

CHAPTER 7

Centrifugal Pumps, Fans and Compressors

And to thy speed add wings. (MILTON, *Paradise Lost*.)

Introduction

This chapter is concerned with the elementary flow analysis and preliminary design of *radial-flow* work-absorbing turbomachines comprising pumps, fans and compressors. The major part of the discussion is centred around the compressor since the basic action of all these machines is, in most respects, the same.

Turbomachines employing centrifugal effects for increasing fluid pressure have been in use for more than a century. The earliest machines using this principle were, undoubtedly, hydraulic pumps followed later by ventilating fans and blowers. Cheshire (1945) recorded that a centrifugal compressor was incorporated in the build of the whittle turbojet engine.

For the record, the first successful test flight of an aircraft powered by a turbojet engine was on August 27, 1939 at Marienebe Airfield, Waruемunde, Germany (Gas Turbine News (1989). The engine, designed by Hans von Ohain, incorporated an axial flow compressor. The Whittle turbojet engine, with the centrifugal compressor, was first flown on May 15, 1941 at Cranwell, England (see Hawthorne 1978).

Development of the centrifugal compressor continued into the mid-1950s but, long before this, it had become abundantly clear (Campbell and Talbert 1945, Moulton and Pearson 1951) that for the increasingly larger engines required for aircraft propulsion the axial flow compressor was preferred. Not only was the frontal area (and drag) smaller with engines using axial compressors but also the efficiency for the same duty was better by as much as 3 or 4%. However, at very low air mass flow rates the efficiency of axial compressors drops sharply, blading is small and difficult to make accurately and the advantage lies with the centrifugal compressor.

In the mid-1960s the need for advanced military helicopters powered by small gas turbine engines provided the necessary impetus for further rapid development of the centrifugal compressor. The technological advances made in this sphere provided a spur to designers in a much wider field of existing centrifugal compressor applications, e.g. in small gas turbines for road vehicles and commercial helicopters as well as for diesel engine turbochargers, chemical plant processes, factory workshop air supplies and large-scale air-conditioning plant, etc.

Centrifugal compressors were the reasoned choice for refrigerating plants and compression-type heat pumps used in district heating schemes described by Hess (1985). These compressors with capacities ranging from below 1 MW up to nearly 30 MW were preferred because of their good economy, low maintenance and absolute reliability. Dean (1973) quoted total-to-static efficiencies of 80–84 per cent for small single-stage centrifugal compressors with pressure ratios of between 4 and 6. Higher pressure ratios than this have been achieved in single stages, but at reduced efficiency and a very limited airflow range (i.e. up to surge). For instance, Schorr *et al.* (1971) designed and tested a single-stage centrifugal compressor which gave a pressure ratio of 10 at an efficiency of 72 per cent but having an airflow range of only 10 per cent at design speed.

Came (1978) described a design procedure and the subsequent testing of a 6.5 pressure ratio centrifugal compressor incorporating 30 deg *back swept vanes*, giving an isentropic total-to-total efficiency for the impeller of over 85 per cent. The overall total-to-total efficiency for the stage was 76.5 per cent and, with a stage pressure ratio of 6.8 a surge margin of 15 per cent was realised. The use of back swept vanes and the avoidance of high vane loading were factors believed to have given a significant improvement in performance compared to an earlier unswept vane design.

Palmer and Waterman (1995) gave some details of an advanced two-stage centrifugal compressor used in a helicopter engine with a pressure ratio of 14, a mass flow rate of 3.3 kg/s and an overall total-to-total efficiency of 80 per cent. Both stages employed back swept vanes (approximately 47 deg) with a low aerodynamic loading achieved by having a relatively large number of vanes (19 full vanes and 19 splitter vanes).

An interesting and novel compressor is the “axi-fuge”, a mixed flow design with a high efficiency potential, described by Wiggins (1986) and giving on test a pressure ratio of 6.5 at an isentropic efficiency (undefined) of 84 per cent. Essentially, the machine has a typical short centrifugal compressor annulus but actually contains six stages of rotor and stator blades similar to those of an axial compressor. The axi-fuge is claimed to have the efficiency and pressure ratio of an axial compressor of many stages but retains the compactness and structural simplicity of a centrifugal compressor.

Some definitions

Most of the pressure-increasing turbomachines in use are of the radial-flow type and vary from fans that produce pressure rises equivalent to a few millimetres of water to pumps producing heads of many hundreds of metres of water. The term *pump* is used when referring to machines that increase the pressure of a flowing liquid. The term *fan* is used for machines imparting only a small increase in pressure to a flowing gas. In this case the pressure rise is usually so small that the gas can be considered as being incompressible. A *compressor* gives a substantial rise in pressure to a flowing gas. For purposes of definition, the boundary between fans and compressors is often taken as that where the density ratio across the machine is 1.05. Sometimes, but more rarely nowadays, the term *blower* is used instead of fan.

A centrifugal compressor or pump consists essentially of a rotating *impeller* followed by *diffuser*. Figure 7.1 shows diagrammatically the various elements of a centrifugal compressor. Fluid is drawn in through the *inlet casing* into the *eye* of the impeller. The function of the impeller is to increase the energy level of the fluid by whirling it outwards, thereby increasing the angular momentum of the fluid. Both the static pressure and the velocity are increased within the impeller. The purpose of the diffuser is to convert the kinetic energy of the fluid leaving the impeller into pressure energy. This process can be accomplished by free diffusion in the annular space surrounding the impeller or, as indicated in Figure 7.1, by incorporating a row of fixed diffuser vanes which allows the diffuser to be made very much smaller. Outside the diffuser is a *scroll* or *volute* whose function is to collect the flow from the diffuser and deliver it to the outlet pipe. Often, in low-speed compressors and pump applications where simplicity and low cost count for more than efficiency, the volute follows immediately after the impeller.

The *hub* is the curved surface of revolution of the impeller $a - b$; the *shroud* is the curved surface $c - d$ forming the outer boundary to the flow of fluid. Impellers may be enclosed by having the shroud attached to the vane ends (called shrouded impellers) or unenclosed with a small clearance gap between the vane ends and the stationary wall. Whether or not the impeller is enclosed the surface, $c - d$ is generally called the shroud. Shrouding an impeller has the merit of eliminating

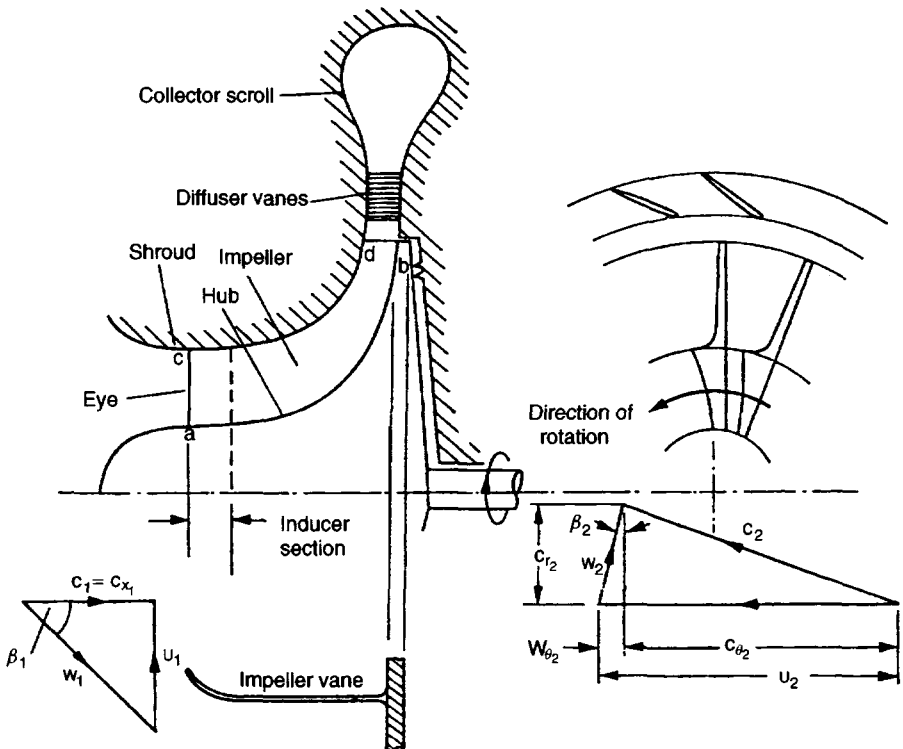


FIG. 7.1. Centrifugal compressor stage and velocity diagrams at impeller entry and exit.

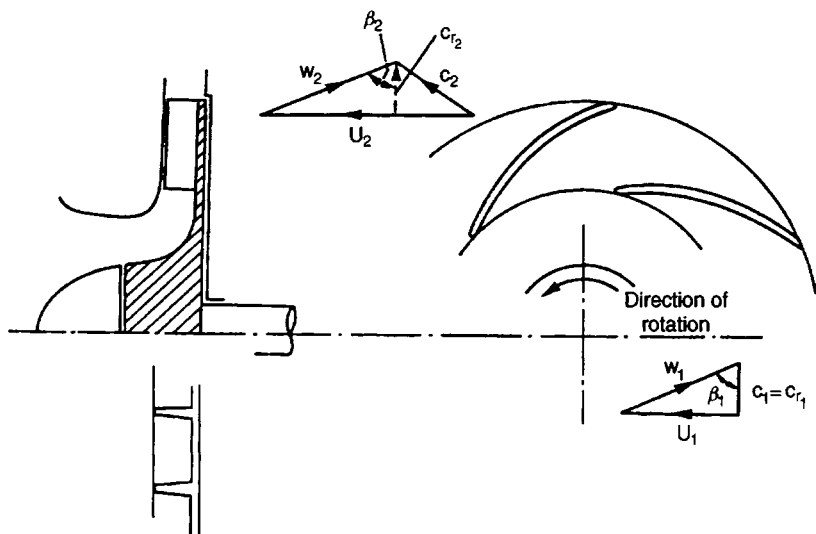


FIG. 7.2. Radial-flow pump and velocity triangles.

tip leakage losses but at the same time increases friction losses. NACA tests have demonstrated that shrouding of a single impeller appears to be detrimental at high speeds and beneficial at low speeds. At entry to the impeller the relative flow has a velocity w_1 at angle β_1 to the axis of rotation. This relative flow is turned into the axial direction by the *inducer section* or *rotating guide vanes* as they are sometimes called. The inducer starts at the eye and usually finishes in the region where the flow is beginning to turn into the radial direction. Some compressors of advanced design extend the inducer well into the radial flow region apparently to reduce the amount of relative diffusion.

To simplify manufacture and reduce cost, many fans and pumps are confined to a two-dimensional radial section as shown in Figure 7.2. With this arrangement some loss in efficiency can be expected. For the purpose of greatest utility, relations obtained in this chapter are generally in terms of the three-dimensional compressor configuration.

Theoretical analysis of a centrifugal compressor

The flow through a compressor stage is a highly complicated, three-dimensional motion and a full analysis presents many problems of the highest order of difficulty. However, we can obtain approximate solutions quite readily by simplifying the flow model. We adopt the so-called *one-dimensional* approach which assumes that the fluid conditions are uniform over certain flow cross-sections. These cross-sections are conveniently taken immediately before and after the impeller as well as at inlet and exit of the entire machine. Where inlet vanes are used to give prerotation to the fluid entering the impeller, the one-dimensional treatment is no longer valid and an extension of the analysis is then required (see Chapter 6).

From the velocity triangle, Figure 7.1, $U - c_\theta = w_\theta$ and together with $w^2 = c_r^2 + w_\theta^2 + c_x^2$, eqn. (7.1) becomes

$$I = h + \frac{1}{2}(w^2 - U^2)$$

or

$$I = h_{0\text{rel}} - \frac{1}{2}U^2,$$

since

$$h_{0\text{rel}} = h + \frac{1}{2}w^2.$$

Since $I_1 = I_2$ across the impeller, then

$$h_2 - h_1 = \frac{1}{2}(\dot{U}_2^2 - U_1^2) + \frac{1}{2}(w_1^2 - w_2^2). \quad (7.2)$$

The above expression provides the reason why the static enthalpy rise in a centrifugal compressor is so large compared with a single-stage axial compressor. On the right-hand side of eqn. (7.2), the second term $\frac{1}{2}(w_2^2 - w_1^2)$, is the contribution from the diffusion of relative velocity and was obtained for axial compressors also. The first term, $\frac{1}{2}(U_2^2 - U_1^2)$, is the contribution due to the centrifugal action which is zero if the streamlines remain at the same radii before and after the impeller.

The relation between state points 1 and 2 in Figure 7.3 can be easily traced with the aid of eqn. (7.2)

Referring to Figure 7.1, and in particular the inlet velocity diagram, the absolute flow has no whirl component or angular momentum and $c_{\theta 1} = 0$. In centrifugal compressors and pumps this is the normal situation where the flow is free to enter axially. For such a flow the specific work done on the fluid, from eqn. (2.12c), is written as

$$\Delta W = U_2 c_{\theta 2} = h_{02} - h_{01} \quad (7.3a)$$

in the case of compressors, and

$$\Delta W = U_2 c_{\theta 2} = gH_i \quad (7.3b)$$

in the case of pumps, where H_i (the “ideal” head) is the total head rise across the pump excluding all internal losses. In high pressure ratio compressors it may be necessary to impart *prerotation* to the flow entering the impeller as a means of reducing a high relative inlet velocity. The effects of high relative velocity at the impeller inlet are experienced as Mach number effects in compressors and cavitation effects in pumps. The usual method of establishing prerotation requires the installation of a row of inlet guide vanes upstream of the impeller, the location depending upon the type of inlet. Unless contrary statements are made it will be assumed for the remainder of this chapter that there is no prerotation (i.e. $c_{\theta 1} = 0$).

Conservation of rothalpy

A cornerstone of the analysis of steady, relative flows in rotating systems has, for many years, been the immutable nature of the fluid mechanical property *rothalpy*.

The conditions under which the rothalpy of a fluid is conserved in the flow through impellers and rotors have been closely scrutinised by several researchers. Lyman (1993) reviewed the equations and physics governing the constancy of rothalpy in turbomachine fluid flows and found that *an increase* in rothalpy was possible for steady, viscous flow without heat transfer or body forces. He proved mathematically that the rothalpy increase was generated mainly by the fluid friction acting on the *stationary* shroud of the compressor considered. From his analysis, and put in the simplest terms, he deduced that:

$$h_{02} - h_{01} = (Uc_\theta)_2 - (Uc_\theta)_1 + W_f/\dot{m}, \quad (7.4)$$

where $W_f = \dot{m}(I_2 - I_1) = \int \mathbf{n} \cdot \boldsymbol{\tau} \cdot \mathbf{W} dA$ is the power loss due to fluid friction on the stationary shroud, $\mathbf{n}$ is a unit normal vector, $\boldsymbol{\tau}$ is a viscous stress tensor, $\mathbf{W}$ is the relative velocity vector and dA is an element of the surface area. Lyman did not give any numerical values in support of his analysis.

In the discussion of Lyman's paper, Moore disclosed that earlier viscous flow calculations of the flow in centrifugal flow compressors (see Moore *et al.* 1984) of the power loss in a centrifugal compressor had shown a rothalpy production amounting to 1.2 per cent of the total work input. This was due to the shear work done at the impeller shroud and it was acknowledged to be of the same order of magnitude as the work done overcoming disc friction on the back face of the impeller. Often disc friction is ignored in preliminary design calculations.

A later, careful, order-of-magnitude investigation by Bosman and Jadayel (1996) showed that the change in rothalpy through a centrifugal compressor impeller would be negligible under typical operating conditions. They also believed that it was not possible to *accurately* calculate the change in rothalpy because the effects due to inexact turbulence modelling and truncation error in computation would far exceed those due to non-conservation of rothalpy.

Diffuser

The fluid is decelerated adiabatically from velocity c_2 to a velocity c_3 , the static pressure rising from p_2 to p_3 as shown in Figure 7.3. As the volute and outlet diffuser involve some further deceleration it is convenient to group the whole diffusion together as the change of state from point 2 to point 3. As the stagnation enthalpy in steady adiabatic flow without shaft work is constant, $h_{02} = h_{03}$ or $h_2 + \frac{1}{2}c_2^2 = h_3 + \frac{1}{2}c_3^2$. The process 2 to 3 in Figure 7.3 is drawn as irreversible, there being a loss in stagnation pressure $p_{02} - p_{03}$ during the process.

Inlet velocity limitations

The inlet eye is an important critical region in centrifugal pumps and compressors requiring careful consideration at the design stage. If the relative velocity of the inlet flow is too large in pumps, cavitation may result with consequent blade erosion or even reduced performance. In compressors large relative velocities can cause an increase in the impeller total pressure losses. In high-speed centrifugal compressors Mach number effects may become important with high relative velocities in the

inlet. By suitable sizing of the eye the maximum relative velocity, or some related parameter, can be minimised to give the optimum inlet flow conditions. As an illustration the following analysis shows a simple optimisation procedure for a low-speed compressor based upon incompressible flow theory.

For the inlet geometry shown in Figure 7.1, the absolute eye velocity is assumed to be uniform and axial. The inlet relative velocity is $w_1 = (c_{x1}^2 + U^2)^{1/2}$ which is clearly a maximum at the inducer tip radius r_{s1} . The volume flow rate is

$$Q = c_{x1}A_1 = \pi(r_{s1}^2 - r_{h1}^2)(w_{s1}^2 - \Omega^2 r_{s1}^2)^{1/2}. \quad (7.5)$$

It is worth noticing that with both Q and r_{h1} fixed:

- (i) if r_{s1} is made large then, from continuity, the axial velocity is low but the blade speed is high,
- (ii) if r_{s1} is made small the blade speed is small but the axial velocity is high.

Both extremes produce large relative velocities and there must exist some optimum radius r_{s1} for which the relative velocity is a minimum.

For maximum volume flow, differentiate eqn. (7.5) with respect to r_{s1} (keeping w_{s1} constant) and equate to zero,

$$\frac{1}{\pi} \frac{\partial Q}{\partial r_{s1}} = 0 = 2r_{s1}(w_{s1}^2 - \Omega^2 r_{s1}^2)^{1/2} - (r_{s1}^2 - r_{h1}^2)\Omega^2 r_{s1}/(w_{s1}^2 - \Omega^2 r_{s1}^2)^{1/2}$$

After simplifying,

$$2(w_{s1}^2 - \Omega^2 r_{s1}^2) = (r_{s1}^2 - r_{h1}^2)\Omega^2, \\ \therefore 2c_{x1}^2 = kU_{s1}^2,$$

where $k = 1 - (r_{h1}/r_{s1})^2$ and $U_{s1} = \Omega r_{s1}$. Hence, the optimum inlet velocity coefficient is

$$\phi = c_{x1}/U_{s1} = \cot \beta_{s1} = (k/2)^{1/2}. \quad (7.6)$$

Equation (7.6) specifies the optimum conditions for the inlet velocity triangles in terms of the hub/tip radius ratio. For typical values of this ratio (i.e. $0.3 \leq r_{h1}/r_{s1} \leq 0.6$) the optimum relative flow angle at the inducer tip β_{s1} lies between 56 deg and 60 deg.

Optimum design of a pump inlet

As discussed in Chapter 1, cavitation commences in a flowing liquid when the decreasing local static pressure becomes approximately equal to the vapour pressure, p_v . To be more precise, it is necessary to assume that gas cavitation is negligible and that sufficient nuclei exist in the liquid to initiate vapour cavitation.

The pump considered in the following analysis is again assumed to have the flow geometry shown in Figure 7.1. Immediately upstream of the impeller blades the static pressure is $p_1 = p_{01} - \frac{1}{2}\rho c_{x1}^2$ where p_{01} is the stagnation pressure and c_{x1} is the axial velocity. In the vicinity of the impeller blades leading edges on the

suction surfaces there is normally a rapid velocity increase which produces a further decrease in pressure. At cavitation inception the dynamic action of the blades causes the *local* pressure to reduce such that $p = p_v = p_1 - \sigma_b(1/2\rho w_1^2)$. The parameter σ_b which is the *blade cavitation coefficient* corresponding to the cavitation inception point, depends upon the blade shape and the flow incidence angle. For conventional pumps (see Pearsall 1972) operating normally this coefficient lies in the range $0.2 \leq \sigma_b \leq 0.4$. Thus, at cavitation inception.

$$p_1 = p_{01} - \frac{1}{2}\rho c_{x1}^2 = p_v + \sigma_b(\frac{1}{2}\rho w_1^2)$$

$$\therefore gH_s = (p_{01} - p_v)/\rho = \frac{1}{2}c_{x1}^2 + \sigma_b(\frac{1}{2}w_1^2) = \frac{1}{2}c_{x1}^2(1 + \sigma_b) + \frac{1}{2}\sigma_b U_{s1}^2$$

where H_s is the net positive suction head introduced earlier and it is implied that this is measured at the shroud radius $r = r_{s1}$.

To obtain the optimum inlet design conditions consider the suction specific speed defined as $\Omega_{ss} = \Omega Q^{1/2}/(gH_s)^{3/4}$, where $\Omega = U_{s1}/r_{s1}$ and $Q = c_{x1}A_1 = \pi k r_{s1}^2 c_{x1}$. Thus,

$$\frac{\Omega_{ss}^2}{\pi k} = \frac{U_{s1}^2 c_{x1}}{\{\frac{1}{2}c_{x1}^2(1 + \sigma_b) + \frac{1}{2}\sigma_b U_{s1}^2\}^{3/2}} = \frac{\phi}{\{\frac{1}{2}(1 + \sigma_b)\phi^2 + \frac{1}{2}\sigma_b\}^{3/2}} \quad (7.7)$$

where $\phi = c_{x1}/U_{s1}$. To obtain the condition of maximum Ω_{ss} , eqn. (7.7) is differentiated with respect to ϕ and the result set equal to zero. From this procedure the optimum conditions are found:

$$\phi = \left\{ \frac{\sigma_b}{2(1 + \sigma_b)} \right\}^{1/2}, \quad (7.8a)$$

$$gH_s = \frac{3}{2}\sigma_b(\frac{1}{2}U_{s1}^2), \quad (7.8b)$$

$$\Omega_{ss}^2 = \frac{2\pi k(2/3)^{1.5}}{\sigma_b(1 + \sigma_b)^{0.5}} = \frac{3.420k}{\sigma_b(1 + \sigma_b)^{0.5}}. \quad (7.8c)$$

EXAMPLE 7.1. The inlet of a centrifugal pump of the type shown in Figure 7.1 is to be designed for optimum conditions when the flow rate of water is $25 \text{ dm}^3/\text{s}$ and the impeller rotational speed is 1450 rev/min . The maximum suction specific speed $\Omega_{ss} = 3.0 \text{ (rad)}$ and the inlet eye radius ratio is to be 0.3 . Determine

- (i) the blade cavitation coefficient,
- (ii) the shroud diameter at the eye,
- (iii) the eye axial velocity, and
- (iv) the NPSH.

Solution. (i) From eqn. (7.8c),

$$\sigma_b^2(1 + \sigma_b) = (3.42k)^2/\Omega_{ss}^4 = 0.1196$$

with $k = 1 - (r_{h1}/r_{s1})^2 = 1 - 0.3^2 = 0.91$. Solving iteratively (e.g. using the Newton–Raphson approximation), $\sigma_b = 0.3030$.

(ii) As $Q = \pi k r_{s1}^2 c_{x1}$ and $c_{x1} = \phi \Omega r_{s1}$ then $r_{s1}^3 = Q/(\pi k \Omega \phi)$ and $\Omega = 1450\pi/30 = 151.84 \text{ rad/s}$.

From eqn. (7.8a), $\phi = \{0.303/(2 \times 1.303)\}^{0.5} = 0.3410$,

$$\therefore r_{s1}^3 = 0.025/(\pi \times 0.91 \times 151.84 \times 0.341) = 1.689 \times 10^{-4},$$

$$\therefore r_{s1} = 0.05528 \text{ m.}$$

The required diameter of the eye is 110.6 mm.

$$(iii) c_{x1} = \phi \Omega r_{s1} = 0.341 \times 151.84 \times 0.05528 = 2.862 \text{ m/s.}$$

(iv) From eqn. (7.8b),

$$H_s = \frac{0.75 \sigma_b c_{x1}^2}{g \phi^2} = \frac{0.75 \times 0.303 \times 2.862^2}{9.81 \times 0.341^2} = 1.632 \text{ m.}$$

Optimum design of a centrifugal compressor inlet

To obtain high efficiencies from high pressure ratio compressors it is necessary to limit the relative Mach number at the eye.

The flow area at the eye can be written as

$$A_1 = \pi r_{s1}^2 k, \text{ where } k = 1 - (r_{h1}/r_{s1})^2.$$

$$\text{Hence } A_1 = \pi k U_{s1}^2 / \Omega^2 \quad (7.9)$$

$$\text{with } U_{s1} = \Omega r_{s1}.$$

With uniform axial velocity the continuity equation is $\dot{m} = \rho_1 A_1 c_{x1}$.

Noting from the inlet velocity diagram (Figure 7.1) that $c_{x1} = w_{s1} \cos \beta_{s1}$ and $U_{s1} = w_{s1} \sin \beta_{s1}$, then, using eqn. (7.9),

$$\frac{\dot{m} \Omega^2}{\rho_1 k \pi} = w_{s1}^3 \sin^2 \beta_{s1} \cos \beta_{s1}. \quad (7.10)$$

For a perfect gas it is most convenient to express the static density ρ_1 in terms of the stagnation temperature T_{01} and stagnation pressure p_{01} because these parameters are usually constant at entry to the compressor. Now,

$$\frac{\rho}{\rho_0} = \frac{p}{p_0} \frac{T_0}{T}.$$

$$\text{With } C_p T_0 = C_p T + \frac{1}{2} c^2 \text{ and } C_p = \gamma R / (\gamma - 1)$$

$$\text{then } \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = \frac{a_0^2}{a^2}$$

where the Mach number, $M = c/(\gamma RT)^{1/2} = c/a$, a_0 and a being the stagnation and local (static) speeds of sound. For isentropic flow,

$$\frac{p}{p_0} = \left(\frac{T}{T_0} \right)^{\gamma/(\gamma-1)}$$

Thus,

$$\frac{\rho_1}{\rho_0} = \left(\frac{T_1}{T_0} \right)^{1-\gamma/(\gamma-1)} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{-1/(\gamma-1)}$$

where

$$\rho_0 = p_0/(RT_0).$$

The absolute Mach number M_1 and the relative Mach number M_{r1} are defined as

$$M_1 = c_{x1}/a_1 = M_{r1} \cos \beta_{s1} \text{ and } w_{s1} = M_{r1} a_1.$$

Using these relations together with eqn. (7.10)

$$\frac{\dot{m}\Omega^2 RT_{01}}{k\pi p_{01}} = \frac{M_{r1}^3 a_1^3}{\left[1 + \frac{1}{2}(\gamma - 1)M_1^2\right]^{1/(\gamma-1)}} \sin^2 \beta_{s1} \cos \beta_{s1}$$

Since $a_{01}/a_1 = \left[1 + \frac{1}{2}(\gamma - 1)M_1^2\right]^{1/2}$ and $a_{01} = (\gamma RT_{01})^{1/2}$ the above equation is rearranged to give

$$\frac{\dot{m}\Omega^2}{\pi k \gamma p_{01} (\gamma RT_{01})^{1/2}} = \frac{M_{r1}^3 \sin^2 \beta_{s1} \cos \beta_{s1}}{\left[1 + \frac{1}{2}(\gamma - 1)M_{r1}^2 \cos^2 \beta_{s1}\right]^{1/(\gamma-1)+3/2}} \quad (7.11)$$

This equation is extremely useful and can be used in a number of different ways. For a particular gas and known inlet conditions one can specify values of γ , R , p_{01} and T_{01} and obtain $\dot{m}\Omega^2/k$ as a function of M_{r1} and β_{s1} . By specifying a particular value of M_{r1} as a limit, the optimum value of β_{s1} for maximum mass flow can be found. A graphical procedure is the simplest method of optimising β_{s1} as illustrated below.

Taking as an example air, with $\gamma = 1.4$, eqn. (7.11) becomes

$$f(M_{r1}) = \dot{m}\Omega^2/(\pi k \rho_{01} a_{01}^3) = \frac{M_{r1}^3 \sin^2 \beta_{s1} \cos \beta_{s1}}{\left(1 + \frac{1}{5}M_{r1}^2 \cos^2 \beta_{s1}\right)^4}. \quad (7.11a)$$

The rhs of eqn. (7.11a) is plotted in Figure 7.4 as a function of β_{s1} for $M_{r1} = 0.8$ and 0.9. These curves are a maximum at $\beta_{s1} = 60$ deg (approximately).

Shepherd (1956) considered a more general approach to the design of the compressor inlet which included the effect of a free-vortex prewhirl or prerotation. The effect of prewhirl on the mass flow function is easily determined as follows. From the velocity triangles in Figure 7.5,

$$c_1 = c_x / \cos \alpha_1 = w_1 \cos \beta_1 / \cos \alpha_1,$$

$$M_1 = \frac{c_1}{a_1} = \frac{w_1 \cos \beta_1}{a_1 \cos \alpha_1} = M_{r1} \left(\frac{\cos \beta_1}{\cos \alpha_1} \right).$$

Also,

$$U_1 = w_1 \sin \beta_1 + c_1 \sin \alpha_1 = w_1 \cos \beta_1 (\tan \beta_1 + \tan \alpha_1),$$

and

$$\dot{m} = \rho_1 A_1 c_{x1},$$

$$\therefore \dot{m} = \frac{\pi k}{\Omega^2} \rho_1 U_1^2 w_1 \cos \beta_1 = \left(\frac{\pi k \rho_1}{\Omega^2} \right) w_1^3 \cos^3 \beta_1 (\tan \beta_1 + \tan \alpha_1)^2. \quad (7.11b)$$

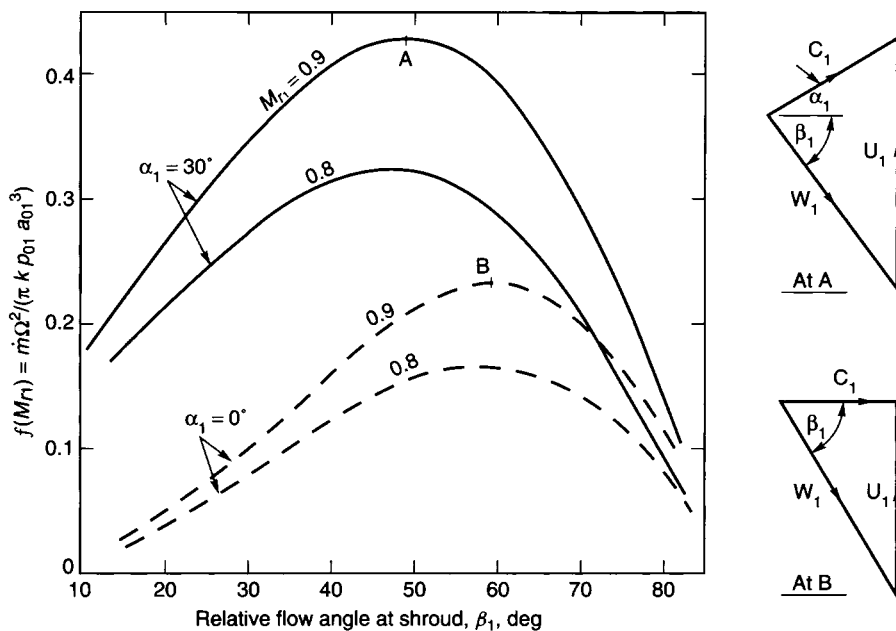


FIG. 7.4. Variation of mass flow function for the inducer of a centrifugal compressor with and without guide vanes ($\gamma = 1.4$). For comparison both velocity triangles are drawn to scale for $M_{r1} = 0.9$ the peak values or curves.

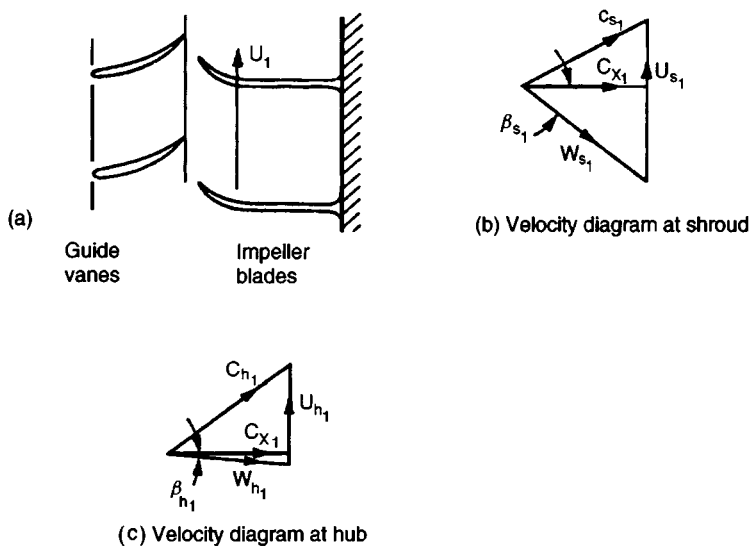


FIG. 7.5. Effect of free-vortex prewhirl vanes upon relative velocity at impeller inlet.

Thus, using the relations developed earlier for T_{01}/T_1 , p_{01}/p_1 and ρ_{01}/ρ_1 , we obtain

$$f(M_{r1}) = \frac{\dot{m}\Omega^2}{\pi k \rho_{01} a_{01}^3} = \frac{M_{r1}^3 \cos^3 \beta_1 (\tan \beta_1 + \tan \alpha_1)^2}{\left(1 + \frac{\gamma-1}{2} M_{r1}^2 \cos^2 \beta_1 / \cos^2 \alpha_1\right)^{\frac{1}{\gamma-1} + \frac{3}{2}}}. \quad (7.12)$$

Substituting $\gamma = 1.4$ for air into eqn. (7.12) we get:

$$f(M_{r1}) = \frac{\Omega^2 \dot{m}}{\pi k \rho_{01} a_{01}^3} = \frac{M_{r1}^3 \cos^3 \beta_1 (\tan \beta_1 + \tan \alpha_1)^2}{\left(1 + \frac{1}{5} M_{r1}^2 \cos^2 \beta_1 / \cos^2 \alpha_1\right)^4}. \quad (7.12a)$$

The rhs of eqn. (7.12a) is plotted in Figure 7.4 with $\alpha_{1s} = 30$ deg for $M_{r1} = 0.8$ and 0.9 , showing that the peak values of $\dot{m}\Omega^2/k$ are significantly increased and occur at much lower values of β_1 .

EXAMPLE 7.2. The inlet of a centrifugal compressor is fitted with free-vortex guide vanes to provide a positive prewhirl of 30 deg at the shroud. The inlet hub/shroud radius ratio is 0.4 and a requirement of the design is that the relative Mach number does not exceed 0.9. The air mass flow is 1 kg/s, the stagnation pressure and temperature are 101.3 kPa and 288 K. For air take $R = 287$ J/(kg K) and $\gamma = 1.4$.

Assuming optimum conditions at the shroud, determine:

- (1) the rotational speed of the impeller;
- (2) the inlet static density downstream of the guide vanes at the shroud and the axial velocity;
- (3) the inducer tip diameter and velocity.

Solution. (1) From Figure 7.4, the peak value of $f(M_{r1}) = 0.4307$ at a relative flow angle $\beta_1 = 49.4$ deg. The constants needed are $a_{01} = \sqrt{(\gamma RT_{01})} = 340.2$ m/s, $\rho_{01} = p_{01}/(RT_{01}) = 1.2255$ kg/m³ and $k = 1 - 0.4^2 = 0.84$. Thus, from eqn. (7.12a), $\Omega^2 = \pi f k \rho_{01} a_{01}^3 = 5.4843 \times 10^7$. Hence,

$$\Omega = 7405.6 \text{ rad/s and } N = 70\,718 \text{ rev/min.}$$

$$(2) \rho_1 = \frac{\rho_{01}}{\left[1 + \frac{1}{5}(M_{r1} \cos \beta_1)^2\right]^{2.5}} = \frac{1.2255}{1.06973^{2.5}} = 0.98464 \text{ kg/m}^3.$$

The axial velocity is determined from eqn. (7.11b):

$$(w_1 \cos \beta_1)^3 = c_x^3 = \frac{\Omega^2 \dot{m}}{\pi k \rho_1 (\tan \beta_1 + \tan \alpha_1)^2} = \frac{5.4843 \times 10^7}{\pi \times 0.84 \times 0.98464 \times 3.0418},$$

$$= 6.9388 \times 10^6,$$

$$\therefore c_x = 190.73 \text{ m/s.}$$

$$(3) A_1 = \frac{\dot{m}}{\rho_1 c_x} = \pi k r_{s1}^2,$$

$$\therefore r_{s1}^2 = \frac{\dot{m}}{\pi \rho_1 c_x k} = \frac{1}{\pi \times 0.98464 \times 190.73 \times 0.84} = 2.0178 \times 10^{-3},$$

$$\therefore r_{s1} = 0.04492 \text{ m and } d_{s1} = 8.984 \text{ cm,}$$

$$U = \Omega r_{s1} = 7405.6 \times 0.04492 = 332.7 \text{ m/s.}$$

Use of prewhirl at entry to impeller

Introducing positive prewhirl (i.e. in the direction of impeller rotation) can give a significant reduction of the inlet Mach number M_{r1} but, as seen from eqn. (2.12c), it reduces the specific work done on the gas. As will be seen later, it is necessary to increase the blade tip speed to maintain the same level of impeller pressure ratio as was obtained without prewhirl.

Preshirl is obtained by fitting guide vanes upstream of the impeller. One arrangement for doing this is shown in Figure 7.5a. The velocity triangles, Figure 7.5b and c, show how the guide vanes reduce the relative inlet velocity. Guide vanes are designed to produce either a free-vortex or a forced-vortex velocity distribution. In Chapter 6 it was shown that for a free-vortex flow the axial velocity c_x is constant (in the ideal flow) with the tangential velocity c_θ varying inversely with the radius. It was shown by Wallace *et al.* (1975) that the use of free-vortex prewhirl vanes leads to a significant increase in incidence angle at low inducer radius ratios. The use of some forced-vortex velocity distribution does alleviate this problem. Some of the effects resulting from the adoption of various forms of forced-vortex of the type

$$c_\theta = A \left(\frac{r}{r_{s1}} \right)^n$$

have been reviewed by Whitfield and Baines (1990). Figure 7.6a shows, for a particular case in which $\alpha_{1s} = 30$ deg, $\beta_{1s} = 60$ deg and $\beta'_{1s} = 60$ deg, the effect of prewhirl on the variation of the incidence angle, $i = \beta_1 - \beta'_1$ with radius ratio, r/r_{1s} , for various whirl distributions. Figure 7.6b shows the corresponding variations of the absolute flow angle, α_1 . It is apparent that a high degree of prewhirl vane twist is required for either a free-vortex design or for the quadratic ($n = 2$) design. The advantage of the quadratic design is the low variation of incidence with radius, whereas it is evident that the free-vortex design produces a wide variation of incidence. Wallace *et al.* (1975) adopted the simple untwisted blade shape ($n = 0$) which proved to be a reasonable compromise.

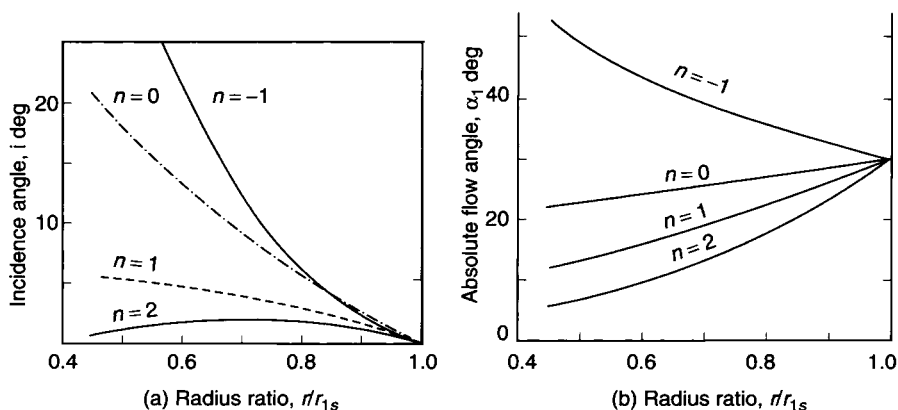


FIG. 7.6. Effect of prewhirl vanes on flow angle and incidence for $\alpha_{1s} = 30$ deg, $\beta_{1s} = 60$ deg and $\beta'_{1s} = 60$ deg. (a) Incidence angle. (b) Inducer flow angle.

Slip factor

Introduction

Even under ideal (frictionless) conditions the relative flow leaving the impeller of a compressor or pump will receive less than perfect guidance from the vanes and the flow is said to *slip*. If the impeller could be imagined as being made with an infinite number of infinitesimally thin vanes, then an ideal flow would be perfectly guided by the vanes and would leave the impeller at the vane angle. Figure 7.7 compares the relative flow angle, β_2 , obtained with a finite number of vanes, with the vane angle, β'_2 . A *slip factor* may be defined as

$$\sigma = \frac{c_{\theta 2}}{c'_{\theta 2}}, \quad (7.13a)$$

where $c_{\theta 2}$ is the tangential component of the absolute velocity and related to the relative flow angle β_2 . The *hypothetical* tangential velocity component $c'_{\theta 2}$ is related to the vane angle β'_2 . The *slip velocity* is given by $c_{\theta s} = c'_{\theta 2} - c_{\theta 2}$ so that the slip factor can be written as

$$\sigma = 1 - \frac{c_{\theta s}}{c'_{\theta 2}}. \quad (7.13b)$$

The slip factor is a vital piece of information needed by pump and compressor designers (also by designers of radial turbines) as its accurate estimation enables the correct value of the energy transfer between impeller and fluid to be made. Various attempts to determine values of slip factor have been made and numerous research papers concerned solely with this topic have been published. Wiesner (1967) has given an extensive review of the various expressions used for determining slip factors. Most of the expressions derived relate to radially vaned impellers ($\beta'_2 = 0$) or to mixed flow designs, but some are given for backward swept vane (b s v) designs. All of these expressions are derived from inviscid flow theory even though the real flow is far from ideal. However, despite this lack of realism in the flow modelling, the fact remains that good results are still obtained with the various theories.

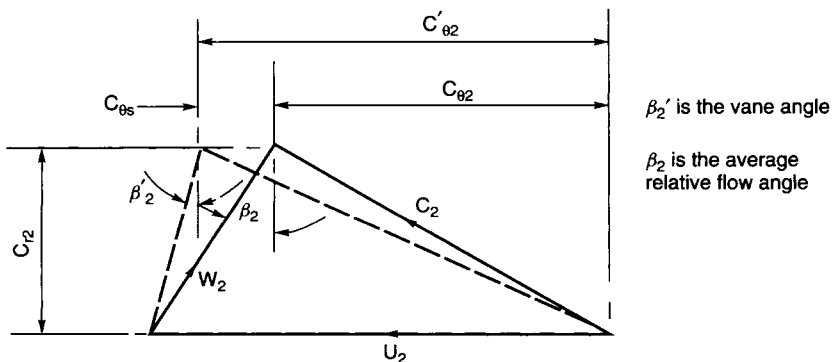


FIG. 7.7. Actual and hypothetical velocity diagrams at exit from an impeller with back swept vanes.

The relative eddy concept

Suppose that an irrotational and frictionless fluid flow is possible which passes through an impeller. If the absolute flow enters the impeller without spin, then at outlet the spin of the absolute flow *must still be zero*. The impeller itself has an angular velocity Ω so that, relative to the impeller, the fluid has an angular velocity of $-\Omega$; this is termed the *relative eddy*. A simple explanation for the slip effect in an impeller is obtained from the idea of a relative eddy.

At outlet from the impeller the relative flow can be regarded as a through-flow on which is superimposed a relative eddy. The net effect of these two motions is that the average relative flow emerging from the impeller passages is at an angle to the vanes and in a direction opposite to the blade motion, as indicated in Figure 7.8. This is the basis of the various theories of slip.

Slip factor correlations

One of the earliest and simplest expressions for the slip factor was obtained by Stodola (1927). Referring to Figure 7.9 the *slip velocity*, $c_{\theta s} = c'_{\theta 2} - c_{\theta 2}$, is considered to be the product of the relative eddy and the radius $d/2$ of a circle which can be inscribed within the channel. Thus $c_{\theta s} = \Omega d/2$. If the number of vanes is denoted by Z then an approximate expression, $d \simeq (2\pi r_2/Z) \cos \beta'_2$ can be written if Z is not small. Since $\Omega = U_2/r_2$ then

$$c_{\theta s} = \frac{\pi U_2 \cos \beta'_2}{Z}. \quad (7.13c)$$

Now as $c'_{\theta 2} = U_2 - c_{r2} \tan \beta'_2$ the Stodola slip factor becomes

$$\sigma = \frac{c_{\theta 2}}{c'_{\theta 2}} = 1 - \frac{c_{\theta s}}{U_2 - c_{r2} \tan \beta'_2} \quad (7.14)$$

or,

$$\sigma = 1 - \frac{(\pi/Z) \cos \beta'_2}{1 - \phi_2 \tan \beta'_2} \quad (7.15)$$

where $\phi_2 = c_{r2}/U_2$.

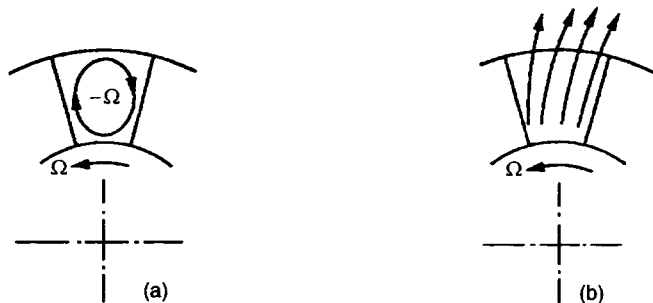


FIG. 7.8. (a) Relative eddy without any throughflow. (b) Relative flow at impeller exit (throughflow added to relative eddy).

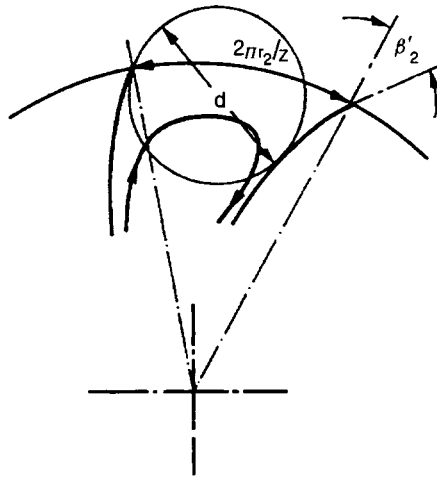
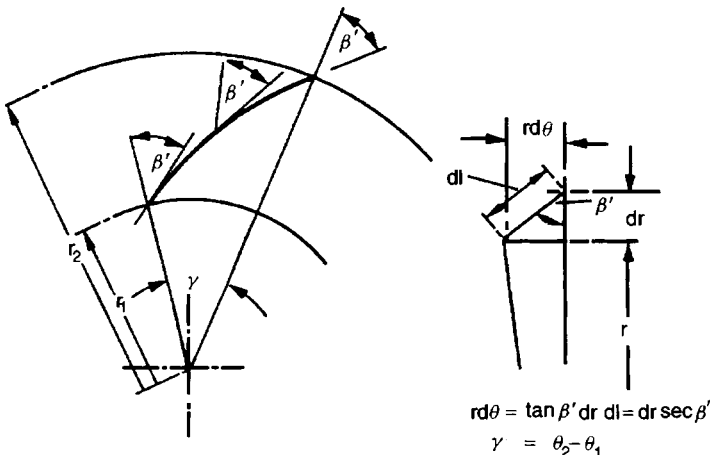


FIG. 7.9. Flow model for Stodola slip factor.

A number of more refined (mathematically exact) solutions have been evolved of which the most well known are those of Busemann, discussed at some length by Wislicenus (1947) and Stanitz (1952) mentioned earlier. The volume of mathematical work required to describe these theories is too extensive to justify inclusion here and only a brief outline of the results is presented.

Busemann's theory applies to the special case of two-dimensional vanes curved as logarithmic spirals as shown in Figure 7.10. Considering the geometry of the vane element shown it should be an easy task for the student to prove that,

$$\gamma = \tan \beta' \ln(r_2/r_1) \quad (7.17a)$$

FIG. 7.10. Logarithmic spiral vane. Vane angle β' is constant for all radii.

that the ratio of vane length to equivalent blade pitch is

$$\frac{l}{s} = \frac{Z}{2\pi \cos \beta'} \ln \left(\frac{r_2}{r_1} \right) \quad (7.17b)$$

and that the equivalent pitch is

$$s = \frac{2\pi(r_2 - r_1)}{Z \ln(r_2/r_1)}.$$

The equi-angular or logarithmic spiral is the simplest form of radial vane system and has been frequently used for *pump impellers* in the past. The Busemann slip factor can be written as

$$\sigma_B = (A - B\phi_2 \tan \beta'_2)/(1 - \phi_2 \tan \beta'_2), \quad (7.16)$$

where both A and B are functions of r_2/r_1 , β'_2 and Z . For typical pump and compressor impellers the dependence of A and B on r_2/r_1 is negligible when the equivalent l/s exceeds unity. From eqn. (7.17b) the requirement for $l/s \geq 1$, is that the radius ratio must be sufficiently large, i.e.

$$r_2/r_1 \geq \exp(2\pi \cos \beta'/Z). \quad (7.17c)$$

This criterion is often applied to other than logarithmic spiral vanes and then β'_2 is used instead of β' . Radius ratios of typical centrifugal pump impeller vanes normally exceed the above limit. For instance, blade outlet angles of impellers are usually in the range $50 \leq \beta'_2 \leq 70$ deg with between 5 and 12 vanes. Taking representative values of $\beta'_2 = 60$ deg and $Z = 8$ the rhs of eqn. (7.17c) is equal to 1.48 which is not particularly large for a pump.

So long as these criteria are obeyed the value of B is constant and practically equal to unity for all conditions. Similarly, the value of A is independent of the radius ratio r_2/r_1 and depends on β'_2 and Z only. Values of A given by Csanady (1960) are shown in Figure 7.11 and may also be interpreted as the value of σ_B for zero through flow ($\phi_2 = 0$).

The exact solution of Busemann makes it possible to check the validity of approximate methods of calculation such as the Stodola expression. By putting $\phi_2 = 0$ in eqns. (7.15) and (7.16) a comparison of the Stodola and Busemann slip factors at the zero through flow condition can be made. The Stodola value of slip comes close to the exact correction if the vane angle is within the range $50 \leq \beta'_2 \leq 70$ deg and the number of vanes exceeds 6.

Stanitz (1952) applied relaxation methods of calculation to solve the potential flow field between the blades (blade-to-blade solution) of eight impellers with blade tip angles β'_2 varying between 0 and 45 deg. His main conclusions were that the computed slip velocity $c_{\theta s}$ was independent of vane angle β'_2 and depended only on blade spacing (number of blades). He also found that compressibility effects did not affect the slip factor. Stanitz's expression for slip velocity is,

$$c_{\theta s} = 0.63U_2\pi/Z \quad (7.18)$$

and the corresponding slip factor σ_s using eqn. (7.14) is

$$\sigma_s = 1 - \frac{0.63\pi/Z}{1 - \phi_2 \tan \beta'_2}. \quad (7.18a)$$

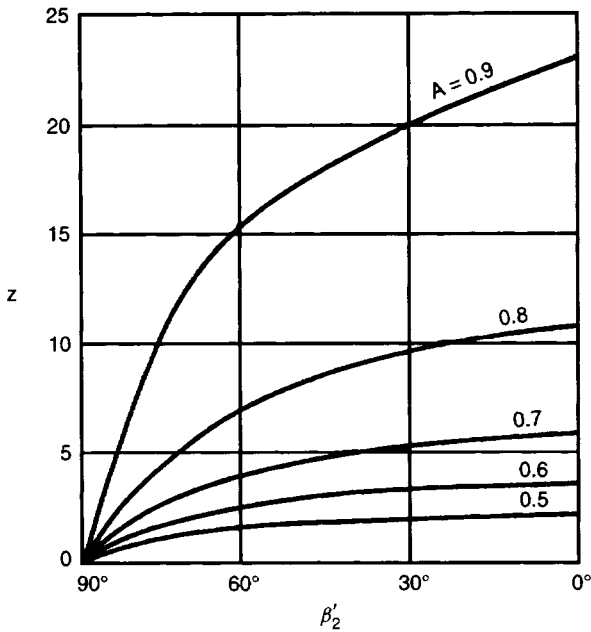


FIG. 7.11. Head correction factors for centrifugal impellers (adapted from Csanady (1960)).

For radial vaned impellers this becomes $\sigma_s = 1 - 0.63\pi/Z$ but is often written for convenience and initial approximate calculations as $\sigma_s = 1 - 2/Z$.

Ferguson (1963) has usefully compiled values of slip factor found from several theories for a number of blade angles and blade numbers and compared them with known experimental values. He found that for pumps, with β'_2 between 60 deg and 70 deg, the Busemann or Stodola slip factors gave fairly good agreement with experimental results. For radial vaned impellers on the other hand, the Stanitz expression, eqn. (7.18a) agreed very well with experimental observations. For intermediate values of β'_2 the Busemann slip factor gave the most consistent agreement with experiment.

Wiesner (1967) reviewed all the available methods for calculating values of slip factor and compared them with values obtained from tests. He concluded from all the material presented that Busemann's procedure was still the most generally applicable predictor for determining the basic slip factor of centripetal impellers. Wiesner obtained the following simple empirical expression for the slip velocity:

$$c_{\theta s} = \frac{U_2 \sqrt{\cos \beta'_2}}{Z^{0.7}} \quad (7.19a)$$

and the corresponding slip factor

$$\sigma_w = 1 - \frac{\sqrt{\cos \beta'_2 / Z^{0.7}}}{(1 - \phi_2 \tan \beta'_2)} \quad (7.19b)$$

which, according to Wiesner, fitted the Busemann results "extremely well over the whole range of practical blade angles and number of blades".

The above equation is applicable to a limiting mean radius ratio for the impeller given by the empirical expression:

$$\varepsilon = \left(\frac{r_1}{r_2} \right)_{\lim} = \exp \left(\frac{-8.16 \cos \beta'_2}{Z} \right). \quad (7.19c)$$

For values of $r_1/r_2 > \varepsilon$ the slip factor is determined from the empirical expression:

$$\sigma'_w = \sigma_w \left[1 - \left(\frac{r_1/r_2 - \varepsilon}{1 - \varepsilon} \right)^3 \right]. \quad (7.19d)$$

Head increase of a centrifugal pump

The actual delivered head H measured as the *head difference* between the inlet and outlet flanges of the pump and sometimes called the *manometric head*, is less than the ideal head H_i defined by eqn. (7.3b) by the amount of the internal losses. The hydraulic efficiency of the pump is defined as

$$\eta_h = \frac{H}{H_i} = \frac{gH}{U_2 c_{\theta 2}}. \quad (7.20)$$

From the velocity triangles of Figure 7.2

$$c_{\theta 2} = U_2 - c_{r2} \tan \beta_2.$$

$$\text{Therefore } H = \eta_h U_2^2 (1 - \phi_2 \tan \beta_2) / g \quad (7.20a)$$

where $\phi_2 = c_{r2}/U_2$ and β_2 is the actual averaged relative flow angle at impeller outlet.

With the definition of slip factor, $\sigma = c_{\theta 2}/c'_{\theta 2}$, H can, more usefully, be directly related to the impeller vane outlet angle, as

$$H = \eta_h \sigma U_2^2 (1 - \phi_2 \tan \beta'_2) / g. \quad (7.20b)$$

In general, centrifugal pump impellers have between five and twelve vanes inclined backwards to the direction of rotation, as suggested in Figure 7.2, with a vane tip angle β'_2 of between 50 and 70 deg. A knowledge of blade number, β'_2 and ϕ_2 (usually small and of the order 0.1) generally enables σ to be found using the Busemann formula. The effect of slip, it should be noted, causes the relative flow angle β_2 to become larger than the vane tip angle β'_2 .

EXAMPLE 7.3 A centrifugal pump delivers $0.1 \text{ m}^3/\text{s}$ of water at a rotational speed of 1200 rev/min. The impeller has seven vanes which lean backwards to the direction of rotation such that the vane tip angle β'_2 is 50 deg. The impeller has an external diameter of 0.4 m, an internal diameter of 0.2 m and an axial width of 31.7 mm. Assuming that the diffuser efficiency is 51.5%, that the impeller head losses are 10% of the ideal head rise and that the diffuser exit is 0.15 m in diameter, estimate the slip factor, the manometric head and the hydraulic efficiency.

Solution. Equation (7.16) is used for estimating the slip factor. Since $\exp(2\pi \cos \beta'_2 / Z) = \exp(2\pi \times 0.643/7) = 1.78$, is less than $r_2/r_1 = 2$, then $B = 1$ and $A \simeq 0.77$, obtained by replotting the values of A given in Figure 7.11 for $\beta'_2 = 50$ deg and interpolating.

The vane tip speed, $U_2 = \pi N D_2 / 60 = \pi \times 1200 \times 0.4 / 60 = 25.13$ m/s.

The radial velocity, $c_{r2} = Q / (\pi D_2 b_2) = 0.1 / (\pi \times 0.4 \times 0.0317)$
 $= 2.51$ m/s.

Hence the Busemann slip factor is

$$\sigma_B = (0.77 - 0.1 \times 1.192) / (1 - 0.1 \times 1.192) = 0.739.$$

Hydraulic losses occur in the impeller and in the diffuser. The kinetic energy leaving the diffuser is not normally recovered and must contribute to the total loss, H_L . From inspection of eqn. (2.45b), the loss in head in the diffuser is $(1 - \eta_D)(c_2^2 - c_3^2)/(2g)$. The head loss in the impeller is $0.1 \times U_2 c_{\theta 2} / g$ and the exit head loss is $c_3^2/(2g)$. Summing the losses,

$$H_L = 0.485(c_2^2 - c_3^2)/(2g) + 0.1 \times U_2 c_{\theta 2} / g + c_3^2/(2g).$$

Determining the velocities and heads needed,

$$c_{\theta 2} = \sigma_B U_2 (1 - \phi_2 \tan \beta'_2) = 0.739 \times 25.13 \times 0.881 = 16.35 \text{ m/s.}$$

$$H_i = U_2 c_{\theta 2} / g = 25.13 \times 16.35 / 9.81 = 41.8 \text{ m.}$$

$$c_2^2/(2g) = (16.35^2 + 2.51^2)/19.62 = 13.96 \text{ m.}$$

$$c_3 = 4Q/(\pi d^2) = 0.4/(\pi \times 0.15^2) = 5.65 \text{ m/s.}$$

$$\text{Therefore } c_3^2/(2g) = 1.63 \text{ m.}$$

$$\text{Therefore } H_L = 4.18 + 0.485(13.96 - 1.63) + 1.63 = 11.8 \text{ m.}$$

The manometric head is

$$H = H_i - H_L = 41.8 - 11.8 = 30.0 \text{ m}$$

and the hydraulic efficiency

$$\eta_h = H/H_i = 71.7\%.$$

Performance of centrifugal compressors

Determining the pressure ratio

Consider a centrifugal compressor having zero inlet swirl, compressing a perfect gas. With the usual notation the energy transfer is

$$\Delta W = \dot{W}_c / \dot{m} = h_{02} - h_{01} = U_2 c_{\theta 2}.$$

The overall or total-to-total efficiency η_c is

$$\begin{aligned}\eta_c &= \frac{h_{03ss} - h_{01}}{h_{03} - h_{01}} = \frac{C_p T_{01} (T_{03ss}/T_{01} - 1)}{h_{02} - h_{01}} \\ &= C_p T_{01} (T_{03ss}/T_{01} - 1) / (U_2 c_{\theta 2}).\end{aligned}\quad (7.21)$$

Now the overall pressure ratio is

$$\frac{p_{03}}{p_{01}} = \left(\frac{T_{03ss}}{T_{01}} \right)^{\gamma/(\gamma-1)}.\quad (7.22)$$

Substituting eqn. (7.21) into eqn. (7.22) and noting that $C_p T_{01} = \gamma R T_{01} / (\gamma - 1) = a_{01}^2 / (\gamma - 1)$, the pressure ratio becomes

$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{(\gamma - 1)\eta_c U_2 c_{r2} \tan \alpha_2}{a_{01}^2} \right]^{\gamma/(\gamma-1)}\quad (7.23)$$

From the velocity triangle at impeller outlet (Figure 7.1)

$$\phi_2 = c_{r2}/U_2 = (\tan \alpha_2 + \tan \beta_2)^{-1}$$

and, therefore,

$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{(\gamma - 1)\eta_c U_2^2 \tan \alpha_2}{a_{01}^2 (\tan \alpha_2 + \tan \beta_2)} \right]^{\gamma/(\gamma-1)}.\quad (7.24a)$$

This formulation is useful if the flow angles can be specified. Alternatively, and more usefully, as $c_{\theta 2} = \sigma c'_{\theta 2} = \sigma(U_2 - c_{r2} \tan \beta'_2)$, then

$$\frac{p_{03}}{p_{01}} = [1 + (\gamma - 1)\eta_c \sigma (1 - \phi_2 \tan \beta'_2) M_u^2]^{\gamma/(\gamma-1)}\quad (7.24b)$$

where $M_u = U_2/a_{01}$, is now defined as a blade Mach number.

It is of interest to calculate the variation of the pressure ratio of a radially vaned ($\beta'_2 = 0$) centrifugal air compressor to show the influence of blade speed and efficiency on the performance. With $\gamma = 1.4$ and $\sigma = 0.9$ (i.e. using the Stanitz slip factor, $\sigma = 1 - 1.98/Z$ and assuming $Z = 20$, the results evaluated are shown in Figure 7.12. It is clear that both the efficiency and the blade speed have a strong effect on the pressure ratio. In the 1970s the limit on blade speed due centrifugal stress was about 500 m/s and efficiencies seldom exceeded 80 per cent giving, with a slip factor of 0.9, radial vanes and an inlet temperature of 288 K, a pressure ratio just above 5. In recent years significant improvements in the performance of centrifugal compressors have been obtained, brought about by the development of computer-aided design and analysis techniques. According to Whitfield and Baines (1990) the techniques employed consist of "a judicious mix of empirical correlations and detailed modelling of the flow physics". It is possible to use these computer packages and arrive at a design solution without any real appreciation of the flow phenomena involved. In *all* compressors the basic flow process is one of diffusion; boundary layers are prone to separate and the flow is extremely complex. With separated wakes in the flow, unsteady flow downstream of the impeller can occur. It must

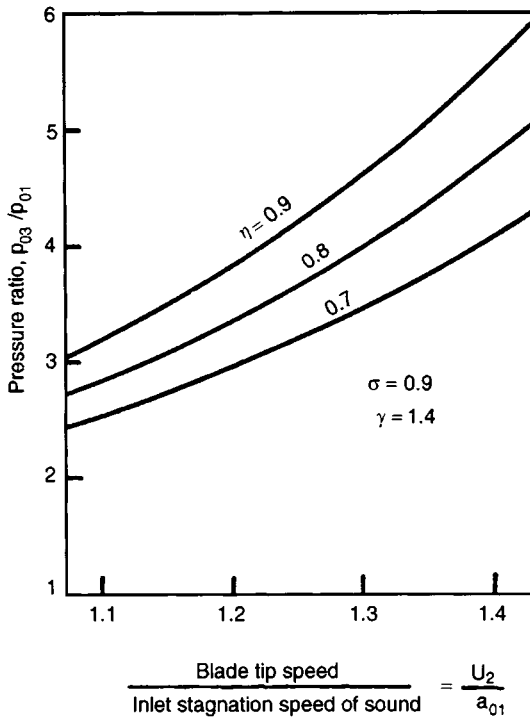


FIG. 7.12. Variation of pressure ratio with blade speed for a radial-bladed compressor ($\beta'_2 = 0$) at various values of efficiency.

be stressed that a broad understanding of the flow processes within a centrifugal compressor is still a vital requirement for the more advanced student and for the further progress of new design methods.

A characteristic of all high performance compressors is that as the design pressure ratio has increased, so the range of mass flow between surge and choking has diminished. In the case of the centrifugal compressor, choking can occur when the Mach number entering the diffuser passages is just in excess of unity. This is a severe problem which is aggravated by shock-induced separation of the boundary layers on the vanes which worsens the problem of flow blockage.

Effect of backswept vanes

Came (1978) and Whitfield and Baines (1990) have commented upon the trend towards the use of higher pressure ratios from single-stage compressors leading to more highly stressed impellers. The increasing use of back swept vanes and higher blade tip speeds result in higher direct stress in the impeller and bending stress in the non-radial vanes. However, new methods of computing the stresses in impellers are being implemented (Calvert and Swinhoe 1977), capable of determining both the direct and the bending stresses caused by impeller rotation.

The effect of using back swept impeller vanes on the pressure ratio is shown in Figure 7.13 for a range of blade Mach number. It is evident that the use of back

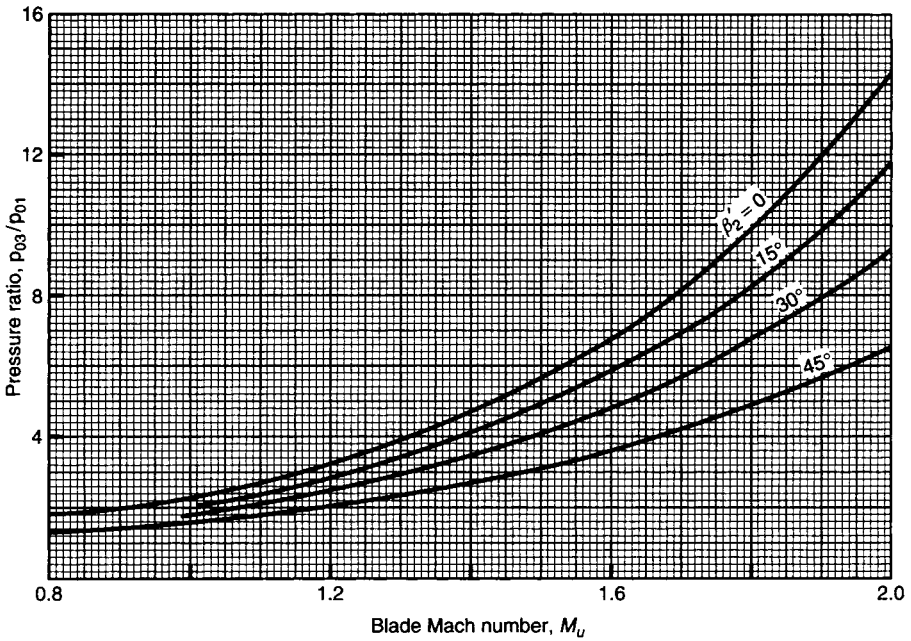


FIG. 7.13. Variation of pressure ratio vs blade Mach number of a centrifugal compressor for selected back sweep angles ($\gamma = 1.4$, $\eta_c = 0.8$, $\sigma = 0.9$, $\phi_2 = 0.375$).

sweep of the vanes at a given blade speed causes a loss in pressure ratio. In order to maintain a given pressure ratio it would be necessary to increase the design speed which, it has been noted already, increases the blade stresses.

With high blade tip speeds the absolute flow leaving the impeller may have a Mach number well in excess of unity. As this Mach number can be related to the Mach number at entry to the diffuser vanes, it is of some advantage to be able to calculate the former.

Assuming a perfect gas the Mach number at impeller exit M_2 can be written as

$$M_2^2 = \frac{c_2^2}{a_2^2} = \frac{c_2^2}{T_{01}} \cdot \frac{T_{01}}{T_2} \cdot \frac{T_2}{a_2^2} = \frac{c_2^2}{a_{01}^2} \frac{T_{01}}{T_2}, \quad (7.25)$$

since $a_{01}^2 = \gamma R T_{01}$ and $a_2^2 = \gamma R T_2$.

Referring to the outlet velocity triangle, Figure (7.7)

$$c_2^2 = c_{r2}^2 + c_{\theta 2}^2 = c_{r2}^2 + (\sigma c'_{\theta 2})^2,$$

where

$$c'_{\theta 2} = U_2 - c_{r2} \tan \beta_2',$$

$$\left(\frac{c_2}{U_2} \right)^2 = \phi_2^2 + \sigma^2 (1 - \phi_2 \tan \beta_2')^2. \quad (7.26)$$

From eqn. (7.2), assuming that rothalpy remains essentially constant,

$$h_2 - h_1 = \frac{1}{2}(U_2^2 - U_1^2) + \frac{1}{2}(w_1^2 - w_2^2),$$

$$\therefore h_2 = (h_1 + \frac{1}{2}w_1^2 - \frac{1}{2}U_1^2) + \frac{1}{2}(U_2^2 - w_2^2) = h_{01} + \frac{1}{2}(U_2^2 - w_2^2)$$

hence,

$$\frac{T_2}{T_{01}} = 1 + \frac{(U_2^2 - w_2^2)}{a_{01}^2/(\gamma - 1)} = 1 + (\gamma - 1)M_u^2 \left(1 - \frac{w_2^2}{U_2^2}\right), \quad (7.27)$$

since $h_{01} = C_p T_{01} = a_{01}^2/(\gamma - 1)$.

From the exit velocity triangle, Figure 7.7,

$$\begin{aligned} w_2^2 &= c_{r2}^2 + (U_2 - c_{\theta 2})^2 = c_{r2}^2 + (U_2 - \sigma c'_{\theta 2})^2 \\ &= c_{r2}^2 + [U_2 - \sigma(U_2 - c_{r2} \tan \beta'_2)]^2, \end{aligned}$$

$$1 - \left(\frac{w_2}{U_2}\right)^2 = 1 - \phi_2^2 - [1 - \sigma(1 - \phi_2 \tan \beta'_2)]^2. \quad (7.28)$$

Substituting eqns. (7.26), (7.27) and (7.28) into eqn. (7.25), we get:

$$M_2^2 = \frac{M_u^2 [\sigma^2 (1 - \phi_2 \tan \beta'_2)^2 + \phi_2^2]}{1 + \frac{1}{2}(\gamma - 1)M_u^2 [1 - \phi_2^2 [1 - \sigma(1 - \phi_2 \tan \beta'_2)]^2]}. \quad (7.29)$$

Although eqn. (7.29) at first sight looks complicated it reduces into an easily managed form when constant values are inserted. Assuming the same values used previously, i.e. $\gamma = 1.4$, $\sigma = 0.9$, $\phi_2 = 0.375$ and $\beta'_2 = 0, 15, 30$ and 45 deg, the solution for M_2 can be written as

$$M_2 = \frac{AM_u}{\sqrt{(1 + BM_u^2)}}, \quad (7.29a)$$

where the constants A and B are as shown in Table 7.1, and, from which the curves of M_2 against M_u in Figure 7.14 have been calculated.

According to Whitfield and Baines (1990) the two most important aerodynamic parameters at impeller exit are the magnitude and direction of the absolute Mach number M_2 . If M_2 has too high a value, the process of efficient flow deceleration within the diffuser itself is made more difficult leading to high friction losses as well as the increased possibility of shock losses. If the flow angle α_2 is large the flow path in the vaneless diffuser will be excessively long resulting in high friction losses

TABLE 7.1. Constants used to evaluate M_2

Constant	β'_2 (degrees)			
	0	15	30	45
A	0.975	0.8922	0.7986	0.676
B	0.2	0.199	0.1975	0.1946

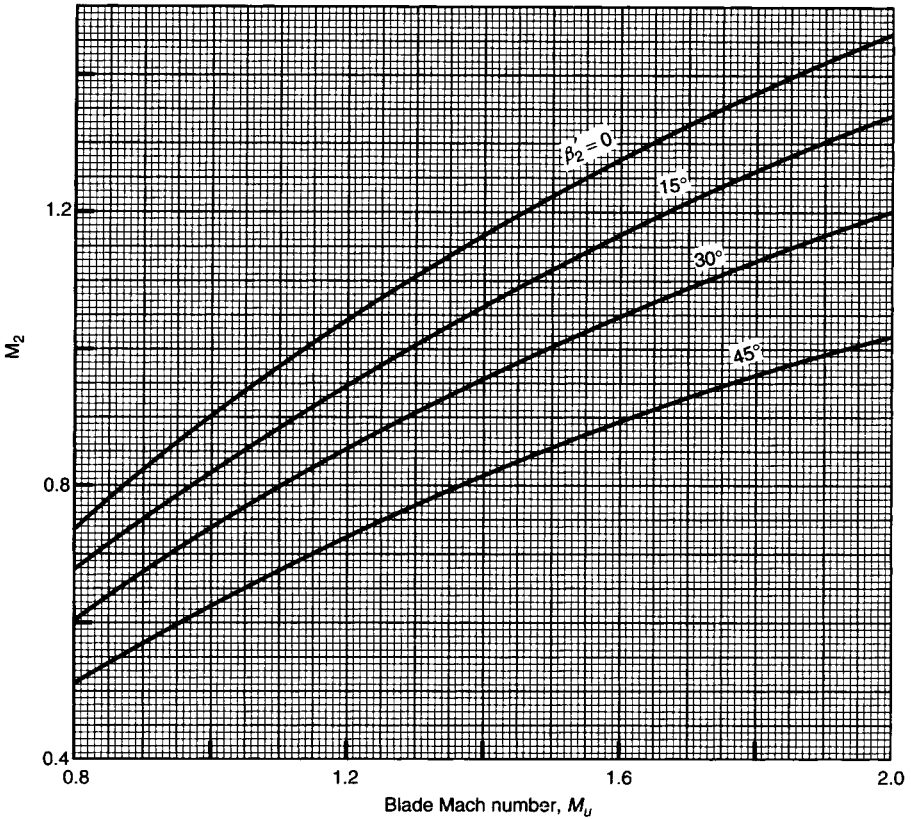


FIG. 7.14. Variation of impeller exit Mach number vs blade Mach number of a centrifugal compressor for selected back sweep angles ($\gamma = 1.4$, $\sigma = 0.9$, $\phi_2 = 0.375$).

and possible stall and flow instability. Several researchers (e.g. Rodgers and Sapiro 1972) have shown that the optimum flow angle is in the range $60^\circ < \alpha_2 < 70^\circ$.

Backswept vanes give a reduction of the impeller discharge Mach number, M_2 , at any given tip speed. A designer making the change from radial vanes to back swept vanes will incur a reduction in the design pressure ratio if the vane tip speed remains the same. To recover the original pressure ratio the designer is forced to increase the blade tip speed which increases the discharge Mach number. Fortunately, it turns out that this increase in M_2 is rather less than the reduction obtained by the use of backsweep.

Illustrative Exercise. Consider a centrifugal compressor design which assumes the previous design data (Figures 7.13 and 7.14), together with $\beta'_2 = 0^\circ$ and a blade speed such that $M_u = 1.6$. From Figure 7.13 the pressure ratio at this point is 6.9 and, from Figure 7.14, the value of $M_2 = 1.27$. Choosing an impeller with a back sweep angle, $\beta'_2 = 30^\circ$, the pressure ratio is 5.0 from Figure 7.13 at the same value of M_u . So, to restore the original pressure ratio of 6.9 the blade Mach number must be increased to $M_u = 1.8$. At this new condition a value of $M_2 = 1.125$ is obtained from Figure 7.14, a significant reduction from the original value.

The absolute flow angle can now be found from the exit velocity triangle, Figure 7.7:

$$\tan \alpha_2 = \frac{c_{\theta 2}}{c_{r 2}} = \frac{\sigma(U_2 - c_{r 2} \tan \beta'_2)}{c_{r 2}} = \sigma \left(\frac{1}{\phi_2} - \tan \beta'_2 \right).$$

With $\sigma = 0.9$, $\phi_2 = 0.375$ then, for $\beta'_2 = 0^\circ$, the value of $\alpha_2 = 67.38^\circ$. Similarly, with $\beta'_2 = 30^\circ$, the value of $\alpha_2 = 62^\circ$, i.e. both values of α_2 are within the prescribed range.

Kinetic energy leaving the impeller

According to Van den Braembussche (1985) "the kinetic energy available at the diffuser inlet easily amounts to more than 50 per cent of the total energy added by the impeller". From the foregoing analysis it is not so difficult to determine whether or not this statement is true. If the magnitude of the kinetic energy is so large then the importance of efficiently converting this energy into pressure energy can be appreciated. The conversion of the kinetic energy to pressure energy is considered in the following section on diffusers.

The fraction of the kinetic energy at impeller exit to the specific work input is

$$r_E = \frac{1}{2} c_2^2 / \Delta W, \quad (7.30)$$

where

$$\begin{aligned} \Delta W &= \sigma U_2^2 (1 - \phi_2 \tan \beta'_2) \text{ and } \left(\frac{c_2}{U_2} \right)^2 = \left(\frac{c_2}{a_2} \cdot \frac{a_2}{a_{01}} \cdot \frac{a_{01}}{U_2} \right)^2 \\ &= \left(\frac{M_2}{M_u} \right)^2 \left(\frac{a_2}{a_{02}} \cdot \frac{a_{02}}{a_{01}} \right)^2. \end{aligned} \quad (7.31)$$

Define the total-to-total efficiency of the impeller as

$$\eta_1 = \frac{h_{02ss} - h_{01}}{h_{02} - h_{01}} = \frac{h_{01} \left(\frac{T_{02ss}}{T_{01}} - 1 \right)}{h_{02} - h_{01}} = \frac{h_{01} \left(p_r^{(\gamma-1)/\gamma} - 1 \right)}{\Delta W}.$$

where p_r is the total-to-total pressure ratio across the impeller, then

$$\left(\frac{a_{02}}{a_{01}} \right)^2 = \frac{T_{02}}{T_{01}} = 1 + \frac{\Delta T_0}{T_{01}} = 1 + \frac{\Delta W}{C_p T_{01}} = 1 + \frac{1}{\eta_1} (p_r^{(\gamma-1)/\gamma} - 1), \quad (7.32)$$

$$\left(\frac{a_{02}}{a_2} \right)^2 = \frac{T_{02}}{T_2} = 1 + \frac{1}{2} (\gamma - 1) M_2^2. \quad (7.33)$$

Substituting eqns. (7.30), (7.31) and (7.32) into eqn. (7.30) we get

$$r_E = \frac{c_2^2 / U_2^2}{2\sigma(1 - \phi_2 \tan \beta'_2)} = \frac{(M_2/M_u)^2 \left[1 + \frac{1}{\eta_1} (p_r^{(\gamma-1)/\gamma} - 1) \right]}{2\sigma(1 - \phi_2 \tan \beta'_2) \left[1 + \frac{1}{2} (\gamma - 1) M_2^2 \right]}. \quad (7.34)$$

Exercise. Determine r_E assuming that $\beta'_2 = 0$, $\sigma = 0.9$, $\eta_l = 0.8$, $p_r = 4$ and $\gamma = 1.4$.

N B. It is very convenient to assume that Figures 7.13 and 7.14 can be used to derive the values of the Mach numbers M_u and M_2 . From Figure 7.13 we get $M_u = 1.3$ and from Figure 7.14, $M_2 = 1.096$. Substituting values into eqn. (7.34),

$$r_E = \frac{1}{2 \times 0.9} \left(\frac{1.096}{1.3} \right)^2 \frac{\left[1 + \frac{1}{0.8} (4^{1/35} - 1) \right]}{1 + \frac{1}{5} \times 1.096^2} = 0.512.$$

Calculations of r_E at other pressure ratios and sweepback angles show that its value remains about 0.51 provided that σ and η_l do not change.

EXAMPLE 7.4. Air at a stagnation temperature of 22°C enters the impeller of a centrifugal compressor in the axial direction. The rotor, which has 17 radial vanes, rotates at 15,000 rev/min. The stagnation pressure ratio between diffuser outlet and impeller inlet is 4.2 and the overall efficiency (total-to-total) is 83%. Determine the impeller tip radius and power required to drive the compressor when the mass flow rate is 2 kg/s and the mechanical efficiency is 97%. Given that the air density at impeller outlet is 2 kg/m^3 and the axial width at entrance to the diffuser is 11 mm, determine the absolute Mach number at that point. Assume that the slip factor $\sigma_s = 1 - 2/Z$, where Z is the number of vanes.

(For air take $\gamma = 1.4$ and $R = 0.287 \text{ kJ/(kg K)}$.)

Solution. From eqn. (7.1a) the specific work is

$$\Delta W = h_{02} - h_{01} = U_2 c_{\theta 2} = \sigma_s U_2^2$$

since $c_{\theta 1} = 0$. Combining eqns. (7.20) and (7.21) with the above and rearranging gives

$$U_2^2 = \frac{C_p T_{01} (r^{(y-1)/y} - 1)}{\sigma_s \eta_c}$$

where $r = p_{03}/p_{01} = 4.2$; $C_p = \gamma R/(\gamma - 1) = 1.005 \text{ kJ/kg K}$; $\sigma_s = 1 - 2/17 = 0.8824$.

$$\text{Therefore } U_2^2 = \frac{1005 \times 295 (4.2^{0.286} - 1)}{0.8824 \times 0.83} = 20.5 \times 10^4.$$

Therefore $U_2 = 452 \text{ m/s}$.

The rotational speed is

$$\Omega = 15,000 \times 2\pi/60 = 1570 \text{ rad/s}.$$

Thus, the impeller tip radius is

$$r_t = U_2/\Omega = 452/1570 = 0.288 \text{ m}.$$

The actual shaft power is obtained from

$$\begin{aligned} \dot{W}_{\text{act}} &= \dot{W}_c/\eta_m = \dot{m} \Delta W/\eta_m = 2 \times 0.8824 \times 452^2/0.97 \\ &= 373 \text{ kW}. \end{aligned}$$

Although the absolute Mach number at the impeller tip can be obtained almost directly from eqn. (7.28) it may be instructive to find it from

$$M_2 = \frac{c_2}{a_2} = \frac{c_2}{(\gamma RT_2)^{1/2}}$$

where

$$c_2 = (c_{\theta 2}^2 + c_{r2}^2)^{1/2}$$

$$c_{r2} = \dot{m}/(\rho_2 2\pi r_2 b_2) = 2/(2 \times 2\pi \times 0.288 \times 0.011) = 50.3 \text{ m/s}$$

$$c_{\theta 2} = \sigma_s U_2 = 400 \text{ m/s.}$$

Therefore $c_2 = \sqrt{(400^2 + 50.3^2)} = 402.5 \text{ m/s.}$

Since $h_{02} = h_{01} + \Delta W$

$$h_2 = h_{01} + \Delta W - \frac{1}{2}c_2^2.$$

Therefore $T_2 = T_{01} + (\Delta W - \frac{1}{2}c_2^2)/C_p = 295 + (18.1 - 8.1)10^4/1005$
 $= 394.5 \text{ K.}$

Hence,

$$M_2 = \frac{402.5}{\sqrt{(402 \times 394.5)}} = 1.01.$$

The diffuser system

Centrifugal compressors and pumps are, in general, fitted with either a vaneless or a vaned diffuser to transform the kinetic energy at impeller outlet into static pressure.

Vaneless diffusers

The simplest concept of diffusion in a radial flow machine is one where the swirl velocity is reduced by an increase in radius (conservation of angular momentum) and the radial velocity component is controlled by the radial flow area. From continuity, since $\dot{m} = \rho A c_r = 2\pi r b \rho c_r$, where b is the width of passage, then

$$c_r = \frac{r_2 b_2 \rho_2 c_{r2}}{r b \rho}. \quad (7.30)$$

Assuming the flow is frictionless in the diffuser, the angular momentum is constant and $c_\theta = c_{\theta 2} r_2 / r$. Now the tangential velocity component c_θ is usually very much larger than the radial velocity component c_r ; therefore, the ratio of inlet to outlet diffuser velocities c_2/c_3 is approximately r_3/r_2 . Clearly, to obtain useful reductions in velocity, vaneless diffusers must be large. This may not be a disadvantage in industrial applications where weight and size may be of secondary importance compared with the cost of a vaned diffuser. A factor in favour of vaneless diffusers is the wide operating range obtainable, vaned diffusers being more sensitive to flow variation because of incidence effects.

For a parallel-walled radial diffuser in incompressible flow, the continuity of mass flow equation requires that rc_r is constant. Assuming that rc_θ remains constant, then

the absolute flow angle $\alpha_2 = \tan^{-1}(c_\theta/c_r)$ is also constant as the fluid is diffused outwards. Under these conditions the flow path is a *logarithmic spiral*. The relationship between the change in the circumferential angle $\Delta\theta$ and the radius ratio of the flow in the diffuser can be found from consideration of an element of the flow geometry shown in Figure 7.15. For an increment in radius dr we have, $r d\theta = dr \tan \alpha_2$ which, upon integration, gives:

$$\Delta\theta = \theta_3 - \theta_2 = \tan \alpha_2 \ln \left(\frac{r_3}{r_2} \right). \quad (7.31)$$

Values of $\Delta\theta$ are shown in Figure 7.16 plotted against r_3/r_2 for several values of α_2 . It can be readily seen that when $\alpha_2 > 70^\circ$, rather long flow paths are implied, friction losses will be significant and the diffuser efficiency will be low.

Vaned diffusers

In the vaned diffuser the vanes are used to remove the swirl of the fluid at a higher rate than is possible by a simple increase in radius, thereby reducing the length of flow path and diameter. The vaned diffuser is advantageous where small size is important.

There is a clearance between the impeller and vane leading edges amounting to about $0.04D_2$ for pumps and between $0.1D_2$ to $0.2D_2$ for compressors. This space

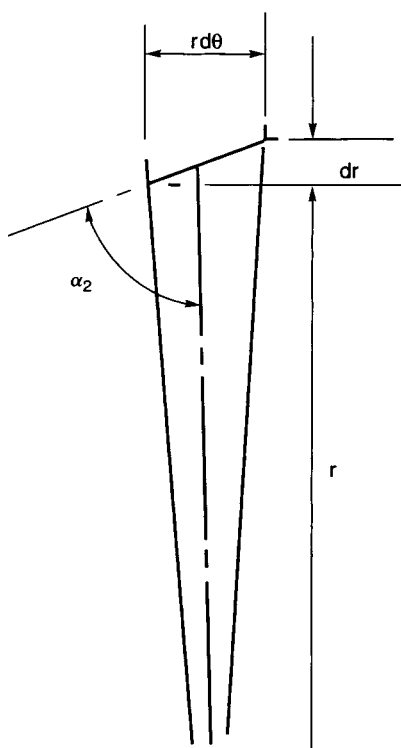


FIG. 7.15. Element of flow path in radial diffuser.

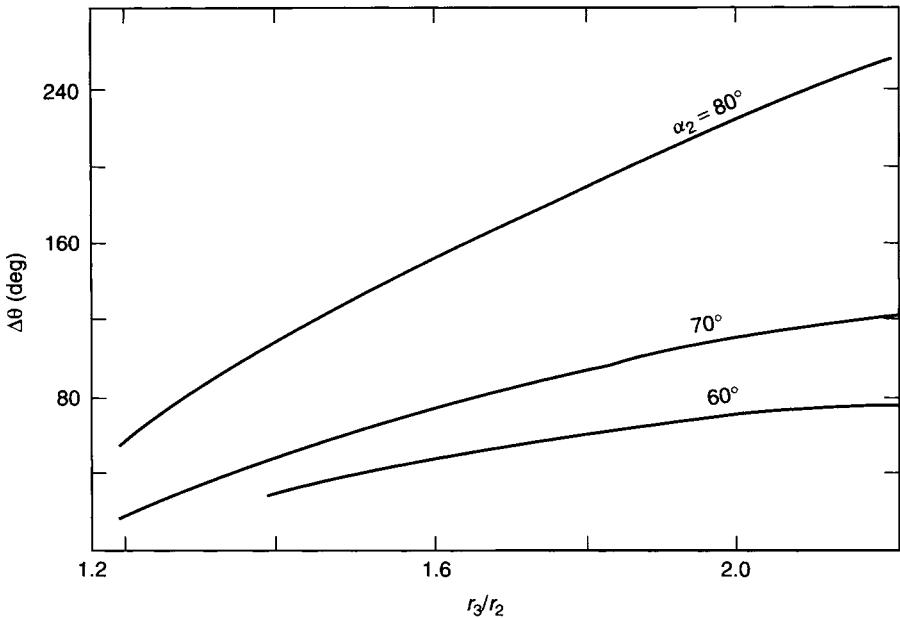


FIG. 7.16. Flow path data for parallel-walled radial diffuser (incompressible flow).

constitutes a vaneless diffuser and its functions are (i) to reduce the circumferential pressure gradient at the impeller tip, (ii) to smooth out velocity variations between the impeller tip and vanes, and (iii) to reduce the Mach number (for compressors) at entry to the vanes.

The flow follows an approximately logarithmic spiral path to the vanes after which it is constrained by the diffuser channels. For rapid diffusion the axis of the channel is straight and tangential to the spiral as shown. The passages are generally designed on the basis of simple channel theory with an equivalent angle of divergence of between 8 deg and 12 deg to control separation. (See remarks in Chapter 2 on straightwalled diffuser efficiency.)

In many applications of the centrifugal compressor, size is important and the outside diameter must be minimised. With a vaned diffuser the channel length can be crucial when considering the final size of the compressor. Clements and Artt (1988) considered this and performed a series of experiments aimed at determining the optimum diffuser channel length to width ratio, L/W . They found that, on the compressor they tested, increasing L/W beyond 3.7 did not produce any improvement in the performance, the pressure gradient at that point having reached zero. Another significant result found by them was that the pressure gradient in the diffuser channel when $L/W > 2.13$ was no greater than that which could be obtained in a vaneless diffuser. Hence, removing completely that portion of the diffuser after this point would yield the same pressure recovery as with the full diffuser.

The number of diffuser vanes can also have a direct bearing on the efficiency and surge margin of the compressor. It is now widely accepted that surge occurs at higher flow rates when vaned diffusers are used than when a simple vaneless diffuser design is adopted. Came and Herbert (1980) quoted an example where a reduction

of the number of diffuser vanes from 29 to 13 caused a significant improvement in the surge margin. Generally, it is accepted that it is better to have fewer diffuser vanes than impeller vanes in order to achieve a wide range of surge-free flow.

With several adjacent diffuser passages sharing the gas from one impeller passage, the uneven velocity distribution from that passage results in alternate diffuser passages being either starved or choked. This is an unstable situation leading to flow reversal in the passages and to surge of the compressor. When the number of diffuser passages is *less* than the number of impeller passages a more uniform total flow results.

Choking in a compressor stage

When the through flow velocity in a passage reaches the speed of sound at some cross-section, the flow *chokes*. For the stationary inlet passage this means that no further increase in mass flow is possible, either by decreasing the back pressure or by increasing the rotational speed. Now the choking behaviour of rotating passages differs from that of stationary passages, making separate analyses for the inlet, impeller and diffuser a necessity. For each component a simple, one-dimensional approach is used assuming that all flow processes are adiabatic and that the fluid is a perfect gas.

Inlet

Choking takes place when $c^2 = a^2 = \gamma RT$. Since $h_0 = h + \frac{1}{2}c^2$, then $C_p T_0 = C_p T + \frac{1}{2}\gamma RT$ and

$$\frac{T}{T_0} = \left(1 + \frac{\gamma R}{2C_p}\right)^{-1} = \frac{2}{\gamma + 1}. \quad (7.32)$$

Assuming the flow in the inlet is isentropic,

$$\frac{\rho}{\rho_0} = \frac{p}{p_0} \frac{T_0}{T} = \left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{1-\gamma/(\gamma-1)}$$

and when $c = a$, $M = 1$, so that

$$\frac{\rho}{\rho_0} = \left(\frac{2}{\gamma + 1}\right)^{1/(\gamma-1)} \quad (7.33)$$

Substituting eqns. (7.31), (7.32) into the continuity equation, $\dot{m}/A = \rho c = \rho(\gamma RT)^{1/2}$, then

$$\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)} \quad (7.34)$$

Thus, since ρ_0 , a_0 refer to inlet stagnation conditions which remain unchanged, the mass flow rate at choking is constant.

Impeller

In the rotating impeller passages, flow conditions are referred to the factor $I = h + \frac{1}{2}(w^2 - U^2)$, which is constant according to eqn. (7.2). At the impeller inlet and for the special case $c_{\theta 1} = 0$, note that $I_1 = h_1 + \frac{1}{2}c_1^2 = h_{01}$. When choking occurs in the impeller passages it is the *relative velocity* w which equals the speed of sound at some section. Now $w^2 = a^2 = \gamma RT$ and $T_{01} = T + (\gamma RT/2C_p) - (U^2/2C_p)$, therefore

$$\frac{T}{T_{01}} = \left(\frac{2}{\gamma + 1} \right) \left(1 + \frac{U^2}{2C_p T_{01}} \right). \quad (7.34)$$

Assuming isentropic flow, $\rho/\rho_{01} = (T/T_{01})^{1/(\gamma-1)}$. Using the continuity equation,

$$\begin{aligned} \frac{\dot{m}}{A} &= \rho_{01} a_{01} \left(\frac{T}{T_{01}} \right)^{(\gamma+1)/2(\gamma-1)} \\ &= \rho_{01} a_{01} \left[\frac{2}{\gamma + 1} \left(1 + \frac{U^2}{2C_p T_{01}} \right) \right]^{(\gamma+1)/2(\gamma-1)} \\ &= \rho_{01} a_{01} \left[\frac{2 + (\gamma - 1)U^2/a_{01}^2}{\gamma + 1} \right]^{(\gamma+1)/2(\gamma-1)} \end{aligned} \quad (7.36)$$

If choking occurs in the rotating passages, eqn. (7.36) indicates that the mass flow is dependent on the blade speed. As the speed of rotation is increased the compressor can accept a *greater* mass flow, unless choking occurs in some other component of the compressor. That the choking flow in an impeller can vary, depending on blade speed, may seem at first rather surprising; the above analysis gives the *reason* for the variation of the choking limit of a compressor.

Diffuser

The relation for the choking flow, eqn. (7.34) holds for the diffuser passages, it being noted that stagnation conditions now refer to the diffuser and not the inlet. Thus

$$\frac{\dot{m}}{A_2} = \rho_{02} a_{02} \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}. \quad (7.37)$$

Clearly, stagnation conditions at diffuser inlet are dependent on the impeller process. To find how the choking mass flow limit is affected by blade speed it is necessary to refer back to inlet stagnation conditions.

Assuming a radial bladed impeller of efficiency η_i then,

$$T_{02s} - T_{01} = \eta_i (T_{02} - T_{01}) = \eta_i \sigma U_2^2 / C_p.$$

Hence

$$p_{02}/p_{01} = (T_{02s}/T_{01})^{\gamma/(\gamma-1)} = [1 + \eta_i \sigma U_2^2 / C_p T_{01}]^{\gamma/(\gamma-1)}$$

and

$$T_{02}/T_{01} = [1 + \sigma U_2^2 / (C_p T_{01})].$$

Now

$$\begin{aligned}\rho_{02} a_{02} &= \rho_{01} a_{01} (\rho_{02} / \rho_{01}) (a_{02} / a_{01}) \\ &= \rho_{01} a_{01} [p_{02} / p_{01} (T_{01} / T_{02})^{1/2}],\end{aligned}$$

therefore,

$$\frac{\dot{m}}{A_2} = \rho_{01} a_{01} \frac{[1 + (\gamma - 1) \eta_i \sigma U_2^2 / a_{01}^2]^{\gamma/(\gamma-1)}}{[1 + (\gamma - 1) \sigma U_2^2 / a_{01}^2]^{1/2}} \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)} \quad (7.38)$$

In this analysis it should be noted that the diffuser process has been assumed to be isentropic but the impeller has not. Eqn. (7.38) indicates that the choking mass flow can be varied by changing the impeller rotational speed.

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Problems

NOTE. In problems 2 to 6 assume γ and R are 1.4 and $287 \text{ J/(kg}^\circ\text{C)}$ respectively. In problems 1 to 4 assume the stagnation pressure and stagnation temperature at compressor entry are 101.3 kPa and 288 K respectively.)

1. A cheap radial-vaned centrifugal fan is required to provide a supply of pressurised air to a furnace. The specification requires that the fan produce a total pressure rise equivalent to 7.5 cm of water at a volume flow rate of $0.2 \text{ m}^3/\text{s}$. The fan impeller is fabricated from 30 thin sheet metal vanes, the ratio of the passage width to circumferential pitch at impeller exit being specified as 0.5 and the ratio of the radial velocity to blade tip speed as 0.1.

Assuming that the overall isentropic efficiency of the fan is 0.75 and that the slip can be estimated from Stanitz’s expression, eqn. (7.18a), determine

- (1) the vane tip speed;
- (2) the rotational speed and diameter of the impeller;
- (3) the power required to drive the fan if the mechanical efficiency is 0.95;
- (4) the specific speed.

For air assume the density is 1.2 kg/m^3 .

2. The air entering the impeller of a centrifugal compressor has an absolute axial velocity of 100 m/s . At rotor exit the relative air angle measured from the radial direction is $26^\circ 36'$, the radial component of velocity is 120 m/s and the tip speed of the radial vanes is 500 m/s . Determine the power required to drive the compressor when the air flow rate is 2.5 kg/s and

the mechanical efficiency is 95%. If the radius ratio of the impeller eye is 0.3, calculate a suitable inlet diameter assuming the inlet flow is incompressible. Determine the overall total pressure ratio of the compressor when the total-to-total efficiency is 80%, assuming the velocity at exit from the diffuser is negligible.

3. A centrifugal compressor has an impeller tip speed of 366 m/s. Determine the absolute Mach number of the flow leaving the radial vanes of the impeller when the radial component of velocity at impeller exit is 30.5 m/s and the slip factor is 0.90. Given that the flow area at impeller exit is 0.1 m^2 and the total-to-total efficiency of the impeller is 90%, determine the mass flow rate.

4. The eye of a centrifugal compressor has a hub/tip radius ratio of 0.4, a maximum relative flow Mach number of 0.9 and an absolute flow which is uniform and completely axial. Determine the optimum speed of rotation for the condition of maximum mass flow given that the mass flow rate is 4.536 kg/s. Also, determine the outside diameter of the eye and the ratio of axial velocity/blade speed at the eye tip. Figure 7.4 may be used to assist the calculations.

5. An experimental centrifugal compressor is fitted with free-vortex guide vanes in order to reduce the relative air speed at inlet to the impeller. At the outer radius of the eye, air leaving the guide-vanes has a velocity of 91.5 m/s at 20 deg to the axial direction. Determine the inlet relative Mach number, assuming frictionless flow through the guide vanes, and the impeller total-to-total efficiency.

Other details of the compressor and its operating conditions are:

Impeller entry tip diameter, 0.457 m
 Impeller exit tip diameter, 0.762 m
 Slip factor 0.9
 Radial component of velocity at impeller exit, 53.4 m/s
 Rotational speed of impeller, 11 000 rev/min
 Static pressure at impeller exit, 223 kPa (abs.)

6. A centrifugal compressor has an impeller with 21 vanes, which are radial at exit, a vaneless diffuser and no inlet guide vanes. At inlet the stagnation pressure is 100 kPa abs. and the stagnation temperature is 300 K.

- (i) Given that the mass flow rate is 2.3 kg/s, the impeller tip speed is 500 m/s and the mechanical efficiency is 96%, determine the driving power on the shaft. Use eqn. (7.18a) for the slip factor.
- (ii) Determine the total and static pressures at diffuser exit when the velocity at that position is 100 m/s. The total to total efficiency is 82%.
- (iii) The reaction, which may be defined as for an axial flow compressor by eqn. (5.10b), is 0.5, the absolute flow speed at impeller entry is 150 m/s and the diffuser efficiency is 84%. Determine the total and static pressures, absolute Mach number and radial component of velocity at the impeller exit.
- (iv) Determine the total-to-total efficiency for the impeller.
- (v) Estimate the inlet/outlet radius ratio for the diffuser assuming the conservation of angular momentum.
- (vi) Find a suitable rotational speed for the impeller given an impeller tip width of 6 mm.

7. A centrifugal pump is used to raise water against a static head of 18.0 m. The suction and delivery pipes, both 0.15 m diameter, have respectively, friction head losses amounting to 2.25 and 7.5 times the dynamic head. The impeller, which rotates at 1450 rev/min, is 0.25 m diameter with 8 vanes, radius ratio 0.45, inclined backwards at $\beta_2 = 60$ deg. The axial width of the impeller is designed so as to give constant radial velocity at all radii and is 20 mm at impeller exit. Assuming an hydraulic efficiency of 0.82 and an overall efficiency of 0.72, determine

- (i) the volume flow rate;
- (ii) the slip factor using Busemann's method;
- (iii) the impeller vane inlet angle required for zero incidence angle;
- (iv) the power required to drive the pump.

8. A centrifugal pump delivers $50 \text{ dm}^3/\text{s}$ of water at an impeller speed of 1450 rev/min . The impeller has eight vanes inclined backwards to the direction of rotation with an angle at the tip of $\beta'_2 = 60^\circ$. The diameter of the impeller is twice the diameter of the shroud at inlet and the magnitude of the radial component of velocity at impeller exit is equal to that of the axial component of velocity at the inlet. The impeller entry is designed for the optimum flow condition to resist cavitation (see eqn. (7.8)), has a radius ratio of 0.35 and the blade shape corresponds to a well tested design giving a cavitation coefficient $\sigma_b = 0.3$.

Assuming that the hydraulic efficiency is 70 per cent and the mechanical efficiency is 90 per cent, determine:

- (1) the diameter of the inlet;
- (2) the net positive suction head;
- (3) the impeller slip factor using Wiesner's formula;
- (4) the head developed by the pump;
- (5) the power input.

Also calculate values for slip factor using the equations of Stodola and Busemann, comparing the answers obtained with the result found from Wiesner's equation.

9. (a) Write down the advantages and disadvantages of using free-vortex guide vanes upstream of the impeller of a high pressure ratio centrifugal compressor. What other sorts of guide vanes can be used and how do they compare with free-vortex vanes?

(b) The inlet of a centrifugal air compressor has a shroud diameter of 0.2 m and a hub diameter of 0.105 m. Free-vortex guide vanes are fitted in the duct upstream of the impeller so that the flow on the shroud at the impeller inlet has a relative Mach number, $M_{r,1} = 1.0$, an absolute flow angle of $\alpha_1 = 20^\circ$ and a relative flow angle $\beta_1 = 55^\circ$. At inlet the stagnation conditions are 288 K and 10^5 Pa .

Assuming frictionless flow into the inlet, determine:

- (1) the rotational speed of the impeller;
- (2) the air mass flow.

(c) At exit from the radially vaned impeller, the vanes have a radius of 0.16 m and a design point slip factor of 0.9. Assuming an impeller efficiency of 0.9, determine:

- (1) the shaft power input;
- (2) the impeller pressure ratio.