17

Design of Absorption

Teruo Obata
Teikyo University

Summary

This chapter presents the basics of designing devices for sound absorption. The absorption coefficient and acoustic impedance are introduced. Characteristic properties and parameters of sound absorption material and basic elements are presented. Acoustic modeling, analysis, and design considerations of components, such as ducts, and noise attenuation devices, such as mufflers, are presented. A practical design example is presented for illustration of the concepts presented in the chapter.

17.1 Introduction

Sound-absorption equipment is used for multiple purposes in architectural acoustics, mechanical noise countermeasures, and so on. In this context, the necessity for designing sound-absorption equipment from the viewpoint of noise control is explained. Architectural acoustics is an important area of study, which involves architecture, sound-absorption design, and sound-measurement facility.

In the area of noise reduction, the characteristics of sound are important, and proper sound-absorbing material should be selected based on how much attenuation is necessary for each frequency of sound. In particular, acoustic characteristics of sound-absorbing material such as the type of material and the sound-absorption mechanism are important. In this chapter, the basics of sound absorption are given, and the prediction and calculation methods for attenuation of lined or dissipative mufflers are outlined.

17.2 Fundamentals of Sound Absorption

17.2.1 Attenuation of Sound

When an acoustic wave propagates in a medium, the sound energy attenuates due to such reasons as viscosity, heat conduction, and the effects of molecular absorption. In a medium of small volume surrounded by a boundary surface, the attenuation is particularly considerable, for example, when the medium is a thin tube. This is because there is the dissipation of the energy controlled by the viscosity of the medium and heat conduction between the material and the medium of tube wall. A sound-absorbing material may be utilized to adjust such dissipation of acoustic energy.

17.2.1.1 Absorption Coefficient and Normal Acoustic Impedance

Some amount of energy is lost when an acoustic wave hits the surface of a sound-absorbing material. Figure 17.1 illustrates an infinite medium of absorbing material separated by air and the reflected wave (sound pressure p_r) from the boundary surface with the air where a plane wave of sound pressure p_i is emitted in the direction indicated by an arrow, at an angle θ. When $\theta = 0$, sound pressure p in air is given by

$$p = p_i + p_r = (Ae^{-jkx} + Be^{jkx})e^{j\omega t} \qquad (17.1)$$

where

$A, B =$ the amplitude of sound pressure of incident and reflected waves (in Pa),
$j = \sqrt{-1}$,
$k = 2\pi f/c$; wave number (1/m),
$\omega =$ angular frequency (rad/sec).

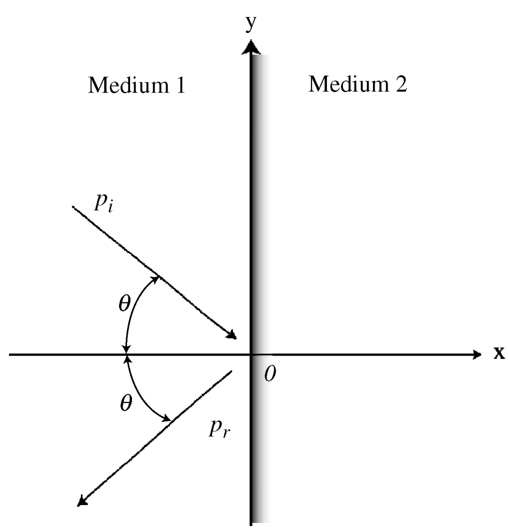

FIGURE 17.1 Plane wave incidence on an infinite absorbing material.

The sound pressure, p_m, in the absorbing material may be expressed using a complex propagation constant, by the equation:

$$p_m = p_{x=0}e^{-\gamma x}e^{j\omega t} \qquad (17.2)$$

where

$\gamma =$ the propagation constant in the absorbing material (m^{-1}). *Note*: $\gamma = \delta + j\beta$. γ is a property of the material itself and is not dependent on the mounting conditions when large areas of material are considered.
$\delta =$ attenuation constant. *Note*: δ tells us how much of the sound wave will be reduced as it travels through the material.
$\beta =$ phase constant. *Note*: β is a measure of the velocity of propagation of the sound wave through the material.

The relation for determining the velocity of sound in the material is given by

$$c_m = \omega/\beta \qquad (17.3)$$

Boundary conditions must be satisfied on the boundary surface. The acoustic impedance of a unit area of air and of absorbing material are, respectively, denoted by z and z_a. The pressure and the particle velocity on both sides of the boundary are equal. We have

$$\left.\begin{aligned} p_i + p_r &= p_{x=0} \\ \frac{p_i - p_r}{z_a} &= \frac{p_{x-0}}{z} \end{aligned}\right\} \tag{17.4}$$

The amplitude of reflectance of sound pressure, r, is obtained from Equation 17.4, and is given by

$$r = \frac{p_r}{p_i} = \frac{z_a - z}{z_a + z} \tag{17.5}$$

The reflectivity is the energy reflection rate. The absorption coefficient, α, of an absorbing material is defined as

$$\alpha = 1 - |r^2| \tag{17.6}$$

The impedance, z_n, through a surface is the quantity that represents the dissipation of energy of sound as well as the absorption coefficient. It is given as a ratio between sound pressure and particle velocity on boundary surface in the reflecting acoustic wave:

$$z_n = \left(\frac{p}{u}\right)_{x=0} = \left(\frac{\rho c}{\cos\theta}\right)\frac{p_i + p_r}{p_i - p_r} \tag{17.7}$$

Note that z_n is a complex quantity and involves both amplitude and phase, both of which depend on the sound pressure at the boundary surface in the reflecting acoustic wave.

In the case of oblique incidence, the surface impedance can be expressed by following equation:

$$z_n = Z\gamma z/q \tag{17.8}$$

where $z =$ the acoustic impedance (Pa sec/m^3). Here,

$$Z = \frac{z_l \cosh(ql) + (\gamma z/q)\sinh(ql)}{z_l \sinh(ql) + (\gamma z/q)\cosh(ql)}$$

$$q = \sqrt{\gamma^2 + k^2 \sin^2\theta}$$

The absorption coefficient, $\alpha(\theta)$, for an oblique incidence with angle θ may be expressed by

$$\alpha(\theta) = 1 - \left|\frac{z_n \cos\theta - \rho c}{z_n \cos\theta + \rho c}\right|^2 \tag{17.9}$$

17.3 Sound-Absorbing Materials

17.3.1 Porous Material

Porous acoustical materials are a special category of a more general class of gas–solid mixtures. They range from porous solids, for example, porous rocks, fibrous granular solids, expanded plastics, and form materials, to porous or turbid gases, for example, suspensions and emulsions. Sound is attenuated in a gas-saturated porous solid due to the restriction on the gas movement within it. A convenient microstructure model for such materials is one of a rigid solid matrix through which run cylindrical, capillary pores (tubing) with constant radius, normal to its surface. This model enables the use of Kirchoff's theory of sound propagation in narrow tubes with rigid walls. Accordingly, this mechanism of dissipation may be identified as (1) a viscous loss in the boundary layer at the wall of each capillary tube associated with the relative motion between the viscous gas and the solid wall, or (2) heat conduction between compressions and rarefactions of the gas and the conducting solid walls.

17.3.2 Tubular Material

Consider the absorption of low-frequency sound using the tubular absorbing material. By itself, sound absorption is not satisfactory with the tubular absorbing material. The material produces bending vibration due to an acoustic wave through it, and sound absorption occurs by the internal friction of the material. For hard plywood and gypsum boards, there is a natural frequency in the range 100 to 200 Hz, and the absorption coefficient ranges from 0.3 to 0.5. It is possible to increase the absorption coefficient by coating the board surface with fibrous absorbing material.

17.3.3 Membrane Material

For membrane material, the sound-absorption mechanism makes use of resonant vibration. Hence, resonant frequency is a governing parameter. The imaginary part (the reactance term) of the acoustic impedance of a membrane gives rise to a resonance. The associated natural frequency is given by

$$f_r = \frac{1}{2}\left\{\frac{1}{m}\left(\frac{1.4 \times 10^5}{L}\right) + K_m\right\} \tag{17.10}$$

where

f_r = natural frequency (Hz)
m = surface density (kg/m^2)
L = thickness of air space (m)
K_m = board rigidity (kg/m^2 sec^2)

The K_m values of some boards are shown in Figure 17.2. The absorption coefficient is approximately 0.3 to 0.4 in the frequency range of 300 to 1000 Hz, when the thickness of the air space between the membrane and the rigid wall behind it is 50 to 100 mm.

17.3.4 Perforated Plate

A perforated board of sound absorbing material (i.e., a board with holes) is placed over a rigid wall at a fixed clearance, as shown in Figure 17.3. The sound-absorption characteristics depend on the board thickness, t, the hole diameter of the perforations, d, the clearance, L, between the perforated board and the rigid wall, and so on. The absorption coefficient becomes a maximum at resonant frequency. In the present case, the resonant frequency is given by

$$f_r = \frac{c}{2\pi}\sqrt{\frac{\varepsilon}{(t + 0.8d)L}} \tag{17.11}$$

where sound speed is c, the airspace thickness is L (typically, 300 mm or less), and the ratio of the total area of holes to the total area of the board is ε. The absorption coefficient is approximately 0.3 to 0.4.

17.3.5 Acoustic Resonator

Yet another method of achieving sound absorption is using an acoustic resonator of Helmholtz type, which consists of a vessel of any shape containing a volume air, as shown in Figure 17.4. The air volume is in direct communication with the ambient air in the room through an interconnecting tube, which may be long or short and of any cross-sectional shape. An example of a resonator of Helmholtz type may be a 1 gal jar. When a sound wave impinges on the aperture of neck of the jar, the air in the neck will be set in oscillation, periodically expanding and compressing the air in the vessel.

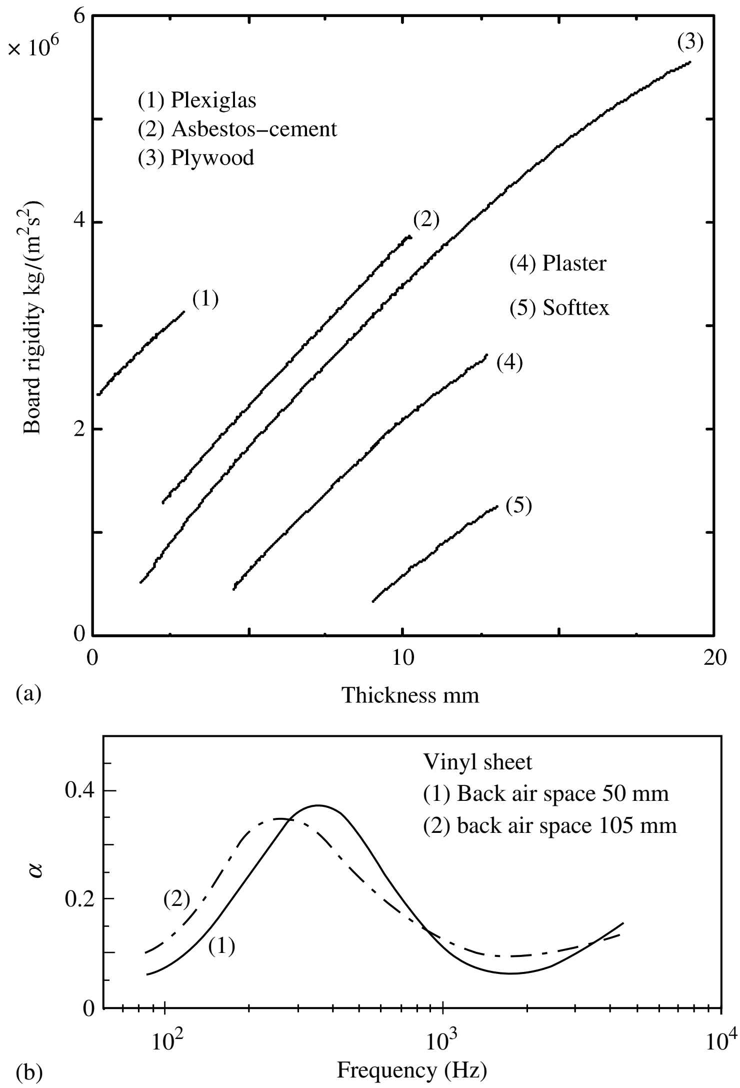

FIGURE 17.2 Some K_m values for membrane absorbing materials.

The resulting amplified motion of the air particles in the neck of the jar, due to phase cancellation between the air plug in the neck and the air volume in the vessel, causes energy dissipation due to friction in and around the neck. This type of absorber can be designed to produce maximum absorption over a very narrow frequency band or even a wide frequency band. The resonant frequency of a Helmholtz resonator may be expressed as

$$f_r = \frac{c}{2m}\sqrt{\frac{\varepsilon}{(t + 0.8d)L}} \tag{17.12}$$

where

$c =$ speed of sound (m/s)
$S_n =$ cross-sectional area of neck of jar (m^2)
$d_n =$ diameter of neck of jar (m)
$V =$ volume of vessel (m^3)

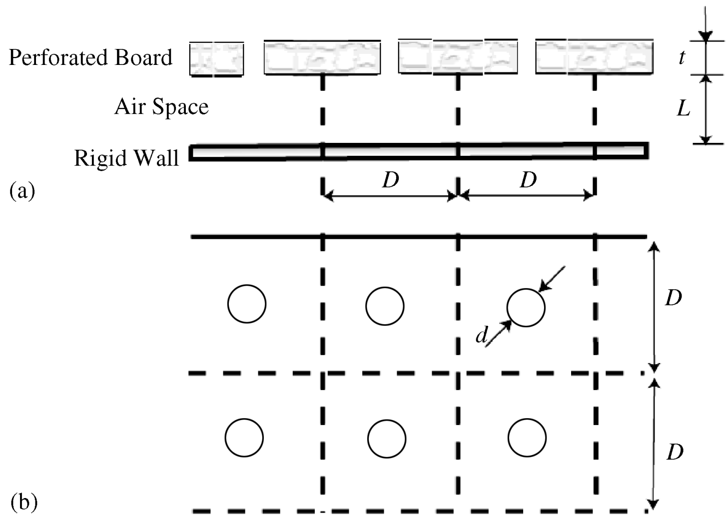

FIGURE 17.3 Sound-absorption characteristics of a perforated plate structure: (a) cross sectional view; (b) plan view.

17.4 Acoustic Characteristic Computation of Compound Wall

17.4.1 Absorption Coefficient of Combined Plate with Porous Blanket

A common form of problem in noise control is the need to reduce the sound radiated from a duct or some other object. A way to achieve this is by lining the duct with several centimeters of porous acoustic material, and covering it with a solid plate of some type, as indicated in Figure 17.5.

Consider the case of normal incidence with the sound-absorbing structure of Figure 17.5. Assume that the boundary conditions for the sound pressure and the volume flow-rate are identical. For plane wave incidence on the hard wall, the magnitude of reflection coefficient is -1 [1]. The following equation is obtained:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & -m_1 & m_1 & 0 \\ 0 & e^{-\gamma l_1} & e^{\gamma l_1} & -(1+e^{-2jkl_2}) \\ 0 & m_2 e^{-\gamma l_1} & m_2 e^{\gamma l_1} & -(1-e^{-2jkl_2}) \end{bmatrix} \begin{bmatrix} B_1 \\ A_1 \\ B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \tag{17.13}$$

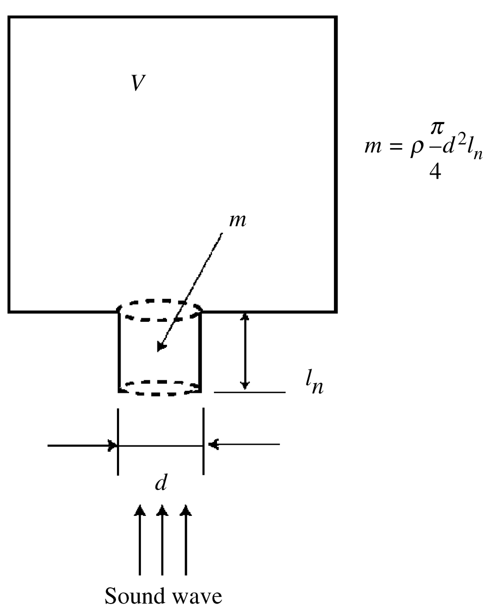

FIGURE 17.4 Geometry of a Helmholtz resonator. Volume, V, is connected to an infinitely open area by a neck tube of diameter d and length l_n.

where

$j = \sqrt{-1}$
$m_1 = z_0/z_1$, $m_2 = z_1/z_2$
z_0, z_1, z_2: acoustic impedance of each medium (Pa s/m^3)
γ = complex propagation constant (1/m)

The absorption coefficient for normal incidence is given by the following equation:

$$\alpha_0 = 1 - \left|\frac{B_0}{A_0}\right|^2 = 1 - |B_0|^2 \qquad (17.14)$$

The absorption coefficient for random incidence may be approximated by

$$\alpha = \frac{1}{n}\sum_{i=1}^{n} \alpha(\theta)_i \qquad (17.15)$$

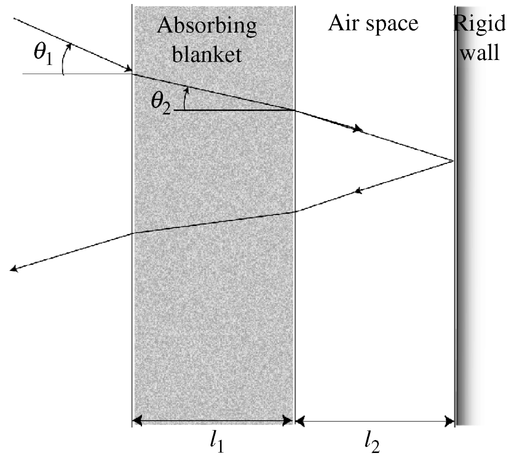

FIGURE 17.5 Structure for sound absorption using a blanket and an air space showing angles θ_1 in the air and θ_2 in the blanket.

where θ = the incident angle of sound, $0 < \theta < \pi/2$.

It is known that the propagation speed of the sound in fibrous materials changes with air, and the following equation holds on the boundary surface:

$$\sin\theta/\sin\theta' = c/c_m \qquad (17.16)$$

Here, c_m is the sound speed in fibrous materials, which is calculated from the imaginary part of Equation 17.18, given later. The angle of reflection, θ', in the boundary surface of the back air space is obtained in a similar way. Hence, the following equation is substituted in Equation 17.13 instead of the thickness of the absorber, l_1, and the thickness of the air space, l_2, to obtain the absorption coefficient in oblique incidence:

$$l_1' = l_1/\cos\theta', \quad l_2' = l_2/\cos\theta'' \qquad (17.17)$$

The complex propagation constant, γ, is an important physical quantity in absorbing material of propagated sound, which is given per unit length of acoustic attenuations, and phase changes. Between the aeroelasticity rate, K_a, of absorbing material and the bulk modulus, Q, of absorbing material, γ is given by the following equation, for $K_a > 20\,Q$ [2,3]:

$$\gamma = j\omega\sqrt{Y/K}\sqrt{\langle\rho_1\rangle - j\langle R_1\rangle/\omega} \qquad (17.18)$$

$$\langle R_1\rangle = \frac{R_1[1-\rho_0(1-Y)/\rho_m]}{\left[1+\dfrac{\rho_0(\kappa-1)}{\rho_m}\right]^2\left[1+\dfrac{R_1^2}{\rho_m^2\omega^2[1+\rho_0(\kappa-1)/\rho_m]^2}\right]}$$

$$\langle\rho_1\rangle = \rho_0\kappa - \frac{\dfrac{R_1^2(Y/\kappa+\rho_m/\rho_0\kappa)}{\rho_m^2\omega^2[1+\rho_0(\kappa-1)/\rho_m]^2} + \dfrac{1+\rho_0 Y(\kappa-1)/\rho_m\kappa}{1+\rho_0(\kappa-1)/\rho_m}}{1+\dfrac{R_1^2}{\rho_m^2\omega^2[1+\rho_0(\kappa-1)/\rho_m]^2}}$$

where

ρ_m = density of acoustical material (kg/m^3)
ρ_0 = density of air (kg/m^3)
c_0 = speed of sound in air (m/s)

K = volume coefficient of elasticity of air (N/m^2)
R_1 = alternating flow resistance for unit thickness of material due to the difference between the velocity of the skeleton and the velocity of air in the interstices (Pa s/m^2). R_1 values are given in Table 17.1
Y = porosity = the ratio of the volume of the voids in the material to the total volume; porosity equals the total volume minus the fiber volume, all divided by total volume
$\kappa = 5.5 - 4.5Y$, the structure factor of the interstices in the skeleton
$\omega = 2\pi f$, the angular frequency (radians/s)

The acoustic impedance, z_1, of absorbing material is given by

$$z_1 = R + jX = -\frac{jK\gamma}{\omega Y} \tag{17.19}$$

in which

$$R = \rho_0 c_0\left\{1 + 0.0571(\rho_0 f/R_f)^{-0.754}\right\}$$
$$X = -\rho_0 c_0\left\{0.0870(\rho_0 f/R_f)^{-0.732}\right\}$$

17.4.2 Transmission Loss through a Single Porous Board

Assume that a sound wave impinges on the left side of a porous board at normal incidence and emerges with a reduced amplitude from the right side. The associated transmission loss of the porous board is obtained from

$$\left.\begin{aligned} TL_0 &= 10\log_{10}(X+Y) \\ X &= \left\{1 + \frac{\omega^2 m^2 P R_f}{2\rho_0 c_0(\omega^2 m^2 P^2 + R_f^2)}\right\}^2 \\ Y &= \left\{\frac{\omega m R_f^2}{2\rho_0 c_0(\omega^2 m^2 P^2 + R_f^2}\right\}^2 \end{aligned}\right\} \tag{17.20}$$

where

m = surface density of the blanket (kg/m^2)
P = porosity of the blanket (porosity = the total volume minus the fiber volume, all divided by the total volume)
R_f = specific flow resistance of material (Pa s/m)

TABLE 17.1 Flow Resistance Values of Glass-Wool Board (Quality Regulation Range by JIS)

Board Type	K value	Gross Specific Gravity (kg/m^3)	Specific Flow Resistance ($\times 10^{-3}$ N s/m^4)	Standard of JIS for Glass Wool
#1 Glass-wool board	8	8 ± 2	1.5 ~ 7.0	JIS A 9505-A
	12	12 ± 2	2.5 ~ 12.0	
	16	16 ± 2	4.7 ~ 17.0	
	20	20 ± 3	5.0 ~ 22.0	
	24	24 ± 3	6.5 ~ 27.0	
#2 Glass-wool board	12	12 ± 2	1.5 ~ 7.0	JIS A 9505-B
	16	16 ± 2	2.5 ~ 10.0	
	20	20 ± 3	3.0 ~ 13.0	
	24	24 ± 3	4.0 ~ 16.0	
	32	32 ± 4	6.0 ~ 22.0	
	48	48 ± 5	11.0 ~ 38.0	
	64	64 ± 6	18.0 ~ 60.0	
	96	96 ± 10	27.0 ~ 95.0	
#3 Glass-wool board	96	96 ± 10	15.0 ~ 40.0	JIS A 9505-C

ρ_0 = density of air (kg/m^3)
c_0 = sound speed in air (m/s)

17.4.3 Transmission Loss through a Sandwich Board

Consider a wide wall formed by two panels (sheets) of infinite area separated with a homogeneous filling of fibrous acoustical material, as shown in Figure 17.6. Suppose that a plane wave impinges at an angle θ. As the pressure of both sides of the wall is equal with regard to the amplitude of the progressing wave and the reflected wave in each boundary surface, the following result may be established [4]:

$$\left.\begin{aligned}
A_0 + B_0 &= A_1 + B_1 \\
(A_0 - B_0)/z_0 &= (A_1 - B_1)/z_1 \\
A_1 e^{-jkl'_1} + B_1 e^{jkl'_1} &= A_2 + B_2 \\
(A_1 e^{-jkl'_1} - B_1 e^{jkl'_1})/z_1 &= (A_2 - B_2)/z_2 \\
A_2 e^{-\gamma l'_2} + B_2 e^{\gamma l'_2} &= A_3 + B_3 \\
(A_2 e^{-\gamma l'_2} - B_2 e^{\gamma l'_2})/z_2 &= (A_3 - B_3)/z_3 \\
A_3 e^{-\gamma l'_3} + B_3 e^{\gamma l'_3} &= A_4 + B_4 \\
(A_3 e^{-\gamma l'_2} - B_3 e^{\gamma l'_2})/z_3 &= (A_4 - B_4)/z_0
\end{aligned}\right\} \quad (17.21)$$

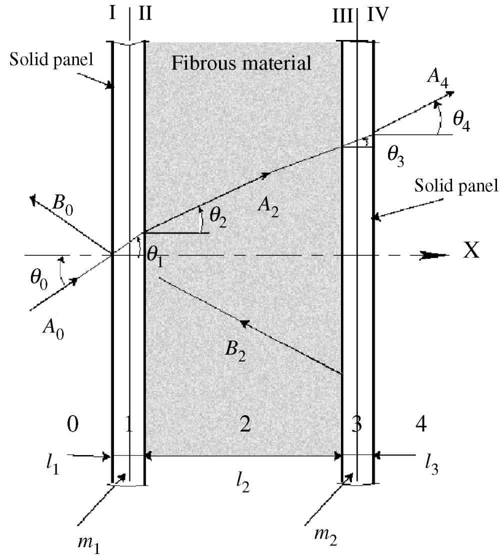

FIGURE 17.6 Cross-sectional view of a sandwich panel.

where A and B are the amplitude of sound pressures.

From Equation 17.17, $l'_1 = l_1/\cos\theta_1$, $l'_2 = l_2/\cos\theta'_2$, and l'_1 and l'_2 may be calculated.

The speed of sound in the walls is given by the following equation in terms of the modulus of longitudinal elasticity, E_i:

$$c_i = \sqrt{E_i/\rho_i} \quad (17.22)$$

The real part of acoustic impedance, z_i (i = 1, 3), is given by $R_i = r_i/\cos\theta$, and of the imaginary part is given at $X_i = m_i\omega$. The internal resistances, r_i, are functions of such factors as the material, frequency, temperature, and density. Some typical values are given in Table 17.2.

If the space of the transmission side is infinite, B_4 in Equation 17.21 becomes equal to zero. Then, the transmission loss is given is given by

$$\mathrm{TL}(\theta) = 10\log_{10}\left|\frac{A_4}{A_0}\right|^2 \quad (17.23)$$

TABLE 17.2 Internal Resistance Values of Several Useful Materials

Material	Thickness (mm)	Internal Resistance (Pa sec/m^3)
Aluminum	0.4	3.0
Plywood	3.0	7.5
Plaster board	7.0	15.0

17.5 Attenuation of Lined Ducts

17.5.1 Computation of Attenuation in a Lined Duct

A lined duct is an air passage with one or more of the interior surfaces covered with an acoustical material such as a glass or mineral fiber blanket. The parallel baffles are merely a series of side-by-side ducts that generally have a rectangular or round cross section. If the walls are covered with absorptive material, attenuation will occur because of the viscous motion of the air in and out of the porous of blanket.

Figure 17.7 shows an isometric illustration of a lined duct. The attenuation of sound for a lined duct is dependent primarily on the duct length, l_e, the thickness of the lining, b, the density of the lining, ρ, the width of the air passage, l, and the wavelength of sound, λ. At low frequencies ($l/\lambda < 0.1$), the attenuation of sound in a lined duct may be calculated from the following empirical formula:

$$\text{ATT} = K_l P/S \qquad (17.24)$$

where

K_l = the coefficient, which is determined from the random incidence absorption coefficient of lined material, given in the chart of Figure 17.8
P = acoustically lined perimeter of duct (m)
S = cross-sectional open area of duct (m^2)

If the absorbing material is lined in the rectangular cross section as shown in Figure 17.9 to Figure 17.11, the attenuation can be estimated using the formulas given in Table 17.3 [5].

17.5.2 Attenuation in a Lined Bend

A lined bend duct is shown in Figure 17.12. The insertion loss, IL, of a lined bend results from two mechanisms: the reflection of sound back toward the source side, and the scattering of sound energy into the high-frequency region is rapidly attenuated by the lining beyond the bend. Higher-frequency modes will be attenuated by even an unlined duct for frequencies below the ratio of the air passage between the linings to the wavelength of sound equal to 0.5. At frequencies well above this ratio, the insertion loss of a lined bend is expected to be comparable to the reverberant-field

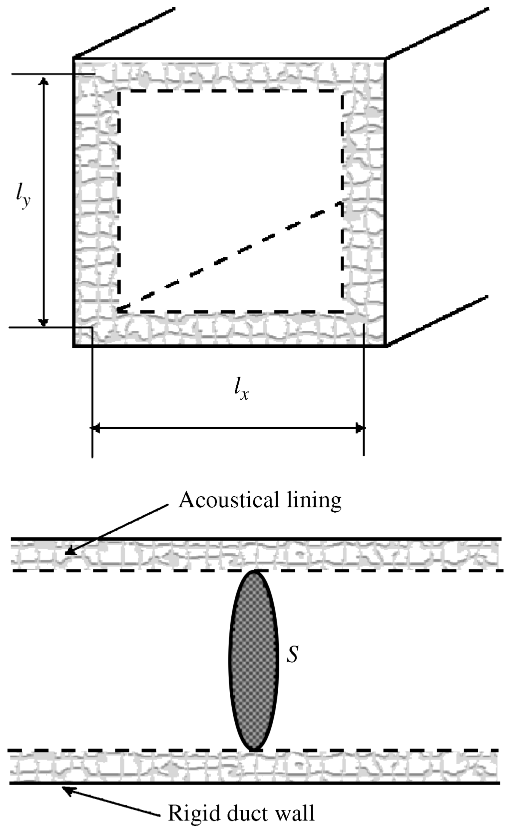

FIGURE 17.7 Illustration of a lined duct.

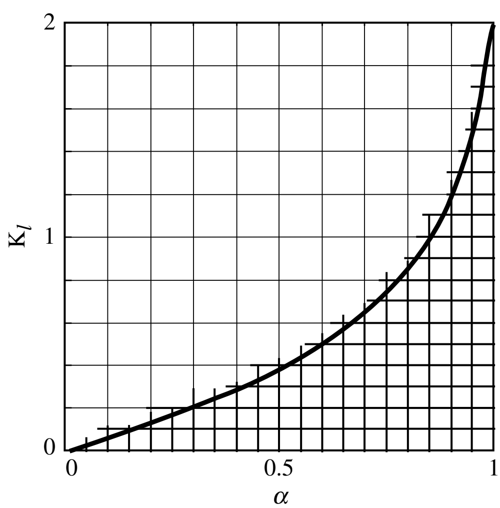

FIGURE 17.8 K_l value for sound-absorption coefficient by reverberation room method.

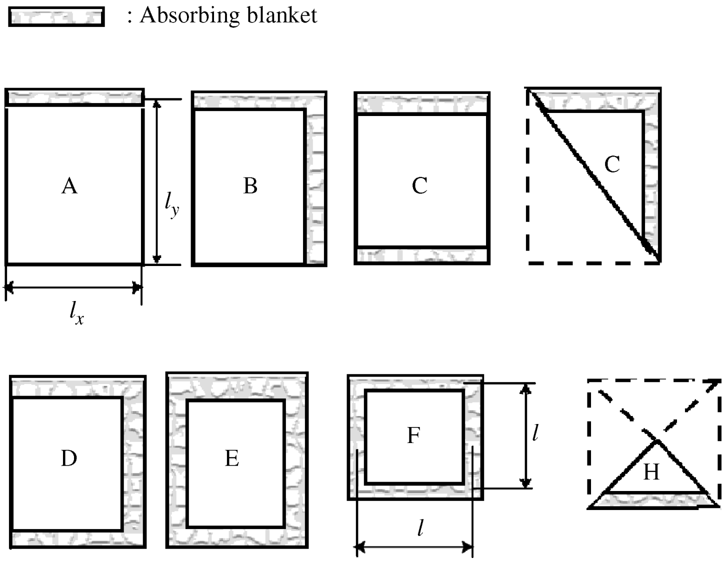

FIGURE 17.9 Duct-liner configurations corresponding to Table 42.3.

end correction derived for the duct. The insertion loss of a lined bend may be obtained as following equation [6]:

$$\text{IL} = \frac{K_1 P}{S} + (l_1 + l_2) + \Phi \tag{17.25}$$

where Φ is obtained from Figure 17.13.

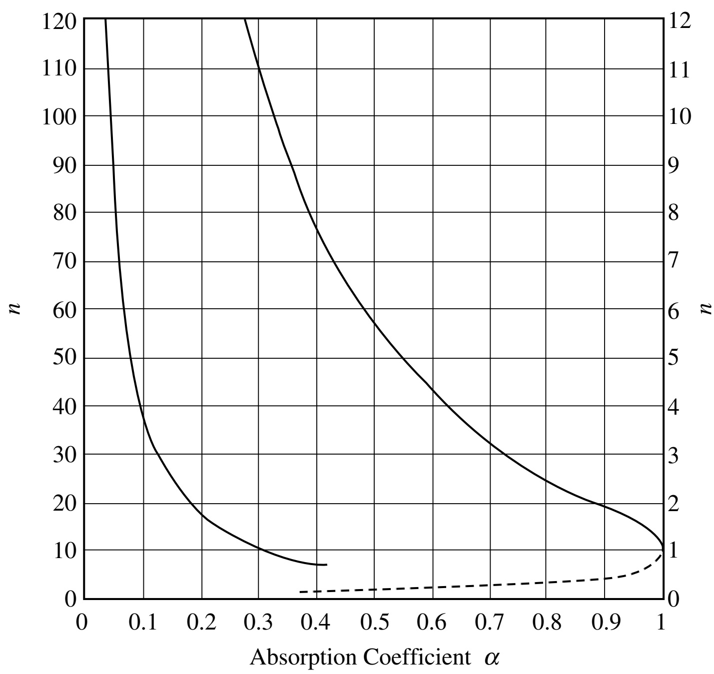

FIGURE 17.10 Relationship between absorption coefficient and stationary wave factor, *n*.

The total insertion loss for a lined bend is given in Figure 17.13 along with the attenuation of the lining beyond the bend.

17.5.3 Attenuation in Splitter Lined Duct

The use of parallel or zigzag baffle-type separators (splitters) to increase the perimeter–area ratio results in more compact attenuators. In rock-wool blankets, the attenuation of a parallel type splitter duct may be obtained directly from Figure 17.14. The peak value of the attenuation is related to wavelength of sound and the splitter interval. With the zigzag arrangement of acoustic blankets, the attenuation of high frequencies is improved over that of the parallel splitter [7].

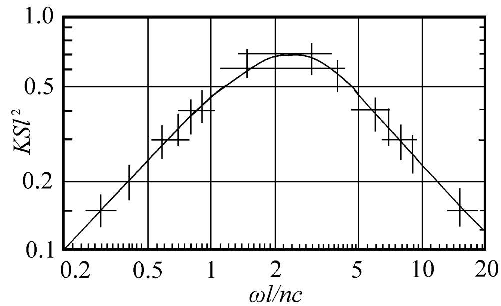

FIGURE 17.11 Damping function KSL^2 as a function of dimensionless frequency, $\omega l/nc$.

17.6 Attenuation of Dissipative Mufflers

17.6.1 Transmission Loss of Lined Expansion Chamber

The geometry and nomenclature for a dissipative muffler are given in Figure 17.15. For $f < 1.2c/D$, the assumption of plane wave is acceptable where $D =$ the diameter of the muffler.

The transmission loss for the light lining in the chamber may be obtained using [8,9]:

$$\mathrm{TL} = 10\log_{10}\left[\left\{\cosh(\delta_e l_e/2) + \frac{m+1}{2m}\sinh(\delta_e l_e/2)\right\}^2 \cos^2 kl_e + \left\{\sinh(\delta_e l_e/2) + \frac{m+1}{2m}\cosh(\delta_e l_e/2)\right\}^2 \sin^2 kl_e\right] \tag{17.26}$$

TABLE 17.3 Formulas for Attenuation of Several Lined Ducts

See Figure 17.7	Low-Frequencies Range: $\frac{\omega l}{nc} < 1$	Middle-Frequencies Range ($K_y S_y l_y$; see Figure 17.7)	High-Frequencies Range: $\frac{\omega l}{nc} > 5$
(A)	$\beta = \frac{4.34}{n l_y}$	$\beta = \frac{8.7c}{l_y^2 \omega}(K_y S_y l_y^2)$	$\beta = 21.4\frac{c^2 n}{\omega^2 l_y^3}$
(B)	$\beta = 4.34\left(\frac{1}{n_y l_y} + \frac{1}{n_x l_x}\right)$	$\beta = \frac{8.7c}{\omega}\left(\frac{K_y S_y l_y^2}{l_y^2} + \frac{K_x S_x l_x^2}{l_x^2}\right)$	$\beta = 21.4\frac{c^2}{\omega^2}\left(\frac{n_y}{l_y^3} + \frac{n_x}{l_x^3}\right)$
(C)	$\beta = \frac{8.7}{n l_y}$	$\beta = \frac{34.7c}{l_y^2 \omega}\left(\frac{K_y S_y l_y^2}{4}\right)$	$\beta = 171\frac{c^2}{\omega^2}\frac{n_y}{l_y^3}$
(D)	$\beta = 4.34\left(\frac{2}{n_y l_y} + \frac{1}{n_x l_x}\right)$	$\beta = \frac{8.7c}{\omega}\left(\frac{K_y S_y l_y^2}{l_y^2} + \frac{K_x S_x l_x^2}{l_x^2}\right)$	$\beta = 21.4\frac{c^2}{\omega^2}\left(\frac{8n_y}{l_y^3} + \frac{n_x}{l_x^3}\right)$
(E)	$\beta = 8.7\left(\frac{1}{n_y l_y} + \frac{1}{n_x l_x}\right)$	$\beta = \frac{34.7c}{\omega}\left(\frac{K_y S_y l_y^2}{4 l_y^2} + \frac{K_x S_x l_x^2}{4 l_x^2}\right)$	$\beta = \frac{171 c^2}{\omega^2}\left(\frac{n_y}{l_y^3} + \frac{n_x}{l_x^3}\right)$
(F)	$\beta = \frac{17.4}{nl}$	$\beta = \frac{69.5c}{4 l^2 \omega} KSl^2$	$\beta = \frac{341 c^2 n}{\omega^2 l^3}$

β is attenuation (dB/m), n is absorbing factor plotted in Figure 17.7, $K_y S_y l_y$ is damping function, plotted in Figure 17.8, c is sound speed, l is the width of the duct, $\omega = 2\pi f$: angular frequency, x, y: coordinates, see Figure 17.7.
Source: Brüel, P.V. 1951. *Sound Insulation and Room Acoustics*, Chapman & Hall, London, p.159. With permission.

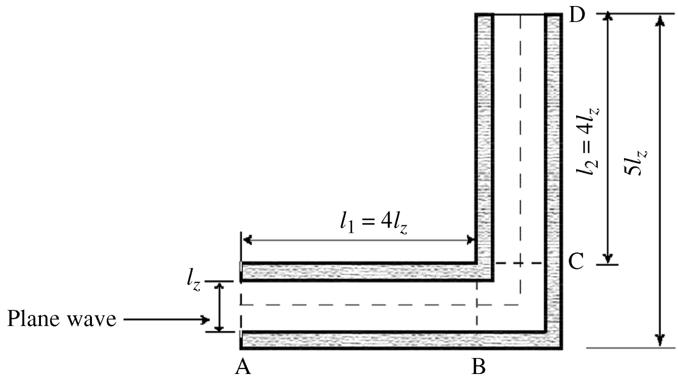

FIGURE 17.12 Sketch of a typical lined bend with plane wave incidence. (*Source*: Beranek, L.L. *Noise Reduction*, McGraw-Hill, 1960. With permission.)

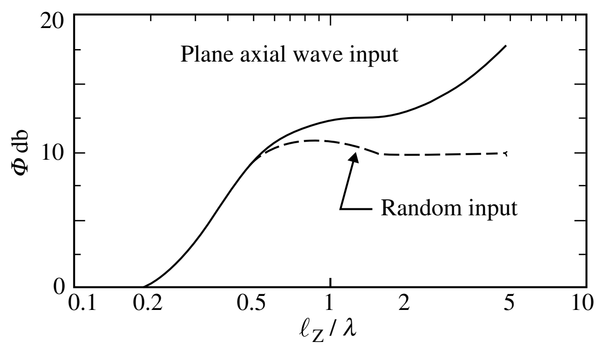

FIGURE 17.13 Insertion loss for lined bend. (The lining must extend two to four duct widths beyond the bend for this data to be valid.) (*Source*: Beranek, L.L. *Noise Reduction*, McGraw-Hill, 1960. With permission.)

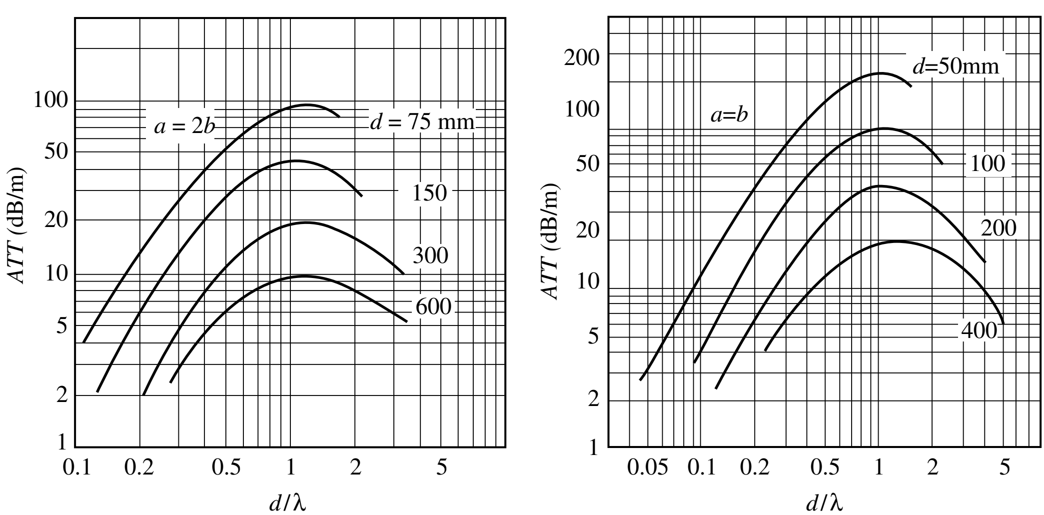

FIGURE 17.14 Sound attenuation for a splitter duct. Each baffle is constructed with two sheets of perforated metal filled with mineral wool, with about 100 to 140 kg/m^3 gross density; $a =$ the width of the open space, $b =$ the width of the baffle, $d =$ the center-to-center distance of baffles, $\lambda =$ the wavelength of the sound.

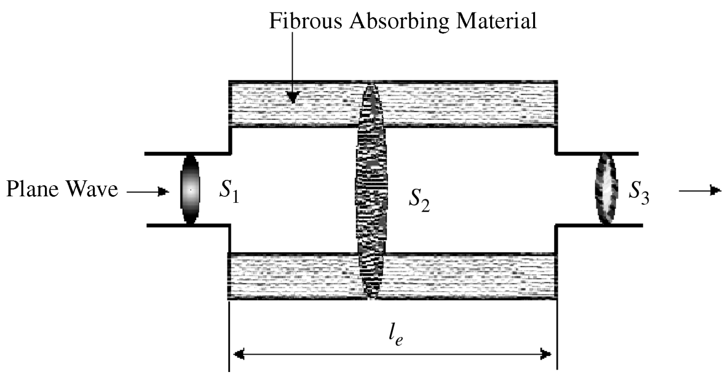

FIGURE 17.15 A dissipative muffler.

TABLE 17.4 Filled up Factor of Glass Wool, α_g

V_g/V	0.05	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
α_g	0.106	0.124	0.288	0.365	0.529	0.677	0.794	0.885	0.935	0.960	0.987	1.0

V_g = Filled up volume (factors of 100 kg/m^3), V = Volume of chamber

in which δ_e = the attenuation per unit length for the lined duct, which is given by the following equation:

$$20\log_{10}(\delta_e l_e) = \frac{K_l P l_e}{S} \tag{17.27}$$

The K_l values are obtained from the absorption coefficient, as shown earlier (see Figure 17.8). In particular, δ_e is given by

$$\delta_e = \frac{1}{l_e} 10^{0.05 K_l P l_e / S} \tag{17.28}$$

where m = the ratio of the area of expanded or lined sections to the area of inlet or outlet sections of muffler; $k = 2\pi f/c$, and l_e = the length of the muffler.

The transmission loss for the case of a thick lining of glass wool in the chamber is obtained using the empirical formula [10]

$$\text{TL} = 10\log_{10}\left[1 + \left\{\frac{1}{2}\alpha_g m k l_e\right\}^2\right] \tag{17.29}$$

where

α_g = the coefficient, which is obtained from Table 17.4, using the filling volume and the density of glass wool
m = the ratio of the area of expanded or lined sections to the area of inlet or outlet sections of muffler
$k = 2\pi f/c$
l_e = the length of muffler

17.6.2 Transmission Loss of a Plenum Chamber

The geometry and nomenclature for a plenum chamber are given in Figure 17.16. A plenum chamber is similar in many ways to a lined expansion chamber. The main difference is that the inlet and outlet of a plenum chamber are not located in line. Generally, there is an offset to direct transmission of sound. Sound is reflected at the square-cornered bend as the cross section dimension of the duct is sufficiently large. Particularly at high frequencies, almost all of the sound energy may reflect many times off the lined sides when propagating from the inlet to the outlet. The transmission loss of a single plenum chamber can be obtained approximately from [11]:

$$\mathrm{TL} = 10 \log_{10}\left\{S_{\mathrm{w}}\left(\frac{\cos\theta}{2\pi d^2} + \frac{1}{R}\right)\right\} \quad (17.30)$$

where

$S_{\mathrm{w}} = lW =$ area of the inlet and outlet
$d = \{(L-l)^2 + H^2\}^{1/2} =$ the slant distance from inlet to outlet
$\cos\theta = H/d$
$R = a/(1-\alpha_{\mathrm{m}})$
$a =$ the total lined area in chamber times absorption coefficient
$\alpha_{\mathrm{m}} =$ the statistical absorption coefficient of the lining

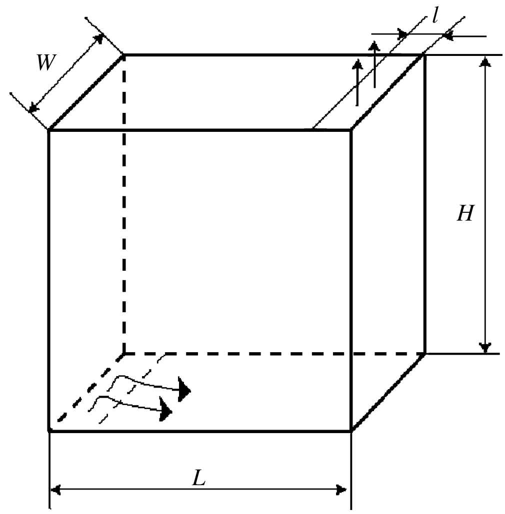

FIGURE 17.16 A single-plenum chamber showing the nomenclature used in Equation 17.26.

17.7 General Considerations

In order to carry out the design of noise-control measures for a particular problem, we must consider not only the fundamental acoustical properties of the material as discussed before, but also such practical aspects of the problem as (1) gas flow velocities, (2) temperature of gas, (3) moisture exposure, and (4) head losses for gas-flow. The client depends heavily on the expertise of the designer to realize adequate protection of the noise-control equipment under operating conditions.

17.7.1 Surface Treatment with Lining of Acoustic Material

Fibrous material in the market has some form of resin binder. Comparatively long fiber flocculent and comparatively short fiber are available. The packaging density of flocculent is about 60 to 100 $\mathrm{kg/m^3}$. It is necessary to cover with perforated thin metal or wire netting so that an arbitrary shape may be maintained in the absorbing material. The perforated metal does not take into account the numerical aperture, hole shape, hole diameter, and metal thickness. From the acoustic viewpoint, a suitable numerical aperture is given in Table 17.5.

17.7.2 Gas Flow Velocity

Noise control problems often involve the use of an acoustical material in high-velocity gas-flow such as those found in the exhaust of engines or ventilating systems. Deterioration of the acoustical

TABLE 17.5 Perforated Metal for Treatment of Absorbing Material (Gas Flow Velocity is 25 m/sec or Less)

Perforation rate: 30 to 50%
Hole diameter: 5 to 10 mm
Hole shape: round, plus, slit and interminglement
Metal: iron, stainless steel (used in case of the corrosive gas)

material due to high-velocity gas flowing past it can be a serious problem. In addition, turbulence in the gas flow subjects the materials to vibration and can cause further deterioration. One solution to this problem is to install the acoustical material behind some form of protective facing, which will vary in complexity depending on the gas velocity.

A limited amount of information on this subject is available through field experience, as shown in Figure 17.17 [12]. However, the parameters of the treatment structure are not well established, for example, those concerning perforated metal, wire net, absorbing material, and gas flow. Multiple layers are used under conditions of flow velocity exceeding 25 m/sec, and the associated performance analysis can become rather complex.

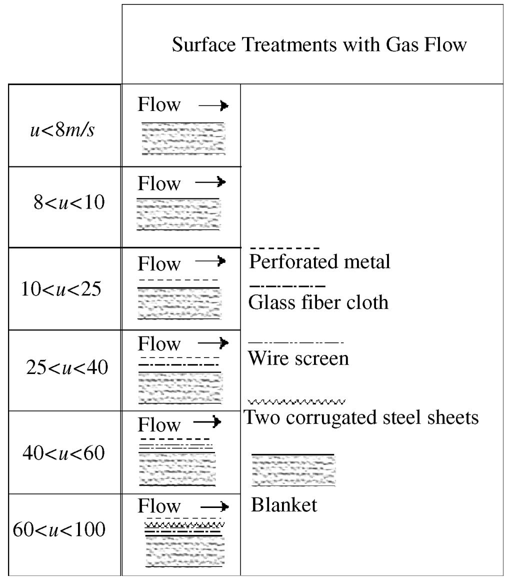

FIGURE 17.17 Protective surface for absorbing material subjected to high-velocity gas flow.

17.7.3 Gas Temperature

In many noise-control problems, temperature is a very important factor. Sometimes high-temperature ducts that are radiating noise, for example, in diesel engines, and induced draft fans, must be wrapped. With a proper choice, it is possible to combine thermal and acoustical insulations using one single material. Under extremely high temperatures, the tensile strength of materials tends to decrease, and the material may be subjected to thermal shock.

Examples of absorbing materials that are currently available for use where temperature is an important consideration are given in Table 17.6.

17.7.4 Dust and Water Exposure

The holes of perforated metal can be blocked if a dust treatment is not carried out, and the sound absorption performance will deteriorate with adhesion to the surface of the absorbing material. Methods of dust accumulation and removal may be designed into cavity type mufflers used on the sound absorption equipment.

A fan of a cooling tower, for example, experiences a considerable amount of moisture. Precautions must be taken so that water droplets are not deposited on the sound absorbing material. The underside of the equipment should be treated with rust prevention material. Figure 17.18 shows the degradation

TABLE 17.6 Fibrous Materials of Use in Hot Gas Flows

Materials	Maximum Allowable Temperature (°C)
Glass fibers with binder	320 ~ 360
Glass wool	960 ~ 1060
Mineral wool felts	1160
Mineral wool	1660
Asbestos fibers	760
Alumina-silica	1900

of the acoustic characteristic of absorbing material due to moisture, using the normal incidence absorption coefficient [13].

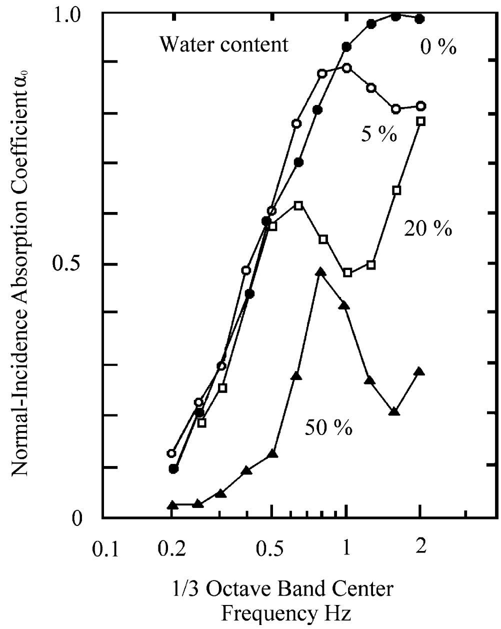

FIGURE 17.18 Degradation of the absorption coefficient by water content.

17.8 Practical Example of Dissipative Muffler

An example is given on the design of a dissipative muffler for noise reduction in an axial-flow fan for a ventilation system.

1. Specification of the axial-flow fan
   - Volume flow rate: $Q = 125\ m^3/min$
   - Wind pressure: $p = 80$ mm Aq
   - Rotor blade number: $Z = 10$
   - Stationary blade number: $Z_s = 5$
   - Rotational speed: $N = 2580$ rpm
   - Shaft horsepower: $P = 3.75$ kW
2. The desired values of attenuation and head loss with the muffler installation follow. The noise of the fan propagates both intake and discharge sides. A performance level (noise reduction) of about 37.5 dB is required, when specific sound level K_s is obtained on the basis of the axial-flow fan specification given by

$$K_s = L_A - 10 \log_{10}(p^2 Q) \tag{17.31}$$

3. Figure 17.19 gives the noise spectrum for the axial-flow fan. The blade passing frequency, BPF, is a fundamental component of the velocity fluctuation as the flow passes the blades. It is seen in the noise spectrum in Figure 17.19 at 430 Hz ($Q \times Z = (2580/60) \times 10$). By adding the background noise spectrum to this spectrum, it is seen that a muffler that provides an attenuation over 20 dB near 430 Hz, and about 15 dB in the frequency range of 800 to 1000 Hz is necessary.
   The head loss value of the muffler is to be maintained within 4 mm Aq.

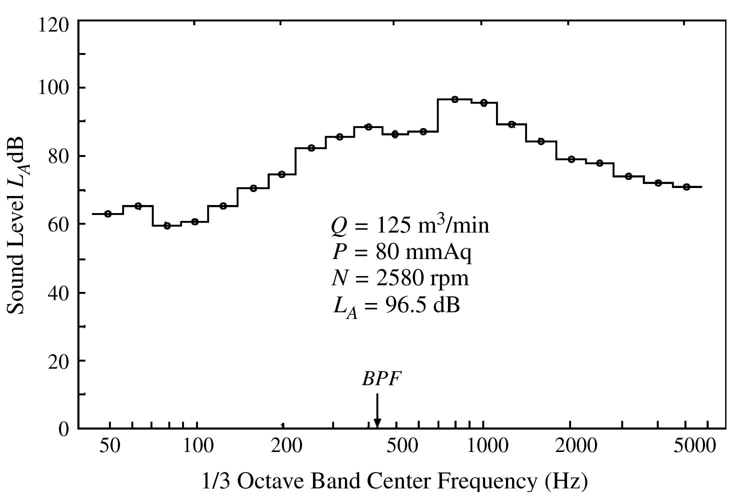

FIGURE 17.19 Noise spectrum of the axial flow fan of a factory ventilation system.

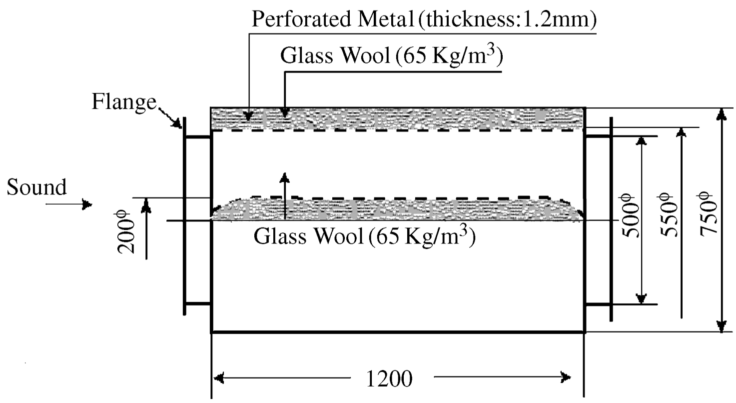

FIGURE 17.20 Half cross-sectional view of dissipative muffler for the axial flow fan.

4. The structure of the dissipative muffler is shown in Figure 17.20. The maximum value of outer diameter of the muffler is 750 mm, and the length chosen to optimize the performance.
5. The packing density of glass wool is chosen as 65 kg/m^3. The surface treatment of glass wool uses perforated metal with 1 mm thickness, 36% open area with 6 mm hole diameter. A sound absorption body of 200 mm diameter is supported in the center part, and it is welded to the outside cylinder by three props in the flow direction, and two in the circumferential direction.
6. The attenuation characteristics of the dissipative muffler may be calculated using Equation 17.27

$$\mathrm{TL} = 10\log_{10}\left[1 + \left\{\frac{1}{2}\alpha_g m k l_e\right\}^2\right]$$

The proportion of the volume of glass wool filled into the muffler is approximately $(0.75^2 - 0.55^2 + 0.2^2)/0.75^2 = 0.53$. For a packing density of 100 kg/m^3, we have $0.53 \times 65/100 = 0.347$. The value of $\alpha_g \approx 0.6$ is obtained from Table 17.2.

The required expansion ratio, m, length, l_e, and wave number, k, are given by

$$m = (750/500)^2 = 2.25, \quad l_e = 1.2\ \mathrm{m}, \quad k = 2\pi f/c$$

The speed of sound c depends on the environmental temperature. For a temperature of 25°C, we get 346.5 m/sec (= 331.5 + 0.6 × 25). The TL values at 100 to 1000 Hz are calculated. We have

$$f = 100\ \mathrm{Hz};\quad \mathrm{TL} = 5.0\ \mathrm{dB}$$

$$f = 430\ \mathrm{Hz};\quad \mathrm{TL} = 16.1\ \mathrm{dB}$$

$$f = 1000\ \mathrm{Hz};\quad \mathrm{TL} = 23.4\ \mathrm{dB}$$

The flow velocity satisfies desired value of head loss p_{loss} (in mm Aq), and is calculated by following empirical equation [12]:

$$p_{\mathrm{loss}} = \left\{0.142 m_f^{-0.1}\left(\frac{l_e}{d_1}\right)^{3/4}\left(\frac{d_m}{d_1}\right)^{-1/3}\right\}\frac{u^2}{g} \tag{17.32}$$

Use the numerical values as follows:

- $m_f = (550/500)^2 = 1.21$, ratio of cross-sectional area between air passage and muffler.
- $d_1 = 500$ mm, diameter of inlet.
- $l_e = 1.2$ m, length of muffler.
- $d_m = 200$ mm, diameter of absorption body.
- $u = 10$ m/sec or less, flow velocity at inlet.
- $g = 9.8$ m/sec^2, acceleration of gravity force.

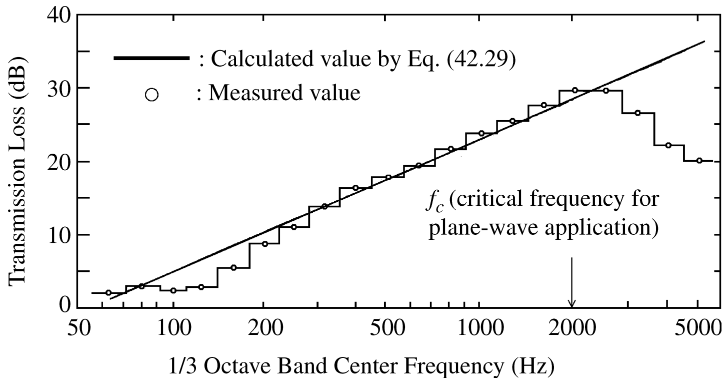

FIGURE 17.21 Noise-reduction characteristics of the designed dissipative muffler.

The corresponding head loss is 3.25 mm Aq, which corresponds to JIS B 833, and nearly agrees with the predicted value. Specifically, the condition of 4 mm Aq or less of the designed value is satisfied. The connection of axial-flow fan and the muffler uses vibration isolation, using the thick synthetic rubber.

7. The result of the attenuation realized from the spectrum after the muffler installation is shown in Figure 17.21. It is proven that the attenuation characteristics almost parallel the designed value. The frequency range where the approximation is valid is given by

$$f < c/d = 346.5/0.175 = 1980 \text{ Hz}$$

For frequencies below 250 Hz, the estimated result of the attenuation becomes slightly overestimated.

References

1. Obata, T., Hirata, M., Nishiwaki, N., Ohnaka, I., and Kato, K., Noise reduction characteristics of dissipative mufflers, 1st report, acoustical characteristics of fibrous materials, *Trans. Japan Soc. Mech. Eng.*, 42, 363, 3500, 1976.
2. Zwikker, C. and Kosten, C.W. 1949. *Sound Absorbing Materials*, Elsevier, New York.
3. Beranek, L.L., Acoustical properties of homogeneous isotropic rigid tiles and flexible blankets, *J. Acoust. Soc. Am.*, 19, 4, 556, 1947.
4. Obata, T. and Hirata, M., Estimation of acoustical transmission loss for combined walls, *Proc. Japan Soc. Mech. Eng. Annu. Meet.*, 780, 1, 42, 1978.
5. Brüel, P.V. 1951. *Sound Insulation and Room Acoustics*, Chapman & Hall, London, p. 159.
6. Beranek, L.L. 1971. *Noise and Vibration Control*, McGraw-Hill, New York, chap. 17, p. 390.
7. King, A.J., Attenuation of lined ducts, *J. Acoust. Soc. Am.*, 30, 6, 505, 1958.
8. Davis, D.D. Jr. and Stokes, G.M., 1954. *Natl. Advisory Comm. Aeronaut. Ann. Rept.*, 1192.
9. Davis, D.D. Jr. 1957. *Acoustical Filters and Mufflers. Handbook of Noise Control*, C.M. Harris, Ed., McGraw-Hill, New York, chap. 21.
10. Hagi, S., Studies on Silencer for Ventilating System, A Doctoral Thesis of University of Tokyo, 1961.
11. Wells, R.J., Acoustical plenum chambers, *Noise Control*, 4, 4, 9, 1958.
12. Obata, T. and Hirata, M., Estimation of acoustic power of flow-generated noise within silencer and head losses, *J. Acoust. Soc. Japan*, 34, 9, 532, 1978.
13. Koyasu, M., Acoustical properties of fibrous materials, personal letter, RC-SC35, *Japan Soc. Mech. Eng. Div. Meet.*, 15, 1975.