Chapter 9

Vibration Control

9.1 $\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{400 \times 10^3}{1500}\right)^{\frac{1}{2}} = 16.3299$ rad/s

If v = speed of the automobile in km/hr,

$\omega = 2\pi f = 2\pi \left\{\frac{v\,(1000)}{3600}\right\} \frac{1}{5} = 0.3491\, v$ rad/s

$r = \frac{\omega}{\omega_n} = \frac{0.3491\, v}{16.3299} = 0.02138\, v$

$Y = 1$ mm $= 10^{-3}$ m

If X = displacement of mass (passengers), we have

$$\frac{X}{Y} = \frac{1}{1-r^2} = \frac{1}{1 - 4.5693 \times 10^{-4}\, v^2}$$

$$X = \frac{10^{-3}}{1 - 4.5693 \times 10^{-4}\, v^2} = \frac{10}{10^4 - 4.5693\, v^2}$$

$$= \infty \quad \text{at} \quad v = \left(\frac{10000}{4.5693}\right)^{\frac{1}{2}} = 46.7816 \text{ km/hr}$$

Thus passengers will perceive vibration in the neighborhood of 46.7816 km/hr speed.

Possible methods of improving the design:

1. Change the stiffness of the system by changing tire pressure, or springs of the suspension.
2. change the mass of the system by adding more mass (dead weight).
3. Add damping to the system by using better shock absorbers.

9.2 $x(t) = X \cos \omega t$, $\quad x^2(t) = X^2 \cos^2 \omega t$

$$\int_0^T x^2(t)\, dt = \int_0^T X^2 \cos^2 \omega t\, dt = X^2 \left\{ \frac{T}{2} + \frac{1}{4\omega} \sin 2\omega T \right\}$$

$$x^2_{rms} = \lim_{T\to\infty} \left\{ \frac{1}{T} \int_0^T x^2(t)\, dt \right\}$$

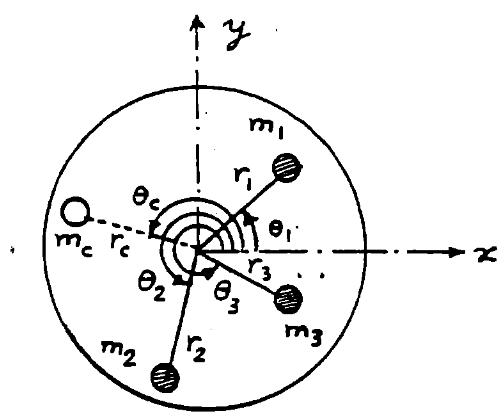
$$= X^2 \lim_{T\to\infty} \left\{ \frac{1}{2} + \frac{1}{4\omega} \frac{\sin 2\omega T}{T} \right\}$$

$$= X^2 \left\{ \frac{1}{2} + 0 \right\} = \frac{X^2}{2}$$

$$\therefore\ x_{rms} = X/\sqrt{2}$$

9.3 For static balance:

$\Sigma F_x = 0$, $\Sigma F_y = 0$



Here

$$\Sigma F_x = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_c r_c \cos\theta_c = 0$$

$\Rightarrow 35(110)\cos 40^\circ + 15(90)\cos 220^\circ + 25(130)\cos 290^\circ + m_c r_c \cos\theta_c = 0$

$\Rightarrow 2949.1 - 1034.1 + 1111.5 + m_c r_c \cos\theta_c = 0$

$\Rightarrow m_c r_c \cos\theta_c = -3026.5$

$\Sigma F_y = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_c r_c \sin\theta_c = 0$

$\Rightarrow 35(110)\sin 40^\circ + 15(90)\sin 220^\circ + 25(130)\sin 290^\circ + m_c r_c \sin\theta_c = 0$

$\Rightarrow 2474.78 - 867.78 - 3054.025 + m_c r_c \sin\theta_c = 0$

$\Rightarrow m_c r_c \sin\theta_c = 1447.025$

$$m_c r_c = \left[(-3026.5)^2 + (1447.025)^2\right]^{1/2} = 3354.6361 \text{ g-mm}$$

$$\theta_c = \tan^{-1}\left(\frac{1447.025}{-3026.5}\right) = -25.5525^\circ$$

9.4 Mass removed from holes 1 and 2 is 100 grams, and from 3 and 4 is 150 grams, radius for holes 1 to 4 is 0.1 m, 5th hole at a radius of 0.12 m.

Solution:

Unbalance due to hole is proportional to (r.m)

Let m_5 = mass removed from 5th hole

r_5 = radius at which 5th hole is drilled = 0.12 m.

θ_5 = angle at which 5th hole is drilled.

$$\Sigma F_x = \sum_{i=1}^{5} r_i m_i \cos\theta_i = 0$$

$\Rightarrow$ $0.1\,(0.1)\cos 0° + 0.1\,(0.1)\cos 60° + 0.1\,(0.15)\cos 120° + 0.1\,(0.15)\cos 180° + r_5\, m_5 \cos\theta_5 = 0$

$\Rightarrow$ $0.01 + 0.005 - 0.0075 - 0.015 + 0.12\, m_5 \cos\theta_5 = 0$

$0.12\, m_5 \cos\theta_5 = 0.0075$

$$\Sigma F_y = \sum_{i=1}^{5} r_i m_i \sin\theta_i = 0$$

$\Rightarrow$ $0.1\,(0.1)\sin 0° + 0.1\,(0.1)\sin 60° + 0.1\,(0.15)\sin 120° + 0.1\,(0.15)\sin 180° + r_5\, m_5 \sin\theta_5 = 0$

$\Rightarrow$ $0 + 0.00866 + 0.01299 + 0 + r_5\, m_5 \sin\theta_5 = 0$

$0.12\, m_5 \sin\theta_5 = -0.02165$

$$m_5 = \frac{1}{0.12}\sqrt{(0.0075)^2 + (-0.02165)^2} = 0.1909\ \text{(kg)}$$

$$\theta_5 = \tan^{-1}\left(\frac{-0.02165}{0.0075}\right) = -70.8929°$$

9.5

$$\Sigma F_x = \sum_{i=1}^{4} m_i r_i \cos\theta_i = 0$$

since all r_1 are same, we have

$0.5\cos 10° + 0.7\cos 100° + 1.2\cos 190° + m_4 \cos\theta_4 = 0$

$\Rightarrow$ $0.4924 - 0.12152 - 1.18176 + m_4 \cos\theta_4 = 0$

$\Rightarrow$ $m_4 \cos\theta_4 = 0.81088$

$$\Sigma F_y = \sum_{i=1}^{4} m_i r_i \sin\theta_i = 0$$

$\Rightarrow$ $0.5\sin 10° + 0.7\sin 100° + 1.2\sin 190° + m_4 \sin\theta_4 = 0$

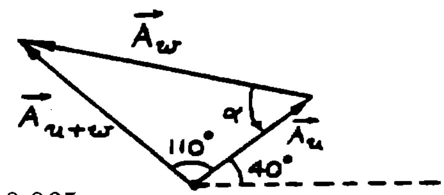
$\Rightarrow$ $0.0868 + 0.6894 - 0.2083 + m_4 \sin\theta_4 = 0$

$\Rightarrow$ $m_4 \sin\theta_4 = -0.56788$

$\therefore$ $m_4 = [(0.81088)^2 + (-0.56788)^2]^{\frac{1}{2}} = 0.98996$ kg.

$$\theta_4 = \tan^{-1}\left(\frac{-0.56788}{0.81088}\right) = -35.0045°$$

9.6 Amplitude of unbalance 0.2 mm,
phase angle of unbalance 40°. ↻,
trial weight M = 0.2 kg, at a radius 0.065 m.,
new amplitude of unbalance 0.5 mm.,
new phase angle 150°. ↻,
balancing weight to be located at a radial distance of 0.065 m.



Solution:

$\vec{A}_u = (0.2,\ 40°\ \text{ccw})$

$\vec{A}_{u+w} = (0.5,\ 150°\ \text{ccw})$

$$A_w = [A_u^2 + A_{u+w}^2 - 2\,A_u\,A_{u+w}\cos(\phi - \theta)]^{\frac{1}{2}}$$

$$= (0.2^2 + 0.5^2 - 2\,(0.2)\,(0.5)\cos 110°)^{\frac{1}{2}} = 0.5987$$

$$M_0 = \text{original unbalance} = \left(\frac{A_u}{A_w}\right) M = \left(\frac{0.2}{0.5987}\right) 0.2 = 0.06681\ \text{kg.}$$

$$\alpha = \cos^{-1}\left[\frac{A_u^2 + A_w^2 - A_{u+w}^2}{2\,A_u\,A_w}\right] = \cos^{-1}\left[\frac{0.2^2 + 0.5987^2 - 0.5^2}{2\,(0.2)\,(0.5987)}\right] = \cos^{-1}(0.6199)$$

$$= 51.6912°\ \text{ccw}$$

Grinding wheel will be balanced if a weight of 0.06681 kg is added at 51.6912° clockwise from the position of the trial weight or (65 + 51.6912°) = 116.6912° clockwise from the phase mark.

9.7 Unbalance amplitude 0.15 mm, trial weight of mass 50 grams, new unbalance amplitude 0.22 mm.

Solution:

$\vec{A}_u = (0.15,\ 15°\ \text{cw})$

$\vec{A}_{u+w} = (0.22,\ 35°\ \text{ccw})$

$$A_w = [A_u^2 + A_{u+w}^2 - 2\,A_u\,A_{u+w}\cos(\phi - \theta)]^{\frac{1}{2}}$$

$$= [0.15^2 + 0.22^2 - 2\,(0.15)\,(0.22)\cos(50°)]^{\frac{1}{2}}$$

$$= 0.1687\ \text{mm}$$

$$M_0 = \text{original unbalance} = \left(\frac{A_u}{A_w}\right) M = \left(\frac{0.15}{0.1687}\right) 0.05 = 0.0444\ \text{kg.}$$

$$\alpha = \cos^{-1}\left[\frac{A_u^2 + A_w^2 - A_{u+w}^2}{2\,A_u\,A_w}\right] = \cos^{-1}\left[\frac{0.15^2 + 0.1687^2 - 0.22^2}{2\,(0.15)\,(0.1687)}\right]$$

$$= \cos^{-1}(0.05058) = 87.1009°\ \text{ccw}$$

Flywheel will be balanced if a mass of 0.0444 kg is added at 87.1009° cw from the position of the trial mass or (–45 + 87.1009) = 42.1009° cw from the phase mark.

9.8 Original unbalance amplitude 0.1 mm, trial weight of mass M = 100 grams, amplitude 0.2 mm.

Solution:

$\vec{A}_u = (0.1,\ 45°\ \text{ccw})$

$\vec{A}_{u+w} = (0.2,\ 145°\ \text{ccw})$

$$A_w = [A_u^2 + A_{u+w}^2 - 2\,A_u\,A_{u+w}\cos(\phi - \theta)]^{\frac{1}{2}}$$

$$= (0.1^2 + 0.2^2 - 2\,(0.1)\,(0.2)\cos 100°)^{\frac{1}{2}} = 0.2386 \text{ mm.}$$

$$M_0 = \text{original unbalance} = \left(\frac{A_u}{A_w}\right) M = \left(\frac{0.1}{0.2386}\right) 0.1 = 0.0419 \text{ kg.}$$

$$\alpha = \cos^{-1}\left[\frac{A_u^2 + A_w^2 - A_{u+w}^2}{2\,A_u\,A_w}\right] = \cos^{-1}\left(\frac{0.1^2 + 0.2386^2 - 0.2^2}{2 \times 0.1 \times 0.2386}\right)$$

$$= \cos^{-1}(0.5643) = 55.6440°\ \text{ccw}$$

Grinding wheel will be balanced if a mass of 0.0419 kg is added at 55.6440° cw from the position of trial weight or (20° + 55.6440°) = 75.6440° cw from the phase mark.

9.9 Figure 3.11 (b) shows that the phase angle between the displacement and the driving force is 90° at a frequency ratio of r = 1. The driving force leads the displacement. Since the driving force is due to the centrifugal force of the eccentric mass, the direction of the unbalanced mass is 90° ahead of the displacement. Hence the mass has to be removed at 229° + 90° = 319° as indicated by the protractor.

9.10 For static balance, sum of all inertia forces must be zero:

$$m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + m_{b1} \vec{r}_{b1} + m_{b2} \vec{r}_{b2} = \vec{0} \qquad (E_1)$$

which can be written in scalar form as

$$m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_{b1} r_{b1} \cos\theta_{b1} + m_{b2} r_{b2} \cos\theta_{b2} = 0 \qquad (E_2)$$

$$m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_{b1} r_{b1} \sin\theta_{b1} + m_{b2} r_{b2} \sin\theta_{b2} = 0 \qquad (E_3)$$

For dynamic balance, sum of moments due to inertia forces must be zero about any point. The moments about the point, defined by the intersection of z-axis with plane A, gives

$$l_1 m_1 \omega^2 \vec{r}_1 + l_2 m_2 \omega^2 \vec{r}_2 + l_3 m_3 \omega^2 \vec{r}_3 + \overset{\nearrow 0}{l_{b1}} m_{b1} \omega^2 \vec{r}_{b1} + l_{b2} m_{b2} \omega^2 \vec{r}_{b2} = \vec{0} \qquad (E_4)$$

or

$$l_1 m_1 r_1 \cos\theta_1 + l_2 m_2 r_2 \cos\theta_2 + l_3 m_3 r_3 \cos\theta_3 + l_{b2} m_{b2} r_{b2} \cos\theta_{b2} = 0 \qquad (E_5)$$

$$l_1 m_1 r_1 \sin\theta_1 + l_2 m_2 r_2 \sin\theta_2 + l_3 m_3 r_3 \sin\theta_3 + l_{b2} m_{b2} r_{b2} \sin\theta_{b2} = 0 \qquad (E_6)$$

Eqs. (E_5) and (E_6) give

$$m_{b2} r_{b2} = \frac{1}{l_{b2}} \left\{ \left(l_1 m_1 r_1 \cos\theta_1 + l_2 m_2 r_2 \cos\theta_2 + l_3 m_3 r_3 \cos\theta_3 \right)^2 + \left(l_1 m_1 r_1 \sin\theta_1 + l_2 m_2 r_2 \sin\theta_2 + l_3 m_3 r_3 \sin\theta_3 \right)^2 \right\}^{1/2} \qquad (E_7)$$

$$\theta_{b2} = \tan^{-1} \left\{ \frac{l_1 m_1 r_1 \sin\theta_1 + l_2 m_2 r_2 \sin\theta_2 + l_3 m_3 r_3 \sin\theta_3}{l_1 m_1 r_1 \cos\theta_1 + l_2 m_2 r_2 \cos\theta_2 + l_3 m_3 r_3 \cos\theta_3} \right\} \qquad (E_8)$$

Once $m_{b2} r_{b2}$ and θ_{b2} are found, Eqs. (E_2) and (E_3) can be used to determine

$$m_{b1} r_{b1} = \left\{ \left(m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_{b2} r_{b2} \cos\theta_{b2} \right)^2 + \left(m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_{b2} r_{b2} \sin\theta_{b2} \right)^2 \right\}^{1/2} \qquad (E_9)$$

$$\theta_{b1} = \tan^{-1} \left\{ \frac{m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_{b2} r_{b2} \sin\theta_{b2}}{m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_{b2} r_{b2} \cos\theta_{b2}} \right\} \qquad (E_{10})$$

9.11 $M_1 = M_2 = 0.1$ kg, radius 0.08 m.
$M_3 = M_4 = 0.1$ kg., holes drilled at 0.1 m radius.

Solution:
Let M_B and M_C be the amount (masses) of the material removed in planes B and C at angular locations θ_B and θ_C, respectively. The material removed must balance the masses added (temporarily) in planes A and D. Measuring angles counter clockwise from the horizontal (x-axis), we need to satisfy the following equations for static balancing of the rotor:

$$\sum_{i=1}^{4} M_i r_i \cos\theta_i + M_B r_B \cos\theta_B + M_C r_C \cos\theta_C = 0 \quad (E_1)$$

$$\sum_{i=1}^{4} M_i r_i \sin\theta_i + M_B r_B \sin\theta_B + M_C r_C \sin\theta_C = 0 \quad (E_2)$$

Since $M_1 = M_2 = M_3 = M_4 = 0.1$ kg, $\theta_1 = 90°$, $\theta_2 = -60°$, $\theta_3 = 120°$, $\theta_4 = -120°$.
$r_1 = r_2 = r_3 = r_4 = 0.08$ m, $r_B = r_C = 0.1$ m.
Eqs. (E_1) and (E_2) yield:

$$0.1\,(0.08)\cos 90° + 0.1\,(0.08)\cos(-60°) + 0.1\,(0.08)\cos(120°) + 0.1\,(0.08)\cos(-120°) + 0.1\,M_B\cos\theta_B + 0.1\,M_C\cos\theta_C = 0$$

$$\Rightarrow \quad M_B\cos\theta_B + M_C\cos\theta_C = 0.04 \quad (E_3)$$

$$0.1\,(0.08)\sin 90° + 0.1\,(0.08)\sin(-60°) + 0.1\,(0.08)\sin(120°) + 0.1\,(0.08)\sin(-120°) + 0.1\,M_B\sin\theta_B + 0.1\,M_C\sin\theta_C = 0$$

$$\Rightarrow \quad M_B\sin\theta_B + M_C\sin\theta_C = -0.0107 \quad (E_4)$$

For dynamic balancing (taking moments from plane A),

$$M_3 r_3 S_3 \cos\theta_3 + M_4 r_4 S_4 \cos\theta_4 + M_B r_B S_B \cos\theta_B + M_C r_C S_C \cos\theta_C = 0$$
$$M_3 r_3 S_3 \sin\theta_3 + M_4 r_4 S_4 \sin\theta_4 + M_B r_B S_B \sin\theta_B + M_C r_C S_C \sin\theta_C = 0$$

i.e.,

$$0.1\,(0.08)\,(0.6)\cos 120° + 0.1\,(0.08)\,(0.6)\cos(-120°) + M_B\,(0.1)\,(0.1)\cos\theta_B + M_C\,(0.1)\,(0.5)\cos\theta_C = 0$$

$$0.1\,(0.08)\,(0.6)\sin 120° + 0.1\,(0.08)\,(0.6)\sin(-120°) + M_B\,(0.1)\,(0.1)\sin\theta_B + M_C\,(0.1)\,(0.5)\sin\theta_C = 0$$

i.e.,

$$0.01\,M_B\cos\theta_B + 0.05\,M_C\cos\theta_C = 0.0048 \quad (E_5)$$

$$0.01\,M_B\sin\theta_B + 0.05\,M_C\sin\theta_C = 0 \quad (E_6)$$

Eqs. (E_3) and (E_5) give

$$4M_C\cos\theta_C = 0.44 \quad \text{or} \quad M_C\cos\theta_C = 0.11 \quad (E_7)$$

Eqs. (E_4) and (E_6) give

$4\ M_C \sin\theta_C = 0.0107$ or $M_C \sin\theta_C = 0.002675$ (E$_8$)

Eqs. (E$_7$) and (E$_8$) yield

$$\left.\begin{aligned} M_C &= [0.11^2 + 0.002675^2]^{\frac{1}{2}} = 0.1100\ (\text{kg}) \\ \theta_C &= \tan^{-1}\left(\frac{0.002675}{0.11}\right) = 1.3931° \end{aligned}\right\} \quad (E_9)$$

$$\left.\begin{aligned} &\text{Eqs. (E}_3\text{) and (E}_9\text{) give } M_B \cos\theta_B = -0.06997 \\ &\text{Eqs. (E}_4\text{) and (E}_9\text{) give } M_B \sin\theta_B = -0.01337 \end{aligned}\right\} \quad (E_{10})$$

$$\therefore \quad M_B = [(-0.06997)^2 + (-0.01337)^2]^{\frac{1}{2}} = 0.07124\ (\text{kg})$$

$$\theta_B = \tan^{-1}\left(\frac{-0.01337}{-0.06997}\right) = 10.8178°$$

∴ Amount of material to be removed:

0.07124 kg at 10.8178° ccw at radius 0.1 m in plane B and 0.11 kg at 1.3931° ccw at radius 0.1 m in plane C.

9.12 Masses 1 kg, 2 kg, 1.5 kg are located at radii 0.5 m, 0.8 m and 0.3 m.

Solution:
$M_C = 1$ kg, $M_D = 2$ kg, $M_E = 1.5$ kg.
$r_C = 0.5$ m, $r_D = 0.8$ m, $r_E = 0.3$ m.
$\theta_C = 90°$, $\theta_D = 220°$, $\theta_E = -30°$.

Let M_A, r_A, θ_A and M_G, r_G, θ_G denote the masses added in planes A and G, respectively.

For static balancing,

$$M_C r_C \cos\theta_C + M_D r_D \cos\theta_D + M_E r_E \cos\theta_E + M_A r_A \cos\theta_A + M_G r_G \cos\theta_G = 0$$

$$\Rightarrow \quad M_A r_A \cos\theta_A + M_G r_G \cos\theta_G = 0.8360 \quad (E_1)$$

$$M_C r_C \sin\theta_C + M_D r_D \sin\theta_D + M_E r_E \sin\theta_E + M_A r_A \sin\theta_A + M_G r_G \sin\theta_G = 0$$

$$\Rightarrow \quad M_A r_A \sin\theta_A + M_G r_G \sin\theta_G = 0.2465 \quad (E_2)$$

For dynamic balancing, we take moments about the left bearing (plane B):

$$M_C r_C S_C \cos\theta_C + M_D r_D S_D \cos\theta_D + M_E r_E S_E \cos\theta_E + M_A r_A S_A \cos\theta_A + M_G r_G S_G \cos\theta_G = 0$$

$$\Rightarrow \quad -0.4\, M_A r_A \cos\theta_A + 2.2\, M_G r_G \cos\theta_G = 0.6021 \qquad (E_3)$$

$$M_C r_C S_C \sin\theta_C + M_D r_D S_D \sin\theta_D + M_E r_E S_E \sin\theta_E + M_A r_A S_A \sin\theta_A + M_G r_G S_G \sin\theta_G = 0$$

$$\Rightarrow \quad -0.4\, M_A r_A \sin\theta_A + 2.2\, M_G r_G \sin\theta_G = 1.0885 \qquad (E_4)$$

Eqs. (E_1) and (E_3) yield

$$6.5\, M_G r_G \cos\theta_G = 2.3413 \quad \text{or} \quad M_G r_G \cos\theta_G = 0.3602 \qquad (E_5)$$

Eqs. (E_2) and (E_4) give

$$6.5\, M_G r_G \sin\theta_G = 2.4748 \quad \text{or} \quad M_G r_G \sin\theta_G = 0.3807 \qquad (E_6)$$

Eqs. (E_5) and (E_6) give

$$\left.\begin{aligned} M_G r_G &= [(0.3602)^2 + (0.3807)^2]^{\frac{1}{2}} = 0.5241 \text{ kg}\cdot\text{m} \\ \theta_G &= \tan^{-1}\left(\frac{0.3807}{0.3602}\right) = 46.5849^\circ \end{aligned}\right\} \qquad (E_7)$$

Eqs. (E_1), (E_2) and (E_7) yield

$$\left.\begin{aligned} M_A r_A \cos\theta_A &= 0.8360 - (0.5241)\cos 46.5849^\circ = 0.4758 \\ M_A r_A \sin\theta_A &= -0.2465 - (0.5241)\sin 46.5849^\circ = -0.6272 \end{aligned}\right\} \qquad (E_8)$$

Eq. (E_8) provides

$$\left.\begin{aligned} M_A r_A &= [(0.4758)^2 + (-0.6272)^2]^{\frac{1}{2}} = 0.7873 \\ \theta_A &= \tan^{-1}\left(\frac{-0.6272}{0.4758}\right) = -52.8157^\circ \end{aligned}\right\} \qquad (E_9)$$

If the balancing weights are placed at a radial distance of 0.05 m in planes A and G, we have $r_A = r_G = 0.5$ m, and hence

$M_A = 1.5746$ (kg), $\theta_A = -52.8157^\circ$

$M_G = 1.0482$ (kg), $\theta_G = 46.5849^\circ$

9.13

$\vec{V}_A = 5\angle 100°$

$= -0.8682 + i\,4.9240$

$\vec{V}_B = 4\angle 180°$

$= -4.0 - i\,0.0$

$\vec{V}'_A = 6.5\angle 120°$

$= -3.25 + i\,5.6292$

$\vec{V}'_B = 4.5\angle 140°$

$= -3.4472 + i\,2.8925$

$\vec{V}''_A = 6.0\angle 90°$

$= 0.0 + i\,6.0$

$\vec{V}''_B = 7.0\angle 60°$

$= 3.5 + i\,6.0622$

$\vec{M}_L = 2.0\angle 30°$

$= 1.7321 + i\,1.0$

$\vec{M}_R = 2.0\angle 0°$

$= 2.0 + i\,0.0$

$$\vec{A}_{AL} = \frac{\vec{V}'_A - \vec{V}_A}{\vec{M}_L}$$

$= 1.2420\angle -46.4914°$

$$\vec{A}_{AR} = \frac{\vec{V}''_A - \vec{V}_A}{\vec{M}_R}$$

$= 0.6913\angle 51.0984°$

$$\vec{A}_{BL} = \frac{\vec{V}'_B - \vec{V}_B}{\vec{M}_L}$$

$= 4.2315\angle -49.7058°$

$$\vec{A}_{BR} = \frac{\vec{V}''_B - \vec{V}_B}{\vec{M}_R}$$

$= 2.1217\angle -39.2682°$

$$\vec{U}_L = \frac{\vec{A}_{BR}\vec{V}_A - \vec{A}_{AR}\vec{V}_B}{\vec{A}_{BR}\vec{A}_{AL} - \vec{A}_{AR}\vec{A}_{BL}}$$

$$\vec{U}_R = \frac{\vec{A}_{BL}\vec{V}_A - \vec{A}_{AL}\vec{V}_B}{\vec{A}_{BL}\vec{A}_{AR} - \vec{A}_{AL}\vec{A}_{BR}}$$

$\vec{B}_L = -\vec{U}_L = 4.2315\angle -49.7058°$

$\vec{B}_R = -\vec{U}_R = 2.1217\angle -39.2682°$

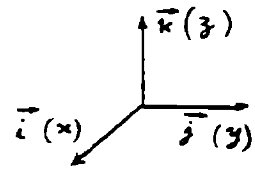
9.14 (a) $\omega = 1000 \times \frac{2\pi}{60} = 104.72$ rad/s

Centrifugal forces due to rotating masses (all parallel to yz-plane) are

$F_1 = m_1 r_1 \omega^2 = (50/1000)(8/100)(104.72)^2 = 43.8651$ N

$F_2 = m_2 r_2 \omega^2 = (20/1000)(5/100)(104.72)^2 = 10.9663$ N

$F_3 = m_3 r_3 \omega^2 = (40/1000)(6/100)(104.72)^2 = 26.3191$ N



These can be written in vector form as

$\vec{F_1} = F_1 \angle\theta_1 = 43.8651 \angle 0^\circ = 43.8651 \ \vec{j}$

$\vec{F_2} = F_2 \angle\theta_2 = 10.9663 \angle 120^\circ = -5.4832 \ \vec{j} + 9.4971 \ \vec{k}$

$\vec{F_3} = F_3 \angle\theta_3 = 26.3191 \angle 200^\circ = -24.7319 \ \vec{j} - 9.0017 \ \vec{k}$

The moments of these forces taken about the bearing at A must be balanced by the moment of the bearing reaction at B. Hence

$$\sum \vec{M_A} = 0.2\,\vec{i} \times 43.8651\,\vec{j} + 0.3\,\vec{i} \times (-5.4832\,\vec{j} + 9.4971\,\vec{k}) + 0.9\,\vec{i} \times (-24.7319\,\vec{j} - 9.0017\,\vec{k}) + 1.1\,\vec{i} \times \vec{R_B} = \vec{0}$$

where $\vec{R_B}$ = reaction at bearing B and '×' denotes the cross product. This gives

$$-15.1307\,\vec{k} + 5.2524\,\vec{j} + 1.1\,\vec{i} \times \vec{R_B} = \vec{0} \quad \text{--- } (E_1)$$

Let $\vec{R_B} = (a\,\vec{j} + b\,\vec{k})$. Then $1.1\,\vec{i} \times (a\,\vec{j} + b\,\vec{k}) = 1.1a\,\vec{k} - 1.1b\,\vec{j}$ --- (E_2)

(E_1) and (E_2) give $a = 13.7552$, $b = 4.7749$

$\therefore \vec{R_B} = 13.7552\,\vec{j} + 4.7749\,\vec{k}$

Similarly, by taking moments about B,

$$-0.2\,\vec{i} \times (-24.7319\,\vec{j} - 9.0017\,\vec{k}) - 0.6\,\vec{i} \times (-5.4832\,\vec{j} + 9.4971\,\vec{k})$$

$$- 0.9\,\vec{i} \times (43.8651\,\vec{j}) - 1.1\,\vec{i} \times \vec{R_A} = \vec{0}$$

where $\vec{R_A}$ = reaction at bearing A = $c\,\vec{j} + d\,\vec{k}$.

$$-31.2423\,\vec{k} + 3.898\,\vec{j} - 1.1c\,\vec{k} + 1.1d\,\vec{j} = \vec{0}$$

$c = -28.4021$, $d = -3.5436$

$\therefore \vec{R_A} = -28.4021\,\vec{j} - 3.5436\,\vec{k}$

Note that these are rotating vectors.

(b) Since the planes L and R pass through the bearings A and B, the balancing forces are given by

$$\vec{B_R} = -\vec{R_B} = -13.7552\,\vec{j} - 4.7749\,\vec{k} = m_R r \omega^2 (\cos\theta_R\,\vec{j} + \sin\theta_R\,\vec{k})$$

i.e. $m_R (0.25)(104.72)^2 \cos\theta_R = -13.7552$

$m_R (0.25)(104.72)^2 \sin\theta_R = -4.7749$

i.e. $m_R \cos\theta_R = -0.005017$, $m_R \sin\theta_R = -0.001742$

$$m_R = \sqrt{(-0.005017)^2 + (-0.001742)^2} = 0.005311 \text{ kg} = 5.311 \text{ g}$$

$$\theta_R = \tan^{-1}\left(-0.001742/-0.005017\right) = 19.1480° + 180° = 199.1480°$$

$$\vec{B}_L = -\vec{R}_A = 28.4021\,\vec{j} + 3.5436\,\vec{k} = m_L\, r\,\omega^2(\cos\theta_L\,\vec{j} + \sin\theta_L\,\vec{k})$$

i.e.
$$m_L \cos\theta_L = \frac{28.4021}{(0.25)(104.72)^2} = 0.01036$$

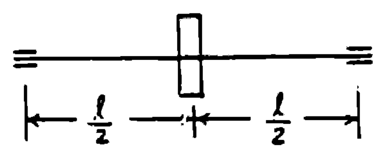
$$m_L \sin\theta_L = \frac{3.5436}{(0.25)(104.72)^2} = 0.001293$$

$$m_L = \sqrt{(0.01036)^2 + (0.001293)^2} = 0.01044 \text{ kg} = 10.44 \text{ g}$$

$$\theta_L = \tan^{-1}(0.001293/0.01036) = 7.1141°$$

Note: Angles are measured clockwise from z-axis while looking from A towards B.

9.15 Stiffness of steel shaft between bearings = $k = \dfrac{48\,E\,I}{\ell^3}$



$$k = \frac{48\,(200 \times 10^9)}{(0.75)^3}\left(\frac{\pi}{64}(0.02)^4\right) = 178.722 \times 10^3 \text{ N/m}$$

(a) Critical seep = $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{178.722 \times 10^3}{45}} = 63.0206$ rad/sec

(b) Vibration amplitude of the rotor (steady state value):

E (9.39) gives the amplitude of the rotor in x-direction as

$$X = D\, m\,\omega^2 e \text{ when damping is zero} = \frac{m\,\omega^2\, e}{|k - m\,\omega^2|}$$

Here $\omega = 1200 \text{ rpm} = 1200\left(\dfrac{2\pi}{60}\right) = 125.664$ rad/sec

$e = 0.01$ m, $m\,\omega^2 = (45)\,(125.664)^2 = 710.615 \times 10^3$

$$X = (45)\,(125.664)^2\,(0.01)\,\frac{1}{|178.722 \times 10^3 - 710.615 \times 10^3|} = 13.3601 \times 10^{-3} \text{ m}$$

Similarly the amplitude in y-direction is given by

$Y = X = 13.3601 \times 10^{-3}$ m

Resultant amplitude of the flywheel = $R = \sqrt{X^2 + Y^2} = 18.894 \times 10^{-3}$ m

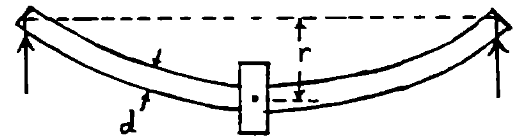
(c) Force transmitted to the bearing supports

$= kR = 178.722 \times 10^3 \times 18.894 \times 10^{-3} = 3376.782$ N

9.16 Considering bearings as simple supports, the spring constant of the beam is $k = \frac{48EI}{l^3}$ where l = distance between bearings.

Let r = variable position of center of mass, and

δ_{st} = static radial displacement of center of mass.



Then equation of motion is

$$m r \omega^2 = k(r - \delta_{st}) \quad \text{or} \quad \frac{k}{k - m\omega^2} = \frac{r}{\delta_{st}} \quad \text{or} \quad r = \frac{k \cdot \delta_{st}}{k - m\omega^2}$$

$$\text{Dynamic force} = F = m r \omega^2 = \frac{m\omega^2 k\, \delta_{st}}{k - m\omega^2}$$

Since F acts at the middle of the beam, $\sigma_{max} = \frac{M \cdot y_{max}}{I} = \frac{Fl}{4} \cdot \frac{d}{2I}$

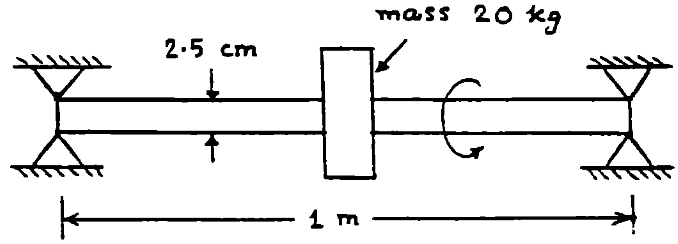
where d = diameter of shaft.

$$\sigma_{max} = \frac{Fld}{8I} = \frac{m\omega^2 k\, \delta_{st}\, l\, d}{8I\,(k - m\omega^2)}$$

Substituting the expression for k,

$$\sigma_{max} = \frac{m\omega^2 \delta_{st}\, l\, d}{8I} \left(\frac{48EI}{l^3}\right) \left\{ \frac{1}{\left(\frac{48EI}{l^3} - m\omega^2\right)} \right\}$$

9.17



Stiffness of a simply supported beam:

$$k = \frac{48\,EI}{\ell^3} = \frac{48\,(207\,(10^9))\left(\frac{\pi}{64}\,(0.025^4)\right)}{1^3} = 19.0521\,(10^4)\ \text{N/m}$$

Natural frequency of the system:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{19.0521\,(10^4)}{20}} = 97.6014\ \text{rad/sec}$$

Frequency of rotor (speed of shsft): $\omega = \frac{6000}{60}\,(2\,\pi) = 628.32$ rad/sec

Whirl amplitude of the disc: $A = \frac{a\, r^2}{\sqrt{(1 - r^2)^2 + (2\,\varsigma\, r)^2}}$

(a) At operating speed:

$$r = \frac{\omega}{\omega_n} = \frac{628.32}{97.6014} = 6.4376$$

$$A = \frac{(0.005)\,(6.4376^2)}{\sqrt{(1-6.4376^2)^2 + (2\,(0.01)\,(6.4376))^2}} = 0.005124 \text{ m}$$

(b) At critical speed (Eq. 9.41):

Critical speed:

$$\omega = \omega_{cri} = \frac{\omega_n}{\left\{1 - \frac{1}{2}\,(\frac{c}{\omega_n})^2\right\}^{\frac{1}{2}}} = \frac{97.6014}{\left\{1 - \frac{1}{2}\,(\frac{39.0406}{97.6014})^2\right\}^{\frac{1}{2}}} = 101.7565 \text{ rad/sec}$$

where $c = 2\sqrt{k\,m}\;\varsigma = 2\sqrt{(19.0521\,(10^4))\,(20)}\;(0.01) = 39.0406$ N–s/m.

$$r = \frac{\omega}{\omega_n} = \frac{101.7565}{97.6014} = 1.0426$$

$$A = \frac{(0.005)\,(1.0426^2)}{\sqrt{(1-1.0426^2)^2 + (2\,(0.01)\,(1.0426))^2}} = 0.06074 \text{ m}$$

(c) At 1.5 times critical speed:

$$r = \frac{1.5\,\omega_{cri}}{\omega_n} = \frac{152.6347}{97.6014} = 1.5638$$

$$A = \frac{(0.005)\,(1.5638^2)}{\sqrt{(1-1.5638^2)^2 + (2\,(0.01)\,(1.5638))^2}} = 0.008457 \text{ m}$$

9.18 (a) At operating speed:

$r = 6.4376$; $\omega = 628.32$ rad/sec. Deflection of mass center:

$$R = a\left\{\frac{1 + (2\,\varsigma\,r)^2}{(1-r^2)^2 + (2\,\varsigma\,r)^2}\right\}^{\frac{1}{2}}$$

$$= (0.005)\left\{\frac{1 + (2\,(0.01)\,(6.4376))^2}{(1-6.4376^2)^2 + (2\,(0.01)\,(6.4376))^2}\right\}^{\frac{1}{2}} = 1.2465\,(10^{-4}) \text{ m}$$

Centrifugal force: $m\,\omega^2\,R = (20)\,(628.32^2)\,(1.2465\,(10^{-4})) = 984.2015$ N

Bearing reactions: $R_1 = R_2 = \dfrac{m\,\omega^2\,R}{2} = 492.1007$ N

$$I = \frac{\pi}{64}\,d^4 = \frac{\pi}{64}\,(0.025^4) = 1.9175\,(10^{-8}) \text{ m}^4$$

Maximum bending stress:

$$\frac{(R_1 \frac{\ell}{2}) \frac{d}{2}}{I} = \frac{492.1007 \, (\frac{1}{2}) \, (\frac{0.025}{2})}{1.9175 \, (10^{-8})} = 1.6040 \, (10^8) \text{ N/m}^2$$

(b) At critical speed:

$r = 1.0426$; $\omega = 101.7565$ rad/sec

$$R = (0.005) \left\{ \frac{1 + (2 \, (0.01) \, (1.0426))^2}{(1 - 1.0426^2)^2 + (2 \, (0.01) \, (1.0426))^2} \right\}^{\frac{1}{2}} = 0.05589 \text{ m}$$

Centrifugal force: $m \, \omega^2 \, R = (20) \, (101.7565^2) \, (0.05589) = 11574.1319$ N

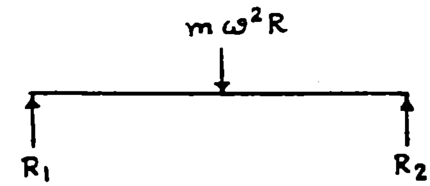
$$R_1 = R_2 = 5787.0659 \text{ N}$$

Maximum bending stress:

$$= \frac{R_1 \, (\frac{\ell}{2}) \, (\frac{d}{2})}{I} = \frac{5787.0659 \, (\frac{1}{2}) \, (\frac{0.025}{2})}{1.9175 \, (10^{-8})} = 18.8627 \, (10^8) \text{ N/m}^2$$

(c) At 1.5 times critical speed:

$r = 1.5638$; $\omega = 152.6347$ rad/sec



$$R = (0.005) \left\{ \frac{1 + (2 \, (0.01) \, (1.5638))^2}{(1 - 1.5638^2)^2 + (2 \, (0.01) \, (1.5638))^2} \right\}^{\frac{1}{2}} = 0.003460 \text{ m}$$

Centrifugal force:

$$m \, \omega^2 \, R = (20) \, (152.6347^2) \, (0.003460) = 1612.1767 \text{ N}$$

$$R_1 = R_2 = 806.0884 \text{ N}$$

Maximum bending stress:

$$= \frac{(806.0884) \, (\frac{1}{2}) \, (\frac{0.025}{2})}{1.9175 \, (10^{-8})} = 2.6274 \, (10^8) \text{ N/m}^2$$

9.19 Stiffness of beam (k):

$$k = \frac{48 \, EI}{\ell^3} = \frac{48 \, (71 \, (10^9)) \, (1.9175 \, (10^{-8}))}{1^3} = 65347.7344 \text{ N/m}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{65347.7344}{20}} = 57.1611 \text{ rad/sec}$$

(a) At operating speed:

$$\omega = \frac{6000}{60}\,(2\,\pi) = 628.32 \text{ rad/sec}; \; r = \frac{\omega}{\omega_n} = \frac{628.32}{57.1611} = 10.9921$$

$$\text{Whirl amplitude of rotor:} \quad A = \frac{a\,r^2}{\left\{(1-r^2)^2 + (2\,\varsigma\,r)^2\right\}^{\frac{1}{2}}}$$

$$= \frac{(0.005)\,(10.9921^2)}{\left\{(1-10.9921^2)^2 + (2\,(0.01)\,(10.9921))^2\right\}^{\frac{1}{2}}} = 0.005041 \text{ m}$$

(b) At critical speed:

$$c = 2\sqrt{k\,m}\;\varsigma = 2\sqrt{(65347.7344)\,(20)}\;(0.01) = 22.8644 \text{ N–s/m}$$

$$\omega_{cri} = \omega = \frac{\omega_n}{\left\{1 - \frac{1}{2}\left(\frac{c}{\omega_n}\right)^2\right\}^{\frac{1}{2}}} = \frac{57.1611}{\left\{1 - \frac{1}{2}\left(\frac{22.8644}{57.1611}\right)^2\right\}^{\frac{1}{2}}} = 59.5946 \text{ rad/sec}$$

$$r = \frac{\omega}{\omega_n} = \frac{59.5946}{57.1611} = 1.0426$$

$$A = \frac{(0.005)\,(1.0426^2)}{\left\{(1-1.0426^2)^2 + (2\,(0.01)\,(1.0426))^2\right\}^{\frac{1}{2}}} = 0.06074 \text{ m}$$

(c) At 1.5 times critical speed:

$$r = \frac{1.5\,\omega_{cri}}{\omega_n} = \frac{89.3919}{57.1611} = 1.5638$$

$$A = \frac{(0.005)\,(1.5638^2)}{\left\{(1-1.5638^2)^2 + (2\,(0.01)\,(1.5638))^2\right\}^{\frac{1}{2}}} = 0.008457 \text{ m}$$

9.20 (a) At operating speed:

$r = 10.9921$; $\omega = 628.32$ rad/sec. Deflection of mass center of disc:

$$R = \frac{(0.005)\left\{1 + (2\,(0.01)\,(10.9921))^2\right\}^{\frac{1}{2}}}{\left\{(1 - 10.9921^2)^2 + (2\,(0.01)\,(10.9921))^2\right\}^{\frac{1}{2}}} = 0.4272\,(10^{-4})\ \text{m}$$

Centrifugal force: $m\,\omega^2\,R = 20\,(628.32^2)\,(0.4272\,(10^{-4})) = 337.3052\ \text{N}$

Bearing reactions: $R_1 = R_2 = \dfrac{m\,\omega^2\,R}{2} = 168.6526\ \text{N}$

Maximum bending stress: $\dfrac{R_1\,(\frac{\ell}{2})\,(\frac{d}{2})}{I} = \dfrac{(168.6526)(\frac{1}{2})\,(\frac{0.025}{2})}{1.9175\,(10^{-8})} = 0.5497\,(10^8)\ \text{N/m}^2$

(b) At critical speed:

$r = 1.0426$; $\omega = 59.5946$ rad/sec

$$R = \frac{(0.005)\left\{1 + (2\,(0.01)\,(1.0426))^2\right\}^{\frac{1}{2}}}{\left\{(1 - 1.0426^2)^2 + (2\,(0.01)\,(1.0426))^2\right\}^{\frac{1}{2}}} = 0.05589\ \text{m}$$

Centrifugal force: $m\,\omega^2\,R = (20)\,(59.5946^2)\,(0.05589) = 3969.8850\ \text{N}$

$$R_1 = R_2 = 1984.9425\ \text{N}$$

Maximum bending stress:

$$\frac{R_1\,(\frac{\ell}{2})\,(\frac{d}{2})}{I} = \frac{(1984.9425)\,(\frac{1}{2})\,(\frac{0.025}{2})}{1.9175\,(10^{-8})} = 6.4698\,(10^8)\ \text{N/m}^2$$

(c) At 1.5 times critical speed:

$r = 1.5638$; $\omega = 89.3919$ rad/sec

$$R = \frac{(0.005)\left\{1 + (2\,(0.01)\,(1.5638))^2\right\}^{\frac{1}{2}}}{\left\{(1 - 1.5638^2)^2 + (2\,(0.01)\,(1.5638))^2\right\}^{\frac{1}{2}}} = 0.003460\ \text{m}$$

Centrifugal force: $m\,\omega^2\,R = (20)\,(89.3919^2)\,(0.003460) = 552.9711\ \text{N}$

$R_1 = R_2 = 276.4855\ \text{N}$

Maximum bending stress:

$$\frac{R_1\,(\frac{\ell}{2})\,(\frac{d}{2})}{I} = \frac{(276.4855)\,(\frac{1}{2})\,(\frac{0.025}{2})}{1.9175\,(10^{-8})} = 0.9012\,(10^8)\ \text{N/m}^2$$

9.21 $k = 3.75\,(10^6)$ N/m ; $\varsigma = 0.05$; $\omega = \frac{3600}{60}\,(2\,\pi) = 376.992$ rad/sec

$m = 60$ kg ; $a = 2000\,(10^{-6})$ m

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.75\,(10^6)}{60}} = 250.0\ \text{rad/sec}$$

$$r = \frac{\omega}{\omega_n} = \frac{376.992}{250.0} = 1.5080$$

(a) Steady state whirl amplitude:

$$A = \frac{a\,r^2}{\left\{(1-r^2)^2 + (2\,\varsigma\,r)^2\right\}^{\frac{1}{2}}} \tag{1}$$

$$= \frac{(2000\,(10^{-6}))\,(1.5080^2)}{\left\{(1-1.5080^2)^2 + (2\,(0.05)\,(1.5080))^2\right\}^{\frac{1}{2}}} = 0.003545\ \text{m}$$

(b) During start-up and stopping conditionss, rotor passes through the natural frequency of the system. Thus, using $r = 1$ in Eq. (1), we obtain the whirl amplitude as

$$A\,\big|_{r=1} = \frac{a}{2\,\varsigma} = \frac{0.005}{2\,(0.05)} = 0.05\ \text{m} \tag{2}$$

9.22 Let $t = 0$

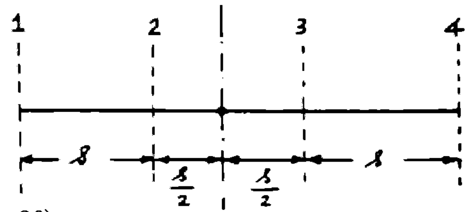
Unbalanced forces:

$$F_{xp} = m\,r\,\omega^2\,(\cos\alpha_1 + \cos\alpha_2 + \cos\alpha_3 + \cos\alpha_4)$$
$$= m\,r\,\omega^2\,(\cos 0^\circ + \cos 180^\circ + \cos 180^\circ + \cos 0^\circ)$$
$$= 0$$

$$F_{xs} = m\,\frac{r^2\,\omega^2}{\ell}\sum_{i=1}^{4}\cos 2\,\alpha_i = \frac{m\,r^2\,\omega^2}{\ell}\,(\cos 0^\circ + \cos 360^\circ + \cos 360^\circ + \cos 0^\circ)$$

$$= \frac{4\,m\,r^2\,\omega^2}{\ell} = \frac{4}{0.25}\,(1)\,(0.1)^2\left(\frac{3000\,(2\,\pi)}{60}\right)^2 = 15791.37\ \text{N}$$

Unbalanced moments:



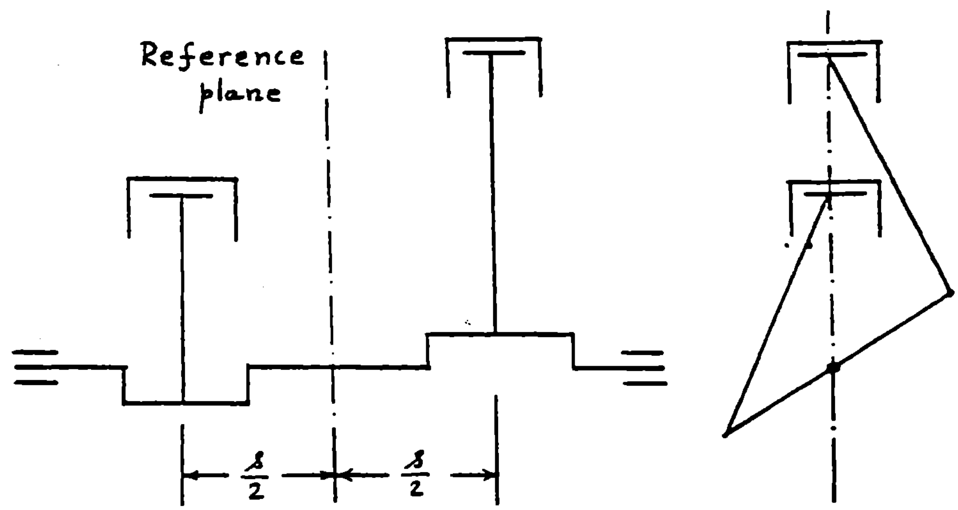
$$M_{zp} = F_{xp_1}\left(\frac{3\,s}{2}\right) + F_{xp_2}\left(\frac{s}{2}\right) - F_{xp_3}\left(\frac{s}{2}\right) - F_{xp_4}\left(\frac{3\,s}{2}\right)$$

$$M_{zp} = \frac{m\,r\,\omega^2\,s}{2}\,(3\cos 0° + \cos 180° - \cos 180° - 3\cos 0°)$$

$$M_{zs} = F_{xs_1}\left(\frac{3\,s}{2}\right) + F_{xs_2}\left(\frac{s}{2}\right) - F_{xs_3}\left(\frac{s}{2}\right) - F_{xs_4}\left(\frac{3\,s}{2}\right)$$

$$= \frac{m\,r^2\,\omega^2\,s}{2\,\ell}\,(3\cos 0° + \cos 360° - \cos 360° - 3\cos 0°) = 0$$

9.23



Let the cylinders be separated axially by a distance s.

$$F_{xp} = m r \omega^2 (\cos\alpha_1 + \cos\alpha_2) = m r \omega^2 (\cos 0° + \cos 180°) = 0$$

$$F_{xs} = \frac{m r^2 \omega^2}{\ell}(\cos 2\alpha_1 + \cos 2\alpha_2) = \frac{m r^2 \omega^2}{\ell}(\cos 0° + \cos 360°) = \frac{2 m r^2 \omega^2}{\ell}$$

Moments about the reference plane:

$$M_{zp} = F_{xp1}\cdot\frac{s}{2} - F_{xp2}\cdot\frac{s}{2} = \frac{s\,m r \omega^2}{2}(\cos 0° - \cos 180°) = s\,m r \omega^2$$

$$M_{zs} = F_{xs1}\cdot\frac{s}{2} - F_{xs2}\cdot\frac{s}{2} = \frac{s\,m r^2 \omega^2}{2\ell}(\cos 0° - \cos 360°) = 0$$

$\therefore$ Secondary forces and primary couple are unbalanced.

9.24 $r = 0.075$ m, $m = 1.5$ kg, $\ell = 0.25$ m, $\omega = 1500$ rpm $= 157.08$ rad/sec,
$\alpha_1 = 0°, \alpha_2 = 180°, \alpha_3 = 90°, \alpha_4 = 270°$

Assume $m_c = 0$ and $m_p = m = 1.5$ kg

Consider the vertical and horizontal components of the inertia forces at $t = 0$. This gives

$$F_{xp} = (m_p + m_c)\, r\, \omega^2 \sum_{i=1}^{4} \cos \alpha_i$$
$$= (m_p + m_c)\, r\, \omega^2 (\cos 0° + \cos 180° + \cos 90° + \cos 270°)$$
$$= 0$$

$$F_{yp} = -m_c\, r\, \omega^2 \sum_{i=1}^{4} \sin \alpha_i = -m_c\, r\, \omega^2 (\sin 0° + \sin 180° + \sin 90° + \sin 270°) = 0$$

$$F_{xs} = m_p \frac{r^2 \omega^2}{\ell} \sum_{i=1}^{4} \cos 2\alpha_i = \frac{m_p r^2 \omega^2}{\ell} (\cos 0° + \cos 360° + \cos 180° + \cos 540°) = 0$$

Primary and secondary forces are balanced.

Moments about the reference plane:

$$M_{zp} = F_{xp_1} (0.15) + F_{xp_2} (0.05) - F_{xp_3} (0.05) - F_{xp_4} (0.15)$$
$$= (m_p + m_c)\, r\, w^2 [0.15 \cos 0° + 0.05 \cos 180° - 0.05 \cos 90° - 0.15 \cos 270°]$$
$$= 0.1\, r\, \omega^2 (m_p + m_c) = 0.1 (0.075) (157.08)^2 (1.5) = 277.584 \text{ N}\cdot\text{m}$$
= unbalanced primary couple

$$M_{zs} = F_{xs_1} (0.15) + F_{xs_2} (0.05) - F_{xs_3} (0.05) - F_{xs_4} (0.15)$$
$$= \frac{m_p r^2 \omega^2}{\ell} [0.15 \cos 2\alpha_1 + 0.05 \cos 2\alpha_2 - 0.05 \cos 2\alpha_3 - 0.15 \cos 2\alpha_4]$$
$$= \frac{m_p r^2 \omega^2}{\ell} (0.4) = \frac{1.5 (0.075)^2 (157.08)^2}{0.25} \times 0.4 = 333.101 \text{ N}\cdot\text{m}$$

$$M_{xp} = F_{yp_1} (0.15) + F_{yp_2} (0.05) - F_{yp_3} (0.05) - F_{yp_4} (0.15)$$
$$= -m_c\, r\, \omega^2 (0.15 \sin \alpha_1 + 0.05 \sin \alpha_2 - 0.05 \sin \alpha_3 - 0.15 \sin \alpha_4)$$
$$= -m_c\, r\, \omega^2 (0.1) = 0 \quad \text{since} \quad m_c = 0$$

9.25 Primary unbalanced forces are given by

$$F_{xp} = \sum_{i=1}^{6} (F_x)_{p_i} = \sum (m_p + m_c)_i\, r\, \omega^2 \cos(\omega t + \alpha_i) \qquad (E_1)$$

$$F_{yp} = \sum_{i=1}^{6} (F_y)_{p_i} = \sum -(m_c)_i\, r\, \omega^2 \sin(\omega t + \alpha_i) \qquad (E_2)$$

Secondary unbalanced force is given by

$$F_{xs} = \sum_{i=1}^{6} (F_x)_{s_i} = \sum (m_p)_i \frac{r^2 \omega^2}{\ell} \cos(2\omega t + 2\alpha_i) \qquad (E_3)$$

Primary and secondary unbalanced moments are given by

$$(M_z)_p = \sum_{i=2}^{6} (F_x)_{pi}\, l_i \qquad (E_4)$$

$$(M_z)_s = \sum_{i=2}^{6} (F_x)_{si}\, l_i \qquad (E_5)$$

$$(M_x)_p = \sum_{i=2}^{6} (F_y)_{pi}\, l_i \qquad (E_6)$$

Eq. (E_1) gives

$$(m_p + m_c)\, r\omega^2 \sum_{i=1}^{6} \cos\alpha_i = (m_p+m_c)\, r\omega^2 \left(2\cos 0^\circ + 2\cos 120^\circ + 2\cos 240^\circ\right) = 0$$

Eq. (E_2) gives

$$-m_c r\omega^2 \sum_{i=1}^{6} \sin\alpha_i = -m_c r\omega^2 \left(2\sin 0^\circ + 2\sin 120^\circ + 2\sin 240^\circ\right) = 0$$

Eq. (E_3) gives

$$\frac{m_p r^2\omega^2}{l} \sum_{i=1}^{6} \cos 2\alpha_i = \frac{m_p r^2 \omega^2}{l}\left(2\cos 0^\circ + 2\cos 240^\circ + 2\cos 480^\circ\right) = 0$$

Eq. (E_4) gives

$$(m_p+m_c)\, r\omega^2 \sum_{i=2}^{6} l_i \cos\alpha_i = (m_p+m_c)\, r\omega^2 a \left(\cos 120^\circ + 2\cos 240^\circ + 3\cos 240^\circ + 4\cos 120^\circ + 5\cos 0^\circ\right) = 0$$

Eq. (E_5) gives

$$\frac{m_p r^2\omega^2}{l} \sum_{i=2}^{6} l_i \cos 2\alpha_i = \frac{m_p r^2\omega^2 a}{l}\left[\cos 240^\circ + 2\cos 480^\circ + 3\cos 480^\circ + 4\cos 240^\circ + 5\cos 0^\circ\right] = 0$$

Eq. (E_6) gives

$$-m_c r\omega^2 \sum_{i=2}^{6} l_i \sin\alpha_i = -m_c r\omega^2 a \left(\sin 120^\circ + 2\sin 240^\circ + 3\sin 240^\circ + 4\sin 120^\circ + 5\sin 0^\circ\right) = 0$$

∴ Engine is completely force and moment balanced.

9.26 (a) $\omega_n = \sqrt{\frac{k}{M}} \simeq 0$; $\omega = 600$ rpm $= 600 \times \frac{2\pi}{60} = 62.832$ rad/s

$\frac{\omega}{\omega_n} \simeq \infty$, $c = 0$

$F_0 = m\omega^2 r = (2.5)(62.832)^2\left(\frac{7.5}{100}\right) = 740.2238$ N

Eq. (9.90) ⇒ $X = \frac{F_0}{(k - m\omega^2)} = \frac{F_0}{k\left(1 - \frac{\omega^2}{\omega_n^2}\right)} = 0$ for $\frac{\omega}{\omega_n} \simeq \infty$

(b) $\omega_n = \infty$, $\frac{\omega}{\omega_n} = 0$

Eq.(9.93) $\Rightarrow$ $F_T = \frac{F_0\, k}{k - m\omega^2} = \frac{F_0}{1 - \left(\frac{\omega}{\omega_n}\right)^2} = F_0 = 740.2238$

9.27 $\omega =$ 25 Hz to 35 Hz $= 157.08$ rad/sec to 219.912 rad/sec

$m = 85/9.81 = 8.6646$ kg

Transmissibility of an undamped isolator is given by Eq.(9.94):

$$T_r = \frac{1}{\left|1 - (\omega/\omega_n)^2\right|} \qquad (E_1)$$

For 80% vibration isolation, Eq. (E_1) gives

$$0.2 = \frac{1}{\left|1 - (\omega/\omega_n)^2\right|} \qquad \text{i.e.,} \qquad \left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right| = 5$$

or $\frac{\omega}{\omega_n} = \sqrt{6} = 2.4495$

At $\omega =$ 25 Hz, $\omega_n = 157.08/2.4495 = 64.1274$ rad/sec

At $\omega =$ 35 Hz, $\omega_n = 219.912/2.4495 = 89.7783$ rad/sec

But $\omega_n = \sqrt{k/m} = \sqrt{kg/W} = \sqrt{g/\delta_{st}} = \sqrt{9.81/\delta_{st}}$

or $\delta_{st} = \frac{9.81}{\omega_n^2}$

At $\omega =$ 25 Hz, $\delta_{st} = 9.81/(64.1274)^2 = 0.002385$ m

At $\omega =$ 35 Hz, $\delta_{st} = 9.81/(89.7783)^2 = 0.001217$ m

$\therefore$ Select static deflection of isolator as 0.002385 m.

checking the performance at $\omega =$ 35 Hz :

$\omega_n = 64.1274$ rad/sec. At $\omega =$ 35 Hz,

$$T_r = \frac{1}{\left|1 - (219.912/64.1274)^2\right|} = 0.0850$$

$\Rightarrow$ 91.5% isolation, better than the required amount.

$\therefore$ δ_{st} of isolator $= 0.2385$ mm

9.28 $mg = 800$ N, $\omega_0 = 600$ rpm $=$ operating speed $= 62.832$ rad/sec

$T_r = 2.5$ at $\omega = \omega_n$

Eq. (9.94) gives

$$T_r^2 = \frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2} \quad \text{with} \quad r = \frac{\omega}{\omega_n} \qquad (E_1)$$

At $r=1$, $\quad T_r^2 = \frac{1+4\zeta^2}{4\zeta^2}$; $\quad 6.25 = \frac{1+4\zeta^2}{4\zeta^2} \Rightarrow \zeta = 0.2182$

At operating speed, $T_r = 0.1$ and $\zeta = 0.2182$; Eq. (E_1) gives

$$(0.1)^2 = \frac{1+4(0.2182)^2 r^2}{(1-r^2)^2+4(0.2182)^2 r^2}$$

which, upon simplification, becomes

$$r^4 - 20.8595\, r^2 - 99 = 0 \quad \Rightarrow \quad r^2 = 24.8443$$

or $\quad r = \frac{\omega_0}{\omega_n} = 4.9844$

Since $\omega_0 = 62.832$ rad/sec, $\quad \omega_n = 12.6057$ rad/sec $= \sqrt{\frac{k}{m}}$

$$k = \omega_n^2\, m = (12.6057)^2 \left(\frac{800}{9.81}\right) = 12958.5054 \text{ N/m}$$

$\therefore$ Isolator is defined by

$k = 12\,958.5054$ N/m

$c = 2\, m\, \omega_n\, \zeta = 2\left(\frac{800}{9.81}\right)(12.6057)(0.2182) = 448.6139 \ \frac{N\text{-}s}{m}$

9.29 $M = 500$ kg, $me = 50$ kg-cm, $\omega = 300$ rpm $= 31.416$ rad/sec

F_0 = steady state force magnitude $= me\,\omega^2$, $\quad \omega_n = \sqrt{k/M}$

Static deflection of compressor $= \delta_{st} = \frac{F_0}{k} = \frac{me\,\omega^2}{k} \qquad (E_1)$

Transmission ratio $= T_r = \frac{F_t}{F_0} = \left\{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right\}^{1/2} \qquad (E_2)$

with $r = \omega/\omega_n$.

Amplitude of vibration of compressor

$$= X = F_0 \Big/ \left[(k - M\omega^2)^2 + \omega^2 c^2\right]^{1/2} = \frac{me}{M}\,\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \qquad (E_3)$$

For a good design, T_r must be small. Also X should be small for smaller dynamic stress.

Isolator with k

Eq. (E_2):

$$T_r = \frac{1}{|1 - r^2|}$$

For small T_r, $r = \frac{\omega}{\omega_n}$ should be large or ω_n small

Let $T_r = 0.1$ so that $|1 - r^2| = 10$.

$$r = \frac{\omega}{\omega_n} = \sqrt{11} = 3.3166$$

$$\omega_n = \omega/3.3166 = 31.416/3.3166$$

$$= 9.4724 \text{ rad/sec}$$

$$= \sqrt{\frac{k}{500}}$$

$$k = (9.4724)^2 (500)$$

$$= 44\,863.1809 \text{ N/m}$$

$$\delta_{st} = \frac{m e \omega^2}{k} = \frac{\left(\frac{50}{100}\right)(31.416)^2}{44\,863.1809}$$

$$= 0.0110 \text{ m}$$

$$r^2 = 11, \quad (1 - r^2)^2 = 100$$

$$(2 \zeta r)^2 = (0.2 r)^2 = (0.2 \times 3.3166)^2$$

$$= 0.44$$

Eq. (E_3):

$$X = \frac{\left(\frac{50}{100}\right)}{500} \cdot \frac{11}{\sqrt{100 + 0.44}}$$

$$= 0.001098 \text{ m}$$

Shock absorber with ζ and k

Eq. (E_2) for $T_r = 0.1$ and $\zeta = 0.1$:

$$0.1 = \left\{ \frac{1 + (0.2 r)^2}{(1 - r^2)^2 + (0.2 r)^2} \right\}^{1/2}$$

or $\quad r^4 - 5.96 r^2 - 99 = 0$

or $\quad r^2 = 13.3665$

or $\quad r = \omega/\omega_n = 3.6560$

$$\omega_n = 31.416/3.656$$

$$= 8.5929 \text{ rad/sec}$$

$$k = (8.5929)^2 (500)$$

$$= 36\,919.2174 \text{ N/m}$$

$$\delta_{st} = \frac{m e \omega^2}{k} = \frac{\left(\frac{50}{100}\right)(31.416)^2}{36\,919.2174}$$

$$= 0.0134 \text{ m}$$

$$r^2 = 13.3665, \quad (1 - r^2)^2 = 152.9303$$

$$(2 \zeta r)^2 = (0.2 \times 3.656)^2$$

$$= 0.5347$$

Eq. (E_3):

$$X = \frac{\left(\frac{50}{100}\right)}{500} \cdot \frac{13.3665}{\sqrt{152.9303 + 0.5347}}$$

$$= 0.001079 \text{ m}$$

Since X is smaller in the case of shock absorber, it is to be preferred. In this case, a smaller value of k will be sufficient; this leads to a cheaper design.

9.30 $m = 200$ kg, $k = 10000$ N/m, $\zeta = 0.15$, $\omega_n = \sqrt{\frac{10^4}{200}} = 7.0711$ rad/s

(a) $\frac{F_t}{F_0} > 1$:

From Eq. (9.94), $\left\{ \frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right\}^{\frac{1}{2}} > 1$; $r = \frac{\omega}{\omega_n}$

i.e. $1 + (2\zeta r)^2 > (1 - r^2)^2 + (2\zeta r)^2$

i.e. $1 > (1 - r^2)^2$

This gives $1 > 1 + r^4 - 2r^2$; $r^4 - 2r^2 < 0$

$r^2 (r^2 - 2) < 0$

$r^2 < 0$ or $r^2 < 2$

Physically possible solution is $\omega < \sqrt{2}\, \omega_n$

$< \sqrt{2}\,(7.0711) = 10$ rad/s

< 95.4927 rpm

(b) $\frac{F_t}{F_0} < 0.1$:

From Eq. (9.94), $1 + (2\zeta r)^2 < 0.01 \{(1-r^2)^2 + (2\zeta r)^2\}$

$1 + 0.09\, r^2 < 0.01 + 0.01\, r^4 - 0.02\, r^2 + 0.0009\, r^2$

$0.01\, r^4 - 0.1091\, r^2 - 0.99 > 0$

$(r^2 - 16.8021)(r^2 + 5.8920) > 0$

i.e. $r^2 - 16.8021 > 0$, $r^2 + 5.8920 > 0$ or $r^2 - 16.8021 < 0$, $r^2 + 5.892 < 0$

i.e. $r^2 > 16.8021$, $r^2 > -5.892$ or $r^2 < 16.8021$, $r^2 < -5.892$ (not possible)

$\therefore$ $r^2 > 16.8021$, $r > 4.0990$

$\omega > 4.0990(7.0711) = 28.9844$ rad/s $= 276.7803$ rpm

9.31 For undamped system, transmission ratio is $T_r = \frac{F_T}{F_0} = \frac{k}{k - m\,\omega^2}$

Since isolation is 60%, we have

$$\frac{F_t}{F_0} = 0.4 = \pm \left\{ \frac{k}{k - (68)\left(\frac{300}{60}(2\pi)\right)^2} \right\} = \pm \left(\frac{k}{k - 67113.3} \right)$$

or $0.4\,k - 26845.3 = -k$ or $k = 19175.23$ N/m

Thus k has to be less than 19175.23 N/m to provide more than 60% isolation.

$$k = \frac{m\,g}{\delta_{st}} \text{ or } (\delta_{st})_{min} = \frac{m\,g}{k_{max}} = \frac{(68)\,(981)}{19175.23} = 0.034786 \text{ m}$$

9.32 $m = 50$ kg ; $\omega = \frac{1200}{60}(2\pi) = 125.664$ rad/sec ; $\varsigma = 0.07$

$$T_r = \frac{F_T}{F_0} = \left\{ \frac{1 + (2 \varsigma r)^2}{(1 - r^2)^2 + (2 \varsigma r)^2} \right\}^{\frac{1}{2}} \quad (1)$$

For 75% isolation, Eq. (1) gives

$$0.25^2 = \frac{1 + (2\,(0.07)\,r)^2}{(1 - r^2)^2 + (2\,(0.07)\,(r))^2} \quad \text{or} \quad 0.0625\, r^4 - 0.143375\, r^2 - 0.9375 = 0 \quad (2)$$

The solution of Eq. (2) is given by:

$$r^2 = 5.186255 \text{ (positive value)} \quad \text{or} \quad r = \frac{\omega}{\omega_n} = 2.2773$$

This gives $\omega_n = \frac{\omega}{2.2773} = \frac{125.664}{2.2773} = 55.1803$ rad/sec

Maximum stiffness: $k = m\,\omega_n^2 = (50)\,(55.1803^2) = 152{,}243.1865$ N/m

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9.33 $m = 80$ kg ; $\omega = \frac{1000}{60}(2\pi) = 104.72$ rad/sec

$$\text{(a)} \quad \frac{F_T}{F_0} = \frac{2000}{10000} = 0.2 = \pm \frac{k}{k - m\,\omega^2} = \pm \frac{k}{k - (80)\,(104.72^2)} \quad (1)$$

Using the negative sign in Eq. (1), we find

$$0.2\,k - 17.5460\,(10^4) = -k$$

$$\text{Maximum stiffness} = k_{max} = \frac{17.5460\,(10^4)}{1.2} = 146217.0453 \text{ N/m}$$

(b) Steady state amplitude:

$$X = \left| \frac{F_0}{k - m\,\omega^2} \right| = \left| \frac{10000}{146217.0453 - 87.7302\,(10^4)} \right| = 0.01368 \text{ m}$$

(c) Maximum amplitude of fan during start-up:

$$X = \left| \frac{F_0}{k - m\,\omega^2} \right| = \left| \frac{F_0}{k\left(1 - \frac{\omega^2}{\omega_n^2}\right)} \right|$$

$$\text{where} \quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{146217.0453}{80}} = 42.7518 \text{ rad/sec}$$

Using $\omega = \omega_n$, $X \to \infty$. Hence an undamped isolator must pass through resonance very quickly to avoid damage.

9.34 $m = 300 \text{ kg} \; ; \; \omega = \frac{3000}{60} (2\,\pi) = 314.16 \text{ rad/sec} \; ; \; F_0 = 30{,}000 \text{ N}$

Requirements:

1. $\delta_{st} = \frac{m\,g}{k} = \text{small} \;\rightarrow\; k = \text{large}$ (1)

2. $X = \frac{F_0}{\left\{(k-\omega^2)^2 + \omega^2\,c^2\right\}^{\frac{1}{2}}} \leq 2.5\,(10^{-3})\text{ m}$ (2)

3. $X\,|_{r=1} = \frac{F_0}{k\left\{\left(1-r^2\right)^2 + \left(\frac{\omega^2\,c^2}{k^2}\right)\right\}^{\frac{1}{2}}}\,|_{r=1} < 2\,(10^{-2})\text{ m}$ (3)

where $r = \frac{\omega}{\omega_n}$

4. $\frac{F_T}{F_0} = \left\{\frac{k^2 + \omega^2\,c^2}{(k - m\,\omega^2)^2 + \omega^2\,c^2}\right\}^{\frac{1}{2}} \leq \frac{10000}{30000} = \frac{1}{3}$ (4)

5. For achieving isolation, $r > \sqrt{2}$. Hence

$$k < \frac{m\,\omega^2}{2} \quad \text{or} \quad k < 14.8045\,(10^6)\text{ N/m} \qquad (5)$$

Since four inequalities, Eqs. (2) to (5), are to be satisfied, in general, we need to use an iterative process. From Eq. (3), we obtain:

$$\frac{F_0}{k\,(\frac{\omega\,c}{k})} = \frac{F_0}{\omega\,c} = \frac{30000}{314.16\,c} \leq 0.02 \quad \text{or} \quad c \geq 4774.6371 \text{ N–s/m} \qquad (6)$$

We assume $c = 5000.0$ N-s/m and $k = 6\,(10^6)$ N/m as trial values. These values satisfy Eqs. (5) and (6) and give:

$$\left\{(k - m\,\omega^2)^2 + \omega^2\,c^2\right\}^{\frac{1}{2}} = \left\{(6\,(10^6) - (300)\,(314.16^2))^2 + (314.16\,(5000))^2\right\}^{\frac{1}{2}}$$
$$= 23.6611\,(10^6)$$

and the left hand side of Eq. (2) becomes:

$$\frac{30000.0}{23.6611\,(10^6)} = 0.001268 \text{ m} < 0.0025 \text{ m}$$

and hence Eq. (2) is satisfied. The numerator on the left hand side of Eq. (4) is:

$$\left\{k^2 + \omega^2\,c^2\right\}^{\frac{1}{2}} = \left\{36\,(10^{12}) + (314.16\,(5000.0))^2\right\}^{\frac{1}{2}} = 6.1844\,(10^6)$$

and the left hand side of Eq. (4) thus becomes:

$$\frac{6.1844\ (10^6)}{23.6611\ (10^6)} = 0.2614$$

which is less than the value on the right hand side. Thus Eq. (4) is satisfied. The final design is given by k = 6.0 (10^6) N/m and c = 5000.0 N-s/m.

(9.35) m = 120 kg ; me = 0.2 kg-m ; k = 0.5 (10^6) N/m ; $\varsigma = 0.06$

$$\omega_n = \left\{\frac{k}{m}\right\}^{\frac{1}{2}} = \left\{\frac{0.5\ (10^6)}{120}\right\}^{\frac{1}{2}} = 64.5497 \text{ rad/sec} \quad ; \quad F_T < 2500 \text{ N}$$

Eq. (9.104) gives:

$$r^2 \left\{\frac{1 + (2\,(0.06)\,(r))^2}{(1-r^2)^2 + (2\,(0.06)\,(r))^2}\right\}^{\frac{1}{2}} = \frac{F_T}{m\,e\,\omega_n^2} < \frac{2500}{(0.2)\,(64.5497^2)} < 3$$

$$\text{or} \quad \left(\frac{1 + 0.0144\,r^2}{1 + r^4 - 2\,r^2 + 0.0144\,r^2}\right) < \frac{9}{r^4} \tag{1}$$

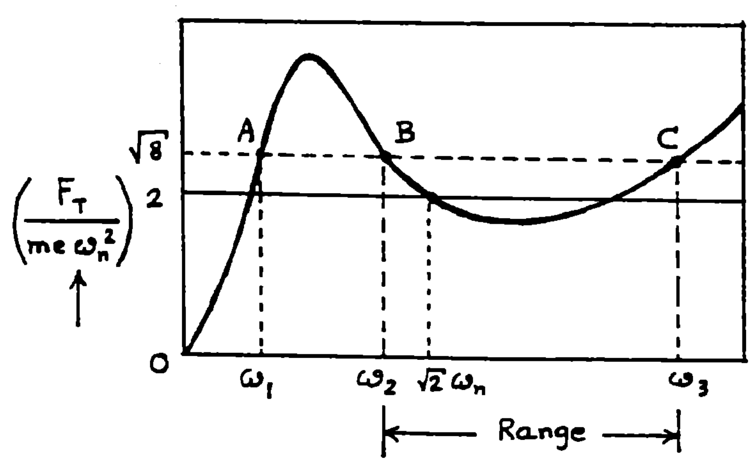
Setting the left hand side of Eq. (1) equal to $\frac{8}{r^4}$, we obtain

$$r^6 - 486.1111\,r^4 + 1103.1111\,r^2 - 555.5555 = 0 \tag{2}$$

The roots of this cubic equation are given by

$$r_1^2 = 0.751358 \quad ; \quad r_1 = 0.866809 \quad ; \quad \omega_1 = 55.9523 \text{ rad/sec}$$
$$r_2^2 = 1.52760 \quad ; \quad r_2 = 1.23596 \quad ; \quad \omega_2 = 79.7808 \text{ rad/sec}$$
$$r_3^2 = 483.834 \quad ; \quad r_3 = 21.9962 \quad ; \quad \omega_3 = 1419.8481 \text{ rad/sec}$$

From Fig. 3.16, the values of ω_1, ω_2 and ω_3 can be interpreted as shown in the following figure. It can be seen that the force transmitted to the foundation will be less than 2500 N (actually, less than $2500\sqrt{8}/3 = 2357.0226$ N) over the frequency range $\omega_2 - \omega_3$ (i.e., 79.7808 rad/sec - 1419.8481 rad/sec).



9.36 me = 1.0 kg-m ; ω = 800 to 2000 rpm = 83.776 to 209.44 rad/sec
F_0 = 7018 N at 800 rpm and 43865 N at 2000 rpm
$F_T \leq 6000$ N over the speed range ; $\varsigma = 0.08$

To find k.

$$\text{Relation be satisfied:} \quad \frac{F_T}{m\,e\,\omega^2} = \left\{\frac{1+(2\,\varsigma\,r)^2}{(1-r^2)^2+(2\,\varsigma\,r)^2}\right\}^{\frac{1}{2}} \leq \frac{6000}{m\,e\,\omega^2}$$

$$\text{or} \quad \left\{\frac{1+0.0256\,r^2}{(1-r^2)^2+0.0256\,r^2}\right\}^{\frac{1}{2}} \leq \frac{6000}{7018} = 0.8549 \text{ at } \omega = 800 \text{ rpm}$$

$$\text{and} \quad \leq \frac{6000}{43865} = 0.1368 \text{ at } \omega = 2000 \text{ rpm} \qquad (1)$$

Equating the lest side of Eq. (1) to 0.85 at ω = 800 rpm, we obtain

$$\frac{1+0.0256\,r_1^2}{1+r_1^4-2\,r_1^2+0.0256\,r_1^2} = 0.7225$$

$$\text{or} \quad r_1^4 - 2.00983\,r_1^2 - 0.3841 = 0 \quad \text{or} \quad r_1^2 = 2.1856 \text{ (positive root)}$$
$$\text{or} \quad r_1 = 1.4784$$

Equating the left side of Eq. (1) to 0.135 at ω = 2000 rpm, we obtain

$$\frac{1+0.0256\,r_2^2}{1+r_2^4-2\,r_2^2+0.0256\,r_2^2} = 0.018225$$

$$\text{or} \quad r_2^4 - 3.3791\,r_2^2 - 53.8697 = 0 \quad \text{or} \quad r_2^2 = 9.2211 \text{ (positive root)}$$
$$\text{or} \quad r_2 = 3.0366$$

By selecting r_2 = 3.0366, we obtain $\omega_n = \frac{\omega}{r_2} = \frac{209.44}{3.0366}$ = 68.9713 rad/sec. If r_1 = 1.4784 is selected, we obtain $\omega_n = \frac{\omega}{r_1} = \frac{83.776}{1.4784}$ = 56.6667 rad/sec. Thus ω_n = 56.6667 rad/sec satisfies the transmitted force requirement at both ends of the operating speed.

Verification:

At the speed 2000 rpm , the value of $r = \frac{\omega}{\omega_n}$ is: $r = \frac{209.44}{56.6667} = 3.6960$

This gives r^2 = 13.6604 and

$$\left\{\frac{1+0.3497}{(1-13.6604)^2+0.3497}\right\}^{\frac{1}{2}} = 0.09166 < 0.1368 \text{ of Eq. (1).}$$

Stiffness of the isolator:

$$k = M\,\omega_n^2 = (200)\,(56.6667^2) = 64.2223\,(10^4) \text{ N/m}$$

9.37 $m = 100$ kg ; $\omega = \frac{600}{60}(2\pi) = 62.832$ rad/sec ; Isolation = 90%

$$0.1 = \frac{F_T}{F_0} = \left|\frac{k}{k - m\,\omega^2}\right| = \left|\frac{k}{k - 100\,(62.832^2)}\right|$$

or $0.1\,k - 39478.6022 = -k$ or $k = \frac{39478.6022}{1.1} = 35889.6384$ N/m

Static deflection of isolator: $\delta_{st} = \frac{m\,g}{k} = \frac{100\,(9.81)}{35889.6384} = 0.02733$ m

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9.38 $m = 300$ kg ; $\omega = \frac{1800}{60}(2\pi) = 188.496$ rad/sec. Unbalance $= m\,e = 1$ kg-m

F_T = maximum permissible force transmitted to floor = 8000 N

Force transmissibility: $T_r = \frac{F_T}{m\,e\,\omega^2} = \frac{8000}{1\,(188.496^2)} = 0.2251$

$$= \left\{\frac{1 + (2\,\zeta\,r)^2}{(1 - r^2)^2 + (2\,\zeta\,r)^2}\right\}^{\frac{1}{2}} \qquad (1)$$

Frequency ratio (r) that satisfies Eq. (1) can be found as

$$(0.2251^2) = \frac{1 + (2\,(0.05)\,(r))^2}{(1 - r^2)^2 + (2\,(0.05)\,(r))^2}$$

or $r^4 - 2.1872\,r^2 - 18.7239 = 0$ or $r^2 = 5.5568$ (positive root)

or $r = \frac{\omega}{\omega_n} = 2.3573$

Necessary natural frequency: $\omega_n = \frac{\omega}{r} = \frac{188.496}{2.3573} = 79.9633$ rad/sec

Possible solutions:

(a) If the available isolator is used,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1\,(10^6)}{300}} = 57.7350 \text{ rad/sec}$$

(smaller than the necessary value of 79.9633 rad/sec).

If two identical isolators are used in parallel, $k_{eq} = 2\,(10^6)$ N/m and

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2\,(10^6)}{300}} = 81.6496 \text{ rad/sec.}$$

(approximately equal to the necessary value of 79.9633 rad/sec).

(b) If the isolator is available in the form of a helical spring, it can be cut into two halves and one of them can be used for isolation to achieve a value of $\omega_n = 81.6496$ rad/sec.

(9.39) Mass of engine = m = 500 kg
Force transmitted with out isolator = F_T = (18000 cos 300 t + 3600 cos 600 t) N
Maximum magnitude of force transmitted:

$$F_{01} = 18000 \text{ N at } \omega = 300 \text{ rad/sec}$$
$$F_{02} = 3600 \text{ N at } \omega = 600 \text{ rad/sec}$$

The maximum possible force transmitted will be the sum of the magnitudes of the two harmonics:

$$F_0 = F_{01} + F_{02} = 18000 + 3600 = 21600 \text{ N}$$

Since F_T = 12000 N, we use the relation

$$\frac{F_T}{F_0} = \frac{12000}{21600} = \left|\frac{1}{1-r^2}\right| \quad \text{or} \quad 0.5556 = \frac{1}{r^2-1} \quad \text{or} \quad r = 1.6733$$

At $\omega = 300$ rad/sec:

$$\omega_n = \frac{\omega}{r} = \frac{300}{1.6733} = 179.2843 \text{ rad/sec} = \sqrt{\frac{k}{500}}$$

$$\text{or} \quad k = m\,(\omega_n^2) = (500)\,(179.2843^2) = 16.0714\,(10^6) \text{ N/m}$$

With this value of k, the value of $\frac{F_{T1}}{F_{01}}$ at 300 rad/sec is:

$$\frac{F_{T1}}{F_{01}} = \left|\frac{1}{1-r_1^2}\right| = \left|\frac{1}{1-\left(\frac{300}{179.2843}\right)^2}\right| = 0.5556$$

$$\text{or} \quad F_{T1} = 0.5556\,(F_{01}) = 0.5556\,(18000) = 10000 \text{ N}$$

The value of $\frac{F_{T2}}{F_{02}}$ at $\omega = 600$ rad/sec is:

$$\frac{F_{T2}}{F_{02}} = \left|\frac{1}{1-r_2^2}\right| = \left|\frac{1}{1-\left(\frac{600}{179.2843}\right)^2}\right| = 0.0980$$

$$\text{or} \quad F_{T2} = 0.0980\,(F_{02}) = 0.0980\,(3600) = 352.8 \text{ N}$$

Since $F_{T1} + F_{T2} = 10000 + 352.8 = 10352.8 \text{ N} < 12000$ N (permitted value), the stiffness of the isolator can be taken as $k = 16.0714\,(10^6)$ N/m.

At $\omega = 600$ rad/sec:

$$\omega_n = \frac{\omega}{r} = \frac{600}{1.6733} = 358.5729 \text{ rad/sec}$$

$$k = m\,\omega_n^2 = (500)\,(358.5729^2) = 64.2872\,(10^6) \text{ N/m} \tag{1}$$

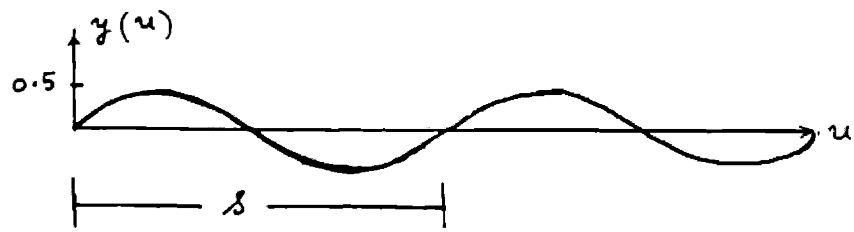
With this value of k, the value of $\frac{F_{T1}}{F_{01}}$ at $\omega = 300$ rad/sec is:

$$\frac{F_{T1}}{F_{01}} = \left|\frac{1}{1 - r_1^2}\right| = \left|\frac{1}{1 - \left(\frac{300}{358.5729}\right)^2}\right| = 3.3331$$

This corresponds to a larger value of F_{T1} than F_{01} and hence the value of k given by Eq. (1) is not suitable for isolation.

9.40 Speed range: 65 to 130 kph (18.0556 to 36.1112 m/s)

Road surface is given by y (u) = 0.15 sin 0.6 u m (1)



where u = horizontal distance (m) = v t, v = velocity (m/s) and t = time (sec).

Since $\omega = 2\pi\rho = \frac{2\pi v}{\zeta}$ where w = frequency of road waviness in rad/sec and s = one wave length (m), Eq. (1) can be expressed as

$$y(t) = 0.15 \sin 0.6\, u = Y \sin \Omega t \tag{2}$$

where $Y = 0.15$ m and $\Omega = 7v$.

Steady state response of the system subjected to the base excitation, $y(t) = Y \sin \Omega t$, is given by Eq. (3.67):

$$x_p(t) = X \sin(\Omega t - \phi) \tag{3}$$

$$\text{where} \quad X = Y \sqrt{\left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\}} \tag{4}$$

Maximum acceleration of mass (driver) is given by

$$\ddot{x}_p(t)\big|_{max} = |-\Omega^2 x| = \Omega^2 Y \left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\}^{\frac{1}{2}}$$

$$= 2g = 2(9.81) = 19.62 \text{ m/sec}^2 \tag{5}$$

At 65 kph:

$\Omega = 6.7\, v = 6.7\,(18.0556) = 120.973$ rad/sec, Eq. (5) gives

$$\Omega^4 Y^2 \left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\} = 19.62^2$$

$$\text{or} \quad (120.973)^4 (0.15)^2 \left\{ \frac{1 + (2\,(0.05)\,r)^2}{(1 - r^2)^2 + (2\,(0.05)\,r)^2} \right\} = 19.62^2$$

$$\text{or} \quad r^4 - 127.171\, r^2 - 12517.1 = 0 \tag{6}$$

The solution of Eq. (6) gives (with positive value or r^2)

$$r = 13.8662 = \frac{\Omega}{\omega_n},\ \omega_n = \frac{\Omega}{13.8662} = 8.7243 \text{ rad/sec} = \sqrt{\frac{k}{m}}$$

Stiffness of isolator (suspension) = $k = m\ \omega_n^2 = (680)\ (8.7243)^2 = 65.4575$ kN/m

Check for acceleration at 130 kph:

$\Omega_2 = 6.7\ v_2 = 6.7\ (36.1112) = 241.945$ rad/sec, $\omega_n = 8.7243$ rad/sec, and

$$r_2 = \frac{\Omega_2}{\omega_n} = 27.7323,\ r_2^2 = 769.081, \text{ and } (2\,\zeta\, r_2)^2 = 7.69081$$

$$\ddot{x}_p(t)\,|_{max} = (\Omega_2)^2\, Y \left\{ \frac{1 + (2\,\zeta\, r_2)^2}{(1 - r_2^2)^2 + (2\,\zeta\, r_2)^2} \right\}^{\frac{1}{2}}$$

$$= (241.945)^2\, (0.15) \left\{ \frac{1 + 7.69081}{(1 - 769.081)^2 + 7.69081} \right\}^{\frac{1}{2}}$$

$$= 33.701159 \text{ m/s}^2 > 2\text{ g}$$

Hence k = 65.4575 kN/m is not acceptable.

At 130 kph:

$\Omega = 6.7\ v = 6.7\ (36.1112) = 241.945$ m/s, and Eq. (5) gives:

$$\Omega^4\, (Y^2) \left\{ \frac{1 + (2\,\zeta\, r)^2}{(1 - r^2)^2 + (2\,\zeta\, r)^2} \right\} = 19.62^2$$

$$\text{or} \quad (241.945)^4\, (0.15)^2 \left\{ \frac{1 + 0.01\, r^2}{(r^4 - 1.99\, r^2 + 1)} \right\} = 19.62^2$$

$$\text{or} \quad r^4 - 2004.852\, r^2 - 200285.24 \tag{7}$$

The Eq. (7) gives (with positive value of r^2): $r = 45.828$. Hence,

$$\omega_n = \frac{\Omega}{45.828} = 5.2794 \text{ rad/sec} = \sqrt{\frac{k}{m}}$$

Stiffness of the isolator (suspension) is: $k = m\ \omega_n^2 = 18.9530$ kN/m

Check for acceleration at 130 kph:

$\Omega_1 = 6.7\ v_1 = 120.973$ rad/sec, $\omega_n = 5.2794$ rad/sec, and $r_1 = \frac{\Omega_1}{\omega_n} = 22.9142$

$r_1^2 = 525.059$, $(2\,\zeta\, r_1)^2 = 5.25058$, $(1 - r_1^2)^2 = 274637.4$ and hence,

$$\ddot{x}_p(t)\,|_{max} = \Omega_1^2\, Y \left\{\frac{1 + (2\,\zeta\, r_1)^2}{(1 - r_1^2)^2 + (2\,\zeta\, r_1)^2}\right\}^{\frac{1}{2}} = 10.4724\ \text{m/sec}^2 < 2\ g$$

Hence k = 18.9530 kN/m is acceptable.

9.41 **Force transmitted to base in case of Coulomb damping can be found using the equivalent viscous damping constant:**

$$F_T = \left\{(k\,x)^2 + (c_{eq}\,\dot{x})^2\right\}^{\frac{1}{2}} = X\left\{k^2 + \omega^2\, c_{eq}^2\right\}^{\frac{1}{2}}$$

$$= \left\{k^2 + \omega^2\, c_{eq}^2\right\}^{\frac{1}{2}} \frac{F_0}{k}\left[\frac{1 - \left(\frac{4\,\mu\, N}{\pi\, F_0}\right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}\right]^{\frac{1}{2}}$$

with $c_{eq} = \left(\frac{4\,\mu\, N}{\pi\,\omega\, X}\right)$ **and** $\frac{\omega^2\, c_{eq}^2}{k^2} = \frac{(4\,\mu\, N)^2}{\pi^2\, F_0^2}\ \frac{(1 - r^2)^2}{\left\{1 - \left(\frac{4\,\mu\, N}{\pi\, F_0}\right)^2\right\}}$

$$\textbf{Thus}\quad F_T = F_0\left\{1 + \frac{(1 - r^2)^2\,(4\,\mu\, N)^2}{\pi^2\, F_0^2\left\{1 - \left(\frac{4\,\mu\, N}{\pi\, F_0}\right)^2\right\}}\right\}^{\frac{1}{2}}\left[\frac{1 - \left(\frac{4\,\mu\, N}{\pi\, F_0}\right)^2}{(1 - r^2)^2}\right]^{\frac{1}{2}}$$

$$= F_0\left\{\frac{1 + r^2\,(r^2 - 2)\left(\frac{4\,\mu\, N}{\pi\, F_0}\right)^2}{(1 - r^2)^2}\right\}^{\frac{1}{2}}$$

Thus the force transmissibility is given by

$$T_r = \frac{F_T}{F_0} = \left\{\frac{1 + r^2\,(r^2 - 2)\left(\frac{4\,\mu\, N}{\pi\, F_0}\right)^2}{(1 - r^2)^2}\right\}^{\frac{1}{2}}$$

(9.42) Under base excitation, the displacement transmissibility is given by (similar to that of a viscously damped system, Eq. 3.68):

$$\frac{X}{Y} = \left\{ \frac{1 + \left(\frac{\omega^2 c_{eq}^2}{k^2} \right)}{\left(1 - \frac{\omega^2}{\omega_n^2} \right)^2 + \left(\frac{\omega^2 c_{eq}^2}{k^2} \right)} \right\}^{\frac{1}{2}} \qquad (1)$$

$$\text{But} \quad \frac{\omega^2 c_{eq}^2}{k^2} = \left(\frac{4 \mu N}{k \pi} \right)^2 \frac{1}{X^2} \qquad (2)$$

Substituting Eq. (2) into (1), we obtain

$$\frac{X}{Y} = \left\{ \frac{1 + \left(\frac{4 \mu N}{\pi k X} \right)^2}{(1 - r^2)^2 + \left(\frac{4 \mu N}{\pi k X} \right)^2} \right\}^{\frac{1}{2}} \qquad (3)$$

Relative displacement transmissibility is given by (similar to Eq. 3.77):

$$\frac{Z}{Y} = \frac{m \omega^2}{\left\{ (k - m \omega^2)^2 + \omega^2 c_{eq}^2 \right\}^{\frac{1}{2}}} = \frac{\left(\frac{\omega}{\omega_n} \right)^2}{\left\{ \left(1 - \frac{\omega^2}{\omega_n^2} \right)^2 + \left(\frac{\omega^2 c_{eq}^2}{k^2} \right) \right\}^{\frac{1}{2}}} \qquad (4)$$

Using Eq. (2), (4) can be written as

$$\frac{Z}{Y} = \frac{r^2}{\left\{ (1 - r^2)^2 + \left(\frac{4 \mu N}{\pi k X} \right)^2 \right\}^{\frac{1}{2}}} \qquad (5)$$

(9.43) $M = 200$ kg ; $me = 0.02$ kg-m ; $\delta_{st} = \frac{M g}{k} = 0.005$ m

$$k = \frac{M g}{0.005} = \frac{(200)(9.81)}{0.005} = 39.24 \, (10^4) \text{ N/m}$$

$$\omega = \frac{1200}{60} (2 \pi) = 125.664 \text{ rad/sec}$$

$$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{39.24 \, (10^4)}{200}} = 44.2944 \text{ rad/sec}$$

(a) Assume $\varsigma = 0$ for the isolator.

$$r = \frac{\omega}{\omega_n} = \frac{125.664}{44.2944} = 2.8370$$

Amplitude of washing machine (from Eq. 3.82):

$$X = \left(\frac{m\,e}{M}\right) \frac{r^2}{\left\{(1-r^2)^2\right\}^{\frac{1}{2}}} = \frac{0.02}{200} \frac{(2.8370^2)}{\left|1 - 2.8370^2\right|} = 11.4188\ (10^{-5})\ \mathrm{m}$$

(b) Force transmitted to foundation (given by Eq. 9.104):

$$F_T = m\,e\,\omega_n^2\,r^2 \frac{1}{|1-r^2|} = (0.02)\,(44.2944^2)\,\frac{(2.8370^2)}{|1-2.8370^2|} = 44.8069\ \mathrm{N}$$

9.44 $M = 60$ kg ; $me = 0.002$ kg-m ; $\omega = \dfrac{3000}{60}(2\,\pi) = 314.16$ rad/sec

$$T_r = \frac{F_T}{m\,e\,\omega^2} < 0.25$$

Let $\varsigma = 0$ for the isolator. From the relation:

$$T_r = \frac{F_T}{m\,e\,r^2\,\omega_n^2} = \frac{1}{|1-r^2|} < 0.25$$

we obtain $|1-r^2| > 4$ or $r > 2.2361$

Let $r = 0.25$.

(a) $r = \dfrac{\omega}{\omega_n} = 2.5$; $\omega_n = \dfrac{314.16}{2.5} = 125.664$ rad/sec.

$k = M\,\omega_n^2 = (60)\,(125.664^2) = 9.4749\ (10^5)$ N/m.

(b) $X = \dfrac{m\,e}{M}\dfrac{r^2}{|1-r^2|}$ (given by Eq. 3.82)

$$= \left(\frac{0.002}{60}\right)\frac{2.5^2}{|1-2.5^2|} = 39.6825\ (10^{-6})\ \mathrm{m}$$

(c) $F_T = m\,e\,\omega_n^2 \dfrac{r^2}{|1-r^2|} = (0.002)\,(125.664^2)\,\dfrac{6.25}{5.25} = 37.5987$ N

(9.45) Using N = 3000 rpm and $\delta_{st} = 0.01$ m, Eq. (9.101) yields

$$N = 29.9092 \sqrt{\frac{2-R}{0.01\,(1-R)}}$$

or $$\sqrt{\frac{2-R}{1-R}} = \frac{\sqrt{0.01}\,(3000)}{29.9092} = 10.0304$$

or $$\frac{2-R}{1-R} = 100.6081$$

or $R = 0.98996$

Thus the reduction in the transmitted force is 98.996%

(9.46) $m = 30$ kg , $\omega = 10 - 75$ Hz $= 62.832 - 471.240$ rad/s

$\zeta = 0.25$, $\frac{X}{Y} \le \frac{15}{100} = 0.15$

Using the displacement transmissibility as $\frac{X}{Y} = 0.15$, we obtain

$$0.15 = \left\{ \frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right\}^{\frac{1}{2}}$$

Squaring this equation, we find

$$0.0225 = \frac{1 + 0.25\, r^2}{(1-r^2)^2 + 0.25\, r^2}$$

or

$$0.0225 \left(1 + r^4 - 2r^2 + 0.25\, r^2 \right) = 1 + 0.25\, r^2$$

or

$$0.0225\, r^4 - 0.289375\, r^2 - 0.9775 = 0$$

Solution of this equation is:

$$r^2 = 15.639056 \,;\; -2.777945$$

Thus $$r = 3.954625 = \frac{\omega}{\omega_n} = \frac{\omega\sqrt{m}}{\sqrt{k}}$$

or $$\sqrt{k} = \frac{\omega\sqrt{m}}{3.954625} = \frac{\omega\sqrt{30}}{3.954625} = 1.385018\ \omega$$

or $k = 1.918274\ \omega^2$

Thus $$k = \begin{cases} 7{,}573.0776 \text{ N/m} & \text{when } \omega = 62.832 \text{ rad/s} \\ 425{,}985.6163 \text{ N/m} & \text{when } \omega = 471.240 \text{ rad/s} \end{cases}$$

Analysis:

When $k = 7,573.0776$ N/m	When $k = 425,985.6163$ N/m
When $\omega = 62.832$ rad/s:	When $\omega = 62.832$ rad/s:
$\omega_n = \sqrt{\frac{k}{m}} = 15.8882$ rad/s since $m = 30$ kg	$\omega_n = \sqrt{\frac{k}{m}} = 119.1617$ rad/s since $m = 30$ kg
$r = 3.9546$	$r = 0.5273$
$(2\zeta r)^2 = 3.9098$	$(2\zeta r)^2 = 0.06951$
$1 - r^2 = -14.6389$	$1 - r^2 = 0.7219$
$T_r = \left\{\frac{1 + 3.9098}{(-14.6389)^2 + 3.9098}\right\}^{\frac{1}{2}}$	$T_r = \left\{\frac{1 + 0.06951}{(0.7219)^2 + 0.06951}\right\}^{\frac{1}{2}}$
$= 0.15 \Rightarrow$ Acceptable	$= 1.3455 \Rightarrow$ Not acceptable
When $\omega = 471.240$ rad/s:	When $\omega = 471.240$ rad/s:
$r = 29.6597$	$r = 3.9546$
$(2\zeta r)^2 = 219.9251$	$(2\zeta r)^2 = 3.9098$
$1 - r^2 = -878.6978$	$1 - r^2 = -14.6389$
$T_r = \left\{\frac{1 + 219.9251}{(-878.6978)^2 + 219.9251}\right\}^{\frac{1}{2}}$	$T_r = \left\{\frac{1 + 3.9098}{(-14.6389)^2 + 3.9098}\right\}^{\frac{1}{2}}$
$= 0.01691 \Rightarrow$ Acceptable	$= 0.15 \Rightarrow$ Acceptable

$\therefore$ stiffness of the suspension $= k = 7,573.0776$ N/m.

9.47 $\omega = \frac{600\,(2\,\pi)}{60} = 62.832$ rad/sec

$$F = m\,r\,\omega^2 = (30)\left(\frac{0.38}{2}\right)(62.832)^2 = 22502.80 \text{ N}$$

(a) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.8 \times 10^6}{1200}} = 38.7298$ rad/sec

$$r = \frac{\omega}{\omega_n} = \frac{62.832}{38.7298} = 1.64814$$

Since $\omega > \omega_n$, force transmitted to the foundation (F_T) is given by

$$F_T = \frac{F}{r^2 - 1} = \frac{22502.80}{1.64814^2 - 1} = 13110.73 \text{ N}$$

(b) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4.4 \times 10^6}{1200}} = 60.553$ rad/sec

$$r = \frac{\omega}{\omega_n} = \frac{62.832}{60.553} = 1.03764$$

F_T is given by $F_T = \frac{F}{r^2 - 1} = \frac{22502.8}{1.03764^2 - 1} = 293.429$ N

9.48 For harmonic base motion, the displacement transmissibility is given by

$$\frac{X}{Y} = \left\{\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right\}^{\frac{1}{2}} \qquad (1)$$

For an undamped isolator, Eq.(1) becomes

$$\frac{X}{Y} = \left|\frac{1}{1-r^2}\right|$$

In the present case,

$$\frac{X}{Y} = \frac{1}{20} = \left|\frac{1}{1-r^2}\right| \quad \text{or} \quad |1-r^2| = 20 \quad \text{or} \quad r^2 = 21$$

Thus $r = \sqrt{21} = 4.5826 = \frac{\omega}{\omega_n} = \frac{2(2\pi)}{\omega_n}$

or $\omega_n = \frac{4\pi}{4.5826} = 2.7422$ rad/s

$$= \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{1}}$$

$\therefore$ $k = (2.7422)^2 = 7.5197$ N/m = stiffness of isolator.

9.49 Let the shock isolator be undamped.

$$\dot{x}_{max} = X\,\omega_n \qquad (1)$$

$$\ddot{x}_{max} = -X\,\omega_n^2 \qquad (2)$$

where X is the amplitude of the displacement of the mass. Since the maximum step velocity is specified as 0.01 m/s, the maximum allowable value of X is given by Eq.(1):

$$X = \frac{\dot{x}_{max}}{\omega_n} < 0.01$$

$$\text{or} \quad \omega_n > \frac{\dot{x}_{max}}{X} = \frac{0.01}{0.01} = 1 \text{ rad/s} \qquad (3)$$

Similarly, using the maximum specified value of $\ddot{x}_{max}$ as $20\,g = 196.2 \text{ m/s}^2$, Eq.(2) gives

$$X\,\omega_n^2 \le 196.2 \quad \text{or} \quad \omega_n \le \sqrt{\frac{\ddot{x}_{max}}{X}} = \sqrt{\frac{196.2}{0.01}}$$

$$\text{or} \quad \omega_n \le 140.0714 \text{ rad/s} \qquad (4)$$

Eqs.(3) and (4) give:

$$1 \text{ rad/s} \le \omega_n \le 140.0714 \text{ rad/s}$$

By selecting the value of ω_n in the middle of the range, we find the stiffness of the isolator pad (k) as

$$k = m\,\omega_n^2 = 10\,(70.5357)^2 = 49{,}752.8570 \text{ N/m}$$

9.50 $m = 10^5$ kg , maximum deflection = 0.5 m

From the response spectrum, the peak value of $\left(\frac{x_{max}\,k}{F_0}\right)$ can be seen to be approximately 1.75 at a value of $\frac{t_0\,\omega_n}{2\pi} \approx 0.75$.

Using $x_{max} = 0.5$, $\frac{x_{max}\,k}{F_0} = 1.75$ gives

$$k = \frac{1.75\,F_0}{x_{max}} = \frac{1.75\,(10{,}000)}{0.5} = 70{,}000.0 \text{ N/m}$$

9.51 $m_1 = 200$ kg , $\omega_1 = 1200(2\pi/60) = 40\pi$ rad/s $= \sqrt{\frac{k_1}{m_1}}$

k_1 = equivalent spring constant of air compressor $= \omega_1^2 m_1 = 3.1583$ MN/m

Let the absorber be tuned so that $\frac{\omega_2}{\omega_1} = 1$.

Natural frequencies of the combined system are given by the roots of Eq.(9.139), which for $(\omega_2/\omega_1) = 1$ becomes

$$\left(\frac{\omega}{\omega_2}\right)^4 - \left(2+\frac{m_2}{m_1}\right)\left(\frac{\omega}{\omega_2}\right)^2 + 1 = 0$$

or, $r^4 - \left(2+\frac{m_2}{m_1}\right) r^2 + 1 = 0$ --- (E_1)

Now $r_1 = \frac{\Omega_1}{\omega_2} = 0.8$. Eq. (E_1) gives

$$(0.8)^4 - \left(2+\frac{m_2}{m_1}\right)(0.8)^2 + 1 = 0 \Rightarrow \frac{m_2}{m_1} = 0.2025$$

$$m_2 = 40.5 \text{ kg}$$

As $\frac{k_2}{k_1} = \frac{m_2}{m_1}\left(\frac{\omega_2}{\omega_1}\right)^2 = \frac{m_2}{m_1}$, $k_2 = k_1 (0.2025) = 0.6396$ MN/m

If we use $r_2 = 1.2$, Eq. (E_1) gives

$$(1.2)^4 - \left(2+\frac{m_2}{m_1}\right)(1.2)^2 + 1 = 0 \Rightarrow \frac{m_2}{m_1} = 0.1344$$

Since this value of $\frac{m_2}{m_1}$ is smaller, we have to use the values of m_2 and k_2 given by $r_1 = 0.8$.

(9.52) Beam: $\omega_1 = \omega_n = 1500\left(\frac{2\pi}{60}\right) = 157.08$ rad/sec

$m_1 = M = 300$ kg = mass of motor

$k_1 = k_{beam} = \omega_1^2 m_1 = (157.08)^2 (300) = 7.4022 \times 10^6$ N/m

$\omega_1 = \sqrt{k_1/m_1} = \omega_2 = \sqrt{k_2/m_2}$

$\therefore k_2 = m_2 \omega_2^2 = 24\,674.1264\ m_2$ (E_1)

(a) Beam with absorber

$\Omega_1 = 0.75\ \omega_2$ or $r_1 = \Omega_1/\omega_2 = 0.75$

For a tuned absorber, $\omega_1/\omega_2 = 1$, and

$$r_1^2 = \left(1+\frac{\mu}{2}\right) - \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1}$$

or $\mu = \left(\frac{r_1^4 + 1}{r_1^2}\right) - 2 = \frac{0.3164+1}{0.5625} - 2 = 0.3403 = \frac{m_2}{m_1}$

$\therefore$ Ratio of absorber mass to the mass of the motor $= \mu = 0.3403$

(b) Mass of the absorber $= m_2 = \mu m_1 = (0.3403)(300) = 102.09$ kg

Stiffness of the absorber $= k_2 = m_2 \omega_2^2$ from Eq. (E_1)

$k_2 = 24674.1264\ (102.09) = 2.519 \times 10^6$ N/m

(c) Amplitude of vibration of the absorber mass (X_2):

Eq. (9.136) gives $X_2 = -\frac{F_0}{k_2} = -\frac{m e \omega^2}{k_2} = -\frac{\left(\frac{2}{100}\right)(157.08)^2}{2.519 \times 10^6}$

$= -0.1959$ mm

9.53 Forcing frequency $= 800\,(2\pi)/60 = 83.776$ rad/sec $= \omega_1 = \omega_2$

$m_1' = 1$ kg

For $\omega_2/\omega_1 = 1$, $\quad r_1^2, r_2^2 = \left(1+\frac{\mu}{2}\right) \mp \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1}$

where $r_1 = \Omega_1/\omega_2$, $r_2 = \Omega_2/\omega_2$, $\mu = m_2'/m_1$

Here $\Omega_1 = 750$ rpm $= 78.54$ rad/sec, $\Omega_2 = 1000$ rpm $= 104.72$ rad/sec,

$r_1 = 750/800 = 0.9375$, and $r_2 = 1000/800 = 1.2500$.

$r_1^2 = \left(1+\frac{\mu}{2}\right) - \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1} \;\Rightarrow\; \left(1+\frac{\mu}{2}\right)^2 - 1 = \left\{\left(1+\frac{\mu}{2}\right) - r_1^2\right\}^2$

$\Rightarrow\; 1+\frac{\mu}{2} = \left(1+r_1^4\right)/2 r_1^2$

$\Rightarrow\; \mu = \left(r_1^4+1\right)/r_1^2 - 2$

$= \frac{0.7725+1}{0.8789} - 2 = 0.0167$

$\therefore\; m_1 = \frac{m_2'}{0.0167} = 59.8861$ kg.

New required value of Ω_1 is 700 rpm $= 73.304$ rad/sec

which corresponds to $r_1 = 700/800 = 0.875$

$\mu = \frac{r_1^4+1}{r_1^2} - 2 = \frac{1.5862}{0.7656} - 2 = 0.07182$

$m_2 = \mu m_1 = 0.07182\,(59.8861) = 4.3010$ kg

With these values, Ω_2 can be found as

$r_2 = \left(1+\frac{\mu}{2}\right) + \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1} = (1+0.03591) + 0.2704 = 1.3063$

$\Omega_2 = r_2\,\omega_2 = 1.3063\,(83.776) = 109.437$ rad/sec $= 1045.04$ rpm

This is larger than the desired upper value of 1040 rpm.

Spring stiffness of the absorber $= k_2 = m_2\,\omega_2^2$

$= (4.301)(83.776)^2 = 30186.2166$ lb/in.

9.54 Original system: $m_1 = 900$ kg, $\omega_1 = 600$ rpm $= 62.832$ rad/sec $= \sqrt{\frac{k_1}{m_1}}$

$$k_1 = m_1\ \omega_1^2 = 900\ (62.832)^2 = 3.55307 \times 10^6 \text{ N/m}$$

(a) Absorber

For tuned absorber, $\omega_2 = \omega_1 = 62.832$ rad/sec, $k_2 = 900 \times 10^3$ N/m,

$$m_2 = \frac{k_2}{\omega_2^2} = \frac{(900 \times 10^3)}{62.832^2} = 227.972 \text{ kg.}$$

Weight of absorber = 2236.40 N

(b) New system

Natural frequencies of the new system, Ω_1 and Ω_2, are given by (for $\omega_2/\omega_1 = 1$):

$$r_1^2,\ r_2^2 = \left(1 + \frac{\mu}{2}\right) \mp \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1}$$

where $r_1 = \frac{\Omega_1}{\omega_2}$, $r_2 = \frac{\Omega_2}{\omega_2}$ and $\mu = \frac{m_2}{m_1}$,

here $\mu = 0.25330$ and hence

$r_1^2 = 0.60767$, $r_1 = 0.77953$

$r_2^2 = 1.64563$, $r = 1.28282$

$\Omega_1 = r_1\ \omega_2 = (0.77953)\ (62.832) = 48.979$ rad/sec $= 467.7191$ rpm

$\Omega_2 = r_2\ \omega_2 = (1.28282)\ (62.832) = 80.602$ rad/sec $= 769.6938$ rpm

---

9.55 Natural frequencies of the combined systems are

$\Omega_1 = 0.7\ \omega_2 = 0.7\ (62.832) = 43.9824$ rad/sec

$\Omega_2 = 1.3\ \omega_2 = 1.3\ (62.832) = 81.6816$ rad/sec

$r_1 = 0.7$, $r_2 = 1.3$ $\sqrt{\frac{k_2}{m_2}} = \omega_2 = \omega_1 = 62.832$ rad/sec (for tuned absorber)

$$k_2 = m_2\ \omega_2^2 = (62.832)^2\ m_2 \qquad (E_1)$$

$$r_1^2 = \left(1 + \frac{\mu}{2}\right) - \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} \quad \text{or} \quad \mu = \frac{r_1^4 + 1}{r_1^2} - 2$$

$$\mu = \frac{(0.7)^4 + 1}{(0.7)^2} - 2 = 0.5308 = \frac{m_2}{m_1} \Rightarrow m_2 = 0.5308\ (900) = 477.72 \text{ kg}$$

From Eq. (E_1), $k_2 = (62.832)^2\ (477.72) = 1.885972 \times 10^6$ N/m

Verification of r_2:

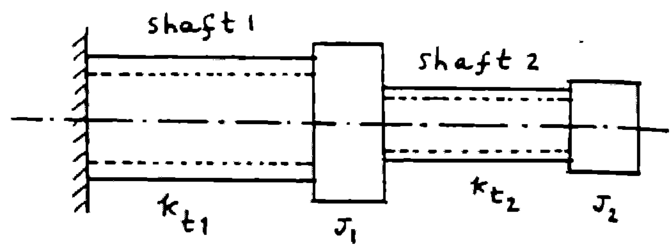
$$r_2^2 = \left(1 + \frac{\mu}{2}\right) + \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} = \left(1 + \frac{0.5308}{2}\right) + \sqrt{\left(1 + \frac{0.5308}{2}\right)^2 - 1}$$

$= 2.0408$ or $r_2 = 1.4286 > 1.3$ (desired value)

Hence $m_2 = 477.72$ kg (weight of absorber $= m_2\, g = 4686.4332$ N)

$k_2 = 1.885972 \times 10^6$ N/m

9.56 $G = 80 \times 10^9$Pa, $J = \dfrac{m\, d^2}{8}$



For shaft 1:

$$k_{t_1} = \frac{\pi G}{32\, \ell}(D^4 - d^4)$$

$$= \frac{\pi\,(80 \times 10^9)}{32\,(0.75)}(0.05^4 - 0.038^4)$$

$= 43.6144 \times 10^3$ N·m/rad

$$J_1 = (45)\left(\frac{0.38^2}{8}\right) = 0.81225 \text{ N-m-sec}^2$$

$$\omega_n = \omega_1 = \sqrt{\frac{k_{t_1}}{J_1}} = \sqrt{\frac{43.6144 \times 10^3}{0.81225}} = 231.7235 \text{ rad/sec}$$

For shaft 2:

$$k_{t_2} = \frac{\pi\,(80 \times 10^9)}{32\,(0.75)}(D^4 - d^4)$$

Assuming $\dfrac{D}{d} = \dfrac{4}{3}$ (D and d in metres)

$$k_{t_2} = \frac{\pi\,(80 \times 10^9)}{32\,(0.75)}\, d^4 \left\{\left(\frac{4}{3}\right)^4 - 1\right\} = 2.26246 \times 10^{10}\, d^4 \text{ N·m/rad}$$

$$J_2 = 9\left(\frac{0.15^2}{8}\right) = 0.0253125 \text{ N-m-sec}^2$$

For $\omega_2 = \omega_1$ and $\omega_2 = \sqrt{\frac{k_{t2}}{J_2}}$, we get

$$231.7235 = \sqrt{\frac{8.14487 \times 10^9 \; d^4}{0.0253125}} = 945416.8 d^2$$

$\therefore$ $d = 0.015656$ m and $D = 0.0208743$ m

9.57 $J_1 = 15$ kg-m^2, $k_{t1} = 0.6 \times 10^6$ N-m/rad

$\omega_1 = \sqrt{k_{t1}/J_1} = \sqrt{0.6 \times 10^6/15} = 200$ rad/sec

Absorber:

k_{t2}, J_2, $\omega_2 = \sqrt{k_{t2}/J_2}$

For the combined system with tuned absorber, the natural frequencies Ω_1 and Ω_2 are given by an equation similar to Eq. (9.140) as

$$r_1^2,\ r_2^2 = \left(1 + \frac{\mu}{2}\right) \mp \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} \qquad (E_1)$$

where $\mu = J_2/J_1$, $r_1 = \Omega_1/\omega_2$ and $r_2 = \Omega_2/\omega_2$.

Let Ω_1 be 25% less than ω_1. Then $r_1 = \frac{150}{200} = 0.75$

$\therefore$ Eq. (E_1) gives

$$(0.75)^2 = \left(1 + \frac{\mu}{2}\right) - \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} \quad \Rightarrow \quad \mu = 0.3403$$

$\therefore$ $J_2 = \mu J_1 = 5.1045$ kg-m^2

Since $\omega_2 = \sqrt{k_{t2}/J_2} = 200$, $k_{t2} = 4 \times 10^4 J_2$

$= 0.2042 \times 10^6$ N-m/rad

Since Ω_2 has to be at least 20% greater than ω_1,

$$r_2 = \frac{\Omega_2}{\omega_2} \geq 1.2$$

Eq. (E_1) gives, for $\mu = 0.3403$,

$$r_2^2 = \left(1 + \frac{0.3403}{2}\right) + \sqrt{\left(1 + \frac{0.3403}{2}\right)^2 - 1} = 1.7779 \Rightarrow r_2 = 1.3333$$

Hence $J_2 = 5.1045$ kg-m^2 and $k_{t2} = 0.2042$ MN-m/rad are acceptable.

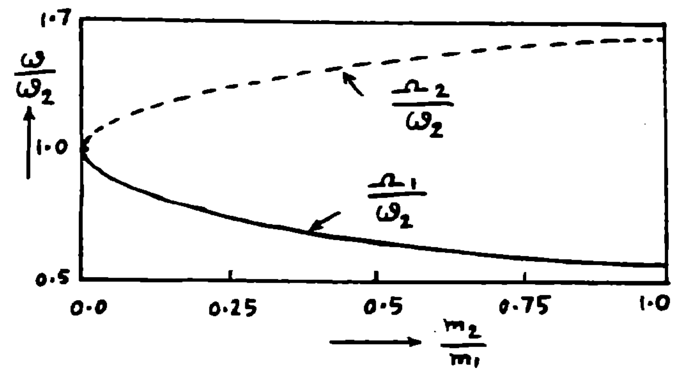
9.58 $\left(\frac{\omega}{\omega_2}\right)^4 - (2+\mu)\left(\frac{\omega}{\omega_2}\right)^2 + 1 = 0$ where $\mu = \frac{m_2}{m_1}$

or, $r^4 - (2+\mu) r^2 + 1 = 0$ where $r = \frac{\omega}{\omega_2} = \frac{\Omega_1}{\omega_2}$ or $\frac{\Omega_2}{\omega_2}$

$$\therefore \quad r^2 = \left\{ \frac{(2+\mu) \pm \sqrt{(2+\mu)^2 - 4}}{2} \right\}$$

μ	$r_1 = \frac{\Omega_1}{\omega_2}$	$r_2 = \frac{\Omega_2}{\omega_2}$
0	1.0	1.0
0.1	0.8543	1.1705
0.2	0.8011	1.2483
0.3	0.7630	1.3107
0.4	0.7326	1.3650
0.5	0.7071	1.4142
0.6	0.6851	1.4597
0.7	0.6656	1.5023
0.8	0.6482	1.5427
0.9	0.6325	1.5811
1.0	0.6180	1.6180

Plot:



9.59 $\left|\frac{X_1}{\delta_{st}}\right| \le 0.5$, $\frac{\omega_2}{\omega_1} = 1$, $\frac{m_2}{m_1} = 0.1$

Eq. (9.134) gives, for $\frac{\omega_2}{\omega_1} = 1$, $\frac{m_2}{m_1} = 0.1$ and $\frac{k_2}{k_1} = \frac{m_2}{m_1}\left(\frac{\omega_2}{\omega_1}\right)^2 = 0.1$,

$$\pm 0.5 = \frac{1 - \left(\frac{\omega}{\omega_2}\right)^2}{\left[1 + 0.1 - \left(\frac{\omega}{\omega_1}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] - 0.1}$$

But $\frac{\omega}{\omega_1} = \frac{\omega}{\omega_2} \cdot \frac{\omega_2}{\omega_1} = \frac{\omega}{\omega_2} = r_2$ (say)

$$\therefore \pm 0.5 = \frac{1 - r_2^2}{(1.1 - r_2^2)(1 - r_2^2) - 0.1} = \frac{1 - r_2^2}{r_2^4 - 2.1\, r_2^2 + 1}$$

For $+0.5$, we get $r_2^4 - 2.1\, r_2^2 + 1 = 2 - 2\, r_2^2$

i.e. $r_2^2 = 1.05125$ (+ value) ; $r_2 = 1.0253$

For -0.5, we get $r_2^4 - 2.1\, r_2^2 + 1 = -2 + 2\, r_2^2$

i.e. $r_2^2 = 0.9534,\ 3.1466$

$r_2 = 0.9764,\ 1.7739$ (above the upper resonance point)

operating range is $0.9764 \le \frac{\omega}{\omega_2} \le 1.05125$

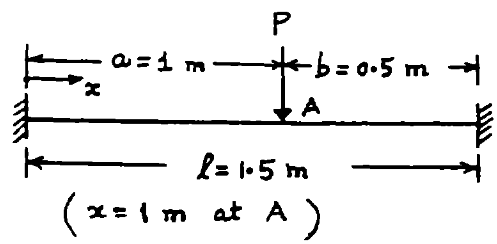
9.60 $\omega_1 = \sqrt{\dfrac{k_1}{m_1}} = \sqrt{\dfrac{10^5}{40}} = 50 \text{ rad/sec}$

Assuming that $\omega_2 = \omega_1$, we obtain $\omega_2 = \left\{\dfrac{k_2}{m_2}\right\}^{\frac{1}{2}} = \left\{\dfrac{k}{30}\right\}^{\frac{1}{2}} = 50$

$k_2 = 75000$ N/m , and

$$X_2 = -\frac{F_0}{m_2\,\omega^2} \approx -\frac{F_0}{m_2\,\omega_1^2} = -\frac{300}{30\,(50^2)} = -0.004 \text{ m}$$

9.61



Motor: $m_1 = 20$ kg, $\omega = 1350$ rpm $= 141.372$ rad/sec, me $= 0.1$ kg-m.
From Appendix B, we have

$$y_A = \frac{P\,b^2\,a^2\,(3\,a\,\ell - a\,(3\,a + b))}{6\,E\,I\,\ell^3}$$

$$\text{where } I = \frac{1}{12}\,w\,d^3 = \frac{1}{12}\,(0.15)\,(0.012^3) = 2.16\,(10^{-8})\text{ m}^4$$

$$\text{Hence}\quad k_1 = \frac{P}{y_A} = \frac{(207\,(10^9))\,(2.16\,(10^{-8}))\,(1.5^3)}{(0.5^2)\,(1^2)\,[3\,(1)\,(1.5) - 1\,(3\,(1) + 0.5)]} = 362167.2 \text{ N/m}$$

Natural frequency of the motor on the beam:

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{362167.2}{20}} = 134.5678 \text{ rad/sec}$$

Natural frequency of the absorber:

$$\omega_2 = \sqrt{\frac{k_2}{m_2}} = \omega = 141.372 \text{ rad/sec}$$

Selecting $m_2 = 10$ kg, we obtain

$$k_2 = 10\,(141.372^2) = 19.9860\,(10^4)\text{ N/m}$$

Amplitude of the absorber at forcing frequency ω:

$$X_2 = -\frac{F_0}{m_2\,\omega^2}$$

where F_0 is the amplitude of the forcing function $= m\,e\,\omega^2$. Hence

$$X_2 = -\frac{m\,e\,\omega^2}{m_2\,\omega^2} = -\frac{m\,e}{m_2} = -\frac{0.1}{10} = -0.01\text{ m}$$

9.62 $m_1 = 15{,}000$ kg; $k_1 = 2\,(10^6)$ N/m; $F_1(t) = 600\,\cos\omega t$

Assume that the forcing frequency coincides with the natural frequency of the bridge.

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{2\,(10^6)}{15000}} = 11.5470\text{ rad/sec}$$

For a tuned absorber, $\omega_2 = \sqrt{\dfrac{k_2}{m_2}} = \omega_1 = 11.5470$ rad/sec

Choose $m_2 = 10$ kg. This gives

$$k_2 = m_2\,\omega_2^2 = 10\,(11.5470^2) = 1333.3333\text{ N/m}$$

Amplitude of bridge will be zero at the forcing frequency, $\omega = 11.5470$ rad/sec.

9.63 $\omega_1 = 100$ rad/sec. To suppress the vibration of the motor, the absorber should have the natural frequency:

$\omega_2 = \sqrt{\dfrac{k_2}{m_2}} = 80$ rad/sec (operating frequency)

or $k_2 = m_2\,(80^2) = 5\,(80^2) = 32$ kN/m

9.64 Equations of motion:

$$\sum F = m\ddot{x} \quad m\ddot{x} + c\dot{x} + (k + K_2)\,x - K_2\,R\,\theta = F_0 \sin\omega t \tag{1}$$
$$\sum M_0 = I\ddot{\theta} \quad (I + M R^2)\,\ddot{\theta} + (K_1 + K_2)\,R^2\,\theta - K_2\,R\,x = 0 \tag{2}$$

Treating the forcing function as the imaginary component of $F_0\,e^{i\omega t}$, we assume the solution as:

$$x = X\,e^{i(\omega t - \phi)} \tag{3}$$
$$\theta = \Theta\,e^{i(\omega t - \phi)} \tag{4}$$

Substitution of Eqs. (3) and (4) into (1) and (2) gives

$$\left[-m\,\omega^2 + (k + K_2) + i\,c\,\omega\right] X - (K_2\,R)\,\Theta = F_0 \tag{5}$$
$$-(K_2\,R)\,X + \left[-(I + M R^2)\,\omega^2 + R^2\,(K_1 + K_2)\right]\Theta = 0 \tag{6}$$

Solution of Eqs. (5) and (6) yields:

$$\Theta = \left\{\frac{-K_2\,R\,F_0}{(-K_2\,R)^2 - \left[m\,\omega^2 + (k + K_2) + i\,\omega\,c\right]\left[-I_0\,\omega^2 + R^2\,(K_1 + K_2)\right]}\right\} \tag{7}$$

$$X = \left\{\frac{\left[-I_0\,\omega^2 + R^2\,(K_1 + K_2)\right] F_0}{\left[-m\,\omega^2 + (k + K_2) + i\,\omega\,c\right]\left[-I_0\,\omega^2 + R^2\,(K_1 + K_2)\right] - (-K_2\,R)^2}\right\} \tag{8}$$

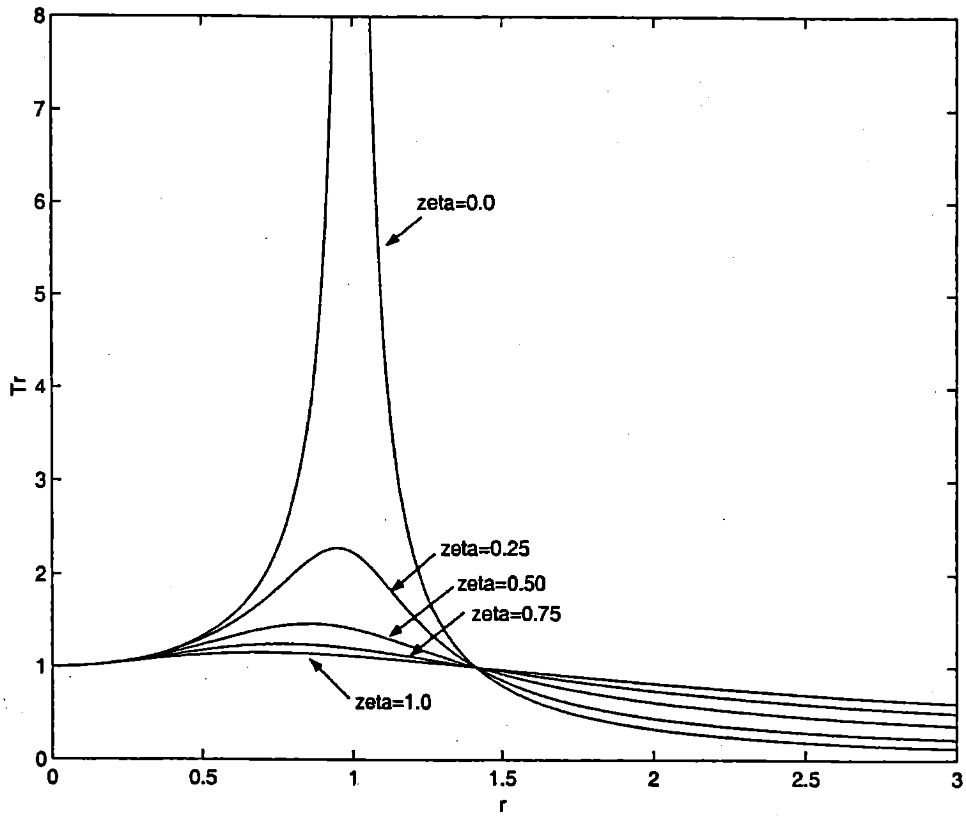
$$\text{where} \quad I_0 = I + M R^2 \tag{9}$$

Equation (8) shows that the steady state displacement of mass m (X) will be zero if

$$I_0\,\omega^2 = R^2\,(K_1 + K_2) \tag{10}$$

9.65

```
%Ex9_65.m
for j = 1 : 5
    zeta = (j-1 )* 0.25;
    for i = 1 : 1001
        r(i) = 3 * (i - 1)/1000;
        Tr(i) = sqrt( (1 + (2 * zeta * r(i))^2)/((1 - r(i)^2) ^ 2 ...
            + (2 * zeta * r(i)) ^ 2));
    end;
    plot(r, Tr);
    hold on;
end
axis([0 3 0 8]);
xlabel('r');
ylabel('Tr')
gtext('zeta=0.0');
gtext('zeta=0.25');
gtext('zeta=0.50');
gtext('zeta=0.75');
gtext('zeta=1.0'); % Click to put the text beside the curve you like
```

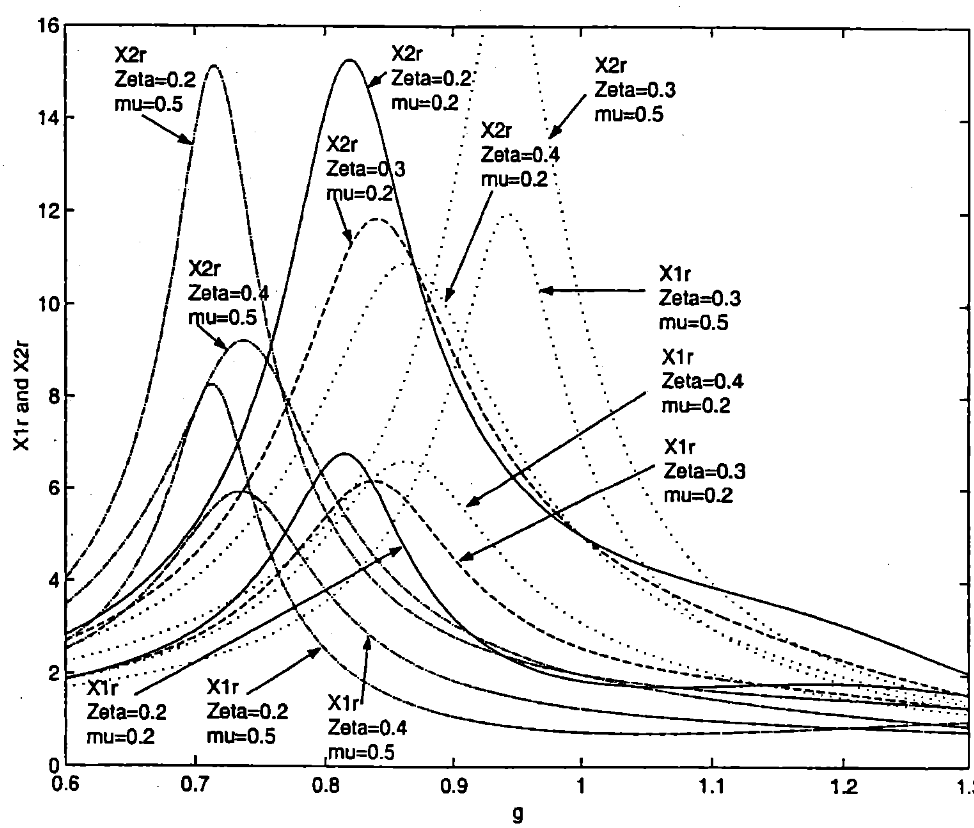


9.66

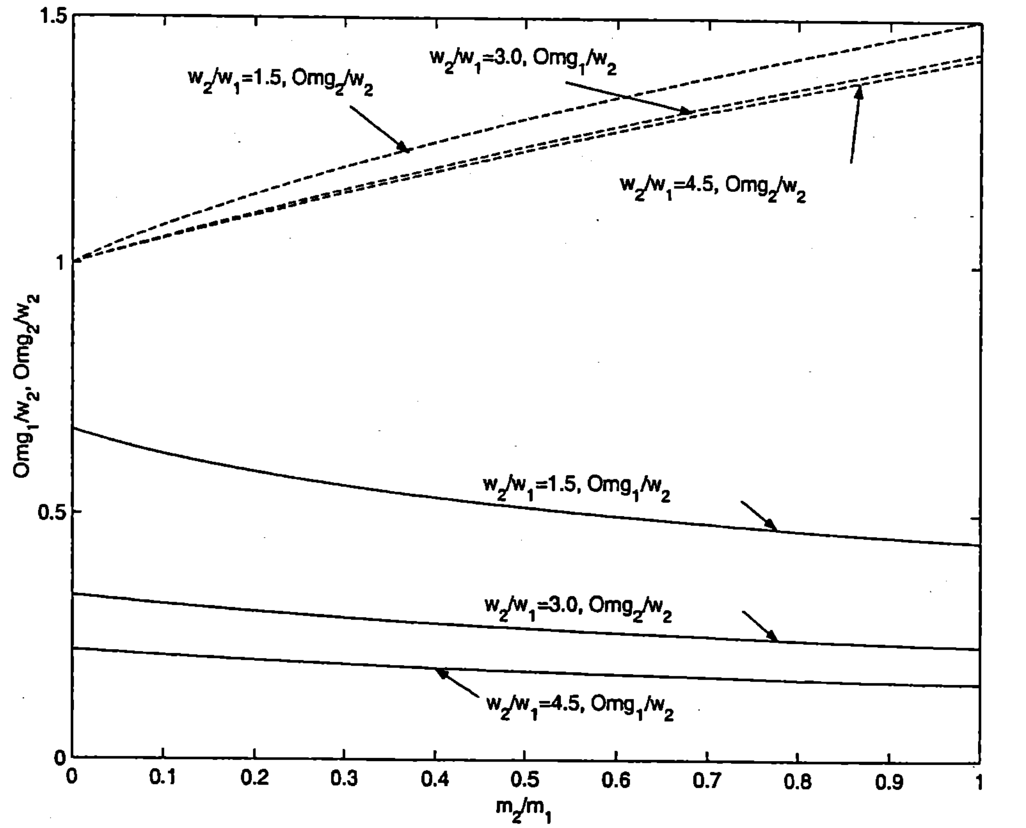
```
% program name: PR966.m

f = 1;
%------  zeta = 0.2, mu=0.2 ---------------------------------------------
------
zeta = 0.2;
mu = 0.2;
g = 0.6 : 0.001 : 1.3;

tzg2 = (2.*zeta.*g).^2 ;%---  tzg2 = (2*zeta*g)^2
g2_f2_2 = (g.^2-f.^2).^2 ;% g2_f2_2 = (g^2-f^2)^2
g2_1mug2_2 = (g.^2-1+mu.*g.^2).^2 ;
muf2g2 = mu.*f.^2*g.^2 ;
g2_1 = g.^2-1 ;
g2_f2 = g.^2-f.^2 ;

x1r =sqrt((tzg2+g2_f2_2)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
x2r =sqrt((tzg2+f.^4)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
plot(g,x1r)
hold on
plot(g,x2r);
hold on
%------  zeta = 0.2, mu=0.5 ---------------------------------------------
------
zeta = 0.2;
mu = 0.5;
g = 0.6 : 0.001 : 1.3;

tzg2 = (2.*zeta.*g).^2 ;%---  tzg2 = (2*zeta*g)^2
g2_f2_2 = (g.^2-f.^2).^2 ;% g2_f2_2 = (g^2-f^2)^2
g2_1mug2_2 = (g.^2-1+mu.*g.^2).^2 ;
muf2g2 = mu.*f.^2*g.^2 ;
```

```
g2_1 = g.^2-1 ;
g2_f2 = g.^2-f.^2 ;

x1r =sqrt((tzg2+g2_f2_2)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
x2r =sqrt((tzg2+f.^4)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
plot(g,x1r,'-.');
hold on
plot(g,x2r,'-.');
hold on
%------  zeta = 0.3, mu=0.2 ---------------------------------------------↙
------
zeta = 0.3;
mu = 0.2;
g = 0.6 : 0.001 : 1.3;

tzg2 = (2.*zeta.*g).^2 ;%---  tzg2 = (2*zeta*g)^2
g2_f2_2 = (g.^2-f.^2).^2 ;% g2_f2_2 = (g^2-f^2)^2
g2_1mug2_2 = (g.^2-1+mu.*g.^2).^2 ;
muf2g2 = mu.*f.^2*g.^2 ;
g2_1 = g.^2-1 ;
g2_f2 = g.^2-f.^2 ;

x1r =sqrt((tzg2+g2_f2_2)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
x2r =sqrt((tzg2+f.^4)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
plot(g,x1r,'--');
hold on
plot(g,x2r,'--');
hold on
%------  zeta = 0.3, mu=0.5 ---------------------------------------------↙
------
zeta = 0.5;
mu = 0.1;
g = 0.6 : 0.001 : 1.3;

tzg2 = (2.*zeta.*g).^2 ;%---  tzg2 = (2*zeta*g)^2
g2_f2_2 = (g.^2-f.^2).^2 ;% g2_f2_2 = (g^2-f^2)^2
g2_1mug2_2 = (g.^2-1+mu.*g.^2).^2 ;
muf2g2 = mu.*f.^2*g.^2 ;
g2_1 = g.^2-1 ;
g2_f2 = g.^2-f.^2 ;

x1r =sqrt((tzg2+g2_f2_2)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
x2r =sqrt((tzg2+f.^4)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));

plot(g,x1r,':');
hold on
plot(g,x2r,':');
hold on

%------  zeta = 0.4, mu=0.2 ---------------------------------------------↙
------
zeta = 0.4;
mu = 0.2;
g = 0.6 : 0.001 : 1.3;

tzg2 = (2.*zeta.*g).^2 ;%---  tzg2 = (2*zeta*g)^2
g2_f2_2 = (g.^2-f.^2).^2 ;% g2_f2_2 = (g^2-f^2)^2
g2_1mug2_2 = (g.^2-1+mu.*g.^2).^2 ;
muf2g2 = mu.*f.^2*g.^2 ;
g2_1 = g.^2-1 ;
g2_f2 = g.^2-f.^2 ;
```

```
x1r =sqrt((tzg2+g2_f2_2)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
x2r =sqrt((tzg2+f.^4)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
plot(g,x1r,':');
hold on
plot(g,x2r,':');
hold on

%------  zeta = 0.4, mu=0.5 -----------------------------------------------
------
zeta = 0.4;
mu = 0.5;
g = 0.6 : 0.001 : 1.3;

tzg2 = (2.*zeta.*g).^2 ;%---  tzg2 = (2*zeta*g)^2
g2_f2_2 = (g.^2-f.^2).^2 ;% g2_f2_2 = (g^2-f^2)^2
g2_1mug2_2 = (g.^2-1+mu.*g.^2).^2 ;
muf2g2 = mu.*f.^2*g.^2 ;
g2_1 = g.^2-1 ;
g2_f2 = g.^2-f.^2 ;

x1r =sqrt((tzg2+g2_f2_2)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
x2r =sqrt((tzg2+f.^4)./(tzg2.*g2_1mug2_2+(muf2g2-g2_1.*g2_f2).^2));
plot(g,x1r,'-.');
hold on
plot(g,x2r,'-.');

xlabel('g')
ylabel('X1r and X2r')
axis([0.6 1.3 0 16])
```



9.67




```
% Ex9_67.m
w2_1 = 1.5;
for i = 1: 101
    m2_1(i) = (i-1)/100;
    Omg1_w2(i) = sqrt( ( (1+(1+m2_1(i))*w2_1^2) - ...
    sqrt( ( 1+(1+m2_1(i))*w2_1^2 )^2 - 4*w2_1^2 ) ) / (2*w2_1^2) );
    Omg2_w2(i) = sqrt( ( (1+(1+m2_1(i))*w2_1^2) + ...
    sqrt( ( 1+(1+m2_1(i))*w2_1^2 )^2 - 4*w2_1^2 ) ) / (2*w2_1^2) );
end
w2_1 = 3.0;
for i = 1: 101
    m2_1(i) = (i-1)/100;
    Omg1_w2_b(i) = sqrt( ( (1+(1+m2_1(i))*w2_1^2) - ...
    sqrt( ( 1+(1+m2_1(i))*w2_1^2 )^2 - 4*w2_1^2 ) ) / (2*w2_1^2) );
    Omg2_w2_b(i) = sqrt( ( (1+(1+m2_1(i))*w2_1^2) + ...
    sqrt( ( 1+(1+m2_1(i))*w2_1^2 )^2 - 4*w2_1^2 ) ) / (2*w2_1^2) );
end
w2_1 = 4.5;
for i = 1: 101
    m2_1(i) = (i-1)/100;
    Omg1_w2_c(i) = sqrt( ( (1+(1+m2_1(i))*w2_1^2) - ...
    sqrt( ( 1+(1+m2_1(i))*w2_1^2 )^2 - 4*w2_1^2 ) ) / (2*w2_1^2) );
    Omg2_w2_c(i) = sqrt( ( (1+(1+m2_1(i))*w2_1^2) + ...
    sqrt( ( 1+(1+m2_1(i))*w2_1^2 )^2 - 4*w2_1^2 ) ) / (2*w2_1^2) );
end
plot(m2_1, Omg1_w2);
hold on;
plot(m2_1, Omg2_w2,'--');
gtext('w_2/w_1=1.5, Omg_1/w_2');
gtext('w_2/w_1=1.5, Omg_2/w_2');
hold on;
plot(m2_1, Omg1_w2_b);
```

```
hold on;
plot(m2_1, Omg2_w2_b,'--');
gtext('w_2/w_1=3.0, Omg_1/w_2');
gtext('w_2/w_1=3.0, Omg_2/w_2');
plot(m2_1, Omg1_w2_c);
hold on;
plot(m2_1, Omg2_w2_c,'--');
gtext('w_2/w_1=4.5, Omg_1/w_2');
gtext('w_2/w_1=4.5, Omg_2/w_2');
xlabel('m_2/m_1');
ylabel('Omg_1/w_2, Omg_2/w_2');
```

9.68

```
Results of Ex9_68
*****************
>> program13
                 Results of two-plane balancing

Left-plane balancing weight          Right-plane balancing weight

Magnitude=4.231537                  Magnitude=2.121730

Angel=130.294244                     Angel=140.731862
```

9.69

```
Results of Ex9_69
*****************

                 RESULTS OF TWO-PLANE BALANCING

LEFT-PLANE BALANCING WEIGHT          RIGHT-PLANE BALANCING WEIGHT

MAGNITUDE = 4.23153662               MAGNITUDE = 2.12172985
ANGLE = 130.29424410                 ANGLE = 140.73186247
```

(9.70) The main program and output are given below:

```
C ======================================================================
C
C PROGRAM 13.F
C TWO-PLANE BALANCING
C
C ======================================================================
      DIMENSION VA(2),VB(2),VAP(2),VBP(2),VAPP(2),VBPP(2),WL(2),WR(2),
     2 BL(2),BR(2)
C FOLLOWING 8 LINES CONTAIN PROBLEM-DEPENDENT DATA
      DATA VA/5.0,100.0/
      DATA VB/4.0,180.0/
      DATA WL/2.0,30.0/
      DATA VAP/6.5,120.0/
      DATA VBP/4.5,140.0/
      DATA WR/2.0,0.0/
      DATA VAPP/6.0,90.0/
      DATA VBPP/7.0,60.0/
C END OF PROBLEM-DEPENDENT DATA
      CALL BALAN (VA,VB,VAP,VBP,VAPP,VBPP,WL,WR,BL,BR)
      PRINT 10
 10   FORMAT (//,31H RESULTS OF TWO-PLANE BALANCING)
      PRINT 20, BL(1),BL(2)
 20   FORMAT (//,28H LEFT-PLANE BALANCING WEIGHT,//,11H MAGNITUDE=,
     2 E15.8,/,7H ANGLE=,4X,E15.8)
      PRINT 30, BR(1),BR(2)
 30   FORMAT (//,29H RIGHT-PLANE BALANCING WEIGHT,//,
     2 11H MAGNITUDE=,E15.8,/,7H ANGLE=,4X,E15.8)
      STOP
      END
```

```
 RESULTS OF TWO-PLANE BALANCING


 LEFT-PLANE BALANCING WEIGHT

 MAGNITUDE= 0.42315292E+01
 ANGLE=    -0.49705841E+02


 RIGHT-PLANE BALANCING WEIGHT

 MAGNITUDE= 0.21217260E+01
 ANGLE=    -0.39268173E+02
```

9.71 From Eq. (9.145),

$$\frac{X_2}{X_1} = \frac{k_2 + i\omega c_2}{k_2 - m_2\omega^2 + i\omega c_2} = \left\{\frac{k_2^2 + \omega^2 c_2^2}{(k_2 - m_2\omega^2)^2 + \omega^2 c_2^2}\right\}^{1/2}$$

i.e.

$$\frac{X_2}{\delta_{st}} = \frac{X_1}{\delta_{st}} \left\{\frac{(f^2)^2 + (2\zeta g)^2}{(f^2 - g^2)^2 + (2\zeta g)^2}\right\}^{1/2}$$

$\frac{X_1}{\delta_{st}}$ is given by Eq. (9.146).

The program for generating the values of $\frac{X_1}{\delta_{st}}$ and $\frac{X_2}{\delta_{st}}$ for $\frac{m_2}{m_1} = \mu = 1/20$, $f = 1$, $\zeta = 0.1, 0, \infty$ as $\frac{\omega}{\omega_n} = g$ varies between 0.6 and 1.3 is given below.

```
C ==========================================================================
C
C PROBLEM 9.71
C
C ==========================================================================
      DIMENSION X(2)
      REAL MU
      MU=0.05
      DO 100 II=1,3
      G=0.4
      DO 90 JJ=1,15
      G=G+0.1
      F=1.0
      IF (II .EQ. 1) ZETA=0.0
      IF (II .EQ. 2) ZETA=0.1
      IF (II .EQ. 3) ZETA=10.0
      XN=(2.0*ZETA*G)**2+(G**2-F**2)**2
      XD=((2.0*ZETA*G)**2)*((G**2-1.0+MU*G*G)**2)+
     2 (MU*F*F*G*G-(G*G-1.0)*(G*G-F*F))**2
      X(1)=SQRT(XN/XD)
      XN=(F**4)+(2.0*ZETA*G)**2
      XD=(F*F-G*G)**2+(2.0*ZETA*G)**2
      X(2)=X(1)*SQRT(XN/XD)
      PRINT 50, MU,ZETA,G,F
 50   FORMAT (/,2X,7H MU    =,E15.8,2X,7H ZETA =,E15.8,/,
     2 2X,7H G     =,E15.8,2X,7H F    =,E15.8)
      PRINT 60, X(1),X(2)
 60   FORMAT (2X,7H X(1) =,E15.8,2X,7H X(2) =,E15.8)
 90   CONTINUE
 100  CONTINUE
      STOP
      END
```

```
 MU   = 0.50000001E-01   ZETA = 0.00000000E+00
 G    = 0.50000000E+00   F    = 0.10000000E+01
 X(1) = 0.13636364E+01   X(2) = 0.18181819E+01

 MU   = 0.50000001E-01   ZETA = 0.00000000E+00
 G    = 0.60000002E+00   F    = 0.10000000E+01
 X(1) = 0.16343209E+01   X(2) = 0.25536263E+01
```

```
MU   = 0.50000001E-01   ZETA = 0.00000000E+00
G    = 0.70000005E+00   F    = 0.10000000E+01
X(1) = 0.21646862E+01   X(2) = 0.42444835E+01
  ⋮
MU   = 0.50000001E-01   ZETA = 0.10000000E+02
G    = 0.17000003E+01   F    = 0.10000000E+01
X(1) = 0.19167000E+00   X(2) = 0.49113098E+00

MU   = 0.50000001E-01   ZETA = 0.10000000E+02
G    = 0.18000003E+01   F    = 0.10000000E+01
X(1) = 0.31646856E+00   X(2) = 0.41582501E+00

MU   = 0.50000001E-01   ZETA = 0.10000000E+02
G    = 0.19000003E+01   F    = 0.10000000E+01
X(1) = 0.35850242E+00   X(2) = 0.35778359E+00
```

9.72 Crane location: $x_c(t) = A_c\, e^{-\omega_c \zeta_c t} \sin \omega_c t$ (E₁)

Forging press location: $x_f(t) = A_f \sin \omega_f t$ (E₂)

Air compressor location: $x_a(t) = A_a \sin \omega_a t$ (E₃)

$A_c = 20\ \mu m, \quad \omega_c = 10\ \text{Hz}, \quad \zeta_c = 0.1$

Attenuation law:

$A_r = A_0\, e^{-0.005\, r}$ where A_0 = amplitude at source and r = distance from source (E₄)

Application of Eq. (E₄) gives

$A_c = 20\mu m$ reduces to $20\, e^{-0.005(60)} = 14.8164\ \mu m$

$A_f = 30\ \mu m$ reduces to $30\, e^{-0.005(80)} = 20.1096\ \mu m$

$A_a = 25\ \mu m$ reduces to $25\, e^{-0.005(40)} = 20.4683\ \mu m$

Disturbances at site of milling machine are

$\tilde{x}_c(t) = 14.8164\, e^{-2\pi t} \sin 20\pi t \quad \mu m$ (E₅)

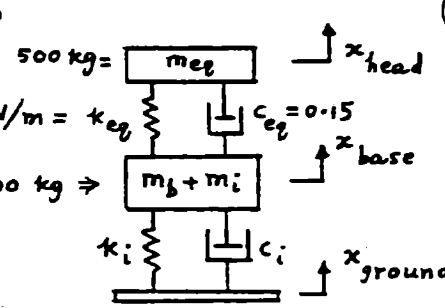
$\tilde{x}_f(t) = 20.1096 \sin 30\pi t \quad \mu m$ (E₆)

$\tilde{x}_a(t) = 20.4683 \sin 40\pi t \quad \mu m$ (E₇)

Find m_i, k_i and c_i such that

$|x_{cutter}|_{max} \le 2.5\ \mu m$

when $x_{ground} = \tilde{x}_c + \tilde{x}_f + \tilde{x}_a$ all acting simultaneously



Equations of motion:

$$(m_b + m_i)\ddot{x}_b - k_i(x_g - x_b) + k_{eq}(x_b - x_h) - c_i(\dot{x}_g - \dot{x}_b) + c_{eq}(\dot{x}_b - \dot{x}_h) = 0 \quad (E_8)$$

$$m_{eq}\ddot{x}_h - k_{eq}(x_b - x_h) - c_{eq}(\dot{x}_b - \dot{x}_h) = 0 \quad (E_9)$$

i.e.,

$$\begin{bmatrix} (m_b + m_i) & 0 \\ 0 & m_{eq} \end{bmatrix} \begin{Bmatrix} \ddot{x}_b \\ \ddot{x}_h \end{Bmatrix} + \begin{bmatrix} c_i + c_{eq} & -c_{eq} \\ -c_{eq} & c_{eq} \end{bmatrix} \begin{Bmatrix} \dot{x}_b \\ \dot{x}_h \end{Bmatrix} + \begin{bmatrix} k_i + k_{eq} & -k_{eq} \\ -k_{eq} & k_{eq} \end{bmatrix} \begin{Bmatrix} x_b \\ x_h \end{Bmatrix} = \begin{Bmatrix} k_i x_g + c_i \dot{x}_g \\ 0 \end{Bmatrix} \quad (E_{10})$$

The right hand side of Eq. (E_{10}) gives, for each type of disturbance,

When $x_g = \underset{\sim}{x}_c(t)$:

$$k_i x_g + c_i \dot{x}_g = k_i \left(14.8164\, e^{-2\pi t} \sin 20\pi t \times 10^{-6}\right) + c_i \left\{14.8164(-2\pi)\, e^{-2\pi t} \sin 20\pi t + 14.8164(20\pi)\, e^{-2\pi t} \cos 20\pi t\right\} 10^{-6} \quad (E_{11})$$

When $x_g = \underset{\sim}{x}_f(t)$:

$$k_i x_g + c_i \dot{x}_g = k_i(20.1096 \sin 30\pi t) + c_i(20.1096 \times 30\pi \times \cos 30\pi t) \quad (E_{12})$$

When $x_g = \underset{\sim}{x}_a(t)$:

$$k_i x_g + c_i \dot{x}_g = k_i(20.4683 \sin 40\pi t) + c_i(20.4683 \times 40\pi \times \cos 40\pi t) \quad (E_{13})$$

Procedure:

Let $\zeta_i = \dfrac{c_i}{2 m_i \omega_n} = \dfrac{c_i \sqrt{m_i}}{2 m_i \sqrt{k_i}} = \dfrac{c_i}{2\sqrt{m_i k_i}} = 0.1$ or $c_i = 0.2\sqrt{m_i k_i}$

(1) Assume m_i

(2) Assume k_i

(3) Find $c_i = 0.2\sqrt{m_i k_i}$

(4) Solve Eq. (E_{10}) numerically with $x_g = \underset{\sim}{x}_c(t)$. Find max $\underset{\sim}{x}_{hc}(t)$.

(5) Solve Eq. (E_{10}) numerically with $x_g = \underset{\sim}{x}_f(t)$. Find max $\underset{\sim}{x}_{hf}(t)$.

(6) Solve Eq. (E_{10}) numerically with $x_g = \underset{\sim}{x}_a(t)$. Find max $\underset{\sim}{x}_{ha}(t)$.

(7) Find max $x_h(t) = \max \left| \underset{\sim}{x}_{hc}(t) \right| + \max \left| \underset{\sim}{x}_{hf}(t) \right| + \max \left| \underset{\sim}{x}_{ha}(t) \right|$

(8) If max $x_h(t) \leq 2.5\ \mu m$, current values of m_i, k_i and c_i constitute the desired design.

(9) Otherwise, increment m_i and/or k_i, and go to step (3).