CHAPTER 9

TRANSIENT RESPONSE: MODAL FORM

9.1 Introduction

The transient response example shown in Figure 9.1 will be solved by hand, using the modal analysis method derivation from Chapter 7. As in the frequency response analysis in the previous chapter, we will again start with the eigenvalues and eigenvectors from Chapter 7. We will use them to transform initial conditions and forces to principal coordinates and write the equations of motion in principal coordinates. Laplace transforms will be used to solve for the motions in principal coordinates and we will then back transform to physical coordinates. Once again, the individual mode contributions to the overall transient response of each of the masses will be evident. The closed form solution is then coded in MATLAB and the results plotted, highlighting the individual mode contributions.

9.2 Review of Previous Results

The applied step forces are as shown in Figure 9.1 and the initial conditions of position and velocity for each of the three masses are shown in Table 9.1.

From previous results, (7.86) to (7.88), we know the eigenvalues and eigenvectors normalized with respect to mass, $\mathbf{z}_n$:

$$\omega_1 = 0, \quad \omega_2 = \sqrt{\frac{k}{m}}, \quad \omega_3 = \sqrt{\frac{3k}{m}} \tag{9.1}$$

$$\mathbf{z}_n = \frac{1}{\sqrt{m}} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \tag{9.2}$$

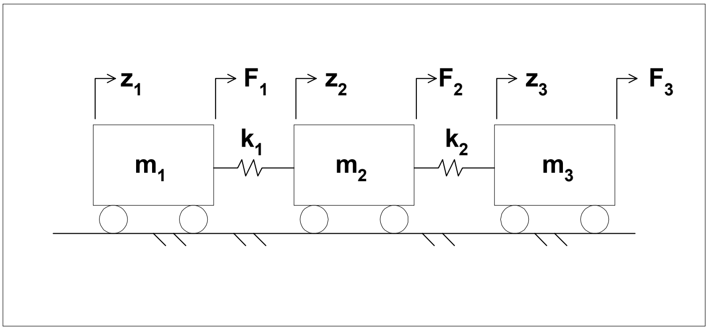

Figure 9.1: Step forces applied to tdof system.

Mass 1	Mass 2	Mass 3
$z_{o1} = 0$	$z_{o2} = -1$	$z_{o3} = 1$
$\dot{z}_{o1} = -1$	$\dot{z}_{o2} = 2$	$\dot{z}_{o3} = -2$

Table 9.1: Initial conditions applied to tdof system.

By inspection, the mass and stiffness matrices in principal coordinates can be written as:

$$\mathbf{m}_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{k}_p = \left(\frac{k}{m}\right)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \tag{9.3}$$

9.3 Transforming Initial Conditions and Forces

9.3.1 Transforming Initial Conditions

The initial condition vectors are transformed to principal coordinates by:

$$\begin{aligned} \dot{\mathbf{z}}_{po} &= \mathbf{z}_n^{-1}\dot{\mathbf{z}}_o \\ \mathbf{z}_{po} &= \mathbf{z}_n^{-1}\mathbf{z}_o \end{aligned} \tag{9.4}$$

The inverse of $\mathbf{z}_n$, found using a symbolic algebra program:

$$\mathbf{z}_n^{-1} = \sqrt{m}\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & 0 & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & \frac{-\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \end{bmatrix} \tag{9.5}$$

$$\mathbf{z}_{po} = \mathbf{z}_n^{-1}\mathbf{z}_o = \sqrt{m}\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & 0 & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & \frac{-\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \sqrt{m}\begin{bmatrix} 0 \\ \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{6}}{2} \end{bmatrix} \tag{9.6}$$

$$\dot{\mathbf{z}}_{po} = \mathbf{z}_n^{-1}\dot{\mathbf{z}}_o = \sqrt{m}\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & 0 & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & \frac{-\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix} = \sqrt{m}\begin{bmatrix} \frac{-\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} \\ \frac{-7\sqrt{6}}{6} \end{bmatrix} \tag{9.7}$$

9.3.2 Transforming Forces

The force vector in principal coordinates is:

$$\mathbf{F}_p = \mathbf{z}_n^T\mathbf{F} = \frac{1}{\sqrt{m}}\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \frac{1}{\sqrt{m}}\begin{bmatrix} \frac{-\sqrt{3}}{3} \\ \frac{3\sqrt{2}}{2} \\ \frac{-\sqrt{6}}{6} \end{bmatrix} \tag{9.8}$$

9.4 Complete Equations of Motion in Principal Coordinates

Now the equations in principal coordinates can be written in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{z}_{p1} \\ \ddot{z}_{p2} \\ \ddot{z}_{p3} \end{bmatrix} + \left(\frac{k}{m}\right) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} z_{p1} \\ z_{p2} \\ z_{p3} \end{bmatrix} = \begin{bmatrix} \frac{-\sqrt{3}}{3} \\ \frac{3\sqrt{2}}{2} \\ \frac{-\sqrt{6}}{6} \end{bmatrix} \frac{1}{\sqrt{m}} \tag{9.9}$$

With initial conditions:

$$\mathbf{z}_{po} = \sqrt{m} \begin{bmatrix} 0 \\ \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{6}}{2} \end{bmatrix}, \quad \dot{\mathbf{z}}_{po} = \sqrt{m} \begin{bmatrix} \frac{-\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} \\ \frac{-7\sqrt{6}}{6} \end{bmatrix} \tag{9.10}$$

Summarizing the equations in tabular form:

Equations of Motion, Principal Coordinates	Displacement Initial Conditions: Principal Coordinates	Velocity Initial Conditions: Principal Coordinates
$\ddot{z}_{p1} = \frac{-\sqrt{3}}{3\sqrt{m}}$	$z_{p1o} = 0$	$\dot{z}_{p1o} = \frac{-\sqrt{3m}}{3}$
$\ddot{z}_{p2} + \left(\frac{k}{m}\right) z_{p2} = \frac{3\sqrt{2}}{2\sqrt{m}}$	$z_{p2o} = \frac{-\sqrt{2m}}{2}$	$\dot{z}_{p2o} = \frac{\sqrt{2m}}{2}$
$\ddot{z}_{p3} + \left(\frac{3k}{m}\right) z_{p3} = \frac{-\sqrt{6}}{6\sqrt{m}}$	$z_{p3o} = \frac{\sqrt{6m}}{2}$	$\dot{z}_{p3o} = \frac{-7\sqrt{6m}}{6}$

Table 9.2: Equations of motion and initial conditions in principal coordinates.

9.5 Solving Equations of Motion Using Laplace Transform

We will now take the Laplace transform of each equation and solve for the transient response resulting from a combination of the forcing function and the initial conditions.

Note that taking the Laplace transform of first and second order differential equations (DE) with initial conditions is (Appendix 2):

First Order DE: $$\mathcal{L}\{\dot{x}(t)\} = sX(s) - x(0) \tag{9.11}$$

Second Order DE: $$\mathcal{L}\{\ddot{x}(t)\} = s^2X(s) - sx(0) - \dot{x}(0) \tag{9.12}$$

Solving for z_{p1} using Laplace transforms:

$$\ddot{z}_{p1} = \frac{-\sqrt{3}}{3\sqrt{m}} \tag{9.13}$$

$$s^2 z_{p1}(s) - s z_{p1}(0) - \dot{z}_{p1}(0) = \frac{-\sqrt{3}}{s3\sqrt{m}} \tag{9.14}$$

$$s^2 z_{p1}(s) - s(0) - \left(\frac{-\sqrt{3m}}{3}\right) = \frac{-\sqrt{3}}{s3\sqrt{m}} \tag{9.15}$$

$$s^2 z_{p1}(s) = \frac{-\sqrt{3}}{s3\sqrt{m}} - \frac{\sqrt{3m}}{3} \tag{9.16}$$

$$z_{p1}(s) = \frac{-\sqrt{3}}{s^3 3\sqrt{m}} - \frac{\sqrt{3m}}{3s^2}$$

$$= \frac{-1}{s^3\sqrt{3m}} - \frac{\sqrt{3m}}{3s^2}$$

$$= \frac{-1}{s^3\sqrt{3m}} - \frac{\sqrt{3m}}{3s^2} \tag{9.17}$$

Back-transforming to time domain, noting that:

$$t^n \rightarrow \frac{n!}{s^{n+1}} \quad \text{or} \quad t^2 = \frac{2!}{s^{(2+1)}} \tag{9.18}$$

$$z_{p1}(t) = \frac{-t^2}{2\sqrt{3m}} \qquad \text{Forced Response}$$

$$+0 \qquad \text{Initial Displacement} \tag{9.19}$$

$$-\frac{\sqrt{3m}}{3}t \qquad \text{(Initial Velocity) x (Time)}$$

Substituting m = 1, k = 1:

$$z_{p1} = \frac{-t^2}{2\sqrt{3}} - \frac{\sqrt{3}\ t}{3} \tag{9.20}$$

Solving for z_{p2} using Laplace transforms:

$$\ddot{z}_{p2} + \left(\frac{k}{m}\right) z_{p2} = \frac{3\sqrt{2}}{2\sqrt{m}} \tag{9.21}$$

$$s^2 z_{p2}(s) - s z_{p2}(0) - \dot{z}_{p2}(0) + \left(\frac{k}{m}\right) z_{p2}(s) = \frac{3\sqrt{2}}{2\sqrt{ms}} \tag{9.22}$$

$$s^2 z_{p2}(s) - s\left(\frac{-\sqrt{2m}}{2}\right) - \frac{\sqrt{2m}}{2} + \left(\frac{k}{m}\right) z_{p2}(s) = \frac{3\sqrt{2}}{2\sqrt{ms}} \tag{9.23}$$

$$z_{p2}(s)\left[s^2 + \left(\frac{k}{m}\right)\right] = \frac{3\sqrt{2}}{s2\sqrt{m}} - \frac{s\sqrt{2m}}{2} + \frac{\sqrt{2m}}{2} = \frac{3\sqrt{2}}{s2\sqrt{m}} + \frac{\sqrt{2m}}{2}(-s+1) \tag{9.24}$$

$$z_{p2}(s) = \frac{\dfrac{3\sqrt{2}}{2\sqrt{m}}}{s\left(s^2 + \dfrac{k}{m}\right)} - \frac{s\left(\dfrac{\sqrt{2m}}{2}\right)}{s^2 + \dfrac{k}{m}} + \frac{\dfrac{\sqrt{2m}}{2}}{s^2 + \dfrac{k}{m}}, \qquad \omega_{3,4}^2 = \frac{k}{m} \tag{9.25}$$

Back-transforming to the time domain:

$$z_{p2}(t)=\frac{3\sqrt{2}}{2\sqrt{m}}\left[\frac{1}{\omega_2^2}\left(1-\cos\omega_2 t\right)\right]-\frac{\sqrt{2m}}{2}\left(\frac{\omega_2}{\omega_2}\right)\underbrace{\sin\left(\omega_2 t+90^{\circ}\right)}_{\cos(\omega_2 t)}+\frac{\sqrt{2m}}{2}\frac{1}{\omega_2}\sin\left(\omega_2 t\right) \tag{9.26}$$

Substituting m = k = 1, $\omega_2 = 1$:

$$z_{p2}(t)=\frac{3\sqrt{2}}{2}-\frac{3\sqrt{2}}{2}\cos(t)-\frac{\sqrt{2}}{2}\cos(t)+\frac{\sqrt{2}}{2}\sin(t) \tag{9.27}$$

Solving for z_{p3} using Laplace transforms:

$$\ddot{z}_{p3}+\omega_3^2 z_{p3}=\frac{-\sqrt{6}}{6\sqrt{m}} \tag{9.28}$$

$$s^2 z_{p3}(s)-s z_{p3}(0)-\dot{z}_{p3}(0)+\omega_3^2 z_{p3}(s)=\frac{-\sqrt{6}}{6s\sqrt{m}} \tag{9.29}$$

$$s^2 z_{p3}(s)+\omega_3^2 z_{p3}(s)-s\left(\frac{\sqrt{6m}}{2}\right)-\left(\frac{-7\sqrt{6m}}{\sqrt{6}}\right)=\frac{\sqrt{6}}{6s\sqrt{m}} \tag{9.30}$$

$$z_{p3}(s)\left(s^2+\omega_3^2\right)=\frac{-\sqrt{6}}{6s\sqrt{m}}+\frac{s\sqrt{6m}}{2}-\frac{7\sqrt{6m}}{\sqrt{6}} \tag{9.31}$$

$$z_{p3}(s)=\frac{-\sqrt{6}}{6\sqrt{m}}\left[\frac{1}{s\left(s^2+\omega_3^2\right)}\right]+\frac{\sqrt{6m}}{2}\left[\frac{s}{\left(s^2+\omega_3^2\right)}\right]-\frac{7\sqrt{6m}}{\sqrt{6}\left(s^2+\omega_3^2\right)} \tag{9.32}$$

Back-transforming to the time domain:

$$z_{p3}(t)=\frac{-\sqrt{6}}{6\sqrt{m}}\left[\frac{1}{\omega_3^2}\left(1-\cos\omega_3 t\right)\right]+\frac{\sqrt{6m}}{2}\left(\frac{\omega_3}{\omega_3}\right)\underbrace{\sin\left(\omega_3 t+90^{\circ}\right)}_{\cos\,(\omega_3 t)}-\frac{7\sqrt{6m}}{\sqrt{6}}\frac{1}{\omega_3}\sin\omega_3 t \tag{9.33}$$

Substituting $m = k = 1, \ \ \omega_3^2 = \frac{3k}{m}$:

$$z_{p3}(t) = \frac{-\sqrt{6}}{18} + \frac{\sqrt{6}}{18}\cos(\sqrt{3}t) + \frac{\sqrt{6}}{2}\cos(\sqrt{3}t) - \frac{7}{\sqrt{6}}\frac{\sqrt{6}}{\sqrt{3}}\sin(\sqrt{3}t)$$

$$= \frac{\sqrt{6}}{6}\left[\frac{-1}{3} + \frac{1}{3}\cos(\sqrt{3}t) + 3\cos(\sqrt{3}t) - \frac{7}{\sqrt{3}}\sin(\sqrt{3}t)\right] \qquad (9.34)$$

Note: $\frac{-\sqrt{2}6}{\sqrt{6}} - \frac{-\sqrt{2}\sqrt{3}\sqrt{2}\sqrt{3}\sqrt{2}}{\sqrt{3}\sqrt{2}} = -2\sqrt{3}$ and $\frac{\sqrt{6}}{\sqrt{3}} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{3}} = \sqrt{2}$

Now that the displacements in principal coordinates are available, they can be plotted to see the motions of each individual mode of vibration.

Displacements in principal coordinates can be back-transformed to physical coordinates:

$$\mathbf{z} = \mathbf{z}_n \mathbf{z}_p \qquad (9.35)$$

$$\mathbf{z}_p = \begin{bmatrix} z_{p1} \\ z_{p2} \\ z_{p3} \end{bmatrix} = \begin{bmatrix} \frac{-t^2}{2\sqrt{3}} - \frac{\sqrt{3}t}{3} \\ \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}\cos t - \frac{\sqrt{2}}{2}\cos t + \frac{\sqrt{2}}{2}\sin t \\ -\frac{\sqrt{6}}{18} + \frac{\sqrt{6}}{18}\cos\sqrt{3}t + \frac{\sqrt{6}}{2}\cos\sqrt{3}t - \frac{7}{\sqrt{3}}\sin\sqrt{3}t \end{bmatrix} \qquad (9.36)$$

$$\mathbf{z} = \mathbf{z}_n \mathbf{z}_p = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{-t^2}{2\sqrt{3}} - \frac{\sqrt{3}t}{3} \\ \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}\cos t - \frac{\sqrt{2}}{2}\cos t + \frac{\sqrt{2}}{2}\sin t \\ -\frac{\sqrt{6}}{18} + \frac{\sqrt{6}}{18}\cos\sqrt{3}t + \frac{\sqrt{6}}{2}\cos\sqrt{3}t - \frac{7}{\sqrt{3}}\sin\sqrt{3}t \end{bmatrix}$$

(9.37)

Rewriting the equations to highlight the contributions to the total motion in physical coordinates of each mode:

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} z_{n11} & z_{n12} & z_{n13} \\ z_{n21} & z_{n22} & z_{n23} \\ z_{n31} & z_{n32} & z_{n33} \end{bmatrix} \begin{bmatrix} z_{p1} \\ z_{p2} \\ z_{p3} \end{bmatrix} \tag{9.38}$$

$$z_1 = \underbrace{z_{n11} z_{p1}}_{\text{1st mode}} + \underbrace{z_{n12} z_{p2}}_{\text{2nd mode}} + \underbrace{z_{n13} z_{p3}}_{\text{3rd mode}}$$ Mode contributions to total z_1 motion

$$z_2 = \underbrace{z_{n21} z_{p1}}_{\text{1st mode}} + \underbrace{z_{n22} z_{p2}}_{\text{2nd mode}} + \underbrace{z_{n23} z_{p3}}_{\text{3rd mode}}$$ Mode contributions to total z_2 motion

$$z_3 = \underbrace{z_{n31} z_{p1}}_{\text{1st mode}} + \underbrace{z_{n32} z_{p2}}_{\text{2nd mode}} + \underbrace{z_{n33} z_{p3}}_{\text{3rd mode}}$$ Mode contributions to total z_3 motion

(9.39a,b,c)

Because the first mode motion for each degree of freedom is rigid body, and its displacement eventually goes to infinity, it masks the vibration motion of the second and third modes for long time period simulations. If the first mode (rigid body) motion is subtracted from the total motion of z_1, z_2, and z_3, the motion due to the vibration can be seen, as shown in Figure 9.8.

9.6 MATLAB code tdof_modal_time.m – Time Domain Displacements in Physical/Principal Coordinates

9.6.1 Code Description

The MATLAB code **tdof_modal_time.m** is used to plot the displacements versus time in principal coordinates using (9.19), (9.27) and (9.34) with $m = k = 1$. Displacements in physical coordinates are obtained by premultiplying principal displacements by the modal matrix.

9.6.2 Code Results

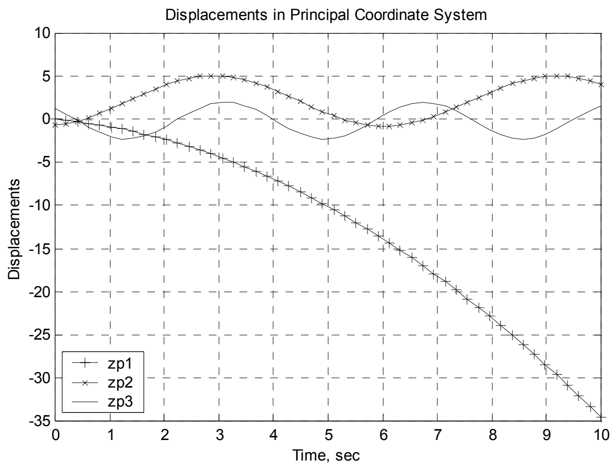

Figure 9.2: Displacements in principal coordinates, motion of the three modes of vibration.

The initial conditions in principal coordinates were 0, -0.707 and 1.225 for z_{p1}, z_{p2} and z_{p3}, respectively, which match the results shown in Figure 9.3.

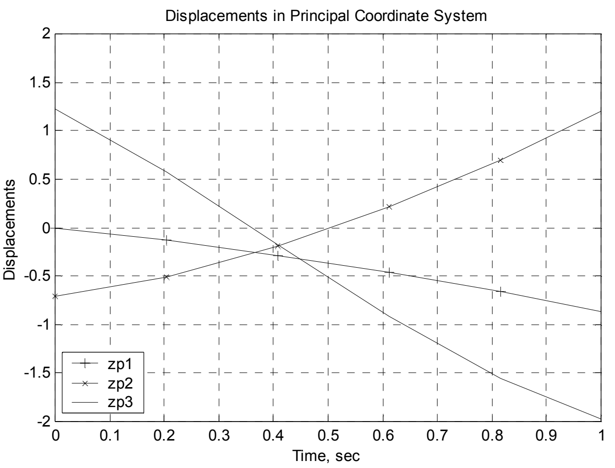

Figure 9.3: Displacements in principal coordinates, expanded vertical scale to check initial conditions.

Plotting the displacements in physical coordinates, where the initial displacement conditions in physical coordinates were 0, -1 and 1 for z_1, z_2 and z_3, respectively.

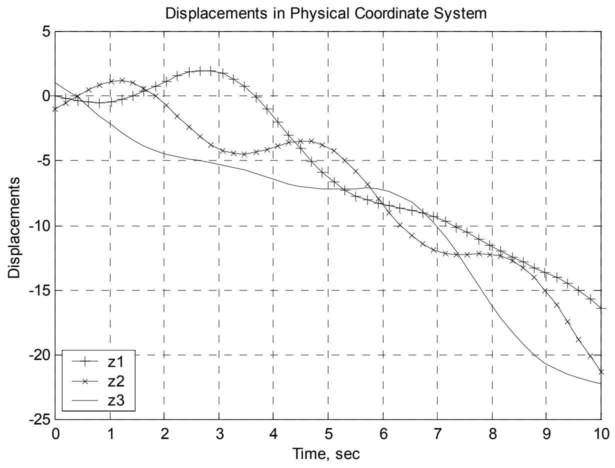

Figure 9.4: Displacement in physical coordinates.

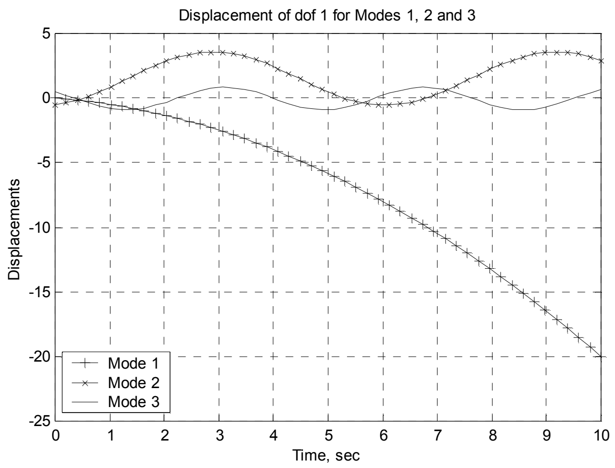

Figure 9.5: Displacements of mass 1 for all three modes of vibration.

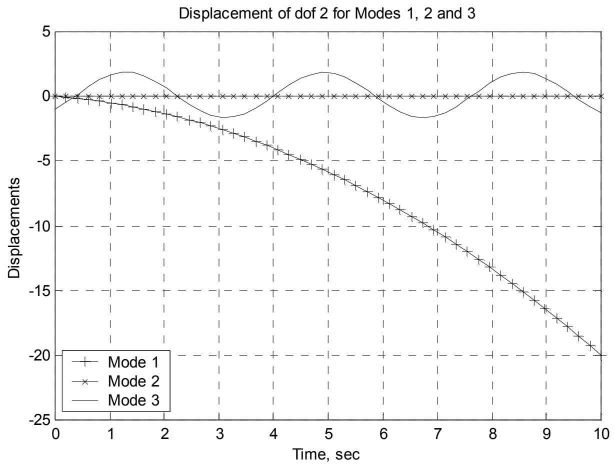

Figure 9.6: Displacements of mass 2 for all three modes of vibration.

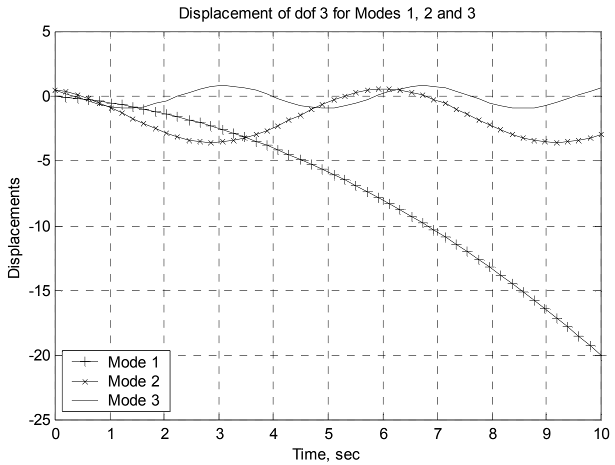

Figure 9.7: Displacements of mass 3 for all three modes of vibration.

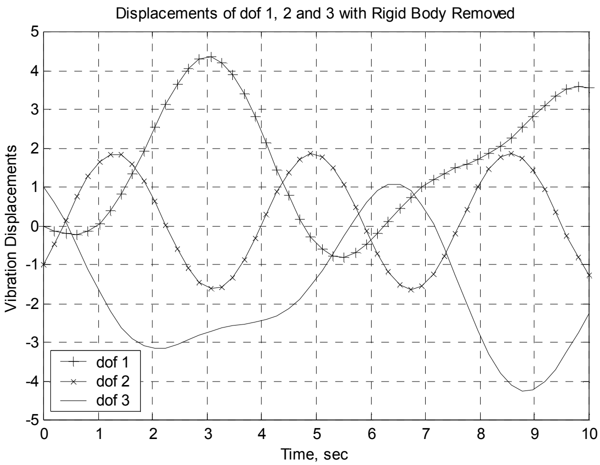

Figure 9.8: Displacements in physical coordinates, with the rigid body motion removed to show more clearly the oscillatory motion of the three masses.

9.6.3 Code Listing

```
%       tdof_modal_time.m    hand solution of modal equations

        clf;

        clear all;

%       define time vector for plotting responses

        t = linspace(0,10,50);

%       solve for and plot the modal displacements

        zp1 = (-t.^2/(2*sqrt(3))) - sqrt(3)*t/3;

        zp2 = 3*sqrt(2)/2 - (3*sqrt(2)/2)*cos(t) - (sqrt(2)/2)*cos(t) + (sqrt(2)/2)*sin(t);

        zp3 = (sqrt(6)/6)*((-1/3) + (1/3)*cos(sqrt(3)*t) + 3*cos(sqrt(3)*t) - …
                        (7/sqrt(3))*sin(sqrt(3)*t));

        plot(t,zp1,'k+-',t,zp2,'kx-',t,zp3,'k-')
        title('Displacements in Principal Coordinate System')
        xlabel('Time, sec')
        ylabel('Displacements')
        legend('zp1','zp2','zp3',3)
        grid

        disp('execution paused to display figure, "enter" to continue'); pause
```

```
            axis([0 1 -2 2])

            disp('execution paused to display figure, "enter" to continue'); pause

%           define the normalized modal matrix, m = 1

            zn = [1/sqrt(3)  1/sqrt(2)  1/sqrt(6)
                  1/sqrt(3)     0          -2/sqrt(6)
                  1/sqrt(3) -1/sqrt(2)  1/sqrt(6)];

%           define the principal displacement matrix, column vectors of principal displacements
%           at each time step

            zp = [zp1; zp2; zp3];

%           multiply zn times zp to get z

            z = zn*zp;

            z1 = z(1,:);
            z2 = z(2,:);
            z3 = z(3,:);

            plot(t,z1,'k+-',t,z2,'kx-',t,z3,'k-')
            title('Displacements in Physical Coordinate System')
            xlabel('Time, sec')
            ylabel('Displacements')
            legend('z1','z2','z3',3)
            grid

            disp('execution paused to display figure, "enter" to continue'); pause

%           define the motion of each each dof for each mode, zij below refers to the
%           motion of dof "i" due to mode "j"

            z11 = zn(1,1)*zp1;

            z12 = zn(1,2)*zp2;

            z13 = zn(1,3)*zp3;

            z21 = zn(2,1)*zp1;

            z22 = zn(2,2)*zp2;

            z23 = zn(2,3)*zp3;

            z31 = zn(3,1)*zp1;

            z32 = zn(3,2)*zp2;

            z33 = zn(3,3)*zp3;

            plot(t,z11,'k+-',t,z12,'kx-',t,z13,'k-')
            title('Displacement of dof 1 for Modes 1, 2 and 3')
```

```
          xlabel('Time, sec')
          ylabel('Displacements')
          legend('Mode 1','Mode 2','Mode 3',3)
          grid

          disp('execution paused to display figure, "enter" to continue'); pause

          plot(t,z21,'k+-',t,z22,'kx-',t,z23,'k-')
          title('Displacement of dof 2 for Modes 1, 2 and 3')
          xlabel('Time, sec')
          ylabel('Displacements')
          legend('Mode 1','Mode 2','Mode 3',3)
          grid

          disp('execution paused to display figure, "enter" to continue'); pause

          plot(t,z31,'k+-',t,z32,'kx-',t,z33,'k-')
          title('Displacement of dof 3 for Modes 1, 2 and 3')
          xlabel('Time, sec')
          ylabel('Displacements')
          legend('Mode 1','Mode 2','Mode 3',3)
          grid

          disp('execution paused to display figure, "enter" to continue'); pause

%         define the motion of each each dof with the rigid body motion for that
%         mode subtracted

          z1vib = z1 - z11;

          z2vib = z2 - z21;

          z3vib = z3 - z31;

          plot(t,z1vib,'k+-',t,z2vib,'kx-',t,z3vib,'k-')
          title('Displacements of dof 1, 2 and 3 with Rigid Body Removed')
          xlabel('Time, sec')
          ylabel('Vibration Displacements')
          legend('dof 1','dof 2','dof 3',3)
          grid

          disp('execution paused to display figure, "enter" to continue'); pause

          tplot = t;

          plot(tplot,z1,'k+-',t,z2,'kx-',t,z3,'k-')
          title('Displacements of dof 1, 2 and 3')
          xlabel('Time, sec')
          ylabel('Vibration Displacements')
          legend('dof 1','dof 2','dof 3',3)
          grid

          disp('execution paused to display figure, "enter" to continue'); pause

          save tdof_modal_time_z1z2z3  tplot z1 z2 z3
```

Problems

Note: All the problems refer to the two dof system shown in Figure P2.2.

P9.1 Using the equations, initial conditions and forcing functions from P7.4, solve for the closed form time domain response in principal coordinates using Laplace transforms. Back transform to physical coordinates and identify the components of the response associated with each mode.

P9.2 (MATLAB) Modify the **tdof_modal_time.m** code for the two dof system and solve for the time domain responses in both principal and physical coordinates using the equations, initial conditions and forcing functions from P7.4.