CHAPTER 4

ZEROS IN SISO MECHANICAL SYSTEMS

4.1 Introduction

Chapters 2 and 3 discussed poles and zeros of SISO systems and their relationship to transfer functions. The origin and influence of poles are clear. They represent the resonant frequencies of the system, and for each resonant frequency a mode shape can be defined to describe the motion at that frequency. We have seen from our frequency response analyses in Chapter 3 that at the frequencies of the zeros, motions approach or go to zero, depending on the amount of damping present. In Chapters 8 and 11 we will illustrate how all the individual modes of vibration can combine at specific frequencies to create zeros of the overall transfer function.

This chapter will expand on analyses shown in Miu [1993] to develop an intuitive understanding for when to expect zeros in Single Input Single Output (SISO) simple mechanical systems and how to predict the frequencies at which they will occur. We will not cover the theory, but will state the conclusions from Miu and show how the conclusions relate to two example systems.

We will start by defining a series arrangement lumped spring/mass system. We will develop guidelines for defining the number of zeros that should be seen and show how to predict their frequencies. A MATLAB model is used to illustrate the guidelines for various combinations of input and output degrees of freedom. Only the MATLAB code results are discussed; the code itself is not listed or discussed as it uses techniques found later in the book. However, the reader is encouraged to run the code and experiment with various values of the input and number of masses in the model to become familiar with the concept.

Next, an ANSYS finite element model of a tip-excited cantilever is analyzed. The resulting transfer function magnitude is plotted using MATLAB to show an overlay of the poles of the "constrained" system and their relationship with the zeros of the original model.

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4.2 “n” dof Example

Figure 4.1 shows a series arrangement of masses and springs, with a total of “n” masses and “n+1” springs. The degrees of freedom are numbered from left to right, z_1 through z_n .

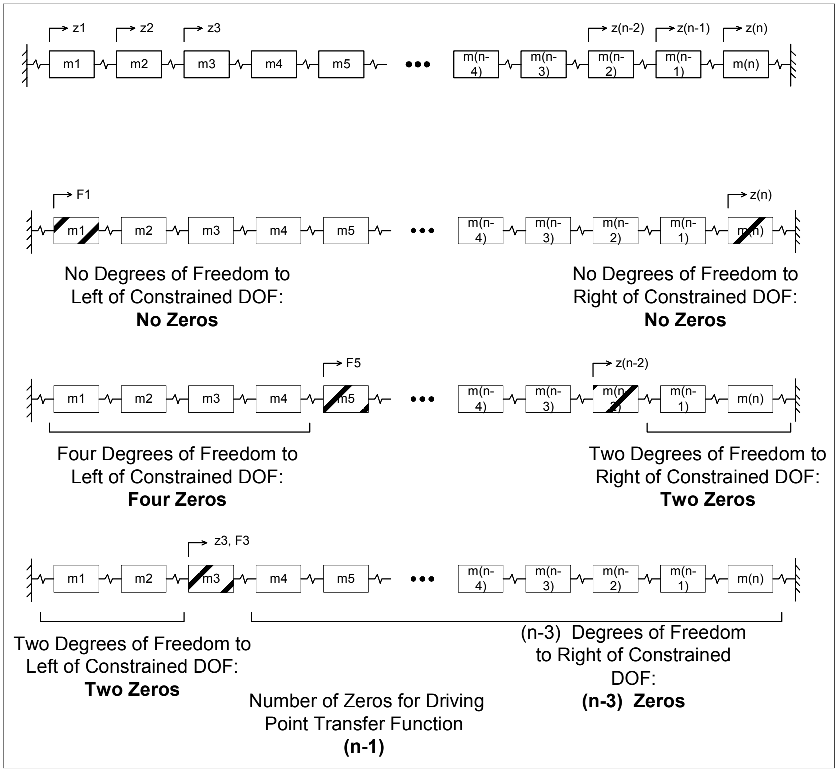

Figure 4.1a,b,c,d: “n” dof system showing various SISO input/output configurations.

Miu [1993] shows that the **zeros** of any particular transfer function are the **poles of the constrained system(s)** to the left and/or right of the system defined by constraining the one or two dof’s defining the transfer function. **The resonances of the “overhanging appendages” of the constrained system create the zeros.**

Two limiting cases are immediately available in (1) and (3) below:

1) For the transfer function from one end of the structure to the other, Figure 4.1b, there are no overhanging appendage

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structures to the left or right of the constrained structure, so there are no zeros.

2) For an arbitrary transfer function, Figure 4.1c, there will be a structure to the left and/or to the right of the constrained dof's. The total degrees of freedom of the overhanging appendage(s) will give the total number of zeros in the transfer function.

3) For the driving point transfer function, Figure 4.1d, the force and displacement are measured at the same dof, so there are a total of $(n-1)$ degrees of freedom left, hence $(n-1)$ zeros of the transfer function. All but one of the masses are overhanging appendages.

In the analysis that follows, we will calculate frequency responses and pole/zero plots for various transfer functions using the MATLAB code **ndof_numzeros.m**.

4.2.1 MATLAB Code ndof_numzeros.m, Usage Instructions

The MATLAB code is based on the ndof series system in Figure 4.1. The code allows one to choose the total number of masses in the problem and sets the values of the masses and stiffnesses randomly between the values of 1 and 2. The program then allows one to choose which transfer function to calculate, and shows the pole/zero plots for the original system as well as the poles for the two structures to the left and/or right. For now, the reader should not worry about the details of the code, which will be covered in later chapters, but should use the code to study the pole/zero patterns in systems with different numbers of degrees of freedom and for different input/output dof's. Sometimes the random values chosen for stiffnesses and damping will cause the poles and zeros to be so close together that they will cancel each other. If this is the case and the number of poles and zeros do not match the expected number, rerun the code until more widely spaced poles/zeros are randomly chosen and the required poles and zeros are apparent.

4.2.2 Seven dof Model – z7/F1 Frequency Response

Taking a seven-mass model as an example, the resulting frequency responses and pole-zero plots are displayed on the following pages. In all cases, the random distribution of masses and spring stiffnesses is used, resulting in a different set of variables for each run.

Figure 4.2 shows the frequency response for applying a force at the first mass and looking at the output at the last (seventh) mass. Note that in accordance

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with the prior analysis, there should be no zeros as there are no "overhanging" appendages. Since there are seven masses, there should be seven poles. Since each mass provides an attenuation of –40db/decade, after the last of seven poles the slope of the curve is 7*(–40 db/decade) = –280 db/decade.

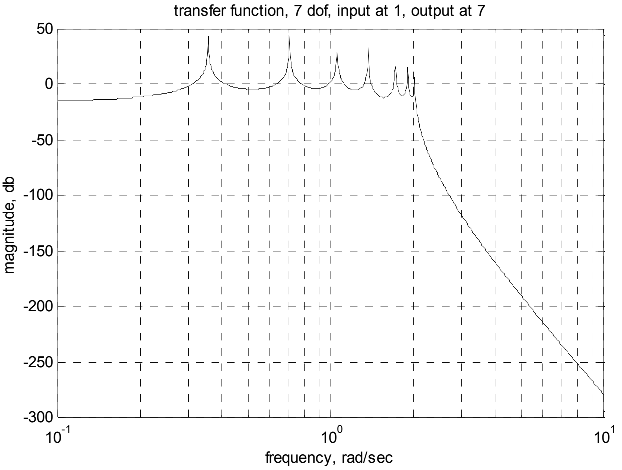

Figure 4.2: z17 transfer function frequency response, seven poles, no zeros.

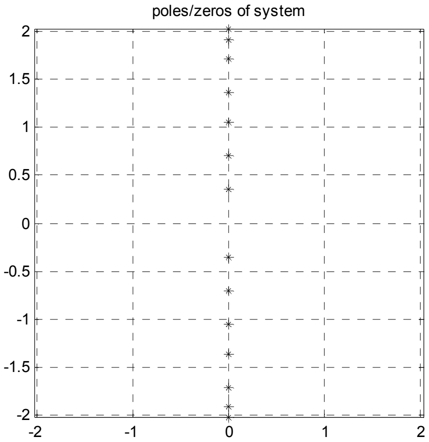

Figure 4.3: z17 pole/zero plot showing only seven poles.

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4.2.3 Seven dof Model – z3/F4 Frequency Response

The same seven dof system provides the following frequency response when the force is applied at mass 3 and the output is taken at mass 4. There are two "overhanging" appendages to the left of mass 3, masses 1 and 2, and there are three "overhanging" appendages to the right of mass 4, masses 5, 6 and 7. These masses should combine to give a total of five zeros and once again, seven poles as shown below.

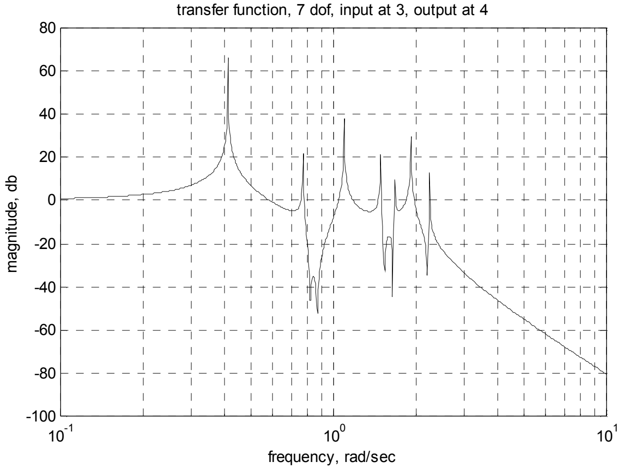

Figure 4.4: z34 transfer function frequency response, seven poles and five zeros.

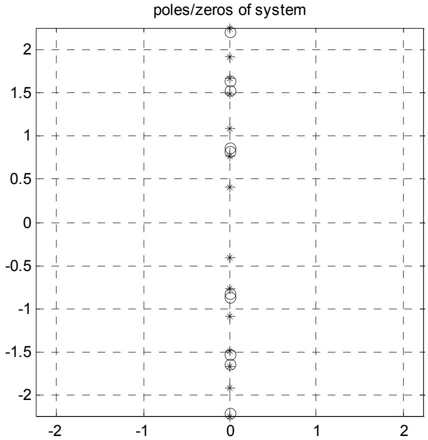

Figure 4.5: z34 pole/zero plot showing seven poles and five zeros.

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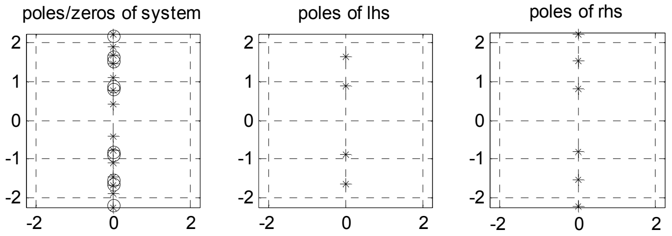

Figure 4.6: z34 poles and zeros; poles of left-hand and right-hand constrained systems are the same as the zeros of the unconstrained system.

The left-hand plot in Figure 4.6 displays the z34 poles and zeros. The middle plot shows the poles of the system to the left of mass 3. The right plot shows the poles of the system to the right of mass 4. It is clear that the poles of the two right plots are the zeros of the z34 system.

4.2.4 Seven dof Model – z3/F3, Driving Point Frequency Response

For the same seven dof system with force and output taken at the same node (driving point transfer function), there should be six "overhanging" masses providing zeros. Therefore the frequency response plot in Figure 4.7 shows six zeros, with alternating pole/zero pairs.

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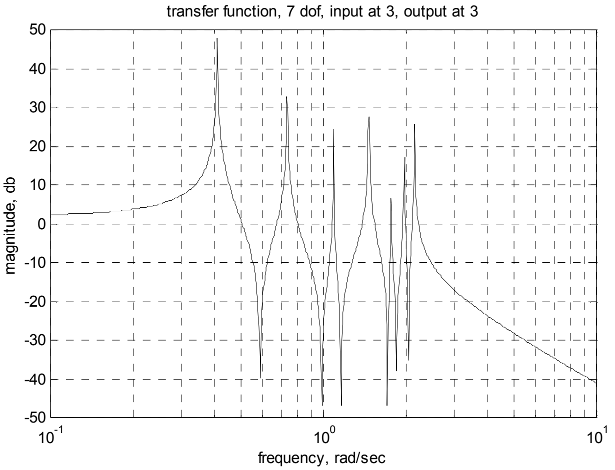

Figure 4.7: z33 transfer function frequency response, seven poles and the expected six zeros.

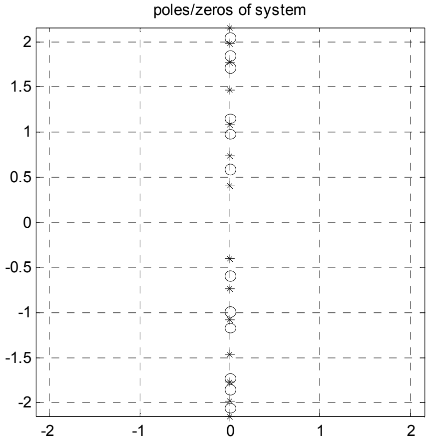

Figure 4.8: z33 pole/zero plot showing seven poles and six zeros.

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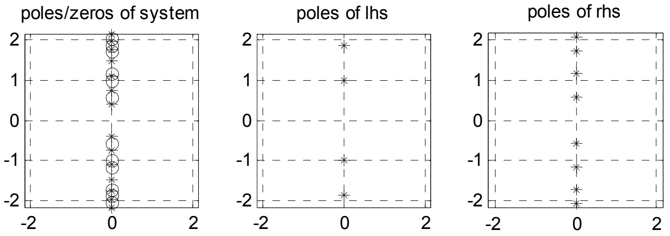

Figure 4.9: z33 poles and zeros. Poles of left-hand and right-hand constrained systems are the same as the zeros of the unconstrained system.

4.3 Cantilever Model – ANSYS

4.3.1 Introduction

Now that we have seen how the "constrained" system artifice works for a simple lumped parameter system, it is interesting to consider how the artifice would work for a continuous system, such as a cantilever beam.

The finite element program ANSYS is used to analyze a cantilever beam with a driving point transfer function at the tip. The transfer function we are interested in is the displacement at the tip, z, due to a vertical force at the tip, F, as shown in Figure 4.10. The "constrained" structure whose poles should define the zero locations for the unconstrained system is the original cantilever with the addition of a simple support at the tip.

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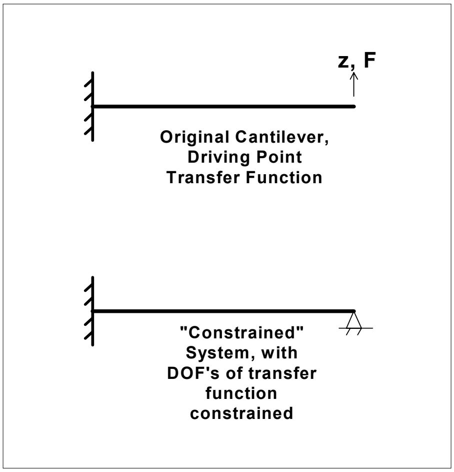

Figure 4.10: Unconstrained and constrained cantilevers used for driving point transfer function example.

4.3.2 ANSYS Code cantfem.inp Description and Listing

The input listings for the ANSYS models of the cantilever and simply supported tip cantilever are below. The cantilever input program is **cantfem.inp** and the supported tip input program is **cantzero.inp**. Both programs can be run if one has access to ANSYS by typing "/input,cantfem,inp" or "/input,cantzero,inp" at the ANSYS program command prompt. The programs will run with no further input and will output graphs of the mode shapes and frequency response. Both programs build the model, and calculate and output the eigenvalues (natural frequencies) and eigenvectors (mode shapes). **Cantfem.inp** then calculates and outputs the frequency response. The mode shapes are shown in cantfem.grp and cantzero.grp and the frequency response is shown in cantfem.grp2. They can all be viewed by using the ANSYS Display program and loading the appropriate file.

```
/title, cantfem.inp, 0.05 x 1 x 20mm aluminum cantilever beam, 20 elements

/prep7

et,1,4                 ! element type for beam

! aluminum

ex,1,71e6                        ! mN/mm^2
dens,1,2.77e-6                   ! kg/mm^3
```

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```
nuxy,1,.345

! real value to define beam characteristics

r,1,1,.00001041,.004166,.05,1   ! area, moments of inertia, thickness

! define plotting characteristics

/view,1,1,-1,1    ! iso view
/angle,1,-60      ! iso view
/pnum,mat,1       ! color by material
/num,1            ! numbers off
/type,1,0         ! hidden plot
/pbc,all,1        ! show all boundary conditions

csys,0                          ! define global coordinate system

! nodes

n,1,0,0,0                       ! left-hand node
n,21,20,0,0                     ! right-hand node

fill,1,21                       ! interior nodes

nall
nplo

! elements

type,1
mat,1
real,1
e,1,2
egen,20,1,-1

! constrain left-hand end

nall
d,1,all,0                       ! constrain node 1, all dof's

! constrain all but uz and roty for all other nodes to allow only those dof's
! this will give 20 nodes, node 2 through node 21, each with 2 dof, giving a total of 40 dof
! can calculate a maximum of 40 eigenvalues if don't use Guyan reduction to reduce size of
! eigenvalue problem

nall
nsel,s,node,,2,21
d,all,ux
d,all,uy
d,all,rotx
d,all,rotz

nall
eall
nplo
```

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```
eplo

! *******************  eigenvalue run  *******************

fini         ! fini just in case not in begin

/solu        ! enters the solution processor, needs to be here to do editing below

allsel       ! default selects all items of specified entity type, typically nodes, elements

antype,modal,new
modopt,reduc,20        ! method - reduced Householder, number of modes to extract
expass,off             ! key = off, no expansion pass, key = on, do expansion
mxpand,20,,,no         ! nummodes to expand
total,20,0             ! total masters, 20 to be used, 1 to exclude rotational dofs

allsel

solve                  ! starts the solution of one load step of a solution sequence, modal here

fini

! plot first mode

/post1

set,1,1

pldi,1

! *****************  output frequencies  *******************

/output,cantfem,frq      ! write out frequency list to ascii file .frq

set,list

/output,term           ! returns output to terminal

! *******************  output eigenvectors  ********************

! define nodes for output:  forces applied or output displacements

nsel,s,node,,21        ! cantilever tip

/output,cantfem,eig      ! write out eigenvectors to ascii file .eig

*do,i,1,20
         set,,i
         prdisp
*enddo

/output,term

! *******************  plot modes  *******************
```

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```
! pldi plots

/show,cantfem,grp,0
allsel

/view,1,,-1,,                    ! side view for plotting
/angle,1,0
/auto

*do,i,1,20
          set,1,i
   pldi
*enddo

/show,term

! ***************  calculate and plot transfer functions  ****************

fini

/assign,rst,junk,rst             ! reassigns a file name to an ANSYS identifier

/solu

dmprat,0.01            ! sets a constant damping ratio for all modes, zeta = 0.01

allsel
eplo                   ! show forces applied

f,21,fz,1              ! 1 mn force applied to node 21, tip node

/title, cantilever with tip load

antype,harmic          ! harmonic (frequency response) analysis

hropt,msup,20          ! mode superposition method, nummodes modes used

harfrq,100,1000000     ! frequency range, hz, for solution, -1 to 10 rad/sec

hrout,off,off          ! amplitude/phase, cluster off

kbc,1

nsubst,10000           ! 10000 frequency points for very fine resolution

outres,nsol,all,       ! controls solution set written to database, nodal dof solution, all
                       ! frequencies, component name for selected set of nodes

solve

fini

/post26

file,,rfrq             ! frequency response results
```

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```
xvar,0               ! display versus frequency

lines,10000          ! specifies the length of a printed page for frequency response listing

nsol,2,21,u,z        ! specifies nodal data to be stored in results file
                     ! u - displacement, z direction
                     ! note that nsol,1 is frequency vector

! plot magnitude

plcplx,0
/grid,1
/axlab,x,frequency, hz
/axlab,y,amplitude, mm
/gropt,logx,1                   ! log plot for frequency
/gropt,logy,1                   ! log plot for amplitude

/show,cantfem,grp1              ! file name for storing
plvar,2
/show,term

! plot phase

plcplx,1
/grid,1
/axlab,x,freq
/axlab,y,phase, deg             ! label for y axis
/gropt,logx,1                   ! log plot for frequency
/gropt,logy,0                   ! linear plot for phase

/show,cantfem,grp1
plvar,2
/show,term

! save ascii data to file

prcplx,1                        ! stores phase angle in asci file .dat

/output,cantfem,dat
prvar,2
/output,term

fini
```

4.3.3 ANSYS Code cantzero.inp Description and Listing

```
/title, cantzero.inp, 0.05 x 1 x 20mm aluminum tip constrained cantilever beam, 20 elements

/prep7

et,1,4               ! element type for beam

! aluminum
```

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```
ex,1,71e6                      ! mN/mm^2
dens,1,2.77e-6                 ! kg/mm^3
nuxy,1,.345

! real value to define beam characteristics

r,1,1,.00001041,.004166,.05,1  ! area, moments of inertia, thickness

! define plotting characteristics

/view,1,1,-1,1     ! iso view
/angle,1,-60       ! iso view
/pnum,mat,1        ! color by material
/num,1             ! numbers off
/type,1,0          ! hidden plot
/pbc,all,1         ! show all boundary conditions

csys,0                         ! define global coordinate system

! nodes

n,1,0,0,0                      ! left-hand node
n,21,20,0,0                              ! right-hand node

fill,1,21                      ! interior nodes

nall
nplo

! elements

type,1
mat,1
real,1
e,1,2
egen,20,1,-1

! constrain left-hand end

nall
d,1,all,0                      ! constrain node 1, all dof's
d,21,uz,0                      ! constrain tip

! constrain all but uz and roty for all other nodes to allow only those dof's
! this will give 20 nodes, node 2 through node 21, each with 2 dof, giving a total of 40 dof
! can calculate a maximum of 40 eigenvalues if don't use Guyan reduction to reduce size of
! eigenvalue problem

nall
nsel,s,node,,2,21
d,all,ux
d,all,uy
d,all,rotx
d,all,rotz
```

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```
nall
eall
nplo
eplo

! *****************  eigenvalue run  *******************

fini                    ! fini just in case not in begin

/solu                   ! enters the solution processor, needs to be here to do editing below

allsel                  ! default selects all items of specified entity type, typically nodes, elements

antype,modal,new
modopt,reduc,20         ! method - reduced Householder, number of modes to extract
expass,off              ! key = off, no expansion pass, key = on, do expansion
mxpand,20,,,no          ! nummodes to expand
total,20                ! total masters, 20 to be used, exclude rotational dofs

allsel

solve                   ! starts the solution of one load step of a solution sequence, modal here

fini

! plot first mode

/post1

set,1,1

pldi,1

! *******************  output frequencies  **********************

/output,cantzero,frq      ! write out frequency list to ascii file .frq

set,list

/output,term            ! returns output to terminal

! *****************  output eigenvectors  ********************

! define nodes for output:  forces applied or output displacements

nsel,s,node,,10         ! cantilever midpoint

/output,cantzero,eig      ! write out eigenvectors to ascii file .eig

*do,i,1,20
          set,,i
          prdisp
*enddo
```

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```
/output,term

! ******************  plot modes  ********************

! pldi plots

/show,cantzero,grp,0
allsel

/view,1,,-1,,                ! side view for plotting
/angle,1,0
/auto

*do,i,1,20
            set,1,i
   pldi
*enddo

/show,term
```

4.3.4 ANSYS Results, cantzero.m

The driving point frequency response for cantfem.inp is shown in Figure 4.11. The ANSYS frequency and magnitude output results are read into MATLAB and plotted in order to be able to overlay the resonances from the cantzero.inp ANSYS run. The MATLAB code to plot the overlay is **cantzero.m**, which reads in two input programs, **cantfem_magphs.m** and **cantzero_freq.m**.

The resonant frequencies (poles) of the cantilever and constrained tip cantilever models are listed in Table 4.1.

According to the guidelines for zeros discussed earlier in the chapter, the poles of the frequency response plot should be the same frequencies as shown in the "cantfem freq" column above. The zeros of the frequency response should be the same frequencies as shown in the "cantzero freq" column above.

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mode	cantfem freq, hz	cantzero freq, hz
1	457.14	2004.6
2	2864.4	6495.0
3	8018.8	13548.
4	15709.	23162.
5	25961.	35336.
6	38771.	50071.
7	54147.	67380.
8	72102.	87291.
9	92672.	0.10985E+06
10	0.11592E+06	0.13520E+06
11	0.14196E+06	0.16337E+06
12	0.17098E+06	0.19495E+06
13	0.20323E+06	0.22951E+06
14	0.23907E+06	0.26909E+06
15	0.27885E+06	0.31129E+06
16	0.32274E+06	0.35968E+06
17	0.37012E+06	0.40928E+06
18	0.41860E+06	0.45602E+06
19	0.46289E+06	0.49344E+06
20	0.49490E+06	0.89212E+06

Table 4.1: Unconstrained (cantfem) and constrained tip (cantzero) cantilever resonances.

The constrained system poles in Figure 4.11 are shown below the curve with "o" symbols. Note that the "o's" align with the zeros of the unconstrained system.

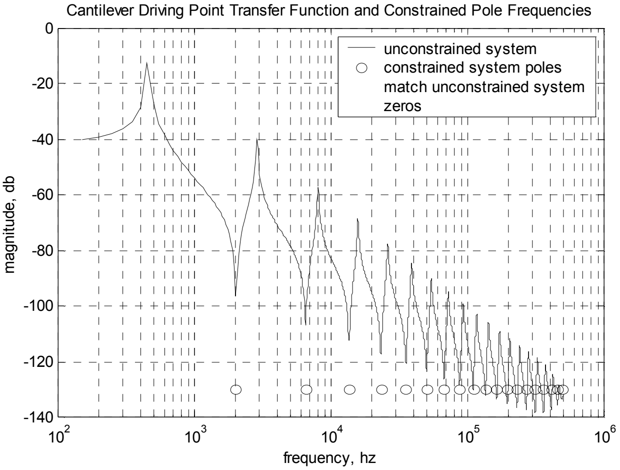

Figure 4.11: Cantilever driving point transfer function frequency response plot with overlaid frequencies of contrained-tip cantilever poles – which should match the unconstrained system zeros.

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Problem

Note: The problem refers to the two dof system shown in Figure P2.2.

P4.1 Use the MATLAB code **ndof_numzeros.m** to identify the number of poles and zeros for a five dof system for the following: z11, z23, z33. Correlate the poles of the constrained system with the zeros of the original system.

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