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Statistical Energy Analysis

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Summary

This chapter presents the basics of statistical energy analysis (SEA) as applied to acoustic problems in structural systems. Power flow equations for structures consisting of two or more subsystems are described. The modeling and analysis procedures for the structural subsystems and acoustic subsystems are given. An estimation procedure of the necessary SEA parameters is given. The practical application of the SEA procedure in structures is illustrated using a two-story building as an example.

20.1 Introduction

This chapter describes the basic concepts of the method of statistical energy analysis (SEA) and presents its application to structures. The analysis and computation techniques for vibration response and radiating sound in instruments and structures vary according to the characteristics of the physical object and the frequency range of interest. Here, we analyze vibration and noise in relation to a rather large-scale structure over a wide frequency band. Extensive computations are usually required, when, for example, the finite element method is used for the computations, with respect to a given oscillation mode. In particular, when the computations must be performed in the high-frequency range and when many modes are included in the frequency band, the level of computation becomes considerable, generally resulting in reduced computational accuracy. To supplement the weak point of the traditional approach, it is necessary to redistribute statistically the energy equally from all modes in the analytical frequency band. This allows computed results to be compared with experimental results for a structure across a wide frequency band. This is the SEA method [1]. Early in its development, the objective of this analytical method was to predict the vibration response of artificial satellites and rockets that receive sound excitation when the jet discharges, and to predict the response of vibration stress in the boundary layer noise of an aircraft's airframe. It also became a model that allows an exciting force to be statistically (randomly) diffused (distributed) over a wide frequency band. This technique considers energy of excitation of a diffused (distributed) sound field and its variables that represent the sound pressure, acceleration, and force. Thus, it can be applied to problems of solid-borne sound in which vibration propagates through each element [2] and problems of air-borne sound in which multiple barriers exist [3], even when more excitation points than one are present.

20.2 Power Flow Equations

With the SEA method, we do not deal with specific characteristic modes of the analyzed structure. Instead, we consider the structural components as a set of equivalent vibrating elements, and evaluate the vibration condition of the components as a macroscopic quantity averaged statistically over the frequency band and space (by describing the energy). We assume that the vibration modes within a given frequency band are distributed uniformly and are excited to the same degree.

Using the SEA method, we can formalize the relationships of power flows between subsystems, and by solving these relationships, we can compute the energy stored in each subsystem. Next, the equations of such basic power flow [4] are explained.

20.2.1 Power Flow Equations of a Two-Subsystem Structure

The power flow relationships of a structure consisting of a two-subsystem structure are shown in Figure 20.1. The equations for the power flows between subsystem 1 and subsystem 2 under typical conditions are expressed as

$$\text{Subsystem 1: } P_{i1} = P_{l1} + P_{l2} \tag{20.1}$$

$$\text{Subsystem 2: } P_{i2} = P_{l2} + P_{21} \tag{20.2}$$

where P_{i1} is the input power to subsystem 1 from outside, P_{l1} is the internal power loss of subsystem 1, and P_{12} is the transmitted power from subsystem 1 to subsystem 2.

The internal power loss, P_{l1}, is written as

$$P_{l1} = \omega\eta_1 E_1 \tag{20.3}$$

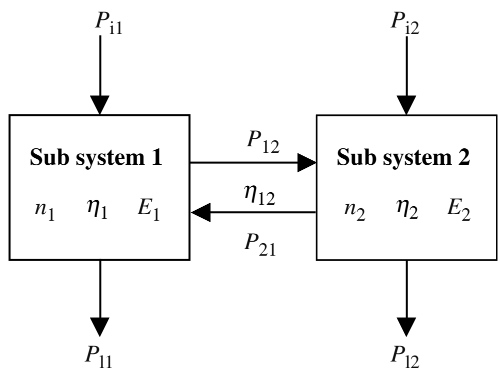

FIGURE 20.1 Power flow relationships between two subsystems.

where ω is the central angular frequency in the band, E_1 is the energy in the bandwidth $\Delta\omega$ of subsystem 1, and η_1 is the internal loss factor (ILF).

The average modal energy E_{m1} in subsystem 1, and E_{m2} in subsystem 2, are given by

$$E_{m1} = \frac{E_1}{N_1}, \quad E_{m2} = \frac{E_2}{N_2} \tag{20.4}$$

where N_1 is number of modes in the bandwidth $\Delta\omega$ of subsystem 1, and N_2 is number of modes in the bandwidth $\Delta\omega$ of subsystem 2.

The transferred power, P_{12}, between subsystems 1 and 2 is expressed as

$$P_{12} = -P_{21} = P'_{12} - P'_{21} \tag{20.5}$$

$$P'_{12} = \omega\eta_{12}E_1 = \omega\eta_{12}N_1E_{m1} \tag{20.6}$$

$$P'_{21} = \omega\eta_{21}E_2 = \omega\eta_{21}N_2E_{m2} \tag{20.7}$$

where η_{12} and η_{21} are the coupling loss factors (CLFs) from subsystem 1 to subsystem 2, and from subsystem 2 to subsystem 1. They satisfy the reciprocity relationship $\eta_{12}n_1 = \eta_{21}n_2$. Therefore, transferred power, P_{12}, becomes

$$P_{12} = \omega\eta_{12}N_1(E_{m1} - E_{m2}) = \omega\eta_{12}N_1\left(\frac{E_1}{N_1} - \frac{E_2}{N_2}\right) \tag{20.8}$$

Consequently, the power flow equations (Equation 20.1 and Equation 20.2) can be expressed as follows:

$$P_{i1} = \omega\eta_1 E_1 + \omega\eta_{12}N_1\left(\frac{E_1}{N_1} - \frac{E_2}{N_2}\right) \tag{20.9}$$

$$P_{i2} = \omega\eta_2 E_2 + \omega\eta_{21}N_2\left(\frac{E_2}{N_2} - \frac{E_1}{N_1}\right) \tag{20.10}$$

If the SEA parameters (i.e., the modal density, intrinsic loss factor, CLF, and input power) are given, then each subsystem's energy condition can be easily computed.

20.2.2 Power Flow Equations of a Multiple Subsystem Structure

By expanding the formulation in the previous section, it is possible to formalize the power flow relationships of a structure composed of multiple subsystems in the same way. The power flow equation for a structure composed of N subsystems is expressed by the following equation in the matrix form:

$$\omega\begin{bmatrix} \left(\eta_1 + \sum_{i\neq 1}^{N}\eta_{1i}\right)n_1 & -\eta_{12}n_1 & \cdots & -\eta_{1N}n_1 \\ -\eta_{21}n_2 & \left(\eta_2 + \sum_{i\neq 2}^{N}\eta_{2i}\right)n_2 & \cdots & -\eta_{2N}n_2 \\ \vdots & \vdots & \ddots & \vdots \\ -\eta_{N1}n_n & \cdots & \cdots & \left(\eta_N + \sum_{i\neq N}^{N-1}\eta_{Ni}\right)n_N \end{bmatrix} \times \begin{bmatrix} E_1/n_1 \\ E_2/n_2 \\ \vdots \\ E_N/n_N \end{bmatrix} = \begin{bmatrix} P_{i1} \\ P_{i2} \\ \vdots \\ P_{iN} \end{bmatrix} \tag{20.11}$$

From Equation 20.11, if the SEA parameters are given in the same way as for the structure of two subsystems, then the energy equation of each subsystem can be obtained.

The average energy of a subsystem is expressed by the following equations by using the vibration velocity and sound pressure:

$$E = M\langle v^2\rangle \tag{20.12}$$

$$E = \frac{M\langle p^2\rangle}{Z_0^2} \tag{20.13}$$

where M is the mass of the subsystem, $\langle v^2\rangle$ is the average spatial square of the vibration velocity, $\langle p^2\rangle$ is the average spatial square of the sound pressure, and Z_0 is the specific acoustic impedance of air.

Accordingly, if each condition of component's energy is determined from Equation 20.11, it is possible to compute the vibration variable and the sound pressure with Equation 20.12 and Equation 20.13.

20.3 Estimation of SEA Parameters

To solve the power flow equations, it is necessary to determine the SEA parameters (i.e., the modal density, ILF, CLF, and input power). In this subsection, a method is given for computing the SEA parameters.

20.3.1 Modal Density

20.3.1.1 Structural Subsystem

Modal density is a key parameter for determining the dynamic characteristic of a structure. The number of modes, N, included in the frequency band (for estimation), is a factor denoting how easily energy, in transferring between subsystems, can be obtained. To determine N in the prescribed frequency band, it is first necessary to determine the modal density $n(f)$, that is, the gradient of N in the frequency band.

The modal density of a structural subsystem is computed by using the following equation [4,5]:

$$n(f) = \frac{\mathrm{d}N}{\mathrm{d}f} = \frac{1}{f_0} = \frac{A}{2t}\sqrt{\frac{12\rho(1-\nu^2)}{E}} \tag{20.14}$$

where A is the area of cross section, t is the thickness of the structural subsystem, ρ is the mass density, ν is the Poisson's ratio, E is the Young's modulus, and f_0 is the fundamental natural frequency of the structural subsystem.

20.3.1.2 Acoustic Subsystem

The modal density of an acoustic subsystem is determined by the following analytical equation [6]:

$$n(f) = \frac{\mathrm{d}N}{\mathrm{d}f} = \frac{4\pi V}{c^3}f^2 \tag{20.15}$$

where c is the speed of sound propagation within the acoustic subsystem, and V denotes the volume of the acoustic subsystem.

Modal density of the cavity in the low frequency band is deduced in a similar manner to that in the two-dimensional space. Define the depth of the cavity by d, and the frequency of the standing wave in the cavity by $f_\mathrm{d} = c/2d$. If $f < f_\mathrm{d}$, then the modal density is assumed to be uniformly distributed, and is estimated by

$$n(f) = \frac{2\pi S}{c^2}f \tag{20.16}$$

where S is the area of the cavity.

If $f > f_\mathrm{d}$, modal density can be estimated using Equation 20.15, because the cavity is designated as acoustically three dimensional.

20.3.2 Internal Loss Factor

20.3.2.1 Structural Subsystem

The ILF, η_1, of a subsystem gives the loss ratio when the input power to the subsystem from the outside is converted to kinetic energy of the subsystem. An excitation test to measure the damping ratio is employed to estimate the ILF of the structural subsystem. There are several methods for estimating the internal loss factor. The ILF applied in the SEA method is estimated by measuring input energy and output energy simultaneously, or by measuring the attenuation ratio within a given period of time. Both methods require the same setup to conduct an excitation test, and one is able to improve the measurement precision by conducting both methods.

With the energy measuring methods mentioned above, the ILF can be estimated by

$$\eta = \frac{\int_{f_1}^{f_2} \mathrm{Re}(Y)F^2 \mathrm{d}f}{\omega_0 M \left\langle \int_{f_1}^{f_2} v^2 \mathrm{d}f \right\rangle} \tag{20.17}$$

where Y is the complex mobility at the driving point in the range of f_1 to f_2, F^2 is the power spectrum of the input vibration force, and v^2 is the power spectrum of the response speed. In addition, $\langle\ \rangle$ denotes the space average operator.

20.3.2.2 Acoustic Subsystem

The ILF of an acoustic subsystem is determined by [7]

$$\eta = \frac{cS\bar{\alpha}}{4V\omega} \tag{20.18}$$

where $\bar{\alpha}$ is the average acoustic absorption coefficient, V is the volume of the acoustic subsystem, and S denotes the surface area. The acoustic absorption coefficient can be estimated by measuring the reverberation time.

Both the ILF of the cavity in the low-frequency band and the modal density are deduced similar to that in two-dimensional spaces. For $f < f_\mathrm{d}$, the ILF is estimated using

$$\eta = \frac{cS_\mathrm{p}\alpha_\mathrm{p}}{\pi\omega V_\mathrm{c}} \tag{20.19}$$

where α_p is the acoustic absorption coefficient in the cavity, S_p is the peripheral area of the cavity, and V_c is the volume of the cavity.

For $f > f_\mathrm{d}$, the modal density can be estimated using Equation 20.18 because the cavity is taken as an acoustic subsystem.

20.3.3 Coupling Loss Factor

20.3.3.1 Between Structural Subsystems

The CLF η_{ij} gives the loss ratio when power transmits between two subsystems [4]. For example, the CLF between two flat plates can be estimated using

$$\eta_{ij} = \frac{c_{\mathrm{g}i} L_\mathrm{c} \tau}{\pi\omega S_i} \tag{20.20}$$

where $c_{\mathrm{g}i}$ is the group velocity of the bending waves, L_c is the coupled length, S_i is the surface area, and τ_{ij} is the energy transmission factor from subsystem i to subsystem j. The transmission factor varies with the type of coupling, for example, I-type, L-type, or T-type shown in Figure 20.2. In this section, we use energy transmission efficiency of vertical incidence, reported by Cremer [7].

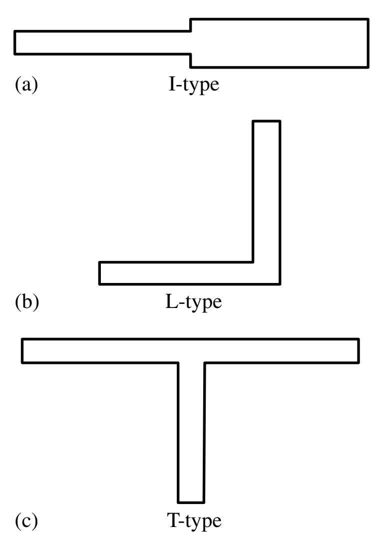

FIGURE 20.2 Coupled type.

20.3.3.2 Between a Structural Subsystem and an Acoustic Subsystem

Coupling power between a structural subsystem and an acoustic subsystem is the power flow based on resonance at transmission. The CLF between a structural subsystem and an acoustic subsystem is given by

$$\eta_{ij} = \frac{Z_0 S_c \sigma}{\omega M_i} \tag{20.21}$$

where Z_0 is the specific acoustic impedance of air, S_c is the surface area of coupling, σ is the acoustical radiation efficiency, and M_i is the mass of the structural subsystem.

20.3.3.3 Between an Acoustic Subsystem and a Cavity

Coupling power between an acoustic subsystem and a cavity is power flow based on resonance in transmission. The CLF between an acoustic subsystem and a cavity is given by [8]

$$\eta_{sc} = \frac{c_s S_{cs} \tau_m}{4\omega V_s} \tag{20.22}$$

where c_s is the sound velocity in the acoustic space, V_s is the volume in the acoustic space, τ_m is the transmission factor at random incidence depending on mass flow through the partition, and S_{cs} is the coupling area.

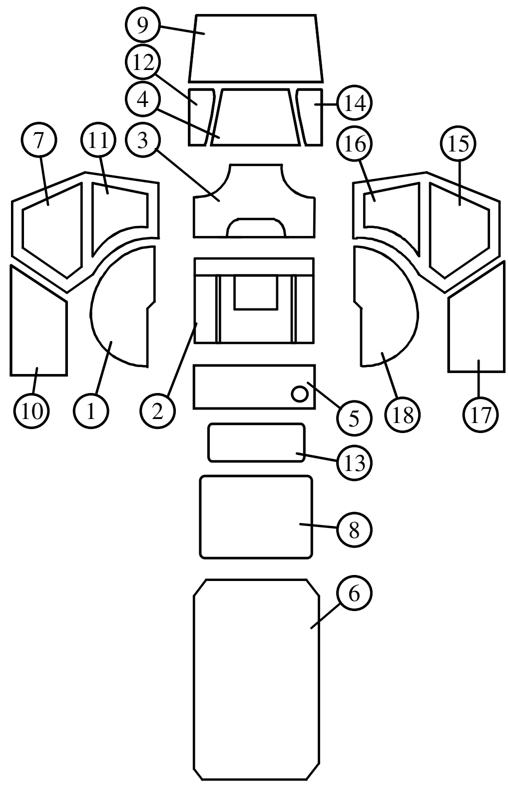

FIGURE 20.3 Modeling of the tractor cabin.

20.3.4 Input Power

20.3.4.1 Vibration Input Power

The vibration input power P_{iN} is given by

$$P_{iN} = \omega M_i \langle v_i^2 \rangle \tag{20.23}$$

where M_i is the equivalent mass, $\langle v_i^2 \rangle$ is the spatial average of square of the vibration velocity, and ω is the central angular frequency.

20.3.4.2 Acoustical Input Power

The acoustical input power P_s is given by

$$P_s = \frac{\langle p^2 \rangle S^2 n(f)}{4M} \sigma_{rad} \frac{c^2}{2\pi S f^2} \tag{20.24}$$

where $\langle p^2 \rangle$ is the square average of the input sound pressure, S is the surface area of the component, M is the mass of the component, $n(f)$ is the modal density, and σ_{rad} is the sound radiation factor [2].

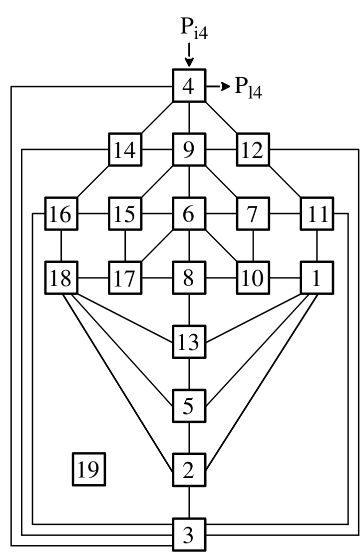

FIGURE 20.4 Power flows in the tractor cabin.

20.4 Application in Structures

In this section, we present an example of modeling with significant analytical accuracy, and discuss the application of the SEA method for structures.

20.4.1 Application for Prediction of Noise in a Tractor Cabin

Figure 20.3 shows a model of the tractor cabin. This figure shows that the cabin consists of a floor, a door, a ceiling, and other components. Figure 20.4 presents the power flow relationships within the cabin [9,10].

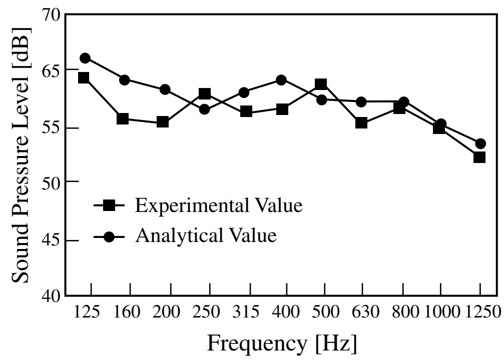

FIGURE 20.5 Results of estimating the sound pressure level in a cabin.

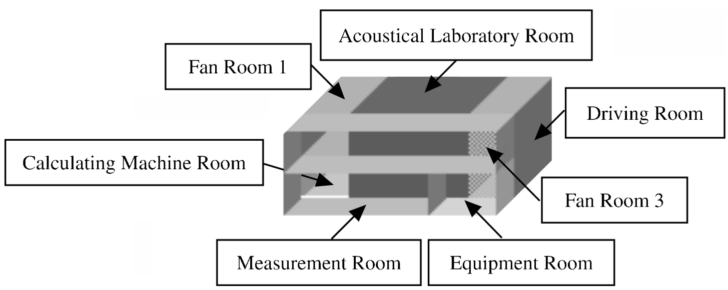

FIGURE 20.6 The configuration of the building.

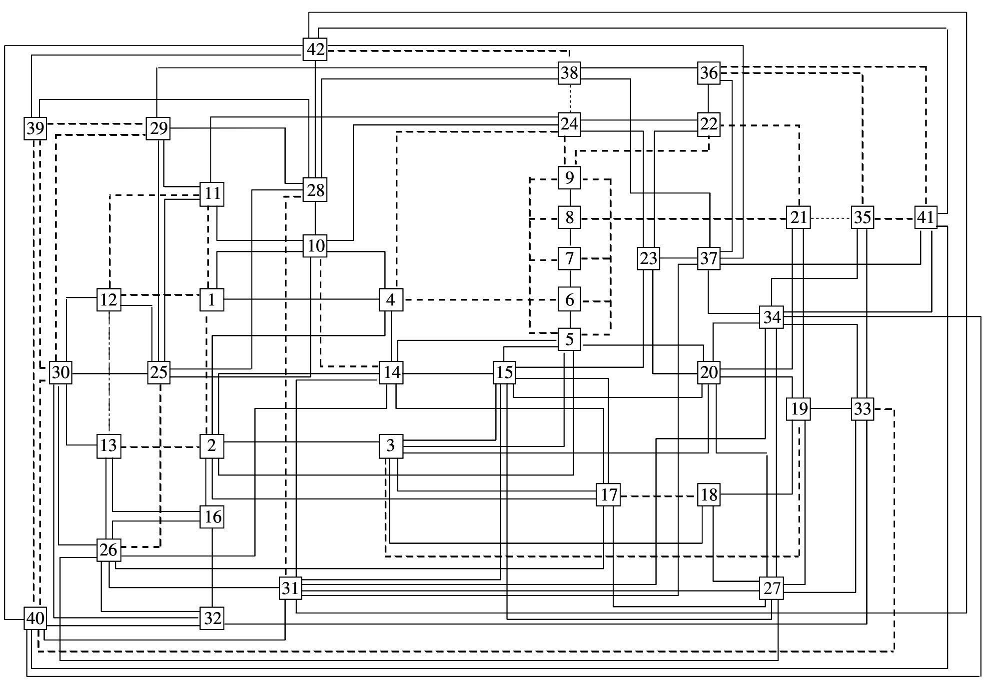

FIGURE 20.7 The power flow relationships between structural subsystems in the entire building.

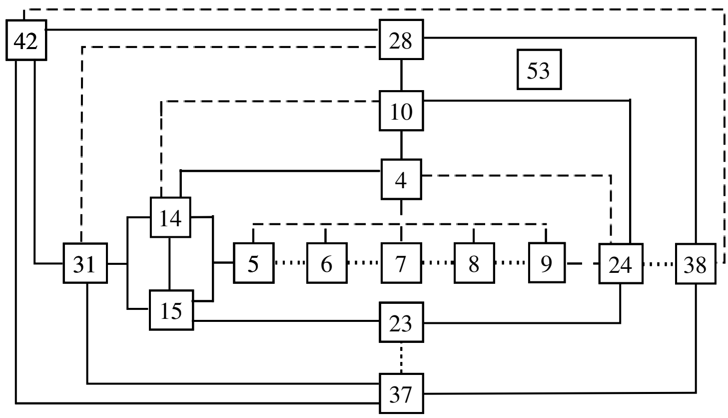

FIGURE 20.8 The power flow relationships in the acoustical laboratory room.

The results obtained for the cabin are shown in Figure 20.5. According to this figure, the disagreement between the computation and the measurement was found about 2 dB in the medium- to high-frequency band.

20.4.2 Application for Prediction of Noise and Vibration in a Building

Consider a two-story laboratory reinforced with concrete [11]. The building configuration is shown in Figure 20.6. This building comprises a driving room, an acoustical laboratory room, a computer room, a measurement room, an equipment room, and others.

We modeled the structural subsystem using I-, L-, or T-type connected points, and the acoustic subsystem as an element shown in Figure 20.6.

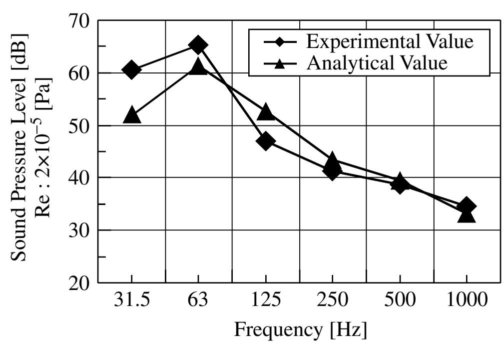

FIGURE 20.9 Estimated sound pressure level results for other rooms.

The SEA model constructed in this manner is composed of 61 elements, and has 244 connecting points. Subsystems 1 to 17 and subsystems 19 to 42 are concrete components. Subsystem 18 and subsystems 43 to 48 are plasterboard components; subsystems 49 to 55 are room components; and subsystems 56 to 61 are cavity components. For example, Figure 20.7 shows the power flow relationships between structural subsystems in the entire building, while Figure 20.8 shows them in the acoustical laboratory room. Here, the thin-dotted, dotted, and solid lines indicate the I-, L-, and T-type combinations, respectively. Subsystem 53 is the room component, and it is connected with all structural components shown in Figure 20.8. The plasterboards located between the computer room and the measurement room are considered as a partition; therefore, connections between subsystem 49 and subsystems 56 to 59 (cavity components), and subsystem 50 and subsystems 60 and 61 (cavity components) are derived from nonresonant modes.

The results obtained for some other rooms are shown in Figure 20.9. Computing accuracy in this building is worse than in the cabin because the structure of this building is complicated, although the differences between the computed values and the measured values were approximately 4 dB in the medium- to high-frequency band.

The computations take approximately 10 sec, so the workload on the personal computer is quite light.

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