

www.FluidMechanics.ir

مکانیک سیالات

اولین دانشنامه مکانیک



مرجع اصلی کتب های مهندسی زبان اصلی و فارسی

مرجع آموزش نرم افزارهای تخصصی

مرجع جزوات اساتید و دانشجویان

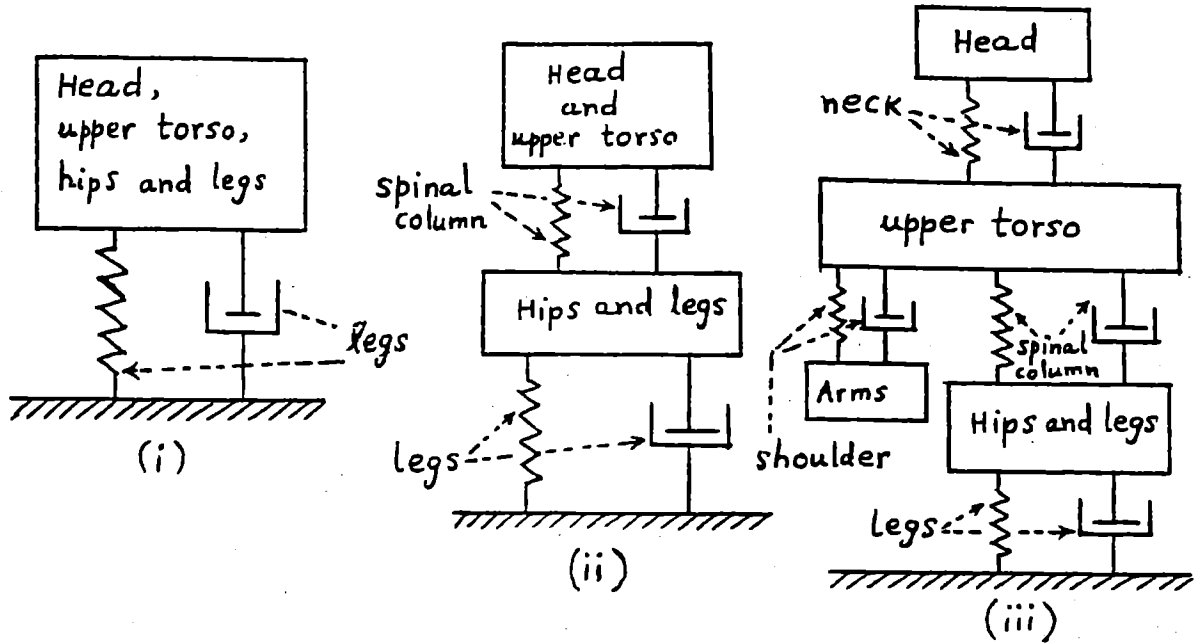
مرجع دروس آزمایشگاهی

و هر آنچه شما نیاز دارید

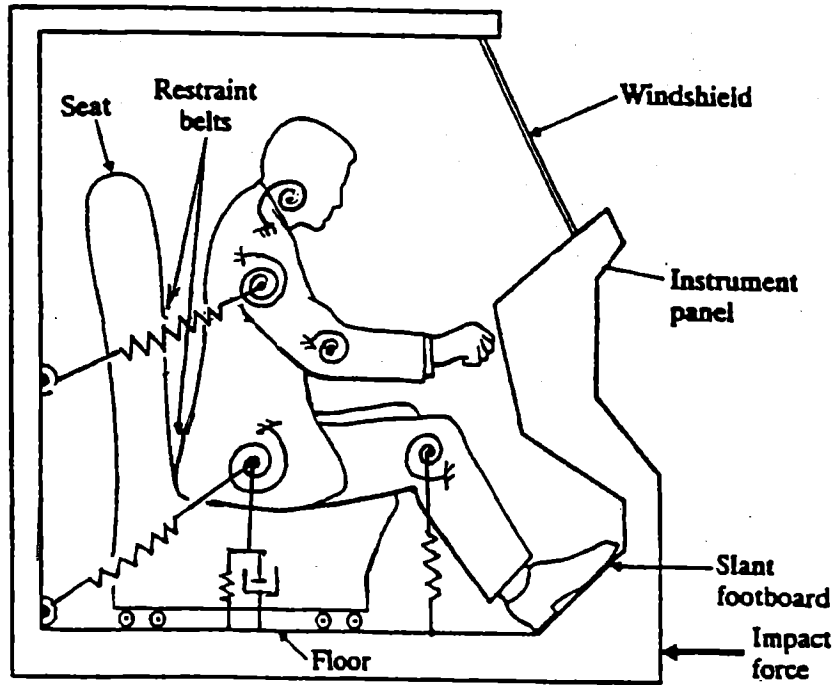
Chapter 1

Fundamentals of Vibration

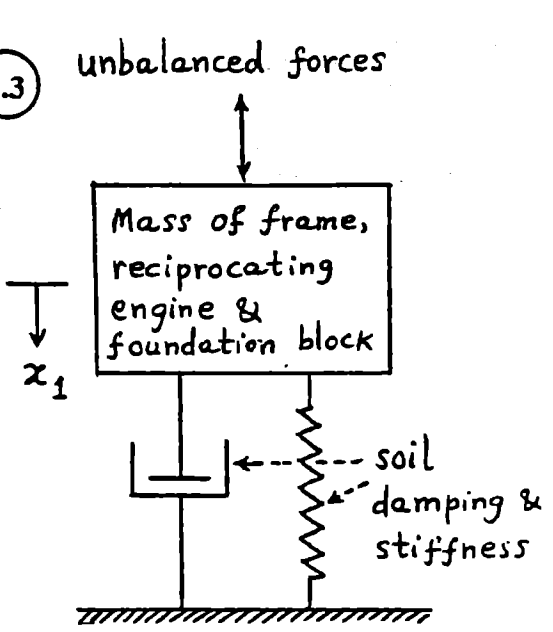
1.1



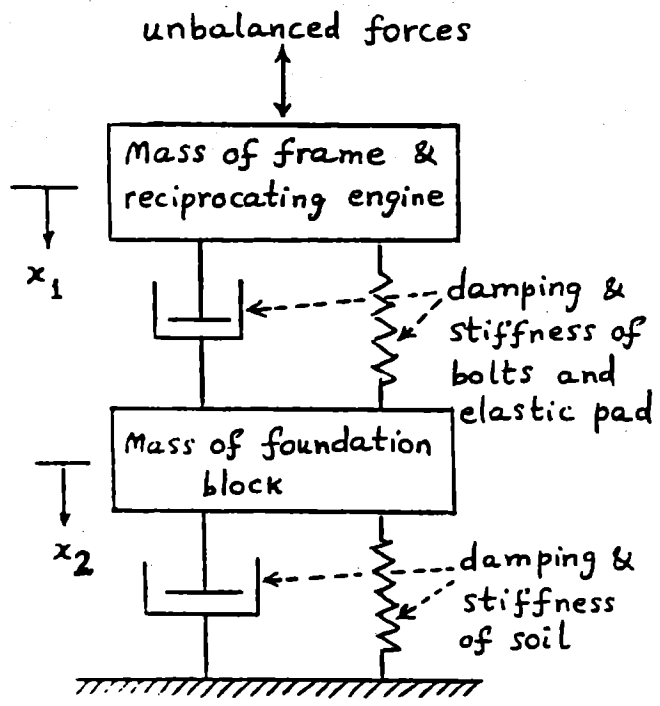
1.2



1.3

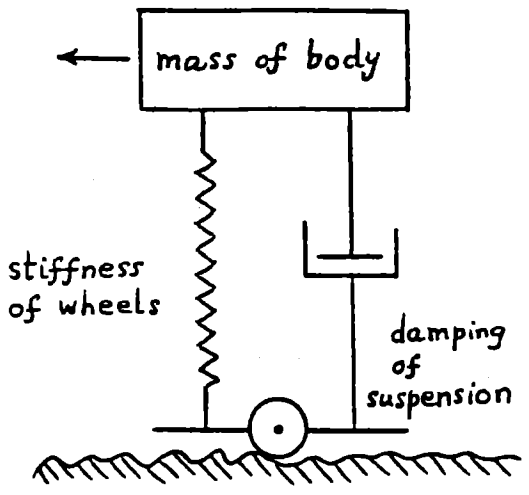


(a) one degree of freedom model

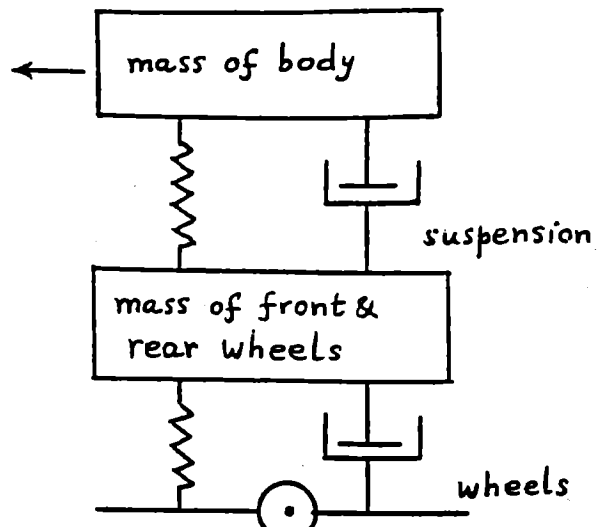


(b) Two degree of freedom model

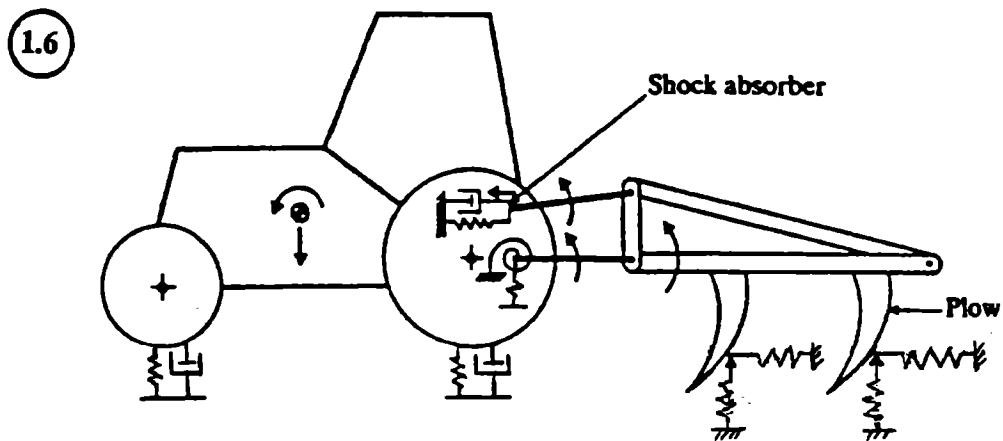
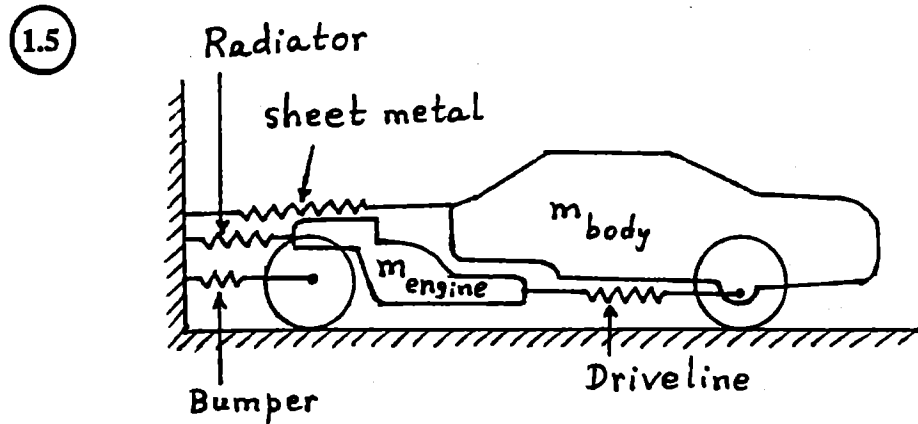
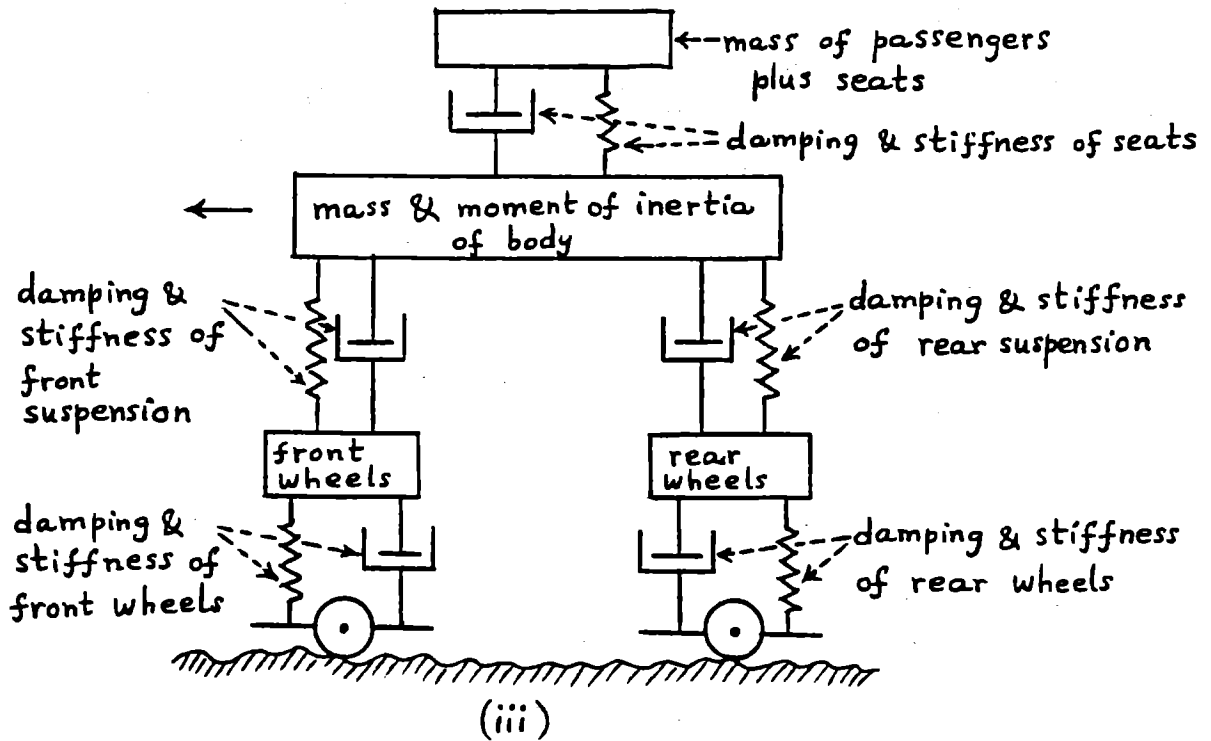
1.4



(i)



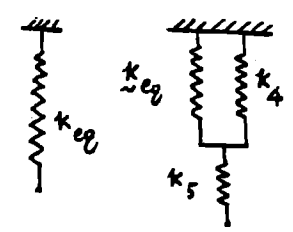
(ii)



1.7

$$\frac{1}{k_{eq}} = \frac{1}{2k_1} + \frac{1}{k_2} + \frac{1}{2k_3} ; \quad k_{eq} = \left(\frac{2k_1 k_2 k_3}{k_2 k_3 + 2k_1 k_3 + k_1 k_2} \right)$$

$$\frac{1}{k_{eq}} = \frac{1}{k_{eq} + k_4} + \frac{1}{k_5}$$



$$k_{eq} = \frac{k_5 (k_{eq} + k_4)}{k_5 + k_4 + k_{eq}} = \frac{k_2 k_3 k_4 k_5 + 2k_1 k_3 k_4 k_5 + k_1 k_2 k_4 k_5 + 2k_1 k_2 k_3 k_5}{k_2 k_3 k_4 + k_2 k_3 k_5 + 2k_1 k_3 k_4 + 2k_1 k_3 k_5 + k_1 k_2 k_4 + k_1 k_2 k_5 + 2k_1 k_2 k_3}$$

1.8

Equivalence of potential energies gives

$$\frac{1}{2} k_{t1} \theta^2 + \frac{1}{2} k_{t2} \theta^2 + \frac{1}{2} k_1 (\theta l_1)^2 + \frac{1}{2} k_2 (\theta l_1)^2 + \frac{1}{2} k_3 (\theta l_2)^2 = \frac{1}{2} k_{eq} \theta^2$$

$$\therefore k_{eq} = k_{t1} + k_{t2} + k_1 l_1^2 + k_2 l_1^2 + k_3 l_2^2$$

1.9

k_{123} = for series springs k_1, k_2 and k_3 :

$$\frac{1}{k_{123}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} ; \quad k_{123} = \frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1}$$

Using energy equivalence,

$$\frac{1}{2} k_{eq} \theta^2 = \frac{1}{2} k_4 \theta^2 + \frac{1}{2} k_{123} \theta^2 + \frac{1}{2} k_5 (\theta R)^2 + \frac{1}{2} k_6 (\theta R)^2$$

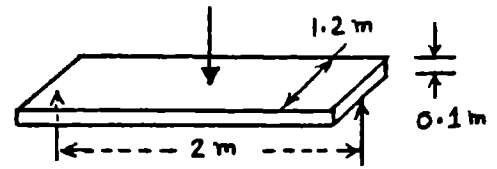
$$\begin{aligned} \therefore k_{eq} &= k_4 + k_{123} + R^2 k_5 + R^2 k_6 \\ &= k_4 + \left(\frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1} \right) + R^2 (k_5 + k_6) \end{aligned}$$

1.10

For simply supported beam, for load at middle,

$$k_1 = \frac{48EI}{l^3} = \frac{48(2.06 \times 10^{11})(10^{-4})}{8} = 12.36 \times 10^7 \text{ N/m}$$

where $I = \frac{1}{12} (1.2)(0.1)^3 = 10^{-4} \text{ m}^4$.



$$\delta_1 = \text{original deflection} = \frac{mg}{k_1} = \frac{500 \times 9.81}{12.36 \times 10^7} = 396.8447 \times 10^{-7} \text{ m}$$

When spring k is added, $k_{eq} = k + k_1$

$$\begin{aligned} \text{(a) New deflection} &= \frac{mg}{k_{eq}} = \frac{\delta_1}{4} ; \quad k_{eq} = \frac{4mg}{\delta_1} = 4k_1 \\ &= k + k_1 \end{aligned}$$

$$\therefore k = 3k_1 = 37.08 \times 10^7 \text{ N/m}$$

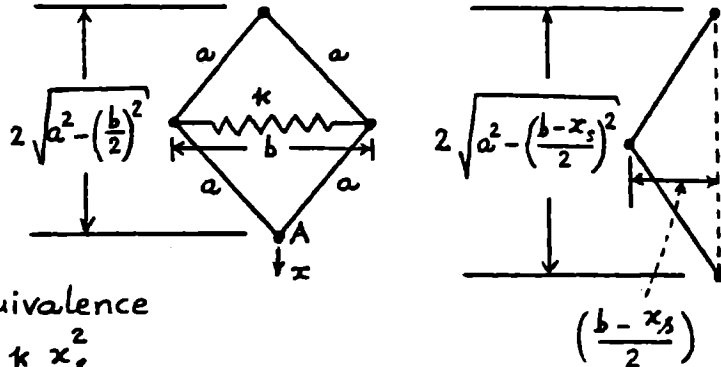
$$(b) \text{ New deflection} = \frac{mg}{k_{eq}} = \frac{\delta_1}{2} ; k_{eq} = \frac{2mg}{\delta_1} = 2k_1$$

$$\therefore k = k_1 = 12.36 \times 10^7 \text{ N/m} = k + k_1$$

$$(c) \text{ New deflection} = \frac{mg}{k_{eq}} = \frac{3}{4} \delta_1 ; k_{eq} = \frac{4mg}{3\delta_1} = \frac{4}{3} k_1$$

$$\therefore k = \frac{1}{3} k_1 = 4.12 \times 10^7 \text{ N/m} = k + k_1$$

1.11 (a) x = downward deflection of point A,
 x_s = resulting deformation of spring



Potential energy equivalence gives $\frac{1}{2} k_{eq} x^2 = \frac{1}{2} k x_s^2$

$$k_{eq} = k \left(\frac{x_s}{x} \right)^2$$

$$\begin{aligned} \text{But } x &= 2 \left[\sqrt{a^2 - \left(\frac{b-x_s}{2}\right)^2} - \sqrt{a^2 - \left(\frac{b}{2}\right)^2} \right] \\ &= 2 \sqrt{a^2 - \left(\frac{b}{2}\right)^2} \left[\left\{ \frac{a^2 - \left\{ \frac{b}{2} \left(1 - \frac{x_s}{b}\right) \right\}^2}{a^2 - \left(\frac{b}{2}\right)^2} \right\}^{1/2} - 1 \right] \\ &= 2 \sqrt{a^2 - \frac{b^2}{4}} \left[\left\{ \frac{\left(a^2 - \frac{b^2}{4} - \frac{x_s^2}{4} + \frac{b x_s}{2}\right)}{\left(a^2 - \frac{b^2}{4}\right)} \right\}^{1/2} - 1 \right] \\ &= 2 \sqrt{a^2 - \frac{b^2}{4}} \left[\left\{ 1 - \frac{x_s^2}{4\left(a^2 - \frac{b^2}{4}\right)} + \frac{b x_s}{2\left(a^2 - \frac{b^2}{4}\right)} \right\}^{1/2} - 1 \right] \end{aligned}$$

Using the relation $(1 + \theta)^{1/2} \approx 1 + \frac{\theta}{2}$, we obtain

$$x = 2 \left(a^2 - \frac{b^2}{4}\right)^{1/2} \left[1 + \frac{b x_s}{4\left(a^2 - \frac{b^2}{4}\right)} - 1 \right] = \frac{b x_s}{2 \left(a^2 - \frac{b^2}{4}\right)^{1/2}}$$

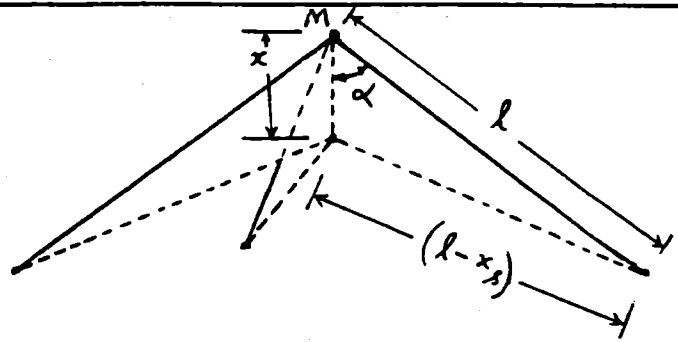
$$\therefore k_{eq} = k \left(\frac{x_s}{x} \right)^2 = 4k \left(\frac{a^2 - \frac{b^2}{4}}{b^2} \right) = k \left(\frac{4a^2 - b^2}{b^2} \right)$$

(b) Here $x = x_s$ (spring deflection)

$$\therefore k_{eq} = k$$

1.12

Let x = vertical displacement of mass M ,
 x_s = resulting deformation of each inclined spring.



From equivalence of potential energy,

$$\frac{1}{2} k_{eq} x^2 = 3 \left(\frac{1}{2} k x_s^2 \right) \quad ; \quad k_{eq} = 3k \left(\frac{x_s}{x} \right)^2$$

From geometry, $(l - x_s)^2 = l^2 + x^2 - 2lx \cos \alpha$
 $x^2 - 2x l \cos \alpha + 2l x_s - x_s^2 = 0$ (E₁)

Solving (E₁), $x = l \cos \alpha \left[1 \pm \left\{ 1 - \frac{(2l x_s - x_s^2)}{l^2 \cos^2 \alpha} \right\}^{1/2} \right]$ (E₂)

Using the relation $\sqrt{1 - \theta} \approx 1 - \frac{\theta}{2}$, (E₂) can be rewritten as

$$x = l \cos \alpha \left[1 \pm \left\{ 1 - \left(\frac{2l x_s - x_s^2}{2l^2 \cos^2 \alpha} \right) \right\} \right] \quad (E_3)$$

Assuming x to be small, we use minus sign and neglect x_s^2 compared to $2l x_s$ in (E₃). This gives

$$x = \frac{x_s}{\cos \alpha}$$

$$\therefore k_{eq} = 3k \cos^2 \alpha$$

In a similar manner, $c_{eq} = 3c \cos^2 \alpha$

1.13

$$k_{23} = \frac{k_2 k_3}{k_2 + k_3}$$

$$k_4 = A \rho g = \frac{\pi d^2}{4} \rho g$$

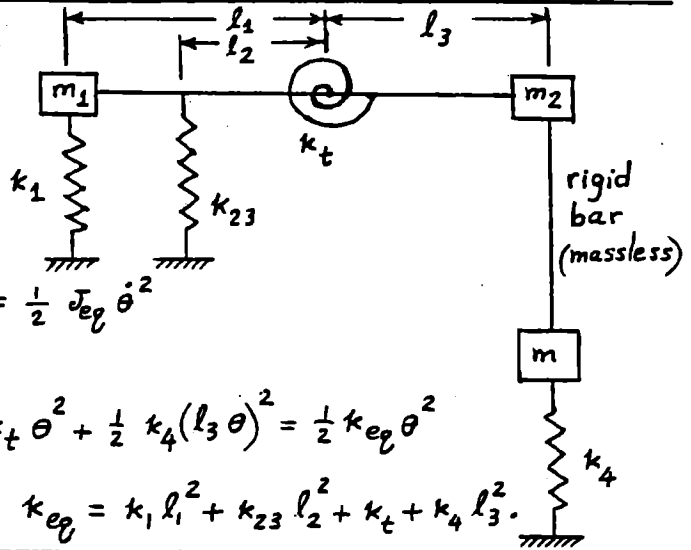
From kinetic energy,

$$\frac{1}{2} m_1 (l_1 \dot{\theta})^2 + \frac{1}{2} (m_2 + m) (l_3 \dot{\theta})^2 = \frac{1}{2} J_{eg} \dot{\theta}^2$$

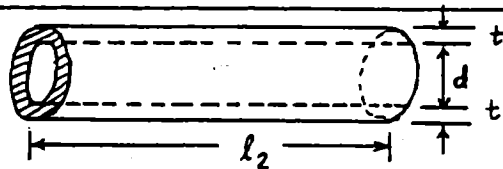
From potential energy,

$$\frac{1}{2} k_1 (l_1 \theta)^2 + \frac{1}{2} k_{23} (l_2 \theta)^2 + \frac{1}{2} k_t \theta^2 + \frac{1}{2} k_4 (l_3 \theta)^2 = \frac{1}{2} k_{eq} \theta^2$$

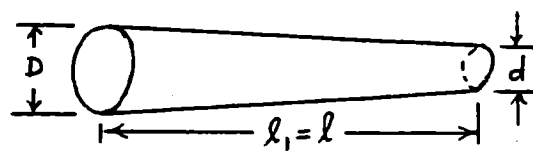
$$\therefore J_{eg} = m_1 l_1^2 + (m_2 + m) l_3^2 ; \quad k_{eq} = k_1 l_1^2 + k_{23} l_2^2 + k_t + k_4 l_3^2$$



1.14



$$k_2 = \frac{EA}{l_2} = \frac{\pi E t (d+t)}{l_2}$$



$$k_1 = \frac{\pi E D d}{4 l}$$

$$k_2 = k_1 \text{ gives } l_2 = \frac{4 t (d+t)}{D d}$$

$$1.15 \text{ (a) } F \approx F|_{x_0} + \left. \frac{dF}{dx} \right|_{x_0} (x - x_0) = \left(500x + 2x^3 \right)_{x=10} + \left(500 + 6x^2 \right)_{x=10} (x - 10) \\ \approx 1100x - 4000$$

(b) at $x = 9$ mm:

$$\text{Exact } F_9 = 500 \times 9 + 2(9)^3 = 5958 \text{ N}$$

$$\text{Approximate } F_9 = 1100 \times 9 - 4000 = 5900 \text{ N}$$

$$\text{Error} = -0.9735\%$$

(c) at $x = 11$ mm:

$$\text{Exact } F_{11} = 500 \times 11 + 2(11)^3 = 8162 \text{ N}$$

$$\text{Approximate } F_{11} = 1100 \times 11 - 4000 = 8100 \text{ N}$$

$$\text{Error} = +0.7596\%$$

$$1.16 \quad p v^\gamma = \text{constant} \dots (E_1) ; \text{ Differentiation of } (E_1) \text{ gives} \\ dp v^\gamma + p \gamma v^{\gamma-1} dv = 0$$

$$dp = - \frac{p \gamma}{v} dv \dots (E_2)$$

change in volume when mass moves by dx , $dv = -A \cdot dx \dots (E_3)$

$$\text{Eqs. } (E_2) \text{ and } (E_3) \text{ give } dp = \frac{p \gamma A}{v} dx$$

$$\text{Force due to pressure change} = dF = dp \cdot A = \frac{p \gamma A^2}{v} dx$$

$$\text{spring constant of air spring} = k = \frac{dF}{dx} = \left(\frac{p \gamma A^2}{v} \right).$$

1.17 Equivalent spring constants in different directions are

$$k_{e1} = \left(\frac{k_5 k_6 k_7}{k_5 k_6 + k_5 k_7 + k_6 k_7} \right), \quad k_{e2} = \left(\frac{k_8 k_9}{k_8 + k_9} \right),$$

$$k_{e3} = \left(\frac{k_1 k_2}{k_1 + k_2} \right), \quad k_{e4} = \left(\frac{k_3 k_4}{k_3 + k_4} \right)$$

If the force P moves by x , spring located at θ_i undergoes a displacement of $x_i = x \cos \theta_i$ (derivation as in problem 1.12).

$$\text{Equivalence of potential energy gives } \frac{1}{2} k_{eq} x^2 = \frac{1}{2} \sum_{i=1}^4 k_{ei} x_i^2$$

$$k_{eq} = \sum_{i=1}^4 (k_{ei} \cos^2 \theta_i)$$

1.18 From problem 1.16, $k = \frac{p \gamma A^2}{v}$ with $\gamma = 1.4$ for air

Let $p = 1.4$ MPa

$$k = 13,000 \text{ N/m} = \frac{(1.4 \times 10^6) (1.4) A^2}{v} \Rightarrow \frac{A^2}{v} = 6.6326 \times 10^{-3}$$

$$\text{Let diameter of a piston} = d = 5 \text{ cm} \quad A = \frac{\pi}{4} (5 \times 10^{-2})^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$v = \frac{A}{6.6326 \times 10^{-3}} = 5.8127 \times 10^{-4} \text{ m}^3$$

$$\text{Let } h = 5 \text{ cm}; \quad \frac{\pi}{4} D^2 (5 \times 10^{-2}) = v \quad \Rightarrow D = 0.12166 \text{ m}$$

1.19 $F = a x + b x^3 = 2 (10^4) x + 4 (10^7) x^3$

$$\text{Around } x^*: F(x) \approx F(x^*) + \left. \frac{dF}{dx} \right|_{x^*} (x - x^*)$$

$$\text{When } x^* = 10^{-2} \text{ m, } F(x^*) = 2 (10^4) (10^{-2}) + 4 (10^7) (10^{-6}) = 240 \text{ N}$$

$$\left. \frac{dF}{dx} \right|_{x^*} = a + 3 b x^2 = 2 (10^4) + 3 (4) (10^7) (10^{-4}) = 32000$$

$$\text{Hence } F(x) = 240 + 32000 (x - 0.01) = (32000 x - 80) \text{ N}$$

Since the linearized spring constant is given by $F(x) = k_{\text{eq}} x$, we have $k_{\text{eq}} = 32,000$ N/m.

1.20

$$F_i = a_i x_i + b_i x_i^3 ; i = 1, 2$$

Springs in series:

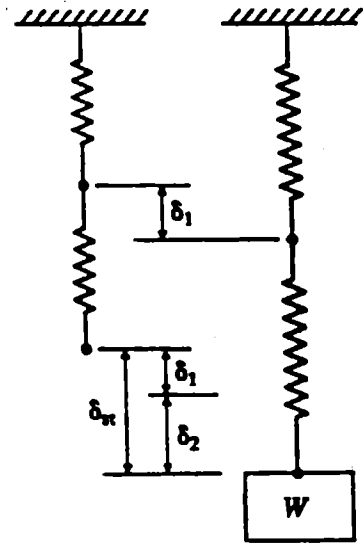
$$W = a_1 \delta_1 + b_1 \delta_1^3 \quad (1)$$

$$W = a_2 \delta_2 + b_2 \delta_2^3 \quad (2)$$

$$W = k_{eq} \delta_{st} \quad (3)$$

$$\delta_{st} = \delta_1 + \delta_2 \quad (4)$$

Solve Eqs. (1) and (2) for δ_1 and δ_2 , respectively. Substitute the result in Eq. (4) and then in Eq. (3) to find k_{eq} .



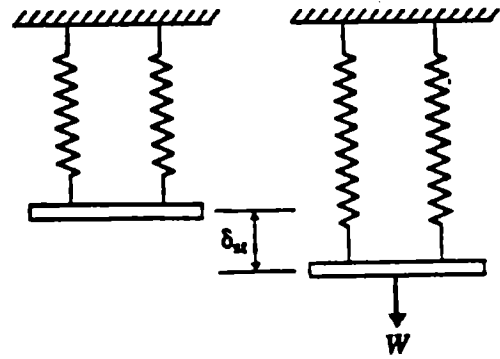
Springs in parallel:

$$W = F_1 + F_2$$

$$= a_1 \delta_{st} + b_1 \delta_{st}^3 + a_2 \delta_{st} + b_2 \delta_{st}^3$$

$$= k_{eq} \delta_{st}$$

$$k_{eq} = a_1 + b_1 \delta_{st}^2 + a_2 + b_2 \delta_{st}^2$$



$$(1.21) \quad k = \frac{G d^4}{8 D^3 N} \geq 8 \times 10^6 \text{ N/m} ; \quad \frac{D}{d} \geq 6 ; \quad N \geq 10$$

$$W = \pi D N \rho \left(\frac{\pi d^2}{4} \right) \quad \text{where } \rho = \text{weight per unit volume}$$

$$f_1 = \frac{1}{2} \sqrt{\frac{k g}{W}} = \frac{1}{2} \sqrt{\frac{G d^2 g}{2 \pi^2 D^4 N^2 \rho}} \geq 0.4 \text{ Hz}$$

Using $G = 73.1 \times 10^9 \text{ N/m}^2$, $\rho = 76000 \text{ N/m}^3$, $g = 9.81 \text{ m/sec}^2$,

$\frac{D}{d} = 6, 8, 10$; $N = 10, 15, 20$; $d = 0.4, 0.6, \dots$, values of

k and f_1 are computed.

Combination of $\frac{D}{d} = 6$, $N = 10$ and $d = 2.0 \text{ m}$, corresponding

to $k = 8.4606 \times 10^6 \text{ N/m}$ and $f_1 = 0.4801 \text{ Hz}$, can be taken as an acceptable design.

(1.22) Total elongation (strain) is same in each material:

$$\epsilon_s = \epsilon_a = \frac{x}{\ell} \quad (1)$$

where x is the total elongation. Equation (1) can be expressed as

$$\frac{\sigma_s}{E_s} = \frac{\sigma_a}{E_a} = \frac{x}{\ell} \quad (2)$$

$$\text{or } \sigma_s = \frac{E_s x}{\ell} \quad (3)$$

$$\sigma_a = \frac{E_a x}{\ell} \quad (4)$$

Total axial force is:

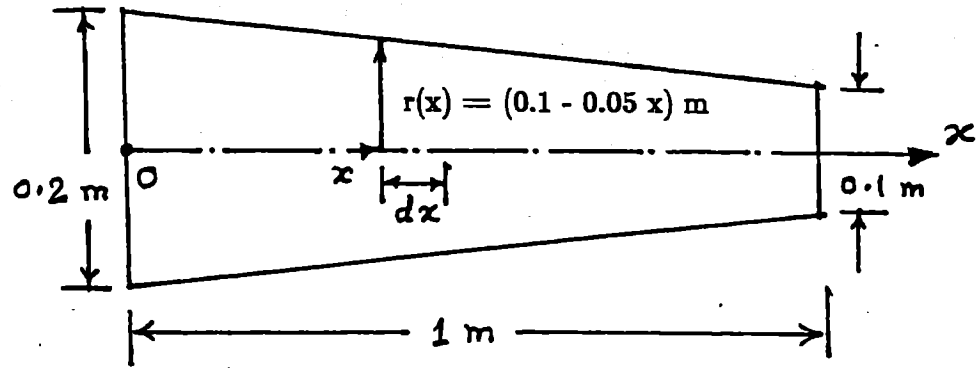
$$F = F_s + F_a = \sigma_s A_s + \sigma_a A_a \quad (5)$$

where F_s and F_a denote the axial forces acting on steel and aluminum, respectively, and A_s and A_a represent the cross-sectional areas of the two materials. Equating F to $k_{eq} x$ where k_{eq} denotes the equivalent spring constant of the bimetallic bar, we obtain from Eqs. (3) to (5):

$$F = k_{eq} x = \left(\frac{E_s x}{\ell} \right) A_s + \left(\frac{E_a x}{\ell} \right) A_a$$

$$\text{or } k_{eq} = \frac{E_s A_s}{\ell} + \frac{E_a A_a}{\ell} \quad (6)$$

1.23



$$J = \frac{\pi}{2} r^4 = \text{area polar moment of inertia at section } x = 1.5708 (0.1 - 0.05x)^4 \text{ m}^4$$

Knowing that the angle of twist, θ , between the ends of a uniform shaft of length ℓ under a torque T is given by $\theta = \frac{T \ell}{GJ}$, the angle of twist for an element of length dx can be expressed as

$$d\theta = \frac{T dx}{GJ} = \frac{T dx}{(80 (10^9)) 1.5708 (0.1 - 0.05x)^4} \quad (1)$$

The total angle of twist can be determined by integrating Eq. (1) from $x=0$ to 1 as:

$$\theta = \int_0^1 \frac{T dx}{(12.5664 (10^{10})) (0.1 - 0.05x)^4} = \left(\frac{T}{12.5664 (10^{10})} \right) \int_0^1 \frac{dx}{(0.1 - 0.05x)^4} \quad (2)$$

$$\begin{aligned} \text{But } \int_0^1 \frac{dx}{(0.1 - 0.05x)^4} &= -\frac{1}{0.05} \int_0^1 \frac{(-0.05 dx)}{(0.1 - 0.05x)^4} = -20 \int_0^{-0.05} \frac{dy}{(0.1 + y)^4} \\ &= 4.6667 (10^4) \quad \text{where } y = -0.05x \end{aligned}$$

$$\text{Hence } \theta = \frac{T (4.6667) (10^4)}{12.5664 (10^{10})} = T (0.3714 (10^{-6})) \text{ rad}$$

$$\text{This gives } k_t = \frac{T}{\theta} = 2.6925 (10^6) \text{ N-m/rad}$$

1.24

The steel and aluminum hollow shafts can be treated as two torsional springs in parallel. For a hollow shaft,

$$k_t = \frac{\pi G}{32 \ell} (D^4 - d^4)$$

For the steel shaft, $G = 80 (10^9) \text{ Pa}$, $\ell = 5 \text{ m}$, $D = 0.25 \text{ m}$, $d = 0.15 \text{ m}$, and hence

$$k_{t_1} = \frac{\pi (8 (10^{10}))}{32 (5)} (0.25^4 - 0.15^4) = 5.34072 (10^6) \text{ N-m/rad}$$

(a) For the aluminum shaft, $G = 26 (10^9) \text{ Pa}$, $\ell = 5 \text{ m}$, $D = 0.15 \text{ m}$, $d = 0.1 \text{ m}$, and hence

$$k_{t_2} = \frac{\pi (26 (10^9))}{32 (5)} (0.15^4 - 0.10^4) = 0.207395 (10^6) \text{ N-m/rad}$$

$$k_{eq} = k_{t_1} + k_{t_2} = 5.34072 (10^6) + 0.20739 (10^6) = 5.54811 (10^6) \text{ N-m/rad}$$

(b) With $G = 26 (10^9) \text{ Pa}$, $\ell = 5 \text{ m}$, $D = 0.15 \text{ m}$ and $d = 0.05 \text{ m}$,

$$k_{t_2} = \frac{\pi (26 \times 10^3)}{32 (5)} (0.15^4 - 0.05^4) = 0.255255 \times 10^6 \text{ N-m/rad}$$

$$k_{eq} = k_{t_1} + k_{t_2} = 5.34072 \times 10^6 + 0.255255 \times 10^6 = 5.595975 \times 10^6 \text{ N-m/rad}$$

1.25 For Helical spring $k = \frac{G d^4}{64 n R^3}$

$$\text{Spring 1: } k_1 = \frac{(82 \times 10^9) (5 \times 10^{-2})^4}{(64) (10) (15 \times 10^{-2})^3} = 237.2685 \text{ kN/m}$$

$$\text{Spring 2: } k_2 = \frac{(27 \times 10^9) (2.5 \times 10^{-2})^4}{(64) (10) (12.5 \times 10^{-2})^3} = 8.4375 \text{ kN/m}$$

(a) Spring 2 inside spring 1 (parallel): $k_{eq} = k_1 + k_2 = 245,706.0 \text{ N/m}$

(b) Spring 2 on top of spring 1 (series):

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_2 + k_1}{k_1 k_2}$$

which gives $k_{eq} = 8147.76 \text{ N/m}$

1.26 For a helical spring, $k = \frac{G d^4}{64 n R^3}$

$$k_1 = \frac{(82 \times 10^9) (2.5 \times 10^{-2})^4}{(64) (10) (15 \times 10^{-2})^3} = 14,829.28 \text{ N/m}$$

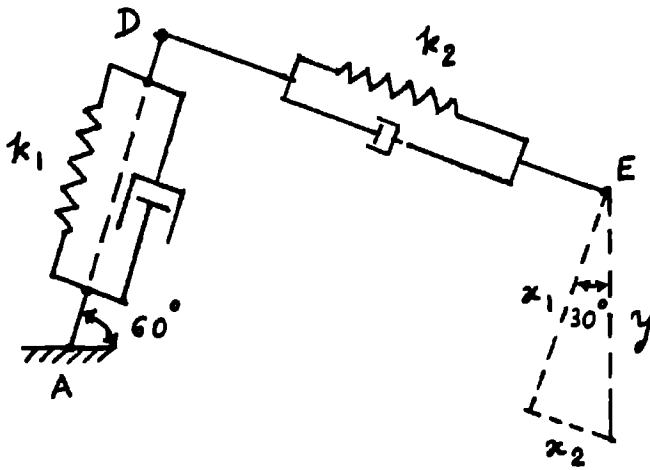
$$k_2 = \frac{(27 \times 10^9) (1.25 \times 10^{-2})^4}{64 (10) (12.5 \times 10^{-2})^3} = 527.34 \text{ N/m}$$

(a) Spring 2 inside spring 1: $k_{eq} = k_1 + k_2 = 15,356.63 \text{ N/m}$

(b) Spring 2 on top of spring 1: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$

$$\text{or } k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{(14,829.28) (527.34)}{14,829.28 + 527.34} = 509.23 \text{ N/m}$$

1.27



$$x_2 = y \sin 30^\circ$$

$$x_1 = y \cos 30^\circ$$

Equivalence of strain energies:

$$\frac{1}{2} k_{\text{eq}} y^2 = \frac{1}{2} k_2 x_2^2 + \frac{1}{2} k_1 x_1^2 = \frac{1}{2} k_1 y^2 \cos^2 30^\circ + \frac{1}{2} k_2 y^2 \sin^2 30^\circ$$

$$\text{i.e., } k_{\text{eq}} = \frac{3}{4} k_1 + \frac{1}{4} k_2$$

$$\text{with } k_1 = \frac{A_1 E_1}{\ell_1} = \frac{\frac{\pi}{4} (0.25^2 - 0.24^2) (200 \times 10^9)}{25} = 307,876.1 \text{ kN/m}$$

$$\text{and } k_2 = \frac{A_2 E_2}{\ell_2} = \frac{\frac{\pi}{4} (0.18^2 - 0.17^2) (200 \times 10^9)}{1.9} = 289,357.2 \text{ kN/m}$$

$$k_{\text{eq}} = \frac{3}{4} k_1 + \frac{1}{4} k_2 = 303,246.4 \text{ kN/m}$$

Similarly, the equivalent damping constant can be found as (using equivalence of kinetic energy):

$$C_{\text{eq}} = \frac{3}{4} c_1 + \frac{1}{4} c_2 = 65.5 \text{ N} \cdot \text{s/m}$$

1.28

Outer diameter: 7.5 mm

Inner diameter: 7.3 mm

Length: 1 m

Solution: stainless steel:

$E = 200 \text{ GPa}$,

$G = 83 \text{ GPa}$,

for each tube

$D = 7.5 \text{ mm}$, $d = 7.3 \text{ mm}$, $\ell_s = 1 \text{ m}$

$$\begin{aligned} \text{Axial stiffness} &= \frac{A E}{\ell} = \frac{\pi}{4} (D^2 - d^2) \frac{E}{\ell} \\ &= \frac{\pi}{4} (0.0075^2 - 0.0073^2) \frac{200 \times 10^9}{1} = 464956 \text{ N/m} = K_a \end{aligned}$$

$$\begin{aligned} \text{Torsional stiffness} &= \frac{\pi G}{32 \ell} (D^4 - d^4) \\ &= \frac{\pi (83 \times 10^9)}{32 (1)} (0.0075^4 - 0.0073^4) = 2.6421 \text{ N} \cdot \text{m/rad} = K_t \end{aligned}$$

for heat exchanger with 6 tubes:

$$\text{Axial stiffness} = 6K_a = 2789736 \text{ N/m}$$

$$\text{Torsional stiffness} = 6K_t = 15.8524 \text{ N.m/rad}$$

- 1.29 Assume small angles θ_1 and θ_2 ; $\theta_2 = \left(\frac{p_1}{p_2}\right) \theta_1$
 $x_1 =$ horizontal displacement of c.G. of mass $m_1 = \theta_1 r_1$
 $x_2 =$ vertical displacement of c.G. of mass $m_2 = \theta_2 r_2 = p_1 \theta_1 r_2 / p_2$
 $y_1 =$ horizontal displacement of springs k_1 and $k_2 = \theta_1 (r_1 + l_1)$
 $y_2 =$ vertical displacement of springs k_3 and $k_4 = \theta_2 l_2 = p_1 l_2 \theta_1 / p_2$

Equivalence of kinetic energies gives

$$\frac{1}{2} J_{eq} (\dot{\theta}_1)^2 = \frac{1}{2} J_1 (\dot{\theta}_1)^2 + \frac{1}{2} J_2 (\dot{\theta}_2)^2 + \frac{1}{2} m_1 (\dot{x}_1)^2 + \frac{1}{2} m_2 (\dot{x}_2)^2$$

$$\therefore J_{eq} = J_1 + J_2 \left(\frac{p_1}{p_2}\right)^2 + m_1 r_1^2 + m_2 r_2^2 \left(\frac{p_1}{p_2}\right)^2$$

Equivalence of potential energies gives

$$\frac{1}{2} k_{eq} \theta_1^2 = \frac{1}{2} k_{12} y_1^2 + \frac{1}{2} k_{34} y_2^2 + \frac{1}{2} k_{t1} \theta_1^2 + \frac{1}{2} k_{t2} \theta_2^2$$

$$\text{with } k_{12} = k_1 + k_2, \quad k_{34} = k_3 k_4 / (k_3 + k_4)$$

$$y_1 = \theta_1 (r_1 + l_1), \quad y_2 = p_1 l_2 \theta_1 / p_2 \quad \text{and} \quad \theta_2 = p_1 \theta_1 / p_2$$

$$\therefore k_{eq} = (k_1 + k_2) (r_1 + l_1)^2 + \left(\frac{k_3 k_4}{k_3 + k_4}\right) \frac{p_1^2 l_2^2}{p_2^2} + k_{t1} + k_{t2} \frac{p_1^2}{p_2^2}$$

1.30 $\theta = \frac{x}{b}, \quad x_1 = \frac{x a}{b}$

From equivalence of kinetic energies,

$$\frac{1}{2} m_{eq} \dot{x}^2 = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2$$

$$m_{eq} = m_1 \left(\frac{a}{b}\right)^2 + m_2 + J_0 \left(\frac{1}{b}\right)^2$$

- 1.31 Let $\dot{\theta}_i$ = angular velocity of the motor (input)
Angular velocities of different gear sets are:

J_{motor}, J_1	J_2, J_3	J_4, J_5	...	J_{2N}, J_{load}
$\dot{\theta}_i$	$\dot{\theta}_i \left(\frac{n_1}{n_2}\right)$	$\dot{\theta}_i \left(\frac{n_1}{n_2} \frac{n_3}{n_4}\right)$		$\dot{\theta}_i \left(\frac{n_1}{n_2} \frac{n_3}{n_4} \dots \frac{n_{2N-1}}{n_{2N}}\right)$

Equivalence of kinetic energies gives

$$\frac{1}{2} J_{\text{eq}} \dot{\theta}_i^2 = \frac{1}{2} J_{\text{motor}} \dot{\theta}_i^2 + \frac{1}{2} \sum_{k=1}^{2N} J_k \dot{\theta}_k^2 + \frac{1}{2} J_{\text{load}} \dot{\theta}_{\text{load}}^2$$

$$\therefore J_{\text{eq}} = (J_{\text{motor}} + J_1) + (J_2 + J_3) \left(\frac{n_1}{n_2}\right)^2 + (J_4 + J_5) \left(\frac{n_1}{n_2} \frac{n_3}{n_4}\right)^2 + \dots + (J_{2N} + J_{\text{load}}) \left(\frac{n_1}{n_2} \frac{n_3}{n_4} \dots \frac{n_{2N-1}}{n_{2N}}\right)^2$$

- 1.32 Equivalence of kinetic energies gives

$$\frac{1}{2} J_{\text{eq}} \dot{\theta}_1^2 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 \quad \text{where} \quad \dot{\theta}_2 = \dot{\theta}_1 \left(\frac{n_1}{n_2}\right)$$

$$J_{\text{eq}} = J_1 + J_2 \left(\frac{n_1}{n_2}\right)^2$$

- 1.33 When point A moves by distance $x = x_h$, the walking beam rotates by the angle

$$\theta_b = \frac{x_h}{\ell_3}$$

This corresponds to a linear motion of point B: $x_B = \theta_b \ell_2 = \frac{x_h \ell_2}{\ell_3}$

and the angular rotation of crank can be found from the relation:

$$x_B = r_c \sin \theta_c + \ell_4 \cos \phi = r_c \sin \theta_c + \ell_4 \sqrt{1 - \frac{r_c^2}{\ell_4^2} \sin^2 \theta_c}$$

For large values of ℓ_4 compared to r_c and for small values of x and θ_c , we have

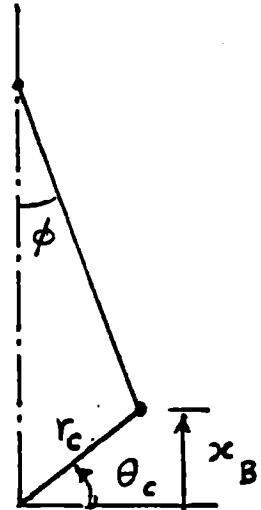
$$x_B \approx r_c \sin \theta_c = r_c \theta_c \quad \text{or} \quad \theta_c = \frac{x_B}{r_c} = \frac{x_h \ell_2}{\ell_3 r_c}$$

The kinetic energy of the system can be expressed as

$$T = \frac{1}{2} m_h \dot{x}_h^2 + \frac{1}{2} J_b \dot{\theta}_b^2 + \frac{1}{2} J_c \dot{\theta}_c^2$$

Equating this to $T = \frac{1}{2} m_{\text{eq}} \dot{x}_h^2 = \frac{1}{2} m_{\text{eq}} \dot{x}_h^2$, we obtain

$$m_{\text{eq}} = m_h + \frac{J_b}{\ell_3^2} + J_c \left(\frac{\ell_2}{\ell_3 r_c}\right)^2$$



- 1.34 When mass m is displaced by x , the bell crank lever rotates by the angle $\theta_b = \frac{x}{\ell_1}$. This

makes the center of the sphere displace by $x_s = \theta_b \ell_2$. Since the sphere rotates with out slip, it rotates by an angle

$$\theta_s = \frac{x_s}{r_s} = \frac{\theta_b l_2}{r_s} = \frac{x l_2}{l_1 r_s}$$

The kinetic energy of the system can be expressed as

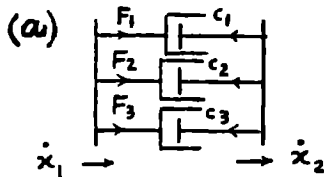
$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 + \frac{1}{2} J_s \dot{\theta}_s^2 + \frac{1}{2} m_s \dot{x}_s^2$$

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \left(\frac{\dot{x}}{l_1} \right)^2 + \frac{1}{2} \left(\frac{2}{5} m_s r_s^2 \right) \dot{x}^2 \left(\frac{l_2}{l_1 r_s} \right)^2 + \frac{1}{2} m_s \left(\frac{\dot{x} l_2}{l_1} \right)^2$$

since for a sphere, $J_s = \frac{2}{5} m_s r_s^2$. Equating this to $T = \frac{1}{2} m_{eq} \dot{x}^2$, we obtain

$$m_{eq} = m + J_0 \frac{1}{l_1^2} + \frac{7}{5} m_s \frac{l_2^2}{l_1^2}$$

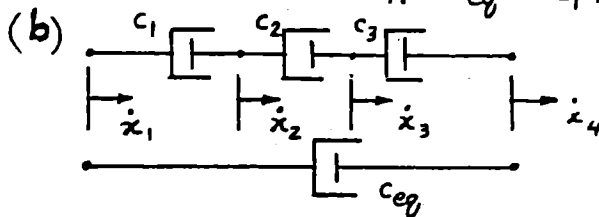
1.35



$$F_i = \text{damping force of } c_i = c_i (\dot{x}_2 - \dot{x}_1); i=1,2,3$$

$$F_{eq} = \text{damping force of } c_{eq} = c_{eq} (\dot{x}_2 - \dot{x}_1) \equiv F_1 + F_2 + F_3$$

$$\therefore c_{eq} = c_1 + c_2 + c_3$$



$$F_1 = c_1 (\dot{x}_2 - \dot{x}_1)$$

$$F_2 = c_2 (\dot{x}_3 - \dot{x}_2)$$

$$F_3 = c_3 (\dot{x}_4 - \dot{x}_3)$$

$$\dot{x}_4 - \dot{x}_1 = \dot{x}_4 - \dot{x}_3 + \dot{x}_3 - \dot{x}_2 + \dot{x}_2 - \dot{x}_1$$

$$\frac{F_{eq}}{c_{eq}} = \frac{F_3}{c_3} + \frac{F_2}{c_2} + \frac{F_1}{c_1}$$

$$\text{Since } F_{eq} = F_1 = F_2 = F_3, \quad \frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$$

(c) Equating the energies dissipated in a cycle,

$$\pi c_{eq} \omega X_1^2 = \pi c_1 \omega X_1^2 + \pi c_2 \omega X_2^2 + \pi c_3 \omega X_3^2$$

$$\text{where } X_1 = \theta l_1, X_2 = \theta l_2 \text{ and } X_3 = \theta l_3$$

$$\therefore c_{eq} = c_1 + c_2 \left(\frac{l_2}{l_1} \right)^2 + c_3 \left(\frac{l_3}{l_1} \right)^2$$

(d) Equating the energies dissipated in a cycle,

$$\pi c_{teq} \omega \theta_1^2 = \pi c_{t1} \omega \theta_1^2 + \pi c_{t2} \omega \theta_2^2 + \pi c_{t3} \omega \theta_3^2$$

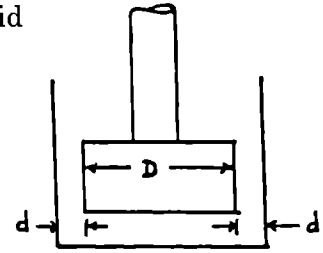
$$\text{where } \theta_2 = \theta_1 \left(\frac{n_1}{n_2} \right) \text{ and } \theta_3 = \theta_1 \left(\frac{n_1}{n_3} \right).$$

$$\therefore c_{teq} = c_{t1} + c_{t2} \left(\frac{n_1}{n_2} \right)^2 + c_{t3} \left(\frac{n_1}{n_3} \right)^2.$$

1.36

Damping constant desired = $C = 180 \text{ N} \cdot \text{s/m}$, viscosity of the fluid
 $= \mu = 30 \times 10^{-3} \text{ N} \cdot \text{s/m}^2$

$$c = \mu \left\{ \frac{3 \pi D^3 \ell \left(1 + \frac{2d}{D} \right)}{4 d^3} \right\} \quad \text{Eq. (1)}$$



Assuming $x = \frac{D}{d}$ as the unknown with $\ell = 5 \text{ cm}$, Eq. (1) can be written as

$$c = \mu \left(\frac{3 \pi \ell \times 3}{4} \right) \left(1 + \frac{2}{x} \right) \text{ or } 180 = (30 \times 10^{-3}) \left(\frac{3 \pi (5 \times 10^{-2})}{4} \right) x^3 \left(1 + \frac{2}{x} \right)$$

This gives $x^3 + 2x^2 - 50,929.58 = 0$

Using a trial and error procedure, the solution of this cubic equation can be found as

$$x \approx 36.41. \text{ Using } D = 7.5 \text{ cm, we get } d = \frac{7.5 \text{ cm}}{36.41} = 0.206 \text{ cm}$$

1.37

Damping constant $20 \times 10^6 \text{ N} \cdot \text{s/m}$
 SAE 30 at 21°C
 diameter of piston $< 65 \text{ mm}$

$$\text{Solution: } c = \mu \left\{ \frac{3 \pi D^3 \ell}{4 d^3} \left(1 + 2 \frac{d}{D} \right) \right\}$$

$D = \text{diameter of piston}$
 $d = \text{radial clearance}$
 $\ell = \text{axial length of piston}$

$$\mu = 0.31 \text{ Pa}\cdot\text{s}$$

Let $d = 0.02 \text{ mm}$, $D = 60 \text{ mm}$, and above equation gives:

$$20 \times 10^6 = 0.31 \left[\frac{3 \pi \ell (60^3)}{4 (0.02)^3} \left(1 + \frac{2 \times 0.02}{60} \right) \right]$$

$$\therefore \ell = 1.0135 \times 10^{-3} \text{ (m)}$$

1.38 Tangential velocity of inner cylinder = $\frac{D}{2} \omega$
 For small d , rate of change of velocity of fluid is

$$\frac{dv}{dr} = \frac{\frac{D}{2} \omega}{d}$$

shear stress between cylinders is

$$\tau = \mu \frac{dv}{dr} = \mu \frac{D\omega}{2d}$$

and shear force is

$$F = \tau \cdot \text{Area} = \tau \pi D(l-h) = \frac{\pi \mu D^2 \omega (l-h)}{2d}$$

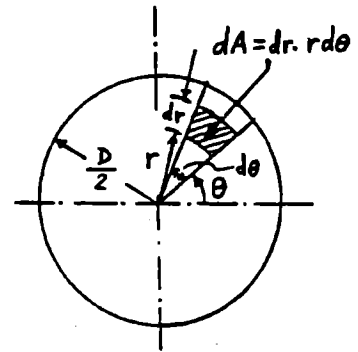
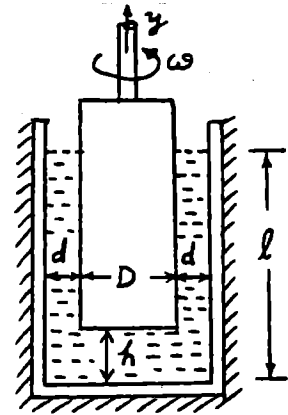
Torque developed = $M_{t1} = F \cdot \frac{D}{2}$

For small h , rate of change of velocity of fluid in vertical direction is

$$\frac{dv}{dy} = \frac{r\omega}{h}$$

Shear stress is $\tau = \mu \frac{dv}{dy} = \frac{\mu r \omega}{h}$

Force on area $dA = dF = \tau dA$



Torque between bottom surfaces of cylinders is

$$M_{t2} = \iint_{\text{area}} dM_{t2} \cdot dA \quad \text{where } dM_{t2} = dF \cdot r = \frac{\mu r^3 \omega}{h} dr d\theta$$

$$\text{i.e., } M_{t2} = \frac{\mu \omega}{h} \int_{r=0}^{D/2} \int_{\theta=0}^{2\pi} r^3 \cdot dr d\theta = \frac{\mu \omega \pi D^4}{64 h}$$

$$\text{Total torque} = M_t = M_{t1} + M_{t2} = \frac{\pi \mu D^3 \omega (l-h)}{4d} + \frac{\pi \mu \omega D^4}{64 h}$$

Expressing M_t as $C_t v = C_t \omega D/2$, we get damping constant:

$$C_t = \frac{\pi \mu D^2 (l-h)}{2d} + \frac{\pi \mu D^3}{32 h}$$

1.39

$$F = a \dot{x} + b \dot{x}^2 = 5 \dot{x} + 0.2 \dot{x}^2$$

$$F(\dot{x}) \approx F(\dot{x}_0) + \left. \frac{dF}{d\dot{x}} \right|_{\dot{x}_0} (\dot{x} - \dot{x}_0)$$

At $\dot{x}_0 = 5$ m/s, $F(\dot{x}_0) = 5(5) + 0.2(25) = 30$ N, $\left. \frac{dF}{d\dot{x}} \right|_{\dot{x}_0} = (5 + 0.4 \dot{x}) \Big|_5 = 7$ and hence

$$F(\dot{x}) = 30 + 7(\dot{x} - 5) = 7 \dot{x} - 5.$$

Thus the linearized damping constant is given by $F(\dot{x}) \approx 7 \dot{x} = c_{eq} \dot{x}$ or $c_{eq} = 7$ N-s/m.

1.40

Damping constant due to skin friction drag is:

$$c = 100 \mu \ell^2 d \quad (1)$$

Damping constant of a plate-type damper is:

$$c_p = \frac{\mu A}{h} \quad (2)$$

where A = area of plates and h = distance between the plates. If the area of plates (A) in Fig. 1.35 is taken to be same as the area of the plate shown in Fig. 1.84, we have $A = \ell d$. Equating (1) and (2) gives

$$100 \mu \ell^2 d = \frac{\mu \ell d}{h} \quad (3)$$

from which the clearance between the plates can be determined as $h = \frac{1}{100 \ell}$.

1.41

$$c = \frac{6 \pi \mu \ell}{h^3} \left\{ \left(a - \frac{h}{2} \right)^2 - r^2 \right\} \left\{ \frac{a^2 - r^2}{a - \frac{h}{2}} - h \right\}$$

When $\mu = 0.3445$ Pa-s, $\ell = 0.1$ m, $h = 0.001$ m, $a = 0.02$ m, and $r = 0.005$ m:

$$c = \frac{6 \pi (0.3445) (0.1)}{(10^{-3})^3} \left\{ (0.02 - 0.0005)^2 - 0.005^2 \right\} \left\{ \frac{0.02^2 - 0.005^2}{0.02 - 0.0005} - 0.001 \right\}$$

$$= 4,205.6394 \text{ N-s/m}$$

1.42

$$c = \frac{6\pi\mu l}{h^3} \left[\left(a - \frac{h}{2} \right)^2 - r^2 \right] \left[\frac{a^2 - r^2}{a - \frac{h}{2}} - h \right]$$

Basic data: $l = 10 \text{ cm}$, $h = 0.1 \text{ cm}$, $a = 2 \text{ cm}$, $r = 0.5 \text{ cm}$,
 $\mu = 0.3445$

Damping constant with basic data :

$$c = 4,205.6230 \text{ N-s/m}$$

(a) r changed to 1 cm ; new $c = 2,617.7920 \text{ N-s/m}$

(b) h changed to 0.05 cm ; new $c = 35,060.8910 \text{ N-s/m}$

(c) a changed to 4 cm ; new $c = 38,754.5860 \text{ N-s/m}$

1.43

$$\vec{x} = 5 + 2i = A e^{i\theta} = A \cos \theta + i A \sin \theta$$

$$A \cos \theta = 5$$

$$A \sin \theta = 2$$

$$A = \sqrt{(A \cos \theta)^2 + (A \sin \theta)^2} = \sqrt{5^2 + 2^2} = 5.3852$$

$$\theta = \tan^{-1} \left(\frac{A \sin \theta}{A \cos \theta} \right) = \tan^{-1} \left(\frac{2}{5} \right) = 21.8014^\circ$$

1.44

$$\vec{x}_1 = 1 + 2i = a_1 + a_2 i, \quad \vec{x}_2 = 3 - 4i = b_1 + b_2 i$$

$$\vec{x} = \vec{x}_1 + \vec{x}_2 = (a_1 + b_1) + i(a_2 + b_2) = 4 - 2i$$

$$= A e^{i\theta} = A \cos \theta + i A \sin \theta$$

$$A = \sqrt{4^2 + (-2)^2} = 4.4721$$

$$\theta = \tan^{-1} \left(\frac{-2}{4} \right) = -26.5651^\circ$$

1.45

$$z_1 = (3 - 4i), z_2 = (1 + 2i)$$

$$z = z_1 - z_2 = (3 - 4i) - (1 + 2i) = 2 - 6i = A e^{i\theta}$$

$$\text{where } A = \sqrt{2^2 + (-6)^2} = 6.3246 \text{ and } \theta = \tan^{-1} \left(\frac{-6}{2} \right) = \tan^{-1} (-3) = -1.2490 \text{ rad}$$

1.46

$$z_1 = 1 + 2i, z_2 = 3 - 4i$$

$$z = z_1 z_2 = (1 + 2i)(3 - 4i) = 11 + 2i = A e^{i\theta}$$

$$\text{where } A = \sqrt{11^2 + 2^2} = 11.1803 \text{ and } \theta = \tan^{-1} (2/11) = 0.1798 \text{ rad}$$

1.47

$$z = \frac{z_1}{z_2} = \frac{1 + 2i}{3 - 4i} = \frac{(1 + 2i)(3 + 4i)}{(3 - 4i)(3 + 4i)} = \frac{-5 + 10i}{25} = -0.2 + 0.4i = A e^{i\theta}$$

$$\text{where } A = \sqrt{(-0.2)^2 + (0.4)^2} = 0.4472$$

$$\text{and } \theta = \tan^{-1} \left(\frac{-0.4}{0.2} \right) = \tan^{-1} (-2) = -1.1071 \text{ rad}$$

1.48 $x(t) = X \cos \omega t$, $y(t) = Y \cos (\omega t + \phi)$

(a) $\frac{x^2}{X^2} = \cos^2 \omega t$, $\frac{y^2}{Y^2} = \cos^2 (\omega t + \phi)$,

$$2 \frac{xy}{XY} \cos \phi = 2 \cos \omega t \cos (\omega t + \phi) \cos \phi$$

$$\begin{aligned} & \frac{x^2}{X^2} + \frac{y^2}{Y^2} - 2 \frac{xy}{XY} \cos \phi \\ &= \cos^2 \omega t + \cos^2 (\omega t + \phi) - 2 \cos \omega t \cos \phi \cos (\omega t + \phi) \end{aligned} \quad (1)$$

Noting that $\cos^2 \alpha = \frac{1}{2} (1 + \cos 2\alpha)$, Eq. (1) can be rewritten as

$$\begin{aligned} & \frac{x^2}{X^2} + \frac{y^2}{Y^2} - 2 \frac{xy}{XY} \cos \phi \\ &= \frac{1}{2} + \frac{1}{2} \cos 2\omega t + \frac{1}{2} + \frac{1}{2} \cos (2\omega t + 2\phi) - 2 \cos \omega t \cos \phi \cos (\omega t + \phi) \\ &= 1 + \frac{1}{2} \left\{ 2 \cos \frac{2\omega t + 2\omega t + 2\phi}{2} \cos \frac{2\omega t - 2\omega t - 2\phi}{2} \right\} \\ & \quad - 2 \cos \omega t \cos \phi \cos (\omega t + \phi) \\ &= 1 + \cos (2\omega t + \phi) \cos \phi - 2 \cos \omega t \cos \phi \cos (\omega t + \phi) \\ &= 1 + \cos (2\omega t + \phi) \cos \phi - 2 \cos \phi \left\{ \frac{1}{2} [\cos (\omega t + \phi - \omega t) + \cos (\omega t + \phi + \omega t)] \right\} \\ &= 1 + \cos \phi \cos (2\omega t + \phi) - \cos \phi \left\{ \cos \phi + \cos (2\omega t + \phi) \right\} \\ &= 1 - \cos^2 \phi = \sin^2 \phi \end{aligned} \quad (2)$$

(b) When $\phi = 0$, Eq. (2) reduces to

$$\frac{x^2}{X^2} + \frac{y^2}{Y^2} - 2 \frac{xy}{XY} = \left(\frac{x}{X} - \frac{y}{Y} \right)^2 = 0$$

which gives $X = \pm \frac{X}{Y} y$. This indicates that the locus of the resultant motion is a straight line. When $\phi = \frac{\pi}{2}$, Eq. (2) reduces to

$$\frac{x^2}{X^2} + \frac{y^2}{Y^2} = 1$$

which denotes an ellipse with its major and minor axes along x and y directions, respectively. When $\phi = \pi$, Eq. (2) reduces to that of a straight line as in the case of $\phi = 0$.

1.49

Equation for resultant motion:

$$\frac{x^2}{X^2} + \frac{y^2}{Y^2} - 2 \frac{xy}{XY} \cos^2 \phi = \sin^2 \phi \quad (1)$$

When $y = 0$, Eq. (1) reduces to $\frac{x^2}{X^2} = \sin^2 \phi$ and hence:

$$x = \pm X \sin \phi = \pm 6.2 = OS \text{ in figure} \quad (2)$$

When $x = 0$, Eq. (1) reduces to $\frac{y^2}{Y^2} = \sin^2 \phi$ and hence:

$$y = \pm Y \sin \phi = \pm 6.0 = OT \text{ in figure} \quad (3)$$

It can be seen that

$$OR = X \cos \phi = 7.6 \text{ in figure} \quad (4)$$

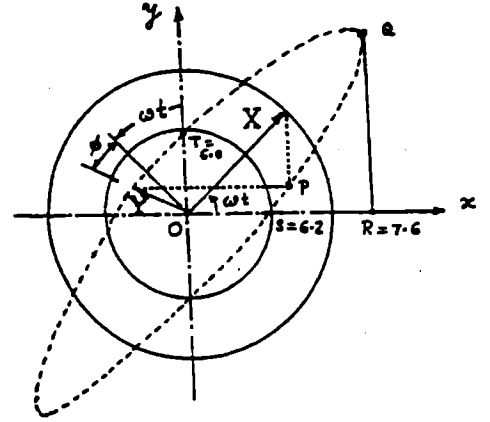
$$\frac{OS}{OR} = \frac{X \sin \phi}{X \cos \phi} = \tan \phi = \frac{6.2}{7.6} = 0.8158 \text{ or } \phi = 39.2072^\circ \quad (5)$$

From Eqs. (2) and (4), we find

$$X = \sqrt{(X \sin \phi)^2 + (X \cos \phi)^2} = \sqrt{(6.2)^2 + (7.6)^2} = 9.8082 \text{ mm}$$

Equations (3) and (5) give

$$Y = \frac{6.0}{\sin \phi} = \frac{6.0}{\sin 39.2072^\circ} = 9.4918 \text{ mm}$$



1.50 (a) $x(t) = \frac{A}{1000} \cos(50t + \alpha)$ m where A is in mm ---- (E₁)

$x(0) = \frac{A}{1000} \cos \alpha = 0.003$, $A \cos \alpha = 3$ ---- (E₂)

$\dot{x}(0) = -\frac{50A}{1000} \sin \alpha = 1$, $A \sin \alpha = -20$ ---- (E₃)

$A = \{(A \cos \alpha)^2 + (A \sin \alpha)^2\}^{1/2} = 20.2237$ mm

$\alpha = \tan^{-1} \left(\frac{A \sin \alpha}{A \cos \alpha} \right) = \tan^{-1} (-6.6667) = -81.4692^\circ = -1.4219$ rad

$x(t) = 20.2237 \cos(50t - 1.4219)$ mm

(b) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

Eq. (E₁) can be expressed as $x(t) = A \cos 50t \cdot \cos \alpha - A \sin 50t \cdot \sin \alpha$
 $= A_1 \cos \omega t + A_2 \sin \omega t$

where $\omega = 50$, $A_1 = A \cos \alpha$, $A_2 = -A \sin \alpha$

$\therefore x(t) = (3 \cos 50t + 20 \sin 50t)$ mm

1.51 $x(t) = A_1 \cos \omega t + A_2 \sin \omega t$

$\frac{dx}{dt}(t) = -A_1 \omega \sin \omega t + A_2 \omega \cos \omega t$, $\frac{d^2x}{dt^2} = -A_1 \omega^2 \cos \omega t - A_2 \omega^2 \sin \omega t$

$\frac{d^2x}{dt^2} = -\omega^2 x(t)$ where ω^2 is a constant

Hence $x(t)$ is a simple harmonic motion.

1.52 (a) Using trigonometric relations:

$x_1(t) = 5 (\cos 3t \cos 1 - \sin 3t \sin 1)$

$x_2(t) = 10 (\cos 3t \cos 2 - \sin 3t \sin 2)$

$x(t) = x_1(t) + x_2(t) = \cos 3t (5 \cos 1 + 10 \cos 2) - \sin 3t (5 \sin 1 + 10 \sin 2)$

If $x(t) = A \cos(\omega t + \alpha) = A \cos \omega t \cos \alpha - A \sin \omega t \sin \alpha$,

$\omega = 3$, $A \cos \alpha = 5 \cos 1 + 10 \cos 2 = -1.4599$,

$A \sin \alpha = 5 \sin 1 + 10 \sin 2 = 13.3003$

$A = \sqrt{(A \cos \alpha)^2 + (A \sin \alpha)^2} = 13.3802$

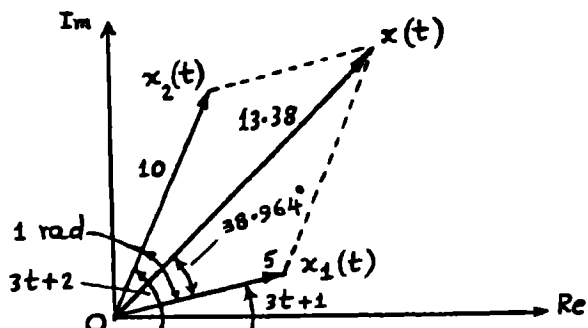
$\alpha = \tan^{-1} \left(\frac{A \sin \alpha}{A \cos \alpha} \right) = \tan^{-1} (-9.1104) = 96.2640^\circ = 1.68$ rad

Angle between $x_1(t)$ and $x(t)$ is $96.2640^\circ - 57.3^\circ = 38.964^\circ$

(b) Using vector addition:

For an arbitrary value of $(\omega t + 1)$, harmonic motions $x_1(t)$ and $x_2(t)$ can be shown as in the figure. From vector addition, we find

$x(t) \approx 13.38 \cos(\omega t + 1.68)$



(C) Using complex numbers:

$$x_1(t) = \operatorname{Re} \{ A_1 e^{i(\omega t + 1)} \} = \operatorname{Re} \{ 5 e^{i(\omega t + 1)} \}$$

$$x_2(t) = \operatorname{Re} \{ A_2 e^{i(\omega t + 2)} \} = \operatorname{Re} \{ 10 e^{i(\omega t + 2)} \}$$

$$\text{If } x(t) = \operatorname{Re} \{ A e^{i(\omega t + \alpha)} \},$$

$$A \cos(3t + \alpha) = A_1 \cos(3t + 1) + A_2 \cos(3t + 2)$$

$$\text{i.e. } A (\cos 3t \cos \alpha - \sin 3t \sin \alpha) = 5 (\cos 3t \cos 1 - \sin 3t \sin 1) + 10 (\cos 3t \cos 2 - \sin 3t \sin 2)$$

$$\text{i.e. } A \cos \alpha = 5 \cos 1 + 10 \cos 2, \quad A \sin \alpha = 5 \sin 1 + 10 \sin 2$$

$$A = 13.3802, \quad \alpha = 1.68 \text{ rad}$$

$$x(t) = \operatorname{Re} \{ 13.3802 e^{i(3t + 1.68)} \}$$

1.53

$$x(t) = 10 \sin(\omega t + 60^\circ) = x_1(t) + x_2(t)$$

$$\text{where } x_1(t) = 5 \sin(\omega t + 30^\circ) \text{ and } x_2(t) = A \sin(\omega t + \alpha^\circ)$$

$$10 (\sin \omega t \cos 60^\circ + \cos \omega t \sin 60^\circ) = 5 (\sin \omega t \cos 30^\circ + \cos \omega t \sin 30^\circ) + A (\sin \omega t \cos \alpha^\circ + \cos \omega t \sin \alpha^\circ)$$

$$10 \cos 60^\circ = 5 \cos 30^\circ + A \cos \alpha^\circ; \quad A \cos \alpha^\circ = 0.6699$$

$$10 \sin 60^\circ = 5 \sin 30^\circ + A \sin \alpha^\circ; \quad A \sin \alpha^\circ = 6.1603$$

$$A = \sqrt{0.6699^2 + 6.1603^2} = 6.1966$$

$$\alpha = \tan^{-1} (6.1603/0.6699) = 83.7938^\circ$$

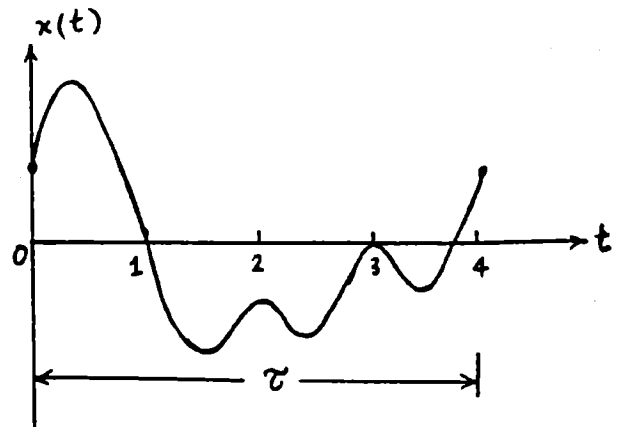
$$x_2(t) = 6.1966 \sin(\omega t + 83.7938^\circ)$$

1.54

$$x(t) = \frac{1}{2} \cos \frac{\pi}{2} t + \sin \pi t$$

$$= \frac{1}{2} \cos \frac{\pi}{2} t (1 + 4 \sin \frac{\pi}{2} t)$$

From the nature of the graph of $x(t)$, it can be seen that $x(t)$ is periodic with a time period of $\tau = 4$.



1.55

If $x(t)$ is harmonic, $\ddot{x}(t) = -\omega^2 x(t)$

$$\text{Here } x(t) = 2 \cos 2t + \cos 3t$$

$$\ddot{x}(t) = -8 \cos 2t - 9 \cos 3t \neq -\text{constant times } x(t)$$

$\therefore x(t)$ is not harmonic

1.56

$$x(t) = \frac{1}{2} \cos \frac{\pi}{2} t - \cos \pi t$$

$$\ddot{x}(t) = -\frac{\pi^2}{8} \cos \frac{\pi}{2} t + \pi^2 \cos \pi t \neq -\text{constant times } x(t)$$

$\therefore x(t)$ is not harmonic

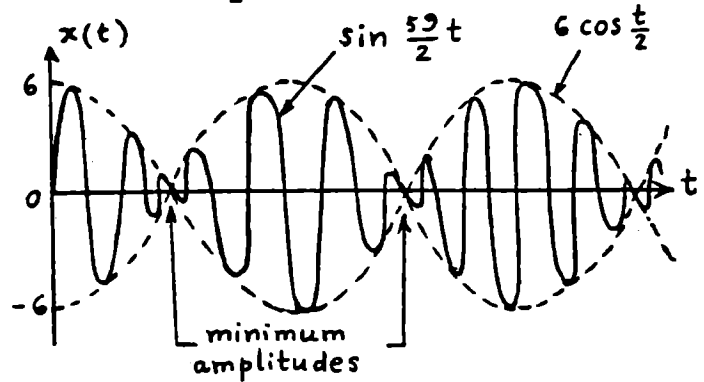
1.57

$$x(t) = x_1(t) + x_2(t) = 3 \sin 30t + 3 \sin 29t$$

Since $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$,

$$x(t) = \left(6 \cos \frac{t}{2}\right) \sin \frac{59}{2} t$$

This equation shows that the amplitude $\left(6 \cos \frac{t}{2}\right)$ varies with time between a maximum value of 6 and a minimum value of 0. The frequency of this oscillation (beat frequency) is $\omega_b = 1$.



Note: Beat frequency is twice the frequency of the term $6 \cos \frac{t}{2}$ since two peaks pass in each cycle of $\left(6 \cos \frac{t}{2}\right)$.

1.58

The resultant motion of two harmonic motions having identical amplitudes (X) but slightly different frequencies (ω and $\omega + \delta\omega$) is given by Eq. (1.67):

$$x(t) = 2X \cos \left(\omega t + \frac{\delta\omega t}{2} \right) \cos \left(\frac{\delta\omega t}{2} \right)$$

Thus the maximum amplitude of the resultant motion is equal to $2X$ and the beat frequency is equal to $\delta\omega$. From Fig. 1.88, we find that $2X \approx 5$ mm or $X = 2.5$ mm and

$$\frac{\delta\omega}{2} = \frac{2\pi}{\tau_{\text{beat}}} = \frac{2\pi}{\tau_{\text{larger}}} = \frac{2\pi}{2(12.8 - 4.2)} = 0.374 \text{ rad/sec}$$

or $\delta\omega = 0.748$ rad/sec and $\omega + \frac{\delta\omega}{2} = \frac{2\pi}{\tau_{\text{smaller}}} = \frac{2\pi}{1} = 6.2832$ rad/sec

Hence $\omega = 6.2832 - 0.3740 = 5.9092$ rad/sec. Thus the amplitudes of the two motions = $X = 2.5$ mm and their frequencies are $\omega = 5.9092$ rad/sec and $\omega + \delta\omega = 5.9092 + 0.7480 = 6.6572$ rad/sec.

1.59

$$A = 0.05 \text{ m}, \quad \omega = 10 \text{ Hz} = 62.832 \text{ rad/sec}$$

$$\text{period} = \tau = \frac{2\pi}{\omega} = \frac{2\pi}{62.832} = 0.1 \text{ sec}$$

$$\text{maximum velocity} = A\omega = 0.05 \times 62.832 = 3.1416 \text{ m/s}$$

$$\text{maximum acceleration} = A\omega^2 = 0.05 (62.832)^2 = 197.393 \text{ m/s}^2$$

1.60 $\omega = 15 \text{ cps} = 94.248 \text{ rad/sec}$
 $\ddot{x}_{\max} = 0.5g = 0.5(9.81) = 4.905 \text{ m/s}^2 = A\omega^2$
 $A = \text{amplitude} = 4.905 / (94.248)^2 = 0.0005522 \text{ m}$
 $\dot{x}_{\max} = \text{max. velocity} = A\omega = 0.05204 \text{ m/s}$

1.61 $x = A \cos \omega t$, $x_{\max} = A = 0.25 \text{ mm}$, $\ddot{x} = -\omega^2 A \cos \omega t$
 $\ddot{x}_{\max} = A\omega^2 = 0.4g = 3924 \text{ mm/s}^2$; $\omega^2 = 2924/A = 15696 \text{ (rad/s)}^2$
operating speed of pump = $\omega = 125.2837 \text{ rad/s} = 19.9395 \text{ rpm}$

1.62 $x(t) = X \sin \frac{2\pi t}{\tau}$; $x_{\text{rms}} = \left[\frac{1}{\tau} \int_0^{\tau} X^2 \sin^2 \frac{2\pi t}{\tau} dt \right]^{\frac{1}{2}}$
Using $\sin^2 \frac{2\pi t}{\tau} = \frac{1 - \cos \frac{4\pi t}{\tau}}{2}$, we obtain
 $x_{\text{rms}} = \left[\frac{X^2}{\tau} \int_0^{\tau} \left(\frac{1}{2} - \frac{1}{2} \cos \frac{4\pi t}{\tau} \right) dt \right]^{\frac{1}{2}} = \left[\frac{X^2}{\tau} \left\{ \frac{t}{2} - \frac{1}{2} \frac{\tau}{4\pi} \sin \frac{4\pi t}{\tau} \right\} \Big|_0^{\tau} \right]^{\frac{1}{2}}$
 $= \left[\frac{X^2}{\tau} \left\{ \frac{\tau}{2} - \frac{\tau}{8\pi} \sin 4\pi - 0 + 0 \right\} \right]^{\frac{1}{2}} = \frac{X}{\sqrt{2}}$

1.63 $x(t) = \frac{A t}{\tau}$; $0 \leq t \leq \tau$
 $x_{\text{rms}} = \left[\frac{1}{\tau} \int_0^{\tau} \frac{A^2}{\tau^2} t^2 dt \right]^{\frac{1}{2}} = \left[\frac{1}{\tau} \frac{A^2}{\tau^2} \left(\frac{t^3}{3} \right) \Big|_0^{\tau} \right]^{\frac{1}{2}} = \left(\frac{A^2}{\tau^3} \frac{\tau^3}{3} \right)^{\frac{1}{2}} = \left(\frac{A^2}{3} \right)^{\frac{1}{2}} = \frac{A}{\sqrt{3}}$

1.64 For even functions, $x(-t) = x(t)$.
From Eq. (1.73), $b_n = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t \cdot dt = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} x(t) \sin n\omega t \cdot dt$
 $= \frac{2}{\tau} \left[\int_{-\tau/2}^0 x(t) \sin n\omega t \cdot dt + \int_0^{\tau/2} x(t) \sin n\omega t \cdot dt \right]$ ----- (E1)

Since $\sin(-n\omega t) = -\sin(n\omega t) = \text{odd function of } t$, the product of $x(t)$ and $\sin n\omega t$ is an odd function.
Further, for an odd function $f(t)$, $f(-t) = -f(t)$, and

$$\int_{-a}^a f(t) dt = \int_{-a}^0 f(t) dt + \int_0^a f(t) dt = \int_0^a f(-t) dt + \int_0^a f(t) dt$$

$$= - \int_0^a f(t) dt + \int_0^a f(t) dt = 0 \quad \text{-----(E}_2\text{)}$$

Equations (E₁) and (E₂) lead to $b_n = 0$.

Also, since $\cos n\omega t$ is an even function, we get

$$a_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} x(t) \cos n\omega t dt = \frac{4}{\tau} \int_0^{\tau/2} x(t) \cos n\omega t dt$$

For odd functions, $x(-t) = -x(t)$.

From Eq. (1.72), $a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} x(t) \cos n\omega t dt$

Since $\cos n\omega t$ is an even function, $\cos(-n\omega t) = \cos(n\omega t)$, the product of $x(t)$ and $\cos n\omega t$ is an odd function.

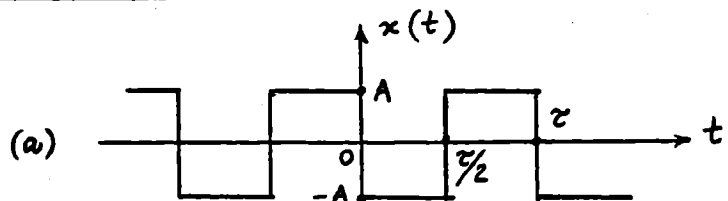
Hence $a_n = 0$.

Further, since $\sin n\omega t$ is an odd function, $x(t) \sin n\omega t$ is an even function and hence

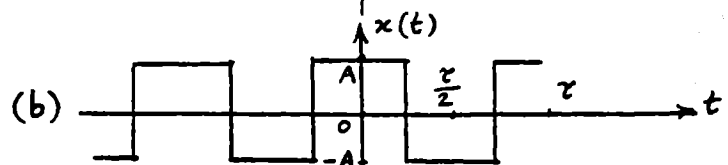
$$b_n = \frac{4}{\tau} \int_0^{\tau/2} x(t) \sin n\omega t dt$$

1.65

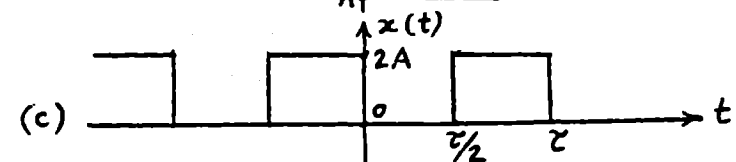
$$x(t) = \begin{cases} -A, & 0 \leq t \leq \frac{\tau}{2} \\ A, & \frac{\tau}{2} \leq t \leq \tau \end{cases}$$



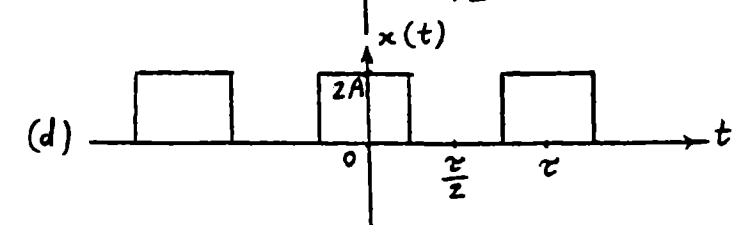
$$x(t) = \begin{cases} A, & 0 \leq t \leq \frac{\tau}{4} \\ -A, & \frac{\tau}{4} \leq t \leq \frac{3\tau}{4} \\ A, & \frac{3\tau}{4} \leq t \leq \tau \end{cases}$$



$$x(t) = \begin{cases} 0, & 0 \leq t \leq \frac{\tau}{2} \\ 2A, & \frac{\tau}{2} \leq t \leq \tau \end{cases}$$



$$x(t) = \begin{cases} 2A, & 0 \leq t \leq \frac{\tau}{4} \\ 0, & \frac{\tau}{4} \leq t \leq \frac{3\tau}{4} \\ 2A, & \frac{3\tau}{4} \leq t \leq \tau \end{cases}$$



(a) $x(-t) = -x(t)$, odd function, hence $a_0 = a_n = 0$.

$$b_n = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t \cdot dt = \frac{2}{\tau} \left[-A \int_0^{\tau/2} \sin n\omega t \cdot dt + A \int_{\tau/2}^{\tau} \sin n\omega t \cdot dt \right]$$

$$= -\frac{2A}{\tau} \left(-\frac{\cos n\omega t}{n\omega} \right)_0^{\tau/2} + \frac{2A}{\tau} \left(-\frac{\cos n\omega t}{n\omega} \right)_{\tau/2}^{\tau}$$

$$= \frac{2A}{\tau n\omega} (2 \cos n\pi - \cos 0 - \cos 2n\pi)$$

$$\therefore x(t) = \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} \sin (2n-1) \omega t$$

(b) $x(-t) = x(t)$, even function, hence $b_n = 0$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2}{\tau} \left[A \cdot (t)_{\tau/4}^{\tau/4} - A (t)_{\tau/4}^{3\tau/4} + A (t)_{3\tau/4}^{\tau} \right] = 0$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t \cdot dt$$

$$= \frac{2A}{\tau n\omega} \left[\sin n\omega t \Big|_0^{\tau/4} - \sin n\omega t \Big|_{\tau/4}^{3\tau/4} + \sin n\omega t \Big|_{3\tau/4}^{\tau} \right]$$

$$= \frac{A}{n\pi} \left[2 \sin \frac{n\pi}{2} - 2 \sin \frac{3n\pi}{2} + \sin 2\pi n \right] = \begin{cases} 4A/n\pi & \text{for } n=1,5,9,\dots \\ -4A/n\pi & \text{for } n=3,7,11,\dots \end{cases}$$

$$\therefore x(t) = \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)} \cos \frac{2\pi(n-1)t}{\tau}$$

(c) $a_0 = \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2}{\tau} \left[0 + 2A (t)_{\tau/2}^{\tau} \right] = 2A$

$$a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt = \frac{4A}{n\omega\tau} (\sin n\omega t)_{\tau/2}^{\tau} = 0$$

$$b_n = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t dt = -\frac{4A}{n\omega\tau} (\cos n\omega t)_{\tau/2}^{\tau}$$

$$= -\frac{4A}{n\omega\tau} (\cos 2\pi n - \cos n\pi)$$

$$\therefore x(t) = -\frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} \sin (2n-1) \omega t \quad \text{with } \omega = 2\pi/\tau.$$

(d) $x(-t) = x(t)$, even function, hence $b_n = 0$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2}{\tau} \left[2A \left(\frac{\tau}{4} - 0 \right) + 2A \left(\tau - \frac{3\tau}{4} \right) \right] = 2A$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt = \frac{4A}{n\omega\tau} \left[(\sin n\omega t)_{\tau/4}^{\tau/4} + (\sin n\omega t)_{3\tau/4}^{\tau} \right]$$

$$= \frac{4A}{n\omega\tau} \left(\sin \frac{n\pi}{2} + \sin 2n\pi - \sin \frac{3n\pi}{2} \right)$$

$$\therefore x(t) = \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)} \cos \frac{2\pi(2n-1)t}{\tau} \quad \text{with } \omega = 2\pi/\tau.$$

1.66 $x(t) = \begin{cases} A \sin \frac{2\pi t}{\tau} & , 0 \leq t \leq \frac{\tau}{2} \\ 0 & , \frac{\tau}{2} \leq t \leq \tau \end{cases}$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} dt = \frac{2A}{\tau} \left(-\frac{\tau}{2\pi} \cos \frac{2\pi t}{\tau} \right)_0^{\tau/2}$$

$$= \frac{2A}{\pi}$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} \cdot \cos n\omega t \cdot dt \quad \dots (E_1)$$

Using the relation $\sin m\omega t \cdot \cos n\omega t = \frac{\sin(m+n)\omega t + \sin(m-n)\omega t}{2}$,

Eg. (E₁) can be rewritten as

$$a_n = \frac{A}{\tau} \int_0^{\pi/\omega} [\sin(1+n)\omega t + \sin(1-n)\omega t] dt$$

When $n=1$, $a_1 = \frac{A}{\tau} \int_0^{\pi/\omega} \sin 2\omega t \cdot dt = 0$

When $n=2, 3, 4, \dots$, $a_n = \frac{A}{\tau} \left[-\frac{\cos(1+n)\omega t}{(1+n)\omega} - \frac{\cos(1-n)\omega t}{(1-n)\omega} \right]_0^{\pi/\omega}$

$$= \frac{A}{2\pi} \left[\frac{1 - \cos(1+n)\pi}{1+n} + \frac{1 - \cos(1-n)\pi}{1-n} \right]$$

$$= \begin{cases} 0 & \text{if } n \text{ is odd} \\ -\frac{2A}{(n-1)(n+1)\pi} & \text{if } n \text{ is even} \end{cases}$$

Similarly $b_n = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} \cos n\omega t dt$

$$= \frac{A}{\tau} \int_0^{\tau/2} [\cos(1-n)\omega t - \cos(1+n)\omega t] dt$$

When $n=1$, $b_1 = \frac{A}{\tau} \int_0^{\pi/\omega} (dt - \cos 2\omega t) dt = \frac{A}{2}$

When $n=2, 3, 4, \dots$, $b_n = \frac{A}{\tau} \left[\frac{\sin(1-n)\omega t}{(1-n)\omega} - \frac{\sin(1+n)\omega t}{(1+n)\omega} \right]_0^{\pi/\omega} = 0$

$$\therefore x(t) = \frac{A}{\pi} + \frac{A}{2} \sin \omega t - \frac{2A}{\pi} \sum_{n=2,4,6,\dots}^{\infty} \frac{\cos n\omega t}{(n^2-1)}$$

$$(1.67) \quad x(t) = \begin{cases} \frac{2At}{\tau} & , \quad 0 \leq t \leq \frac{\tau}{2} \\ -\frac{2At}{\tau} + 2A & , \quad \frac{\tau}{2} \leq t \leq \tau \end{cases}$$

$$\begin{aligned} a_0 &= \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2}{\tau} \left[\int_0^{\tau/2} \frac{2At}{\tau} dt + \int_{\tau/2}^{\tau} \left(-\frac{2At}{\tau} + 2A\right) dt \right] \\ &= \frac{2}{\tau} \left[\frac{2A}{\tau} \cdot \frac{t^2}{2} \Big|_0^{\tau/2} - \frac{2A}{\tau} \cdot \frac{t^2}{2} \Big|_{\tau/2}^{\tau} + 2A \cdot t \Big|_{\tau/2}^{\tau} \right] \\ &= \frac{2}{\tau} \left[\frac{A\tau}{4} - \frac{3A\tau}{4} + A\tau \right] = A \end{aligned}$$

$$\begin{aligned} a_n &= \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt \\ &= \frac{2}{\tau} \left[\int_0^{\tau/2} \frac{2A}{\tau} t \cos n\omega t dt + \int_{\tau/2}^{\tau} \left(-\frac{2A}{\tau} t + 2A\right) \cos n\omega t dt \right] \\ &= \frac{2}{\tau} \left[\frac{2A}{\tau} \left\{ t \frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2} \right\}_0^{\tau/2} \right. \\ &\quad \left. - \frac{2A}{\tau} \left\{ t \frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2} \right\}_{\tau/2}^{\tau} + 2A \left(-\frac{\sin n\omega t}{n\omega} \right)_{\tau/2}^{\tau} \right] \end{aligned}$$

$$\text{As } \tau = \frac{2\pi}{\omega} ,$$

$$\begin{aligned} a_n &= \frac{\omega}{\pi} \left[\frac{A\omega}{\pi n^2\omega^2} \cos n\pi - \frac{A\omega}{\pi n^2\omega^2} - \frac{A\omega}{\pi n^2\omega^2} \cos 2\pi n + \frac{A\omega}{\pi n^2\omega^2} \cos n\pi \right] \\ &= \frac{2A}{n^2\pi^2} (\cos n\pi - 1) = \begin{cases} -\frac{4A}{n^2\pi^2} & , \quad n = 1, 3, 5, \dots \\ 0 & , \quad n = 2, 4, 6, \dots \end{cases} \end{aligned}$$

$$\begin{aligned} b_n &= \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t dt = \frac{2}{\tau} \left[\int_0^{\tau/2} \frac{2A}{\tau} t \sin n\omega t dt \right. \\ &\quad \left. + \int_{\tau/2}^{\tau} \left(-\frac{2A}{\tau} t + 2A\right) \sin n\omega t dt \right] \\ &= \frac{2}{\tau} \left[\frac{2A}{\tau} \left\{ -\frac{t \cos n\omega t}{n\omega} + \frac{\sin n\omega t}{n^2\omega^2} \right\}_0^{\tau/2} \right. \\ &\quad \left. - \frac{2A}{\tau} \left\{ -\frac{t \cos n\omega t}{n\omega} + \frac{\sin n\omega t}{n^2\omega^2} \right\}_{\tau/2}^{\tau} + 2A \left(-\frac{\cos n\omega t}{n\omega} \right)_{\tau/2}^{\tau} \right] \\ &= \frac{\omega}{\pi} \left[-\frac{A}{n\omega} \cos n\pi + \frac{2A}{n\omega} \cos 2\pi n - \frac{A}{n\omega} \cos n\pi - \frac{2A}{n\omega} \cos 2\pi n + \frac{2A}{n\omega} \cos n\pi \right] = 0 \end{aligned}$$

$$\therefore x(t) = \frac{A}{2} - \frac{4A}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \cos n\omega t$$

1.68

$$x(t) = \begin{cases} \frac{4At}{\tau} & , 0 \leq t \leq \frac{\tau}{4} \\ -\frac{4At}{\tau} + 2A & , \frac{\tau}{4} \leq t \leq \frac{3\tau}{4} \\ \frac{4At}{\tau} - 4A & , \frac{3\tau}{4} \leq t \leq \tau \end{cases}$$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2}{\tau} \left[\frac{4A}{\tau} \frac{t^2}{2} \Big|_0^{\tau/4} + \left(-\frac{4A}{\tau} \frac{t^2}{2} + 2A t \right) \Big|_{\tau/4}^{3\tau/4} + \left(\frac{4A}{\tau} \frac{t^2}{2} - 4A t \right) \Big|_{3\tau/4}^{\tau} \right] = 0$$

$$\begin{aligned} a_n &= \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt = \frac{2}{\tau} \left[\frac{4A}{\tau} \int_0^{\tau/4} t \cos n\omega t dt - \frac{4A}{\tau} \int_{\tau/4}^{3\tau/4} t \cos n\omega t dt + 2A \int_{\tau/4}^{3\tau/4} \cos n\omega t dt \right. \\ &\quad \left. + \frac{4A}{\tau} \int_{3\tau/4}^{\tau} t \cos n\omega t dt - 4A \int_{3\tau/4}^{\tau} \cos n\omega t dt \right] \\ &= \frac{2}{\tau} \left[\frac{4A}{\tau} \left\{ t \frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2 \omega^2} \right\} \Big|_0^{\tau/4} - \frac{4A}{\tau} \left\{ t \frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2 \omega^2} \right\} \Big|_{\tau/4}^{3\tau/4} \right. \\ &\quad \left. + 2A \left(\frac{\sin n\omega t}{n\omega} \right) \Big|_{\tau/4}^{3\tau/4} + \frac{4A}{\tau} \left\{ t \frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2 \omega^2} \right\} \Big|_{3\tau/4}^{\tau} - 4A \left(\frac{\sin n\omega t}{n\omega} \right) \Big|_{3\tau/4}^{\tau} \right] \\ &= \frac{\omega}{\pi} \left[\sin \frac{n\pi}{2} \left(\frac{A}{n\omega} + \frac{A}{n\omega} - \frac{2A}{n\omega} \right) + \cos \frac{n\pi}{2} \left(\frac{2A}{\pi n^2 \omega^2} + \frac{2A}{\pi n^2 \omega^2} \right) \right. \\ &\quad \left. + \sin \frac{3n\pi}{2} \left(-\frac{3A}{n\omega} + \frac{2A}{n\omega} - \frac{3A}{n\omega} + \frac{4A}{n\omega} \right) + \cos \frac{3n\pi}{2} \left(-\frac{2A}{\pi n^2 \omega^2} - \frac{2A}{\pi n^2 \omega^2} \right) \right. \\ &\quad \left. + \cos 2\pi n \left(\frac{2A}{\pi n^2 \omega^2} \right) - \cos 0 \left(\frac{2A}{\pi n^2 \omega^2} \right) \right] = 0 \end{aligned}$$

$$\begin{aligned} b_n &= \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t dt = \frac{2}{\tau} \left[\frac{4A}{\tau} \int_0^{\tau/4} t \sin n\omega t dt - \frac{4A}{\tau} \int_{\tau/4}^{3\tau/4} t \sin n\omega t dt \right. \\ &\quad \left. + 2A \int_{\tau/4}^{3\tau/4} \sin n\omega t dt + \frac{4A}{\tau} \int_{3\tau/4}^{\tau} t \sin n\omega t dt - 4A \int_{3\tau/4}^{\tau} \sin n\omega t dt \right] \\ &= \frac{2}{\tau} \left[\frac{4A}{\tau} \left\{ \frac{1}{n^2 \omega^2} \sin n\omega t - \frac{t}{n\omega} \cos n\omega t \right\} \Big|_0^{\tau/4} - \frac{4A}{\tau} \left\{ \frac{1}{n^2 \omega^2} \sin n\omega t \right. \right. \\ &\quad \left. \left. - \frac{t}{n\omega} \cos n\omega t \right\} \Big|_{\tau/4}^{3\tau/4} + 2A \left(-\frac{\cos n\omega t}{n\omega} \right) \Big|_{\tau/4}^{3\tau/4} + \frac{4A}{\tau} \left\{ \frac{1}{n^2 \omega^2} \sin n\omega t \right. \right. \\ &\quad \left. \left. - \frac{t}{n\omega} \cos n\omega t \right\} \Big|_{3\tau/4}^{\tau} - 4A \left(-\frac{\cos n\omega t}{n\omega} \right) \Big|_{3\tau/4}^{\tau} \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{t}{n\omega} \cos n\omega t \left. \begin{array}{l} \tau = \frac{2\pi}{\omega} \\ \frac{3\tau}{4} = \frac{3\pi}{2\omega} \end{array} \right\} -4A \left(-\frac{\cos n\omega t}{n\omega} \right) \left. \begin{array}{l} \tau = \frac{2\pi}{\omega} \\ \frac{3\tau}{4} = \frac{3\pi}{2\omega} \end{array} \right] \\
& = \frac{4A}{\pi^2 n^2} \left(\sin \frac{n\pi}{2} - \sin \frac{3n\pi}{2} \right) = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{8A}{\pi^2 n^2} (-1)^{\frac{n-1}{2}} & \text{if } n \text{ is odd} \end{cases} \\
\therefore x(t) = & \frac{8A}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} (-1)^{\frac{n-1}{2}} \frac{\sin n\omega t}{n^2}
\end{aligned}$$

(1.69) $x(t) = A \left(1 - \frac{t}{\tau}\right), \quad 0 \leq t \leq \tau$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2A}{\tau} \int_0^{\tau} \left(1 - \frac{t}{\tau}\right) dt = \frac{2A}{\tau} \left(t - \frac{t^2}{2\tau}\right)_0^{\tau} = A$$

$$\begin{aligned}
a_n &= \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt = \frac{2A}{\tau} \left(\frac{\sin n\omega t}{n\omega} - \frac{t}{\tau} \frac{\sin n\omega t}{n\omega} - \frac{\cos n\omega t}{\tau n^2 \omega^2} \right)_0^{2\pi/\omega} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
b_n &= \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t dt = \frac{2A}{\tau} \left(-\frac{\cos n\omega t}{n\omega} + \frac{t}{\tau} \frac{\cos n\omega t}{n\omega} - \frac{\sin n\omega t}{\tau n^2 \omega^2} \right)_0^{2\pi/\omega} \\
&= \frac{A}{\pi n}
\end{aligned}$$

$$\therefore x(t) = \frac{A}{2} + \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{\sin n\omega t}{n}$$

(1.70) The truncated series of k terms can be denoted as

$$\bar{x}(t) = \frac{\bar{a}_0}{2} + \sum_{n=1}^k \bar{a}_n \cos n\omega t + \sum_{n=1}^k \bar{b}_n \sin n\omega t \quad (1)$$

with $\bar{x}(t)$ denoting an approximation to the exact $x(t)$ given by Eq. (1.70). The error to be minimized is given by

$$E = \int_{-\pi/\omega}^{\pi/\omega} e^2(t) dt \quad (2)$$

$$\text{where } e(t) = x(t) - \bar{x}(t) \quad (3)$$

and $x(t)$ is the exact value (with infinite series on the right hand side of Eq. (1)). Treating E as a function of the unknowns \bar{a}_n and \bar{b}_n , it can be minimized by setting:

$$\frac{\partial E}{\partial \bar{a}_n} = 2 \int_{-\pi/\omega}^{\pi/\omega} \left\{ x(t) - \bar{x}(t) \right\} \left(-\cos n\omega t \right) dt = 0 \quad (4)$$

$$\frac{\partial E}{\partial \bar{b}_n} = 2 \int_{-\pi/\omega}^{\pi/\omega} \left\{ x(t) - \bar{x}(t) \right\} \left(-\sin n\omega t \right) dt = 0 \quad (5)$$

Rearranging Eq. (4) gives

$$\int_{-\pi/\omega}^{\pi/\omega} x(t) \cos n \omega t dt = \int_{-\pi/\omega}^{\pi/\omega} \bar{x}(t) \cos n \omega t dt \quad (6)$$

Using orthogonality property, the right hand side of Eq. (6) can be expressed as

$$\int_{-\pi/\omega}^{\pi/\omega} \bar{x}(t) \cos n \omega t dt = \begin{cases} 0 & \text{for } m \neq n \\ \frac{\bar{a}_n \pi}{\omega} & \text{for } m = n \end{cases} \quad (7)$$

This leads to

$$\int_{-\pi/\omega}^{\pi/\omega} x(t) \cos n \omega t dt = \frac{\bar{a}_n \pi}{\omega} \quad (8)$$

$$\text{or } \bar{a}_n = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} x(t) \cos n \omega t dt ; n = 0, 1, 2, \dots, k \quad (9)$$

In a similar manner, we can derive:

$$\bar{b}_n = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} x(t) \sin n \omega t dt ; n = 1, 2, \dots, k \quad (10)$$

It can be observed that Eqs. (9) and (10) are similar to those of Eqs. (E.3) and (E.4).

1.71

i	t _i	x _i	n=1		n=2		n=3	
			x _i cos $\frac{2\pi t_i}{0.32}$	x _i sin $\frac{2\pi t_i}{0.32}$	x _i cos $\frac{4\pi t_i}{0.32}$	x _i sin $\frac{4\pi t_i}{0.32}$	x _i cos $\frac{6\pi t_i}{0.32}$	x _i sin $\frac{6\pi t_i}{0.32}$
1	0.02	9	8.3149	3.4442	6.3639	6.3640	3.4441	8.3149
2	0.04	13	9.1924	9.1924	0.0000	13.0000	-9.1924	9.1923
3	0.06	17	6.5056	15.7060	-12.0209	12.0208	-15.7059	-6.5057
4	0.08	29	0.0000	29.0000	-29.0000	0.0000	0.0000	-29.0000
5	0.10	43	-16.4556	39.7267	-30.4053	-30.4059	39.7271	-16.4548
6	0.12	59	-41.7195	41.7191	0.0000	-59.0000	41.7187	41.7199
7	0.14	63	-58.2045	24.1087	44.5482	-44.5472	-24.1101	58.2040
8	0.16	57	-57.0000	0.0000	57.0000	0.0000	-57.0000	0.0000
9	0.18	49	-45.2700	-18.7518	34.6477	34.6487	-18.7505	-45.2705
10	0.20	35	-24.7485	-24.7489	0.0000	35.0000	24.7493	-24.7482
11	0.22	35	-13.3936	-32.3359	-24.7493	24.7482	32.3354	13.3950
12	0.24	41	0.0000	-41.0000	-41.0000	0.0000	0.0000	41.0000
13	0.26	47	17.9866	-43.4221	-33.2333	-33.2347	-43.4229	17.9847
14	0.28	41	28.9917	-28.9911	0.0000	-41.0000	-28.9905	-28.9923
15	0.30	13	12.0105	-4.9747	9.1927	-9.1921	4.9755	-12.0102
16	0.32	7	7.0000	0.0000	7.0000	0.0000	7.0000	0.0000

$$\sum_{i=1}^{16} () \quad 558 \quad -166.7897 \quad -31.3278 \quad -11.6552 \quad -91.5984 \quad -43.2234 \quad 26.8281$$

$$\frac{1}{8} \sum_{i=1}^{16} () \quad 69.75 \quad -20.8487 \quad -3.9160 \quad -1.4569 \quad -11.4498 \quad -5.4029 \quad 3.3535$$

1.72

Speed = 100 rpm

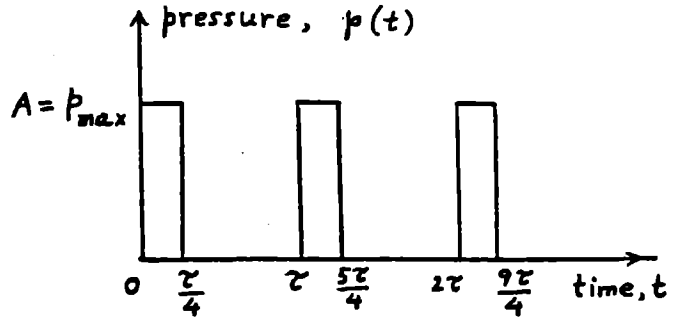
In a minute, a point will be subjected to the maximum pressure,

$$A = p_{\max} = 700 \text{ kPa,}$$

$100 \times 4 = 400$ times.

$$\text{Hence period} = \tau = \frac{60}{400} = 0.15 \text{ sec.}$$

$$p(t) = \begin{cases} A, & 0 \leq t \leq \tau/4 \\ 0, & \tau/4 \leq t \leq \tau \end{cases}$$



$$a_0 = \frac{2}{\tau} \int_0^{\tau} p(t) dt = \frac{2}{\tau} A(t)_0^{\tau/4} = \frac{A}{2} = 350 \text{ kPa}$$

$$a_m = \frac{2}{\tau} \int_0^{\tau} p(t) \cos m \omega t dt = \frac{2A}{\tau} \left(\frac{\sin m \omega t}{m \omega} \right)_0^{\tau/4} = \frac{A}{\pi m} \sin \frac{m \pi}{2}$$

$$b_m = \frac{2}{\tau} \int_0^{\tau} p(t) \sin m \omega t dt = -\frac{2A}{\tau} \left(\frac{\sin m \omega t}{m \omega} \right)_0^{\tau/4} = -\frac{A}{\pi m} \left(\cos \frac{m \pi}{2} - 1 \right)$$

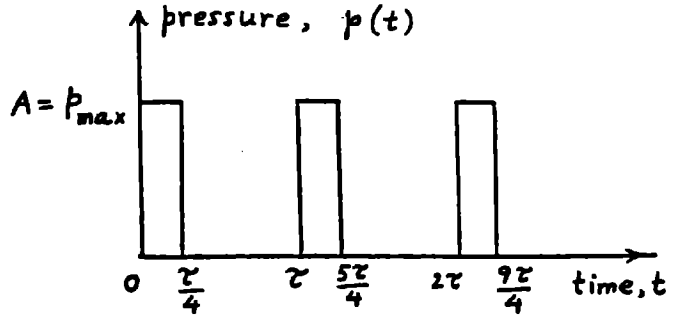
Evaluation of a_m and b_m :

$m = 1$	$m = 2$	$m = 3$
$a_1 = \frac{A}{\pi} \sin \frac{\pi}{2} = \frac{A}{\pi}$ $= 222.817 \text{ kPa}$	$a_2 = \frac{A}{2\pi} \sin \pi = 0$	$a_3 = \frac{A}{3\pi} \sin \frac{3\pi}{2}$ $= -74.272 \text{ kPa}$
$b_1 = -\frac{A}{\pi} \left(\cos \frac{\pi}{2} - 1 \right)$ $= 222.817 \text{ kPa}$	$b_2 = -\frac{A}{2\pi} (\cos \pi - 1)$ $= 222.817 \text{ kPa}$	$b_3 = -\frac{A}{3\pi} \left(\cos \frac{3\pi}{2} - 1 \right)$ $= 74.272 \text{ kPa}$

$$\therefore p(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m \omega t + b_m \sin m \omega t) \text{ kPa}$$

1.73

Speed = 200 rpm
 In a minute, a point will be subjected to the maximum pressure,
 $A = p_{max} = 700 \text{ kPa}$,
 $200 \times 6 = 1200$ times.



Hence period = $\tau = \frac{60}{1200} = 0.05 \text{ sec.}$

$$p(t) = \begin{cases} A, & 0 \leq t \leq \tau/4 \\ 0, & \tau/4 \leq t \leq \tau \end{cases}$$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} p(t) dt = \frac{2}{\tau} A(t)_{\tau/4}^0 = \frac{A}{2} = 350 \text{ kPa}$$

$$a_m = \frac{2}{\tau} \int_0^{\tau} p(t) \cos m \omega t dt = \frac{2A}{\tau} \left(\frac{\sin m \omega t}{m \omega} \right)_0^{\tau/4} = \frac{A}{\pi m} \sin \frac{m \pi}{2}$$

$$b_m = \frac{2}{\tau} \int_0^{\tau} p(t) \sin m \omega t dt = -\frac{2A}{\tau} \left(\frac{\sin m \omega t}{m \omega} \right)_0^{\tau/4} = -\frac{A}{\pi m} \left(\cos \frac{m \pi}{2} - 1 \right)$$

Evaluation of a_m and b_m :

$m = 1$	$m = 2$	$m = 3$
$a_1 = \frac{A}{\pi} \sin \frac{\pi}{2} = \frac{A}{\pi}$ $= 222.817 \text{ kPa}$	$a_2 = \frac{A}{2\pi} \sin \pi = 0$	$a_3 = \frac{A}{3\pi} \sin \frac{3\pi}{2}$ $= -74.272 \text{ kPa}$
$b_1 = -\frac{A}{\pi} \left(\cos \frac{\pi}{2} - 1 \right)$ $= 222.817 \text{ kPa}$	$b_2 = -\frac{A}{2\pi} (\cos \pi - 1)$ $= 222.817 \text{ kPa}$	$b_3 = -\frac{A}{3\pi} \left(\cos \frac{3\pi}{2} - 1 \right)$ $= 74.272 \text{ kPa}$

$$\therefore p(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m \omega t + b_m \sin m \omega t) \text{ kPa}$$

1.74

i	t_i	M_{t_i}	n = 1		n = 2		n = 3	
			$M_{t_i} \cos \frac{2\pi t_i}{0.012}$	$M_{t_i} \sin \frac{2\pi t_i}{0.012}$	$M_{t_i} \cos \frac{4\pi t_i}{0.012}$	$M_{t_i} \sin \frac{4\pi t_i}{0.012}$	$M_{t_i} \cos \frac{6\pi t_i}{0.012}$	$M_{t_i} \sin \frac{6\pi t_i}{0.012}$
1	0.0005	770	743.7627	199.2912	666.8391	385.0010	544.4712	544.4731
2	0.0010	810	701.4802	405.0007	404.9988	701.4812	0.0000	810.0000
3	0.0015	850	601.0398	601.0417	0.0000	850.0000	-601.0442	601.0373
4	0.0020	910	454.9978	788.0845	-455.0041	788.0808	-910.0000	0.0000

5	0.0025	1010	261.4043	975.5859	-874.689	504.995	-714.171	-714.184
6	0.0030	1170	0.0000	1170.0000	-1170.000	0.000	0.000	-1170.000
7	0.0035	1370	-354.5874	1323.3169	-1186.449	-685.010	968.748	-968.725
8	0.0040	1610	-805.0073	1394.2966	-804.987	-1394.309	1610.000	0.000
9	0.0045	1890	-1336.4407	1336.4229	0.000	-1890.000	1336.410	1336.454
10	0.0050	1750	-1515.5491	874.7922	875.019	-1515.534	0.000	1750.000
11	0.0055	1630	-1574.4619	421.8647	1411.634	-814.979	-1152.608	1152.560
12	0.0060	1510	-1510.0000	0.0000	1510.000	0.000	-1510.000	0.000
13	0.0065	1390	-1342.6345	-359.7671	1203.767	695.014	-982.858	-982.898
14	0.0070	1290	-1117.1677	-645.0088	644.982	1117.183	0.000	-1290.000
15	0.0075	1190	-841.4492	-841.4648	0.000	1190.000	841.479	-841.435
16	0.0080	1110	-554.9897	-961.2942	-555.021	961.276	1110.000	0.000
17	0.0085	1050	-271.7498	-1014.2249	-909.337	524.982	742.440	742.485
18	0.0090	990	0.0000	-990.0000	-990.000	0.000	0.000	990.000
19	0.0095	930	240.7123	-898.3081	-805.393	-465.018	-657.633	657.586
20	0.0100	890	445.0095	-770.7571	-444.981	-770.773	-890.000	0.000
21	0.0105	850	601.0478	-601.0337	0.000	-850.000	-601.022	-601.060
22	0.0110	810	701.4868	-404.9895	405.022	-701.468	0.000	-810.000
23	0.0115	770	743.7659	-199.2798	666.851	-384.980	544.500	-544.444
24	0.0120	750	750.0000	0.0000	750.000	0.000	750.000	0.000
$\sum_{i=1}^{24} ()$		27,300	-4,979.3242	1,803.7673	343.270	-1,754.047	428.734	661.855
$\frac{1}{12} \sum_{i=1}^{24}$		2,275	-414.9436	150.3139	28.606	-146.171	35.728	55.155

1.75

i	t _i	x _i	n = 1		n = 2		n = 3	
			x _i cos $\frac{2\pi t_i}{0.6}$	x _i sin $\frac{2\pi t_i}{0.6}$	x _i cos $\frac{4\pi t_i}{0.6}$	x _i sin $\frac{4\pi t_i}{0.6}$	x _i cos $\frac{6\pi t_i}{0.6}$	x _i sin $\frac{6\pi t_i}{0.6}$
1	0.025	9.00	8.69	2.33	7.79	4.50	6.36	6.36
2	0.050	17.00	14.72	8.50	8.50	14.72	0.00	17.00
3	0.075	23.00	16.26	16.26	0.00	23.00	-16.26	16.26
4	0.100	25.00	12.50	21.65	-12.50	21.65	-25.00	0.00
5	0.125	26.00	6.73	25.11	-22.52	13.00	-18.38	-18.38
6	0.150	28.00	0.00	28.00	-28.00	0.00	0.00	-28.00
7	0.175	33.00	-8.54	31.88	-28.58	-16.50	23.33	-23.33
8	0.200	35.00	-17.50	30.31	-17.50	-30.31	35.00	0.00
9	0.225	34.00	-24.04	24.04	0.00	-34.00	24.04	24.04
10	0.250	29.00	-25.11	14.50	14.50	-25.11	0.00	29.00
11	0.275	24.00	-23.18	6.21	20.78	-12.00	-16.97	16.97
12	0.300	26.00	-26.00	0.00	26.00	0.00	-26.00	0.00
13	0.325	32.00	-30.91	-8.28	27.71	16.00	-22.63	-22.63
14	0.350	40.00	-34.64	-20.00	20.00	34.64	0.00	-40.00
15	0.375	18.00	-12.73	-12.73	0.00	18.00	12.73	-12.73
16	0.400	8.00	-4.00	-6.93	-4.00	6.93	8.00	0.00
17	0.425	-5.00	1.29	4.83	4.33	-2.50	-3.54	-3.54
18	0.450	-14.00	0.00	14.00	14.00	0.00	0.00	-14.00
19	0.475	-28.00	-7.25	27.05	24.25	14.00	19.80	-19.80
20	0.500	-37.00	-18.50	32.04	18.50	32.04	37.00	0.00
21	0.525	-33.00	-23.33	23.33	0.00	33.00	23.33	23.34
22	0.550	-29.00	-25.11	14.50	-14.50	25.11	0.00	29.00

23	0.575	-22.00	-21.25	5.69	-19.05	11.00	-15.56	15.56
24	0.600	0.00	0.00	0.00	0.00	0.00	0.00	0.00

$$\sum_{i=1}^{24} () \quad 239.00 \quad -241.90 \quad 282.30 \quad 39.72 \quad 147.18 \quad 45.26 \quad -4.88$$

$$\frac{1}{12} \sum_{i=1}^{24} () \quad 19.92 \quad -20.16 \quad 23.53 \quad 3.31 \quad 12.26 \quad 3.77 \quad -0.41$$

1.76

```

%=====
%
%Program1.m
%Program for calling the subroutine FORIER
%
%=====
%Run "Program1.m" in MATLAB Command Window. Program1.m and forier.m should be
%in the same file folder, and set the path to this folder
%Following 6 lines contain problem-dependent data
n=16;
m=3;
time=0.32;
x=[9 13 17 29 43 59 63 57 49 35 35 41 47 41 13 7];
t=0.02:0.02:0.32;
%end of problem-dependent data
%Following line calls subroutine forier.m
[azero,a,b,xsin,xcos]=forier(n,m,time,x,t);
%following outputs data
fprintf('Fourier series expansion of the function x(t)\n\n');
fprintf('Data:\n\n');
fprintf('Number of data points in one cycle = %3.0f \n',n);
fprintf(' \n');
fprintf('Number of Fourier Coefficients required = %3.0f \n',m);
fprintf(' \n');
fprintf('Time period = %8.6e \n\n',time);
fprintf('Station i      ')
fprintf('Time at station i: t(i)      ')
fprintf('x(i) at t(i)')
for i=1:n
    fprintf('\n %8d%25.6e%27.6e ',i,t(i),x(i));
end
fprintf(' \n\n');
fprintf('Results of Fourier analysis:\n\n');
fprintf('azero=%8.6e \n\n',azero);
fprintf('values of i      a(i)                b(i)\n');
for i=1:m
    fprintf('%10.0g   %8.6e%20.6e \n',i,a(i),b(i));
end

```

```

%=====
%
%Subroutine forier.m
%
%=====
function [azero,a,b,xsin,xcos]=forier(n,m,time,x,t)
pi=3.1416;
sumz=0.0;
for i=1:n
    sumz=sumz+x(i);
end
azero=2.0*sumz/n;
for ii=1:m
    sums=0.0;
    sumc=0.0;
    for i=1:n
        theta=2.0*pi*t(i)*ii/time;
        xcos(i)=x(i)*cos(theta);
        xsin(i)=x(i)*sin(theta);
        sums=sums+xsin(i);
        sumc=sumc+xcos(i);
    end
    a(ii)=2.0*sumc/n;
    b(ii)=2.0*sums/n;
end

>> program1
Fourier series expansion of the function x(t)

Data:

Number of data points in one cycle = 16
Number of Fourier Coefficients required = 3
Time period = 3.200000e-001

Station i      Time at station i: t(i)      x(i) at t(i)
1              2.000000e-002              9.000000e+000
2              4.000000e-002              1.300000e+001
3              6.000000e-002              1.700000e+001
4              8.000000e-002              2.900000e+001
5              1.000000e-001              4.300000e+001
6              1.200000e-001              5.900000e+001
7              1.400000e-001              6.300000e+001
8              1.600000e-001              5.700000e+001
9              1.800000e-001              4.900000e+001
10             2.000000e-001              3.500000e+001
11             2.200000e-001              3.500000e+001
12             2.400000e-001              4.100000e+001
13             2.600000e-001              4.700000e+001
14             2.800000e-001              4.100000e+001
15             3.000000e-001              1.300000e+001
16             3.200000e-001              7.000000e+000

Results of Fourier analysis:

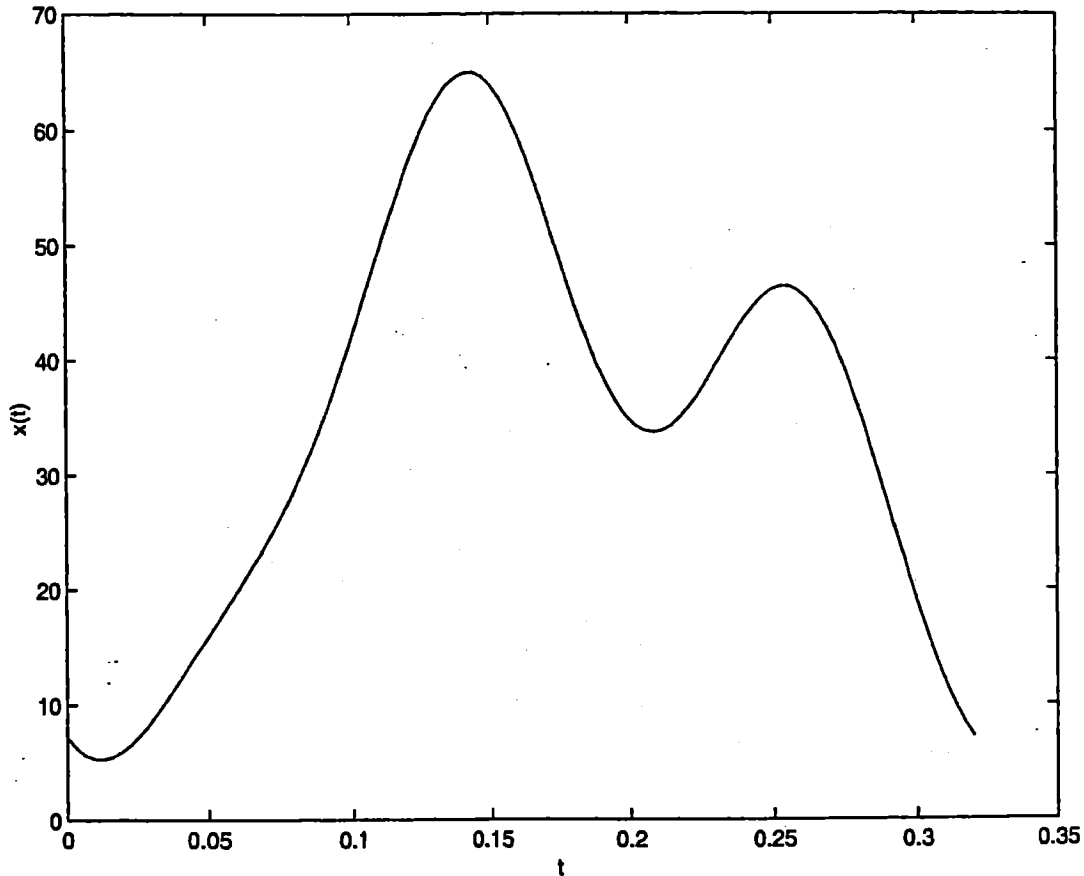
azero=6.975000e+001

values of i      a(i)              b(i)
1              -2.084870e+001    -3.915985e+000
2              -1.456887e+000    -1.144979e+001
3              -5.402900e+000     3.353473e+000

```

1.77

```
% Ex1_77.m
for i = 1: 101
    t(i) = 0.32*(i-1)/100;
    x(i) = 34.875 - 20.8487*cos(19.635*t(i)) - 3.9160*sin(19.635*t(i))...
        - 1.4569*cos(39.27*t(i)) - 11.4498*sin(39.27*t(i))...
        - 5.4029*cos(58.905*t(i)) + 3.3535*sin(58.905*t(i));
end
plot(t,x)
xlabel('t');
ylabel('x(t)');
```



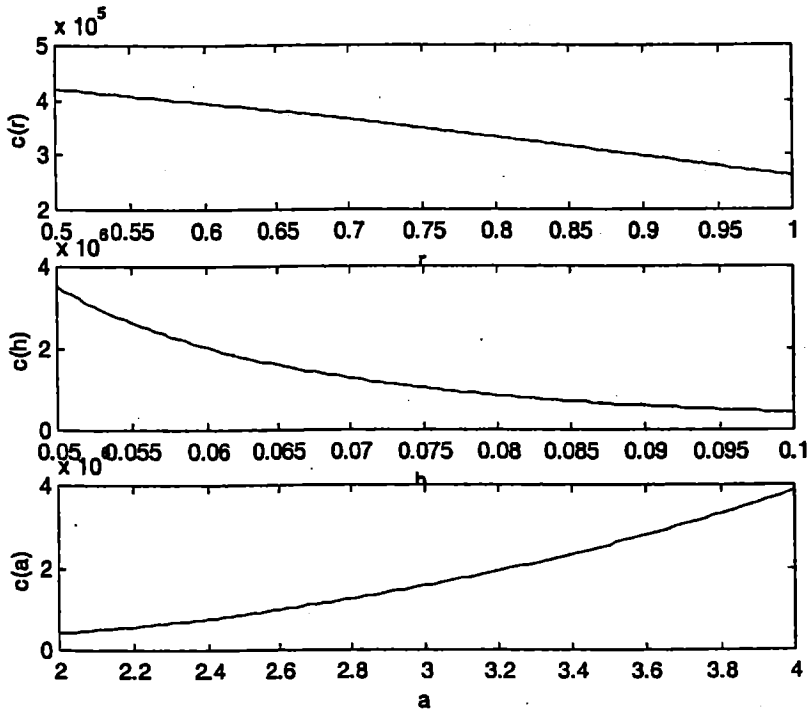
1.78

```
% Ex1_78.m
u = 0.3445;
l = 10;
h0 = 0.1;
a0 = 2;
r0 = 0.5;
% First case, r changes
for i = 1:101
    r(i) = 0.5 + (i-1)*0.5/100;
    c1(i) = ( 6*pi*u*1/(h0^3) ) * ( (a0 - h0/2)^2 - r(i)^2 )...
        * ( (a0^2-r(i)^2)/(a0-h0/2) - h0 );
end
```

```

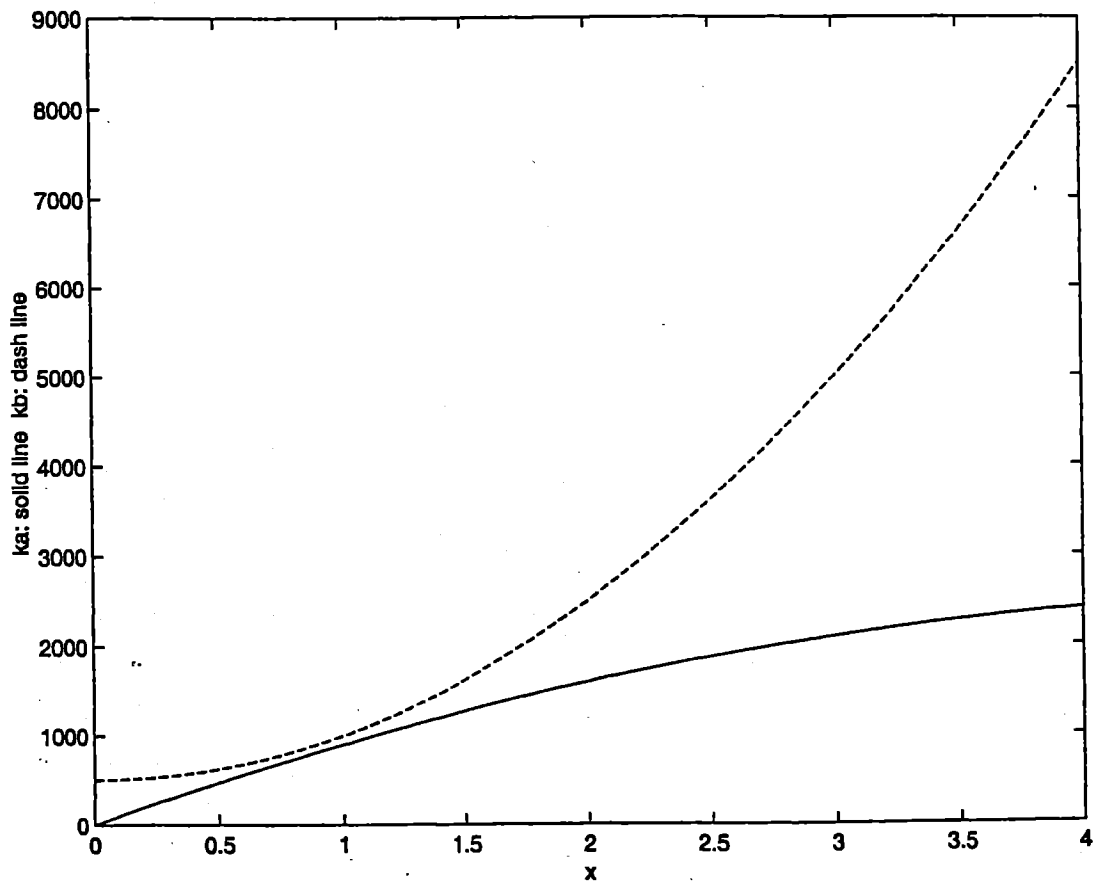
% Second case, h changes
for i = 1:101
    h(i) = 0.05 + (i-1)*0.05/100;
    c2(i) = ( 6*pi*u*1/(h(i)^3) ) * ( (a0 - h(i)/2)^2 - r0^2 )...
            * ( (a0^2-r0^2)/(a0-h(i)/2) - h(i) );
end
% Third case, a changes
for i = 1:101
    a(i) = 2 + (i-1)*2/100;
    c3(i) = ( 6*pi*u*1/(h0^3) ) * ( (a(i) - h0/2)^2 - r0^2 )...
            * ( (a(i)^2-r0^2)/(a(i)-h0/2) - h0 );
end
subplot(311);
plot(r,c1);
xlabel('r');
ylabel('c(r)');
subplot(312);
plot(h,c2);
xlabel('h');
ylabel('c(h)');
subplot(313);
plot(a,c3);
xlabel('a');
ylabel('c(a)');

```



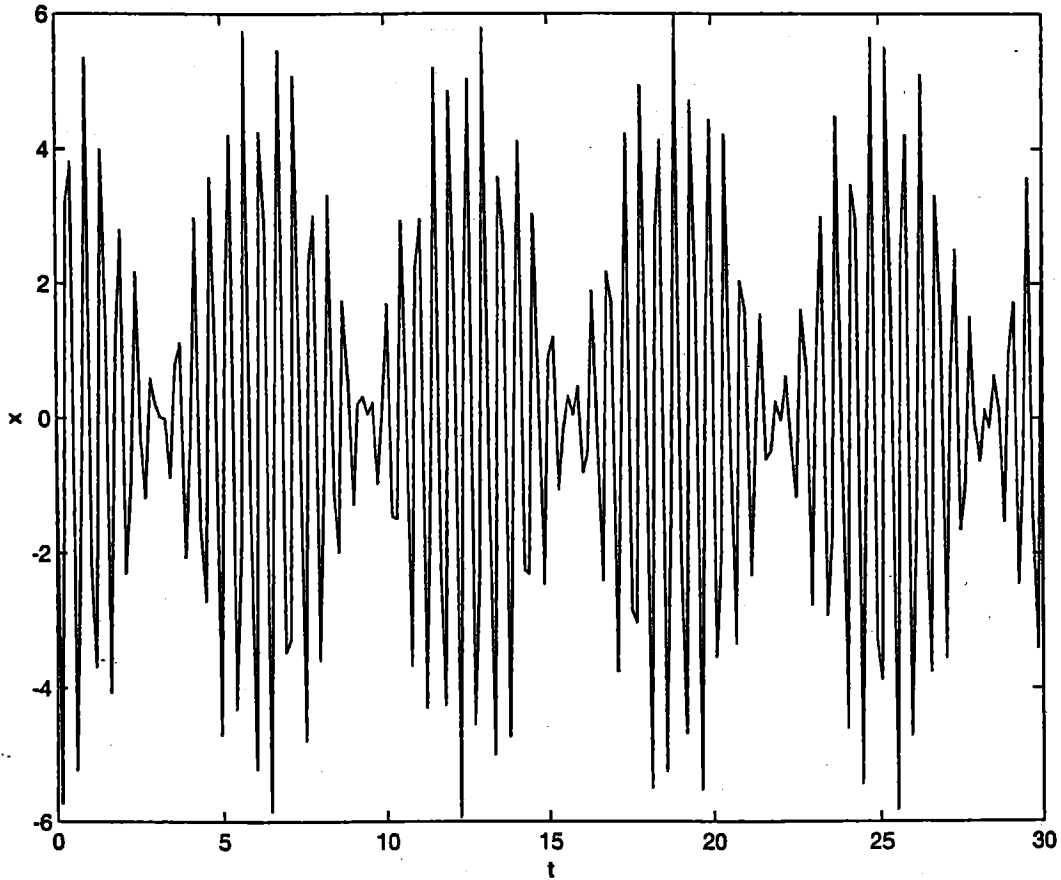
1.79

```
% Ex1_79.m
for i = 1:101
    x(i) = (i-1)*4/100;
    ka(i) = 1000*x(i) - 100*x(i)^2;
    kb(i) = 500 + 500 *x(i)^2;
end
plot(x,ka);
hold on
plot(x,kb, '--');
xlabel('x');
ylabel('ka: solid line kb: dash line');
```



1.80

```
% Ex1_80.m
for i = 1:201
    t(i) = (i-1)*30/200;
    x1(i) = 3*sin(30*t(i));
    x2(i) = 3*sin(29*t(i));
    x(i) = x1(i) + x2(i);
end
plot(t,x);
xlabel('t');
ylabel('x');
```



1.81

Results of Ex1_81

Please input the data:

Please input n:

24

Please input m:

3

Please input time:

0.012

Please input the value of x[i], i = 0, ... n-1:

770 810 850 910 1010 1170 1370 1610 1890 1750 1630 1510 1390 1290 1190
1110

1050 990 930 890 850 810 770 750

Please input the value of t[i], i = 0, ... n-1:

0.0005 0.001 0.0015 0.002 0.0025 0.003 0.0035 0.004
0.0045 0.005 0.0055 0.006 0.0065 0.007 0.0075 0.008
0.0085 0.009 0.0095 0.010 0.0105 0.011 0.0115 0.012

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

NUMBER OF DATA POINTS IN ONE CYCLE = 24

NUMBER OF FOURIER COEFFICIENTS REQUIRED = 3

TIME PERIOD = 1.200000e-002

TIME AT VARIOUS STATIONS, T(I) =

5.000000e-004	1.000000e-003	1.500000e-003	2.000000e-003
2.500000e-003	3.000000e-003	3.500000e-003	4.000000e-003
4.500000e-003	5.000000e-003	5.500000e-003	6.000000e-003
6.500000e-003	7.000000e-003	7.500000e-003	8.000000e-003
8.500000e-003	9.000000e-003	9.500000e-003	1.000000e-002
1.050000e-002	1.100000e-002	1.150000e-002	1.200000e-002

KNOWN VALUES OF X(I) AT T(I) =

7.700000e+002	8.100000e+002	8.500000e+002	9.100000e+002
1.010000e+003	1.170000e+003	1.370000e+003	1.610000e+003
1.890000e+003	1.750000e+003	1.630000e+003	1.510000e+003
1.390000e+003	1.290000e+003	1.190000e+003	1.110000e+003
1.050000e+003	9.900000e+002	9.300000e+002	8.900000e+002
8.500000e+002	8.100000e+002	7.700000e+002	7.500000e+002

RESULTS OF FOURIER ANALYSIS:

AZERO = 2.275000e+003

VALUES OF I, A(I) AND B(I) ARE

1	-4.149436e+002	1.503138e+002
2	2.860518e+001	-1.461703e+002
3	3.572730e+001	5.515471e+001

1.82

Results of Ex1_82

Please input the data:

Please input n:

16

Please input m:

3

Please input time:

0.32

Please input the value of x[i], i = 0, ... n-1:

9 13 17 29 43 59 63 57 49 35 35 41 47 41 13 7

Please input the value of t[i], i = 0, ... n-1:

0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16

0.18 0.20 0.22 0.24 0.26 0.28 0.30 0.32

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

NUMBER OF DATA POINTS IN ONE CYCLE = 16

NUMBER OF FOURIER COEFFICIENTS REQUIRED = 3

TIME PERIOD = 3.200000e-001

TIME AT VARIOUS STATIONS, T(I) =

2.000000e-002	4.000000e-002	6.000000e-002	8.000000e-002
1.000000e-001	1.200000e-001	1.400000e-001	1.600000e-001
1.800000e-001	2.000000e-001	2.200000e-001	2.400000e-001
2.600000e-001	2.800000e-001	3.000000e-001	3.200000e-001

KNOWN VALUES OF X(I) AT T(I) =

9.000000e+000	1.300000e+001	1.700000e+001	2.900000e+001
4.300000e+001	5.900000e+001	6.300000e+001	5.700000e+001
4.900000e+001	3.500000e+001	3.500000e+001	4.100000e+001
4.700000e+001	4.100000e+001	1.300000e+001	7.000000e+000

RESULTS OF FOURIER ANALYSIS:

AZERO = 6.975000e+001

VALUES OF I, A(I) AND B(I) ARE

1	-2.084870e+001	-3.915985e+000
2	-1.456887e+000	-1.144979e+001
3	-5.402900e+000	3.353473e+000

1.83

Results of Ex1_83

Please input the data:

Please input n:

24

Please input m:

6

Please input time:

0.6

Please input the value of x[i], i = 0, ... n-1:

9 17 23 25 26 28 33 35 34 29 24 26 32 40 18 8 -5 -14 -28 -37 -33 -29 -
22 0

Please input the value of t[i], i = 0, ... n-1:

0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.30
0
0.325 0.350 0.375 0.400 0.425 0.450 0.475 0.500 0.525 0.550 0.575 0.60
0

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

NUMBER OF DATA POINTS IN ONE CYCLE = 24

NUMBER OF FOURIER COEFFICIENTS REQUIRED = 6

TIME PERIOD = 6.000000e-001

TIME AT VARIOUS STATIONS, T(I) =

2.500000e-002	5.000000e-002	7.500000e-002	1.000000e-001
1.250000e-001	1.500000e-001	1.750000e-001	2.000000e-001
2.250000e-001	2.500000e-001	2.750000e-001	3.000000e-001
3.250000e-001	3.500000e-001	3.750000e-001	4.000000e-001
4.250000e-001	4.500000e-001	4.750000e-001	5.000000e-001
5.250000e-001	5.500000e-001	5.750000e-001	6.000000e-001

KNOWN VALUES OF X(I) AT T(I) =

9.000000e+000	1.700000e+001	2.300000e+001	2.500000e+001
2.600000e+001	2.800000e+001	3.300000e+001	3.500000e+001
3.400000e+001	2.900000e+001	2.400000e+001	2.600000e+001
3.200000e+001	4.000000e+001	1.800000e+001	8.000000e+000
-5.000000e+000	-1.400000e+001	-2.800000e+001	-3.700000e+001
-3.300000e+001	-2.900000e+001	-2.200000e+001	0.000000e+000

RESULTS OF FOURIER ANALYSIS:

AZERO = 1.991667e+001

VALUES OF I, A(I) AND B(I) ARE

1	-2.015867e+001	2.352528e+001
2	3.309933e+000	1.226463e+001
3	3.771938e+000	-4.063822e-001
4	-9.584350e-001	3.247643e+000
5	1.137750e+000	-1.871580e+000
6	-1.166755e+000	1.249975e+000

1.84

Results of Ex1_84

Please input the data:

Please input n:

14

Please input m:

10

Please input time:

0.35

Please input the value of x[i], i = 0, ... n-1:

0.45 -0.8 -0.9 -0.6 -0.75 -0.7 0.55 1.75 1.65 0.25 -1.1 -1.4 -1.05 0.0

Please input the value of t[i], i = 0, ... n-1:

0.025 0.05 0.075 0.1 0.125 0.15 0.175 0.2 0.225 0.25 0.275 0.3 0.325 0.35

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

NUMBER OF DATA POINTS IN ONE CYCLE = 14

NUMBER OF FOURIER COEFFICIENTS REQUIRED = 10

TIME PERIOD = 3.500000e-001

TIME AT VARIOUS STATIONS, T(I) =

2.500000e-002	5.000000e-002	7.500000e-002	1.000000e-001
1.250000e-001	1.500000e-001	1.750000e-001	2.000000e-001
2.250000e-001	2.500000e-001	2.750000e-001	3.000000e-001
3.250000e-001	3.500000e-001	2.105353e-314	3.416413e-312

KNOWN VALUES OF X(I) AT T(I) =

4.500000e-001	-8.000000e-001	-9.000000e-001	-6.000000e-001
-7.500000e-001	-7.000000e-001	5.500000e-001	1.750000e+000
1.650000e+000	2.500000e-001	-1.100000e+000	-1.400000e+000
-1.050000e+000	0.000000e+000	2.105353e-314	1.190440e-311

RESULTS OF FOURIER ANALYSIS:

AZERO = -3.785714e-001

VALUES OF I, A(I) AND B(I) ARE

1	-6.195109e-001	-3.504626e-001
2	4.624270e-001	9.240901e-001
3	4.149472e-001	-1.712710e-001
4	2.227148e-002	2.469142e-001
5	-4.543377e-002	1.043036e-001
6	-2.043688e-002	5.600801e-002
7	-5.000000e-002	4.775167e-006
8	-2.043367e-002	-5.599619e-002
9	-4.542944e-002	-1.042959e-001
10	2.228333e-002	-2.469100e-001

1.85

The main program and the output are shown below.

```

=====
C
C MAIN PROGRAM FOR CALLING THE SUBROUTINE FORIER
C
C =====
C FOLLOWING 6 LINES NEED TO BE CHANGED FOR A DIFFERENT PROBLEM
  DIMENSION X(24), T(24), XSIN(24), XCOS(24), A(5), B(5)
  DATA N, M, TIME /24, 5, 0.012/
  DATA X/770, 810, 850, 910, 1010, 1170, 1370, 1610, 1890, 1750, 1630, 1510,
  2 1390, 1290, 1190, 1110, 1050, 990, 930, 890, 850, 810, 770, 750/
  DATA T /.0005, .001, .0015, .002, .0025, .003, .0035, .004, .0045, .005,
  2 .0055, .006, .0065, .007, .0075, .008, .0085, .009, .0095, .01, .0105,
  3 .011, .0115, .012/
C END OF PROBLEM-DEPENDENT DATA
  CALL FORIER (N, M, TIME, X, T, AZERO, A, B, XSIN, XCOS)
  PRINT 100
100  FORMAT (//, 46H FOURIER SERIES EXPANSION OF THE FUNCTION X(T), //)
  PRINT 200, N, M, TIME

200  FORMAT (6H DATA: , //, 37H NUMBER OF DATA POINTS IN ONE CYCLE =, I5,
  2 /, 42H NUMBER OF FOURIER COEFFICIENTS REQUIRED =, I5, /,
  3 14H TIME PERIOD =, E15.8)
  PRINT 300, (T(I), I=1, N)
300  FORMAT (/, 33H TIME AT VARIOUS STATIONS, T(I) =, /, (4E15.8, 1X))
  PRINT 400, (X(I), I=1, N)
400  FORMAT (/, 31H KNOWN VALUES OF X(I) AT T(I) =, /, (4E15.8, 1X))
  PRINT 500
500  FORMAT (//, 29H RESULTS OF FOURIER ANALYSIS: //)
  PRINT 600, AZERO
600  FORMAT (8H AZERO =, 2X, E15.8, //, 31H VALUES OF I, A(I) AND B(I) ARE
  2 , /)
  DO 700, I =1, M
700  PRINT 800, I, A(I), B(I)
800  FORMAT (I5, 2X, E15.8, 2X, E15.8)
  STOP
  END

```

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

```

NUMBER OF DATA POINTS IN ONE CYCLE = 24
NUMBER OF FOURIER COEFFICIENTS REQUIRED = 5
TIME PERIOD = 0.11999998E-01

```

TIME AT VARIOUS STATIONS, T(I) =

0.50000008E-03 0.99999993E-03 0.15000000E-02 0.20000001E-02
0.24999999E-02 0.30000000E-02 0.35000001E-02 0.40000007E-02
0.45000017E-02 0.49999990E-02 0.55000000E-02 0.60000010E-02
0.64999983E-02 0.69999993E-02 0.75000003E-02 0.80000013E-02
0.84999986E-02 0.89999996E-02 0.95000006E-02 0.10000002E-01
0.10499999E-01 0.11000000E-01 0.11500001E-01 0.11999998E-01

KNOWN VALUES OF X(I) AT T(I) =

0.77000000E+03 0.81000000E+03 0.85000000E+03 0.91000000E+03
0.10100000E+04 0.11700000E+04 0.13700000E+04 0.16100000E+04
0.18900000E+04 0.17500000E+04 0.16300000E+04 0.15100000E+04
0.13900000E+04 0.12900000E+04 0.11900000E+04 0.11100000E+04
0.10500000E+04 0.99000000E+03 0.93000000E+03 0.89000000E+03
0.85000000E+03 0.81000000E+03 0.77000000E+03 0.75000000E+03

RESULTS OF FOURIER ANALYSIS:

AZERO = 0.22750000E+04

VALUES OF I, A(I) AND B(I) ARE

1	-0.41494360E+03	0.15031395E+03
2	0.28605835E+02	-0.14617058E+03
3	0.35727844E+02	0.55154602E+02
4	-0.40830078E+02	0.14440117E+01
5	0.11577332E+02	-0.16659973E+02

1.86 The main program and the output are given below.

```
-----  
C  
C MAIN PROGRAM FOR CALLING THE SUBROUTINE FORIER  
C  
C =====  
C FOLLOWING 6 LINES NEED TO BE CHANGED FOR A DIFFERENT PROBLEM  
  DIMENSION X(16), T(16), XSIN(16), XCOS(16), A(5), B(5)  
  DATA N, M, TIME /16, 5, 0.32/  
  DATA X /9., 13., 17., 29., 43., 59., 63., 57., 49., 35., 35., 41., 47., 41.,  
  2 13., 7. /  
  DATA T /.02., .04., .06., .08., .1., .12., .14., .16., .18., .2., .22., .24., .26., .28.,  
  2 .3., .32/  
C END OF PROBLEM-DEPENDENT DATA  
  CALL FORIER (N, M, TIME, X, T, AZERO, A, B, XSIN, XCOS)  
  PRINT 100  
100  FORMAT (//, 46H FOURIER SERIES EXPANSION OF THE FUNCTION X(T), //)  
  PRINT 200, N, M, TIME  
200  FORMAT (6H DATA: , //, 37H NUMBER OF DATA POINTS IN ONE CYCLE =, )5,  
  2 /, 42H NUMBER OF FOURIER COEFFICIENTS REQUIRED =, )5, /,  
  3 14H TIME PERIOD =, E15.8)  
  PRINT 300, (T(I); I=1, N)  
300  FORMAT (/, 33H TIME AT VARIOUS STATIONS, T(I) =, /, (4E15.8, 1X))  
  PRINT 400, (X(I), I=1, N)  
400  FORMAT (/, 31H KNOWN VALUES OF X(I) AT T(I) =, /, (4E15.8, 1X))  
  PRINT 500
```

```

500  FORMAT (//,29H RESULTS OF FOURIER ANALYSIS:,/)
      PRINT 600, AZERO
600  FORMAT (8H AZERO =,2X,E15.8,//,31H VALUES OF I, A(I) AND B(I) ARE
2,/)
      DO 700, I =1,M
700  PRINT 800, I, A(I), B(I)
800  FORMAT (I5,2X,E15.8,2X,E15.8)
      STOP
      END

```

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

```

NUMBER OF DATA POINTS IN ONE CYCLE = 16
NUMBER OF FOURIER COEFFICIENTS REQUIRED = 5
TIME PERIOD = 0.31999999E+00

```

TIME AT VARIOUS STATIONS, T(I) =

```

0.20000000E-01 0.39999999E-01 0.59999999E-01 0.79999983E-01
0.10000002E+00 0.12000000E+00 0.13999999E+00 0.16000003E+00
0.18000001E+00 0.19999999E+00 0.22000003E+00 0.24000001E+00
0.25999999E+00 0.27999997E+00 0.30000001E+00 0.31999999E+00

```

KNOWN VALUES OF X(I) AT T(I) =

```

0.90000000E+01 0.13000000E+02 0.17000000E+02 0.29000000E+02
0.43000000E+02 0.59000000E+02 0.63000000E+02 0.57000000E+02
0.49000000E+02 0.35000000E+02 0.35000000E+02 0.41000000E+02
0.47000000E+02 0.41000000E+02 0.13000000E+02 0.70000000E+01

```

RESULTS OF FOURIER ANALYSIS:

AZERO = 0.59750000E+02

VALUES OF I, A(I) AND B(I) ARE

```

1  -0.20848709E+02  -0.39159737E+01
2  -0.14568996E+01  -0.11449797E+02
3  -0.54029312E+01   0.33535175E+01
4  -0.17500381E+01   0.24999523E+01
5  -0.26129246E-01   0.10607624E+01

```

1.87 The main program and the output are shown below.

```

C
C MAIN PROGRAM FOR CALLING THE SUBROUTINE FORIER
C
C ===== : : :
C FOLLOWING 6 LINES NEED TO BE CHANGED FOR A DIFFERENT PROBLEM
  DIMENSION X(24), T(24), XSIN(24), XCOS(24), A(6), B(6)
  DATA N, M, TIME /24, 6, 0.6/
  DATA X/9, 17, 23, 25, 26, 28, 33, 35, 34, 29, 24, 26, 32, 40, 18, 8, -5, -14,
  2 -28, -37, -33, -29, -22, 0/

```

```

DATA T / .025, .05, .075, .1, .125, .15, .175, .2, .225, .25, .275, .3, .325,
2 .35, .375, .4, .425, .45, .475, .5, .525, .55, .575, .6/
C END OF PROBLEM-DEPENDENT DATA
CALL FORIER (N,M,TIME,X,T,AZERO,A,B,XSIN,XCOS)
PRINT 100
100 FORMAT (//,46H FOURIER SERIES EXPANSION OF THE FUNCTION X(T),//)
PRINT 200, N,M,TIME
200 FORMAT (6H DATA:,//,37H NUMBER OF DATA POINTS IN ONE CYCLE =,),,
2 /,42H NUMBER OF FOURIER COEFFICIENTS REQUIRED =,15,/,
3 14H TIME PERIOD =,E15.8)
PRINT 300, (T(I),I=1,N)
300 FORMAT (/,33H TIME AT VARIOUS STATIONS, T(I) =,/, (4E15.8,1X))
PRINT 400, (X(I),I=1,N)
400 FORMAT (/,31H KNOWN VALUES OF X(I) AT T(I) =,/, (4E15.8,1X))
PRINT 500
500 FORMAT (//,29H RESULTS OF FOURIER ANALYSIS:,//)
PRINT 600, AZERO
600 FORMAT (8H AZERO =,2X,E15.8,//,31H VALUES OF I, A(I) AND B(I) ARE:
2 ,/)
DO 700, I =1,M
700 PRINT 800, I, A(I), B(I)
800 FORMAT (15,2X,E15.8,2X,E15.8)
STOP
END

```

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

NUMBER OF DATA POINTS IN ONE CYCLE = 24
NUMBER OF FOURIER COEFFICIENTS REQUIRED = 6
TIME PERIOD = 0.60000002E+00

TIME AT VARIOUS STATIONS, T(I) =
0.24999999E-01 0.50000001E-01 0.74999988E-01 0.10000002E+00
0.12500000E+00 0.14999998E+00 0.17500001E+00 0.19999999E+00
0.22500002E+00 0.25000000E+00 0.27499998E+00 0.30000001E+00
0.32499999E+00 0.35000002E+00 0.37500000E+00 0.39999998E+00
0.42500001E+00 0.44999999E+00 0.47500002E+00 0.50000000E+00
0.52499998E+00 0.55000001E+00 0.57499999E+00 0.60000002E+00

KNOWN VALUES OF X(I) AT T(I) =
0.90000000E+01 0.17000000E+02 0.23000000E+02 0.25000000E+02
0.26000000E+02 0.28000000E+02 0.33000000E+02 0.35000000E+02
0.34000000E+02 0.29000000E+02 0.24000000E+02 0.26000000E+02
0.32000000E+02 0.40000000E+02 0.18000000E+02 0.80000000E+01
-0.50000000E+01 -0.14000000E+02 -0.28000000E+02 -0.37000000E+02
-0.33000000E+02 -0.29000000E+02 -0.22000000E+02 0.00000000E+00

RESULTS OF FOURIER ANALYSIS:

AZERO = 0.19916672E+02

VALUES OF I, A(I) AND B(I) ARE

1	-0.20158676E+02	0.23525284E+02
2	0.33099222E+01	0.12264638E+02
3	0.37719278E+01	-0.40640426E+00
4	-0.95843577E+00	0.32476425E+01
5	0.11377630E+01	-0.18716125E+01
6	-0.11667604E+01	0.12500324E+01

1.88

The main program and the output are given below.

```
C =====
C
C MAIN PROGRAM FOR CALLING THE SUBROUTINE FORIER
C
C =====
C FOLLOWING 6 LINES NEED TO BE CHANGED FOR A DIFFERENT PROBLEM
  DIMENSION X(14), T(14), XSIN(14), XCOS(14), A(10), B(10)
  DATA N, M, TIME /14, 10, 0.35/
  DATA X / .45, -.8, -.9, -.6, -.75, -.7, .55, 1.75, 1.65, .25, -1.1,
2 -1.4, -1.05, 0.0/
  DATA T / .025, .05, .075, .1, .125, .15, .175, .2, .225, .25, .275, .3, .325,
2 .35/
C END OF PROBLEM-DEPENDENT DATA
  CALL FORIER (N, M, TIME, X, T, AZERO, A, B, XSIN, XCOS)
  PRINT 100
100 FORMAT (//, 46H FOURIER SERIES EXPANSION OF THE FUNCTION X(T), //)
  PRINT 200, N, M, TIME
200 FORMAT (6H DATA: , //, 37H NUMBER OF DATA POINTS IN ONE CYCLE =, I5,
2 /, 42H NUMBER OF FOURIER COEFFICIENTS REQUIRED =, I5, /,
3 14H TIME PERIOD =, E15.8)
  PRINT 300, (T(I), I=1, N)
300 FORMAT (/, 33H TIME AT VARIOUS STATIONS, T(I) =, /, (4E15.8, 1X))
  PRINT 400, (X(I), I=1, N)
400 FORMAT (/, 31H KNOWN VALUES OF X(I) AT T(I) =, /, (4E15.8, 1X))
  PRINT 500
500 FORMAT (//, 29H RESULTS OF FOURIER ANALYSIS: , //)
  PRINT 600, AZERO
600 FORMAT (8H AZERO =, 2X, E15.8, //, 31H VALUES OF I, A(I) AND B(I) ARE:
2 , //)
  DO 700, I =1, M
700 PRINT 800, I, A(I), B(I)
800 FORMAT (I5, 2X, E15.8, 2X, E15.8)
  STOP
  END
```

FOURIER SERIES EXPANSION OF THE FUNCTION X(T)

DATA:

NUMBER OF DATA POINTS IN ONE CYCLE = 14
NUMBER OF FOURIER COEFFICIENTS REQUIRED = 10
TIME PERIOD = 0.35000002E+00

TIME AT VARIOUS STATIONS, T(I) =

0.24999999E-01 0.50000001E-01 0.74999988E-01 0.10000002E+00
 0.12500000E+00 0.14999998E+00 0.17500001E+00 0.19999999E+00
 0.22500002E+00 0.25000000E+00 0.27499998E+00 0.30000001E+00
 0.32499999E+00 0.35000002E+00

KNOWN VALUES OF X(I) AT T(I) =

0.44999999E+00 -0.80000001E+00 -0.89999998E+00 -0.60000002E+00
 -0.75000000E+00 -0.69999999E+00 0.55000001E+00 0.17500000E+01
 0.16499996E+01 0.25000000E+00 -0.11000004E+01 -0.13999996E+01
 -0.10500002E+01 0.00000000E+00

RESULTS OF FOURIER ANALYSIS:

AZERO = -0.37857169E+00

VALUES OF I, A(I) AND B(I) ARE

1	-0.61951119E+00	-0.35046142E+00
2	0.46242946E+00	0.92408919E+00
3	0.41494656E+00	-0.17127156E+00
4	0.22273250E-01	0.24691588E+00
5	-0.45435190E-01	0.10430527E+00
6	-0.20434089E-01	0.56009147E-01
7	-0.49999803E-01	-0.35824633E-05
8	-0.20436604E-01	-0.55996872E-01
9	-0.45427930E-01	-0.10429442E+00
10	0.22274703E-01	-0.24690533E+00

1.89 $x_p = r + l - r \cos \theta - l \cos \phi = r + l - r \cos \omega t - l \sqrt{1 - \sin^2 \phi}$ (E₁)

But $l \sin \phi = r \sin \theta$, $\cos \phi = \left(1 - \frac{r^2}{l^2} \sin^2 \omega t\right)^{\frac{1}{2}}$ (E₂)

Using (E₂) in (E₁), $x_p = r + l - r \cos \omega t - l \left(1 - \frac{r^2}{l^2} \sin^2 \omega t\right)^{\frac{1}{2}}$ (E₃)

Let $\frac{r}{l} = \text{small } (< \frac{1}{4})$. Using $\sqrt{1 - \epsilon} \approx 1 - \frac{1}{2} \epsilon$, (E₃) becomes

$$x_p \approx r \left(1 + \frac{r}{2l}\right) - r \left(\cos \omega t + \frac{r}{4l} \cos 2\omega t\right) \quad (E_4)$$

(a) Eq. (E₄) gives $y_p = x_p - r \left(1 + \frac{r}{2l}\right) \approx -r \left(\cos \omega t + \frac{1}{4} \frac{r}{l} \cos 2\omega t\right)$ --- (E₅)

If $\frac{r}{l}$ is very small, $y_p \approx -r \cos \omega t \Rightarrow$ harmonic motion.

(b) To have amplitude of second harmonic smaller than that of first harmonic in Eq. (E₅), we need to have

$$\frac{1}{4} \frac{r}{l} \leq \frac{1}{25}, \quad \text{i.e.,} \quad \frac{r}{l} \leq \frac{4}{25}, \quad \text{i.e.,} \quad \frac{l}{r} \geq 6.25$$

Once the amplitude of second harmonic is smaller by a factor of 25, the amplitudes of higher harmonics arising from the expansion of square-root-term in (E₃) are expected to be still smaller.

1.90 Unbalanced force developed = $P = 2 m \omega^2 r \cos \omega t$, range of force = 0 - 100 N, range of frequency = 25 - 50 Hz = 157.08 - 314.16 rad/sec.

Parameters to be determined: m, r, ω .

Let $r = 0.1$ m. To generate 100 N force at 25 Hz, set:

$$P_{\max} = 100 = 2 m (157.08)^2 (0.1)$$

which gives

$$m = \frac{100}{2 (157.08)^2 (0.1)} = 0.0202641 \text{ kg} = 20.2641 \text{ g}$$

To generate 100 N force at 50 Hz, set:

$$P_{\max} = 100 = 2 m (314.16)^2 (0.1)$$

which yields

$$m = \frac{100}{2 (314.16)^2 (0.1)} = 0.0050660 \text{ kg} = 5.0660 \text{ g}$$

1.91

Goal: Weight to be maintained at 5 ± 0.05 kg/min

Parameters to be determined: Angular velocity of crank (ω), lengths of crank and connecting rod, dimensions of the wedge, dimensions of the orifice in the hopper, dimensions of the actuating rod, and dimensions of the lever arrangement.

Given: Density of the material in the hopper.

Procedure:

Select ω based on available motor. Determine the dimensions of the orifice in the hopper which delivers approximately 5 kg/min (assuming continuous flow of material). For trial dimensions of the wedge, determine the increase/decrease in the size (diameter) of the orifice. Choose the final dimensions of the wedge such that the material flow rate delivered by the orifice lies within the specified range.

1.92

Force to be applied = 1000 N , frequency = 50 Hz = 314.16 rad/sec.

Procedure:

1. Select a motor that provides, either directly or through a gear system, the desired frequency. Assume that it is connected to the cam.
2. Determine the sizes and dimensions of the plate cam and the roller.
3. Choose the dimensions of the follower.
4. Select the weight as 1000 N. From the geometry, determine the range of displacement (vertical motion) of the weight.
5. Determine the force exerted due to the falling weight.

1.93

Considerations to be taken in the design of vibratory bowl feeders:

1. Suitable design of the electromagnet and its coil.
 2. Radius of the bowl and the pitch of the spiral (helical) delivery track.
 3. Tooling to be fixed along the spiral track to reject the defective or out-of-tolerance or incorrectly oriented parts.
 4. Design of elastic supports.
 5. Size and location of the outlet.
-

1.94

Axial spring constant of each tube = $k = \frac{A E}{\ell}$.

Let diameter of each tube be 0.01 m (1 cm) with thickness 0.001 m (1 mm). Then

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (0.01^2 - 0.008^2) = 28.27 (10^{-6}) \text{ m}^2$$

This gives

$$k = \frac{(28.27 (10^{-6})) (2.07 (10^{11}))}{2} = 29.26 (10^5) \text{ N/m}$$

Since 76 tubes are in parallel, we have the total axial stiffness as:

$$k_{\text{eq}} = 76 k = (76) (29.26 (10^5)) = 222.38 (10^6) \text{ N/m}$$

The polar area moment of inertia of each tube is

$$J = \frac{\pi}{32} (D^4 - d^4) = \frac{\pi}{32} (0.01^4 - 0.008^4) = 580 (10^{-8}) \text{ m}^4$$

Torsional stiffness of each tube is given by

$$\frac{G J}{\ell} = \frac{(79.6154 (10^9)) (580 (10^{-8}))}{2} = 231 (10^3) \text{ N-m/rad}$$

For 76 tubes in parallel, equivalent torsional stiffness will be:

$$k_{t_{\text{eq}}} = (76) (231 (10^3)) = 17.56 (10^6) \text{ N-m/rad}$$
