

## CHAPTER 9

# Hydraulic Turbines

*Hear ye not the hum of mighty workings? (KEATS, Sonnet No. 14).*

*The power of water has changed more in this world than emperors or kings.*  
(Leonardo da Vinci).

### Introduction

To put this chapter into perspective some idea of the scale of hydropower development in the world might be useful before delving into the intricacies of hydraulic turbines. A very detailed and authoritative account of virtually every aspect of hydropower is given by Raabe (1985) and this brief introduction serves merely to illustrate a few aspects of a very extensive subject.

Hydropower is the longest established source for the generation of electric power which, starting in 1880 as a small dc generating plant in Wisconsin, USA, developed into an industrial size plant following the demonstration of the economic transmission of high voltage ac at the Frankfurt Exhibition in 1891. Hydropower now has a worldwide yearly growth rate of about five per cent (i.e. doubling in size every 15 years). In 1980 the worldwide installed generating capacity was 460 GW according to the United Nations (1981) so, by the year 2000, at the above growth rate this should have risen to a figure of about 1220 GW. The main areas with potential for growth are China, Latin America and Africa.

Table 9.1 is an extract of data quoted by Raabe (1985) of the distribution of harnessed and harnessable potential of some of the countries with the biggest usable potential of hydro power. From this list it is seen that the People's Republic of China, the country with the largest harnessable potential in the world had, in 1974, harnessed only 4.22 per cent of this. According to Cotillon (1978), with growth rates of 14.2 per cent up to 1985 and then with a growth rate of eight per cent, the PRC should have harnessed about 26 per cent of its harvestable potential by the year 2000. This would need the installation of nearly 4600 MW per annum of new hydropower plant, and a challenge to the makers of turbines around the world! One scheme in the PRC, under construction since 1992 and scheduled for completion in 2009, is the Xanxia (Three Gorges) project on the Yangtse which has a planned installed capacity of 25 000 MW, and which would make it the biggest hydropower plant in the world.

### Features of hydropower plants

The initial cost of hydropower plants may be much higher than those of thermal power plants. However, the present value of total costs (which includes those of

TABLE 9.1. Distribution of harnessed and harnessable potential of hydroelectric power.

| Country         | Usable potential,<br>TWh | Amount of potential<br>used, TWh | Percentage of usable<br>potential |
|-----------------|--------------------------|----------------------------------|-----------------------------------|
| 1 China (PRC)   | 1320                     | 55.6                             | 4.22                              |
| 2 Former USSR   | 1095                     | 180                              | 16.45                             |
| 3 USA           | 701.5                    | 277.7                            | 39.6                              |
| 4 Zaire         | 660                      | 4.3                              | 0.65                              |
| 5 Canada        | 535.2                    | 251                              | 46.9                              |
| 6 Brazil        | 519.3                    | 126.9                            | 24.45                             |
| 7 Malaysia      | 320                      | 1.25                             | 0.39                              |
| 8 Columbia      | 300                      | 13.8                             | 4.6                               |
| 9 India         | 280                      | 46.87                            | 16.7                              |
| Sum 1–9         | 5731                     | 907.4                            | 15.83                             |
| Other countries | 4071                     | 843                              | 20.7                              |
| Total           | 9802.4                   | 1750.5                           | 17.8                              |

TABLE 9.2. Features of hydroelectric powerplants.

| Advantages                                                                                                                                      | Disadvantages                                                                                                                                                 |
|-------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Technology is relatively simple and proven. High efficiency. Long useful life. No thermal phenomena apart from those in bearings and generator. | Number of favourable sites limited and only available in some countries. Problems with cavitation and water hammer.                                           |
| Small operating, maintenance and replacement costs.                                                                                             | High initial cost especially for low head plants compared with thermal power plants.                                                                          |
| No air pollution. No thermal pollution of water.                                                                                                | Inundation of the reservoirs and displacement of the population. Loss of arable land. Facilitates sedimentation upstream and erosion downstream of a barrage. |

fuel) is, in general, lower in hydropower plants. Raabe (1985) listed the various advantages and disadvantages of hydropower plants and a brief summary of these is given in Table 9.2.

## Hydraulic turbines

### Early history of hydraulic turbines

The hydraulic turbine has a long period of development, its oldest and simplest form being the waterwheel, first used in ancient Greece and subsequently adopted throughout medieval Europe for the grinding of grain, etc. It was a French engineer, Benoit Fourneyron, who developed the first commercially successful hydraulic turbine (circa 1830). Later Fourneyron built turbines for industrial purposes that achieved a speed of 2300 rev/min, developing about 50 kW at an efficiency of over 80 per cent.

The American engineer James B. Francis designed the first *radial-inflow* hydraulic turbine which became widely used, gave excellent results and was highly regarded. In its original form it was used for heads of between 10 and 100 m. A simplified form of this turbine is shown in Figure 1.1d. It will be observed that the flow path followed is essentially from a radial direction to an axial direction.

The Pelton wheel turbine, named after its American inventor Lester A. Pelton, was brought into use in the second half of the nineteenth century. This is an impulse turbine in which water is piped at high pressure to a nozzle where it expands completely to atmospheric pressure. The emerging jet impacts onto the blades (or buckets) of the turbine producing the required torque and power output. A simplified diagram of a Pelton wheel turbine is shown in Figure 1.1f. The head of water used originally was between about 90 m and 900 m (modern versions operate up to heads of 2000 m).

The increasing need for more power during the early years of the twentieth century also led to the invention of a turbine suitable for small heads of water, i.e. 3 m to 9 m, in river locations where a dam could be built. It was in 1913 that Viktor Kaplan revealed his idea of the propeller (or Kaplan) turbine, see Figure 1.1e, which acts like a ship's propeller but in reverse. At a later date Kaplan improved his turbine by means of swivelable blades which improved the efficiency of the turbine in accordance with the prevailing conditions (i.e. the available flow rate and head).

### Flow regimes for maximum efficiency

Although there are a large number of turbine types in use, only the three mentioned above and variants of them are considered in this book. The efficiencies of the three types are shown in Figure 9.1 as functions of the power specific speed,  $\Omega_{sp}$  which from eqn. (1.9), is

$$\Omega_{sp} = \frac{\Omega \sqrt{P/\rho}}{(gH_E)^{5/4}} \quad (9.1)$$

where  $P$  is the power delivered by the shaft,  $H_E$  is the effective head at turbine entry and  $\Omega$  is the rotational speed in rad/s.

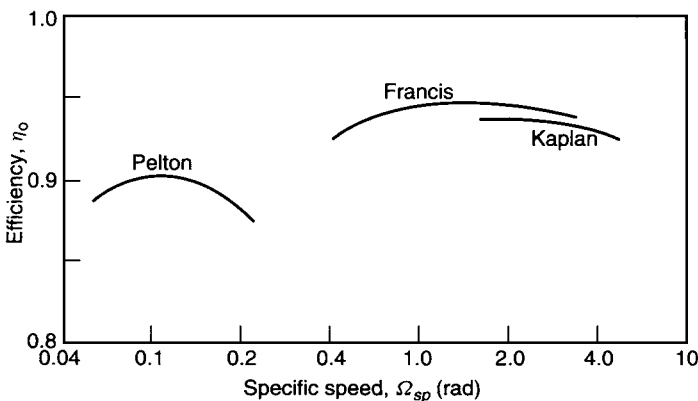


FIG. 9.1. Typical design point efficiencies of Pelton, Francis and Kaplan turbines.

TABLE 9.3. Operating ranges of hydraulic turbines.

|                              | Pelton turbine                   | Francis turbine              | Kaplan turbine                |
|------------------------------|----------------------------------|------------------------------|-------------------------------|
| Specific speed (rad)         | 0.05 – 0.4                       | 0.4 – 2.2                    | 1.8 – 5.0                     |
| Head (m)                     | 100–1770                         | 20–900                       | 6–70                          |
| Maximum power (MW)           | 500                              | 800                          | 300                           |
| Optimum efficiency, per cent | 90                               | 95                           | 94                            |
| Regulation method            | Needle valve and deflector plate | Stagger angle of guide vanes | Stagger angle of rotor blades |

NB. Values shown in the table are only a rough guide and are subject to change.

The regimes of these turbine types are of some significance to the designer as they indicate the most suitable choice of machine for an application once the specific speed has been determined. In general low specific speed machines correspond to low volume flow rates and high heads, whereas high specific speed machines correspond to high volume flow rates and low heads. Table 9.3 summarises the normal operating ranges for the specific speed, the effective head, the maximum power and best efficiency for each type of turbine.

According to the experience of Sulzer Hydro Ltd., of Zurich, the application ranges of the various types of turbines and turbine pumps (including some not mentioned

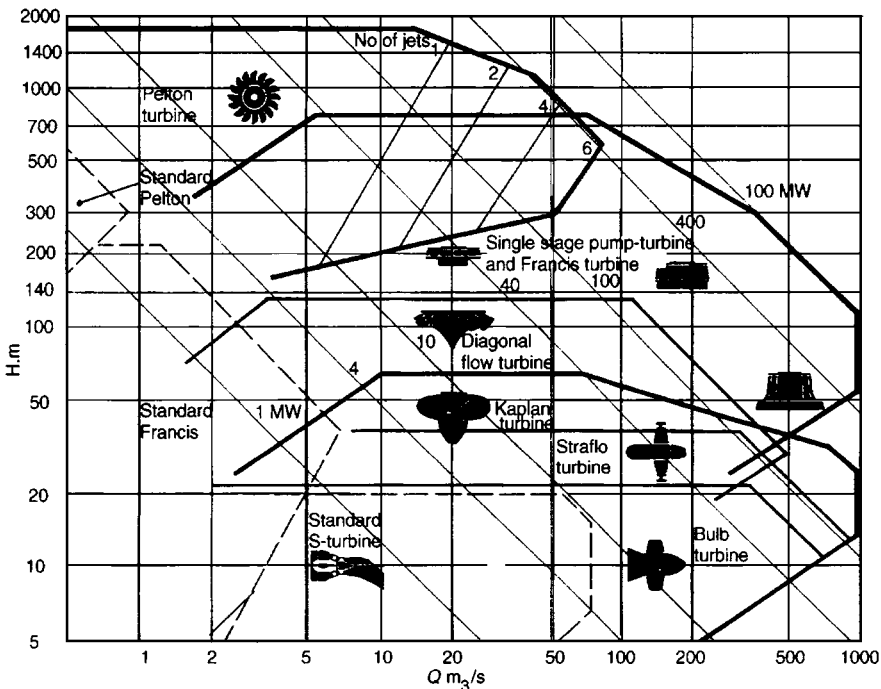


FIG. 9.2. Application ranges for various types of hydraulic turbomachines, as a plot of  $Q$  vs  $H$  with lines of constant power determined assuming  $\eta_0 = 0.8$ . (Courtesy Sulzer Hydro Ltd., Zurich).

here) are plotted in Figure 9.2 on a  $\ln Q$  vs  $\ln H$  diagram, and reflect the present state of the art of hydraulic turbomachinery design. Also in Figure 9.2 lines of constant power output are conveniently shown and have been calculated as the product  $\eta \rho g QH$ , where the efficiency  $\eta$  is accorded the value of 0.8 throughout the chart.

### Capacity of large Francis turbines

The size and capacity of some of the recently built Francis turbines is a source of wonder, they seem so enormous! The size and weight of the runners cause special problems getting them to the site, especially when rivers have to be crossed and the bridges are inadequate.

The largest installation now in North America is at La Grande on James Bay in eastern Canada where 22 units each rated at 333 MW have a total capacity of 7326 MW. For the record, the Itaipu hydroelectric plant on the Paraná river (between Brazil and Paraguay), dedicated in 1982, has the greatest capacity of 12 870 MW in full operation (with a planned value of 21 500 MW) using 18 Francis turbines each sized at over 700 MW.

The efficiency of large Francis turbines has gradually risen over the years and now is about 95 per cent. An historical review of this progress has been given by Danel (1959). There seems to be little prospect of much further improvement in efficiency as skin friction, tip leakage and exit kinetic energy from the diffuser now apparently account for the remaining losses. Raabe (1985) has given much attention to the statistics of the world's biggest turbines. It would appear at the present time that the largest hydroturbines in the world are the three vertical shaft Francis turbines installed at Grand Coulee III on the Columbia River, Washington, USA. Each of these leviathans has been uprated to 800 MW, with the delivery (or effective) head,  $H = 87$  m,  $N = 85.7$  rev/min, the runner having a diameter of  $D = 9.26$  m and weighing 450 ton. Using this data in eqn. (9.1) it is easy to calculate that the power specific speed is 1.74 rad.

### The Pelton turbine

This is the only hydraulic turbine of the impulse type now in common use. It is an efficient machine and it is particularly suited to high head applications. The rotor consists of a circular disc with a number of blades (usually called "buckets") spaced around the periphery. One or more nozzles are mounted in such a way that each nozzle directs its jet along a tangent to the circle through the centres of the buckets. There is a "splitter" or ridge which splits the oncoming jet into two equal streams so that, after flowing round the inner surface of the bucket, the two streams depart from the bucket in a direction nearly opposite to that of the incoming jet.

Figure 9.3 shows the runner of a Pelton turbine and Figure 9.4 shows a six-jet vertical axis Pelton turbine. Considering one jet impinging on a bucket, the appropriate velocity diagram is shown in Figure 9.5. The jet velocity at entry is  $c_1$  and the blade speed is  $U$  so that the relative velocity at entry is  $w_1 = c_1 - U$ . At exit from the bucket one half of the jet stream flows as shown in the velocity diagram, leaving with a relative velocity  $w_2$  and at an angle  $\beta_2$  to the original direction of flow. From the velocity diagram the much smaller absolute exit velocity  $c_2$  can be determined.

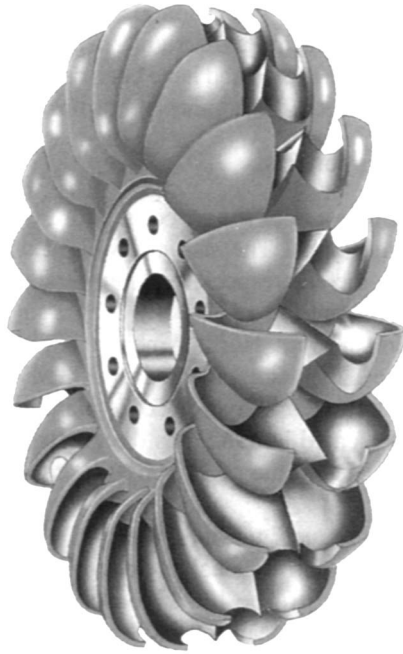


FIG. 9.3. Pelton turbine runner (Courtesy Sulzer Hydro Ltd, Zurich).

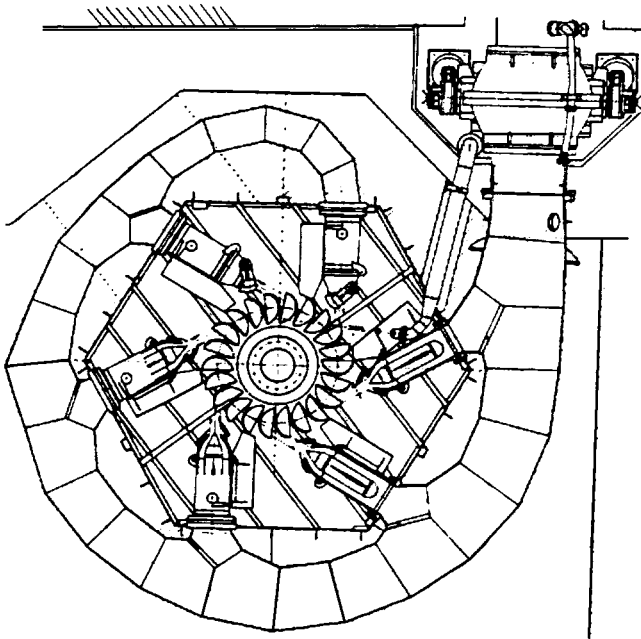


FIG. 9.4. Six-jet vertical shaft Pelton turbine, horizontal section. Power rating 174.4 MW, runner diameter 4.1 m, speed 300 rev/min, head 587 m. (Courtesy Sulzer Hydro Ltd., Zurich).

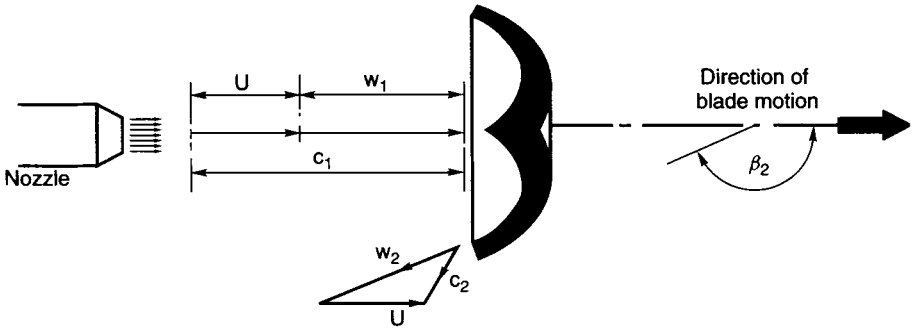


FIG. 9.5. The Pelton wheel showing the jet impinging onto a bucket and the relative and absolute velocities of the flow (only one-half of the emergent velocity diagram is shown).

From Euler's turbine equation, eqn. (2.12b), the specific work done by the water is

$$\Delta W = U_1 c_{\theta 1} - U_2 c_{\theta 2}.$$

For the Pelton turbine,  $U_1 = U_2 = U$ ,  $c_{\theta 1} = c_1$  so we get

$$\Delta W = U[U + w_1 - (U + w_2 \cos \beta_2)] = U(w_1 - w_2 \cos \beta_2)$$

in which the value of  $c_{\theta 2} < 0$ , as defined in Figure 9.5, i.e.  $c_{\theta 2} = U + w_2 \cos \beta_2$ .

The effect of friction on the fluid flowing inside the bucket will cause the relative velocity at outlet to be less than the value at inlet. Writing  $w_2 = kw_1$ , where  $k < 1$ , then,

$$\Delta W = Uw_1(1 - k \cos \beta_2) = U(c_1 - U)(1 - k \cos \beta_2). \quad (9.2)$$

An efficiency  $\eta_R$  for the runner can be defined as the specific work done  $\Delta W$  divided by the incoming kinetic energy, i.e.

$$\eta_R = \Delta W / (\frac{1}{2} c_1^2) \quad (9.3)$$

$$= 2U(c_1 - U)(1 - k \cos \beta_2) / c_1^2$$

$$\therefore \eta_R = 2v(1 - v)(1 - k \cos \beta_2) \quad (9.4)$$

where the blade speed to jet speed ratio,  $v = U/c_1$ .

In order to find the optimum efficiency, differentiate eqn. (9.4) with respect to the blade speed ratio, i.e.

$$\begin{aligned} \frac{d\eta_R}{dv} &= 2 \frac{d}{dv} (v - v^2)(1 - k \cos \beta_2) \\ &= 2(1 - 2v)(1 - k \cos \beta_2) = 0. \end{aligned}$$

Therefore, the maximum efficiency of the runner occurs when  $v = 0.5$ , i.e.  $U = c_1/2$ . Hence,

$$\eta_{R \max} = (1 - k \cos \beta_2)/2. \quad (9.5)$$

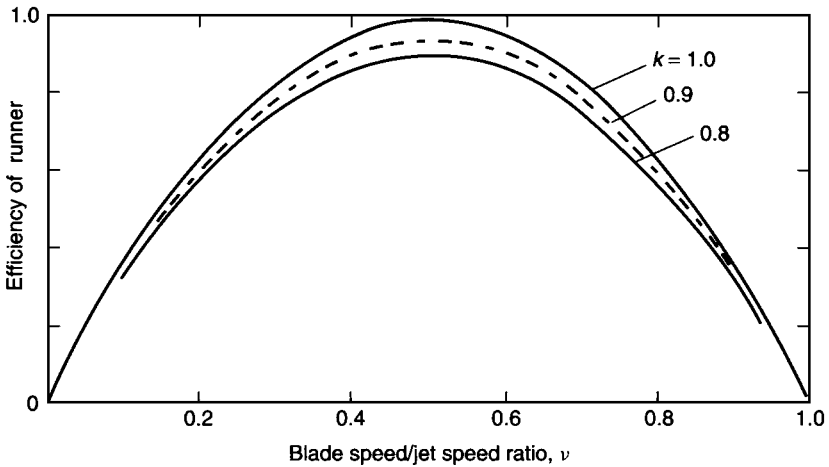


FIG. 9.6. Theoretical variation of runner efficiency for a Pelton wheel with blade speed to jet speed ratio for several values of friction factor  $k$ .

Figure 9.6 shows the variation of the runner efficiency with blade speed ratio for assumed values of  $k = 0.8, 0.9$  and  $1.0$  with  $\beta_2 = 165^\circ$ . In practice the value of  $k$  is usually found to be between  $0.8$  and  $0.9$ .

### A simple hydroelectric scheme

The layout of a Pelton turbine hydroelectric scheme is shown in Figure 9.7. The water is delivered from a constant level reservoir at an elevation  $z_R$  (above sea level) and flows via a pressure tunnel to the penstock head, down the penstock to the turbine nozzles emerging onto the buckets as a high speed jet. In order to reduce the deleterious effects of large pressure surges, a *surge tank* is connected to the flow close to the penstock head which acts so as to damp out transients. The elevation of the nozzles is  $z_N$  and the gross head,  $H_G = z_R - z_N$ .

### Controlling the speed of the Pelton turbine

The Pelton turbine is usually directly coupled to an electrical generator which must run at synchronous speed. With large size hydroelectric schemes supplying electricity to a national grid it is essential for both the voltage and the frequency to closely match the grid values. To ensure that the turbine runs at constant speed despite any load changes which may occur, the rate of flow  $Q$  is changed. A spear (or needle) valve, Figure 9.8a, whose position is controlled by means of a servomechanism, is moved axially within the nozzle to alter the diameter of the jet. This works well for very gradual changes in load. However, when a sudden loss in load occurs a more rapid response is needed. This is accomplished by temporarily deflecting the jet with a deflector plate so that some of the water does not reach the buckets, Figure 9.8b. This acts to prevent overspeeding and allows time for the slower acting spear valve to move to a new position.

It is vital to ensure that the spear valve *does move slowly* as a sudden reduction in the rate of flow could result in serious damage to the system from pressure surges

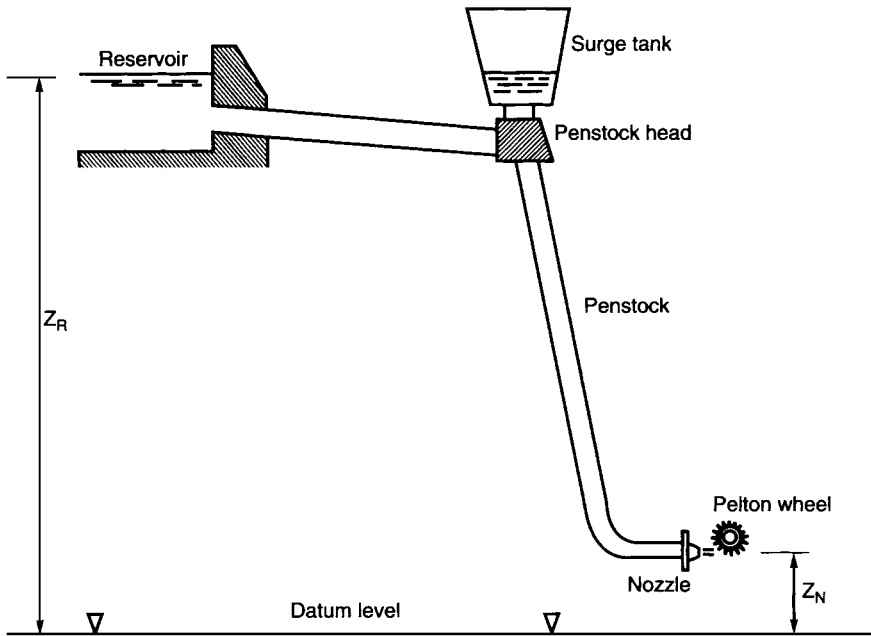


FIG. 9.7. Pelton turbine hydroelectric scheme.

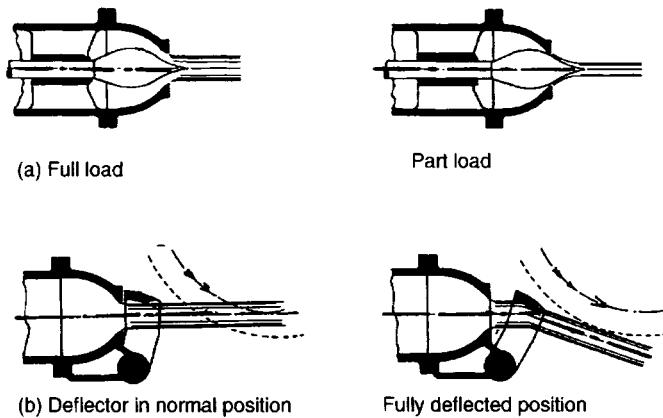


FIG. 9.8. Methods of regulating the speed of a Pelton turbine: (a) with a spear (or needle) valve; (b) with a deflector plate.

(called “water hammer”). If the spear valve did close quickly, all the kinetic energy of the water in the penstock would be absorbed by the elasticity of the supply pipeline (penstock) and the water, creating very large stresses which would reach their greatest intensity at the turbine inlet where the pipeline is already heavily stressed. The surge chamber, shown in Figure 9.7, has the function of absorbing and dissipating some of the pressure and energy fluctuations created by too rapid a closure of the needle valve.

## Sizing the penstock

It is shown in elementary textbooks on fluid mechanics, e.g. Shames (1992), Douglas *et al.* (1995), that the loss in head with incompressible, steady, turbulent flow in pipes of circular cross-section is given by Darcy's equation:

$$H_f = \frac{2flV^2}{gd} \quad (9.6)$$

where  $f$  is the friction factor,  $l$  is the length of the pipe,  $d$  is the pipe diameter and  $V$  is the mass average velocity of the flow in the pipe. It is assumed, of course, that the pipe is running full. The value of the friction factor has been determined for various conditions of flow and pipe surface roughness and the results are usually presented in what is called a "Moody diagram". This diagram gives values of  $f$  as a function of pipe Reynolds number for varying levels of relative roughness of the pipe wall.

The penstock (the pipeline bringing the water to the turbine) is long and of large diameter and this can add significantly to the total cost of a hydroelectric power scheme. Using Darcy's equation it is easy to calculate a suitable pipe diameter for such a scheme if the friction factor is known and an estimate can be made of the allowable head loss. Logically, this head loss would be determined on the basis of the cost of materials, etc. needed for a large diameter pipe and compared with the value of the useful energy lost from having too small a pipe. A commonly used compromise for the loss in head in the supply pipes is to allow  $H_f \leq 0.1H_G$ .

A summary of various factors on which the "economic diameter" of a pipe can be determined is given by Raabe (1985).

From eqn. (9.6), substituting for the velocity,  $V = 4Q/(\pi d^2)$ , we get

$$H_f = \left( \frac{32fl}{\pi^2 g} \right) \frac{Q^2}{d^5} \quad (9.7)$$

**EXAMPLE 9.1.** Water is supplied to a turbine at the rate  $Q = 2.272 \text{ m}^3/\text{s}$  by a single penstock 300 m long. The allowable head loss due to friction in the pipe amounts to 20 m. Determine the diameter of the pipe if the friction factor  $f = 0.1$ .

*Solution.* Rearranging eqn. (9.7):

$$\begin{aligned} d^5 &= \frac{32fl}{gH_f} \left( \frac{Q}{\pi} \right)^2 = \frac{32 \times 0.01 \times 300}{9.81 \times 20} \left( \frac{2.272}{\pi} \right)^2 \\ &= 0.2559 \end{aligned}$$

$$\therefore d = 0.7614 \text{ m.}$$

## Energy losses in the Pelton turbine

Having accounted for the energy loss due to friction in the penstock, the energy losses in the rest of the hydroelectric scheme must now be considered. The effective head,  $H_E$ , (or delivered head) at entry to the turbine is the gross head minus the friction head loss,  $H_f$ , i.e.

$$H_E = H_G - H_f = z_R - z_N - H_f$$

and the spouting (or ideal) velocity,  $c_0$ , is

$$c_0 = \sqrt{2gH_E}.$$

The pipeline friction loss  $H_f$  is regarded as an external loss and is not included in the losses attributed to the turbine system. The efficiency of the turbine is measured against the ideal total head  $H_E$ .

The nozzle velocity coefficient,  $K_N$ , is

$$K_N = \frac{\text{actual velocity at nozzle exit}}{\text{spouting velocity at nozzle exit}} = \frac{c_1}{c_0}.$$

Values of  $K_N$  are normally around 0.98 to 0.99.

Other energy losses occur in the nozzles and also because of windage and friction of the turbine wheel. Let the loss in head in the nozzle be  $\Delta H_N$  then the head available for conversion into power is

$$H_E - \Delta H_N = c_1^2 / (2g). \quad (9.8)$$

$$\text{nozzle efficiency, } \eta_N = \frac{\text{energy at nozzle exit}}{\text{energy at nozzle inlet}} = \frac{c_1^2}{2gH_E} \quad (9.9)$$

Equation (2.23) is an expression for the hydraulic efficiency of a turbine which, in the present notation and using eqns. (9.3) and (9.9), becomes

$$\eta_h = \frac{\Delta W}{gH_E} = \left( \frac{\Delta W}{\frac{1}{2}c_1^2} \right) \left( \frac{\frac{1}{2}c_1^2}{gH_E} \right) = \eta_R \eta_N. \quad (9.10)$$

The efficiency  $\eta_R$  only represents the effectiveness of converting the kinetic energy of the jet into the mechanical energy of the runner. Further losses occur as a result of bearing friction and “windage” losses inside the casing of the runner. In large Pelton turbines efficiencies of around 90 per cent may be achieved but, in smaller units, a much lower efficiency is usually obtained.

### The overall efficiency

In Chapter 2 the overall efficiency was defined as

$$\eta_0 = \frac{\text{mechanical energy available at output shaft in unit time}}{\text{maximum energy difference possible for the fluid in unit time}}$$

$$\eta_0 = \eta_m \eta_h = \eta_m \eta_R \eta_N$$

where  $\eta_m$  is the mechanical efficiency.

The external losses, bearing friction and windage, are chiefly responsible for the energy deficit between the runner and the shaft. An estimate of the effect of the windage loss can be made using the following simple flow model in which the specific energy loss is assumed to be proportional to the square of the blade speed, i.e.

$$\text{loss/unit mass flow} = KU^2$$

where  $K$  is a dimensionless constant of proportionality.

The overall efficiency can now be written as

$$\eta_0 = \frac{\Delta W - KU^2}{gH_E} = \eta_h - \frac{KU^2}{gH_g} = \eta_h - 2K \left( \frac{U}{c_1} \right)^2 \left( \frac{c_1^2}{2gH_E} \right)$$

$$\therefore \eta_0 = \eta_R \eta_N - 2K \eta_N v^2 = \eta_N (\eta_R - 2K v^2). \quad (9.11)$$

Hence, the mechanical efficiency is,

$$\eta_m = 1 - 2K v^2 / \eta_R. \quad (9.12)$$

It can be seen that according to eqn. (9.12), as the speed ratio is reduced towards zero, the mechanical efficiency increases and approaches unity. As there must be *some* bearing friction at all speeds, however small, an additional term is needed in the loss equation of the form  $Ac_0^2 + kU^2$ , where  $A$  is another dimensionless constant. The solution of this is left for the student to solve.

The variation of the overall efficiency based upon eqn. (9.11) is shown in Figure 9.9 for several values of  $K$ . It is seen that the peak efficiency:

- (1) is progressively reduced as the value of  $K$  is increased;
- (2) occurs at lower values of  $v$  than the optimum determined for the runner.

Thus, this evaluation of overall efficiency demonstrates the reason why experimental results obtained of the performance of Pelton turbines always yields a peak efficiency at a value of  $v < 0.5$ .

Typical performance of a Pelton turbine *under conditions of constant head and speed* is shown in Figure 9.10 in the form of the variation of overall efficiency against load ratio. As result of a change in the load the output of the turbine must then be regulated by a change in the setting of the needle valve in order to keep the turbine speed constant. The observed almost constant value of the efficiency over most of the load range is the result of the *hydraulic losses* reducing in proportion to the power output. However, as the load ratio is reduced to even lower values, the windage and bearing friction losses, which have not diminished, assume a relatively greater importance and the overall efficiency rapidly diminishes towards zero.

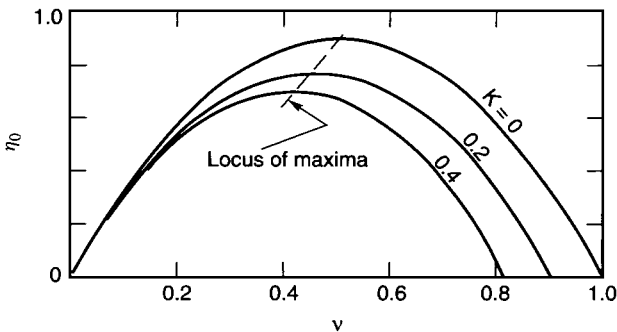


FIG. 9.9. Variation of overall efficiency of a Pelton turbine with speed ratio for several values of windage coefficient,  $K$ .

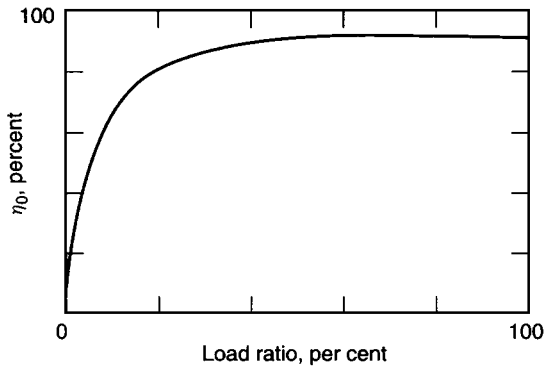


FIG. 9.10. Pelton turbine overall efficiency variation with load under constant head and constant speed conditions.

**EXAMPLE 9.2.** A Pelton turbine is driven by two jets, generating 4.0 MW at 375 rev/min. The effective head at the nozzles is 200 m of water and the nozzle velocity coefficient,  $K_N = 0.98$ . The axes of the jets are tangent to a circle 1.5 m in diameter. The relative velocity of the flow across the buckets is decreased by 15 per cent and the water is deflected through an angle of 165 deg.

Neglecting bearing and windage losses, determine:

- (1) the runner efficiency;
- (2) the diameter of each jet;
- (3) the power specific speed.

*Solution.* (1) The blade speed is:

$$U = \Omega r = (375 \times \pi/30) \times 1.5/2 \\ = 39.27 \times 1.5/2 = 29.45 \text{ m/s.}$$

The jet speed is:

$$c_1 = K_N \sqrt{2gH_e} = 0.98 \times \sqrt{(2 \times 9.81 \times 200)} = 61.39 \text{ m/s} \\ \therefore v = U/c_1 = 0.4798.$$

The efficiency of the runner is obtained from eqn. (9.4):

$$\eta_R = 2 \times 0.4798 \times (1 - 0.4798)(1 - 0.85 \times \cos 165^\circ) \\ = 0.9090.$$

(2) The “theoretical” power is  $P_{th} = P/\eta_R = 4.0/0.909 = 4.40 \text{ MW}$  where  $P_{th} = \rho g Q H_e$

$$\therefore Q = P_{th}/(\rho g H_e) = 4.4 \times 10^6 / (9810 \times 200) = 2.243 \text{ m}^3/\text{s}.$$

Each jet must have a flow area of,

$$A_j = \frac{Q}{2c_1} = 2.243 / (2 \times 61.39) = 0.01827 \text{ m}^2.$$

$$\therefore d_j = 0.1525 \text{ m.}$$

(3) Substituting into eqn. (9.1), the power specific speed is,

$$\begin{aligned}\Omega_{sp} &= 39.27 \times \left( \frac{4.0 \times 10^6}{10^3} \right)^{1/2} / (9.81 \times 200)^{5/4} \\ &= 0.190 \text{ rad.}\end{aligned}$$

## Reaction turbines

The primary features of the reaction turbine are:

- (1) only part of the overall pressure drop has occurred up to turbine entry, the remaining pressure drop takes place in the turbine itself;
- (2) the flow completely fills all of the passages in the runner, unlike the Pelton turbine where, for each jet, only one or two of the buckets at a time are in contact with the water;
- (3) pivotable guide vanes are used to control and direct the flow;
- (4) a draft tube is normally added on to the turbine exit; it is considered as an integral part of the turbine.

The pressure of the water gradually decreases as it flows through the runner and it is the reaction from this pressure change which earns this type of turbine its appellation.

## The Francis turbine

The majority of Francis turbines are arranged so that the axis is vertical (some smaller machines can have horizontal axes). Figure 9.11 illustrates a section through a vertical shaft Francis turbine with a runner diameter of 5 m, a head of 110 m and a power rating of nearly 200 MW. Water enters via a spiral casing called a *volute* or *scroll* which surrounds the runner. The area of cross-section of the volute decreases along the flow path in such a way that the flow velocity remains constant. From the volute the flow enters a ring of stationary guide vanes which direct it onto the runner at the most appropriate angle.

In flowing through the runner the angular momentum of the water is reduced and work is supplied to the turbine shaft. At the design condition the absolute flow leaves the runner axially (although a small amount of swirl may be countenanced) into the *draft tube* and, finally, the flow enters the *tail race*. It is essential that the exit of the draft tube is submerged below the level of the water in the tail race in order that the turbine remains full of water. The draft tube also acts as a diffuser; by careful design it can ensure maximum recovery of energy through the turbine by significantly reducing the exit kinetic energy.

Figure 9.12 shows a section through part of a Francis turbine together with the velocity triangles at inlet to and exit from the runner at mid-blade height. At inlet to the guide vanes the flow is in the radial/tangential plane, the absolute velocity is  $c_1$  and the absolute flow angle is  $\alpha_1$ . Thus,

$$\alpha_1 = \tan^{-1}(c_{\theta 1}/c_{r1}). \quad (9.13)$$

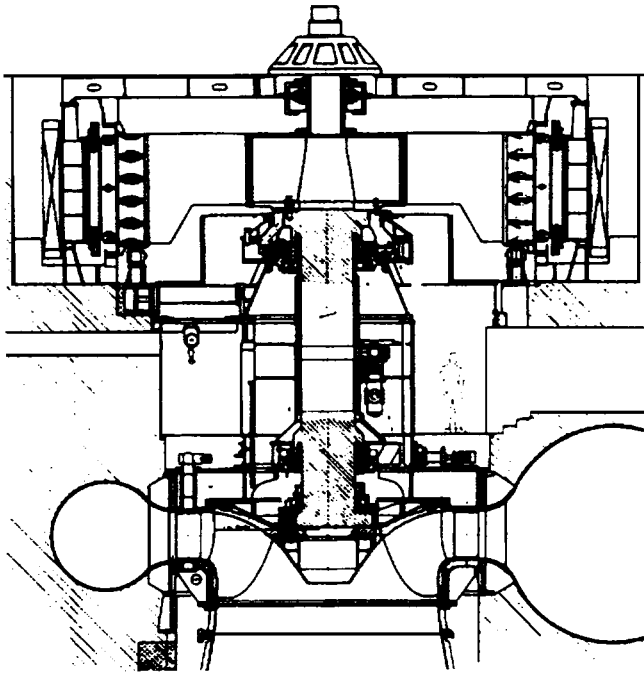


FIG. 9.11. Vertical shaft Francis turbine: runner diameter 5 m, head 110 m, power 200 MW (courtesy Sulzer Hydro Ltd, Zurich).

The flow is turned to angle  $\alpha_2$  and velocity  $c_2$ , the absolute condition of the flow at entry to the runner. By vector subtraction the relative velocity at entry to the runner is found, i.e.  $w_2 = c_2 - U_2$ . The relative flow angle  $\beta_2$  at inlet to the runner is defined as

$$\beta_2 = \tan^{-1} [(c_{\theta 2} - U_2)/c_{r2}]. \quad (9.14)$$

Further inspection of the velocity diagrams in Figure 9.12 reveals that the direction of the velocity vectors approaching both guide vanes and runner blades are tangential to the camber lines at the leading edge of each row. This is the ideal flow condition for “shockless” low loss entry, although an incidence of a few degrees may be beneficial to output without a significant extra loss penalty. At vane outlet some deviation from the blade outlet angle is to be expected (see Chapter 3). For these reasons, in all problems concerning the direction of flow, it is clear that it is the angle of the fluid flow which is important and not the vane angle as is often quoted in other texts.

At outlet from the runner the flow plane is simplified as though it was actually in the radial/tangential plane. This simplification will not affect the subsequent analysis of the flow but it must be conceded that some component of velocity in the axial direction does exist at runner outlet.

The water leaves the runner with a relative flow angle  $\beta_3$  and a relative flow velocity  $w_3$ . The absolute velocity at runner exit is found by vector addition, i.e.

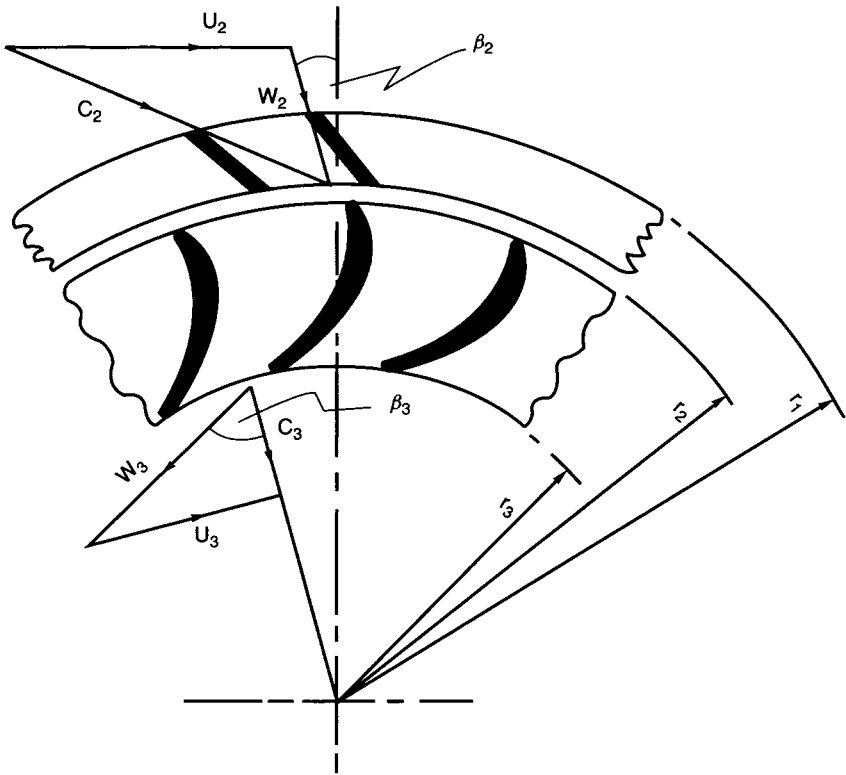


FIG. 9.12. Sectional sketch of blading for a Francis turbine showing velocity diagrams at runner inlet and exit.

$c_3 = w_3 + U_3$ . The relative flow angle,  $\beta_3$ , at runner exit is given by

$$\beta_3 = \tan^{-1} [(c_{\theta 3} + U_3)/c_{r3}]. \quad (9.15)$$

In this equation it is assumed that some residual swirl velocity  $c_{\theta 3}$  is present ( $c_{r3}$  is the radial velocity at exit from the runner). In most simple analyses of the Francis turbine it is assumed that there is no exit swirl. Detailed investigations have shown that some extra *counter-swirl* (i.e. acting so as to increase  $\Delta c_\theta$ ) at the runner exit does increase the amount of work done by the fluid without a significant reduction in turbine efficiency.

When a Francis turbine is required to operate at part load, the power output is reduced by swivelling the guide vanes to restrict the flow, i.e.  $Q$  is reduced, while the blade speed is maintained constant. Figure. 9.13 compares the velocity triangles at full load and at part load from which it will be seen that the relative flow at runner entry is at a high incidence and at runner exit the absolute flow has a large component of swirl. Both of these flow conditions give rise to high head losses. Figure. 9.14 shows the variation of hydraulic efficiency for several types of turbine, including the Francis turbine, over the full load range at constant speed and constant head.

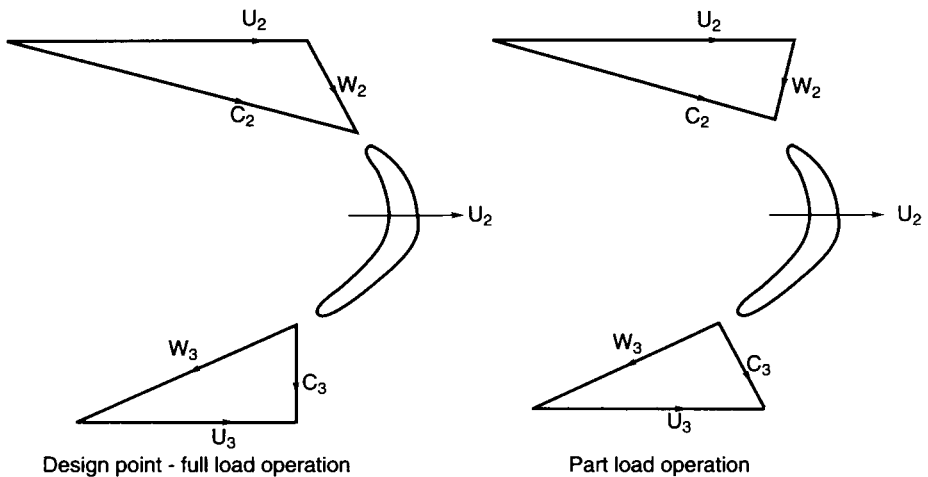


FIG. 9.13. Comparison of velocity triangles for a Francis turbine for full load and at part load operation.

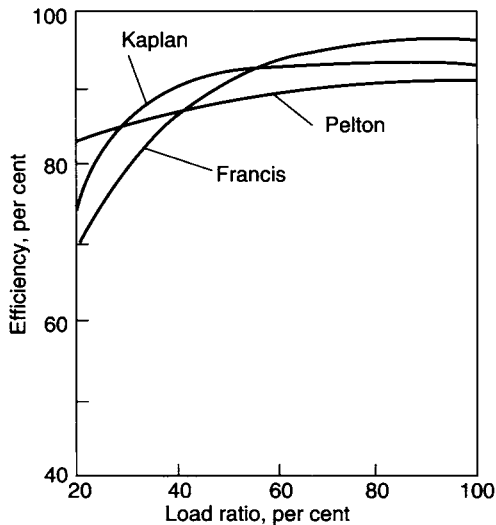


FIG. 9.14. Variation of hydraulic efficiency for various types of turbine over a range of loading, at constant speed and constant head.

It is of interest to note the effect that swirling flow has on the performance of the following diffuser. The results of an extensive experimental investigation made by McDonald *et al.* (1971), showed that swirling inlet flow *does not* affect the performance of conical diffusers which are well designed and give unseparated or only slightly separated flow when the flow through them is entirely axial. Accordingly, part load operation of the turbine is unlikely to give adverse diffuser performance.

## Basic equations

Euler's turbine equation, eqn. (2.12b), in the present notation, is written as

$$\Delta W = U_2 c_{\theta 2} - U_3 c_{\theta 3}. \quad (9.16)$$

If the flow at runner exit is without swirl then the equation reduces to

$$\Delta W = U_2 c_{\theta 2}. \quad (9.16a)$$

The effective head for all reaction turbines,  $H_E$ , is the total head available at the turbine inlet *relative to the surface of the tailrace*. At entry to the runner the energy available is equal to the sum of the kinetic, potential and pressure energies, i.e.

$$g(H_E - \Delta H_N) = \frac{p_2 - p_a}{\rho} + \frac{1}{2}c_2^2 + gz_2, \quad (9.17)$$

where  $\Delta H_N$  is the loss of head due to friction in the volute and guide vanes and  $p_2$  is the *absolute* static pressure at inlet to the runner.

At runner outlet the energy in the water is further reduced by the amount of specific work  $\Delta W$  and by friction work in the runner,  $g\Delta H_R$  and this remaining energy equals the sum of the pressure potential and kinetic energies, i.e.

$$g(H_E - \Delta H_N - \Delta H_R) - \Delta W = \frac{1}{2}c_3^2 + p_3/\rho - p_a/\rho + gz_3 \quad (9.18)$$

where  $p_3$  is the *absolute* static pressure at runner exit.

By differencing eqns. (9.17) and (9.18), the specific work is obtained

$$\Delta W = (p_{02} - p_{03})/\rho - g\Delta H_R + g(z_2 - z_3) \quad (9.19)$$

where  $p_{02}$  and  $p_{03}$  are the absolute total pressures at runner inlet and exit.

Figure 9.15 shows the draft tube in relation to a vertical-shaft Francis turbine. The most important dimension in this diagram is the vertical distance ( $z = z_3$ ) between the

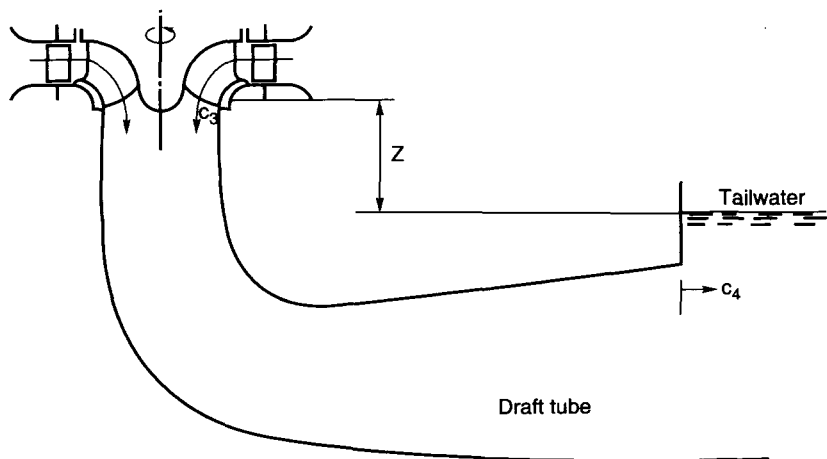


FIG. 9.15. Location of draft tube in relation to vertical shaft Francis turbine.

exit plane of the runner and the free surface of the tailrace. The energy equation between the exit of the runner and the tailrace can now be written as

$$p_3/\rho + \frac{1}{2}c_3^2 + gz - g\Delta H_{DT} = \frac{1}{2}c_4^2 + p_a/\rho, \quad (9.20)$$

where  $\Delta H_{DT}$  is the loss in head in the draft tube and  $c_4$  is the exit velocity.

The hydraulic efficiency is given by

$$\eta_H = \frac{\Delta W}{gH_E} = \frac{U_2c_{\theta 2} - U_3c_{\theta 3}}{gH_E} \quad (9.21)$$

and, if  $c_{\theta 3} = 0$ , then

$$\eta_H = \frac{U_2c_{\theta 2}}{gH_E}. \quad (9.21a)$$

The overall efficiency is given by  $\eta_0 = \eta_m\eta_H$ . For large machines the mechanical losses are relatively small and  $\eta_m \approx 100$  per cent and so  $\eta_0 \approx \eta_H$ .

For the Francis turbine the ratio of the runner speed to the spouting velocity,  $v = U/c_0$ , is not as critical for high efficiency operation as it is for the Pelton turbine and, in practice, it lies within a fairly wide range, i.e.  $0.6 \leq v \leq 0.9$ . In most applications of Francis turbines the turbine drives an alternator and its speed must be maintained constant. The regulation at part load operation is achieved by varying the angle of the guide vanes. The guide vanes are pivoted and, by means of a gearing mechanism, the setting can be adjusted to the optimum angle. However, operation at part load causes a whirl velocity component to be set up downstream of the runner causing a reduction in efficiency. The strength of the vortex can be such that cavitation can occur along the axis of the draft tube (see remarks on cavitation later in this chapter).

**EXAMPLE 9.3.** In a vertical-shaft Francis turbine the available head at the inlet flange of the turbine is 150 m and the vertical distance between the runner and the tailrace is 2.0 m. The runner tip speed is 35 m/s, the meridional velocity of the water through the runner is constant and equal to 10.5 m/s, the flow leaves the runner without whirl and the velocity at exit from the draft tube is 3.5 m/s. The hydraulic energy losses estimated for the turbine are as follows:

$$\Delta H_N = 6.0 \text{ m}, \Delta H_R = 10.0 \text{ m}, \Delta H_{DT} = 1.0 \text{ m}.$$

Determine:

- (1) the pressure head (relative to the tailrace) at inlet to and at exit from the runner;
- (2) the flow angles at runner inlet and at guide vane exit;
- (3) the hydraulic efficiency of the turbine.

If the flow discharged by the turbine is  $20 \text{ m}^3/\text{s}$  and the power specific speed of the turbine is 0.8 (rad), determine the speed of rotation and the diameter of the runner.

**Solution.** From eqn. (9.20)

$$\frac{p_3 - p_a}{\rho g} = H_3 = \frac{1}{2g}(c_4^2 - c_3^2) + \Delta H_{DT} - z.$$

NB. The head  $H_3$  is relative to the tailrace.

$$\therefore H_3 = \frac{(3.5^2 - 10.5^2)}{2 \times 9.81} + 1 - 2 = -6.0 \text{ m},$$

i.e. the pressure at runner outlet is *below* atmospheric pressure, a matter of some importance when we come to consider the subject of cavitation later in this chapter. From eqn. (9.18),

$$H_2 = H_E - \Delta H_N - c_2^2/(2g) = 150 - 6 - 38.73^2/(2 \times 9.81) = 67.22 \text{ m}.$$

From eqn. (9.18),

$$\begin{aligned} \Delta W &= g(H_E - \Delta H_N - \Delta H_{R-z}) - \frac{1}{2}c_3^2 - gH_3 \\ &= 9.81 \times (150 - 6 - 10 - 2) - 10.5^2/2 + 9.81 \times 6 = 1298.7 \text{ m}^2/\text{s}^2 \end{aligned}$$

$$\therefore c_{\theta 2} = \Delta W/U_2 = 1298.7/35 = 37.1 \text{ m/s}$$

$$\alpha_2 = \tan^{-1} \left( \frac{c_{\theta 2}}{c_{r2}} \right) = \tan^{-1} \left( \frac{37.1}{10.5} \right) = 74.2 \text{ deg}$$

$$\beta_2 = \tan^{-1} \left( \frac{c_{\theta 2}}{c_{r2}} \right) = \tan^{-1} \left( \frac{37.1 - 35}{10.5} \right) = 11.31 \text{ deg}.$$

The hydraulic efficiency is

$$\eta_H = \frac{\Delta W}{gH_E} = 1298.7/(9.81 \times 150) = 0.8826.$$

From the definition of the power specific speed, eqn. (9.1),

$$\Omega = \frac{\Omega_{SP}(gH_E)^{5/4}}{\sqrt{Q\Delta W}} = \frac{0.8 \times 9114}{\sqrt{20 \times 1298.7}} = 45.24 \text{ rad/s}.$$

Thus, the rotational speed is  $N = 432 \text{ rev/min}$  and the runner diameter is

$$D_2 = 2U_2/\Omega = 70/45.24 = 1.547 \text{ m}.$$

## The Kaplan turbine

This type of turbine evolved from the need to generate power from much lower pressure heads than are normally employed with the Francis turbine. To satisfy large power demands very large volume flow rates need to be accommodated in the Kaplan turbine, i.e. the product  $QH_E$  is large. The overall flow configuration is from radial to axial. Figure 9.16 is a part sectional view of a Kaplan turbine in which the flow enters from a volute into the inlet guide vanes which impart a degree of swirl to the flow determined by the needs of the runner. The flow leaving the guide vanes is forced by the shape of the passage into an axial direction and the swirl becomes essentially a free vortex, i.e.

$$rc_\theta = \text{a constant}.$$

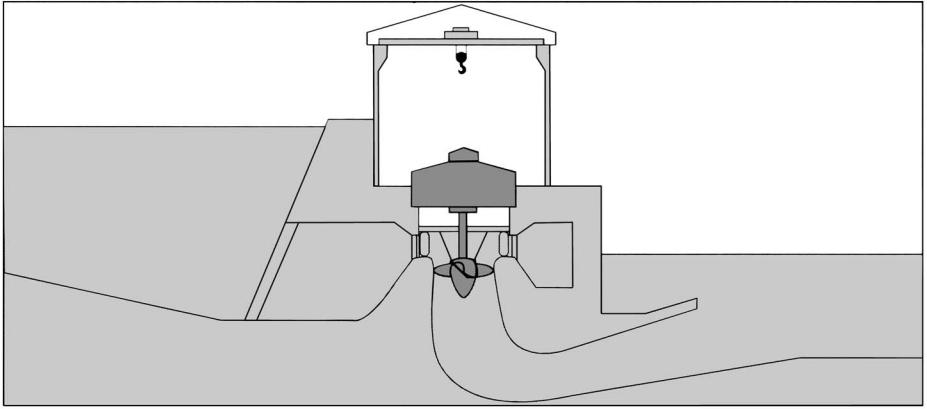


FIG. 9.16. Part section of a Kaplan turbine *in situ* (courtesy Sulzer Hydro Ltd, Zurich).

The vanes of the runner are similar to those of an axial-flow turbine rotor but designed with a twist suitable for the free-vortex flow at entry and an axial flow at outlet. Because of the very high torque that must be transmitted and the large length of the blades, strength considerations impose the need for large blade chords. As a result, pitch/chord ratios of 1.0 to 1.5 are commonly used by manufacturers and, consequently, the number of blades is small, usually 4, 5 or 6. The Kaplan turbine incorporates one essential feature not found in other turbine rotors and that is the setting of the stagger angle can be controlled. At part load operation the setting angle of the runner vanes is adjusted automatically by a servo mechanism to maintain optimum efficiency conditions. This adjustment requires a complementary adjustment of the inlet guide vane stagger angle in order to maintain an absolute axial flow at exit from the runner.

### Basic equations

Most of the equations presented for the Francis turbine also apply to the Kaplan (or propeller) turbine, apart from the treatment of the runner. Figure 9.17 shows the velocity triangles and part section of a Kaplan turbine drawn for the mid-blade height. At exit from the runner the flow is shown leaving the runner without a whirl velocity, i.e.  $c_{\theta 3} = 0$  and constant axial velocity. The theory of free-vortex flows was expounded in Chapter 6 and the main results as they apply to an incompressible fluid are given here. The runner blades will have a fairly high degree of twist, the amount depending upon the strength of the circulation function  $K$  and the magnitude of the axial velocity. Just upstream of the runner the flow is assumed to be a free-vortex and the velocity components are accordingly:

$$c_{\theta 2} = K/r \quad c_x = \text{a constant.}$$

The relations for the flow angles are

$$\tan \beta_2 = U/c_x - \tan \alpha_2 = \Omega r/c_x - K/(rc_x) \quad (9.22a)$$

$$\tan \beta_3 = U/c_x = \Omega r/c_x. \quad (9.22b)$$

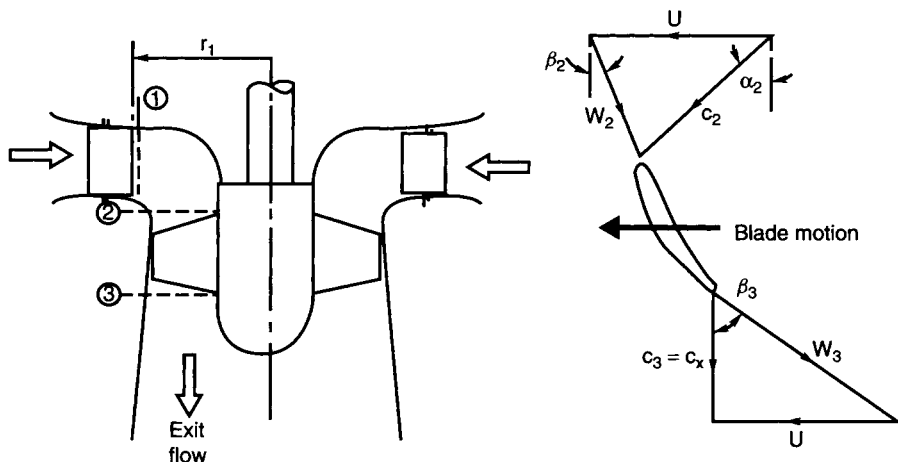


FIG. 9.17. Section of a Kaplan turbine and velocity diagrams at inlet to and exit from the runner.

**EXAMPLE 9.4.** A small-scale Kaplan turbine has a power output of 8 MW, an available head at turbine entry of 13.4 m and a rotational speed of 200 rev/min. The inlet guide vanes have a length of 1.6 m and the diameter at the trailing edge surface is 3.1 m. The runner diameter is 2.9 m and the hub to tip ratio is 0.4.

Assuming the hydraulic efficiency is 92 per cent and the runner design is “free-vortex”, determine:

- (1) the radial and tangential components of velocity at exit from the guide vanes;
- (2) the component of axial velocity at the runner;
- (3) the absolute and relative flow angles upstream and downstream of the runner at the hub, mid-radius and tip.

**Solution.** As  $P = \eta_H \rho g Q H_E$ , then the volume flow rate is

$$Q = P / (\eta_H \rho g H_E) = 8 \times 10^6 / (0.92 \times 9810 \times 13.4) = 66.15 \text{ m}^3/\text{s}$$

$$\therefore c_{r1} = Q / (2\pi r_1 L) = 66.15 / (2\pi \times 1.55 \times 1.6) = 4.245 \text{ m/s}$$

$$c_{x2} = \frac{4Q}{\pi D_2^2 (1 - v^2)} = 4 \times 66.15 / (\pi \times 2.9^2 \times 0.84) = 11.922 \text{ m/s}.$$

As the specific work done is  $\Delta W = U_2 c_{\theta 2}$  and  $\eta_H = \Delta W / (g H_E)$ , then at the tip

$$c_{\theta 2} = \frac{\eta_H g H_E}{U_2} = \frac{0.92 \times 9.81 \times 13.4}{30.37} = 3.892 \text{ m/s},$$

where the blade tip speed is,  $U_2 = \Omega D_2 / 2 = (200 \times \pi / 30) \times 2.9 / 2 = 30.37 \text{ m/s}$

$$c_{\theta 1} = c_{\theta 2} r_2 / r_1 = 3.892 \times 1.45 / 1.55 = 3.725 \text{ m/s}^2$$

$$\alpha_1 = \tan^{-1} \left( \frac{c_{\theta 1}}{c_{r1}} \right) = \tan^{-1} \left( \frac{3.725}{4.245} \right) = 41.26^\circ.$$

TABLE 9.4. Calculated values of flow angles for Example 9.4.

| Parameter          | Ratio $r/r_t$ |        |       |
|--------------------|---------------|--------|-------|
|                    | 0.4           | 0.7    | 1.0   |
| $c_{\theta 2}$ m/s | 9.955         | 5.687  | 3.982 |
| $\tan \alpha_2$    | 0.835         | 0.4772 | 0.334 |
| $\alpha_2$ (deg)   | 39.86         | 25.51  | 18.47 |
| $U/c_{x2}$         | 1.019         | 1.7832 | 2.547 |
| $\beta_2$ (deg)    | 10.43         | 52.56  | 65.69 |
| $\beta_3$ (deg)    | 45.54         | 60.72  | 68.57 |

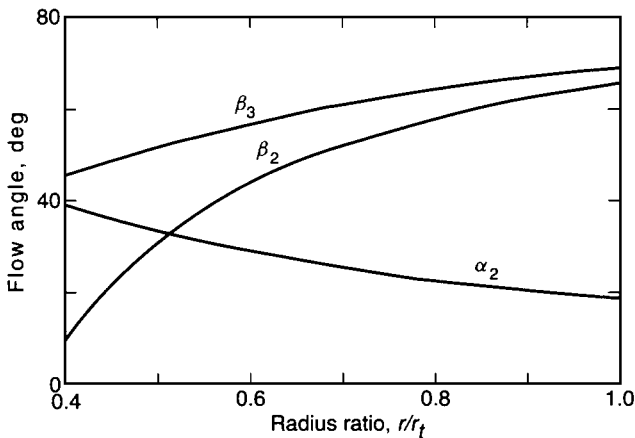


FIG. 9.18. Calculated variation of flow angles for Kaplan turbine of Example 9.4.

Values  $\alpha_2$ ,  $\beta_2$  and  $\beta_3$  shown in Table 9.4 have been derived from the following relations:

$$\alpha_2 = \tan^{-1} \left( \frac{c_{\theta 2}}{c_{x2}} \right) = \tan^{-1} \left( \frac{c_{\theta 2t} r_t}{c_{x2} r} \right)$$

$$\beta_2 = \tan^{-1} \left( \frac{\Omega r}{c_{x2}} - \tan \alpha_2 \right) = \tan^{-1} \left( \frac{U_{2t} r}{c_{x2} r_t} - \tan \alpha_2 \right)$$

$$\beta_3 = \tan^{-1} \left( \frac{U}{c_{x2}} \right) = \tan^{-1} \left( \frac{U_{2t} r}{c_{x2} r_t} \right).$$

Finally, Figure 9.18 illustrates the variation of the flow angles, from which the large amount of blade twist mentioned earlier can be inferred.

## Effect of size on turbomachine efficiency

Despite careful attention to detail at the design stage and during manufacture it is a fact that small turbomachines always have lower efficiencies than larger

geometrically similar machines. The primary reason for this is that it is not possible to establish perfect dynamical similarity between turbomachines of different size. In order to obtain this condition, each of the dimensionless terms in eqn. (1.2) would need to be the same for all sizes of a machine.

To illustrate this consider a family of turbomachines where the loading term,  $\psi = gH/N^2D^2$  is the same and the Reynolds number,  $Re = ND^2/\nu$  is the same for every size of machine, then

$$\psi Re^2 = \frac{gH}{N^2D^2} \cdot \frac{N^2D^4}{\nu^2} = \frac{gHD^2}{\nu^2}$$

must be the same for the whole family. Thus, for a given fluid ( $\nu$  is a constant), a reduction in size  $D$  must be followed by an increase in the head  $H$ . A turbine model of  $\frac{1}{8}$  the size of a prototype would need to be tested with a head 64 times that required by the prototype! Fortunately, the effect on the model efficiency caused by changing the Reynolds number is not large. In practice, models are normally tested at conveniently low heads and an empirical correction is applied to the efficiency.

With model testing other factors effect the results. Exact geometric similarity cannot be achieved for the following reasons:

- (a) the blades in the model will probably be relatively thicker than in the prototype;
- (b) the relative surface roughness for the model blades will be greater;
- (c) leakage losses around the blade tips of the model will be relatively greater as a result of increased relative tip clearances.

Various simple corrections have been devised (see Addison 1964) to allow for the effects of size (or scale) on the efficiency. One of the simplest and best known is that due to Moody and Zowski (1969), also reported by Addison (1964) and Massey (1979), which as applied to the efficiency of reaction turbines is

$$\frac{1 - \eta_p}{1 - \eta_m} = \left( \frac{D_m}{D_p} \right)^n \quad (9.23)$$

where the subscripts  $p$ ,  $m$  refer to prototype and model, and the index  $n$  is in the range 0.2 to 0.25. From comparison of field tests of large units with model tests, Moody and Zowski concluded that the best value for  $n$  was approximately 0.2 rather than 0.25 and for general application this is the value used. However, Addison (1964) reported tests done on a full-scale Francis turbine and a model made to a scale of 1 to 4.54 which gave measured values of the maximum efficiencies of 0.85 and 0.90 for the model and full-scale turbines, respectively, which agreed very well with the ratio computed with  $n = 0.25$  in the Moody formula!

**EXAMPLE 9.5.** A model of a Francis turbine is built to a scale of 1/5 of full size and when tested it developed a power output of 3 kW under a head of 1.8 m of water, at a rotational speed of 360 rev/min and a flow rate of 0.215 m<sup>3</sup>/s. Estimate the speed, flow rate and power of the full-scale turbine when working under dynamically similar conditions with a head of 60 m of water.

By making a suitable correction for scale effects, determine the efficiency and the power of the full-size turbine. Use Moody's formula and assume  $n = 0.25$ .

*Solution.* From the group  $\psi = gH/(ND)^2$  we get:

$$N_p = N_m(D_m/D_p)(H_p/H_m)^{0.5} = (360/5)(60/1.8)^{0.5} = 415.7 \text{ rev/min.}$$

From the group  $\phi = Q/(ND^3)$  we get:

$$Q_p = Q_m(N_p/N_m)(D_p/D_m)^3 = 0.215 \times (360/415.7) \times 5^3 = 23.27 \text{ m}^3/\text{s.}$$

Lastly, from the group  $\hat{P} = P/(\rho N^3 D^5)$  we get:

$$P_p = P_m(N_p/N_m)^3(D_p/D_m)^5 = 3 \times (415.7)^3 \times 5^5 = 14\,430 \text{ kW} = 14.43 \text{ MW.}$$

This result has still to be corrected to allow for scale effects. First we must calculate the efficiency of the model turbine. The efficiency is found from

$$\eta_m = P/(\rho Q g H) = 3 \times 10^3 / (10^3 \times 0.215 \times 9.81 \times 1.8) = 0.79.$$

Using Moody's formula the efficiency of the prototype is determined:

$$(1 - \eta_p) = (1 - \eta_m) \times 0.2^{0.25} = 0.21 \times 0.6687$$

hence

$$\eta_p = 0.8596.$$

The corresponding power is found by an adjustment of the original power obtained under dynamically similar conditions, i.e.

$$\text{Corrected } P_p = 14.43 \times 0.8596/0.79 = 15.7 \text{ MW.}$$

## Cavitation

A description of the phenomenon of cavitation, mainly with regard to pumps, was given in Chapter 1. In hydraulic turbines, where reliability, long life and efficiency are all so very important, the effects of cavitation must be considered. Two types of cavitation may be in evidence,

- (a) on the suction surfaces of the runner blades at outlet which can cause severe blade erosion; and
- (b) a twisting "rope-type" cavity that appears in the draft tube at off-design operating conditions.

Cavitation in hydraulic turbines can occur on the suction surfaces of the runner blades where the dynamic action of the blades acting on the fluid creates low pressure zones in a region where the static pressure is already low. Cavitation will commence when the local static pressure is less than the vapour pressure of the water, i.e. where the head is low, the velocity is high and the elevation,  $z$ , of the turbine is set too high above the tailrace. For a turbine with a horizontal shaft the lowest pressure will be located in the upper part of the runner, which could be of major significance in large machines. Fortunately, the runners of large machines are,

in general, made so that their shafts are orientated vertically, lessening the problem of cavitation occurrence.

The cavitation performance of hydraulic turbines can be correlated with the Thoma coefficient,  $\sigma$ , defined as

$$\sigma = \frac{H_S}{H_E} = \frac{(p_a - p_v)/(\rho g) - z}{H_E}, \quad (9.24)$$

where  $H_S$  is the net positive suction head (NPSH), the amount of head needed to avoid cavitation, the difference in elevation,  $z$ , is defined in Figure 9.15 and  $p_v$  is the vapour pressure of the water. The Thoma coefficient was, strictly, originally defined in connection with cavitation in turbines and its use in pumps is not appropriate (see Yedidiah 1981). It is to be shown that  $\sigma$  represents the fraction of the available head  $H_E$  which is unavailable for the production of work. A large value of  $\sigma$  means that a smaller part of the available head can be utilised. In a pump, incidentally, there is no direct connection between the developed head and its suction capabilities, provided that cavitation does not occur, which is why the use of the Thoma coefficient is not appropriate for pumps.

From the energy equation, eqn. (9.20), this can be rewritten as

$$\frac{p_a - p_3}{\rho g} - z = \frac{1}{2g}(c_3^2 - c_4^2) - \Delta H_{DT}, \quad (9.25)$$

so that when  $p_3 = p_v$ , then  $H_S$  is equal to the rhs of eqn. (9.24).

Figure 9.19 shows a widely used correlation of the Thoma coefficient plotted against specific speed for Francis and Kaplan turbines, approximately defining the boundary between no cavitation and severe cavitation. In fact, there exists a wide range of critical values of  $\sigma$  for each value of specific speed and type of turbine due to the individual cavitation characteristics of the various runner designs. The curves drawn are meant to assist preliminary selection procedures. An alternative method for avoiding cavitation is to perform tests on a model of a particular turbine in which the value of  $p_3$  is reduced until cavitation occurs or, a marked decrease in efficiency becomes apparent. This performance reduction would correspond to the production of large-scale cavitation bubbles. The pressure at which cavitation erosion occurs will actually be at some higher value than that at which the performance reduction starts.

For the centre-line cavitation that appears downstream of the runner at off-design operating conditions, oscillations of the cavity can cause severe vibration of the draft tube. Young reported some results of a ‘‘corkscrew’’ cavity rotating at 4 Hz. Air injected into the flow both stabilizes the flow and cushions the vibration.

**EXAMPLE 9.6.** Using the data in Example 9.3 and given that the atmospheric pressure is 1.013 bar and the water is at 25°C, determine the NPSH for the turbine. Hence, using Thoma’s coefficient and the data shown in Figure 9.19, determine whether cavitation is likely to occur. Also using the data of Wislicenus verify the result.

**Solution.** From tables of fluid properties, e.g. Rogers and Mayhew (1995), or using the data of Figure 9.20, the vapour pressure for water corresponding to a

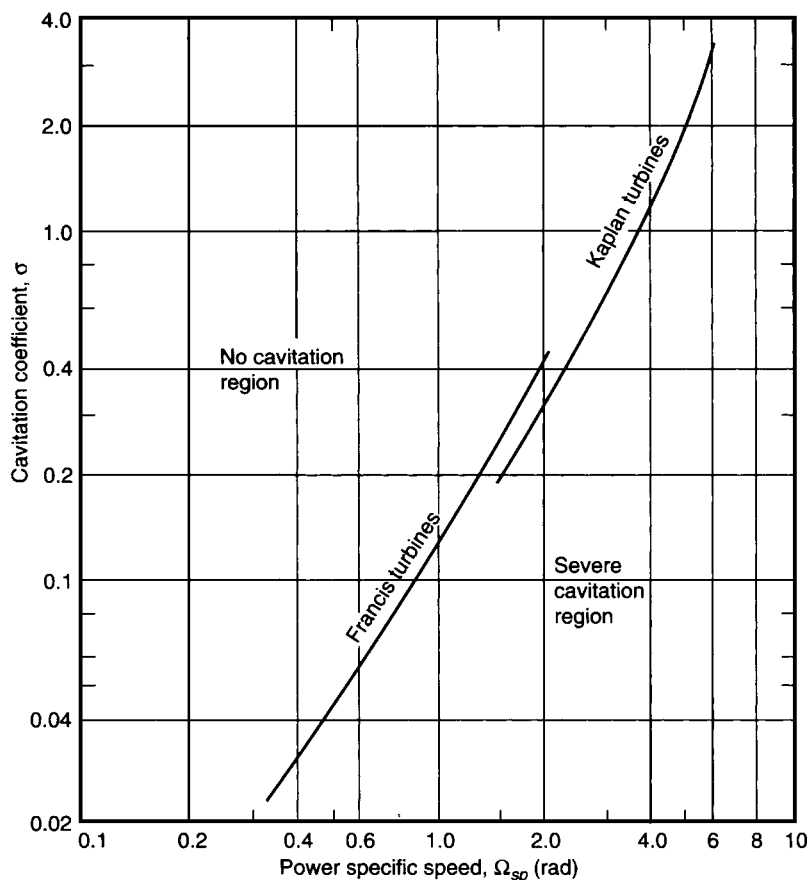


FIG. 9.19. Variation of critical cavitation coefficient with non-dimensional specific speed for Francis and Kaplan turbines (adapted from Moody and Zowski 1969).

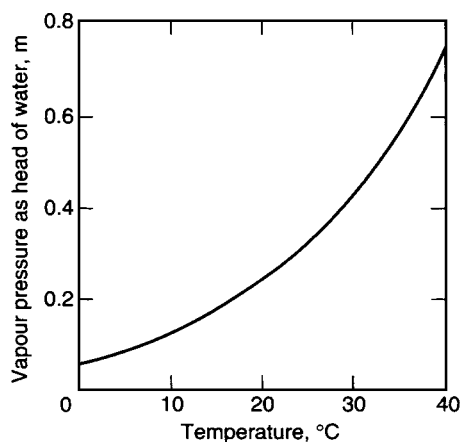


FIG. 9.20. Vapour pressure of water as head (m) versus temperature.

temperature of 25°C is 0.03166 bar. From the definition of NPSH, eqn. (9.24), we obtain:

$$H_s = \frac{p_a - p_v}{\rho g} - z = (1.013 - 0.03166) \times 10^5 / (9810) - 2 = 8.003 \text{ m.}$$

Thus, Thoma's coefficient is,  $\sigma = H_s/H_E = 8.003/150 = 0.05336$ .

At the value of  $\Omega_{SP} = 0.8$  given as data, the value of the critical Thoma coefficient  $\sigma_c$  corresponding to this is 0.09 from Figure 9.19. From the fact that  $\sigma < \sigma_c$ , then the turbine *will* cavitate.

From the definition of the suction specific speed

$$\Omega_{SS} = \frac{\Omega Q^{1/2}}{(gH_s)^{3/4}} = \frac{44.9 \times 20^{1/2}}{(9.81 \times 8.003)^{3/4}} = 200.8/26.375 = 7.613.$$

According to eqn. (1.12b), when  $\Omega_{SS}$  exceeds 4.0(rad) then cavitation can occur, giving further confirmation of the above conclusion.

### Connection between Thoma's coefficient, suction specific speed and specific speed

The definitions of suction specific speed and specific speed are

$$\Omega_{SS} = \frac{\Omega Q^{1/2}}{(gH_s)^{3/4}} \text{ and } \Omega_s = \frac{\Omega Q^{1/2}}{(gH_E)^{3/4}}$$

Combining and using eqn. (9.24), we get:

$$\begin{aligned} \frac{\Omega_s}{\Omega_{SS}} &= \left( \frac{gH_s}{gH_E} \right)^{3/4} = \sigma^{3/4} \\ \therefore \sigma &= \left( \frac{\Omega_s}{\Omega_{SS}} \right)^{4/3}. \end{aligned} \quad (9.26)$$

*Exercise.* Verify the value of Thoma's coefficient in the earlier example using the values of power specific speed, efficiency and suction specific speed given or derived.

We use as data  $\Omega_{SS} = 7.613$ ,  $\Omega_{SP} = 0.8$  and  $\eta_H = 0.896$  so that, from eqn. (1.9c),

$$\Omega_s = \Omega_{SP} / \sqrt{\eta_H} = 0.8 / \sqrt{0.896} = 0.8452$$

$$\therefore \sigma = (0.8452/7.613)^{4/3} = 0.05336.$$

### Avoiding cavitation

By rearranging eqn. (9.24) and putting  $\sigma = \sigma_c$ , a critical value of  $z$  can be derived on the boundary curve between cavitation and no cavitation. Thus,

$$z = z_c = \frac{p_a - p_v}{\rho g} - \sigma_c H_E = (101.3 - 3.17)/9.81 - 0.09 \times 150 = -3.5 \text{ m.}$$

This means that the turbine would need to be submerged at a depth of 3.5 m or more below the surface of the tailwater and, for a Francis turbine, would lead to

problems with regard to construction and maintenance. Equation (9.24) shows that the greater the available head  $H_E$  at which a turbine operates, the lower it must be located relative to the surface of the tailrace.

### Sonoluminescence

The collapse of vapour cavities generates both noise and flashes of light (called *sonoluminescence*). Young (1989) has given an extended and interesting review of experiments on sonoluminescence from hydrodynamic cavitation and its causes. The phenomenon has also been reported by Pearsall (1974) who considered that the collapse of the cavity was so rapid that very high pressures and temperatures were created. Temperatures as high as 10 000 K have been suggested. Shock waves with pressure differences of 4000 atm have been demonstrated in the liquid following the collapse of a cavity. The effect of the thermal and pressure shocks on any material in close proximity causes mechanical failure, i.e. erosion damage.

Light has been reported in large energy distributions in field installations. An example again quoted by Young is that of the easily visible light observed at night in the tailrace at Boulder Dam, USA. This occurs when sudden changes of load necessitate the release of large quantities of high-pressure water into an energy-dissipating structure. Under these conditions the water cavitates severely. In a further example, Young mentions the light (observed at night) from the tailrace of the hydroelectric power station at Erochty, Scotland. The luminescence appeared for up to ten seconds shortly after the relief valve was opened and was seen as a blue shimmering light stretching over an area of the water surface for several square metres.

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## Problems

1. A generator is driven by a small, single-jet Pelton turbine designed to have a power specific speed  $\Omega_{SP} = 0.20$ . The effective head at nozzle inlet is 120 m and the nozzle velocity coefficient is 0.985. The runner rotates at 880 rev/min, the turbine overall efficiency is 88 per cent and the mechanical efficiency is 96 per cent.

If the blade speed to jet speed ratio,  $v = 0.47$ , determine:

- (1) the shaft power output of the turbine;
- (2) the volume flow rate;
- (3) the ratio of the wheel diameter to jet diameter.

2. (a) Water is to be supplied to the Pelton wheel of a hydroelectric power plant by a pipe of uniform diameter, 400 m long, from a reservoir whose surface is 200 m vertically above the nozzles. The required volume flow of water to the Pelton wheel is  $30 \text{ m}^3/\text{s}$ . If the pipe skin friction loss is not to exceed 10% of the available head and  $f = 0.0075$ , determine the minimum pipe diameter.

(b) You are required to select a suitable pipe diameter from the available range of stock sizes to satisfy the criteria given. The range of diameters (m) available are: 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8. For the diameter you have selected, determine:

- (1) the friction head loss in the pipe;
- (2) the nozzle exit velocity assuming no friction losses occur in the nozzle and the water leaves the nozzle at atmospheric pressure;
- (3) the total power developed by the turbine assuming that its efficiency is 75% based upon the energy available at turbine inlet.

3. A multi-jet Pelton turbine with a wheel 1.47 m diameter, operates under an effective head of 200 m at nozzle inlet and uses  $4 \text{ m}^3/\text{s}$  of water. Tests have proved that the wheel efficiency is 88 per cent and the velocity coefficient of each nozzle is 0.99.

Assuming that the turbine operates at a blade speed to jet speed ratio of 0.47, determine:

- (1) the wheel rotational speed;
- (2) the power output and the power specific speed;
- (3) the bucket friction coefficient given that the relative flow is deflected  $165^\circ$ ;
- (4) the required number of nozzles if the ratio of the jet diameter to mean diameter of the wheel is limited to a maximum value of 0.113.

4. A four-jet Pelton turbine is supplied by a reservoir whose surface is at an elevation of 500 m above the nozzles of the turbine. The water flows through a single pipe 600 m long, 0.75 m diameter, with a friction coefficient  $f = 0.0075$ . Each nozzle provides a jet 75 mm diameter and the nozzle velocity coefficient  $K_N = 0.98$ . The jets impinge on the buckets of the wheel at a radius of 0.65 m and are deflected (relative to the wheel) through an angle of  $160^\circ$ . Fluid friction within the buckets reduces the relative velocity by 15 per cent. The blade speed to jet speed ratio,  $v = 0.48$  and the mechanical efficiency of the turbine is 98 per cent.

Calculate, using an iterative process, the loss of head in the pipeline and hence, determine for the turbine:

- (1) the speed of rotation;
- (2) the overall efficiency (based on the effective head);
- (3) the power output;
- (4) the percentage of the energy available at turbine inlet which is lost as kinetic energy at turbine exit.

5. A Francis turbine operates at its maximum efficiency point at  $\eta_0 = 0.94$ , corresponding to a power specific speed of 0.9 rad. The effective head across the turbine is 160 m and the speed required for electrical generation is 750 rev/min. The runner tip speed is 0.7 times the spouting velocity, the absolute flow angle at runner entry is 72 deg from the radial direction and the absolute flow at runner exit is without swirl.

Assuming there are no losses in the guide vanes and the mechanical efficiency is 100 per cent, determine:

- (1) the turbine power and the volume flow rate;
- (2) the runner diameter;
- (3) the magnitude of the tangential component of the absolute velocity at runner inlet;
- (4) the axial length of the runner vanes at inlet.

6. The power specific speed of a 4 MW Francis turbine is 0.8, and the hydraulic efficiency can be assumed to be 90 per cent. The head of water supplied to the turbine is 100 m. The runner vanes are radial at inlet and their internal diameter is three-quarters of the external diameter. The meridional velocities at runner inlet and outlet are equal to 25 and 30 per cent, respectively, of the spouting velocity.

Determine:

- (1) the rotational speed and diameter of the runner;
- (2) the flow angles at outlet from the guide vanes and at runner exit;
- (3) the widths of the runner at inlet and at exit.

Blade thickness effects can be neglected.

7. (a) Review, briefly, the phenomenon of cavitation in hydraulic turbines and indicate the places where it is likely to occur. Describe the possible effects it can have upon turbine operation and the turbine's structural integrity. What strategies can be adopted to alleviate the onset of cavitation?

(b) A Francis turbine is to be designed to produce 27 MW at a shaft speed of 94 rev/min under an effective head of 27.8 m. Assuming that the optimum hydraulic efficiency is 92 per cent and the runner tip speed to jet speed ratio is 0.69, determine:

- (1) the power specific speed;
- (2) the volume flow rate;
- (3) the impeller diameter and blade tip speed.

(c) A 1/10 scale model is to be constructed in order to verify the performance targets of the prototype turbine and to determine its cavitation limits. The head of water available for the model tests is 5.0 m. When tested under dynamically similar conditions as the prototype, the net positive suction head  $H_s$  of the model is 1.35 m.

Determine for the model:

- (1) the speed and the volume flow rate;

- (2) the power output, corrected using Moody's equation to allow for scale effects (assume a value for  $n = 0.2$ );
- (3) the suction specific speed  $\Omega_{ss}$ .

(d) The prototype turbine operates in water at 30°C when the barometric pressure is 95 kPa. Determine the necessary depth of submergence of that part of the turbine mostly likely to be prone to cavitation.

8. The preliminary design of a turbine for a new hydro-electric power scheme has under consideration a vertical-shaft Francis turbine with a hydraulic power output of 200 MW under an effective head of 110 m. For this particular design a specific speed,  $\Omega_s = 0.9$  (rad), is selected for optimum efficiency. At runner inlet the ratio of the absolute velocity to the spouting velocity is 0.77, the absolute flow angle is 68 deg and the ratio of the blade speed to the spouting velocity is 0.6583. At runner outlet the absolute flow is to be without swirl.

Determine:

- (1) the hydraulic efficiency of the rotor;
- (2) the rotational speed and diameter of the rotor;
- (3) the volume flow rate of water;
- (4) the axial length of the vanes at inlet.

9. A Kaplan turbine designed with a *shape factor* (power specific speed) of 3.0 (rad), a runner tip diameter of 4.4 m and a hub diameter of 2.0 m, operates with a net head of 20 m and a shaft speed of 150 rev/min. The absolute flow at runner exit is axial. Assuming that the hydraulic efficiency is 90% and the mechanical efficiency is 99%, determine:

- (1) the volume flow rate and shaft power output;
- (2) the relative flow angles at the runner inlet and outlet at the hub, the mean radius and at the tip.