18

Design of Reactive Mufflers

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Summary

This chapter concerns the design of noise suppression devices such as mufflers. In particular, reactive mufflers that are inserted into long ducts are considered in detail. Analytical and empirical equations and information that are useful in the modeling and analysis of mufflers are presented, with an indication of their application ranges and limitations. A design procedure, complete with the necessary computations, is given. Methodologies and parameters of the performance analysis of acoustic systems with mufflers are indicated. An illustrative example of a muffler for a double-acting reciprocating compressor is presented.

18.1 Introduction

In noise-reduction applications, the need for a reactive muffler usually arises when transporting gas through a duct. For sound transmission through the duct to be minimized, an acoustic suppression device must be incorporated into the duct system. For example, in internal-combustion engines, it is required to reduce the intake and exhaust noise to acceptable levels. This may be accomplished by inserting a muffler in the intake and exhaust ducting to attenuate the pressure pulsations before they reach the environment.

A successful muffler design must satisfy at least the following three criteria: (1) muffler performance as a function of frequency (the maximum permissible noise generated by the gas flow through the muffler may have to be specified as well); (2) the maximum permissible average pressure drop through the muffler at a given temperature and mass flow; (3) the maximum allowable volume and restrictions on space utilization.

The customer may ask for a muffler with unrealistically high noise attenuation, virtually no backpressure, and very small size. In addition, it is important to the customer that the muffler is inexpensive and durable, and presents no maintenance problems. Needless to say, in practice, these criteria for muffler design are unrealistic, and have to be modified to practical levels. In this chapter, we will present some of the analytical and empirical tools that are helpful in muffler design.

18.2 Fundamental Equations

18.2.1 Analytical Model

The physical behavior of a reactive muffler may be adequately modeled by linear differential equations. The law of conservation of mass must hold, while three simultaneous equations, Newton's, Boyle–Charles, and that of conservation, must be satisfied. When these equations are combined, we obtain the wave equation for the plane, one-dimensional sound-pressure wave:

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2} \tag{18.1}$$

$$p = \rho c^2 \frac{\partial \xi}{\partial x} \tag{18.2}$$

where

$\xi =$ displacement of particle motion (m)
$c =$ velocity of sound (m/s)
$p =$ sound pressure (Pa)
$\rho =$ density of air (kg/m^3)
$t =$ time (s)
$x =$ coordinate system along which wave travels (m)

The stationary solutions for angular frequency ω of Equation 18.1 and Equation 18.2 are given by

$$\xi = (A\,\mathrm{e}^{-jkx} - B\,\mathrm{e}^{jkx})\mathrm{e}^{j\omega t} \tag{18.3}$$

$$p = -\rho c^2 k(A\,\mathrm{e}^{-jkx} + B\,\mathrm{e}^{jkx})\mathrm{e}^{j\omega t} \tag{18.4}$$

where A, $B =$ amplitudes of sound pressure or particle motion for traveling and reflecting waves, $k = 2\pi f/c$, wave number, and $j = \sqrt{-1}$.

18.2.2 Boundary Conditions

The boundary conditions are given below.

(1) *Sound source*. The sound source is assumed to be independent of the existence of the muffler, and the volume rate of the particles is assumed constant, as given by

$$S\dot{\xi} = \text{const} \tag{18.5}$$

in which $\cdot$ denotes the time derivative.

(2) *Open end of duct*. The reflection coefficient, R, at the open end of an unflanged circular pipe is available, and is given by

$$R = \frac{B\,\mathrm{e}^{jkx}}{A\,\mathrm{e}^{-jkx}} \tag{18.6}$$

The magnitude of the reflection coefficient, $|R|$, is shown in Figure 18.1a as a function of ka, where a is the pipe radius. The phase shift can be determined from Figure 18.1b, which is a plot of α/a as a function of ka. Also, the reflection coefficient is [1]:

$$R = -|R|\mathrm{e}^{-2jk\alpha} \tag{18.7}$$

For the small values of ka that are most often encountered in reactive muffler design, $|R| \approx 1$ and $\alpha/a = 0.613$.

(3) *Closed end*. The displacement of particle motion is zero at a rigid wall. Hence, we have

$$\xi = 0 \tag{18.8}$$

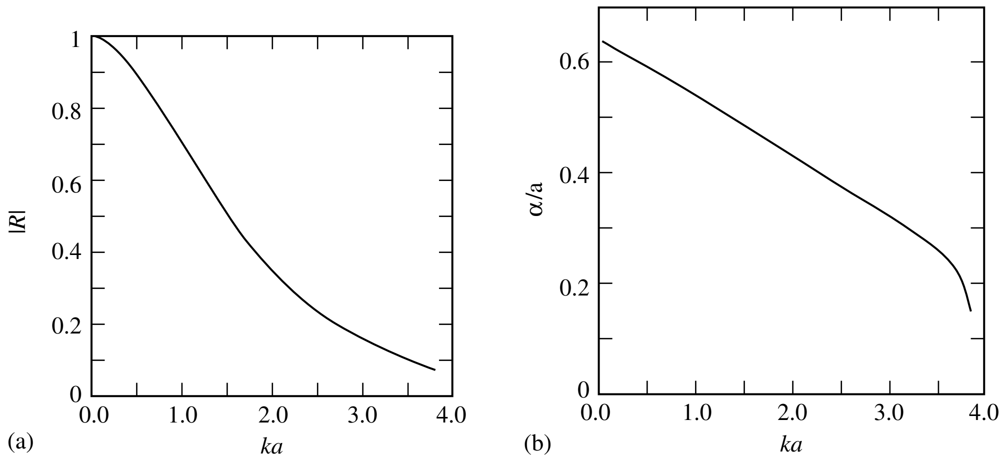

FIGURE 18.1 (a) Magnitude of the reflection coefficient at open end of an unflanged circular pipe; (b) End correction for an unflanged circular pipe. (*Source:* Levine, H. and Schwinger, J., On the radiation of sound from an unflanged circular pipe, *J. Phys. Rev.*, 73, 383, 1948. With permission.)

(4) *Junction conditions*. The following equations correspond to the continuity of volume flow rate of the particles and the continuity of pressure, even if the cross section changes suddenly:

$$S_i \dot{\xi}_i = S_{i+1} \dot{\xi}_{i+1} \tag{18.9}$$

$$p_i = p_{i+1} \tag{18.10}$$

18.3 Effects of Reactive Mufflers

The acoustic behavior of a reactive muffler may be expressed in term of the insertion loss, the difference in the noise levels measured at some external point with and without the muffler in the system. The transmission loss is defined as the insertion loss for a nonreflecting source and the end of exhaust duct.

18.3.1 Insertion Loss

A single expansion-type muffler installation is shown schematically in Figure 18.2. At the open end of a pipe, as in Figure 18.2, the traveled and reflected waves of the source become $A_0\,e^{-jkl}$, $B_0\,e^{jkl}$, over a length l, where the amplitudes are denoted by A_0, B_0. The reflective coefficient for length l_0 is given by

$$R_0 = \frac{B_0\,e^{jkl_0}}{A_0\,e^{-jkl_0}} \tag{18.11}$$

This is obtained from Equation 18.6 with $x = l_0$.

The energy, W_0, of the acoustic wave escaping from the open end of the pipe is given by

$$W_0 \propto \frac{S_0 A_0^2 (1 - R_0^2)}{\rho_0 c_0} \tag{18.12}$$

in which ρ_0 = density of air, and c_0 = speed of sound in air. The equation of the sound-pressure level measured at an open point at some distance is given by

$$p_0 = 10\log_{10}\left(\frac{Q_d}{4\pi r_0^2} + \frac{4}{R_r}\right) + PWL_{r_0} \tag{18.13}$$

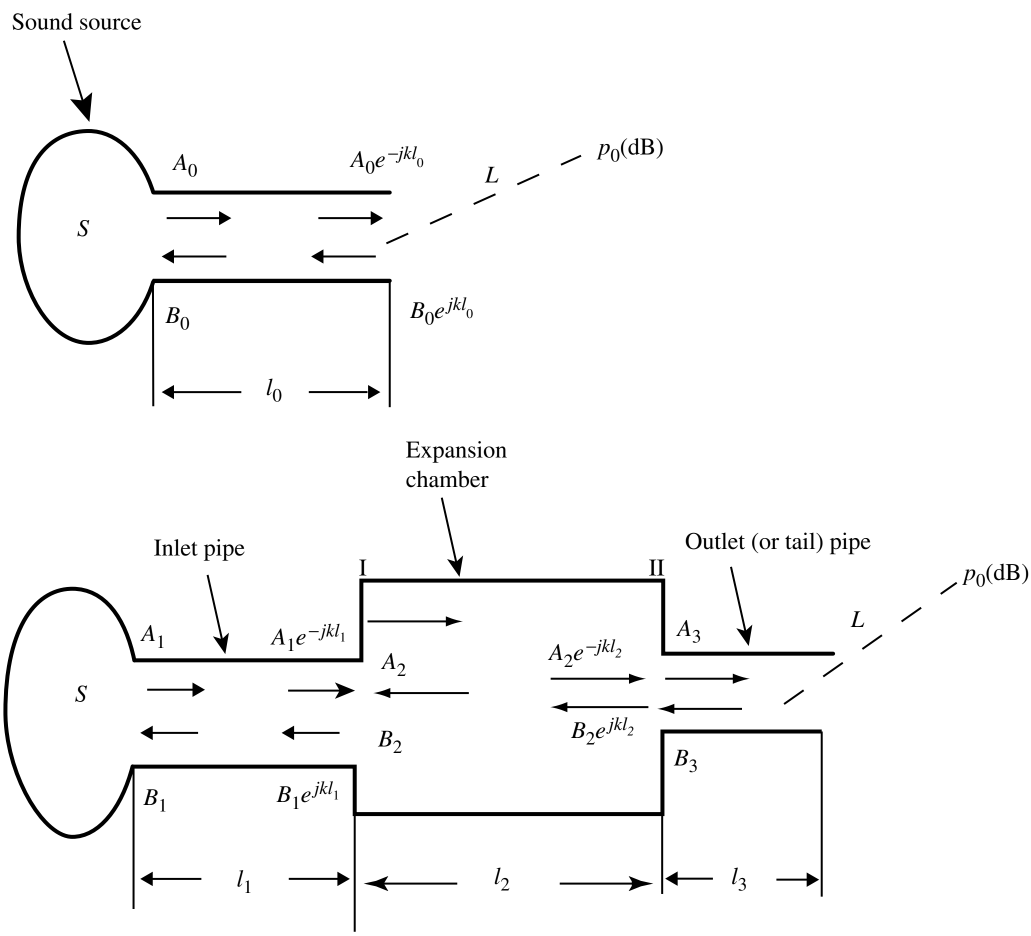

FIGURE 18.2 Measurement of insertion loss.

where

$r_{0,}$ = distance
$R_r = A/(1-\alpha)$, room constant
A = the indoor sound absorbing power (indoor surface area times indoor average absorption coefficient)
α = the indoor average absorption coefficient
Q_d = the directivity factor from the open end

Therefore, the measured value of insertion loss can be obtained from Equation 18.14, when Q_d values are equal. Power level is defined as

$$\mathrm{PWL} = 10\log_{10}\left(\frac{W}{10^{-12}}\right)$$

Now,

$$\mathrm{IL} = \mathrm{PWL}_{r_0} - \mathrm{PWL}_{r} \tag{18.14}$$

Using Equation 18.14, it can be shown that IL can be expressed by

$$\mathrm{IL} = 10\log_{10}\left|\frac{S_0}{S_3}\left|\frac{A_0}{A_3}\right|^2\right|\left|\frac{1-R_0^2}{1-R_3^2}\right| \tag{18.15}$$

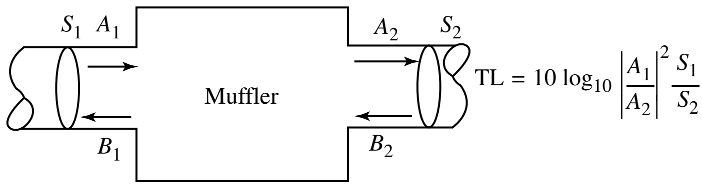

FIGURE 18.3 Definition of transmission loss for a muffler.

3.2 Transmission Loss

It is desirable to eliminate the source and radiation characteristics from the system in Figure 18.3, and to look only at some property of the muffler itself. This may be accomplished by defining a quantity called "transmission loss" (TL) as follows:

$$\mathrm{TL} = 10\log_{10}\left|\frac{A_1}{A_2}\right|^2\frac{S_1}{S_2} \tag{18.16}$$

In the measurement of TL, it is difficult to separate the reflected wave. The theoretical calculation is easy and useful.

18.4 Calculation Procedure

For the reactive muffler shown in Figure 18.2, the following equations are obtained from Equation 18.5 to Equation 18.7 and Equation 18.9. The inlet pipe, cavity, and tail pipe are denoted by suffix in the figure [2].

(1) The sound source:

$$S_0(A_0 - B_0) = S_1(A_1 - B_1) \tag{18.17}$$

(2) The open end of pipes:

$$\left.\begin{aligned} B_0\,\mathrm{e}^{\mathrm{j}kl_0} &= A_0\,\mathrm{e}^{-\mathrm{j}kl_0}R_0 \\ B_3\,\mathrm{e}^{\mathrm{j}kl_3} &= A_3\,\mathrm{e}^{-\mathrm{j}kl_3}R_3 \end{aligned}\right\} \tag{18.18}$$

(3) The sudden expansion (junction I):

$$\left.\begin{aligned} A_1\,\mathrm{e}^{-\mathrm{j}kl_1} + B_1\,\mathrm{e}^{\mathrm{j}kl_1} &= A_2 + B_2 \\ S_1(A_1\,\mathrm{e}^{-\mathrm{j}kl_1} - B_1\,\mathrm{e}^{\mathrm{j}kl_1}) &= S_2(A_2 - B_2) \end{aligned}\right\} \tag{18.19}$$

(4) The sudden contraction (junction II):

$$\left.\begin{aligned} A_2\,\mathrm{e}^{-\mathrm{j}kl_2} + B_2\,\mathrm{e}^{\mathrm{j}kl_2} &= A_3 + B_3 \\ S_2(A_2\,\mathrm{e}^{-\mathrm{j}kl_2} - B_2\,\mathrm{e}^{\mathrm{j}kl_2}) &= S_3(A_3 - B_3) \end{aligned}\right\} \tag{18.20}$$

From these linear equations, the following equation can be obtained:

$$\begin{aligned} \frac{A_0}{B_0} = \frac{m_{10}}{1 - R_0\,\mathrm{e}^{-2\mathrm{j}kl_0}} \Big\{ &\mathrm{j}\left(1 + R_3\,\mathrm{e}^{-2\mathrm{j}kl_3}\right)(\sin kl_1 \cos kl_2 + m_{21}kl_1 \sin kl_2) \\ &+ m_{32}\left(1 - R_3\,\mathrm{e}^{-2\mathrm{j}kl_3}\right)(m_{21}\cos kl_1 \cos kl_2 - \sin kl_1 \sin kl_2)\Big\} \end{aligned} \tag{18.21}$$

When the reflection coefficients are absent, $R_0 = R_3 = 0$, we have

$$IL = 10 \log_{10} m_{03} \left\{ m_{10}^2 (\sin kl_1 \cos kl_2 + m_{21} \cos kl_1 \sin kl_2)^2 + m_{10}^2 m_{32}^2 \times (m_{21} \cos kl_1 \cos kl_2 - \sin kl_1 \sin kl_2)^2 \right\} \quad (18.22)$$

where $m_{10} = S_0/S_1$, $m_{21} = S_2/S_1$, $m_{32} = S_3/S_2$, $m_{03} = S_0/S_3$, which are ratios of cross-sectional areas of the pipes.

For the reflection factors, $R_0 = R_3 = -1$, $S_0 = S_1 = S_3$, and $m_{21} = m$, the equation of insertion loss becomes

$$\mathrm{IL} = 20 \log_{10} \left\| \left[\frac{\cos kl_1}{\cos kl_0} \left\{ \cos kl_{21} \cos kl_3 - m \sin kl_2 \sin kl_3 - \tan kl_1 \left(\cos kl_2 \sin kl_3 + \frac{1}{m} \sin kl_2 \cos kl_3 \right) \right\} \right] \right\| \quad (18.23)$$

The ratio $|A_1/A_3|$ may also be obtained from Equation 18.17 to Equation 18.20. When the magnitude of reflection coefficients $R_0 = R_3 = 0$, $S_0 = S_1 = S_3$, and expansion ratio of the cross-sectional open area of the pipes $m_{21} = m$, the transmission loss is given by

$$\mathrm{TL} = 10 \log_{10} \left\{ 1 + \frac{1}{4} \left(m - \frac{1}{m} \right)^2 \sin^2 kl_2 \right\} \quad (18.24)$$

When $R_0 = R_3 = -1$, $S_0 = S_1 = S_3$, and $m_{21} = m$, TL is obtained from the following equation:

$$\mathrm{TL} = 10 \log_{10} \left| 1 + \frac{1}{m^2} (m^2 - 1) \{ (m^2 + 1) \sin^2 kl_3 - 1 \} \sin^2 kl_2 + \frac{1}{2} \left(-m + \frac{1}{m} \right) \sin 2kl_2 \sin 2kl_3 \right| \quad (18.25)$$

Computation formulas of insertion loss and transmission loss for the case of an expansion chamber with insertion pipe and resonator are shown in Table 18.1.

The principal structures of several reactive mufflers are shown in Figure 18.4.

18.5 Application Range of Model

18.5.1 Condition for Approximation of Plane Wave

The frequency range where the approximation of a plane wave is valid is given by

$$f_c < 1.22 \frac{c}{D} \quad (18.26)$$

where

f_c = critical frequency of plane wave (Hz)
c = speed of sound (m/s)
D = diameter of muffler (m)

It is seen that the expansion ratio of an open area of a pipe increases with IL or TL. However, the application range of the analytical model decreases with increasing diameter of chamber.

18.5.2 Effect of Temperature

Under conditions of high-temperature and high-speed gas flow, as in an engine exhaust system, the primary effect of a change in pipe temperature is the corresponding change in the speed of sound, which is proportional to the square root of the absolute temperature. In the design of a reactive muffler, it is necessary to use the actual speed of sound in the gas inside the pipe. The most accurate values available for density (ρ_0) and the speed of sound (c_0) at each element should be used in calculating the impedance

TABLE 18.1 Transmission Loss of Reactive Mufflers and Insertion Loss of Reactive Mufflers

Muffler (see Figure 18.4)	TL (dB)	Application Limits and Comments
Transmission loss of reactive mufflers		
(a)	$\mathrm{TL} = 10\log_{10}\left\{1 + \left(\frac{kS}{4a}\right)^2\right\}$ a: radius of orifice	$a/\lambda < 0.1$, $R = 0$
(b)	$\mathrm{TL} = 20\log_{10}\left\lvert \begin{matrix} (1 + R_3\,\mathrm{e}^{-2jkl3})(\cos kl_2 + \mathrm{j}m_{21}\sin kl_2) \\ + m_{32}(1 - R_3\,\mathrm{e}^{-2jkl3})(\mathrm{j}\sin kl_2 + m_{21}\cos kl_2) \end{matrix} \right\rvert$ $m_{10} = S_1/S_0$, $m_{21} = S_2/S_1$, $m_{32} = S_3/S_2$, R_0, R_3: the reflection coefficient of the open end, l_3 = length of the tail pipe (1) when $R_0 = R_3 = 0$, $S_0 = S_1 = S_3$ $\mathrm{TL} = 10\log_{10}\left\{1 + \frac{1}{4}\left(m_{21} - \frac{1}{m_{21}}\right)^2 \sin kl_2\right\}$ (2) when $R_0 = R_3 = -1$, $S_0 = S_1 = S_3$ $\mathrm{TL} = 10\log_{10}\left[1 + \left(\frac{m_{21}^2 - 1}{m_{21}^2}\right)\{(m_{21}^2 + 1)\sin^2 kl_3 - 1\}\sin^2 kl_2 + \frac{1}{2}\left(\frac{1}{m_{21}} - m_{21}\right)\sin 2kl_2 \sin 2kl_3\right]$	$f < 1.22c/D$
(c)	$\mathrm{TL} = 10\log_{10}\left[\left\{2\cos k(l_1 - l_{11} - l_{22}) - \frac{m-1}{m}\sin k(l_1 - l_{11} - l_{22})(\tan kl_{11} + \tan kl_{22})\right\}^2 + \left\{\left(m + \frac{1}{m}\right)\sin k(l_1 - l_{11} - l_{22}) + (m-1)\cos(l_1 - l_{11} - l_{22})(\tan kl_{11} + \tan kl_{22}) - \frac{(m-1)^2}{m}\tan kl_{11}\tan kl_{22}\sin k(l_1 - l_{11} - l_{22})\right\}^2\right]$ $M = S_1/S_0$	$R \approx 0$
(d)	$\mathrm{TL} = 10\log_{10}\left\{1 + \frac{1}{4}\left(\frac{m}{\frac{kS_2}{C_0} - \cot kl}\right)^2\right\}$ $C_0 = NC_i$; N: number of holes, $C_i = 2\pi a_i^2/(l_b + \pi a_{i)})$, l_b, l_b: thickness of the pipe, a_i: radius of a hole, $m = S_{12}/S$	$R \approx 0$

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TABLE 18.1 (*continued*)

Muffler (see Figure 18.4)	TL (dB)	Application Limits and Comments
(e)	$\mathrm{TL} = 10\log_{10}\left[1 + \frac{1}{4}\left\lvert \frac{\frac{\sqrt{C_0 V}}{S}}{\frac{f}{f_r} - \frac{f_r}{f}} \right\rvert^2\right]$ $f_r = \frac{c}{2\pi}\sqrt{\frac{C_0}{V}},$ $C_0 = \frac{2\pi a^2}{2l_b + \pi a}$ l_b: length of the neck or thickness of the pipe, a: radius of the neck or hole	$R \approx 0$, $l_b \ll \lambda$ Resonator size $\ll \lambda$
(f)	$\mathrm{TL} = 10\log_{10}\left\{1 + \frac{m^2}{4}\left(\frac{\tan kl_b - \frac{S_b}{kV}}{\frac{S_b}{kV}\tan kl_b + 1}\right)\right\}$ $m = S_b/S$	$R \approx 0$, $l_b \ll \lambda$ Resonator size $\ll \lambda$
(g)	$\mathrm{TL} = 20\log_{10}\frac{1}{16m^2}\Big\lvert[4m(m+1)^2\cos 2k(l+l_c) - 4m(m-1)^2\cos 2k(l-l_c)]$ $+ \mathrm{j}\Big\{2(m^2+1)(m+1)^2\sin 2k(l+l_c) - 2(m^2+1)(m-1)^2\sin 2k(l-l_c)$ $-4(m^2-1)^2\sin 2kl_c\Big\}\Big\rvert$ $m = S_2/S_1$	$R \approx 0$
(h)	$\mathrm{TL} = 10\log_{10}\Big\lvert\{\cos 2kl - (m-1)\sin 2kl_c \tan kl_c\}^2 + \Big\{\frac{\mathrm{j}}{2}\left(m + \frac{1}{m}\right)\sin 2kl$ $+(m-1)\tan kl_c\left(\left(m + \frac{1}{m}\right)\cos 2kl - \left(m - \frac{1}{m}\right)\right)\Big\}^2\Big\rvert$ $m = S_2/S_1$	$R \approx 0$

(i)

$$\mathrm{TL} = 10\log_{10}\left\{\frac{1}{4}\left|\frac{A_1 + \mathrm{j}B_1}{A_2 + \mathrm{j}B_2}\right|^2\right\}$$

$R \approx 0$ Resonator size $\ll \lambda$

$$A_1 = Y_3X_1^2 + Z_0Y_3^2 + Z_0(X_1 + X_3)^2$$

$$B_1 = X_1Y_3^2 + X_1X_3(X_1 + X_3)$$

$$A_2 = Y_3X_1^2 \cos kl + Z_0X_1Y_3 \sin kl$$

$$B_2 = X_1Y_3^2 + X_1X_3(X_1 + X_3)\cos kl - Z_0X_1(X_1 + X_3)\sin kl$$

$$X_1 = \frac{\omega\rho}{C_0} - \frac{\rho c^2}{\omega V_1},$$

$$X_2 = \frac{\omega\rho}{C_0} - \frac{\rho c^2}{\omega V_2}$$

$$X_3 = \frac{Z_0^2\left(X_2 \cos 2kl + \frac{1}{2}Z_0 \sin^2 kl\right)}{(X_2 \sin kl - Z_0 \cos kl)^2 + X_2^2 \cos^2 kl}$$

$$Y_3 = \frac{Z_0X_2^2}{(X_2 \sin kl - Z_0 \cos kl)^2 + X_2^2 \cos^2 kl}$$

$$Z_0 = \frac{\rho c}{S_0},$$

$$C_0 = \frac{2\pi a^2}{2l_b + \pi a}$$

l_b: thickness of the pipe, a: radius of hole

(j)

$$\mathrm{TL} = 10\log_{10}\left\{\left(\cos kl + \frac{\rho c}{4S_0X}\left(m + \frac{1}{m}\right)\sin kl - \frac{\rho c}{4S_0X}\left(m - \frac{1}{m}\right)\cos 2kl_b \sin kl\right)^2 + \left(\frac{1}{2}\left(m + \frac{1}{m}\right)\sin kl + \frac{\rho c}{4S_0X}\left(m - \frac{1}{m}\right)\sin 2kl_b \sin kl - \frac{\rho c}{2S_0X}\cos kl\right)^2\right\}$$

$R \approx 0$

$$X = \frac{\omega\rho}{C_0} - \frac{\rho c^2}{\omega V},$$

$$m = \frac{S}{S_0}$$

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TABLE 18.1 *(continued)*

Muffler (see Figure 18.4)	IL (dB)	Application Limits and Comments
Insertion loss of reactive mufflers		
(b)	$\mathrm{IL} = 10\log_{10}\dfrac{1}{m_{30}} \times \left\lvert \dfrac{m_{10}}{1 - R_0\,\mathrm{e}^{-2jkl_0}}\{\mathrm{j}(1 + R_3\,\mathrm{e}^{-2jkl_3}(\sin kl_1 \cos kl_2 + m_{21}\cos kl_1 \sin kl_2). + m_{32}(1 - R_3\,\mathrm{e}^{-2jkl_3})(m_{21}\cos kl_1 \cos kl_2 - \sin kl_1 \sin kl_2)\} \right\rvert^2$	$f < 1.22c/D$, R is plotted in Fig.18.1
	(1) $\mathrm{IL} = 10\log_{10}\left\lvert \{1 + (m_{21}^2 - 1)\}\left(1 - \dfrac{m_{21}^2 + 1}{m_{21}^2}\sin^2 kl_1\right)\sin^2 kl_2 + \dfrac{1}{2}\left(m_{21} - \dfrac{1}{m_{21}}\right)\sin 2kl_1 \sin 2kl_2 \right\rvert$	$R_0 = R_3 = 0$, $S_0 = S_1 = S_3$
	(2) $\mathrm{IL} = 10\log_{10}\left\lvert \left(\dfrac{\cos kl_1}{\cos kl_0}\{\cos kl_2 \cos kl_3 - m_{21}\sin kl_2 \sin kl_3 - \tan kl_1(\cos kl_2 \sin kl_3 + \dfrac{1}{m_{21}}\sin kl_2 \cos kl_3)\}\right)\right\rvert^2$	$R_0 = R_3 = -1$, $S_0 = S_1 = S_3$
(c)	$\mathrm{IL} = 20\log_{10}\left\lvert \dfrac{\cos kl_1 \cos kl_2 - m \sin kl_1 \sin kl_2}{\cos kl_{11} \cos kl_{22}} \right\rvert$ $m = S_1/S_0$	$R = -1$
(d)	$\mathrm{IL} = 20\log_{10}\left[\cos^2 kl_2 + \dfrac{m}{\dfrac{kS_2}{C_0} - \cot kl}\sin 2kl_2 + \left(\dfrac{m}{\dfrac{kS_2}{C_0} - \cot kl}\right)^2 \sin^2 kl_2\right]$ $C_0 = NC_i$; $C_i = 2\pi a_i^2/(l_b + \pi a_{i)}$, N: number of holes, l_b: thickness of the pipe, a_i: radius of a hole, $m = S_{12}/S$	$R = -1$, $kl_0 << 1$

(e)

$$\mathrm{IL} = 10\log_{10}\left|\frac{\dfrac{\sqrt{C_0 V}}{S}}{\dfrac{f}{f_r}-\dfrac{f_r}{f}}\sin 2kl_2 + \frac{\dfrac{C_0 V}{S^2}}{\left(\dfrac{f}{f_r}-\dfrac{f_r}{f}\right)^2}\sin^2 kl_2 + \cos^2 kl_2\right|$$

$R = -1,\ kl_0 << 1$

$$f_r = \frac{c}{2\pi}\sqrt{\frac{C_0}{V}},$$

$$C_0 = \frac{2\pi a^2}{2l_b + \pi a}$$

l_b: length of the neck or thickness of pipe, a: radius of the neck or hole

(f)

$$\mathrm{IL} = 10\log_{10}\left[\cos^2 kl_2 + m\sin 2kl_2 \frac{\dfrac{S_b}{kV}\tan kl_b + 1}{\tan kl_b - \dfrac{S_b}{kV}} + \left(m\frac{\dfrac{S_b}{kV}\tan kl_b + 1}{\tan kl_b - \dfrac{S_b}{kV}}\right)^2 \sin^2 kl_2\right]$$

$R = -1,\ kl_0 << 1$

$$f_r = \frac{c}{2\pi}\sqrt{\frac{C_0}{V}},$$

$$C_0 = \frac{2\pi a^2}{2l_b + \pi a}$$

l_b: length of the neck or thickness of the pipe, a: radius of the neck or hole

(k)

$$\mathrm{IL} = 20\log_{10}\left|(\cos kl_1 \cos kl_{11} - m_1 \sin kl_1 \sin kl_{11}) + (\cos kl_2 \cos kl_{22} - m_2 \sin kl_2 \sin kl_{22}) + \cdots + (\cos kl_i \cos kl_{ii} - m_i \sin kl_i \sin kl_{ii})\right|$$

$R = -1,\ kl_0 << 1$

(l)

$$\mathrm{IL} = 20\log_{10}\left|\left\{\frac{\cos kL_1 \cos kl_1 - m\sin kL_1 \sin kl_1}{\cos kl_{11}} + \frac{\cos kL_2 \cos kl_2 - m\sin kL_2 \sin kl_2}{\cos kl_{12}\cos kl_{21}} + \cdots + \frac{\cos kL_n \cos kl_n - m\sin kL_n \sin kl_n}{\cos kl_{(n-1)2}}\right\}\right|$$

$R = -1,\ kl_0 << 1$

A is the radius of tube in orifice hole or diameter of side branch, c is the sound speed, C_0 is the conductivity, D is the diameter of chamber, f is the frequency, f_r is the resonant frequency of the resonator, $k = 2\pi f/c$ is the wave number, L is the length, $m = S_i/S_{i+1}$ is the ratio of the cross section, IL is the insertion loss, N is the number of holes, R is the reflection coefficient, S is the cross section, TL is the transmission loss, V is the volume of chamber, Z is the acoustic impedance, ρ is the density, λ is the wavelength, $\omega = 2\pi f$, angular frequency.

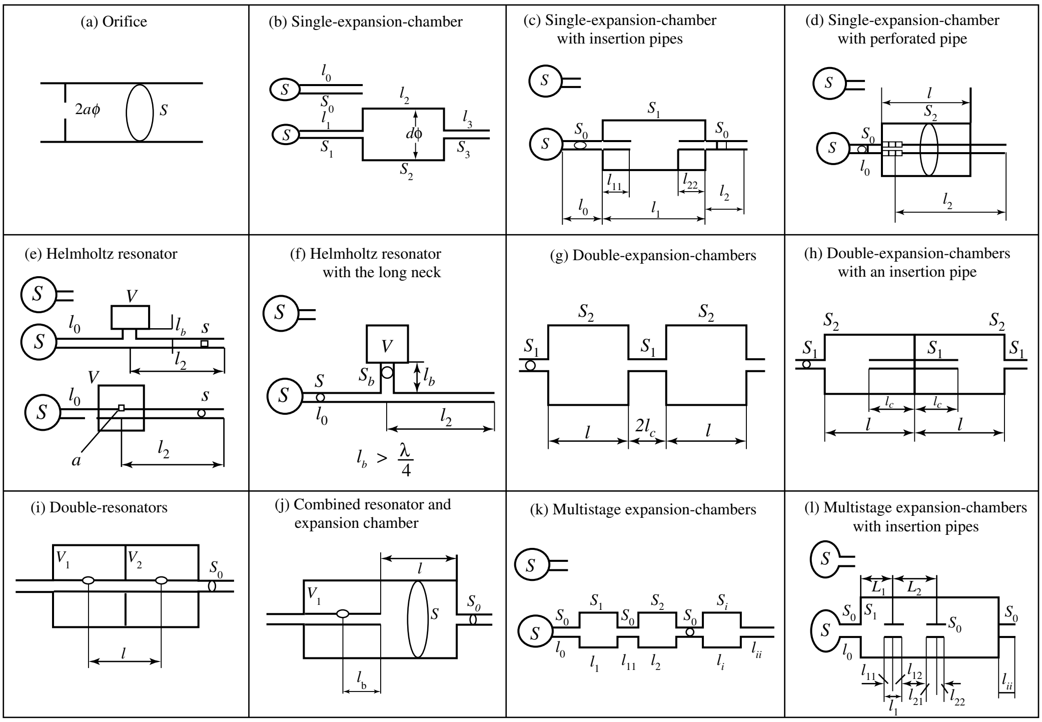

FIGURE 18.4 Sketches of the 12 principal structures of reactive mufflers.

of the elements. The impedance, z, is given by

$$z = -\mathrm{j}\frac{\rho_0 c_0}{S}\frac{1}{kl} \tag{18.27}$$

where

$S =$ the cross-sectional open area of pipe
$k = 2\pi f/c_0$, wave number
$l =$ the length of the pipe element

Note that the impedance of the resonator chamber is proportional to $\rho_0 c_0^2$. However, c_0^2 is proportional to the absolute temperature of gas (T) and ρ_0 is proportional to $1/T$. Hence, the chamber impedance is independent of temperature. The connector impedance is a function of T, but in most cases the connector will be at the pipe temperature. For a resonator-type muffler, a temperature difference between the pipe and chamber is expected to have little effect on the performance of the muffler.

18.5.3 Effect of Gas Flow in Pipe

Under conditions of high-temperature and high-speed gas flow in a pipe, the pressure amplitude in the pipe is large, and is larger than what is predicted by theory. Analysis by the characteristic curve method is desirable under such conditions.

In a reactive muffler where the pipe flow passes through a sudden pipe expansion or an orifice, the computed transmission loss or insertion loss tends to be an overestimate because of new noise that is generated due to the resulting irregular air-flow within the muffler.

18.5.4 Effect of Friction Loss in Pipe

When an acoustic wave propagates in a pipe, it will attenuate due to viscous friction. The effect is large for long pipes of small diameter. Friction damping in a pipe may be incorporated into the propagation constant, γ, such that

$$\gamma = \delta + jk \tag{18.28}$$

where δ is the attenuation constant per unit length of pipe. By substituting Equation 18.28 into Equation 18.3 and Equation 18.4, we obtain

$$\xi = (A\,\mathrm{e}^{-\gamma l} - B\,\mathrm{e}^{\gamma l})\mathrm{e}^{\mathrm{j}\omega t} \tag{18.29}$$

$$p = -\rho c^2 k(A\,\mathrm{e}^{-\gamma l} + B\,\mathrm{e}^{\gamma l})\mathrm{e}^{\mathrm{j}\omega t} \tag{18.30}$$

Empirical formulas are given below for two cases of the attenuation coefficient δ [3].

(1) The formula for seamless steel or chloride-ethylene pipes (regression formula when the inside roughness is 4 to 8 μm and length under 3 m) is

$$\delta = 26,100\lambda^{-0.5}\frac{\mu}{\rho c d} \tag{18.31}$$

where

λ = wavelength of sound (m)
μ = viscosity of gas in the pipe (Pa s)
ρ = density of gas (kg/m^3)
d = diameter of the pipe (m)

(2) The equations for lining with glass wool are

$$\left.\begin{aligned} \delta_2 &= 2491\lambda^{-0.476}\left(\frac{\rho c d}{\mu}\right)^{-1.068} \\ \delta_6 &= 5175\lambda^{-0.476}\left(\frac{\rho c d}{\mu}\right)^{-1.303} \\ \delta_6 &= 11596\lambda^{-0.476}\left(\frac{\rho c d}{\mu}\right)^{-1.270} \end{aligned}\right\} \tag{18.32}$$

The suffix of δ gives the thickness of absorbing material in mm.

18.6 Practical Example

18.6.1 Expansion-Type Muffler for Reciprocating Compressor

Consider a double-acting (i.e., fluid on both sides of the piston in the cylinder) reciprocating compressor for supplying high-pressure air to a machine shop of a factory, for example.

The specifications of the reciprocating compressor follow:

- Delivery pressure: 6.9×10^5 Pa
- Rotational speed of driving shaft: 600 rpm
- Power of driving shaft: 450 kW
- Diameter of inlet pipe: 380 mm

Pressure pulsations of 10 and 20 Hz are produced by the compressor due to the rotational speed, as seen in Figure 18.5. The pressure wave from the inlet propagates the free space and can the damage nearby private houses. Wooden doors with glass paneling, wooden sliding-doors, and leaves of plants and foliage, and have been found to vibrate due to low-frequency audible sound. An attenuation of 15 to 20 dB was

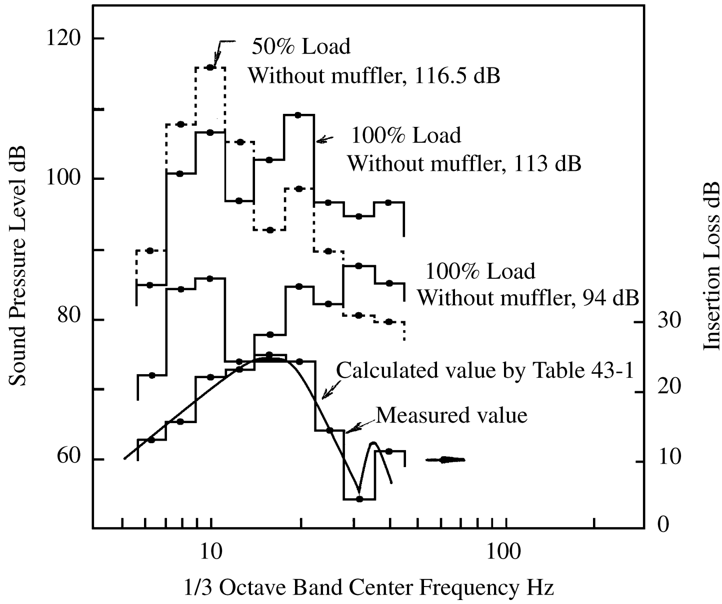

FIGURE 18.5 Noise spectrum at inlet of reciprocating compressor and insertion loss of designed muffler.

necessary at the frequencies 10 and 20 Hz. A muffler using an expansion and tail pipe type was suggested to handle the problem. The reflection coefficient of the tail pipe is approximately $R = -1$ and $ka = 2\pi fa/c = 2\pi \times 20 \times 0.19/345 = 0.0692$.

$$\text{IL} = 20\ \log_{10}|\cos kl_2 \cos kl_3 - m \sin kl_2 \sin kl_3|$$

where

$k = 2\pi f/c$; wave number (1/m)
l_2 = the length of the chamber (m)
l_3 = the length of the tail pipe (m)
m = the expansion ratio of the cross section between the chamber and inlet

With $kl_2 = kl_3 = \pi/2$, we have

$$\text{IL} = 20\ \log_{10}|m|$$

We need $m > 10$ in order to satisfy the desired value of IL. For 20 Hz, we use $kl_2 = kl_3 = \pi$. When $kl_2 = kl_3 = \pi/2$ at $f = 10$ Hz, we have IL = 0. Then, using $kl_2 = kl_3 = \pi/2$ at frequency 15 Hz, we can satisfy the IL condition of 20 dB at both frequencies. Hence, $l_2 = l_3 = 345/(4 \times 15) = 5.75$ (m) is chosen at a speed of sound $c = 345$ m/s.

The noise spectrum at the inlet of the reciprocating compressor under study and insertion loss of the muffler design in this example are shown in Figure 18.5.

The diameters or the lengths of the chamber and the tail pipe are properly selected, as shown and in Figure 18.6. At 10 Hz, IL is determined as

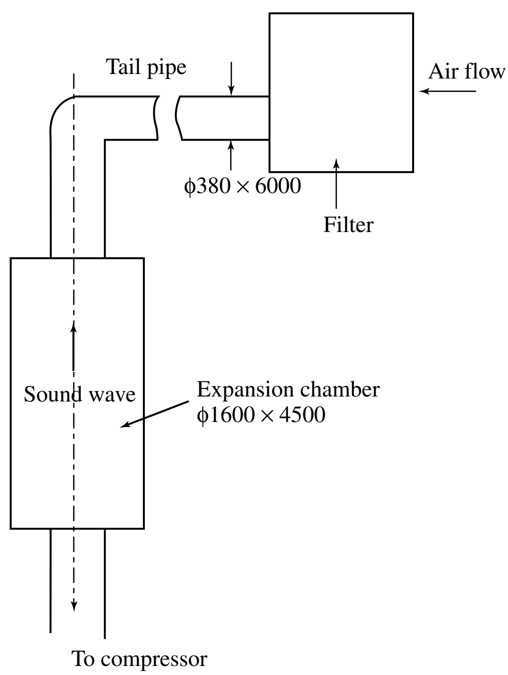

FIGURE 18.6 The muffler designed for the noise control of a reciprocating compressor.

indicated below:

$$\begin{aligned}\mathrm{IL} &= 20\log_{10}|\cos(2\pi\times10\times4.5/345)\cos(2\pi\times10\times6/345)-(1.6/0.38)^2\sin(2\pi\times10\times4.5/345)\\&\quad\times\sin(2\pi\times10\times6/345)|\\&=20\log_{10}|0.6825\times0.4600-17.728\times0.7308\times0.8879|=21.0\ (\mathrm{dB})\end{aligned}$$

Similarly, at 20 Hz, we have

$$\begin{aligned}\mathrm{IL} &= 20\log_{10}|\cos(2\pi\times20\times4.5/345)\cos(2\pi\times20\times6/345)-(1.6/0.38)^2\sin(2\pi\times20\times4.5/345)\\&\quad\times\sin(2\pi\times20\times6/345)|\\&=20\log_{10}|(-0.0682)\times(-0.5767)-17.728\times0.9977\times0.8170|=23.2\ (\mathrm{dB})\end{aligned}$$

Clearly, the attenuation at both frequencies satisfies the desired lower limit of 20 dB. Calculated values of IL at low frequencies are shown by a curved continuous line in Figure 18.5.

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2. Ohnaka, I., Lecture for noise reduction of machines, no. 2, *J. Marine Eng. Soc. Jpn.*, 4, 179, 1969.
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