CHAPTER 1

INTRODUCTION

This book has three main purposes. The first purpose is to collect in one document the various methods of constructing and representing dynamic mechanical models. The second purpose is to help the reader develop a strong understanding of the modal analysis technique, where the total response of a system can be constructed by combinations of individual modes of vibration. The third purpose is to show how to take the results of large finite element models and reduce the size of the model (model reduction), extracting lower order state space models for use in MATLAB.

1.1 Representing Dynamic Mechanical Systems

We will see that the nature of damping in the system will determine which representation will be required. In lightly damped structures, where the damping comes from losses at the joints and the material losses, we will be able to use "modal analysis," enabling us to restructure the problem in terms of individual modes of vibration with a particular type of damping called "proportional damping." For systems which have significant damping, as in systems with a specific "damper" element, we will have to use the original, coupled differential equations for solution.

The left-hand block in Figure 1.1 represents a damped dynamic model with coupled equations of motion, a set of initial conditions and a definition of the forcing function to be applied. If damping in the system is significant, then the equations of motion need to be solved in their original form. The option of using the normal modes approach is not feasible. The three methods of solving for time and frequency domain responses for highly damped, coupled equations are shown.

1.2 Modal Analysis

Most practical problems require using the finite element method to define a model. The finite element method can be formulated with specific damping elements in addition to structural elements for highly damped systems, but its most common use is to model lightly damped structures.

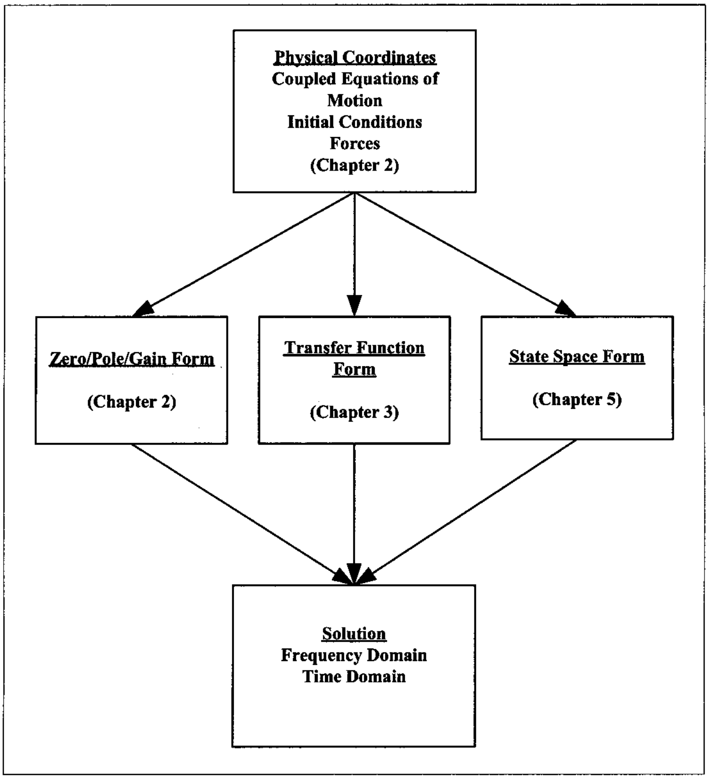

Figure 1.1: Coupled equations of motion flowchart.

The diagram in Figure 1.2 shows the methodology for analyzing a lightly damped structure using normal modes. As with the coupled equation solution above, the solution starts with deriving the undamped equations of motion in physical coordinates. The next step is solving the eigenvalue problem, yielding eigenvalues (natural frequencies) and eigenvectors (mode shapes). This is the most intuitive part of the problem and gives one considerable insight into the dynamics of the structure by understanding the mode shapes and natural frequencies.

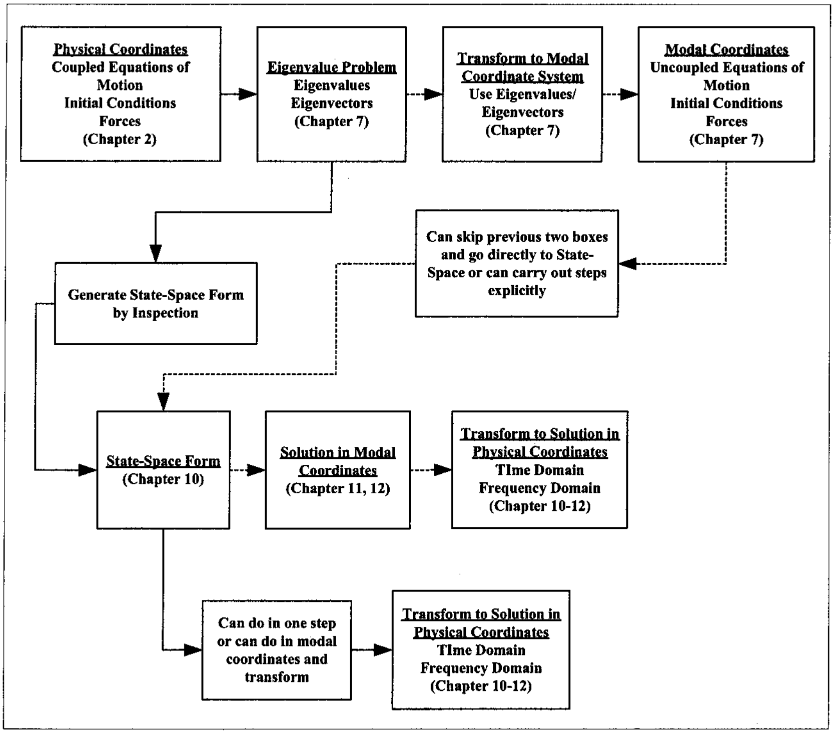

Figure 1.2: Modal analysis method flowchart.

To solve for frequency and time domain responses, it is necessary to transform the model from the original physical coordinate system to a new coordinate system, the modal or principal coordinate system, by operating on the original equations with the eigenvector matrix. In the modal coordinate system the original undamped **coupled** equations of motion are transformed to the same number of undamped **uncoupled** equations. Each uncoupled equation represents the motion of a particular mode of vibration of the system. It is at this step that proportional damping is applied. It is trivial to solve these uncoupled equations for the responses of the modes of vibration to the forcing function and/or initial conditions because each equation is the equation of motion of a simple single degree of freedom system. The desired responses are then back-transformed into the physical coordinate system, again using the eigenvector matrix for conversion, yielding the solution in physical coordinates.

The modal analysis sequence of taking a complicated system, (1) transforming to a simpler coordinate system, (2) solving equations in that coordinate system and then (3) back-transforming into the original coordinate system is analogous to using Laplace transforms to solve differential equations. The original differential equation is (1) transformed to the "s" domain by using a Laplace transform, (2) the algebraic solution is then obtained and is (3) back-transformed using an inverse Laplace transform.

It will be shown that once the eigenvalue problem has been solved, setting up the zero initial condition state space form of the uncoupled equations of motion in principal coordinates can be performed by inspection. The solution and back-transformation to physical coordinates can be performed in one step in the MATLAB solution.

The advantage of the modal solution is the insight developed from understanding the modes of vibration and how each mode contributes to the total solution.

1.3 Model Size Reduction

It is useful to be able to provide a model of the mechanical system to control engineers using the fewest states possible, while still providing a representative model. The mechanical model can then be inserted into the complete mechanical/control system model and be used to define the system dynamics.

Figure 1.3 shows how to convert a large finite element model (and most real finite element models are "large," with thousands to hundreds of thousands of degrees of freedom) to a smaller model which still provides correct responses for the forcing function input and desired output points.

The problem starts out with the finite element model which is solved for its eigenvalues and eigenvectors (resonant frequencies and mode shapes). There are as many eigenvalues and eigenvectors as degrees of freedom for the model, typically too large to be used in a MATLAB model.

Once again, the eigenvalues and eigenvectors provide considerable insight into the system dynamics, but the objective is to provide an efficient, "small" model for inclusion into the mechanical/servo system model. This requires reducing the size of the model while still maintaining the desired input/output relationships.

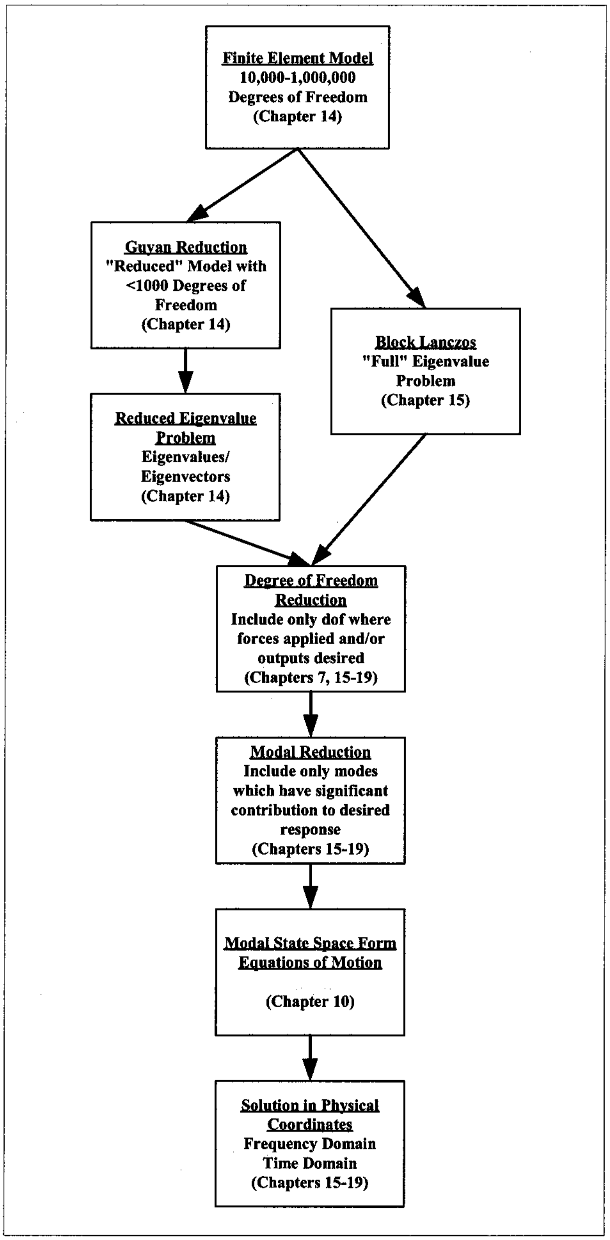

Figure 1.3: Model size reduction flowchart.

The reduction of the size of the model is accomplished in two steps. The first is to reduce the number of degrees of freedom of the model from the original set to a new set which includes only those degrees of freedom where forces are applied and/or where responses are desired.

The second step for Single Input Single Output (SISO) systems is to reduce the number of modes of vibration used for the solution by ranking the relative importance of each mode to the overall response. For Multi Input Multi Output (MIMO) systems, a more sophisticated method of reduction which simultaneously takes into account the controllability and observability of the system is required.

Figure 1.4 shows the overall frequency response for a SISO cantilever beam model discussed in Chapter 15. Superimposed over the overall frequency response is the contribution of each of the individual 10 modes of vibration which make up the overall response.

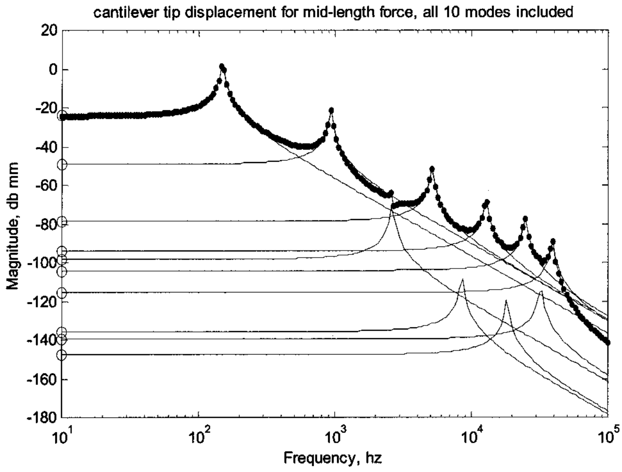

Figure 1.4: Individual mode contribution to overall frequency response.

We will show that modes with little or no displacement at the reduced set of degrees of freedom are candidates for elimination. For example, the three modes which have low frequency magnitudes of less than –120db in Figure 1.4 have no effect on the overall frequency response – their peaks do not show up on the overall frequency response. The less important modes either can be eliminated directly or a more sophisticated method can be used which takes into account the low frequency effects of the removed modes. Both types are discussed in detail, accompanied by examples.

A reduced solution can provide very good results with a significant reduction in number of states – a model which is very amenable to being combined with a servo model for a complete servo mechanical system model.