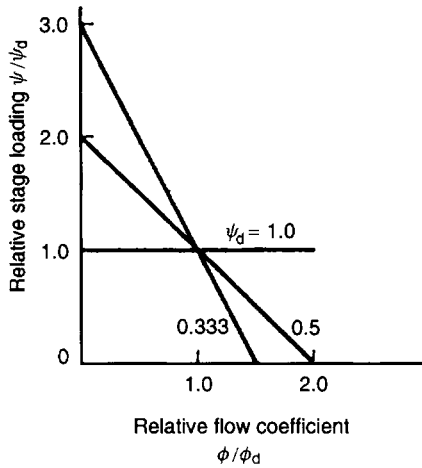


FIG. 5.6. Effect of design stage loading (ψ_d) on simplified off-design performance characteristics (adapted from Horlock 1958).

Stage pressure rise

Consider first the *ideal* compressor stage which has no stagnation pressure losses. Across the rotor row $p_{0\text{rel}}$ is constant and so

$$p_2 - p_1 = \frac{1}{2}\rho(w_1^2 - w_2^2). \quad (5.23a)$$

Across the stator row p_0 is constant and so

$$p_3 - p_2 = \frac{1}{2}\rho(c_2^2 - c_3^2). \quad (5.23b)$$

Adding together the pressure rise for each row and considering a normal stage ($c_3 = c_1$), gives

$$(p_3 - p_1)2/\rho = (c_2^2 - w_2^2) + (w_1^2 - c_1^2). \quad (5.24)$$

For either velocity triangle (Figure 5.2), the *cosine rule* gives $c^2 - U^2 + w^2 = 2Uw \cos(\pi/2 - \beta)$ or

$$c^2 - w^2 = U^2 - 2Uw_y. \quad (5.25)$$

Substituting eqn. (5.25) into the stage pressure rise,

$$\begin{aligned} 2(p_3 - p_1)/\rho &= (U^2 - 2Uw_{y2}) - (U^2 - 2Uw_{y1}) \\ &= 2U(w_{y1} - w_{y2}). \end{aligned}$$

Again, referring to the velocity diagram, $w_{y1} - w_{y2} = c_{y2} - c_{y1}$ and

$$(p_3 - p_1)/\rho = U(c_{y2} - c_{y1}) = h_3 - h_1. \quad (5.26)$$

It is noted that, for an isentropic process, $Tds = 0 = dh - (1/\rho)dp$ and therefore, $\Delta h = (1/\rho)\Delta p$.

The pressure rise in a real stage (involving irreversible processes) can be determined if the stage efficiency is known. Defining the stage efficiency η_s as the ratio of the isentropic enthalpy rise to the actual enthalpy rise corresponding to the same *finite* pressure change, (cf. Figure 2.7), this can be written as

$$\eta_s = (\Delta h_{is})/(\Delta h) = (1/\rho)\Delta p/\Delta h.$$

Thus,

$$(1/\rho)\Delta p = \eta_s \Delta h = \eta_s U \Delta c_y. \quad (5.27)$$

If $c_1 = c_3$, then η_s is a very close approximation of the total-to-total efficiency η_{tt} . Although the above expressions are derived for incompressible flow they are, nevertheless, a valid approximation for compressible flow if the stage temperature (and pressure) rise is small.

Pressure ratio of a multistage compressor

It is possible to apply the preceding analysis to the determination of multistage compressor pressure ratios. The procedure requires the calculation of pressure and temperature changes for a single stage, the stage exit conditions enabling the density at entry to the following stage to be found. This calculation is repeated for each stage in turn until the required final conditions are satisfied. However, for compressors having identical stages it is more convenient to resort to a simple compressible flow analysis. An illustrative example is given below.

EXAMPLE 5.1. A multistage axial compressor is required for compressing air at 293 K, through a pressure ratio of 5 to 1. Each stage is to be 50% reaction and the mean blade speed 275 m/s, flow coefficient 0.5, and stage loading factor 0.3, are taken, for simplicity, as constant for all stages. Determine the flow angles and, the number of stages required if the stage efficiency is 88.8%. Take $C_p = 1.005 \text{ kJ/(kg}^\circ\text{C)}$ and $\gamma = 1.4$ for air.

Solution. From eqn. (5.14a) the stage load factor can be written as,

$$\psi = \phi(\tan \beta_1 - \tan \beta_2).$$

From eqn. (5.11) the reaction is

$$R = \frac{\phi}{2}(\tan \beta_1 + \tan \beta_2).$$

Solving for $\tan \beta_1$ and $\tan \beta_2$ gives

$$\tan \beta_1 = (R + \psi/2)/\phi \text{ and } \tan \beta_2 = (R - \psi/2)/\phi.$$

Calculating β_1 and β_2 and observing for $R = 0.5$ that the velocity diagram is symmetrical,

$$\beta_1 = \alpha_2 = 52.45^\circ \text{ and } \beta_2 = \alpha_1 = 35^\circ.$$

Writing the stage load factor as $\psi = C_p \Delta T_0 / U^2$, then the stage stagnation temperature rise is,

$$\Delta T_0 = \psi U^2 / C_p = 0.3 \times 275^2 / 1005 = 22.5^\circ \text{C}.$$

It is reasonable to take the stage efficiency as equal to the polytropic efficiency since the stage temperature rise of an axial compressor is small. Denoting compressor inlet and outlet conditions by subscripts I and II respectively then, from eqn. (2.33),

$$\frac{T_{0II}}{T_{0I}} = 1 + \frac{N \Delta T_0}{T_{0I}} = \left(\frac{p_{0II}}{p_{0I}} \right)^{(\gamma-1)/\eta_p \gamma},$$

where N is the required number of stages. Thus

$$N = \frac{T_{0I}}{\Delta T_0} \left[\left(\frac{p_{0II}}{p_{0I}} \right)^{(\gamma-1)/\eta_p \gamma} - 1 \right] = \frac{293}{22.5} [5^{1/3.11} - 1] = 8.86.$$

A suitable number of stages is therefore 9.

The overall efficiency is found from eqn. (2.36).

$$\begin{aligned} \eta_{II} &= \left[\left(\frac{p_{0II}}{p_{0I}} \right)^{(\gamma-1)/\gamma} - 1 \right] / \left[\left(\frac{p_{0II}}{p_{0I}} \right)^{(\gamma-1)/\eta_p \gamma} - 1 \right] \\ &= [5^{1/3.5} - 1] / [5^{1/3.11} - 1] = 86.3\%. \end{aligned}$$

Estimation of compressor stage efficiency

In eqn. (5.9) the amount of the actual stage work ($h_{03} - h_{01}$) can be found from the velocity diagram. The losses in total pressure may be estimated from cascade data. This data is incomplete however, as it only takes account of the blade profile loss. Howell (1945) has subdivided the total losses into three categories as shown in Figure 3.11.

- (i) Profile losses on the blade surfaces.
- (ii) Skin friction losses on the annulus walls.
- (iii) "Secondary" losses by which he meant all losses not included in (i) and (ii) above.

In performance estimates of axial compressor and fan stages the *overall* drag coefficient for the blades of each row is obtained from

$$\begin{aligned} C_D &= C_{Dp} + C_{Da} + C_{Ds} \\ &= C_{Dp} + 0.02 s/H + 0.018 C_L^2 \end{aligned} \quad (5.28)$$

using the empirical values given in Chapter 3.

Although the subject matter of this chapter is primarily concerned with two-dimensional flows, there is an interesting three-dimensional aspect which cannot be ignored. In multistage axial compressors the annulus wall boundary layers

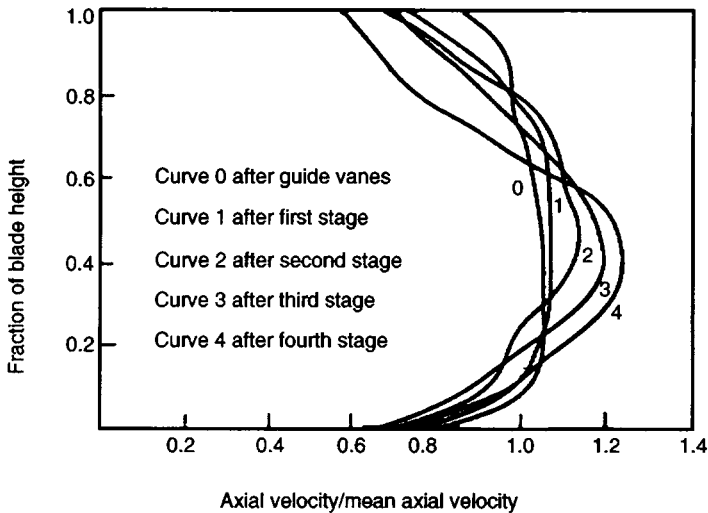


FIG. 5.7. Axial velocity profiles in a compressor (Howell 1945). (Courtesy of the Institution of Mechanical Engineers).

rapidly thicken through the first few stages and the axial velocity profile becomes increasingly peaked. This effect is illustrated in Figure 5.7, from the experimental results of Howell (1945), which shows axial velocity traverses through a four-stage compressor. Over the central region of the blade, the axial velocity is higher than the mean value based on the through flow. The mean blade section (and most of the span) will, therefore, do less work than is estimated from the velocity triangles based on the mean axial velocity. In theory it would be expected that the tip and root sections would provide a compensatory effect because of the low axial velocity in these regions. Due to stalling of these sections (and tip leakage) no such work increase actually occurs, and the net result is that the work done by the whole blade is below the design figure. Howell (1945) suggested that the stagnation enthalpy rise across a stage could be expressed as

$$h_{03} - h_{01} = \lambda U(c_{y2} - c_{y1}), \quad (5.29)$$

where λ is a “work done”. For multistage compressors Howell recommended for λ a mean value of 0.86. Using a similar argument for axial turbines, the increase in axial velocity at the pitch-line gives an *increase* in the amount of work done, which is then roughly cancelled out by the loss in work done at the blade ends. Thus, for turbines, no “work done” factor is required as a correction in performance calculations.

Other workers have suggested that λ should be high at entry (0.96) where the annulus wall boundary layers are thin, reducing progressively in the later stages of the compressor (0.85). Howell (1950) has given mean “work done” factors for compressors with varying numbers of stages, as in Figure 5.9. For a four-stage compressor the value of λ would be 0.9 which would be applied to all four stages.

Smith (1970) commented upon the rather pronounced deterioration of compressor performance implied by the example given in Figure 5.7 and suggested that things are not so bad as suggested. As an example of modern practice he gave the axial

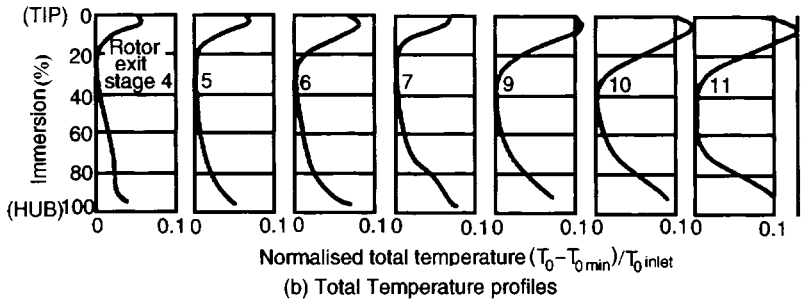
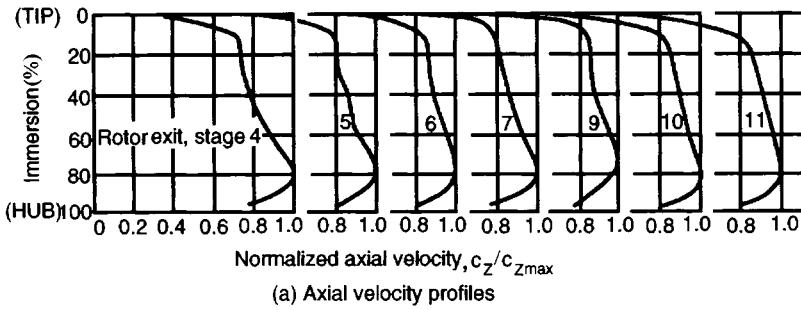


FIG. 5.8. Traverse measurements obtained from a 12-stage compressor (Smith 1970).
(Courtesy of the Elsevier Publishing Co).

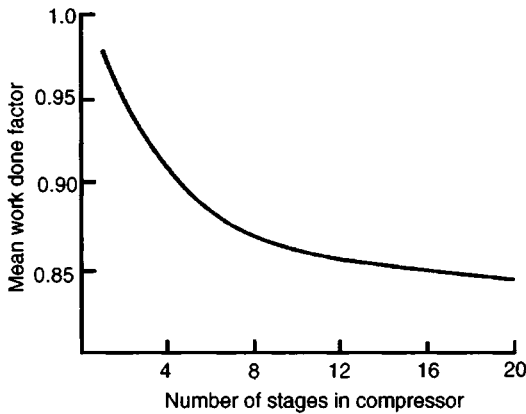


FIG. 5.9. Mean work done factor in compressors (Howell and Bonham 1950). (Courtesy of the Institution of Mechanical Engineers).

velocity distributions through a twelve-stage axial compressor, Figure 5.8(a). This does illustrate that rapid changes in velocity distribution still occur in the first few stages, but that the profile settles down to a fairly constant shape thereafter. This phenomenon has been referred to as *ultimate steady flow*.

Smith also provided curves of the spanwise variation in total temperature, Figure 5.8(b), which shows the way losses increase from midpassage towards

the annulus walls. An inspection of this figure shows also that the excess total temperature near the end walls increases in magnitude and extent as the flow passes through the compressor. Work on methods of predicting annulus wall boundary layers in turbomachines and their effects on performance are being actively pursued in many countries. Although rather beyond the scope of this textbook, it may be worth mentioning two papers for students wishing to advance their studies further. Mellor and Balsa (1972) offer a mathematical flow model based on the pitchwise-averaged turbulent equations of motion for predicting axial flow compressor performance whilst Daneshyar *et al.* (1972) review and compare different existing methods for predicting the growth of annulus wall boundary layers in turbomachines.

EXAMPLE 5.2. The last stage of an axial flow compressor has a reaction of 50% at the design operating point. The cascade characteristics, which correspond to each row at the mean radius of the stage, are shown in Figure 3.12. These apply to a cascade of circular arc camber line blades at a space–chord ratio 0.9, a blade inlet angle of 44.5 deg and a blade outlet angle of -0.5 deg. The blade height–chord ratio is 2.0 and the work done factor can be taken as 0.86 when the mean radius relative incidence $(i - i^*)/\epsilon^*$ is 0.4 (the operating point).

For this operating condition, determine

- (i) the nominal incidence i^* and nominal deflection ϵ^* ;
- (ii) the inlet and outlet flow angles for the rotor;
- (iii) the flow coefficient and stage loading factor;
- (iv) the rotor lift coefficient;
- (v) the overall drag coefficient of each row;
- (vi) the stage efficiency.

The density at entrance to the stage is 3.5 kg/m^3 and the mean radius blade speed is 242 m/s. Assuming the density across the stage is constant and ignoring compressibility effects, estimate the stage pressure rise.

In the solution given below the *relative flow* onto the rotor is considered. The notation used for flow angles is the same as for Figure 5.2. For blade angles, β' is therefore used instead of α' for the sake of consistency.

Solution. (i) The nominal deviation is found using eqns. (3.39) and (3.40). With the camber $\theta = \beta'_1 - \beta'_2 = 44.5^\circ - (-0.5^\circ) = 45^\circ$ and the space/chord ratio, $s/l = 0.9$, then

$$\delta^* = [0.23 + \beta'_2/500]\theta(s/l)^{1/2}$$

$$\text{But } \beta'_2 = \delta^* + \beta'_2 = \delta^* - 0.5$$

$$\begin{aligned} \therefore \delta^* &= [0.23 + (\delta^* + \beta'_2)/500] \times 45 \times (0.9)^{1/2} \\ &= [0.229 + \delta^*/500] \times 42.69 = 9.776 + 0.0854 \delta^* \end{aligned}$$

$$\therefore \delta^* = 10.69^\circ$$

$$\begin{aligned} \therefore \beta'_2 &= \delta^* + \beta'_2 = 10.69 - 0.5 \\ &\simeq 10.2^\circ \end{aligned}$$

The nominal deflection $\varepsilon^* = 0.8\varepsilon_{\max}$ and, from Figure 3.12, $\varepsilon_{\max} = 37.5^\circ$. Thus, $\varepsilon^* = 30^\circ$ and the nominal incidence is

$$\begin{aligned} i^* &= \beta_2^* + \varepsilon^* - \beta_1' \\ &= 10.2 + 30 - 44.5 = -4.3^\circ. \end{aligned}$$

(ii) At the operating point $i = 0.4\varepsilon^* + i^* = 7.7^\circ$. Thus, the actual inlet flow angle is

$$\beta_1 = \beta_1' + i = 52.2^\circ.$$

From Figure 3.12 at $i = 7.7^\circ$, the deflection $\varepsilon = 37.5^\circ$ and the flow outlet angle is

$$\beta_2 = \beta_1 - \varepsilon = 14.7^\circ.$$

(iii) From Figure 5.2, $U = c_{x1}(\tan \alpha_1 + \tan \beta_1) = c_{x2}(\tan \alpha_2 + \tan \beta_2)$. For c_x = constant across the stage and $R = 0.5$

$$\beta_1 = \alpha_2 = 52.2^\circ \text{ and } \beta_2 = \alpha_1 = 14.7^\circ$$

and the flow coefficient is

$$\phi = \frac{c_x}{U} = \frac{1}{\tan \alpha_1 + \tan \beta_1} = 0.644.$$

The stage loading factor, $\psi = \Delta h_0/U^2 = \lambda\phi(\tan \alpha_2 - \tan \alpha_1)$ using eqn. (5.29). Thus, with $\lambda = 0.86$,

$$\psi = 0.568.$$

(iv) The lift coefficient can be obtained using eqn. (3.18)

$$C_L = 2(s/l) \cos \beta_m (\tan \beta_1 - \tan \beta_2)$$

ignoring the small effect of the drag coefficient. Now $\tan \beta_m = (\tan \beta_1 + \tan \beta_2)/2$. Hence $\beta_m = 37.8^\circ$ and so

$$C_L = 2 \times 0.9 \times 0.7902 \times 1.027 = 1.46.$$

(v) Combining eqns. (3.7) and (3.17) the drag coefficient is

$$C_D = \frac{s}{l} \left(\frac{\Delta p_0}{\frac{1}{2}\rho w_1^2} \right) \frac{\cos^3 \beta_m}{\cos^2 \beta_1}.$$

Again using Figure 3.12 at $i = 7.7^\circ$, the profile total pressure loss coefficient $\Delta p_0/(\frac{1}{2}\rho w_1^2) = 0.032$, hence the profile drag coefficient for the blades of either row is

$$C_{D_p} = 0.9 \times 0.032 \times 0.7902^3/0.6129^2 = 0.038.$$

Taking account of the annulus wall drag coefficient C_{Da} and the secondary loss drag coefficient C_{Ds}

$$C_{Da} = 0.02(s/l)(l/H) = 0.02 \times 0.9 \times 0.5 = 0.009$$

$$C_{Ds} = 0.018C_L^2 = 0.018 \times 1.46^2 = 0.038.$$

Thus, the *overall* drag coefficient, $C_D = C_{D_p} + C_{D_a} + C_{D_s} = 0.084$ and this applies to each row of blades. If the reaction had been other than 0.5 the drag coefficients for each blade row would have been computed separately.

(vi) The total-to-total stage efficiency, using eqn. (5.9) can be written as

$$\eta_{tt} = 1 - \frac{\Sigma \Delta p_0 / \rho}{\psi U^2} = 1 - \frac{\Sigma \Delta p_0 / (\frac{1}{2} \rho c_x^2)}{2\psi / \phi^2} = 1 - \frac{(\zeta_R + \zeta_S) \phi^2}{2\psi}$$

where ζ_R and ζ_S are the overall total pressure loss coefficients for the rotor and stator rows respectively. From eqn. (3.17)

$$\zeta_s = (l/s) C_D \sec^3 \alpha_m.$$

Thus, with $\zeta_S = \zeta_R$

$$\begin{aligned} \eta_{tt} &= 1 - \frac{\phi^2 C_D (l/s)}{\psi \cos^3 \alpha_m} \\ &= 1 - \frac{0.644^2 \times 0.084}{0.568 \times 0.7903^3 \times 0.9} = 0.862. \end{aligned}$$

From eqn. (5.27), the pressure rise across the stage is

$$\begin{aligned} \Delta p &= \eta_{tt} \psi \rho U^2 = 0.862 \times 0.568 \times 3.5 \times 242^2 \\ &= 100 \text{ kPa}. \end{aligned}$$

Stall and surge phenomena in compressors

Casing treatment

It was discovered in the late 1960s that the stall of a compressor could be delayed to a lower mass flow by a suitable treatment of the compressor casing. Given the right conditions this can be of great benefit in extending the range of stall-free operation. Numerous investigations have since been carried out on different types of casing configurations under widely varying flow conditions to demonstrate the range of usefulness of the treatment.

Greitzer *et al.* (1979) observed that two types of stall could be found in a compressor blade row, namely, “blade stall” or “wall stall”. Blade stall is, roughly speaking, a two-dimensional type of stall where a significant part of the blade has a large wake leaving the blade suction surface. Wall stall is a stall connected with the boundary layer on the outer casing. Figure 5.10 illustrates the two types of stall. Greitzer *et al.* found that the response to casing treatment depended conspicuously upon the type of stall encountered.

The influence of a grooved casing treatment on the stall margin of a model axial compressor rotor was investigated experimentally. Two rotor builds of different blade solidities, σ , (chord/space ratio) but with all the other parameters identical, were tested. Greitzer emphasised that the motive behind the use of different solidities was simply a convenient way to change the type of stall from a blade stall to a wall stall and that the benefit of casing treatment was unrelated to the amount of solidity

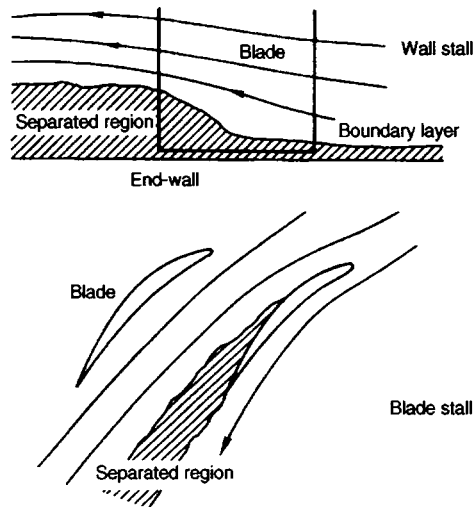


FIG. 5.10. Compressor stall inception (adapted from Greitzer *et al.* 1979).

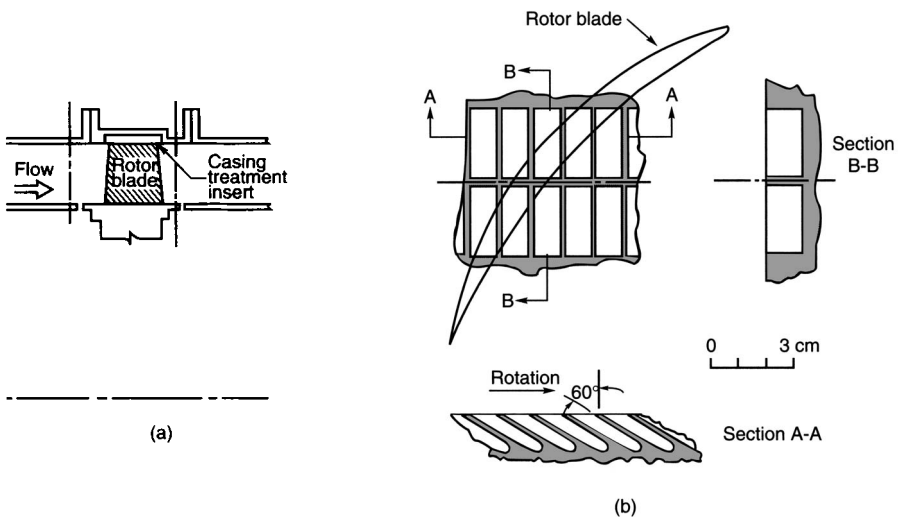


FIG. 5.11. Position and appearance of casing treatment insert (adapted from Greitzer *et al.* 1979).

of the blade row. The position of the casing treatment insert in relation to the rotor blade row is shown in Figure 5.11a and the appearance of the grooved surface used is illustrated in Figure 5.11b. The grooves, described as “axial skewed” and extending over the middle 44% of the blade, have been used in a wide variety of compressors.

As predicted from their design study, the high solidity blading ($\sigma = 2$) resulted in the production of a wall stall, while the low solidity ($\sigma = 1$) blading gave a blade stall. Figure 5.12 shows the results obtained for non-dimensionalised wall static

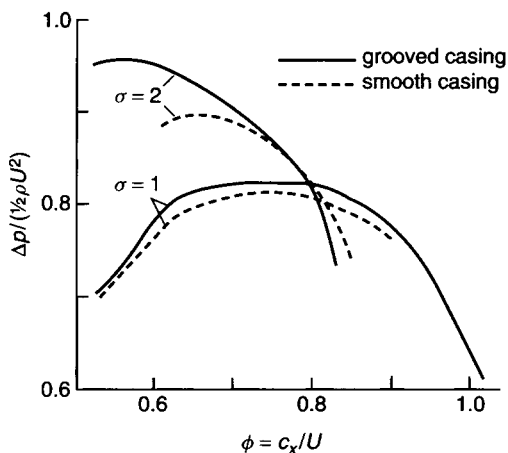


FIG. 5.12. Effects of casing treatment and solidity on compressor characteristics (adapted from Greitzer *et al.* 1979 and data points removed for clarity).

pressure rise, $\Delta p/(\frac{1}{2}\rho U^2)$, across the rotor plotted against the mean radius flow coefficient, $\phi = c_x/U$, for the four conditions plotted. The extreme left end of each curve represents the *onset* of stall. It can be seen that there is a marked difference in the curves for the two solidities. For the high solidity configuration there is a higher static peak pressure rise and the decline does not occur until ϕ is much lower than the low solidity configuration. However, the most important difference in performance is the change in the stall point with and without the casing treatment. It can be seen that with the grooved casing a substantial change in the range of ϕ occurred with the high solidity blading. However, for the low solidity blading there is only a marginal difference in range. The shape of the performance curve is significantly affected for the high solidity rotor blades, with a substantial increase in the peak pressure rise brought about by the grooved casing treatment.

The conclusion reached by Greitzer *et al.* (1979) is that casing treatment is highly effective in delaying the onset of stall when the compressor row is more prone to *wall stall* than *blade stall*. However, despite this advantage casing treatment has not been generally adopted in industry. The major reason for this ostensible rejection of the method appears to be that a performance penalty is attached to it. The more effective the casing treatment, the more the stage efficiency is reduced.

Smith and Cumsty (1984) made an extensive series of experimental investigations to try to discover the reasons for the effectiveness of casing treatment and the underlying causes for the loss in compressor efficiency. At the simplest level it was realised that the slots provide a route for fluid to pass from the pressure surface to the suction surface allowing a small proportion of the flow to be recirculated. The approaching boundary layer fluid tends to have a high absolute swirl and is, therefore, suitably orientated to enter the slots. Normally, with a smooth wall the high swirl would cause energy to be wasted but, with the casing treatment, the flow entering the slot is turned and reintroduced back into the main flow near the blade's leading edge with its absolute swirl direction reversed. The re-entrant flow has, in effect, flowed upstream along the slot to a lower pressure region.

Rotating stall and surge

A salient feature of a compressor performance map, such as Figure 1.10, is the limit to stable operation known as the *surge line*. This limit can be reached by reducing the mass flow (with a throttle valve) whilst the rotational speed is maintained constant.

When a compressor goes into surge the effects are usually quite dramatic. Generally, an increase in noise level is experienced, indicative of a pulsation of the air flow and of mechanical vibration. Commonly, there are a small number of predominant frequencies superimposed on a high background noise. The lowest frequencies are usually associated with a *Helmholtz-type of resonance* of the flow through the machine, with the inlet and/or outlet volumes. The higher frequencies are known to be due to *rotating stall* and are of the same order as the rotational speed of the impeller.

Rotating stall is a phenomenon of axial-compressor flow which has been the subject of many detailed experimental and theoretical investigations and the matter is still not fully resolved. An early survey of the subject was given by Emmons *et al.* (1959). Briefly, when a blade row (usually the rotor of a compressor) reaches the “stall point”, the blades instead of all stalling together as might be expected, stall in separate patches and these stall patches, moreover, travel around the compressor annulus (i.e. they rotate).

That stall patches *must* propagate from blade to blade has a simple physical explanation. Consider a portion of a blade row, as illustrated in Figure 5.13 to be affected by a stall patch. This patch must cause a partial obstruction to the flow which is deflected on both sides of it. Thus, the incidence of the flow on to the blades on the right of the stall cell is reduced but, the incidence to the left is increased. As these blades are already close to stalling, the net effect is for the stall patch to move to the left; the motion is then self-sustaining.

There is a strong practical reason for the wide interest in rotating stall. Stall patches travelling around blade rows load and unload each blade at some frequency related to the speed and number of the patches. This frequency may be close to a natural frequency of blade vibration and there is clearly a need for accurate prediction of the conditions producing such a vibration. Several cases of blade failure due to resonance induced by rotating stall have been reported, usually with serious consequences to the whole compressor.

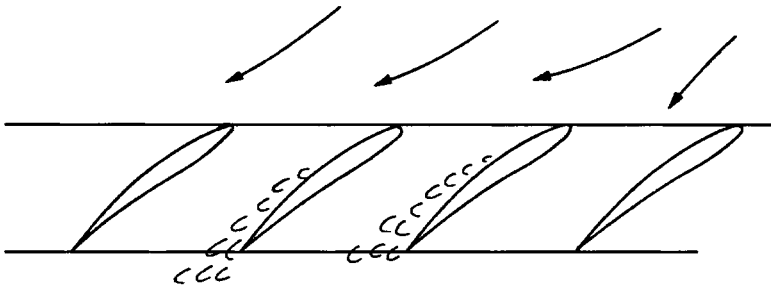


FIG. 5.13. Model illustrating mechanism of stall cell propagation: partial blockage due to stall patch deflects flow, increasing incidence to the left and decreasing incidence to the right.

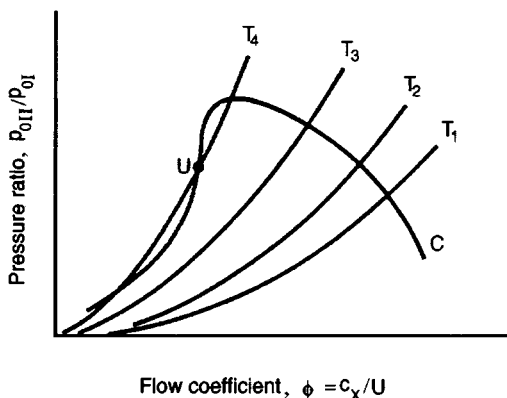


FIG. 5.14. Stability of operation of a compressor (adapted from Horlock 1958).

It is possible to distinguish between surge and propagating stall by the unsteadiness, or otherwise, of the total mass flow. The characteristic of stall propagation is that the flow passing through the annulus, summed over the whole area, is steady with time; the stall cells merely redistribute the flow over the annulus. Surge, on the other hand, involves an axial oscillation of the total mass flow, a condition highly detrimental to efficient compressor operation.

The conditions determining the point of surge of a compressor have not yet been completely determined satisfactorily. One physical explanation of this breakdown of the flow is given by Horlock (1958).

Figure 5.14 shows a constant rotor speed compressor characteristic (C) of pressure ratio plotted against flow coefficient. A second set of curves (T_1 , T_2 , etc.) are superimposed on this figure showing the pressure loss characteristics of the throttle for various fixed throttle positions. The intersection of curves T with compressor curve C denotes the various operating points of the combination. A state of *flow stability* exists if the throttle curves at the point of intersection have a greater (positive) slope than the compressor curve. That this is so may be illustrated as follows. Consider the operating point at the intersection of T_2 with C . If a small reduction of flow should momentarily occur, the compressor will produce a greater pressure rise and the throttle resistance will fall. The flow rate must, of necessity, increase so that the original operating point is restored. A similar argument holds if the flow is temporarily augmented, so that the flow is completely stable at this operating condition.

If, now, the operating point is at point U , unstable operation is possible. A small reduction in flow will cause a greater reduction in compressor pressure ratio than the corresponding pressure ratio across the throttle. As a consequence of the increased resistance of the throttle, the flow will decrease even further and the operating point U is clearly unstable. By inference, neutral stability exists when the slopes of the throttle pressure loss curves equal the compressor pressure rise curve.

Tests on low pressure ratio compressors appear to substantiate this explanation of instability. However, for high rotational speed multistage compressors the above argument does not seem sufficient to describe surging. With high speeds no stable

operation appears possible on constant speed curves of positive slope and surge appears to occur when this slope is zero or even a little negative. A more complete understanding of surge in multistage compressors is only possible from a detailed study of the individual stages performance and their interaction with one another.

Control of flow instabilities

Important and dramatic advances have been made in recent years in the understanding and controlling of surge and rotating stall. Both phenomena are now regarded as the mature forms of the natural oscillatory modes of the compression system (see Moore and Greizer 1986). The flow model they considered predicts that an initial disturbance starts with a very small amplitude but quickly grows into a large amplitude form. Thus, the stability of the compressor is equivalent to the stability of these small amplitude waves that exist just prior to stall or surge (Haynes *et al.* 1994). Only a very brief outline can be given of the advances in the understanding of these unstable flows and the means now available for controlling them. Likewise only a few of the many papers written on these topics are cited.

Epstein *et al.* (1989) first suggested that surge and rotating stall could be prevented by using active feedback control to damp the hydrodynamic disturbances while they were still of small amplitude. Active suppression of surge was subsequently demonstrated on a centrifugal compressor by Ffowcs Williams and Huang (1989), also by Pinsley *et al.* (1991) and on an axial compressor by Day (1993). Shortly after this Paduano *et al.* (1993) demonstrated active suppression of rotating stall in a single-stage low-speed axial compressor. By damping the small amplitude waves rotating about the annulus prior to stall, they increased the stable flow range of the compressor by 25%. The control scheme adopted comprised a circumferential array of hot wires just upstream of the compressor and a set of 12 individually actuated vanes upstream of the rotor used to generate the rotating disturbance structure required for control. Haynes *et al.* (1994), using the same control scheme as Paduano *et al.*, actively stabilised a three-stage, low-speed axial compressor and obtained an 8% increase in the operating flow range.

Gysling and Greitzer (1995) employed a different strategy using aeromechanical feedback to suppress the onset of rotating stall in a low-speed axial compressor. Figure 5.15 shows a schematic of the aeromechanical feedback system they used. An auxiliary injection plenum chamber is fed by a high pressure source so that high momentum air is injected upstream towards the compressor rotor. The amount of air injected at a given circumferential position is governed by an array of locally reacting reed valves able to respond to perturbations in the static pressure upstream of the compressor. The reeds valves, which were modelled as mass-spring-dampers, regulated the amount of high-pressure air injected into the face of the compressor. The cantilevered reeds were designed to deflect upward to allow an increase of the injected flow, whereas a downward deflection decreases the injection.

A qualitative explanation of the stabilising mechanism has been given by Gysling and Greitzer (1995):

Consider a disturbance to an initially steady, axisymmetric flow, which causes a small decrease in axial velocity in one region of the compressor annulus. In this

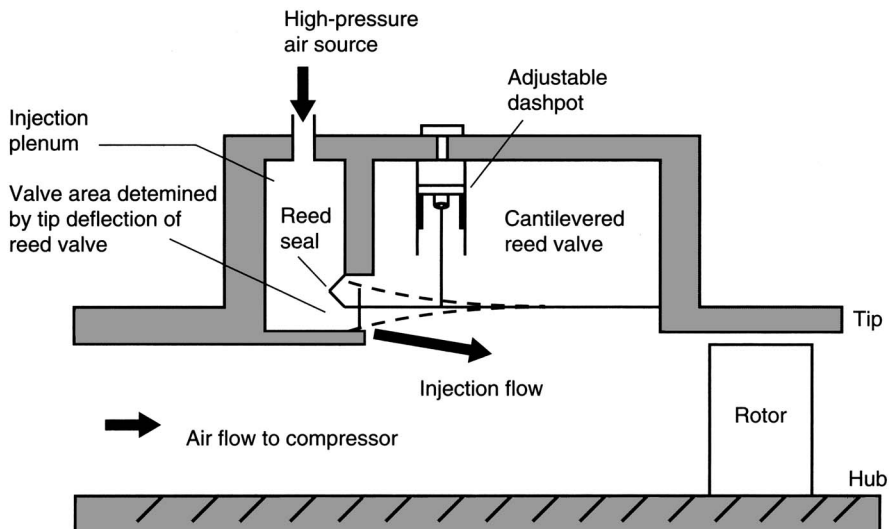


FIG. 5.15. Schematic of the aeromechanical feedback system used to suppress the onset of rotating stall (adapted from Gysling and Greitzer 1995).

region, the static pressure in the potential flow field upstream of the compressor will increase. The increase in static pressure deflects the reed valves in that region, increasing the amount of high momentum fluid injected and, hence, the local mass flow and pressure rise across the compressor. The net result is an increase in pressure rise across the compressor in the region of decreased axial velocity. The feedback thus serves to add a negative component to the real part of the compressor pressure rise versus mass flow transfer function.

Only a small amount (4%) of the overall mass flow through the compressor was used for aeromechanical feedback, enabling the stall flow coefficient of the compression system to be reduced by 10% compared to the stalling flow coefficient with the same amount of steady-state injection.

It is claimed that the research appears to be the first demonstration of dynamic control of rotating stall in an axial compressor using aeromechanical feedback.

Axial-flow ducted fans

In essence, an axial-flow fan is simply a single-stage compressor of low pressure (and temperature) rise, so that much of the foregoing theory of this chapter is valid for this class of machine. However, because of the high space–chord ratio used in many axial fans, a simplified theoretical approach based on *isolated aerofoil theory* is often used. This method can be of use in the design of ventilating fans (usually of high space–chord) in which aerodynamic interference between adjacent blades can be assumed negligible. Attempts have been made to extend the scope of isolated aerofoil theory to less widely spaced blades by the introduction of an *interference factor*; for instance, the ratio k of the lift force of a single blade in a cascade to the

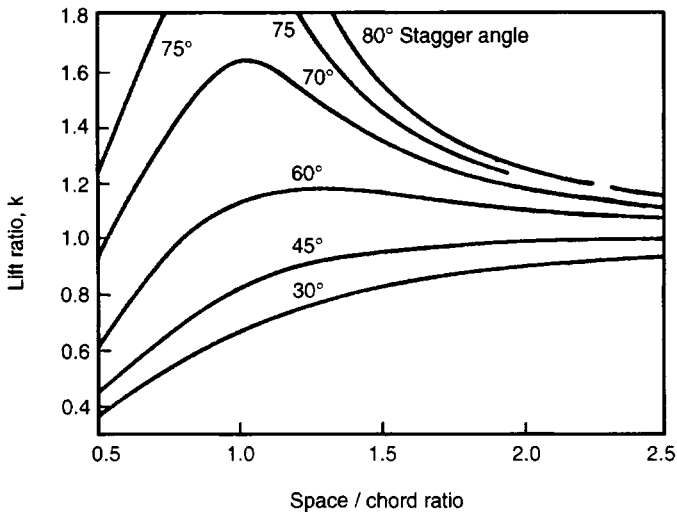


FIG. 5.16. Weinig's results for lift ratio of a cascade of thin flat plates, showing dependence on stagger angle and space/chord ratio (adapted from Wislicenus (1947)).

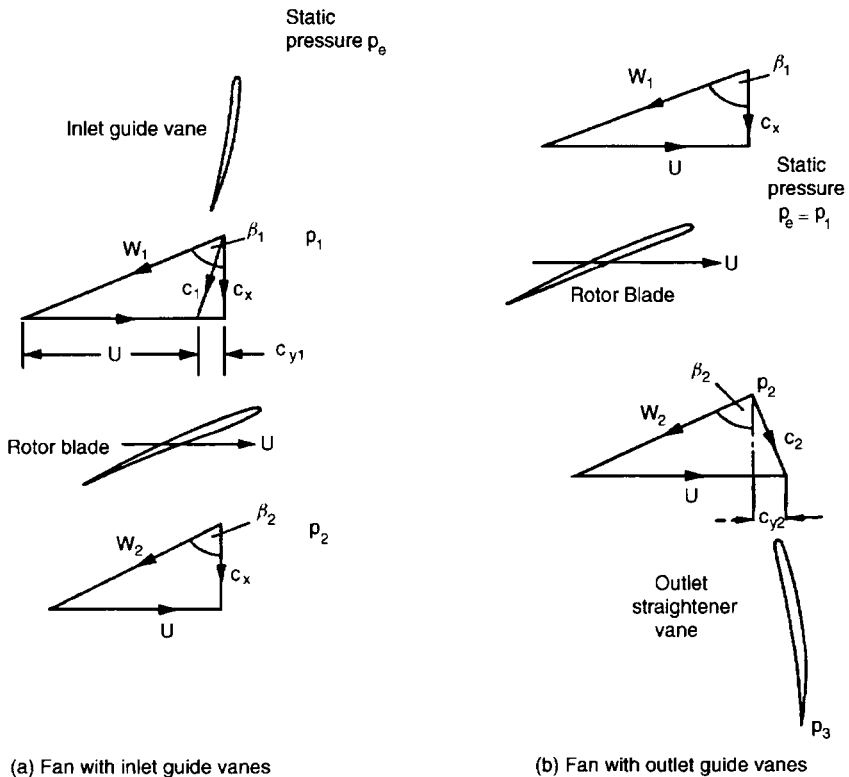


FIG. 5.17. Two simple types of axial-flow fan and their associated velocity diagrams (after Van Niekirk 1958).

lift force of a single isolated blade. As a guide to the degree of this interference, the exact solution obtained by Weinig (1935) and used by Wislicenus (1947) for a row of thin flat plates is of value and is shown in Figure 5.16. This illustrates the dependence of k on space–chord ratio for several stagger angles. The rather pronounced effect of stagger for moderate space–chord ratios should be noted as well as the asymptotic convergence of k towards unity for higher space–chord ratios.

Two simple types of axial-flow fan are shown in Figure 5.17 in which the inlet and outlet flows are entirely axial. In the first type (a), a set of guide vanes provide a contra-swirl and the flow is restored to the axial direction by the rotor. In the second type (b), the rotor imparts swirl in the direction of blade motion and the flow is restored to the axial direction by the action of outlet *straighteners* (or outlet guide vanes). The theory and design of both the above types of fan have been investigated by Van Niekerk (1958) who was able to formulate expressions for calculating the optimum sizes and fan speeds using blade element theory.

Blade element theory

A blade element at a given radius can be defined as an aerofoil of vanishingly small span. In fan-design theory it is commonly assumed that each such element operates as a two-dimensional aerofoil, behaving completely independently of conditions at any other radius. Now the forces impressed upon the fluid by unit span of a single stationary blade have been considered in some detail already, in Chapter 3. Considering an *element of a rotor blade* dr , at radius r , the elementary axial and tangential forces, dX and dY respectively, exerted on the fluid are, referring to Figure 3.5,

$$dX = (L \sin \beta_m - D \cos \beta_m)dr, \quad (5.30)$$

$$dY = (L \cos \beta_m + D \sin \beta_m)dr, \quad (5.31)$$

where $\tan \beta_m = \frac{1}{2} \{\tan \beta_1 + \tan \beta_2\}$ and L, D are the lift and drag on unit span of a blade.

Writing $\tan \gamma = D/L = C_D/C_L$ then,

$$dX = L(\sin \beta_m - \tan \gamma \cos \beta_m)dr.$$

Introducing the lift coefficient $C_L = L/(\frac{1}{2}\rho w_m^2 l)$ for the rotor blade (cf. eqn. (3.16a)) into the above expression and rearranging,

$$dX = \frac{\rho c_x^2 l C_L dr}{2 \cos^2 \beta_m} \cdot \frac{\sin(\beta_m - \gamma)}{\cos \gamma}, \quad (5.32)$$

where $c_x = w_m \cos \beta_m$.

The torque exerted by *one* blade element at radius r is rdY . If there are Z blades the elementary torque is

$$\begin{aligned} d\tau &= rZdY \\ &= rZL(\cos \beta_m + \tan \gamma \sin \beta_m)dr, \end{aligned}$$

after using eqn. (5.31). Substituting for L and rearranging,

$$d\tau = \frac{\rho c_x^2 l Z C_L r dr}{2 \cos^2 \beta_m} \cdot \frac{\cos(\beta_m - \gamma)}{\cos \gamma}. \quad (5.33)$$

Now the work done by the rotor in unit time equals the product of the stagnation enthalpy rise and the mass flow rate; for the elementary ring of area $2\pi r dr$,

$$\Omega d\tau = (C_p \Delta T_0) d\dot{m}, \quad (5.34)$$

where Ω is the rotor angular velocity and the element of mass flow, $d\dot{m} = \rho c_x 2\pi r dr$.

Substituting eqn. (5.33) into eqn. (5.34), then

$$C_p \Delta T_0 = C_p \Delta T = C_L \frac{U_{C_x} l \cos(\beta_m - \gamma)}{2 s \cos^2 \beta_m \cos \gamma}. \quad (5.35)$$

where $s = 2\pi r/Z$. Now the static temperature rise equals the stagnation temperature rise when the velocity is unchanged across the fan; this, in fact, is the case for both types of fan shown in Figure 5.17.

The increase in static pressure of the *whole* of the fluid crossing the rotor row may be found by equating the total axial force on all the blade elements at radius r with the product of static pressure rise and elementary area $2\pi r dr$, or

$$Z dX = (p_2 - p_1) 2\pi r dr.$$

Using eqn. (5.32) and rearranging,

$$p_2 - p_1 = C_L \frac{\rho c_x^2 l \sin(\beta_m - \gamma)}{2 s \cos^2 \beta_m \cos \gamma} \quad (5.36)$$

Note that, so far, all the above expressions are applicable to both types of fan shown in Figure 5.17.

Blade element efficiency

Consider the fan type shown in Figure 5.17a fitted with guide vanes at inlet. The pressure rise across this fan is equal to the rotor pressure rise ($p_2 - p_1$) *minus* the drop in pressure across the guide vanes ($p_e - p_1$). The ideal pressure rise across the fan is given by the product of density and $C_p \Delta T_0$. Fan designers define a blade element efficiency

$$\eta_b = \{(p_2 - p_1) - (p_e - p_1)\} / (\rho C_p \Delta T_0). \quad (5.37)$$

The drop in static pressure across the guide vanes, assuming *frictionless* flow for simplicity, is

$$p_e - p_1 = \frac{1}{2} \rho (c_1^2 - c_x^2) = \frac{1}{2} \rho c_{y1}^2. \quad (5.38)$$

Now since the change in swirl velocity across the rotor is equal and opposite to the swirl produced by the guide vanes, the work done per unit mass flow, $C_p \Delta T_0$ is equal to $U c_{y1}$. Thus the second term in eqn. (5.37) is

$$(p_e - p_1) / (\rho C_p \Delta T_0) = c_{y1} / (2U). \quad (5.39)$$

Combining eqns. (5.35), (5.36) and (5.39) in eqn. (5.37), then

$$\eta_b = (c_x/U) \tan(\beta_m - \gamma) - c_{y1}/(2U). \quad (5.40a)$$

The foregoing exercise can be repeated for the second type of fan having outlet straightening vanes and, assuming frictionless flow through the “straighteners”, the rotor blade element efficiency becomes,

$$\eta_b = (c_x/U) \tan(\beta_m - \gamma) + c_{y2}/(2U). \quad (5.40b)$$

Some justification for ignoring the losses occurring in the guide vanes is found by observing that the ratio of guide vane pressure change to rotor pressure rise is normally small in ventilating fans. For example, in the first type of fan

$$(p_e - p_1)/(p_2 - p_1) \div (\frac{1}{2}\rho c_{y1}^2)/(\rho U c_{y1}) = c_{y1}/2(U),$$

the tangential velocity c_{y1} being rather small compared with the blade speed U .

Lift coefficient of a fan aerofoil

For a specified blade element geometry, blade speed and lift/drag ratio the temperature and pressure rises can be determined if the lift coefficient is known. An estimate of lift coefficient is most easily obtained from two-dimensional aerofoil potential flow theory. Glauert (1959) shows for isolated aerofoils of small camber and thickness, that

$$C_L = 2\pi \sin \chi, \quad (5.41)$$

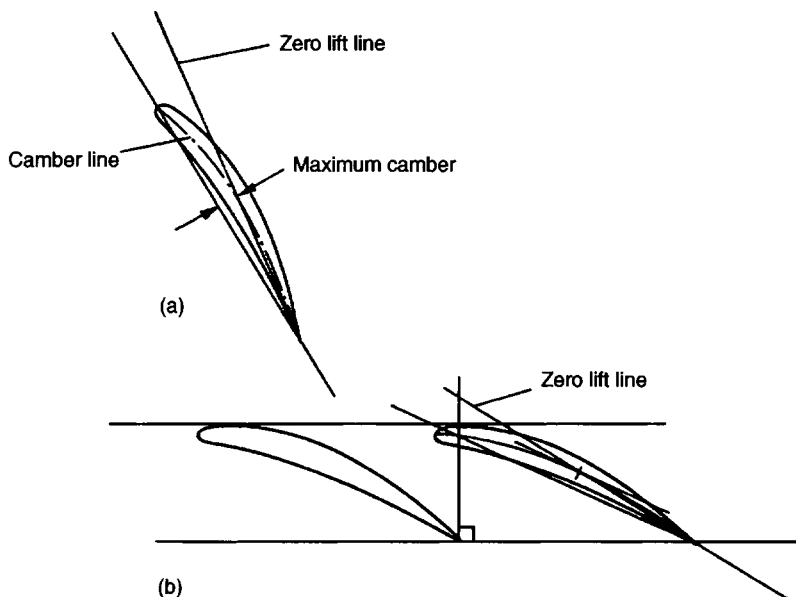


FIG. 5.18. Method suggested by Wislicenus (1947) for obtaining the zero lift line of cambered aerofoils.

where χ is the angle between the flow direction and *line of zero lift* of the aerofoil. For an isolated, cambered aerofoil Wislicenus (1947) suggested that the zero lift line may be found by joining the trailing edge point with the point of maximum camber as depicted in Figure 5.18a. For fan blades experiencing some interference effects from adjacent blades, the modified lift coefficient of a blade may be estimated by assuming that Weinig's results for flat plates (Figure 5.15) are valid for the slightly cambered, finite thickness blades, and

$$C_L = 2\pi k \sin \chi. \quad (5.41a)$$

When the vanes overlap (as they may do at sections close to the hub), Wislicenus suggested that the zero lift line may be obtained by the line connecting the trailing edge point with the maximum camber of that portion of blade which is not overlapped (Figure 5.18b).

The extension of both blade element theory and cascade data to the design of complete fans is discussed in considerable detail by Wallis (1961).

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Problems

(Note: In questions 1 to 4 and 8 take $R = 287 \text{ J/(kg}^\circ\text{C)}$ and $\gamma = 1.4$.)

1. An axial flow compressor is required to deliver 50 kg/s of air at a stagnation pressure of 500 kPa. At inlet to the first stage the stagnation pressure is 100 kPa and the stagnation temperature is 23°C. The hub and tip diameters at this location are 0.436 m and 0.728 m. At the mean radius, which is constant through all stages of the compressor, the reaction is 0.50 and the absolute air angle at stator exit is 28.8 deg for all stages. The speed of the rotor is 8000 rev/min. Determine the number of similar stages needed assuming that the polytropic efficiency is 0.89 and that the axial velocity at the mean radius is constant through the stages and equal to 1.05 times the average axial velocity.

2. Derive an expression for the degree of reaction of an axial compressor stage in terms of the flow angles relative to the rotor and the flow coefficient.

Data obtained from early cascade tests suggested that the limit of efficient working of an axial-flow compressor stage occurred when

- (i) a *relative* Mach number of 0.7 on the rotor is reached;
- (ii) the flow coefficient is 0.5;
- (iii) the relative flow angle at rotor outlet is 30 deg measured from the axial direction;
- (iv) the stage reaction is 50%.

Find the limiting stagnation temperature rise which would be obtained in the first stage of an axial compressor working under the above conditions and compressing air at an inlet stagnation temperature of 289 K. Assume the axial velocity is constant across the stage.

3. Each stage of an axial flow compressor is of 0.5 reaction, has the same mean blade speed and the same flow outlet angle of 30 deg relative to the blades. The mean flow coefficient is constant for all stages at 0.5. At entry to the first stage the stagnation temperature is 278 K, the stagnation pressure 101.3 kPa, the static pressure is 87.3 kPa and the flow area 0.372 m². Using compressible flow analysis determine the axial velocity and the mass flow rate.

Determine also the shaft power needed to drive the compressor when there are 6 stages and the mechanical efficiency is 0.99.

4. A sixteen-stage axial flow compressor is to have a pressure ratio of 6.3. Tests have shown that a stage total-to-total efficiency of 0.9 can be obtained for each of the first six stages and 0.89 for each of the remaining ten stages. Assuming constant work done in each stage and similar stages find the compressor overall total-to-total efficiency. For a mass flow rate of 40 kg/s determine the power required by the compressor. Assume an inlet total temperature of 288 K.

5. At a particular operating condition an axial flow compressor has a reaction of 0.6, a flow coefficient of 0.5 and a stage loading, defined as $\Delta h_0/U^2$ of 0.35. If the flow exit angles for each blade row may be assumed to remain unchanged when the mass flow is throttled, determine the reaction of the stage and the stage loading when the air flow is reduced by 10% at constant blade speed. Sketch the velocity triangles for the two conditions.

Comment upon the likely behaviour of the flow when further reductions in air mass flow are made.

6. The proposed design of a compressor rotor blade row is for 59 blades with a circular arc camber line. At the mean radius of 0.254 m the blades are specified with a camber of 30 deg, a stagger of 40 deg and a chord length of 30 mm. Determine, using Howell's correlation method, the nominal outlet angle, the nominal deviation and the nominal inlet angle. The tangent difference approximation, proposed by Howell for nominal conditions ($0 \leq \alpha_2^* \leq 40^\circ$), can be used:

$$\tan \alpha_1^* - \tan \alpha_2^* = 1.55/(1 + 1.5 s/l).$$

Determine the nominal lift coefficient given that the blade drag coefficient $C_D = 0.017$.

Using the data for relative deflection given in Figure 3.17, determine the flow outlet angle and lift coefficient when the incidence $i = 1.8$ deg. Assume that the drag coefficient is unchanged from the previous value.

7. The preliminary design of an axial flow compressor is to be based upon a simplified consideration of the mean diameter conditions. Suppose that the stage characteristics of a repeating stage of such a design are as follows:

Stagnation temperature rise	25°C
Reaction ratio	0.6
Flow coefficient	0.5
Blade speed	275 m/s

The gas compressed is air with a specific heat at constant pressure of 1.005 kJ/(kg°C). Assuming constant axial velocity across the stage and equal absolute velocities at inlet and outlet, determine the relative flow angles for the rotor.

Physical limitations for this compressor dictate that the space/chord ratio is unity at the mean diameter. Using Howell's correlation method, determine a suitable camber at the mid-height of the rotor blades given that the incidence angle is zero. Use the tangent difference approximation:

$$\tan \beta_1^* - \tan \beta_2^* = 1.55/(1 + 1.5 s/l)$$

for nominal conditions and the data of Figure 3.17 for finding the design deflection. (*Hint.* Use several trial values of θ to complete the solution.)

8. Air enters an axial flow compressor with a stagnation pressure and temperature of 100 kPa and 293 K, leaving at a stagnation pressure of 600 kPa. The hub and tip diameters at entry to the first stage are 0.3 m and 0.5 m. The flow Mach number *after* the inlet guide vanes is 0.7 at the mean diameter. At this diameter, which can be assumed constant for all the compressor stages, the reaction is 50%, the axial velocity to mean blade speed ratio is 0.6 and the absolute flow angle is 30 deg at the exit from all stators. The type of blading used for this compressor is designated “free-vortex” the axial velocity is constant for each stage.

Assuming isentropic flow through the inlet guide vanes and a small stage efficiency of 0.88, determine:

- (1) the air velocity at exit from the IGVs at the mean radius;
- (2) the air mass flow and rotational speed of the compressor;
- (3) the specific work done in each stage;
- (4) the overall efficiency of the compressor;
- (5) the number of compressor stages required and the power needed to drive the compressor;
- (6) consider the implications of rounding the number of stages to an integer value if the pressure ratio *must* be maintained at 6 for the same values of blade speed and flow coefficient.

NB. In the following problems on axial-flow fans the medium is air for which the density is taken to be 1.2 kg/m^3 .

9. (a) The volume flow rate through an axial-flow fan fitted with inlet guide vanes is $2.5 \text{ m}^3/\text{s}$ and the rotational speed of the rotor is 2604 rev/min. The rotor blade tip radius is 23 cm and the root radius is 10 cm. Given that the stage static pressure increase is 325 Pa and the blade element efficiency is 0.80, determine the angle of the flow leaving the guide vanes at the tip, mean and root radii.

(b) A diffuser is fitted at exit to the fan with an area ratio of 2.5 and an effectiveness of 0.82. Determine the overall increase in static pressure and the air velocity at diffuser exit.

10. The rotational speed of a four-bladed axial-flow fan is 2900 rev/min. At the mean radius of 16.5 cm the rotor blades operate at $C_L = 0.8$ with $C_D = 0.045$. The inlet guide vanes produce a flow angle of 20° to the axial direction and the axial velocity through the stage is constant at 20 m/s.

For the mean radius, determine:

- (1) the rotor relative flow angles;
- (2) the stage efficiency;
- (3) the rotor static pressure increase;
- (4) the size of the blade chord needed for this duty.

11. A diffuser is fitted to the axial fan in the previous problem which has an efficiency of 70% and an area ratio of 2.4. Assuming that the flow at entry to the diffuser is uniform and axial in direction, and the losses in the entry section and the guide vanes are negligible, determine:

- (1) the static pressure rise and the pressure recovery factor of the diffuser;
- (2) the loss in total pressure in the diffuser;
- (3) the overall efficiency of the fan and diffuser.