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MISSILE+ CONFIGURATION DESIGN

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To Ruth

PREFACE

The purpose of this book is to present some of the important basic principles involved in configuration design of guided missiles. Aerodynamic considerations are given primary emphasis in this text. Propulsion-system and structural-design aspects are also considered, since they are intimately related to the external aerodynamic configuration of the missile. The material is slanted primarily toward the aerodynamicist in preliminary design studies. It is also intended to serve as a working handbook for the aeronautical engineers in their “first assignments” in the missile industry. Aeronautical engineers who are in the “airplane-to-missile” transition phase will find this book helpful in stepping into the missile field. Finally, this book can be helpful to those engineers who are not directly concerned with external missile design, i.e., electro-mechanical and electronic missile-system engineers, in obtaining a good insight and hence some appreciation for the over-all composite missile design.

This book is an **outgrowth** of a company training course of The Martin Company, given by the author. The **text** is based largely on material developed for the **afore-mentioned** training class taught in several semesters during the period of **1957–1960**.

Chapter 1 states some of the most important **considerations involved** in the over-all guided missile weapon system design. In Chap. 2 a qualitative ‘discussion is presented on the types and classes of missiles and missile configurations together with their associated uses and their advantages and disadvantages. Chapter 3 discusses the aerodynamic characteristics of the various external **configuration** components to aid in the selection of these components for the **design** of an optimum configuration. Missile performance is treated thoroughly in Chap. 4, in which evaluation of the drag of the missile is first presented, followed by a detailed discussion of the methods of analysis of missile performance for a variety of types of missile

trajectories commonly used. The static longitudinal stability and control analysis for a simplified two degrees of freedom is presented in Chap. 5, which includes (1) model build-up of the aerodynamic derivatives involved and (2) the determination of static stability margin and load-factor capability for both forward- and rear-control configurations. Chapter 6 discusses the kinematics of the missile in maneuvering or accelerated flight and shows both analytically and graphically the effect of static stability margin on the load-factor or maneuvering capability of the missile.

Chapters 7 and 8 discuss the static directional and lateral-control design aspects of both cruciform and monowing missiles. The dynamic-stability problems are discussed in Chap. 9, which includes (1) solution of a simplified two-degree-of-freedom analysis using the classical method and (2) development and usage of transfer functions. Chapter 10 presents the method of evaluating component air loads and loading distribution for preliminary analysis. Chapter 11 discusses the general aerodynamic problems associated with missile launchings. The problem of air launching is discussed from the standpoint of safety to the parent aircraft and launch boundaries. Chapter 12 discusses the free-flight dispersion characteristics of the missile during boost and power-off flight. A simplified method is developed to predict missile drift due to cross wind during the latter phase of missile flight. Some dispersion-sensitivity factors in vacuum are derived to indicate the relative order of magnitude of various error effects. Also included in Chap. 12 is a brief discussion of the design aspects of a long-range ballistic-missile reentry body from the standpoint of heating and free-flight dispersion.

Chapter 13 discusses the fundamental principles of propulsion and the design aspects of both liquid-propellant and solid-propellant rocket motors. Finally, Chap. 14 deals with some of the more important structural-design aspects to round out the over-all missile configuration design. Several appendixes included in this volume contain derivations of equations and methods of analysis. Several methods of determining aerodynamic coefficients from flight-test data are presented in Appendix H.

It is not the intention of this book to include comprehensive data to enable the engineer to design his configuration. In the first place, much of the essential data is classified, and secondly, it would require several volumes to include the abundance of data desirable and necessary for a reasonably efficient design. However, sufficient

references are included to assist those who are actively engaged in this challenging field of endeavor. The author believes that one of the most important assets the missile engineer must have in discharging his assignment is a sound understanding and knowledge of the fundamental principles involved in configuration design. In addition, the author also believes that the missile engineer should have a broader view, and hence a better appreciation, of the various design aspects in order to achieve a more efficient design. With these thoughts in mind, the author dedicates this book.

The author would like to express his appreciation to H. C. Sanderson, F. E. Nixon, and C. E. Hogan for their assistance, helpful suggestions, and encouragement during the writing and preparation of this book. He would also like to express his gratitude to the National Aeronautics and Space Administration (NASA) for the liberal use of their references and technical data. Many sincere thanks are due to the following associates at The Martin Company: J. E. Coster, Information Service Division; J. L. Grabber, Legal Division; C. D. Pierson, H. C. Johnson, R. T. Patterson, N. E. Singletary, C. Padera, and C. M. Bowden, Engineering Division. The author would also like to express his gratitude to Mrs. Dorothy Carrigan for her assistance in the numerical calculations and to Miss Peggy Yow, Mrs. Joyce Hartman, and Mrs. Eve Morgan for their assistance in typing the manuscript.

Finally, the author wishes to express his appreciation to his wife, who proofread the manuscript and exhibited much patience during its preparation.

S. S. Chin

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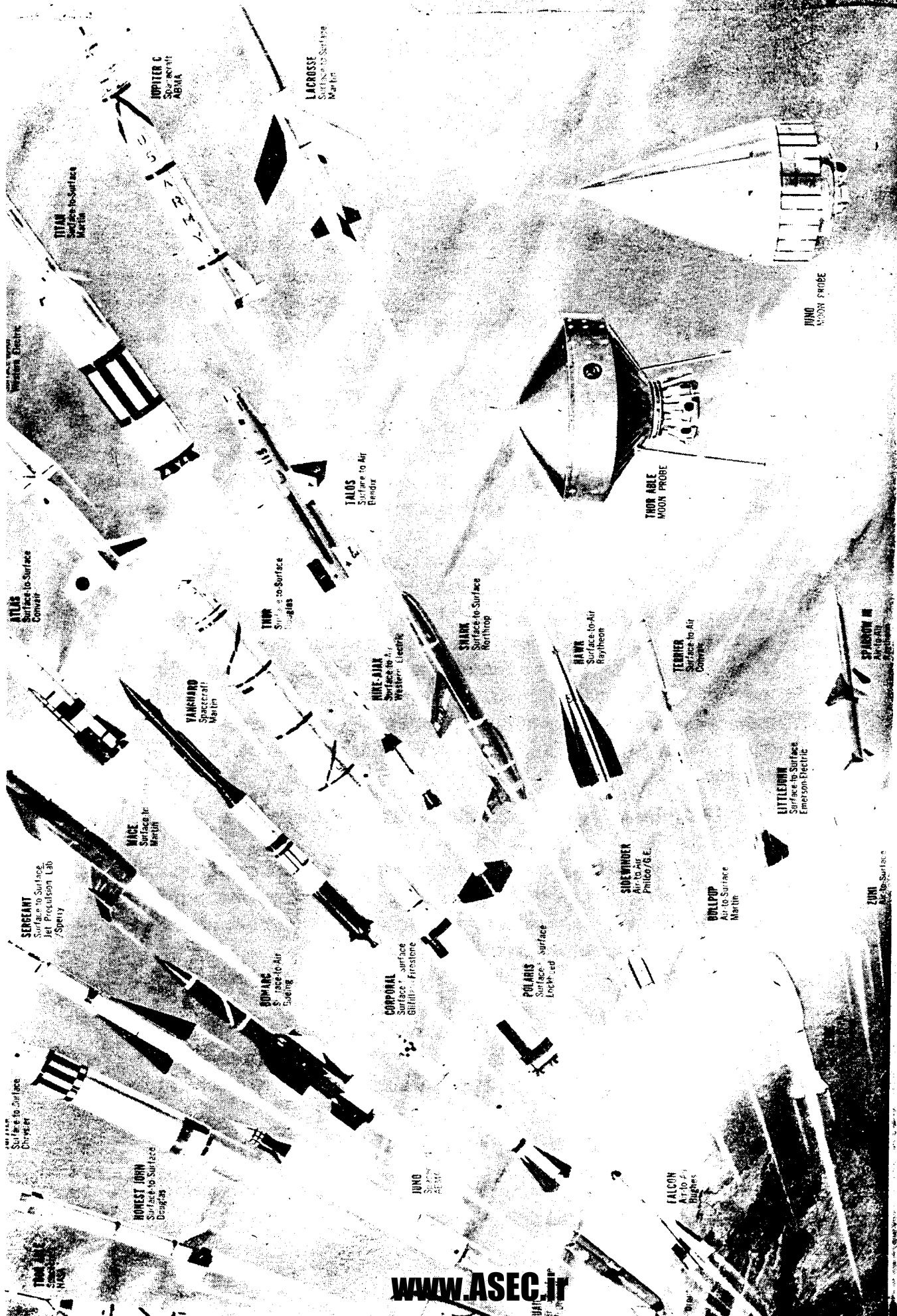
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CHAPTER I

INTRODUCTION

The design of missile configurations is one of the most interesting and challenging fields—and perhaps the most complex—for the aeronautical design engineer. It requires a reasonably broad knowledge of the fundamentals of many technical specialties—aerodynamics, thermodynamics, kinematics, propulsion, structural design, etc. A quick glance at the external configurations of United States missiles indicates that these configurations (Fig. 1-1) would never have been designed if the individual design specialist—aerodynamicist, propulsion engineer, etc.—had had complete control of the final design. More often than not, the missile configuration culminates from many design compromises to achieve a final system that meets the over-all weapon requirements. In general, the optimization of design is gained by careful analysis of the following considerations:

1. Simplicity in external configuration to reduce development time and cost
2. Efficient aerodynamic control surfaces to simplify control and guidance-system circuits and to minimize servo power requirements
3. Missile range, speed, and other performance characteristics that satisfy the mission requirements
4. Adequacy of the airframe from the standpoint of stability, maneuverability, and dynamic responses
5. Simple, efficient, and highly reliable power plant
6. Low cost, producibility, and lightweight airframe construction
7. Accuracy of the control and guidance systems to accomplish the desired mission
8. Reliability of the complete weapon system as well as its individual components
9. Efficiency in packaging the various major components to facilitate check-out and replacement



10. Degree of complexity in the preparation and delivery of the missile to accomplish its mission

It is beyond the scope of a single volume to elaborate upon all the major considerations involved in missile design. Hence this book is restricted primarily to the aerodynamic design considerations. However, since the propulsion-system and structural-design aspects are so intimately related to the external aerodynamic configuration of the missile, a brief treatise on the latter two facets is considered appropriate for the final chapters.



FIG. 1-1. Panorama of missiles (see facing page). (*By permission from Remsen Advertising Agency, Inc., ©April, 1959.*)

CHAPTER 2

GENERAL AERODYNAMIC DESIGN CONSIDERATIONS

2-1. INTRODUCTION

One of the primary goals of the aerodynamics design engineer is to develop a simple and yet efficient external aerodynamic configuration. As pointed out in Chap. 1, he often has to compromise his design because of many other important considerations. Consequently many types of design are evolved, each resulting from a series of design compromises. The purpose of this chapter is to discuss the various missile configurations together with their associated uses, advantages, and disadvantages.

2-2. CLASSES OF MISSILES

Most missiles may be classified into four general classes: AAM, SAM, ASM, and SSM.

The air-to-air missile (AAM) is one which is launched from one airplane against another flying aircraft or air target. This type of missile is generally of the smaller variety because of (1) the carrying capacity of the relatively small fighter aircraft and (2) the relatively short range requirements. Because of the relatively short range involved, the AAM's are usually the boost-glide type; i.e., they are boosted to their maximum speed with their own power plant and glide (or coast) the rest of the way to the target. A solid-propellant rocket motor is most frequently used on this type of missile because of its inherent simplicity, reliability, and attractive handling features. However, technical advances in prepackaged-liquid rocket motors have placed these power plants in a competitive position with the solid-propellant units, as will be discussed in Chap. 13. In some cases a boost-sustainer rocket motor is used to improve the performance characteristics of the missile. The merit of this type of rocket motor will be discussed in Chap. 13.

The aerodynamic problems associated with launching these missiles straight ahead of the parent aircraft are relatively simple. However, caution must be exercised to account for the possibility of airflow interference between the missile and airplane. This interference problem is particularly critical for high-speed launchings from highly swept-wing fighters. This problem of airplane safety and missile-launch dispersion will be discussed in detail in Chap. 11. In the special role of a bomber defense, for which the AAM may be retrolaunched or launched perpendicular to the flight direction of the bomber, detailed analysis is required to determine its launch dispersion characteristics.

The maneuverability requirement of an AAM is generally the most severe in comparison with other classes of missiles. This is due to (1) the relatively high launching speeds, (2) high target velocity and maneuverability, and (3) relatively short flight time for flight-path corrections. In addition to the severe maneuverability requirements the wide range of operating altitude presents the designer a very challenging problem. For example, the missile-maneuverability requirement generally varies linearly from sea level to some altitude and then remains at a constant value up to the maximum design altitude, as shown in Fig. 2-1. However, if the missile is designed to satisfy the low-altitude (say sea-level) maneuvering requirement with maximum control deflection the missile will not meet the high-altitude design requirement as a result of the appreciable reduction in air density, as indicated in Fig. 2-1. Hence the maximum altitude generally dictates the missile design to meet the maneuvering requirement with maximum control deflection. This results in extremely large loading at the low-altitude conditions if full control deflection is used or inadvertently realized. In order to keep the structural weight to a reasonable level and avoid excessive flight loads either the control travel must be limited as a function of operating altitude or some g -sensing device must be used to return the control surfaces to neutral or some low value.

The surface-to-air missile (SAM) is normally designed for area

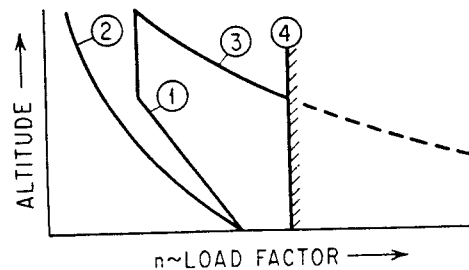


FIG. 2-1. Altitude vs. load factor (constant Mach number). (1) Requirement. (2) $\delta_{\max}$ designed for minimum altitude condition. (3) $\delta_{\max}$ designed for maximum altitude condition. (4) Structural limit.

defense against attacking aircraft or ICBM's. As a result the aerodynamic design depends upon its range, which can vary from a few miles to several hundred miles. For relatively short ranges, a single-stage solid- or liquid-propellant rocket motor is generally used. Again a boost-sustainer trajectory may be used to improve the missile performance characteristics. For longer ranges multistaging is often required in order to reduce the over-all weight of the missile. Ram jets or other air-breathing power plants prove to be advantageous from the economy and weight standpoint. Perhaps one of the chief disadvantages of using an air-breathing ram jet is that very large external and/or droppable boosters are required to boost the missile to ram-jet ignition speeds. The guidance system may vary significantly for different range requirements. For relatively short ranges, beam-riding or even infrared guidance may provide sufficient accuracy for the desired probability of kill. However, for long-range intercepts some form of homing or terminal guidance system is required to ensure an effective kill. The maneuverability requirement for this class of missile can be quite severe, particularly against fast-flying low-altitude targets. Generally this type of missile is launched vertically or at a relatively high elevation to maximize the "effective area" of the missile weapon system.

The air-to-surface missile (ASM) is normally designed as an offensive or tactical weapon to knock out enemy surface targets such as gun emplacements, tanks, ships, or airfields. The launching problems associated with this type of missile are similar to those associated with the straight-forward launchings of the AAM's. Again, the range of this type of missile varies from a few miles to hundreds of miles. For relatively short range missions the ASM is normally boosted off the pylon because of its relatively light weight. For long-range applications the ASM is usually dropped off the pylon, attitude-stabilized, and gradually boosted up to the required speeds. Multistaging may be beneficial for this type of missile if the range is appreciable. Careful consideration must be given to the missile-aircraft installation in order to minimize the performance degradation to the parent aircraft, particularly with the large-sized long-range ASM's. The maneuverability requirement for the ASM's is generally much lower than that for the AAM's because of the slow-moving or stationary ground targets.

The surface-to-surface missile (SSM) may be used in two separate roles in war. One is as a long-range offensive or retaliatory weapon; the other is for use in support of ground troops in the role of an

artillery battery. The former weapon may have a range which varies from one thousand to several thousands of miles and may fly the cruise or ballistic type of trajectory. For the latter type of trajectory, multistaging is mandatory to reduce the size of the rocket motor and the over-all weight of the missile for this mission. An indication of the gains realized by staging will be illustrated in Chap. 4. Because of their weight these missiles are always launched from their upright position on the launch pads. The maneuverability requirement is very low, and the chief control requirement arises from wind-profile (shear) effects on the missile trajectory during its ascent. The reentry body (in MRBM, IRBM, and ICBM applications) of such missiles is a design which merits many special considerations; it will be discussed in Chap. 12.

As an artillery weapon the SSM is normally boosted off a ground launcher and coasts to the target. In general only a moderate degree of maneuverability is required for this type of missile. In the case of an unguided missile (i.e., Honest John) the stability requirement is dictated from launch-dispersion considerations which will be detailed in Chap. 12. A special type of SSM is one which is launched from beneath the water (i.e., Polaris).

2-3. TYPES OF DESIGN AND CONTROL

Many types of external aerodynamic configurations (and modifications thereof) used for the four classes of missiles previously described are shown in Fig. 2-2. Each of these will be discussed briefly to point out the advantages and disadvantages.

1. Wing Control. A wing-control configuration consists of a relatively large wing located close to the center of gravity of the missile and a set of tail or stabilizing surfaces at the aft end of the missile. This type of control is used mostly in an AAM because of its extremely fast response characteristics. As shown in Fig. 2-3, instantaneous lift is developed as a result of wing deflection to provide lift for maneuvering flight. Additional lift is developed as angle of attack builds up. Control effectiveness (pitching or turning moment due to control-surface deflection) $C_{m\delta}$ from the wings is generally very low, since the lift developed is located close to the center of gravity of the missile. However, the resulting downwash has a beneficial effect on the tails since it produces a down load which provides the desired turning moment to develop angle of attack for additional lift previously mentioned. This beneficial downwash

effect on control effectiveness is readily apparent in Fig. 2-4. The wing location for a wing-control design is very critical, particularly for cases where large center-of-gravity travel occurs between launch

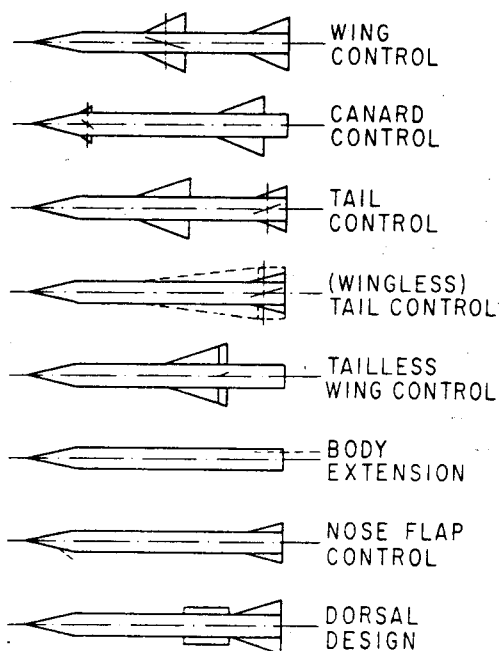


FIG. 2-2. Types of design and control.

and rocket-motor burn-out conditions. Since the wing surfaces are generally mounted very near the launch center of gravity (in order to obtain the desired degree of static stability with a reasonably small-sized tail), the forward shift in center of gravity can result in a substantial reduction in control effectiveness and hence maneuverability per unit wing deflection, as shown in Fig. 2-4.

Because of the added lift due to wing deflection the angle of attack required for a given trim lift coefficient or load factor is generally lower than that encountered in a tail control. This low value of trim angle of attack is advantageous particularly from the standpoint of inlet design for air-breathing power-plant and guidance-seeker designs. The aerodynamic hinge moments are relatively large because of the large wing area. Hence careful analysis must be made to optimize the hinge-line location for the entire range of flight Mach numbers to minimize the servo-power requirements. Lateral control may be obtained by superimposing differential wing deflections on top of the basic pitch-and-yaw deflection, thereby simplifying the servo system. However, several poor features associated with this type of control must be carefully considered before this design is selected for particular application. These are (1) relatively large loss in tail effectiveness due to downwash ϵ_α to static-stability contribution,

and rocket-motor burn-out conditions. Since the wing surfaces are generally mounted very near the launch center of gravity (in order to obtain the desired degree of static stability with a reasonably small-sized tail), the forward shift in center of gravity can result in a substantial reduction in control effectiveness and hence maneuverability per unit wing deflection, as shown in Fig. 2-4.

Because of the added lift due to wing deflection the angle of attack required for a given trim lift coefficient or load factor is generally lower than that encountered in a tail control. This low value of trim angle of attack is advantageous particularly from the stand-

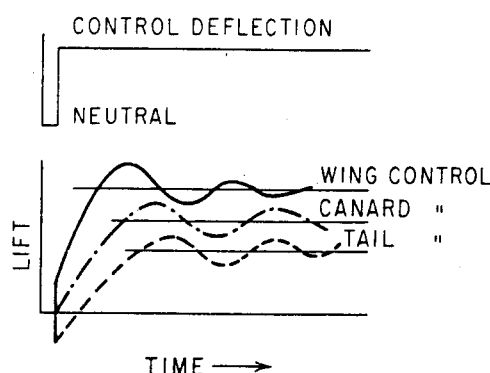


FIG. 2-3. Comparison of missile responses.

(2) nonlinear aerodynamics resulting from downwash due to both wing deflection ϵ_δ and angle of attack ϵ_α , and (3) severe adverse rolling moments induced on the tail surfaces from combined effects of angle of attack and wing deflection. The latter effect will be discussed in more detail in Chap. 8.

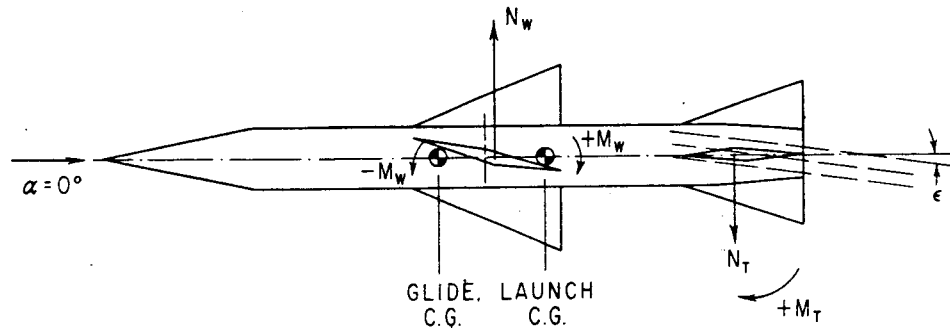


FIG. 2-4. Aerodynamics of wing control.

2. Canard Control. A canard-control configuration consists of a set of small control surfaces called canards located well forward on the body and a set of large surfaces (wing or tail) attached to the middle or aft section of the missile. In contrast to the wing-control configuration, the canards (because of their small size) do not generate a significant amount of downwash to affect the longitudinal-stability characteristics adversely. Thus relatively large static-stability margins can easily be obtained by simple changes in wing

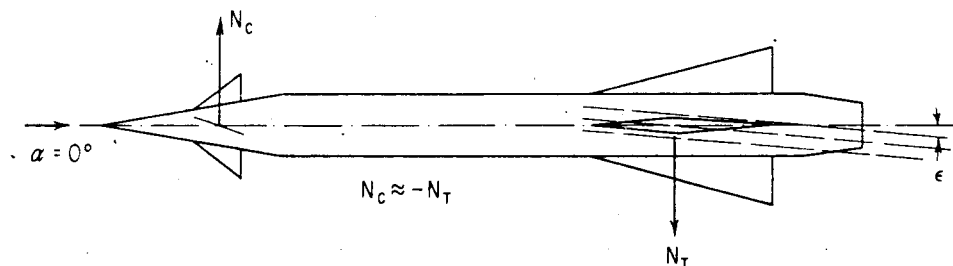


FIG. 2-5. Comparison of canard and tail loads.

location. Lift on this type of configuration is derived almost entirely from angle of attack since the lift of the canards is generally nullified by the down load on the wing caused by downwash, as shown in Fig. 2-5. One of the most attractive features in the canard configuration is the inherent simplicity of packaging due to the small size of the control system and its location in the nose. In addition, changes in center-of-gravity locations due to the design changes may

easily be accommodated by simple relocation of the wings. Since the total lifting-surface area is generally lower, the total drag and weight of the missile are reduced.

Some of the major disadvantages of the canard configuration are: (1) Roll stabilization is difficult if not impossible with the use of the canard surfaces because of their size and downwash effect on the wings. Consequently, a more complex method of providing roll control is required, i.e., wing-tip controls. (2) Relatively high control-surface rates are required to obtain the desired rate of response since angle of attack must be generated before any lift is developed, as shown in Fig. 2-3. As the result of the higher surface-rate requirement, an increase in servo-power requirement is demanded. However, the resultant servo-power requirement is generally lower than that required for the wing-control configuration. Therefore, for relatively small missiles which do not require roll-attitude stabilization, the canard configuration is probably the best over-all configuration.

3. Tail Control. With this type of control, it is obvious that the tail deflection δ_T must be opposite in direction to the angle of attack. This feature results in relatively slow response characteristics, since

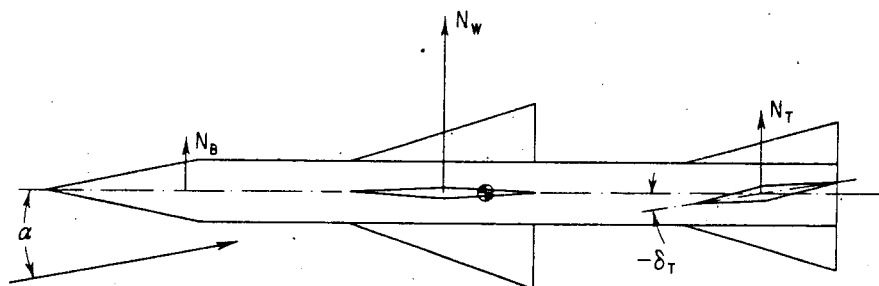


FIG. 2-6. Typical component loading—tail control.

the initial lift is in a direction opposite to the desired one, as shown in Fig. 2-3. However, it is advantageous in that the tail loads and hinge moments are kept relatively low as the total angle of attack on the tail is reduced (i.e., $\alpha_T = \alpha - \epsilon - \delta_T$). This in turn reduces the body bending to a minimum, since most of the resultant load is concentrated on the main wings located near the center of gravity, as shown in Fig. 2-6. In addition, the wing-tail interference effects are reduced, since the forward main lifting surfaces are fixed, i.e., no downwash due to wing deflection ϵ_δ . Therefore, the aerodynamic characteristics are more linear than those for a wing-control design. Major disadvantages associated with this configuration are (1)

limited space at the tail end for a control mechanism if a solid rocket motor is used, and (2) deficiency of tail surfaces to provide the desired lateral control.

A special case of a tail control is one which does not have a wing. This type of configuration is restricted to missiles which require relatively low maneuverability. The body in this case provides a goodly portion of the lift with an attendant large angle of attack. This type of configuration may be used for reentry bodies which may require small flight-path guidance upon entering the sensible atmosphere. The tail size and location may be chosen to provide the desired static- and dynamic-stability characteristics to achieve a satisfactory reentry to low altitudes.

Another special case of a tail control is one in which a very low aspect ratio or arrow wing is employed. The tail in this case is really the trailing part of the wing cut out for control, as shown in Fig. 2-2. For reasons mentioned above, the aerodynamic hinge moments are generally very low. However, the trim angle of attack for this type of configuration is rather high because of the very low aspect ratio surfaces.

4. Tail-less (or Wing Control). A tail-less type of configuration involves one set of surfaces (wings) with control flaps located at the trailing edge. The chief advantage of this type of configuration is the reduced number of surfaces, which results in reduced drag and manufacturing costs. The most obvious disadvantage of this type is that the location of the wing is extremely critical. Locating the wing too far aft would provide such excessive stability as to require very large control-surface size or deflection to get the desired load factor. On the other hand, locating the wing too far forward near the center of gravity would cause a reduction in control effectiveness as well as in aerodynamic damping, which is generally required and/or desirable in most missile systems.

5. Body Extension. Another method of providing control is the profitable use of the base pressure or, rather, the pressure differential between the base and the free stream. A segment extending aft of the body into the region of differential pressure as shown in Fig. 2-7 experiences a lift force for control. Chief advantages associated with this design are (1) it has an extremely low servo-power requirement since there is no hinge moment, and (2) it is simple and compact in design. Disadvantages are (1) low maneuverability, (2) inoperability at subsonic speeds where the base pressure is practically nonexistent, and (3) reversal in control between power-on and power-off

conditions due to jet effects. If positive static stability is required, flare tail configuration is readily adaptable to this type of control, since it also provides additional space at the aft end for installation of the associated control mechanisms.

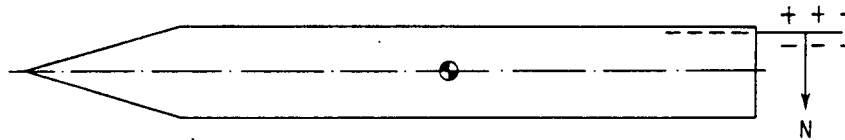


FIG. 2-7. Principle of body extension.

6. Nose-flap Control. This type of control is composed of segments of a nose section or flap extended from each of the four quadrants. When not in use they are retracted to form the nose contour. This type of control, when used with extremely low aspect ratio tail surfaces, may be suitable for air-launched missiles because of its compact design. The degree of stability required from a launching standpoint may be realized with these control surfaces retracted. However, this type of control has a rather low maneuverability characteristic and may be considered only in limited applications.

7. Dorsal. In certain applications where the over-all span of the missile is severely limited the use of "dorsal" may be necessary. Dorsal, when used in missile design, may be considered as an "aerodynamic fix" required to make up for the loss in aerodynamic efficiency due to reduction in tail aspect ratio and/or area. Since this fix is rather inefficient it may be used only in limited and specialized cases.

8. Jet Control. This type of control may be subdivided into four classes: (1) simple auxiliary jet (rocket) reactions, (2) gimbale engine and/or rocket nozzles, (3) jet vanes, and (4) jetavators. The first type of control may be used to good advantage especially on small missiles of relatively short flight duration or on reentry bodies to provide stabilization or spin (rolling velocity) to minimize dispersion in free flight. Jet or air injection over aerodynamic surfaces for augmentation may also be employed. Gimbale liquid-propellant engines have been used successfully for many years. For example, the Viking research vehicles have been successfully flown many times using this type of control during phases of flight wherein aerodynamic control is inadequate (i.e., at lift-off where the dynamic pressure is low or at extremely high altitudes where the air density

is low). Solid-propellant rocket motors, however, ordinarily cannot be gimballed; other means of jet control must therefore be used. Swivel nozzles on this type of rocket motor (as well as on liquid-propellant rocket motors) are practical and may be employed.

In addition to the afore-mentioned jet controls, jet vanes are frequently used to provide or supplement aerodynamic controls. Although the loss of total impulse caused by these vanes submerged in the rocket-motor exhaust is relatively low (i.e., generally less than 5 per cent), the major problem is to develop a material to withstand the high temperatures for a prolonged operating period. Consideration has been given to retraction of the vanes out of the hot jet exhaust when not in operation. However, the temperature shocks associated with such a mode of operation may be equally detrimental. In addition, the complexity and added weight of the retraction mechanism are undesirable from the standpoint of reliability, cost, and missile performance. The heating problem associated with the use of jet vanes is particularly critical when high-energy solid or liquid propellants are used because of their much higher operating temperatures. Jetavators are relatively novel devices and may be used for controls. Jetavator control appears to be adequate and requires relatively low operating forces because of its low hinge moments. Under certain design conditions it may be advantageous to discard all aerodynamic stabilizing and control surfaces and rely completely on the above-mentioned jet controls. However, detailed analysis is required before one of these jet controls is selected as either a supplementary or sole means of control for a given missile-design requirement.

In addition to the above description of the various classes of missiles and types of control a few words on wing arrangements are felt necessary to close this chapter on general design considerations.

9. Monowing. The monowing arrangement is generally used on cruise-type missiles, i.e., missiles designed to cruise for relatively long ranges like manned aircraft. This type of design is generally lighter and has less drag than the cruciform configuration. The wing area and span are, however, somewhat larger. Although the monowing missile must bank to orient its lift vector in the desired direction during maneuvering flights, the response time may be sufficiently fast and acceptable from a guidance-accuracy standpoint.

10. Triform. This type of wing arrangement, which employs three panels or wings generally of equal area spaced 120° apart, is seldom used since no noticeable advantage can be realized. Results

of a brief preliminary analysis indicate that the total wing area of the triform is equal to that used on a cruciform arrangement and that consequently no noticeable change in drag may be realized. In addition, little or no weight saving will be realized, even though one less attachment or fitting is required, because the total load remains the same. Thus the increased load per panel requires heavier fitting for the triform configuration. Also, the triform panels will have a somewhat larger span with a resulting outboard shift in loading; consequently the weight per unit area increases, partially nullifying the gain in the elimination of one attachment. Triform tail arrangements may be preferred over cruciform when used in conjunction with monowing design to provide the desired degree of directional stability (see Chap. 7).

11. Cruciform. Undoubtedly the most commonly used configuration in missile design is the cruciform. There are several major advantages in the use of this type of configuration.

1. Fast response in producing lift in any direction
2. Identical pitch and yaw characteristics
3. Simpler control system as the result of item 2

One of the most important aspects associated with a cruciform design is the orientation of the tail surfaces with respect to the wing planes. Considerable experience was accumulated and experi-

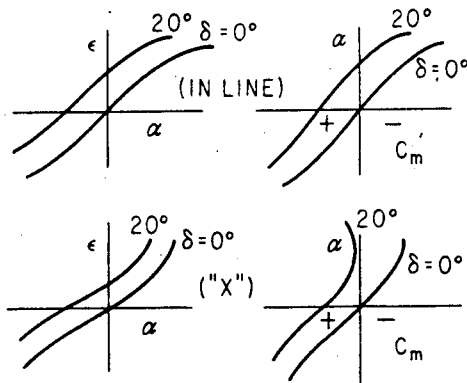


FIG. 2-8. Aerodynamic characteristics of in-line and X tail configurations.

mental data were derived in the late forties and early fifties on this subject. The significant conclusion was that an in-line tail surface provides the best over-all aerodynamic characteristics for most missile applications. This conclusion was made on the basis that:

1. Desirable stable trim conditions at high angles of attack and control-surface deflections are realized, whereas an interdigitated tail configuration results in unstable or divergent conditions.

2. Although an in-line tail configuration provides low stability at low angles of attack, this is a feature which can easily be compensated for by simple changes in the control-system gains.

To appreciate the significance of this important design aspect,

the downwash and resultant pitching-moment curves are illustrated in Fig. 2-8. The variation of downwash angle with free-stream angle of attack is the predominant factor in the determination of the complete model aerodynamic pitching-moment characteristics, since the pitching-moment curves for the body alone, body plus wing, or body plus tail configurations are, in general, relatively linear. The

TABLE 2-1. SUMMARY OF AERODYNAMIC DESIGN CHARACTERISTICS

Type of control	Advantages	Disadvantages
Wing control. . . .	Fast control Low trim α Relatively good packaging feature Beneficial downwash from canard deflection for control	High hinge moments Severe servo power required Nonlinear aerodynamics Large induced rolling moments cg travel critical High drag Large downwash decreases tail contribution to static stability
Canard control. . .	Good packaging feature Low hinge moments Fairly linear aerodynamics cg travel not critical Facilitates design changes Low drag	No simple lateral control Relatively large body bending moments High control rates required Relatively high trim α
Tail control.	Low tail loads Low tail hinge moments Low body bending moments Fairly linear aerodynamics	Slow response Negative C_{N_δ} (initial force in wrong direction) Packaging problems Poor lateral control High trim α

relatively large change in downwash with angle of attack in the low region for the in-line tail configuration results from the fact that the tail is located directly in the downwash region. Consequently relatively large losses in static-stability contribution by the tail surfaces are evident in the complete model pitching moments. As the angle of attack increases to a sufficiently large value, the tail surfaces move out of this downwash region, and an increase in stability results, as shown in Fig. 2-8. For the X, or interdigitated, configuration the reverse situation is evident.

SYMBOLS

C_N	normal-force coefficient
$C_{N\delta}$	normal-force effectiveness (variation of C_N with δ) $\partial C_N / \partial \delta$
C_m	pitching-moment coefficient
$C_{m\delta}$	pitching-moment (or control) effectiveness $\partial C_m / \partial \delta$
M	pitching moment
N	normal force
α	angle of attack
ϵ	downwash angle
ϵ_α	variation of ϵ with α , $d\epsilon/d\alpha$
ϵ_δ	variation of ϵ with δ , $d\epsilon/d\delta$
δ	control-surface deflection

Subscripts

B	body
C	canard
T, t	tail
W	wing

CHAPTER 3

AERODYNAMIC CHARACTERISTICS OF AIRFRAME COMPONENTS

3-1. INTRODUCTION

Now that some of the general design aspects of guided missile configuration design have been discussed, it is necessary to get into a detailed discussion of some of the specific items or components which make up the complete missile.

As a missile flies through the air it experiences aerodynamic forces and moments. These forces, lift and drag, may be classified into two general types: (1) those due to air friction and (2) those due to pressure. The former type, i.e., drag, is created by the shearing action of the air due to its viscosity and the latter by differences in surface pressures which result in the creation of both lift and drag forces. In supersonic missile-design studies it is more convenient to consider normal forces, i.e., forces perpendicular to the missile axis, in lieu of lift forces. The reason for this is that the component section and aerodynamic lifting surfaces are generally symmetrical about the longitudinal axis or chordwise center line; the resultant aerodynamic-pressure forces on these symmetrical sections are thus normal to the longitudinal axis or wing chord. In addition, most wind-tunnel strain-gauge balance systems are arranged to measure forces normal and parallel to the longitudinal axis of the model. An additional advantage to using normal forces is in the stability analysis, as will be discussed in Chaps. 5 and 9.

Before going into the discussion of the aerodynamic characteristics of the airframe components it may be well to show the resolution of these forces. From Fig. 3-1 the following force relations are derived:

$$\begin{aligned} D &= A \cos \alpha + N \sin \alpha \\ L &= N \cos \alpha - A \sin \alpha \end{aligned} \quad (3-1)$$

$$\begin{aligned} A &= D \cos \alpha - L \sin \alpha \\ N &= L \cos \alpha + D \sin \alpha \end{aligned} \quad (3-2)$$

To reduce Eq. (3-2) into coefficient form, divide by qS and Eq. (3-3) is obtained:

$$\begin{aligned} C_A &= C_D \cos \alpha - C_L \sin \alpha \\ C_N &= C_L \cos \alpha + C_D \sin \alpha \end{aligned} \quad (3-3)$$

It is interesting to note that the value of C_A first decreases and then increases with increasing angle of attack whereas C_D always increases with angle of attack, as indicated in Fig. 3-2.

In the following discussions, the airframe is assumed to be composed of the body plus lifting and control surfaces. A brief mention

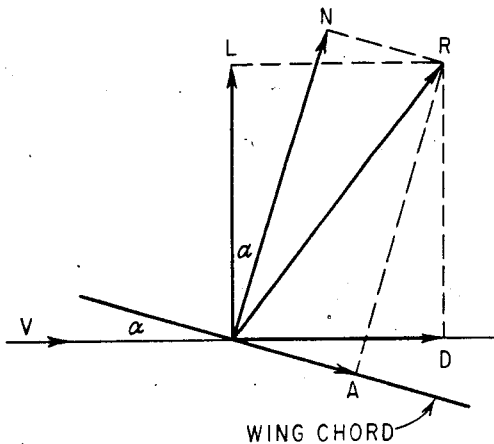


FIG. 3-1. Force resolution.

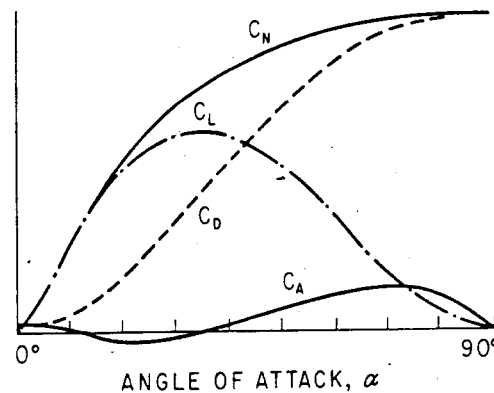


FIG. 3-2. Effect of α on C_N , C_L , C_A , and C_D .

of jet controls will also be included to complete the discussion on control characteristics.

3-2. BODIES OF REVOLUTION

The body of the missile may be divided into three major sections, the forebody, or nose; the mid-section; and the aft, or boattail, section. Forebodies may have many varieties of shapes, most common of which are conical, ogival, "power-series," or hemispherical. These shapes, shown in Figs. 3-7 and 3-11, are used primarily on missiles of supersonic speeds and are generally selected on the basis of combined aerodynamic, guidance, and structural considerations. For example, a hemispherical nose has very high drag from the aerodynamic drag or performance standpoint, but it is excellent from the standpoint of structural integrity, aerodynamic heating, and certain types of guidance (i.e., infrared). Since the pressure or wave drag may be several times that due to friction at supersonic

speeds, careful selection of the nose shape is mandatory to assure satisfactory performance and operation of the over-all system. Hence it is of interest to understand and appreciate the aerodynamic forces of these forebodies at supersonic speeds.

3-3. CONICAL FOREBODY

The supersonic aerodynamic characteristics of a conical forebody will be discussed first since many early theoretical and experimental

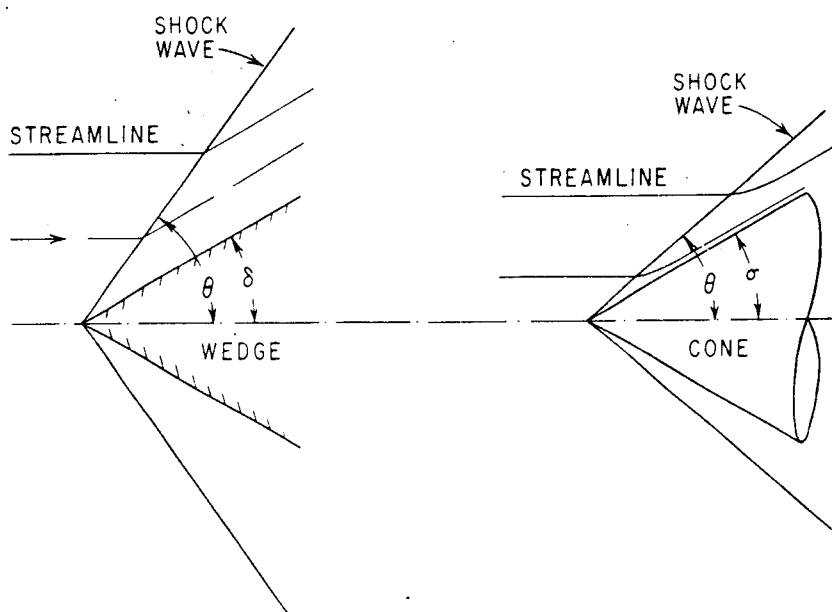


FIG. 3-3. Flow past a wedge and cone.

analyses were concentrated on this simple configuration. The supersonic flow over a cone has characteristics which are similar in appearance but markedly different in nature from those corresponding to two-dimensional flow (i.e., flow over a wedge). The similarity in appearance is that an oblique shock is formed at the tip of the wedge and apex of the cone (see Fig. 3-3). However, the flow characteristics aft of the shock wave, where the air undergoes both aerodynamic and thermodynamic changes, are noticeably different depending upon whether the flow is two-dimensional or three-dimensional as in this case of the conical nose. Briefly, these flow characteristics are (1) the shock angle, (2) streamlines or flow direction, and (3) air properties between the shock wave and surface of the body. Because the air can change direction in all planes normal to the flow instead of being restricted to just one plane as in the

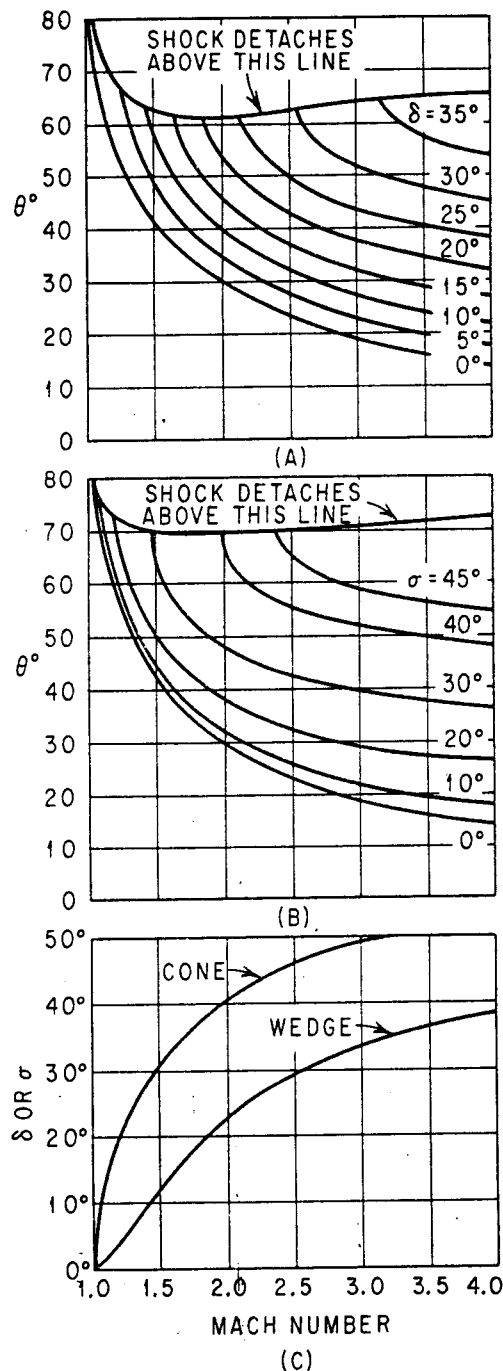


FIG. 3-4. (A) Shock-wave angle vs. Mach number in two-dimensional flow. (B) Shock-wave angle vs. Mach number for cones. (C) Detachment angle vs. Mach number for wedges and cones.

* Superscript numbers indicate references listed at the end of the chapter.

two-dimensional case, the shock angle θ for the same cone vertex angle and free-stream Mach number is noticeably less than that for the two-dimensional case (see Fig. 3-4). This corresponds to a much weaker shock, and consequently the air undergoes a smaller change in property behind the shock. The streamlines in two-dimensional flow change direction abruptly and travel in a direction parallel to the surface of the wedge, whereas in three-dimensional flow these streamlines turn gradually and approach the conical surface direction asymptotically as shown in Fig. 3-3. Finally, the density, pressure, etc., are not constant between the shock wave and the surface of the cone as they are in the case of a wedge. However, the pressure along any ray originating from the apex of the cone and hence along the cone surface is constant. This fundamental characteristic forms the basis for the well-known conical-flow theory.

The supersonic flow of air around cones at zero angle of attack has been thoroughly investigated by many experimenters, notable among whom were Taylor and Maccoll.^{1,2,*} The exact three-dimensional compressible-flow differential equations can be solved only by considering the symmetry of the cone, and thus the problem is reduced to one

in two dimensions. The resultant equation is of the second order and nonlinear and must be solved by means of an iteration process. Results derived may be found in the MIT cone tables^{3,4} and are plotted in *NACA Technical Report 1135*. Figures 3-4 and 3-5 show the shock-angle and pressure-coefficient variations with cone-geometry and Mach-number effects. These results are valid only for conditions under which the shock wave is attached to the cone. For a higher degree of accuracy the reader should consult the aforementioned references.

The pressure change between the free stream and surface of the cone—written in terms of a pressure coefficient [defined as $(p_2 - p_1)/q$ or $\Delta p/q$] as a function of Mach number—is expressed by an empirical equation within 5 per cent accuracy for semivertex cone angles up to 50° .⁶

$$\frac{p_2 - p_1}{q} = \frac{\Delta p}{q} = \left(0.083 + \frac{0.096}{M^2}\right) \left(\frac{\sigma}{10}\right)^{1.69} \quad (3-4)$$

where σ is in degrees

With the pressure defined, the drag (equivalent to axial force at zero lift or angle of attack) is simply related as

$$D = \frac{\Delta p}{q} qS = C_D qS \quad (3-5)$$

or

$$C_D = \frac{\Delta p}{q} \quad (3-6)$$

Equation (3-6) is the drag coefficient due to pressure on the cone alone and does not include base pressure or friction drag, which will be discussed in a later section.

The normal force on the cone is obtained by integrating the pressure forces in a direction normal to the center line of the conical body. The theoretical values obtained by various investigators are as follows:

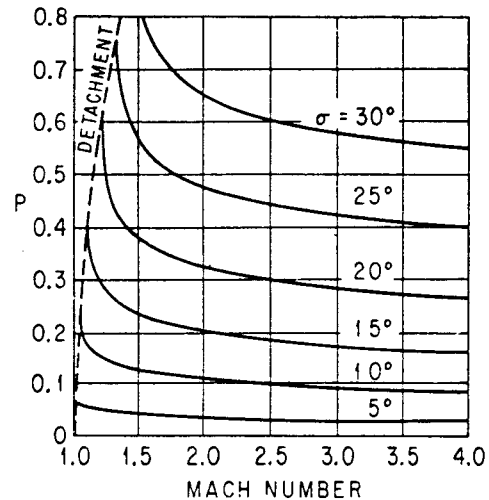


FIG. 3-5. Pressure coefficient vs. Mach number for cones.

From refs. 4 and 5, the values of C_{N_α} at $\alpha = 0^\circ$ for various cone semivertex angles and free-stream Mach numbers are shown in Fig. 3-6. Busemann⁷ in his analysis arrived at a value of $C_{N_\alpha} = 2$ per radian, which is identical to that obtained with the slender-body theory. This value is independent of the free-stream Mach number.

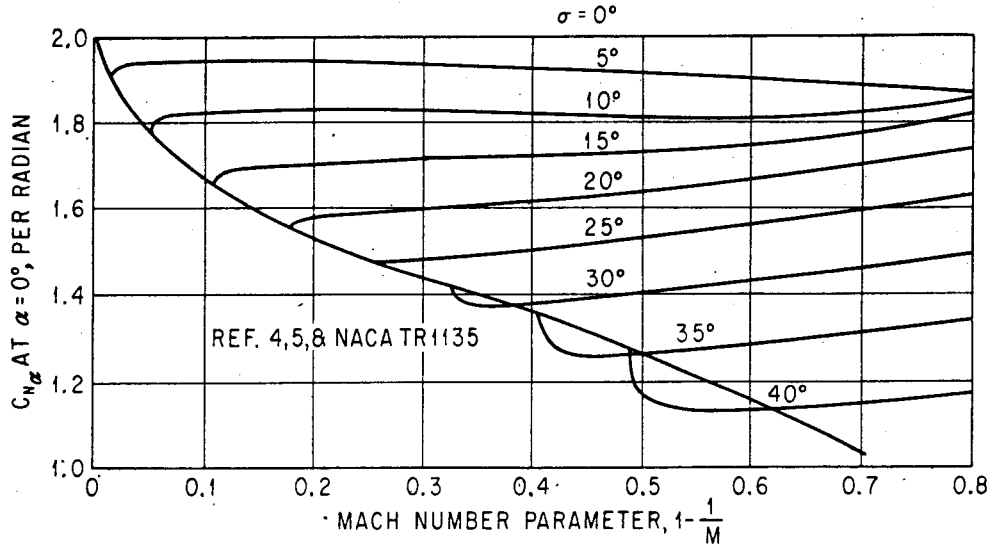


FIG. 3-6. C_{N_α} vs. Mach number parameter for cones.

Refinements made by Ferrari⁸ and Tsien⁹ resulted in the following equations:

$$C_{N_\alpha} = \frac{2 \sqrt{\left(\frac{\cot \sigma}{\sqrt{M^2 - 1}}\right)^2 - 1}}{\frac{\sqrt{M^2 - 1}}{\cot \sigma} \cosh^{-1} \frac{\cot \sigma}{\sqrt{M^2 - 1}}} + \sqrt{\left(\frac{\cot \sigma}{\sqrt{M^2 - 1}}\right)^2 - 1} (1 + 2 \tan^2 \sigma) \quad (3-7)$$

$$C_{N_\alpha} = \frac{2 \sqrt{\left(\frac{\cot \sigma}{\sqrt{M^2 - 1}}\right)^2 - 1}}{\frac{\sqrt{M^2 - 1}}{\cot \sigma} \cosh^{-1} \frac{\cot \sigma}{\sqrt{M^2 - 1}} + \sqrt{\left(\frac{\cot \sigma}{\sqrt{M^2 - 1}}\right)^2 - 1}} \quad (3-8)$$

Since the pressure is constant along the surface of the cone (conical-flow theory) the center of pressure is at the centroid of the planform area of the cone, or two-thirds of the nose length aft of the apex.

3-4. OGIVAL FOREBODY

Next the aerodynamic characteristics of the ogival forebody will be discussed, since this type of nose configuration is used more frequently than the conical nose. An ogive is similar to a cone except that the planform shape is formed by an arc of a circle instead of a straight line as shown in Fig. 3-7. A tangent ogive shown in

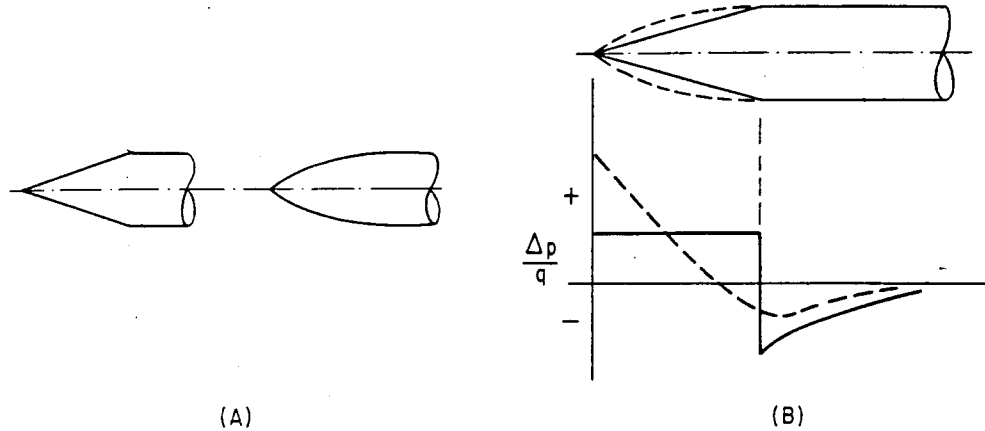


FIG. 3-7. (A) Conical and ogival nose shapes. (B) $\Delta p/q$ over cone and ogive at $\alpha = 0^\circ$.

Fig. 3-7 is one whose base is tangent to the cylindrical mid-section of the body of the missile. The ogival shape has several advantages over the conical section. These advantages are (1) slightly greater volume for a given base and length (l/d ratio), (2) a blunter nose, providing structural superiority, and (3) slightly lower drag.

Semiempirical expressions were derived by E. R. C. Miles¹⁰ for the form or pressure drag and center of pressure of the tangent ogives.

$$C_{D_F} = P \left\{ 1 - \frac{2[196(l/d)^2 - 16]}{28(M + 18)(l/d)^2} \right\} \quad (3-9)$$

$$\frac{cp}{l} = \frac{1}{2} \left[\frac{50(M + 18) + 7M^2 P(5M - 18)}{40(M + 18) + 7M^2 P(4M - 3)} \right] \quad (3-10)$$

where P is the pressure coefficient $\Delta p/q$ as given by Eq. (3-4). To facilitate calculation of the value of P , the ogive semivertex angle σ_o at the tip of the nose is given as

$$\sigma_o = 2 \tan^{-1} \frac{1}{2l/d} \quad (3-11)$$

It is obvious from Eq. (3-11) that the angle of the tip of the ogival nose is twice the conical value for the same length-to-diameter ratio

(see Fig. 3-8). Other characteristics of the ogival nose (i.e., radius and volume) are derived mathematically in Appendix A.

Results of an independent correlation indicate close agreement with those derived in Eq. (3-9). This will be shown quantitatively

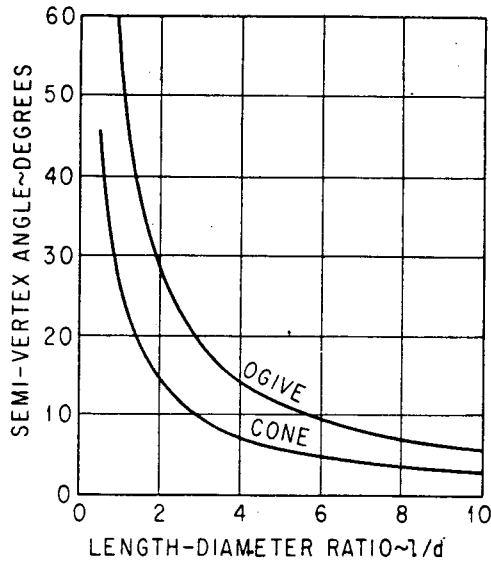


FIG. 3-8. Semivertex angle vs. length-diameter ratio for cones and ogives.

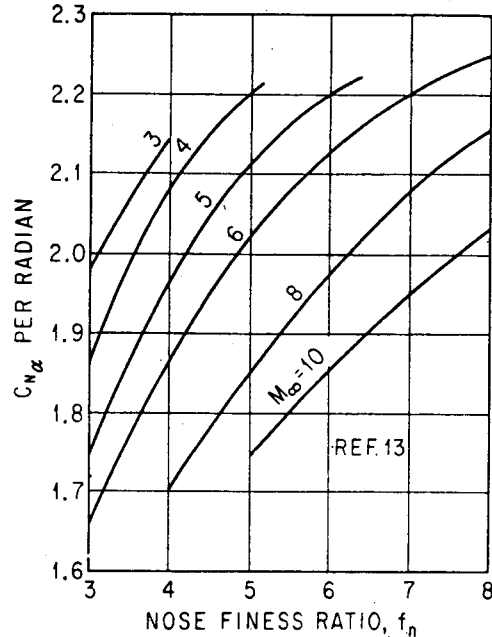


FIG. 3-9. $C_{N\alpha}$ vs. nose fineness ratio.

in Chap. 4. However, the centers of pressure derived by the slender-body theory^{34,35} and by Eq. (3-10) are somewhat different, as shown below for values of l/d of 2.5 and 4.0, which correspond to semivertex angles at the tip of the ogive of 22.6 and 14.6°, respectively.

$$\frac{l}{d} = 2.5 \quad \frac{l}{d} = 4.0$$

Using Eq. (3-10),

$$\begin{aligned} \frac{c_p}{l} &= 0.55 \text{ at } M = 1.5 & 0.59 \text{ at } M = 1.5 \\ &0.47 \text{ at } M = 3.0 & 0.55 \text{ at } M = 3.0 \end{aligned}$$

Using slender-body theory,^{34,35}

$$\frac{c_p}{l} = 0.46 \quad \frac{c_p}{l} = 0.47$$

and is independent of Mach number. Results from experimental tests indicate that the empirical expression yields results which are more accurate than those derived from the slender-body theory.

Many other investigators¹¹⁻²² have studied the problem of airflow over ogives and general bodies of revolutions. Syvertson and Dennis¹³ developed the second-order shock-expansion method for bodies of revolution near zero lift. This method appears to give results consistently more accurate than those obtained from other theories and is applicable over a wide range of Mach numbers and hypersonic similarity parameters, the latter defined as M/f_n where f_n is the fineness ratio of the nose. The variation of C_N with f_n for ogive-cylinder configuration is shown in Fig. 3-9 using the simplest of the three approximations available.

3-5. HEMISPHERICAL FOREBODY

This type of nose is used on some of the missiles, particularly those which use IF (infrared) seekers. This type of nose imposes an

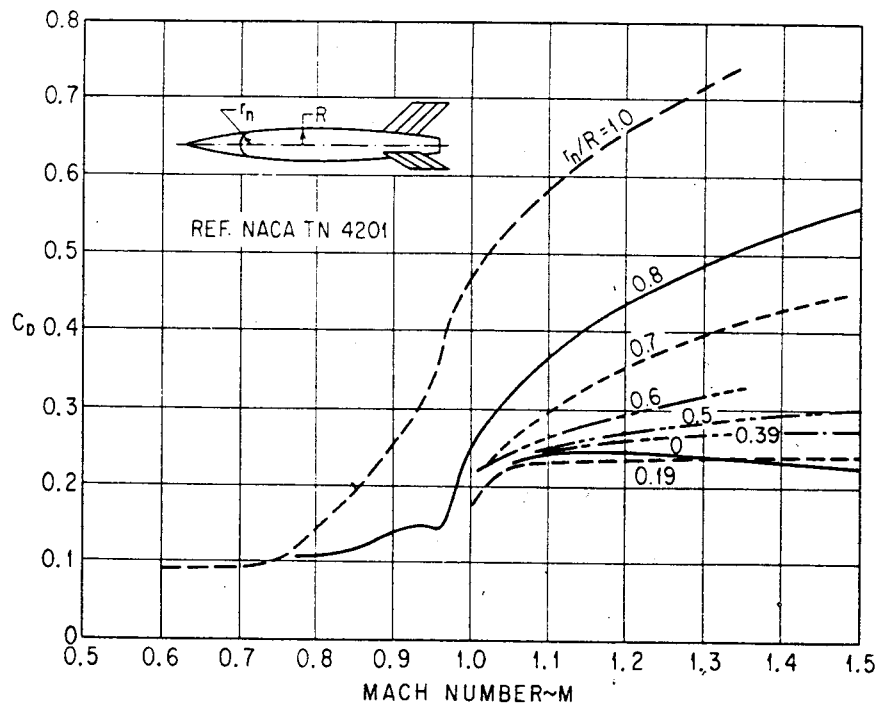


FIG. 3-10. C_D vs. Mach number for rounded-nose parabolic body.

extremely high drag penalty on the missile; i.e., wave drag may be six to seven times that of an ogive (see Fig. 3-10). The use of this

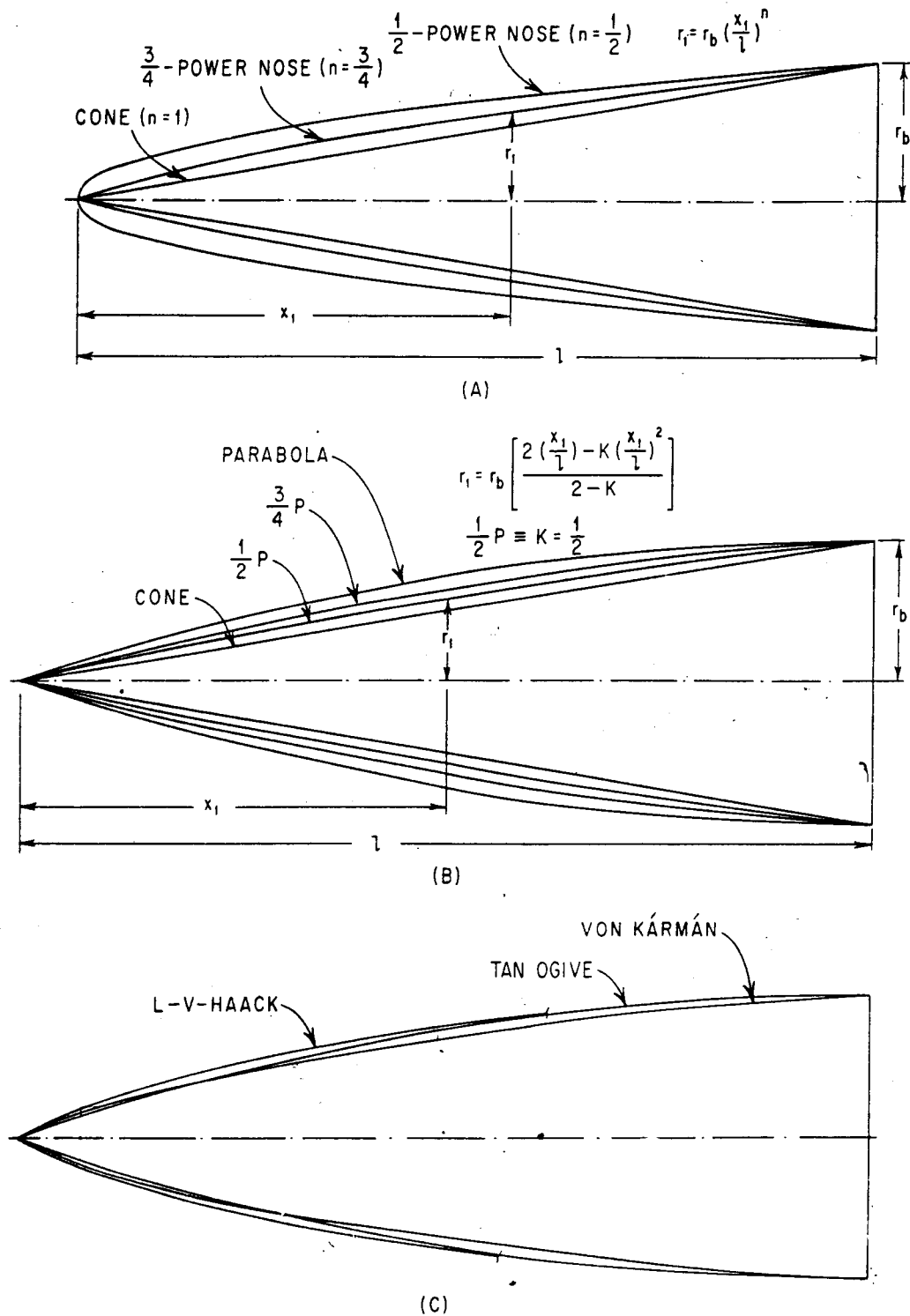


FIG. 3-11. (A) Profiles of fineness ratio, 3 noses—power series. (B) Profiles of fineness ratio, 3 noses—parabolic series. (C) Profiles of fineness ratio, 3 noses—Haack series and tangent ogive.

type of nose on missiles indicates the extent to which an aerodynamicist must compromise to achieve an optimum and feasible missile system.

3-6. OTHER FOREBODY SHAPES

Many other shapes of noses are being used on present-day missiles. The majority of these shapes may be considered as modified ogives. On some missiles, the shape of the nose section may be defined by

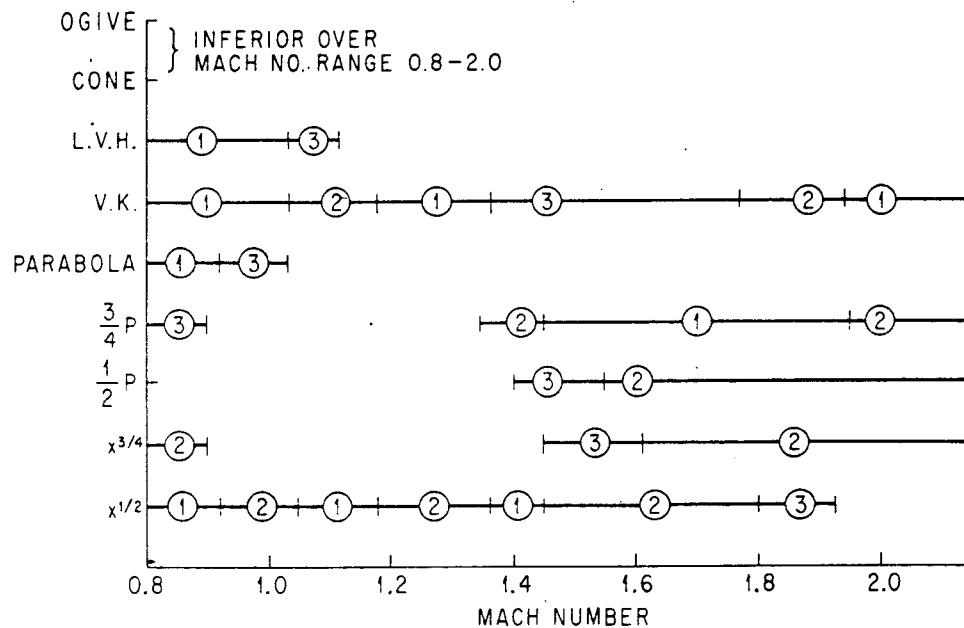


FIG. 3-12. Qualitative drag comparison of nine nose shapes ($l/d = 3.0$). (1) Best (least drag). (2) Better. (3) Good.

some elaborate expression derived purely from radar consideration, i.e., to minimize refraction and radar-beam distortion. For these peculiarly shaped nose sections, the drag may be estimated by mathematical expressions or by "eyeballing" the degree of similarity between the ogive and the shape in question. There are other types of nose shapes which were derived theoretically from aerodynamic drag consideration. Some of these are the "parabolic" or "power-series" noses (see Fig. 3-11). Another well-known one is the "Von Kármán" nose, named after its originator (see Fig. 3-11). The expression for the Von Kármán nose is

$$r = \frac{1}{\sqrt{\pi}} \sqrt{\varphi - (1/2) \sin 2\varphi} \quad (3-12)$$

where $\varphi = \cos^{-1} (1 - 2x)$

The coordinate and expressions for a variety of common nose shapes are included in Appendix B. All these have their best application (drag standpoint) at certain ranges of operating Mach numbers. A qualitative presentation of their applicability is shown in Fig. 3-12. The reader is referred to *NACA Technical Note 4201*, which has a vast collection of zero-lift-drag data on bodies of revolution. This report also contains numerous references which the reader may use to obtain additional data.

Quite frequently the aerodynamicist is asked to determine how much bluntness is acceptable for the nose of a missile, since blunting of the nose is advantageous from the standpoint of realizing better radar tracking characteristics, increasing structural strength, and alleviating the aerodynamic heating problem. The results presented in Fig. 7 of *NACA Technical Note 4201* show the effect of blunting on the drag characteristics at several Mach numbers. It can be said that the nose section may be "rounded off" for r/R values of 0.2 or less, without causing any drag penalty.

3-7. MID-SECTION

In most missile configurations, the mid-section is cylindrical in shape. This shape is advantageous from the standpoints of drag, ease of manufacturing, and load-carrying capability. The zero-lift drag ($\alpha = 0^\circ$) of a cylindrical body is caused by viscous forces (skin friction) only. At low angles of attack, a very small amount of normal force is developed on the body, and this results from the "carry-over" from the nose section. At rather large angles, some amount of normal force is developed because the cross-flow drag acts normal to the body center line. The effect of this viscous cross flow at angle of attack was reported by Allen and Perkins.²² The total normal-force coefficient is approximated by

$$C_N = 2\alpha + C_{D\alpha=90^\circ} \frac{A_p}{S} \alpha^2 \quad (3-13)$$

where A_p = planform area

S = reference area

The effects of mid-section or afterbody extension on the aerodynamic characteristics of conical and ogival nose bodies were systematically investigated experimentally by Buford²³ over a wide range of supersonic Mach numbers. These experimental data show that the effect

of afterbody extension is to increase C_N and move the center of pressure aft as a result of body carry-over and viscous cross-flow effects. It is also noted that Van Dyke's hybrid theory,¹⁹ which combines first- and second-order theories, gives results which are in reasonable agreement with these experimental results.

3-8. BOATTAIL

The tapered portion of the aft section of a body is called the boattail. The purpose of the boattail is to decrease the drag of a body which has a "squared-off" base. The latter feature has relatively large base pressure and, consequently, high drag values because of the large base area. By "boat-tailing" the rear portion of the body, the base area is reduced and thus a decrease in base drag is realized. However, the decrease in base drag may be partially nullified by the boattail drag.

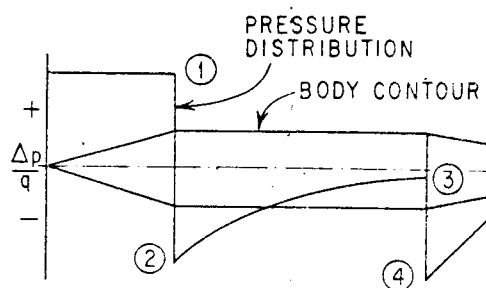


FIG. 3-13. Pressure distribution over body.

Accurate determination of boattail drag is rather difficult because of the dependence on the geometry of the fore and aft portions of the body and real-fluid effects. However, for preliminary design analysis, results from correlation of both theoretical and experimental test data should be used when available. In the absence of experimental test data, the boattail drag may be estimated by theoretical methods. One such method involves the derivation of the pressure distribution over the entire length of the body, as shown schematically in Fig. 3-13. This figure shows the familiar pressure distribution over the forward portions of the body as obtained by the Taylor-Maccoll theory and the Prandtl-Meyer equation for an expansion around a two-dimensional corner, beginning with the Mach number at point 1. The increase in pressure along the cylindrical portion can be approximated by linearized theory²⁴ to obtain the pressure at point 3. The Prandtl-Meyer equation again can be used, as before, to approximate the pressure at point 4. The boattail drag may then be obtained by integrating the pressure shown in Fig. 3-13.

Another method involves the use of the second-order theory¹⁹ to predict the boattail drag. The theoretical pressure distribution and

wave drags for conical boattails were determined with this theory by Jack⁴⁹ using the procedure detailed in *NACA Technical Note 2744*. In his theoretical investigation, he assumed that the boattail was preceded by a cylindrical body section of sufficient length to give uniform flow at the free-stream Mach number at the beginning of the boattail (i.e., free-stream static pressure at point 3 in Fig. 3-13). The results thus obtained appear to be in reasonable agreement with those derived from the theoretical method previously mentioned as well as with experimental test results as shown in Fig. 3-14

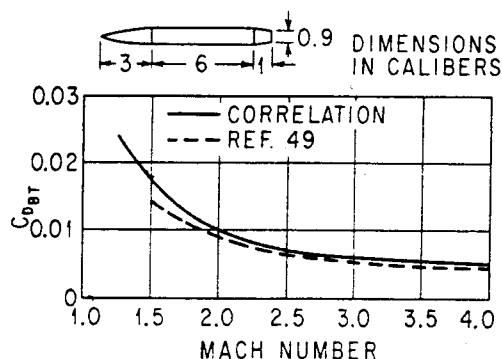


FIG. 3-14. Typical boattail drag coefficient vs. Mach number.

for a typical boattail configuration. For other boattail configurations, the reader is referred to Jack's report, which includes the effects of Mach number, boattail angle, area ratio, and body contours (i.e., boattail shapes).

Although boattailing is generally used, it does have some disadvantages which must be overcome. First, the lift on the boattail is destabilizing, which must be compensated by some increase in tail

area. Next, the center-of-pressure travel is increased with angle of attack. Hence one must weigh these adverse factors with the drag-saving feature to decide whether or not to use boattail in a particular design. Presumably, no boattail was used on the German V-2 because of the undesirable effect of center-of-pressure travel with angle of attack.

3-9. BASE PRESSURE

At supersonic velocities the base of the body experiences a large negative pressure (relative to ambient or free-stream static pressure) resulting in a substantial increase in missile drag. An accurate determination of this base-pressure coefficient is also quite involved since it depends on many parameters, including boattail angle, Mach number, and boattail length. Chapman,²⁵ in his investigations, concluded that a strictly inviscid-fluid theory could not be employed, and consequently he developed an approximate semi-empirical theory for the real fluid-flow case. Others²⁶⁻²⁹ have

analyzed the effects of individual parameters such as Reynolds number, boundary layer, and tail surfaces near the base of the body. The combined effects of several of these important parameters were compiled and correlated with test data with reasonably good success for the power-off condition (i.e., no jet exhausting from the base). For typical applications the base-pressure-coefficient variation with Mach number is shown in Fig. 3-15. The base-pressure drag coefficient C_{D_b} is related to the base-pressure coefficient as follows:

$$C_{D_b} = C_{P_b} \frac{S_b}{S} \quad (3-14)$$

where S_b = base area

S = body cross-sectional area (reference area)

C_{P_b} = base-pressure coefficient

The proper base area must be used in Eq. (3-14). For the power-off condition the total base area is used, whereas with power on, only that portion of the base shown in Fig. 3-16 is used.

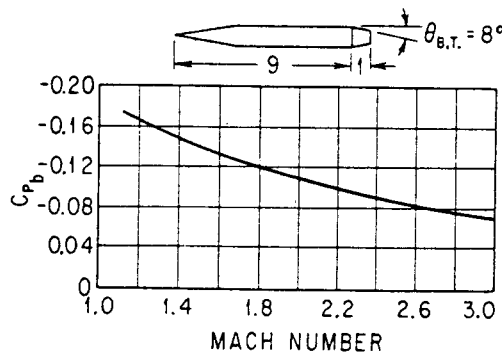


FIG. 3-15. Typical base-pressure coefficient vs. Mach number.

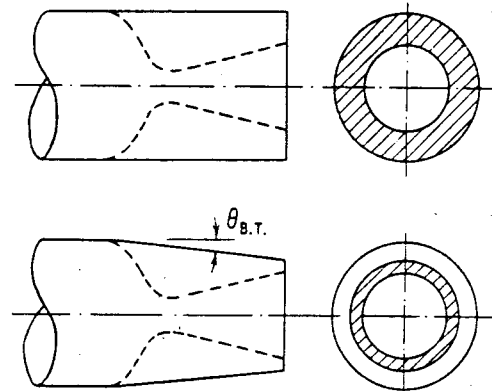


FIG. 3-16. Effective base area.

The base pressure is markedly changed by the presence of a jet emitting from the rocket motor. Limited unclassified data³⁰⁻³³ are available to determine the effects of different jet parameters such as jet pressure ratio (jet static to free-stream static pressure), nozzle angle, and base diameter. However, most of the results on jet effects on base pressure are classified, and they must be analyzed before an accurate determination of these effects can be made.

3-10. SUMMARY OF CHARACTERISTICS OF BODIES OF REVOLUTION

In summing up the aerodynamic characteristics of the complete body, the following general observations are noted:

1. The drag of the body at supersonic speeds is dependent primarily on the nose shape and the amount of boattailing.
2. Base drag is greatly affected by the presence of the jet.
3. The majority of the body lift is on the nose section of the body with a small down load on the boattail.
4. The resultant center of pressure for a conventional body varies from between 15 and 20 per cent of the body length at low angles of attack. At higher angles of attack the center of pressure of the complete body can move forward or aft of the nominal center-of-pressure location, depending on the amount of boattailing.
5. For moderate boattailing, say 7° , the center of pressure tends to shift rearward with angle of attack.
6. The nose may be "rounded off" to a value of r/R of 0.2 or less without causing any drag penalty.
7. For preliminary analysis the value of complete body C_{N_α} may be assumed to be 0.03 to 0.04 per degree for a wide range of subsonic and supersonic Mach numbers.

3-11. GENERAL AERODYNAMICS OF AIRFOIL

The next major important item in the aerodynamic missile configuration is the wing or main lifting surface. A quick glance at the various missile configurations indicates that a great variety of wing planforms or configurations are used. Undoubtedly, the wing configuration for each of these missiles was selected after detailed analyses had been conducted to optimize the configuration to the requirements of each of the complete missile weapon systems. To help in understanding the logic behind the selection of a given configuration design, the following sections are devoted to a brief discussion of the aerodynamics of different wing configurations, including the effects of airfoil section, wing planform, aspect ratio, and wing area. It is beyond the scope of this book to treat the subject of supersonic wing theory; the reader is therefore referred to the listing of the numerous reports on this subject at the end of this chapter.³⁶⁻⁴³

Perhaps one of the most useful and well-known theories used in supersonic flow over wings is the linearized theory. This theory,

derived from the exact differential equation of steady compressible flow, has the basic assumption that the body or airfoil is very thin. Other basic assumptions include the requirements that (1) the shock wave is attached to the airfoil and (2) the flow is two-dimensional, i.e., the air is deflected in only one plane perpendicular to the direction of flow. Using this simplified linear analysis, the value of $\Delta p/q$ is given:

$$\frac{\Delta p}{q} = \frac{2\delta}{\sqrt{M^2 - 1}} \quad (3-15)$$

where δ is the wedge semivertex angle, in radians. The above equation is the well-known "first-order," "linear," or "Ackeret" theory for two-dimensional flow.³⁸ Busemann⁴⁰ derived higher-order terms as constants in a series expansion as

$$\frac{\Delta p}{q} = C_1\delta + C_2\delta^2 + C_3\delta^3 + C_4\delta^4 + \dots \quad (3-16)$$

where $C_1, C_2, \dots$ are known as Busemann constants and are defined as

$$C_1 = \frac{2}{\sqrt{M^2 - 1}} \quad (3-16a)$$

$$C_2 = \frac{2(3M^4 - 5M^2 + 5)}{5(M^2 - 1)^2} \quad (3-16b)$$

$$C_3 = \frac{10}{375(M^2 - 1)^{7/2}} (15M^8 - 68M^6 + 150M^4 - 75M^2 + 50) \quad (3-16c)$$

$$C_4 = \frac{1}{750(M^2 - 1)^5} (60M^{12} - 558M^{10} + 2,222M^8 - 2,870M^6 + 4,200M^4 - 500M^2 + 500) \quad (3-16d)$$

The above constants are for the case of an expansion wave (δ is negative) for air ($\gamma = 1.4$). For an oblique compressive wave, Eq. (3-16) becomes

$$\frac{\Delta p}{q} = C_1\delta + C_2\delta^2 + C_3^*\delta^3 + C_4^*\delta^4 + \dots \quad (3-17)$$

where

$$C_3^* = \frac{1}{75(M^2 - 1)^{7/2}} (27M^8 - 112M^6 + 270M^4 - 150M^2 + 100) \quad (3-18)$$

$$C_4^* = \frac{1}{10(M^2 - 1)C_1} [9M^2C_1(C_1C_3^* + C_2^2) - 10(M^2 - 1) \times C_2C_3^* - 14M^2C_1C_2 + \frac{4}{3}(3M^2 - 5)(3C_3^* - C_1)] \quad (3-19)$$

The numerical values of C_1 , C_2 , C_3 , and C_3^* are tabulated in Table 3-1 and plotted in Fig. 3-17.

Since the normal force on a wing is the result of difference in pressure between the upper and lower surfaces the normal-force

TABLE 3-1. BUSEMANN'S CONSTANTS
For $\gamma = 1.4$

M	C_1	C_2	C_3	C_3^*
1.2	3.015	8.307	54.03	53.22
1.4	2.041	2.919	5.80	6.13
1.6	1.601	1.950	1.937	2.153
1.8	1.336	1.618	1.144	1.280
2.0	1.155	1.467	0.934	1.016
2.5	0.873	1.320	0.943	0.947
3.0	0.707	1.269	1.112	1.155
3.5	0.596	1.245	1.309	1.387
4.0	0.516	1.232	1.513	1.621
4.5	0.456	1.224	1.719	1.854
5.0	0.408	1.219	1.925	2.086

coefficient may easily be derived. For the "first-order" linear theory the normal force is given by

$$N = (\Delta p_L - \Delta p_U) S_W \quad (3-20)$$

$$C_N = \frac{N}{q S_W} = \left(\frac{\Delta p}{q} \right)_L - \left(\frac{\Delta p}{q} \right)_U = \frac{2\delta}{\sqrt{M^2 - 1}} - \frac{-2\delta}{\sqrt{M^2 - 1}} = \frac{4\delta}{\sqrt{M^2 - 1}} \quad (3-20a)$$

For the two-dimensional wing of zero thickness the normal-force coefficient is thus

$$C_N = \frac{4\alpha}{\sqrt{M^2 - 1}} \quad (3-21)$$

or

$$C_{N_\alpha} = \frac{4}{\sqrt{M^2 - 1}} \equiv \frac{4}{\beta} \quad (3-22)$$

Thus the normal-force-curve slope of a wing in supersonic flow is characterized by a decrease in C_N with increasing Mach number. This is in contrast to the variation in subsonic flow where C_{N_α} varies approximately inversely as $\sqrt{1 - M^2}$, known as the Prandtl-Glauert factor.

Using a second-order degree of accuracy, the value of C_N for a two-dimensional airfoil of symmetrical double-wedge shape can readily be derived. For this condition Eq. (3-16) is reduced to

$$\frac{\Delta p}{q} = C_1 \theta + C_2 \theta^2 \quad (3-23)$$

The angle θ in Eq. (3-23) is the total local angle between the wing surface and free-stream direction. From Fig. 3-18 the values of the four regions of flow over the airfoil are

$$\begin{aligned} \theta_1 &= -\alpha + \delta & \theta_2 &= -\alpha - \delta \\ \theta_3 &= \alpha + \delta & \theta_4 &= \alpha - \delta \end{aligned} \quad (3-24)$$

By substituting Eq. (3-24) in Eq. (3-23) the pressure differential (between upper and lower surfaces) for the forward and aft sections of the airfoil as shown in Fig. 3-18 may be written as

$$\begin{aligned} \left(\frac{\Delta p}{q} \right)_F &= \left(\frac{\Delta p}{q} \right)_3 - \left(\frac{\Delta p}{q} \right)_1 = 2C_1 \alpha + 4C_2 \alpha \delta \\ \left(\frac{\Delta p}{q} \right)_A &= \left(\frac{\Delta p}{q} \right)_4 - \left(\frac{\Delta p}{q} \right)_2 = 2C_1 \alpha - 4C_2 \alpha \delta \end{aligned} \quad (3-25)$$

The normal force is the total pressure difference between the upper and lower surfaces

$$N = \left[\left(\frac{\Delta p}{q} \right)_F + \left(\frac{\Delta p}{q} \right)_A \right] S \quad (3-26)$$

where S = twice the planform area S_W . The value of C_N is thus

$$C_N = \frac{N}{q S_W} = 2C_1 \alpha \quad (3-27)$$

or

$$C_{N_\alpha} = 2C_1 = \frac{4}{\sqrt{M^2 - 1}} \equiv \frac{4}{\beta} \quad (3-28)$$

which is identical to the "first-order" linear theory by Ackeret for a two-dimensional airfoil of zero thickness. The value of C_{N_α} determined by Eq. (3-27) is valid for only low angles of attack. At higher

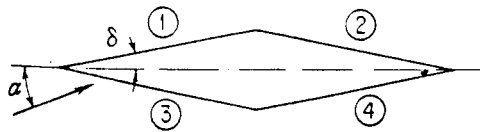


FIG. 3-18. Regions of flow—double-wedge airfoil at an angle of attack.

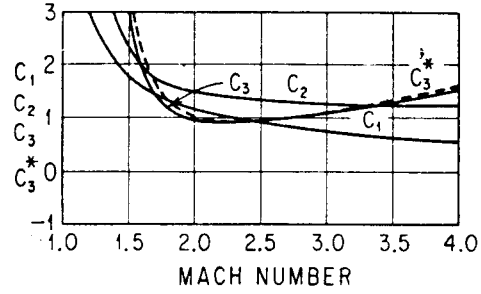


FIG. 3-17. Busemann's constants ($\gamma = 1.4$).

angles of attack this second-order theory must be replaced by the "exact" theory.^{42,43} However, for preliminary design analysis Eqs. (3-27) and (3-28) may be used for α up to approximately 10° .

The form or pressure drag is the component of the pressure force in the chordwise direction at $\alpha = 0^\circ$. The pressure differential between the upper and lower surfaces may be written as

$$\begin{aligned}\left(\frac{\Delta p}{q}\right)_U &= \left(\frac{\Delta p}{q}\right)_1 - \left(\frac{\Delta p}{q}\right)_2 = 2C_1\delta - 4C_2\alpha\delta \\ \left(\frac{\Delta p}{q}\right)_L &= \left(\frac{\Delta p}{q}\right)_3 - \left(\frac{\Delta p}{q}\right)_4 = 2C_1\delta + 4C_2\alpha\delta\end{aligned}\quad (3-29)$$

The total pressure-drag coefficient is obtained by

$$C'_{D_F} = \frac{1}{2} \left[\left(\frac{\Delta p}{q}\right)_U + \left(\frac{\Delta p}{q}\right)_L \right] = 2C_1\delta = \frac{4\delta}{\sqrt{M^2 - 1}} = \frac{4\delta}{\beta} \quad (3-30)$$

On the basis of wing planform area,

$$C_{D_F} = \frac{4\delta^2}{\beta} \quad (3-31)$$

It can be seen that the wave drag of the airfoil varies as the thickness squared. Hence, in designs which require relatively large wings the thickness must be kept to a minimum to reduce the form-drag contribution.

The drag due to normal force is readily determined as follows:

$$C_{D_N} = C_N \sin \alpha \cong C_N \alpha = \frac{4\alpha^2}{\beta} = \frac{C_N^2}{C_{N\alpha}} \quad (3-32)$$

The center of pressure of a two-dimensional airfoil is at the mid-chord when the first-order linear theory is used. However, second-order linear theory must be used to obtain a more accurate center-of-pressure value. This is done first by obtaining an expression for the pitching-moment coefficient about the mid-chord $C_{m_{0.5}}$ and dividing by the normal-force coefficient C_N as shown below.

$$\begin{aligned}C_{m_{0.5}} &= \frac{M_{0.5}}{qSC} = C_N \frac{x}{C} = \left[\left(\frac{\Delta p}{q}\right)_F x_F - \left(\frac{\Delta p}{q}\right)_A x_A \right] \frac{1}{2C} \\ &= \frac{1}{2C} 0.25C(2C_1\alpha + 4C_2\alpha\delta) - 0.25C(2C_1\alpha - 4C_2\alpha\delta) \\ &= C_2\alpha\delta\end{aligned}\quad (3-33)$$

$$cp_{0.5} = \frac{C_{m_c}}{C_N} = \frac{C_2 c \delta}{2C_1} \quad (3-34)$$

ahead of the mid-chord.

The above discussion has been restricted to the most simple and approximate methods for the determination of two-dimensional airfoil characteristics. Other more elaborate methods^{42,43} are available when more accurate results are desired. The latter methods involve stepwise application of the known relations for the compression through the shock wave and the expansion around a convex corner. This procedure is usually termed the "shock-expansion" method.

In most practical designs, finite-aspect-ratio wings and wings with planform other than rectangular are of interest. These features will be discussed in the following sections.

3-12. ASPECT RATIO

The above section dealt with the aerodynamic characteristics of two-dimensional airfoils, i.e., wings of infinite span or aspect ratio. In most practical cases three-dimensional airfoils are employed. In the study of airfoils in three-dimensional flow the concept of the Mach cone, a fundamental result of the linear theory, is employed. According to this concept (see Fig. 3-19) the effect of a given disturbance is felt only within the interior of a circular cone whose vertex is located at the point of disturbance and whose axis extends downstream parallel to the free-stream flow direction. The geometry of the cone is determined by the requirement that the component of the free-stream velocity normal to the surface of the cone be equal to the speed of sound in the undisturbed stream. Hence the semivertex angle of the cone is a function of the free-stream Mach number only and is given by Eq. (3-35) by inspection of Fig. 3-19.

$$\mu = \sin^{-1} \frac{c}{V} = \sin^{-1} \frac{1}{M} \quad (3-35)$$

where c = speed of sound

V = free-stream velocity

For a rectangular wing of finite aspect ratio, there will be a loss of pressure at the tips because air flows from the lower surface onto the upper surface as a result of pressure differences. These tip losses are similar in nature to those realized in subsonic-flow phenomena except that in the supersonic case these tip losses are restricted to the shaded area in Fig. 3-20. Thus it is readily apparent that the effect of aspect ratio is to decrease the over-all lift or normal-force slope of the wing. The pressure within this conical area of influence is

constant along any line radiating from the vertex of the cone created by the leading edge and wing tip and is determined^{44,45} to be

$$\frac{p - p_1}{p_\infty - p_1} = \frac{2}{\pi} \sin^{-1} \sqrt{\frac{\tan \mu'}{\tan \mu}} \quad (3-36)$$

where p = pressure at any point

p_∞ = two-dimensional value at limit of Mach cone

p_1 = free-stream static pressure

μ' = angle of any ray radiating from wing tip (see Fig. 3-21)

The variation of this pressure (loss) at the wing tip is shown in Fig.

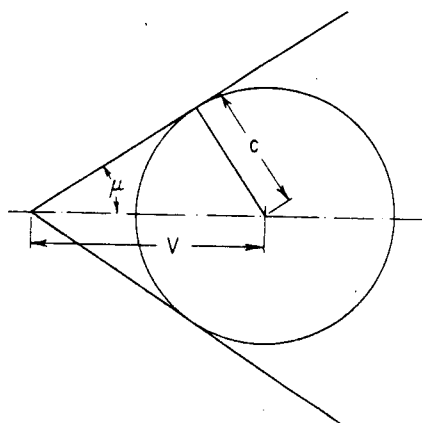


FIG. 3-19. The "Mach triangle."

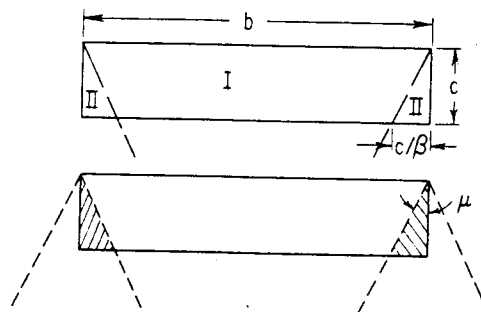


FIG. 3-20. Tip effects.

3-21. The resultant spanwise pressure distribution is shown in Fig. 3-24. Evvard⁴⁶ showed that the loss of lift due to the tips is exactly half the two-dimensional value, which yields essentially the same tip loss as calculated by Eq. (3-36) but differs slightly in the exact nature of the pressure distribution (see Fig. 3-21). Using Evvard's result, the effects of aspect ratio (i.e., tip loss) on the aerodynamic characteristics of a rectangular wing can be readily determined below.

From Fig. 3-20 it can be seen that the value of C_N for area I is given by the two-dimensional value [Eq. (3-21)].

$$C_{N_I} = \frac{4\alpha}{\beta} \quad (3-37)$$

The value of C_N at the tip sections II is

$$C_{N_{II}} = \frac{1}{2} C_{N_I} = \frac{2\alpha}{\beta} \quad (3-38)$$

The total value of C_N based on total wing area bc is therefore given by

$$C_N = \frac{4\alpha}{\beta} \frac{A_I - 2A_{II}}{A_I + 2A_{II}} + \frac{1}{2} \frac{4\alpha}{\beta} \frac{2A_{II}}{A_I + 2A_{II}} \quad (3-39)$$

Since $A_{II} = \frac{c^2}{2\beta}$ $A_I = bc - \frac{c^2}{\beta}$

Therefore, Eq. (3-39) becomes

$$C_N = \frac{4\alpha}{\beta} \frac{bc - c^2/\beta}{bc} + \frac{1}{2} \frac{4\alpha}{\beta} \frac{c^2/\beta}{bc} \quad (3-40)$$

or $C_N = \frac{4\alpha}{\beta} \left(1 - \frac{c}{2\beta b}\right)$ (3-41)

Since $A = b/c$, Eq. (3-41) becomes

$$C_N = \frac{4\alpha}{\beta} \left(1 - \frac{1}{2A\beta}\right) \quad (3-42)$$

or $C_{N_\alpha} = \frac{4}{\beta} \left(1 - \frac{1}{2\beta A}\right)$ (3-43)

Equation (3-43) shows quantitatively the detrimental effect of aspect ratio on the normal-force-curve slope. Consequently "raked tips" (see Fig. 3-25) are sometimes used to reduce the effect of aspect ratio or tip losses. These tip losses may be completely eliminated by cutting the tips at an oblique angle equal to or somewhat greater than the Mach angle. In this case the flow over the entire wing will again be completely two-dimensional.

It is usually more convenient to use the "effective aspect ratio," defined as βA , as a common parameter. For example, the effective aspect ratio of a wing at geometric aspect ratio 3 traveling at Mach 2 is 5.2. For this case the value of C_{N_α} is approximately 90 per cent of the two-dimensional value as shown in Fig. 3-22.

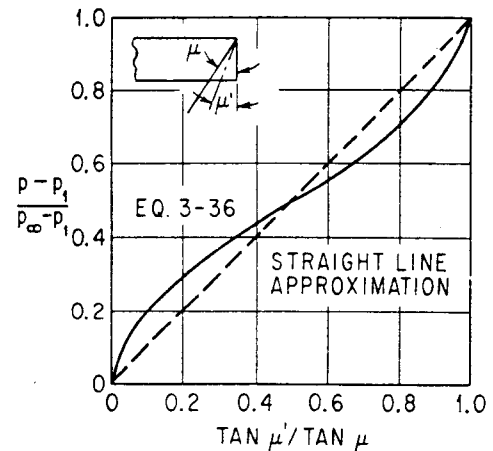


FIG. 3-21. Pressure loss due to tip effects.

By taking moments about the wing leading edge and dividing by the total force given by Eq. (3-43) the center of pressure may be shown to be

$$\frac{cp}{c} = \frac{\beta A - 2/3}{2\beta A - 1} \quad (3-44)$$

The variation of center of pressure for a rectangular wing of finite aspect ratio in supersonic flow as derived by linearized theory [Eq. (3-44)] is shown in Fig. 3-23. Since the pressure is decreased in the

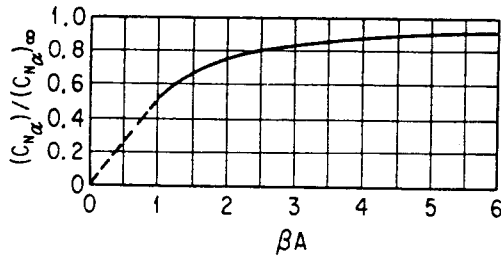


FIG. 3-22. Effect of βA on C_{N_α} —thin rectangular wing.

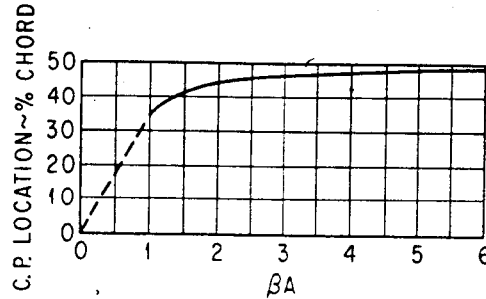


FIG. 3-23. Effect of βA on center-of-pressure location—thin rectangular wing.

region near the tips the drag of a rectangular wing of finite aspect ratio must be corrected as follows:

$$C_D = \frac{4\delta^2}{\beta} \left(1 - \frac{1}{2\beta A} \right) \quad (3-45)$$

By combining Eqs. (3-22) and (3-43) it can be shown that the drag due to normal force is given by

$$C_{D_N} = \frac{4\alpha^2}{\beta} \left(1 - \frac{1}{2\beta A} \right) \quad (3-46)$$

Equation (3-46) indicates that the effect of decreasing aspect ratio is to decrease the magnitude of the induced drag. This is in direct contrast to the case of subsonic flow, as will be shown in a later section. It is also noted that in supersonic flow induced drag is realized with an infinite-aspect-ratio wing, whereas in the subsonic case the induced drag is zero.

Strictly speaking, Eqs. (3-43) through (3-46) are derived for a flat plate (i.e., airfoil of zero thickness). The effect of airfoil thickness

will be discussed in a later section. It has been shown⁴⁷ that the value of $C_{N\alpha}$ calculated by linear theory [Eq. (3-43)] agrees quite closely with experimental data.⁴⁸ However, the chordwise pressure distribution in the two-dimensional-flow region (i.e., area I in Fig. 3-20) determined by linearized theory is noticeably different from that measured from tests particularly in the aft section of the wing, where the pressure over the upper surface is markedly lower than that predicted by theory. This discrepancy in pressure distribution may be attributed to airfoil thickness and viscous or real-fluid effects, the latter of which was assumed to be negligible in the linearized theory. As a consequence of the discrepancy in the afore-mentioned pressure distributions, the center of pressure calculated by Eq. (3-44) will be slightly forward of the actual value. In addition, the wave drag calculated by Eq. (3-45) will be lower than the actual drag, which includes viscous effects (i.e., interaction between the oblique wave and viscous boundary layer, flow separation, etc.). However, since the values of the normal-force-curve slope are in close agreement with test results, the induced drag predicted by theory [Eq. (3-46)] is generally satisfactory^{47,55} for wings with rectangular planform as well as for wings of other planforms with supersonic leading edges, a condition in supersonic flow which will be discussed in the next section.

It should be noted that Eq. (3-43) is restricted to cases where the Mach cones from the tips of the wing do not intersect each other. Hence the limiting aspect ratio is $1/\beta$. For relatively low aspect ratio or low-Mach-number flight conditions, the Mach lines from the two tips will intersect on the wing surface, resulting in an area which is influenced by both forward-tip effects as shown in Fig. 3-24. In this area, IV, the pressure decreases due to the two waves are additive and result in a substantial loss in lift. As shown in Fig. 3-24D, the length of ad equals the sum of ac and ab (i.e., $ab = cd$). The value of

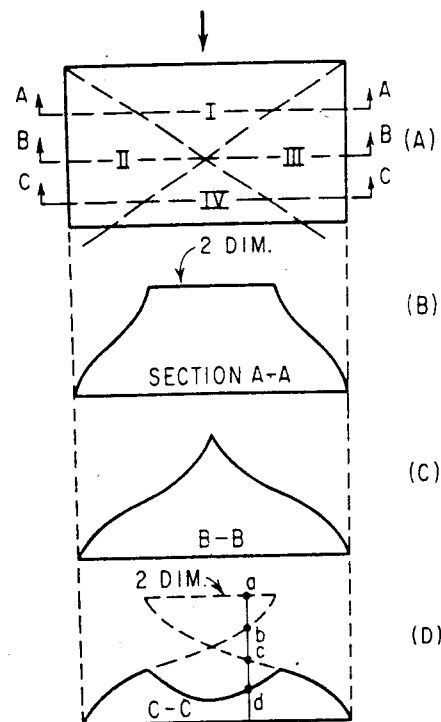


FIG. 3-24. Tip effects on low-aspect-ratio rectangular wing.

C_{N_α} for this condition of low-aspect-ratio Mach number is derived below.

$$C_{N_\alpha} = \left[\frac{4}{\beta} A_T - \frac{2}{\beta} (A_{II} + A_{IV}) - \frac{2}{\beta} (A_{III} + A_{IV}) \right] \frac{1}{A_T} \quad (3-47)$$

$$C_{N_\alpha} = \left[\frac{4}{\beta} (A_I + A_{II} + A_{III} + A_{IV}) - \frac{2}{\beta} (A_{II} + A_{III} + 2A_{IV}) \right] \frac{1}{A_T} \quad (3-47a)$$

or
$$C_{N_\alpha} = \frac{2}{\beta} \frac{2A_I + A_{II} + A_{III}}{A_T} \quad (3-48)$$

where A_T = total wing planform area.

For a given missile configuration design the aspect ratio of the aerodynamic lifting surface is primarily dependent upon the following requirements:

1. Load factor or maximum trimmed lift
2. Wing area
3. Trim angle of attack
4. Lift-to-drag L/D ratio
5. Maximum permissible span

In general, when the maneuverability and range requirements are high, it is advantageous to use a relatively large aspect ratio since the normal-force-curve slope is improved. This results in a reduction in both wing areas and trimmed angle of attack required. In addition, a higher maximum lift-to-drag ratio is realized with a high-aspect-ratio wing since $(L/D)_{\max}$ is proportional to the square root of the lift-curve slope (for a supersonic leading edge) as given below.

$$\left(\frac{L}{D} \right)_{\max} = \frac{1}{2\sqrt{C_{D_0}}} \sqrt{\frac{C_{N_\alpha}}{C_{D_0}}} \quad (3-49)$$

Equation (3-49) will be derived in Appendix C.

In some cases the optimum aspect ratio as determined from purely aerodynamic considerations must be compromised and reduced because of maximum span limitation or severe structural

loading problems. In summary the effects of increasing aspect ratio are as follows:

1. Increase $C_{N\alpha}$, C_{D_0}
2. Increase $(L/D)_{\max}$
3. Increase span
4. Reduce trim angle of attack
5. Reduce wing area
6. Increase structural weight
7. Negligible effect on center-of-pressure travel for $\beta A > 2$

3-13. WING PLANFORM

In addition to the basic straight-wing planform discussed in the previous section two other basic wing planforms are often used: delta and swept-back wings. There are many variations of these

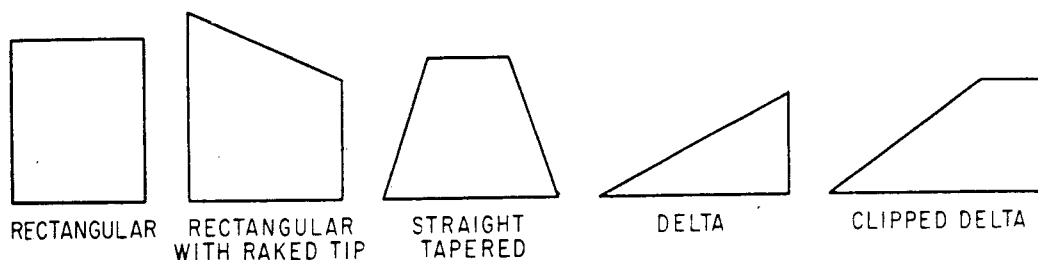


FIG. 3-25. Typical supersonic wing planforms.

three basic planforms, as shown in Fig. 3-25, each optimized for its particular application. Many advantages and disadvantages are associated with each of the three basic planforms used. Consequently a thorough study involving their aerodynamic efficiency, structural weight, and cost of manufacturing must be made before the best planform can be determined. In this section the aerodynamic characteristics of the delta and swept-back planforms are discussed and compared qualitatively with the straight-wing planform in order to provide the engineer a "feel" for selecting a wing planform.

In the analysis of wings of arbitrary planform it is important to know whether the leading (and trailing) edge is "subsonic" or "supersonic" since the pressure distribution is markedly different for each condition. The leading edge is termed "supersonic" if the normal component of the free-stream Mach number is supersonic,

i.e., leading edge ahead of the Mach line as shown in Fig. 3-26. Mathematically, a supersonic leading edge exists when $\mu < \epsilon$ where ϵ is the leading-edge angle. Other edge conditions are shown in Fig. 3-26. The theoretical pressure in the region behind a supersonic leading edge and ahead of the Mach angle increases from the two-dimensional value for no sweep [defined by Eq. (3-15)] to a value of infinity when the leading-edge sweep angle and the complement of the

Mach angle are equal (i.e., a sonic leading edge). The equation for this pressure variation is given as

$$\frac{\Delta p}{q} = \frac{2\delta}{\beta\sqrt{1-n^2}} \quad (3-50)$$

where $n = k/\beta$ as shown in Fig. 3-26. As the leading-edge sweep angle is further increased a "subsonic" leading-edge condition exists and the theoretical pressure at the edge has an infinite value.

Based on the conical-flow theory first conceived by Busemann, the theoretical aerodynamic characteristics of delta or triangular wings have been studied extensively by many authors.⁵⁰⁻⁵⁴ References 6 and 50 through 54 are recommended to the reader for a

complete treatment of this type of wing planform. For wings of arbitrary planform ref. 6 is also highly recommended. Based on the work of refs. 50 to 54 the normal-force-curve slope of the delta wing with a supersonic leading edge is shown to have the two-dimensional value of $4/\beta$. For the case of a subsonic leading edge the value becomes

$$C_{N_\alpha} = \frac{2\pi \tan \epsilon / \tan \mu}{E\beta} \quad (3-51)$$

where E is the elliptic integral of the second kind for

$$\sqrt{1 - (\tan \epsilon / \tan \mu)^2}$$

and ϵ is the triangular-wing semivertex angle shown in Fig. 3-27.

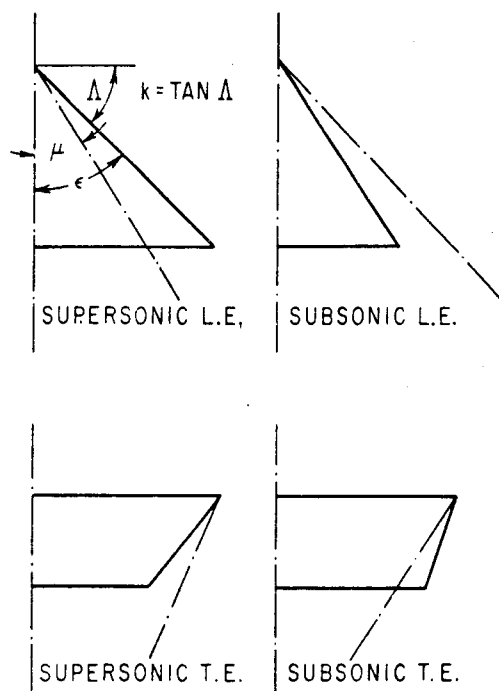


FIG. 3-26. Edge conditions.

In Fig. 3-27 the ratio of $C_{N\alpha}$ to the theoretical two-dimensional value $4/\beta$ is plotted against the parameter $\tan \epsilon/\tan \mu$. Results from an extensive and systematic series of experimental tests⁵⁵ indicate that the theoretical values of $C_{N\alpha}$ are reasonably accurate for values of $\tan \epsilon/\tan \mu$ of 0.6 or below. Above this value the linear theory yields values of $C_{N\alpha}$ substantially higher than those obtained from experiment except at very large values of $\tan \epsilon/\tan \mu$. For design purposes it is recommended that experimental values of $C_{N\alpha}$ of ref. 55 be used in lieu of theoretical values. The experimental results also show that the center of area is a good approximation of the center of pressure for both subsonic and supersonic leading-edge conditions.

The effect of sweepback for a two-dimensional airfoil can be shown theoretically to be somewhat beneficial from the standpoint of improving the value of $C_{N\alpha}$ and reducing the wave drag.

Since the pressure acting on the surface is a function of the component of Mach number normal to the leading edge instead of the free-stream Mach number (i.e., $M \cos \Lambda$, where Λ is the sweep angle), the value of $C_{N\alpha}$ for the two-dimensional swept-back wing may be written as

$$(C_{N\alpha})_{\Lambda} = \frac{4 \cos \Lambda}{M^2 \cos^2 \Lambda - 1} \quad (3-52)$$

The form or wave drag may be shown to be reduced by the following factor:

$$\frac{\cos^3 \Lambda}{\sqrt{M^2 \cos^2 \Lambda - 1}}$$

Extensive experimental investigations have been conducted to determine and compare the aerodynamic characteristics of the three basic planforms (i.e., straight, delta, and swept-back wings) of finite aspect ratios for practical applications. For most preliminary designs sufficient test data and correlation of these data are available^{47,55} to evaluate the quantitative effects of planform systematically, taking into account such factors as Reynolds number and viscous fluid which are not included in the theoretical values. A

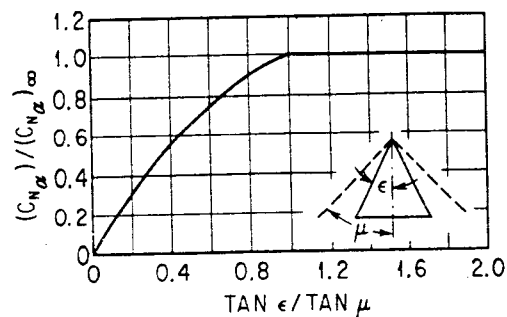


FIG. 3-27. Normal-force characteristics for delta wings.

qualitative summary based on the combined theoretical and experimental results is presented below as a guide in the selection of wing planform for design purposes. Again it must be emphasized that a thorough study involving the important factors previously mentioned (i.e., aerodynamic efficiency, structural weight, cost of manufacturing, etc.) must be conducted before the optimum planform can be determined.*

3-14. AIRFOIL SECTIONS

Since the pressure over an airfoil is primarily a function of the angle between the free-stream air direction and the surface, as previously shown, the airfoil shape or section for supersonic application is noticeably different from those sections used in the subsonic region. In general, sharp-nosed symmetrical airfoil sections of the double-wedge, modified double-wedge, or biconvex variety shown in Fig. 3-28 result in the most efficient aerodynamic design. While a sharp leading edge is desirable from the standpoint of maintaining shock attachment, thereby reducing the magnitude of the wave drag and adverse pressure gradient (conducive to separated flow), it is often necessary to round off the nose of the airfoil to minimize the aerodynamic heating effect as well as provide structural integrity to the section. From theoretical considerations a symmetrical section

* As a matter of interest to the reader, many articles written on the selection of a particular type of wing planform over the past several years may be found in the following issues of *Aviation Week*: Mar. 21, 1949; Aug. 16, 1948; Dec. 20, 1948; July 4, 1949; Sept. 17, 1951; Sept. 22, 1952; Oct. 20, 1952; Dec. 7, 1953; June 24, 1957; and July 15, 1957. Although these articles are primarily concerned with the selection of wings for manned aircraft, the fundamental problems involved in guided missile design are similar.

Supersonic aerodynamic characteristics	Wing planform		
	Straight	Delta	Swept-back
$C_{N\alpha}$	High	Low	Avg
C_{D_0}	High	Low	Avg
$(L/D)_{\max}$	Low	High	Avg
C_{D_i}	Low	High	Avg
$C_{L_{\max}}$	Not noticeably affected by planform ⁶⁰		
cp travel, % MAC	Large	Avg	Small
Aeroelastic effect	Small	Avg	Large

offers the minimum wave drag, but because of real (viscous) fluid effects previously mentioned, minimum wave drag is not always realized on a symmetrical section especially for the case of a relatively thick airfoil section where the linearized theory becomes less accurate.

In the final selection of airfoil shape, one must consider also the structural efficiency and manufacturing cost as well as its aerodynamic efficiency. From the latter standpoint the double wedge has the lowest drag for a given thickness ratio whereas the biconvex section has the lowest drag per unit strength.⁶ From the manufacturing standpoint the modified double wedge is preferred, where solid sections are involved, since it is easier to machine. As mentioned previously, the sharp leading (and trailing) edges may have to be rounded to provide local structural integrity as well as to minimize the aerodynamic heating effects. Generally speaking, the biconvex should be used on large wings which are not solid sections and modified double wedge should be used for smaller-sized solid wings.

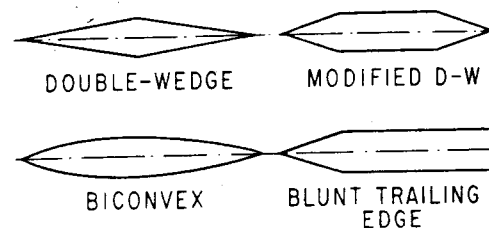


FIG. 3-28. Typical supersonic airfoil sections.

The effects of blunting the trailing edge on the lift and drag characteristics of the airfoil have been investigated by many authors,⁵⁶⁻⁵⁹ notably by Chapman. The latter⁵⁶ showed that a properly designed airfoil with moderately blunt trailing edge resulted in a substantial drag reduction over a double-wedge airfoil of equal section modulus. In addition, beneficial increases in lift-curve slope and maximum L/D were obtained. The stability of the missile (or aircraft) may be improved by the use of blunt-trailing-edge wings or control surfaces since the tendency for flow separation at the aft section is minimized (i.e., adverse pressure build-up is reduced).

3-15. WING AREA

The size of the wing of a missile may be dictated by the following three important considerations:

1. Range and maneuverability requirements
2. Types of design
3. Compatibility with subsystem operation

Case 1 may result in two vastly different wing sizes, depending on whether range or maneuverability is the primary factor. In the case of a long-range missile which is designed to cruise at relatively high altitudes, a relatively large wing is required to provide the desired maximum L/D . For this design the wing area is optimized when the induced drag is equal to the drag at zero lift, i.e., $C_{D_i} = C_{D_0}$ or $C_D = 2C_{D_0}$. It will be shown in Appendix C that, when this condition is realized, maximum L/D is obtained. For preliminary design studies the optimum wing area may be determined by calculating the values of C_{D_0} and C_{D_i} for three different wing sizes and determining the wing area which satisfies the condition of $C_{D_0} = C_{D_i}$. However, if maneuverability is the primary concern, which is generally the case for very short range guided missiles, the wing size will be dictated from stability and control considerations. In this case the wing area and location will be such that adequate trim load factor (defined as ratio of trim normal force to missile weight) is realized. In addition, a positive stability margin and good damping characteristics are desired in order to minimize control-system complexity. For example, consider the design of a wing of a canard-control configuration (see Fig. 2-2) designed for short range and a relatively large degree-of-maneuverability requirement. For a given fixed-wing location, too large a wing area may result in a configuration which has an excessive stability and hence may be unsatisfactory since (1) it does not meet the maneuverability requirement or (2) it requires increased canard size and hence larger and heavier servos to obtain the desired load factor. On the other hand, if the wing area is too small the design configuration may be intolerably unstable or have too low a stability margin.

The type of design also has strong influence on the wing size. For example, a wing-control design generally requires a smaller wing area than a tail-control configuration since the lift due to wing deflection for the former design is in the same direction as the main lifting force, as noted in Chap. 2. The size of the wing may also be governed by the operational requirement of the subsystem of the missile. Perhaps one of the most predominant requirements is the low trim angle of attack desired from the standpoint of inlet efficiency or radar-seeker operation. To meet this requirement the size of the wing may be larger than that required to obtain maximum L/D . In addition, a relatively large wing may be required to obtain a relatively low stall speed, particularly for missiles which are accelerated to flying speeds from a stationary ground launcher, thus reducing the size of the (external)

booster required. For this case the wing area may be determined by the following equation:

$$S = \frac{2W}{\rho V_s^2 C_{L_{\max}}} \quad (3-53)$$

where S = wing area required

W = weight of missile at end of boost (excluding weight of booster case if latter is detachable)

V_s = stall speed (or incremental speed due to booster)

$C_{L_{\max}}$ = maximum lift coefficient

The method of computing the incremental speed will be outlined in Chap. 4. In general it is desirable to accelerate the missile to a speed slightly above V_s or to increase the wing area slightly to reduce the size of the booster required.

3-16. SUBSONIC CHARACTERISTICS OF AIRFOIL

Before leaving the subject of wing and lifting surfaces, it may be of interest to mention some of the more important aerodynamic characteristics of the wing at subsonic and high subsonic speeds. In the above discussion, it was stated that the pressure or wave drag of the wing (or nose of the missile) constitutes the majority of the total drag and is a function of the shape (leading-edge angle) of the aerodynamic surface. In subsonic flow, the friction drag is over 90 per cent of the total drag force and is primarily a function of the type of flow (laminar or turbulent) and the total wetted area.

The equation for the skin-friction coefficient of a flat plate in laminar flow (known as Blasius's equation), derived from the Navier-Stokes differential equation of the motion of a viscous fluid, is

$$C_f = \frac{1.328}{\sqrt{\text{Re}}} \quad (3-54)$$

where C_f = skin-friction coefficient based on wetted area

Re = Reynolds number ($\rho VL/\mu \equiv VL/\nu$)

ρ = density of air, slug/ft³

V = free-stream velocity, fps

μ = viscosity of air, slug/ft-sec

ν = kinematic viscosity of air μ/ρ , ft²/sec

Equation (3-54) may be converted into a drag coefficient by dividing by the ratio of wetted area to reference area:

$$C_D = C_f \frac{S_{\text{wetted}}}{S_{\text{reference}}} \quad (3-55)$$

Equation (3-55) may then be used to determine the drag of any smooth aerodynamic surfaces (i.e., wing or body) in laminar incompressible flow.

For the turbulent incompressible flow case no simple theory is available or can be derived to define the complex flow characteristics in the turbulent boundary layer. Consequently, empirical or semi-empirical expressions are necessary to relate the friction coefficient

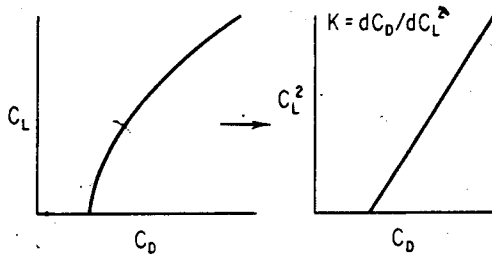


FIG. 3-29. Subsonic drag-rise factor.

with experimental results. The Kármán-Schoenherr equation has been recommended⁶¹ and is given as

$$C_f \log_{10} C_f Re = 0.242 \quad (3-56)$$

Again, Eq. (3-55) may be used to convert the friction coefficient into a drag coefficient using the value of C_f from Eq. (3-56). This will give

the drag of the airframe components in turbulent incompressible flow. The effect of compressibility in turbulent flow will be discussed in Chap. 4, which treats the problem of friction drag more fully in conjunction with the performance of the missile.

The above discussion deals with drag at zero lift or zero angle of attack for the case of a symmetrical airfoil. The induced drag or drag due to lift may be expressed as

$$C_{Di} = KC_L^2 \quad (3-57)$$

where $K = dC_D/dC_L^2$, called the drag-rise factor, may be obtained from a plot of C_L^2 vs. C_D as shown in Fig. 3-29. For an elliptical lift distribution the value of K is $1/\pi R$. For nonelliptical lift distributions the drag-rise factor is usually written as

$$K = \frac{1}{\pi e R} \quad (3-58)$$

where e is the Oswald efficiency factor and has a value less than unity. The theoretical value of e may be evaluated by the method presented in ref. 62. However, correlation of test results of a series of 18 wings indicates that the induced-drag coefficient may be accurately expressed as

$$C_{Di} = \frac{C_L^2}{\pi m^2 R} \quad (3-58a)$$

where m is a constant required to correlate the test results. The value

of m^2 was found to be approximately 13 per cent below the theoretical value of e .

The theoretical lift-curve slope of a thin wing of infinite aspect ratio in inviscid incompressible flow is given as

$$a_0 = 2\pi \text{ per radian} \quad (3-59)$$

The theoretical effects of aspect ratio⁶³ and sweep⁶⁴ are given by Eqs. (3-60) and (3-61) below:

$$(C_{L\alpha})_{\Lambda=0} = \frac{Ra_0}{R+2} = \frac{2\pi}{1+2/R} \quad (3-60)$$

$$(C_{L\alpha})_{\Lambda} = \frac{(R+2) \cos \Lambda}{R+2 \cos \Lambda} (C_{L\alpha})_{\Lambda=0} \quad (3-61)$$

For low-aspect-ratio wings (i.e., wings with aspect ratio less than 2), the variation of C_L with α is nonlinear. A more accurate representation of the variation of C_L with α is given by⁶⁵

$$C_L = k\alpha^n \quad (3-62)$$

where k and n are constants and a function of aspect ratio as shown in Fig. 3-30.

The theoretical effect of compressibility is to increase the incompressible subsonic value of $C_{L\alpha}$ by the Prandtl-Glauert factor $1/\sqrt{1-M^2}$. The compressibility effect on swept-back wings is given by⁶⁶

$$(C_{L\alpha})_M = \frac{R+2 \cos \Lambda}{R\sqrt{1-M^2 \cos^2 \Lambda} + 2 \cos \Lambda} (C_{L\alpha})_{M=0} \quad (3-63)$$

The center of pressure of an airfoil in subsonic flow is generally between 20 and 25 per cent of the mean aerodynamic chord. The effect of compressibility is to shift the center of pressure forward of the quarter chord. Since airfoil thickness and section have a pronounced effect on the aerodynamic characteristics, experimental data should be used whenever possible in lieu of the theoretical results.

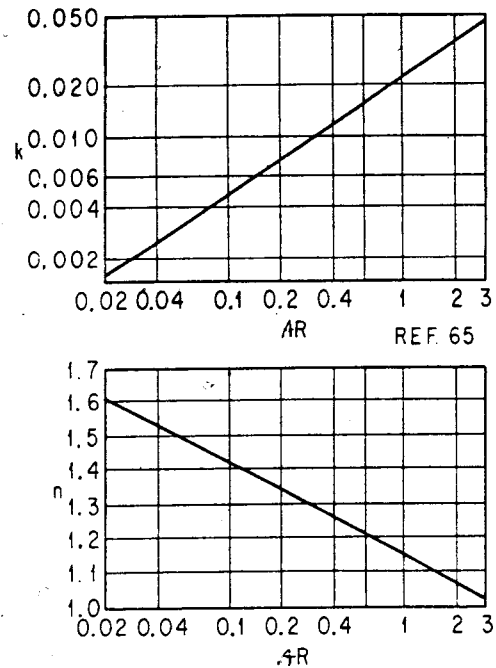


FIG. 3-30. Lift characteristics of low-aspect-ratio wings in subsonic flow.

3-17. AERODYNAMIC CONTROLS

Naturally, the type of control surface is directly associated with the type of configuration (see Fig. 2-2) in question. Perhaps the two most important design features of any type of control surface are (1) its effectiveness and (2) its hinge-moment characteristics. Control effectiveness must be large in order that its area may be kept to a minimum from the standpoint of hinge moments. The latter must be sufficiently low to make unnecessary large, heavy, and costly servos required to obtain the desired response characteristics. These two design features for the wing, canard, and tail control will be briefly discussed qualitatively below.

1. Wing Control. The control effectiveness in terms of pitching moment per unit deflection is perhaps the lowest of all three above-mentioned control surfaces. This results primarily from the extremely small moment arm normally associated with this type of control. The BW (body plus wing) configuration may actually have control reversal under certain flight or loading conditions (i.e., center of gravity ahead of wing center of pressure.) Thus the wing downwash on the rearward surfaces contributes to a large extent the resultant control effectiveness of the complete model configuration. Although the trim angle of attack may be somewhat lower, the net hinge moment of the wing control is the largest. Lateral control by differential wing deflection may be satisfactory, provided careful considerations are given to the aft surface design, as will be discussed in Chap. 8.

2. Canard Control. The control effectiveness is quite large as the result of two obvious features: (1) large canard moment arm and (2) downwash on the wing (or tail) surfaces, which can cause an almost equal amount of pitching-moment effectiveness. In addition, because of the small size, the canard has relatively low hinge moments despite the relatively large total angle of attack ($\alpha + \delta$) on the canards. This, in addition to the other features mentioned in Chap. 2, makes the canard control a very efficient aerodynamic and over-all systems design. Lateral control by differential canard deflection is completely unsatisfactory because of the low forces and moments as well as the adverse effect of downwash on the rear surfaces. Hence lateral-control devices such as wing spoilers, flaps, or tip ailerons are required for roll stabilization.

3. Tail Control. The pitching moment or control effectiveness is quite high in view of the fact that (1) the tail moment arm is generally fairly large and (2) there is no adverse downwash from deflection of

the forward surfaces. The level of hinge moment is low as a result of the relatively low resultant angle of attack. The method of solution of the tail-control effectiveness will be presented in Chap. 5 together with those previously discussed. As mentioned in Chap. 2, the use of tail control may present a critical packaging problem, especially on smaller-sized missiles. In general, lateral control by differential tail deflection is inadequate because of the relatively short moment arms as well as the large magnitude of the aerodynamic induced rolling moments from the large wings associated with this design. Consequently separate lateral-control surfaces are often required when movable tail surfaces are used for pitch control.

4. Lateral Controls. Although the previously mentioned controls (i.e., canards and tails) may be satisfactory for pitch (or yaw) control they are seldom adequate for lateral (or roll) control. Consequently separate aerodynamic control about the x , or roll, axis is required and may consist of the following types: flaps, spoilers, or wing-tip ailerons. The flap-type ailerons are generally located on the main wing-lifting surface. An excellent collection and summary of the lateral-control effectiveness of this type of control, presented in ref. 68 over a Mach-number range of 0.6 to 1.6, is recommended for preliminary design purposes. A complete bibliography⁶⁹ of control research done by the NACA (now the NASA) between 1946 and 1955 is also recommended for further study.

3-18. JET CONTROLS

Jet controls may be divided broadly into two types: those which obtain most of their effectiveness by blowing air or changing the pressure over an aerodynamic surface and those so-called (jet) reaction controls which obtain all their effectiveness by deflecting the jet-exhaust stream. The former type has received somewhat limited attention from both the theoretical and the experimental standpoint, whereas a vast amount of work has been done on the latter. Limited experimental data⁷⁰ are available on the lateral-control effectiveness of jet control using either free-stream or compressed air ejected through orifices over various portions of the wing. For configurations requiring relatively low lateral-control effectiveness this type of jet control may be satisfactorily employed.

Jet-reaction controls have been widely used, particularly on long-range ballistic missiles which, during portions of their flight, must operate at very low dynamic pressure. During lift-off and flights at

extremely high altitudes, jet-reaction control is mandatory to steer the missile on the proper flight path. Among the most common jet-reaction controls in usage are (1) simple auxiliary jets (rocket), (2) gimbaled nozzles, (3) jetavators, and (4) jet vanes. Strictly speaking, the latter should be classified as an aerodynamic control since the control forces and moments are derived principally from the deflected vanes rather than from the deflected jet. The first type has been used primarily in flight testing of experimental models. Small jet rockets are used to "pulse or excite" the model to determine pitching-moment derivatives as well as the model damping characteristics. Jet rockets

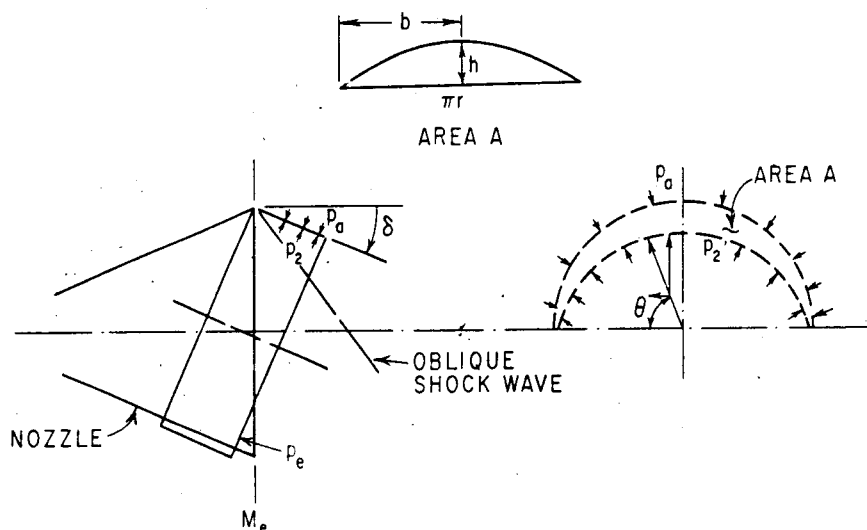


FIG. 3-31. Principle of jetavator operation.

may also be employed to provide velocity control for satellites. Gimbaled nozzles may be used to provide control about all three axes with either liquid- or solid-propellant rockets. The response of this type of control is relatively low because of its inherent large inertia and hinge moments. Gimbaled engines have also been used but have been restricted to liquid-propellant rockets. The disadvantages associated with this type of control are similar to those of the gimbaled nozzles except that their magnitudes are greater.

A jetavator is a relatively simple jet-deflection device consisting of a right circular cylindrical ring around the diverging portion of the nozzle. The hinge line is normal to the nozzle center line when the jetavator is in the neutral position. When the jetavator is deflected, an oblique shock is produced (see Fig. 3-31), causing a large change in pressure in the jet. The pressure rise across the oblique shock p_a/p_e is a function of the jetavator deflection δ and Mach number at the exit

of the nozzle M_e and may be determined from *NACA Technical Report 1135*. The pressure in the deflected jet p_2 acts normal to the ring area A , whereas the ambient pressure p_a acts in the opposite direction. As shown in Fig. 3-31, the pertinent pressure-force components vary as the sine of the angle θ . Consequently the mean pressure force varies as the mean $\sin \theta$ (i.e., $\overline{\sin \theta}$). The resultant force of the jetavator can then be written as

$$F = \overline{\sin \theta} \left(\frac{p_2}{p_e} p_e - p_a \right) A \quad (3-64)$$

The value of mean $\sin \theta$ may be obtained by plotting $\sin \theta$ vs. ring diameter and then taking the area under the sine curve and dividing by the ring diameter. This value is $\pi/4$. The value of p_e is obtained from rocket-motor performance data. The value of p_a is obtained from atmospheric tables and must be corrected for base-pressure and jet effects. From Fig. 3-32 the normal component of the jetavator force F_y is given as

$$F_y = \frac{\pi}{4} \left(\frac{p_2}{p_e} p_e - p_a \right) A \cos \delta \quad (3-64a)$$

and the axial or drag component F_x is

$$F_x = \frac{\pi}{4} \left(\frac{p_2}{p_e} p_e - p_a \right) A \sin \delta \quad (3-64b)$$

Area A may be found by the following relationship:

$$A = \frac{4bh}{3} \quad (3-65)$$

where $b = \pi r/2$.

A single-axis jetavator will be adequate for either pitch or yaw control, whereas a full-swiveling jetavator is required where both pitch and yaw control are desired. For control about all three (pitch, yaw, and roll) axes, two nozzles, each equipped with a full-swiveling jetavator, are required.

Jet vanes are commonly used for controls about all three axes on ballistic missiles. For relatively long periods of continuous operation it is essential to use high-temperature-resistant material over a goodly portion of the vanes. In jet-vane design it is important to keep the thickness ratio to a minimum since it has been shown that the wave drag of an airfoil varies as the square of the thickness ratio. This is particularly important for conditions where the jet vanes are submerged in the jet exhaust throughout rocket-motor operation since

drag losses occur even for zero vane deflection. The aerodynamic forces on the jet vane may be calculated by the methods previously discussed using the environments of the jet (i.e., values of γ , p , and M in the jet to obtain dynamic pressure q).

The lateral (or normal) and axial components of the jet vanes F_y and F_x may be expressed with the aid of Eqs. (3-1) and (3-2) and Fig. 3-1 as follows:

$$F_y = N_{jv} \cos \delta - A_{jv} \sin \delta \quad (3-66)$$

$$F_x = A_{jv} \cos \delta + N_{jv} \sin \delta \quad (3-67)$$

The drag-loss equivalent to an impulse loss may be determined if the vane-deflection history is known during rocket-motor operation. The

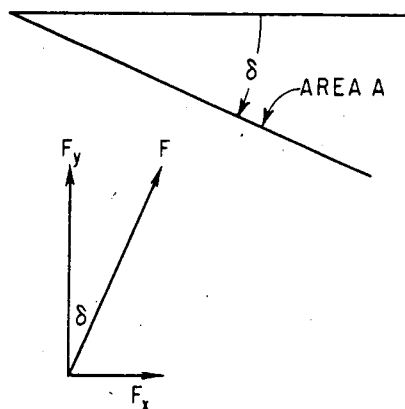


FIG. 3-32. Side view of jet-vator forces.

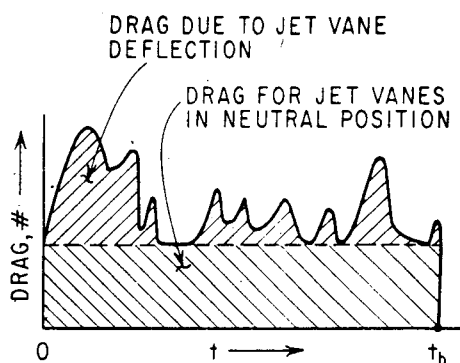


FIG. 3-33. Drag losses caused by jet vanes.

latter information can be obtained from simulation studies which determine the amount of guidance or vane deflection required for several typical flight conditions. The total drag loss I_D is then the total area under the drag-time curve (see Fig. 3-33) and can be expressed as follows:

$$I_D = - \int_0^{t_b} F_x dt \quad (3-68)$$

where t_b = total burning time of rocket motor

SYMBOLS

A	axial force, aspect ratio, area
A_p	planform area
C_A	axial-force coefficient (A/qS)
C_D	drag coefficient (D/qS)

C_{D_F}	force-drag coefficient [see Eq. (3-31)]
C_{D_i}	induced-drag coefficient
C_{D_N}	drag (coefficient) due to normal force
C_{D_0}	drag (coefficient) at zero lift
C_f	skin-friction coefficient
C_L	lift coefficient
C_N	normal-force coefficient
$C_{N\alpha}$	normal-force-curve slope (variation of C_N with α), $\partial C_N / \partial \alpha$
C_p	pressure coefficient $(p - p_0)/q_0$
$C_1, C_2, C_3, \dots$	Busemann constants [see Eqs. (3-16) and (3-17)]
D	drag force
E	elliptic integral of second kind [see Eq. (3-51)]
F	force
I_D	total drag loss [see Eq. (3-68)]
K	drag-rise factor, or term used in the "parabolic-series" nose (see Fig. 3-11B)
L	lift force
L/D	lift-to-drag ratio
M	Mach number
M/f_n	hypersonic similarity parameter
N	normal force
P	pressure coefficient $\Delta p/q$ or C_p [see Eq. (3-9)], or denotes "parabolic-series" nose (see Fig. 3-11B)
R	aspect ratio, resultant force, or radius of body shown in Fig. 3-10
Re	Reynolds number [see Eq. (3-54)]
S	reference area
S_b	area of base of missile
S_w	wing planform area
V	free-stream velocity
V_s	stall speed
W	weight of missile
a_0	theoretical lift-curve slope of a thin wing of infinite aspect ratio in inviscid incompressible flow
b	wing span, or semispan of jetavator (see Fig. 3-31)
c	wing chord, speed of sound
cp	center of pressure
d	body diameter
e	Oswald efficiency factor
f_n	fineness ratio of nose l_n/d

h	projected height of jetavator (see Fig. 3-31)	
k	a constant in Eq. (3-62), or tangent of leading-edge sweep angle shown in Fig. 3-26	
l	length of body or nose section	
l/d	length-to-diameter ratio	
m	a correlation constant used in Eq. (3-58a)	
n	a constant in Eq. (3-50) and defined as k/β as shown in Fig. 3-26, or exponent in defining the "power-series" nose (see Fig. 3-11A), or a constant in Eq. (3-62)	
p	static pressure	
Δp	pressure differential ($p - p_0$ or $p_2 - p_1$)	
p_a	ambient pressure (see Fig. 3-31)	
p_e	pressure at the exit of the nozzle (see Fig. 3-31)	
p_0	free-stream static pressure	
p_1	free-stream static pressure as used in Eq. (3-4)	
p_2	static pressure on the surface of the cone as used in Eq. (3-4), or pressure in the deflected jet as shown in Fig. 3-31	
q, q_c	free-stream dynamic pressure ($\frac{1}{2}\rho V^2$ or $\frac{1}{2}\gamma p M^2$)	
r	local nose radius (see Fig. 3-11A and B), or dimension of the jetavator shown in Fig. 3-31	l
r_b	radius at the base of the nose section (see Fig. 3-11A)	l
r_n	nose radius (see Fig. 3-10)	l
t_b	burning time of the rocket motor	
x	moment arm	
α	angle of attack	
β	$\sqrt{M^2 - 1}$	
γ	specific-heat ratio of air (1.4)	l
δ	wedge semivertex angle (see Fig. 3-3), or jetavator deflection (see Figs. 3-31 and 3-32)	l Z
ϵ	semivertex angle of a triangular wing	F
θ	shock angle, or circumferential angle shown in Fig. 3-31 or boattail angle	a_1
Λ	wing leading-edge sweep angle	B
μ	Mach angle [see Eq. (3-35)]	
μ'	angle of any ray radiating from wing tip (see Fig. 3-21)	th
ν	kinematic viscosity of air	nc
ρ	density of air	
σ	semivertex cone angle	Ae
σ_o	semivertex ogive angle	Ae
φ	a term used in Eq. (3-12)	

Subscripts

A	aft sections of the airfoil
BT	boattail
F	forward sections of airfoil
L	lower surfaces of airfoil
M	effect of compressibility
max	maximum
T	total
U	upper surfaces of airfoil
b	base (section)
e	exit of nozzle
jv	jet vanes
x, y	coordinates shown in Fig. 3-32
Λ	with sweepback
I, II, III, IV	denote area shown in Figs. 3-20 and 3-24
1, 2, 3, 4	denote regions of airfoil in Fig. 3-18
0.5	denotes mid-chord of airfoil
∞	two-dimensional value at limit of Mach cone as used in Eq. (3-36), or two-dimensional value as used in Fig. 3-22

REFERENCES

1. Taylor, G. F., and J. W. Maccoll: The Air Pressure on a Cone Moving at High Speeds, *Proc. Roy. Soc. (London) (A)*, vol. 139, pp. 278-293, 1933.
2. Taylor, G. I., and J. W. Maccoll: The Mechanics of Compressible Fluids, Durand's "Aerodynamic Theory," vol. III, div. H, Springer-Verlag, Berlin, 1934.
3. Staff of the Computing Section, Center of Analysis (under direction of Zdeněk Kopal): Tables of Supersonic Flow around Cones, *MIT Tech. Rept. 1*, 1947.
4. Staff of the Computing Section, Center of Analysis (under direction of Zdeněk Kopal): Tables of Supersonic Flow around Yawing Cones, *MIT Tech. Rept. 3*, 1947.
5. Stone, A. H.: On Supersonic Flow Past a Slightly Yawing Cone, *J. Math. and Phys.*, vol. 27, no. 1, pp. 67-81, April, 1948.
6. Bonney, E. A.: "Engineering Supersonic Aerodynamics," McGraw-Hill Book Company, Inc., New York, 1950.
7. Busemann, A.: Aerodynamic Lift at Supersonic Speeds, lecture given at the 5th Volta Conference at Rome, published in *Luftfahrt-Forsch.*, vol. 12, no. 6, 1935 (*British ARC Rep.* 2844, Feb. 3, 1937).
8. Ferrari, C.: The Fields of Supersonic Flow around a Solid of Revolution, *Aerotecnica*, vol. 19, no. 1, 1937.
9. Tsien, H.: Supersonic Flow over an Inclined Body of Revolution, *J. Aeronaut. Sci.*, vol. 5, no. 2, pp. 480-483, October, 1938.

10. Miles, E. R. C.: "Semi-empirical Formulae for Ogives," CM-505, Applied Physics Laboratory, Johns-Hopkins University.
11. Von Kármán, T., and N. B. Moore: Resistance of Slender Bodies Moving with Supersonic Velocities with Special Reference to Projectiles, *Trans. ASME*, vol. 54, 1932.
12. Von Kármán, T.: The Problem of Resistance in Compressible Fluids, *Proc. 5th Volta Cong. Roy. Acad. Italy*, 1936.
13. Syvertson, C. A., and D. H. Dennis.: A Second-order Shock-expansion Method Applicable to Bodies of Revolution Near Zero Lift, *NACA Tech. Rept.* 1328, 1957.
14. Ferri, A.: Supersonic Flow around Circular Cones at Angles of Attack, *NACA Tech. Rept.* 1045, 1951.
15. Eggers, A. J., Jr., and R. C. Savin: A Unified Two-dimensional Approach to the Calculation of Three-dimensional Hypersonic Flows, with Application to Bodies of Revolution, *NACA Rept.* 1249, 1955 (supersedes *NACA Tech. Note* 2811).
16. Eggers, A. J., Jr., and R. C. Savin: Approximate Methods for Calculating the Flow about Nonlifting Bodies of Revolution at High Supersonic Airspeeds, *NACA Tech. Note* 2579, 1951.
17. Savin, R. C.: Application of the Generalized Shock-expansion Method to Inclined Bodies of Revolution Traveling at High Supersonic Airspeeds, *NACA Tech. Note* 3349, 1955.
18. Eggers, A. J., R. C. Savin, and C. A. Syvertson: The Generalized Shock-expansion Method and Its Application to Bodies Traveling at High Supersonic Airspeeds, *J. Aeronaut. Sci.*, vol. 22, no. 4, pp. 231-238, April, 1955.
19. Van Dyke, M. D.: First- and Second-order Theory of Super-sonic Flow Past Bodies of Revolution, *J. Aeronaut. Sci.*, vol. 18, no. 3, pp. 161-179, March, 1951.
20. Van Dyke, M. D.: A Study of Second-order Supersonic Flow Theory, *NACA Rept.* 1081, 1952 (supersedes *NACA Tech. Note* 2200).
21. Van Dyke, M. D.: A Study of Hypersonic Small-disturbance Theory, *NACA Tech. Note* 3173, 1954 (superseded by *Tech. Rept.* 1194).
22. Allen, H. J., and E. W. Perkins: A Study of Effects of Viscosity on Flow over Slender Inclined Bodies of Revolution, *NACA Tech. Rept.* 1048, 1951.
23. Buford, W. E.: The Effects of Afterbody Length and Mach Number on the Normal Force and Center of Pressure of Conical and Ogival Nose Bodies, *J. Aeronaut. Sci.*, vol. 25, pp. 103-108, February, 1958.
24. Graham, E. W.: Pressure Distribution on a Cylinder Preceded by a Cone in Axial Supersonic Flow, Douglas Aircraft Company, SM-13322, July, 1948.
25. Chapman, D. R.: An Analysis of Base Pressure at Supersonic Velocities and Comparison with Experiment, *NACA Tech. Note* 2137, July, 1950.
26. Spahr, J. R., and R. R. Dickey: Effect of Tail Surfaces on the Base Drag of a Body of Revolution at Mach Numbers of 1.5 and 2.0, *NACA Tech. Note* 2360, 1951.
27. Chapman, D. R., and E. W. Perkins: Experimental Investigation of the Effects of Viscosity on the Drag and Base Pressure of Bodies of Revolution at a Mach Number of 1.5, *NACA Tech. Rept.* 1036, 1951.

28. Bogdonoff, S. M.: A Preliminary Study of Reynolds Number Effects on Base Pressure at $M = 2.95$, *J. Aeronaut. Sci.*, vol. 19, p. 201.

29. Love, E. S.: Base Pressure at Supersonic Speeds on Two-dimensional Airfoils and on Bodies of Revolution with and without Fins Having Turbulent Boundary Layers, *NACA Tech. Note* 3819, January, 1957.

30. Chow, W. L.: On the Base Pressure Resulting from the Interaction of a Supersonic External Stream with a Sonic or Subsonic Jet, *J. Aeronaut. Sci.*, vol. 26, no. 3, pp. 176-180, March, 1959.

31. Cabbage, J., Jr.: Jet Effects on Base and Afterbody Pressures of a Cylindrical Afterbody at Transonic Speeds, *NACA Research Mem.* L56C21, May 23, 1956.

32. Cartright, E. M., Jr., and A. H. Schroeder: Investigation at Mach Number 1.91 of Side and Base Pressure Distributions over Conical Boattails without and with Jet Flow Issuing from Base, *NACA Research Mem.* E51F26, Sept. 12, 1951.

33. Englert, G. W., D. J. Vargo, and R. W. Cubbison: Effect of Jet-nozzle-expansion Ratio on Drag of Parabolic Afterbodies, *NACA Research Mem.* E54B12, Apr. 19, 1954.

34. Pitts, W. C., J. N. Nielsen, and G. E. Kaattari: Lift and Center of Pressure of Wing-body-tail Combinations at Subsonic, Transonic, and Supersonic Speeds, *NACA Tech. Rept.* 1307, 1957.

35. Spreiter, J. R.: The Aerodynamic Forces on Slender Plane- and Cruciform-wing and Body Combinations, *NACA Tech. Rept.* 962, 1950.

36. Von Kármán, T.: Supersonic Aerodynamics—Principles and Applications, *J. Aeronaut. Sci.*, vol. 14, no. 7, pp. 373-402, July, 1947.

37. Ivey, H., G. W. Stickle and A. Schuettler: Charts for Determining the Characteristics of Sharp-nose Air-foils in Two-dimensional Flow at Supersonic Speeds, *NACA Tech. Note* 1143, 1947.

38. Ackeret, J.: Air Forces on Airfoil Moving Faster than Sound, *NACA Tech. Mem.* 317, 1925.

39. Taylor, G. I.: Applications to Aeronautics of Ackeret's Theory of Aerofoils Moving at Speeds Greater than That of Sound, British ARC, *R & M* 1467, 1932.

40. Busemann, A., and O. Walchner: "Airfoil Characteristics at Supersonic Speeds," RTP Translation 1786, British Ministry of Aircraft Production.

41. Lock, C. N. H.: Examples of the Application of Busemann's Formula to Evaluate the Aerodynamic Force Coefficients on Supersonic Aerofoils, British ARC, *R & M* 2101, 1949.

42. Laitone, E. V.: Exact and Approximate Solutions of Two-dimensional Oblique Flow, *J. Aeronaut. Sci.*, vol. 14, no. 1, p. 25, January, 1947.

43. Meyer, T.: Über Zweidimensionale Bewegungsvorgänge in einem Gas, das mit Überschallgeschwindigkeit strömt, *Mitt. Forschungsarb. Gebiete Ingenieurw.*, vol. 62, no. C, Berlin, 1908.

44. Snow, R. M.: Aerodynamics of Thin Quadrilateral Wings at Supersonic Speeds, *Quart. Appl. Math.*, vol. 5, no. 4, 1948.

45. Ward, G. N.: The Pressure Distribution on Some Flat Laminar Aerofoils at Incidence at Supersonic Speeds—I, Scientific Research and Experiment Airflow, Paper 30, February, 1946.

46. Evvard, J. C.: Distribution of Wave Drag and Lift in the Vicinity of Wing Tips at Supersonic Speeds, *NACA Tech. Note* 1382, July, 1947.

47. Vincenti, W.: Comparison between Theory and Experiment for Wings at Supersonic Speeds, *NACA Tech. Rept.* 1033, 1951.
48. Ferri, A.: Experimental Results with Airfoils Tested in the High-speed Tunnel at Guidonia, *NACA Tech. Mem.* 946, 1940.
49. Jack, J. R.: Theoretical Pressure Distribution and Wave Drags for Conical Boattails, *NACA Tech. Note* 2972, July, 1953.
50. Puckett, A. E.: Supersonic Wave Drag of Thin Airfoils, *J. Aeronaut. Sci.*, vol. 13, no. 9, pp. 475-484, September, 1946.
51. Puckett, A. E., and H. J. Stewart: Aerodynamic Performance of Delta Wings at Supersonic Speeds, *J. Aeronaut. Sci.*, vol. 14, no. 10, pp. 567-568, October, 1947.
52. Jones, R. T.: Thin Oblique Airfoils at Supersonic Speeds, *NACA Tech. Note* 1107, September, 1946.
53. Stewart, H. J.: The Lift of a Delta Wing at Supersonic Speeds, *Quart. Appl. Math.*, vol. IV, no. 3, pp. 246-254, October, 1946.
54. Brown, C. E.: Theoretical Lift and Drag of Thin Rectangular Wings at Supersonic Speeds, *NACA Rept.* 839, 1946 (supersedes *Tech. Note* 1183).
55. Love, E. S.: Investigation at Supersonic Speeds of 22 Triangular Wings Representing Two Airfoil Sections for Each of 11 Apex Angles," *NACA Tech. Rept.* 1238, 1955.
56. Chapman, D. R.: Reduction of Profile Drag at Supersonic Velocities by the Use of Airfoil Sections having a Blunt Trailing Edge, *NACA Tech. Note* 3503, September, 1955.
57. Chapman, D. R., and R. H. Kester: Effect of Trailing-edge Thickness on Lift at Supersonic Velocities, *NACA Tech. Note* 3504, June, 1955.
58. Morrow, J. D., and E. Katz: Flight Investigation at Mach Numbers from 0.6 and 1.7 to Determine Drag and Base Pressures on a Blunt-trailing-edge Airfoil and Drag of Diamond and Circular-arc Airfoils at Zero Lift, *NACA Tech. Note* 3548, November, 1958.
59. Morrow, J. D.: Measurements of the Effect of Trailing-edge-thickness on the Zero-lift Drag of Thin Low-aspect-ratio Wings, *NACA Tech. Note* 3550, November, 1958.
60. Gallagher, J. J., and J. N. Mueller: An Investigation of the Maximum Lift of Wings at Supersonic Speeds, *NACA Tech. Rept.* 1227, 1955.
61. Locke, F. W. S., Jr.: Recommended Definition of Turbulent Friction in Incompressible Fluids, *Bureau of Aeronautics Research Division Rept.* 1415, June, 1952.
62. DeYoung, J.: Theoretical Additional Span Loading Characteristics of Wings with Arbitrary Sweep, Aspect Ratio and Taper Ratio, *NACA Tech. Note* 1491, December, 1947.
63. Swanson, R. S., and S. M. Crandall: Lifting Surface Theory Aspect Ratio Corrections to the Lift and Hinge-moment Parameters for Full-span Elevators on Horizontal Tail Surfaces, *NACA Tech. Note* 1175, 1947.
64. Toll, T. A., and J. J. Queijo: Approximate Relations and Charts for Low Speed Stability Derivatives of Swept Wings, *NACA Tech. Note* 1581, May, 1948.
65. Locke, F. W. S.: An Empirical Study of Low Aspect Ratio Lifting Surfaces with Particular Regard to Planing Craft, *J. Aeronaut. Sci.*, March, 1949.

66. Fischer, L. R.: Approximate Corrections for the Effects of Compressibility on the Subsonic Stability Derivatives of Swept Wings, *NACA Tech. Note* 1854, April, 1949.
67. Abbott, I. H., A. E. Von Doenhoff, and L. S. Stiver, Jr.: Summary of Airfoil Data, *NACA Tech. Rept.* 824, 1948.
68. Strass, H. K., E. W. Stephens, E. M. Fields, and E. D. Schult: Collection and Summary of Flap-type-aileron Rolling Effectiveness Data at Zero Lift as Determined by Rocket-powered Model Tests at Mach Numbers between 0.6 and 1.6, *NACA Research Mem.* L55F14, Sept. 2, 1955.
69. Brewer, J. D.: Description and Bibliography of NACA Research on Wing Controls—January 1946–February 1955, *NACA Research Mem.* 54K24, Mar. 24, 1955.
70. Lowry, J. G.: Recent Control Studies, *NACA Research Mem.* L55L22a, Feb. 16, 1956.
71. Krzywoblocki, M. Z.: Aerodynamic Studies: The Forces Acting on an Air Vehicle—A Review of the Literature, *WADC Tech. Note* 56-360, pt. XVI, May, 1957 (*ASTIA AD* 130862).
72. Hall, C. F.: Lift, Drag, and Pitching Moment of Low-aspect Ratio Wings at Subsonic and Supersonic Speeds, *NACA Research Mem.* A53A30, Apr. 14, 1953.

CHAPTER 4

MISSILE PERFORMANCE

4-1. INTRODUCTION

The performance characteristics of a missile can be determined once the magnitude of the pertinent forces (i.e., thrust, drag, weight) and flight conditions are known or specified. The methods of calculating performance parameters such as maximum speed and rate of climb for cruise-type missiles are identical with those used on conventional manned aircraft. For rocket-powered missiles, particularly those which are accelerated to the desired or required speed and then decelerate or glide to their destination, a somewhat different approach to the performance evaluation is used. This chapter will be concerned with a fairly comprehensive treatment of performance of missiles designed for both long and short ranges and different modes of powered flight. In addition, methods of evaluation of drag characteristics of the missile, as well as their application, are presented in connection with missile-performance determination.

4-2. FRICTION DRAG

In incompressible flow the skin-friction coefficient depends primarily on the type of flow (i.e., laminar or turbulent) and the Reynolds number. As noted in Chap. 3, the skin-friction coefficient for laminar and turbulent flow is given as

$$C_{fL} = \frac{1.328}{\sqrt{\text{Re}}} \quad (4-1)$$

and

$$\sqrt{C_{fT}} \log_{10} (C_{fT} \text{Re}) = 0.242 \quad (4-2)$$

Although Eq. (4-1) has been generally accepted as accurate for the determination of skin-friction drag in laminar flow, such flow conditions are seldom realized in actual practice except for very low values of Reynolds number (i.e., less than 1 million). For $\text{Re} > 10^6$, turbulent

flow generally prevails and Eq. (4-2) must be used. It should be noted here that a complete understanding of friction drag in turbulent flow has yet to be achieved. Recently Coles^{1,*} proposed the "universal skin-friction constants" in an attempt to predict skin friction accurately with a method of boundary-layer analysis. Results of limited experimental tests² indicate that the skin-friction coefficient approaches asymptotically a constant value at Reynolds numbers exceeding 21 million. However, until more data are obtained, it is recommended that the turbulent skin-friction coefficient be calculated by Eq. (4-2). Figure 4-1 is a plot of the skin-friction coefficient as a function of Reynolds number for laminar and turbulent flow.

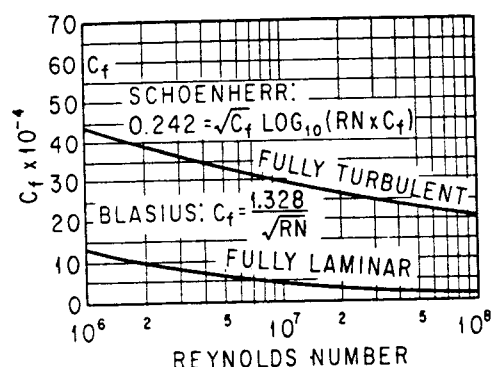


FIG. 4-1. Skin-friction coefficient vs. Reynolds number (incompressible flow).

For most flight conditions laminar flow prevails over the extreme forward portion of the missile wing or body, followed by completely turbulent flow over the remaining portion of the missile. The problem of drag determination has thus resolved into that of determining the transition point where turbulent flow commences. No useful theoretical methods are known to exist to determine the exact location of the transition point because of the complex nature of the flow and the interaction effects of such factors as pressure gradient, surface smoothness, and Reynolds number. Hence experience and judgment must be used to determine the transition point in most preliminary frictional-drag estimates. In general the transition point of a smooth and streamlined body may be located as far back as 30 per cent of its length. For missile application the nose tangency point of the end of the nose or forebody section as a transition point is a fairly valid assumption. For the wings turbulent flow may be assumed in preliminary drag evaluation. Once the transition is known or assumed, the next step is to calculate the skin-friction coefficient for the laminar and turbulent section of the body (or wing) as follows:

$$C_f = C_{f_L} \frac{S_x}{S_l} + C_{f_T} \frac{S_l - S_x}{S_l} \quad (4-3)$$

* Superscript numbers indicate references listed at the end of the chapter.

where C_{f_L} = skin-friction coefficient in laminar flow, Re based on length x shown in Fig. 4-2

C_{f_T} = skin-friction coefficient in turbulent flow, Re based on length l

S_x = wetted area over length x

S_l = total wetted area (over length l)

Although somewhat more exact expressions are available, Eq. (4-3) is sufficiently accurate for most preliminary drag analyses.

The skin-friction-drag coefficients determined above are applicable

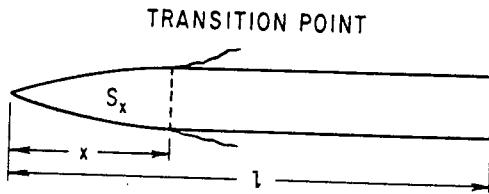


FIG. 4-2. Definition of terms for friction-drag calculation.

for condition of incompressible flow only (the effects of Mach number have been neglected). For subsonic speeds the Mach-number effects are not too pronounced. However, for the transonic and supersonic regions, the Mach-number effects become significant and cannot be ignored. For the lam-

inar flow the effect of compressibility has been shown by a simplified theoretical approach to reduce the skin-friction coefficient by the following expression:

$$\frac{C_f}{C_{f_0}} = \left(\frac{1}{1 + 0.85M^2/5} \right)^{0.1295} \quad (4-4)$$

where C_{f_0} is the skin-friction-drag coefficient in laminar incompressible flow. Results derived by a more comprehensive theory appear to be in good agreement with those defined by Eq. (4-4). In addition, the accuracy derived by both theories is within the scatter of the test data. Figure 4-3 shows the effects of compressibility predicted by the two theories.

For turbulent flow the semiempirical mixing-length theories of Von Kármán and Prandtl have been used successfully in predicting skin friction for a flat plate at low subsonic speeds. Based on the most recent data available, the following expression appears to be most accurate in defining the effect of compressibility in turbulent flow for Mach numbers less than 1:

$$\frac{C_f}{C_{f_0}} = \frac{1}{1 + 0.08M^2} \quad (4-5)$$

When mixing theory is applied to turbulent compressible flow, numerous solutions are obtained that yield results vastly different from each other and from test data. A comprehensive bibliography of theoretical predictions of skin-friction coefficient in turbulent compressible flow may be found in ref. 3. Of all these theoretical predictions, the "extended Frankl-Voishel" theory^{4,5} appears to be one of the first to predict the compressibility effect accurately. Subsequent theories by Cope,⁶ Wilson,⁷ Tucker,⁸ and others also yield results comparable with those predicted by the extended Frankl-Voishel theory. For design purposes the latter theory is recommended, with the results shown in Fig. 4-3. The expression for the compressibility effect is given below.⁵

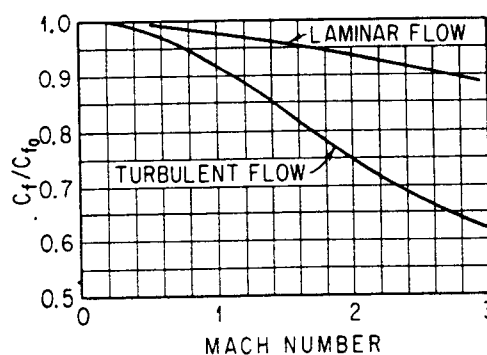


FIG. 4-3. Effect of compressibility on skin-friction coefficient.

$$\frac{C_f}{C_{f_0}} = \frac{1}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{0.467}} \quad (4-6)$$

where C_{f_0} is the skin-friction coefficient in turbulent incompressible flow.

The friction-drag coefficient is obtained by

$$C_D = C_f \frac{S_{\text{wetted}}}{S_{\text{reference}}}$$

In preliminary design studies it is recommended that the friction-drag coefficient be increased by 10 per cent to account for surface roughness and small protuberances.

4-3. PRESSURE DRAG

Pressure or form drag in subsonic flow^{9,10,11} is generally small and may be neglected in preliminary design studies. However, in the transonic and supersonic region pressure drag constitutes a great percentage of the total drag of the missile and hence must be carefully obtained before an accurate determination can be made on missile-performance capabilities. Pressure drag in the transonic region can

best be determined from experimental test data or by the transonic-area rule,¹² which is highly mathematical but can be solved on automatic computing machines. In the supersonic-speed region the pressure drag can be determined by the relationships presented in the preceding chapter.

1. Body. As previously discussed, the body-pressure drag consists of three primary parts—nose-wave drag, boattail-wave drag, and base-pressure drag. For a tangent ogive nose the wave-drag coefficient was shown [Eq. (3-9)] to be

$$C_{D_w} = P \left\{ 1 - \frac{2[196(l/d)^2 - 16]}{28(M + 18)(l/d)^2} \right\} \quad (4-8)$$

A comparison of results obtained by Eq. (4-8) and those derived from correlated test results mentioned in Chap. 3 is shown below. It can be seen that the agreement between the two sets of results is satisfactory for most preliminary design studies.

Mach No.	C_{D_w} from Eq. (4-8)	C_{D_w} from correlated data
1.3	0.115	0.124
1.5	0.101	0.106
2.0	0.095	0.095
2.5	0.094	0.093

The boattail and (power-off) base-drag coefficients may be determined from Chap. 3.

2. Wing. The theoretical wave drag of an airfoil has been shown [Eq. (3-30)] to vary as the thickness squared. For an untapered rectangular wing with a double-wedge airfoil section, the wave-drag coefficient was determined [Eq. (3-45)] to be

$$C_D = \frac{4\delta^2}{\beta} \left(1 - \frac{1}{2\beta A} \right) \quad (4-9)$$

For wings of different planform such as a delta wing, the theoretical wave drag may be obtained from the numerous references (i.e., 50 to 54 of Chap. 3) previously mentioned. However, for practical preliminary design, experimental results such as those contained in refs. 47 and 55 of Chap. 3 should be used.

4-4. INDUCED DRAG

For subsonic speeds the induced-drag coefficient may be obtained by the following empirical [Eq. (3-58a)] expression:

$$C_{Di} = \frac{C_L^2}{\pi R m^2} \quad (4-10)$$

The value of m^2 may be obtained from Chap. 3. For the high subsonic- and transonic-speed regions, experimental or correlated test results must be used in order to obtain reasonably accurate values of C_{Di} . However, for supersonic speeds the induced drag (or drag due to normal force) can be approximated fairly closely as follows:

$$C_{DN} = C_{Di} = KC_N^2 = \frac{C_N^2}{C_{N\alpha}^2} \quad (4-11)$$

The above expression is quite accurate, particularly for straight wings or triangular wings with supersonic leading edge. At lower supersonic speeds where subsonic leading edge is realized, "leading-edge suction" effect exists and results in a lower drag-rise factor K as shown in Fig. 4-4.

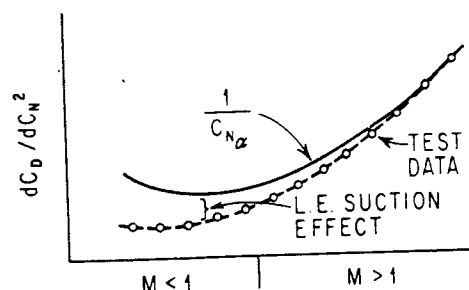


FIG. 4-4. Drag-rise factor vs. Mach number.

4-5. INTERFERENCE DRAG

Interference effects on drag in the transonic- and supersonic-speed regions are not completely understood and hence are difficult to evaluate. Theories such as the transonic- and supersonic-area rules^{12,13} are available to predict trend and with some degree of accuracy the actual magnitude of the drag of composite configurations (i.e., body plus wing, etc.). The methods and procedures on the usage of these highly mathematical theories have been greatly simplified with the aid of automatic computing equipment. In most aircraft companies the area-rule methods are programmed on automatic computers and hence the magnitude of the problem of drag (interference) evaluation is greatly reduced.

Quite frequently one must rely upon experimental results and correlations for an accurate evaluation of this interference drag. Much more effort is still necessary before a quick solution to this problem can be found.

4-6. BOOST-GLIDE TRAJECTORY

Boost-glide trajectory is defined as one in which the missile is rapidly accelerated to its maximum speed and then coasts power-off the remaining portion of its trajectory. For relatively short range missiles (i.e., AAM's, SAM's, SSM's, or ASM's described in Chap. 2), this type of trajectory is very common and warrants some consideration. Several methods are available for computing the range and velocity history of missiles employing this type of trajectory. These are (1) "short-cut" or graphical solution, (2) iteration method by manual calculations, and (3) solution of the equations of motion by automatic computing machines. If the flight path of the missile is nonlinear like that of a short-range surface-to-surface ballistic missile, the last method mentioned is recommended over the first two methods of solution. However, for certain types of missions the missile may follow a relatively linear flight path, for which case the graphical or manual iteration methods may be used. A detailed description of the latter two methods is given below.

1. Graphical Solution. This "short-cut" method has been found to be extremely useful and timesaving in the evaluation of performance characteristics for short-range missiles which follow essentially a straight flight path, i.e., level flight, diving or climbing flight trajectory. It is both desirable and convenient first to calculate the burnout condition by the analytical method and then to use the graphical solution for the power-off portion of the trajectory. The essential data which define the burnout condition are (1) missile weight, (2) velocity, and (3) range or space coordinate of the missile. The weight at burnout is readily known by subtracting the weight of the rocket-motor propellant (for an integral rocket motor) from the launch gross weight of the missile. In case of an external droppable booster the weight of the motor casing and associated attachments must also be subtracted. The velocity at burnout is the sum of the initial launch velocity and the incremental velocity due to boost ΔV_b , which may be calculated by the familiar equation shown below.

$$\Delta V_b = K' I_{sp} g \ln \frac{W_L}{W_E} - g t_b \sin \gamma_L \quad (4-12)$$

where K' = factor to account for drag effects

I_{sp} = rocket-motor specific impulse, lb-sec/lb of propellant, sec

W_L = missile launch weight, lb

W_E = missile launch weight minus propellant weight, lb

γ_L = launch attitude (assumed constant), deg

t_b = rocket-motor burning time, sec

A detailed derivation of Eq. (4-12) is presented in Appendix D. The velocity at the end of boost V_b is given as

$$V_b = V_L + \Delta V_b \quad (4-13)$$

where V_L = initial launch velocity

The distance traveled during boost S_b may be calculated by

$$S_b = \frac{V_L + V_b}{2} t_b \quad (4-14)$$

In order to calculate the value of ΔV_b , the value of K' must first be determined. This can be done by calculating ΔV_b by Eq. (4-12) by assuming $K' = 1$. The uncorrected value of ΔV_b (denoted as $\Delta V_{b_{unc}}$) thus determined is then used to determine the uncorrected end of boost velocity $V_{b_{unc}}$, using Eq. (4-13). The weighted average drag can then be determined for the $V_{b_{unc}}$ and V_L (and altitude) conditions; hence a fairly accurate value of K' is obtained. By substituting this new value of K' into Eq. (4-12) the value of ΔV_b , and hence V_b , can be readily determined. If better accuracy is desired the above process may be repeated using the second approximation of V_b to determine K' and hence ΔV_b and V_b . However, for most practical applications only one approximation is required. Having thus calculated the value of V_b , the distance traveled during boost or accelerated flight can be readily calculated by Eq. (4-14). With the conditions at burnout established one can proceed with the graphical method of evaluating the trajectory characteristics of the remaining portion of flight of the missile.

The simple case of a straight and level flight trajectory is first used to illustrate the graphical solution. The procedure involved in this graphical solution is as follows:

1. Calculate the drag of the missile as a function of velocity by the following relationship:

$$D = (C_{D_0} + KC_L^2 + C_{D_\delta} \delta_{TR}) qS \quad (4-15)$$

where $C_L = W_E/qS$ (W_E is launch weight minus propellant weight, C_{D_δ} is the drag coefficient due to control-surface deflection, and δ_{TR} is the control-surface deflection required for trim).

- $$n = -\frac{\Delta V}{D} = -\frac{g \Delta t}{W} \quad (4-16)$$

4. Impose slope n at point A , the condition at burnout, and establish point B . $\Delta V'_1$ is the first approximation of velocity decrement

$$F = \frac{W_E}{g} a = D$$

$$\Delta V = b \Delta t = \frac{Dg}{W_E} \Delta t$$

FOR $D = W_E$ AS SHOWN,

$$\Delta V = 32.2 \Delta t$$

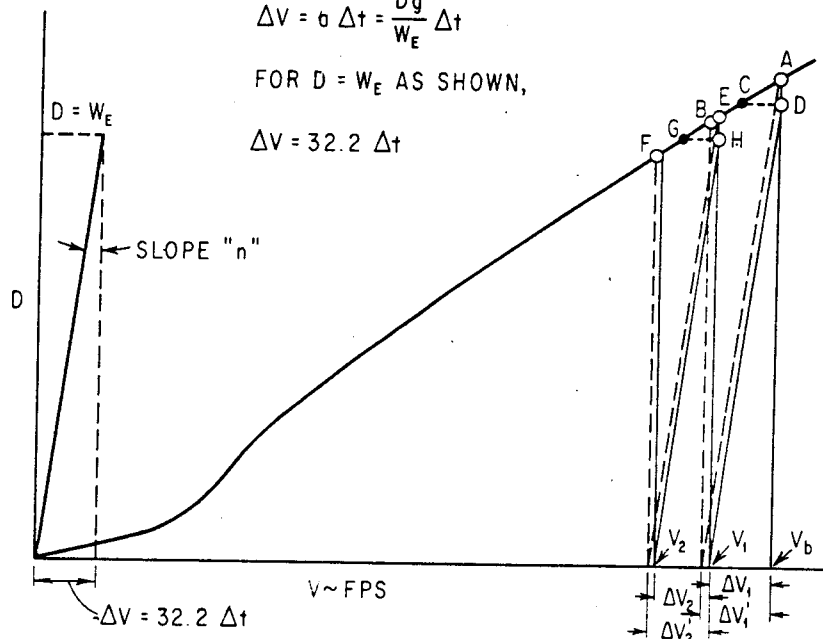


FIG. 4-5. Graphical solution—level-flight condition.

75

C
t
a
c

1
a

$$V_1 = V_b + \Delta V_1 \quad (4-17)$$

$$S_1 = S_b + \frac{V_b + V_1}{2} \Delta t_1 \quad (4-18)$$

TABLE 4-1. SOLUTION BY ITERATION METHOD

	Flight time	1	2	3	4	5	6	7	8	9	10	11	12	13
		Weight of missile	$M = V/c = V/1,107$	$q = \frac{1}{2} \rho V^2 = 1,481 M^2$	$C_{D_0} + K \left(\frac{q S^2}{W^2} \right)$	$D = (C_{D_0} + C_{D_i}) q S^2$	Rocket-motor thrust	$T - D$, lb	a , ft/sec ²	$\Delta V^* = (3a_{n-1} - a_{n-2}) \frac{\Delta t}{2}$	$V = V_{n-1} + \Delta V$	$\bar{V} = \frac{V}{2} (V_{n-1} + V)$	$\Delta S = V \Delta t$	$\Sigma S = S_{n-1} + \Delta S$
	t , sec	W , lb	M	q , psf	$C_{D_0} + C_{D_i}$	D , lb	T , lb			ΔV , fps	V , fps	$\bar{V}$, fps	ΔS , ft	ΣS , ft
A	0	500	0.400	236	0.200	37	10,000	9,963	642	---	447.0	---	---	0
B ₁	0.1	495	0.459	312	0.200	49	10,000	9,951	(646)	64.4	511.4	---	---	48
B	0.1	495	0.459	312	0.200	49	10,000	9,951	648	64.5	512.5	479.8	48	103
C	0.2	490	0.518	397	0.200	63	10,000	9,937	652	65.1	577.6	545.1	55	2,284
D	2.0	400	1.645	4,000	0.320	1,005	10,000	8,995	72.4	---	1,837.0	---	---	2,284
E	2.0	400	1.645	4,000	0.320	1,005	0	-1,005	-81.0	---	1,837.0	---	---	2,284
F ₁	2.0	400	1.575	3,670	0.330	954	0	-954	(-77.0)	-79.0	1,758.0	---	---	2,464
F	3.0	400	1.575	3,670	0.330	954	0	-954	-76.8	-78.9	1,758.1	1,797.6	180	2,636
	4.0	400	1.510	3,370	0.341	905	0	-905	-72.8	-74.7	1,683.4	1,720.8	172	

Flight condition: $M_L = 0.4$ at sea level. $T = 10,000$ lb. $t_b = 2$ sec. $W_L = 500$ lb. $W_p = 100$ lb. $W_E = 400$ lb. $S_\pi = 0.786$ ft². Sea-level flight altitude, c (speed of sound) = 1,117 fps.
 * See Eq. (4-21).

4. Shift the abscissa up by $W_E \sin \gamma_L$ as shown in Fig. 4-6.
5. Calculate conditions at burnout, V_b and S_b , using Eqs. (4-13) and (4-14).
6. Arc off S_b along the flight-path diagram shown in Fig. 4-6 and read the altitude at burnout.
7. Impose slope n at burnout condition, point A as shown.
8. Read ΔV_1 (first approximation) at the intercept of slope n with the new abscissa and calculate S_1 by Eq. (4-18).
9. Arc off S_1 on the flight-path diagram and read the new altitude (point B).
10. Compare the drag at altitude B with that at burnout altitude A . If the drag at B is approximately the same as that at A no further approximation is required, in which case point B may be used as the starting point for the next time interval Δt_2 . However, if the drag at B is significantly different from that at A , an average value of these two drags must be used for a second approximation, as was done in the illustration for the level-flight case shown in Fig. 4-5. In general, for a diving-flight straight-line trajectory no such second approximation is required.
11. Repeat steps 7 through 10 until the missile impacts or reaches the desired altitude and/or velocity.

The accuracy of the above-described graphical method may be improved by using (1) small values of Δt or (2) second or third approximations. In practice the former method is less involved and is therefore recommended.

2. Iteration Method. This method is rather tedious for manual computation. However, where only a limited number of trajectories is involved, this method may be used. A simplified boost-glide constant-altitude trajectory (similar to the first one used to illustrate the graphical method) is used here to illustrate this method of solution. The boost phase of the trajectory may be calculated by Eqs. (4-12) through (4-14). However, for purpose of illustration, the incremental velocity (and hence end of boost velocity) and distance traveled during boost will be calculated by the iteration method. The power-off flight will also be calculated by this method.

Table 4-1 presents the calculations for several time intervals during both phases of flight to illustrate the procedures involved in this method of solution for a typical level-flight trajectory. Row A in Table 4-1 represents the initial launch condition of the missile. In order to start the calculation for the first time interval (i.e., row B) the value of acceleration a in column 8 must be estimated. In the

sample calculation, a value of 646 fps was assumed and was based on the predicted behavior of the missile for the flight conditions involved. Using this assumed value of 646 fps, the pertinent information for row B can be readily determined as follows:

1. Use the average value of a from rows A and B to calculate V for column 9.
2. Next, calculate V , column 10.
3. With V thus determined, calculate M and q , columns 2 and 3.
4. Read the value of C_{D_0} from the plot of C_{D_0} vs. M and calculate C_{D_i} for the appropriate Mach number and q .
5. Calculate the drag, column 5.
6. Calculate the new value of a for row B.

From the sample calculation, the new value of a was determined to be 648 fps as compared with the previously assumed value of 646 fps; this indicates that the assumed value of 646 fps appears to be reasonably accurate. However, if increased accuracy is desired or required, a second approximation should be made using the average value between the first assumed value of 646 fps and the calculated value of 648 fps. In general, the second approximation should yield sufficiently accurate results to permit one to proceed with the iteration method as detailed below:

7. Calculate ΔV , column 9, row C, by the following expression:

$$\Delta V = 3a_{n-1} - a_{n-2} \quad (4-21)$$

where a_{n-1} and a_{n-2} are the value of acceleration for the time interval before and twice the time interval before the point in question, respectively (i.e., $a_{n-1} = 648$ and $a_{n-2} = 642$ fps in the sample calculation). Derivation of Eq. (4-21) is presented in Appendix E.

8. Calculate V , column 10, as shown in Table 4-1.
9. Calculate M and q as before (i.e., step 3).
10. Complete columns 4 through 8.
11. Calculate ΔS and ΣS , columns 12 and 13. This completes the calculations for row C.

The above steps 7 through 11 are repeated for the duration of the boost phase. Row D represents the end-of-boost condition. In order to start the power-off phase of flight, row E must be calculated for $T = 0$ condition. At this point, the time interval Δt may be changed to minimize the number of calculations required. Again, in order to start the calculation for the first time interval (i.e., row F), the value of acceleration a in column 8 must be estimated. The procedure previously outlined is then used to establish the final values for row F.

Steps 7 through 11 may then be used to determine the complete trajectory information. It should be noted that the time interval must be kept constant throughout the calculation. This limitation is readily apparent from the derivation of Eq. (4-21) in Appendix E. However, if it is found desirable to change the time interval, a restart at this point is necessary. In general, the appropriate time interval can be selected at the beginning of the computation and kept constant throughout the trajectory.

4-7. BOOST-SUSTAIN TRAJECTORY

This type of trajectory consists of a relatively short boost period followed by a sustained powered flight. The velocity in the latter phase of flight is generally constant, in which case the trajectory is termed "boost, constant, sustained" flight. Where the velocity during sustaining increases or decreases, the trajectory is termed "boost, oversustained" or "undersustained," respectively. For certain applications, the boost-sustain trajectory is preferred over the boost-glide. The main advantages are (1) reduction in the maximum Mach number required for a given range, (2) alleviation of the aerodynamic heating problem, (3) better missile aerodynamic performance due to the avoidance of high drag, and (4) simplification of the control system in that the aerodynamic gain (for a given altitude condition) is held nearly constant.

For this type of trajectory, the methods outlined in Sec. 4-5 may be used to determine the missile-performance characteristics during the boost phase of flight. For the sustained powered portion of flight either the graphical or iteration method may be used, particularly for the over- or undersustained cases. In the graphical solution the abscissa of the drag vs. velocity plot must be shifted upward by the value of the sustainer thrust (constant thrust is assumed, of course). However, for the common and simple case wherein the missile is sustained at a near constant velocity, a rather simple expression to approximate the range R traveled during this portion of flight can be derived as follows:

$$T = D$$

$$T \times t = \text{total impulse} = I = Dt \quad (4-22)$$

since

$$I = W_p I_{sp} \quad (4-23)$$

where W_p = weight of propellant

I_{sp} = propellant specific impulse

Eq. (4-22) becomes

$$W_p I_{sp} = Dt \quad (4-24)$$

since $t = R/V$ and $D = C_D \frac{1}{2} \rho V^2 S$, Eq. (4-24) can be written as

$$R = \frac{2W_p I_{sp}}{C_D \rho V S} \quad (4-25)$$

For a given sustainer propellant weight W_p the flight time during the sustained portion of flight is

$$t = \frac{2W_p I_{sp}}{C_D \rho V^2 S} \quad (4-26)$$

or simply

$$t = \frac{R}{V} \quad (4-27)$$

The above simplified expressions [Eqs. (4-25) and (4-26)] may be used only for relatively short range missiles (i.e., $W_p \ll$ missile gross weight). For longer-range missiles, such as the conventional cruise type, somewhat more exact expressions must be used to calculate the range and flight time, as will be discussed in the next section.

It is of interest to point out that, if the missile weapon system permits, it is very advantageous from the standpoint of performance first to boost the missile up to speed, let it coast down to some lower velocity, and then sustain for the remaining portion of the trajectory. Using this technique, the high-drag region is avoided and a substantial range improvement consequently results. However, the power-plant design problems associated with this mode of flight are somewhat more severe, particularly when solid-propellant rocket motors are used. These problems will be discussed in Chap. 13.

4-8. LONG-RANGE CRUISE TRAJECTORY

The methods used to determine the performance characteristics of cruise missiles are identical with those used on conventional manned aircraft. Again, it is necessary to know the engine performance (i.e., thrust, fuel consumption, etc.), drag, and operating altitude conditions to determine the various performance parameters such as maximum speed and rate of climb. To help understand the fundamentals behind these calculations, the forces involved are illustrated in Fig. 4-7. The methods used to determine the pertinent performance parameters are discussed below.

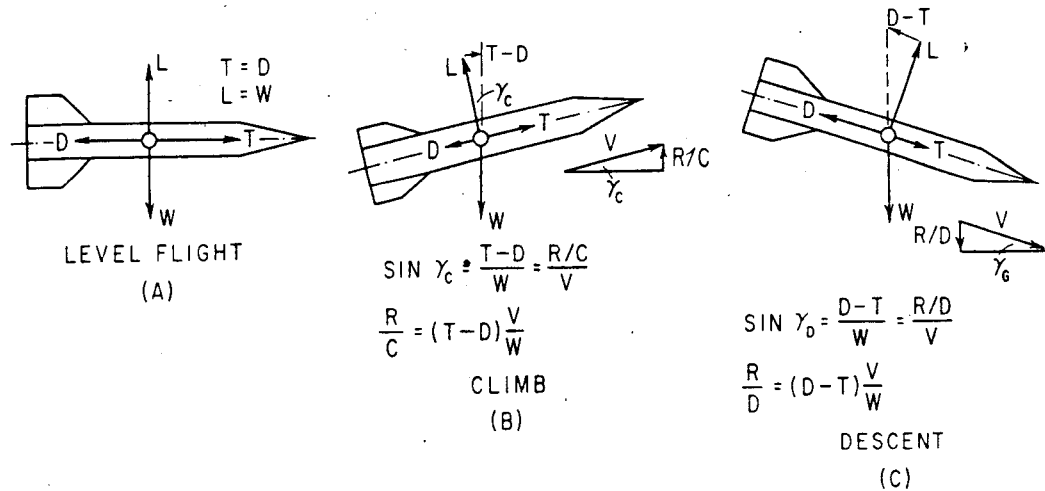
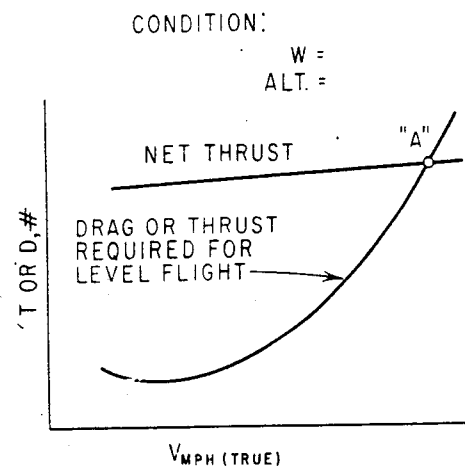


FIG. 4-7. Force diagram for various equilibrium flight conditions.

1. Maximum Speed. The maximum speed of the cruise missile for a given operating altitude is readily determined if the engine performance and missile drag characteristics are known. The latter may be calculated by the following expression:

$$D = \left[C_{D_0} + K \left(\frac{W}{qS} \right)^2 + C_{D_\delta} \delta_{TR} \right] qS \quad (4-28)$$

The value of drag is equivalent to the thrust required for level flight at a given constant altitude and velocity. The net thrust is obtained from the engine specification and must be corrected for installation losses such as inlet and duct losses. The resultant plot of this information is shown in Fig. 4-8 for a given altitude condition. By inspection of Fig. 4-8, the maximum speed of the missile is defined by point A, where the net thrust available is equal to the thrust required to maintain level flight. Point A is really a fictitious or theoretical $V_{\max}$ since it cannot be realized in actual flight. This is readily apparent when one considers the fact that the missile cannot accelerate to this point because the force $T - D$ required approaches zero at this point. Hence in actuality $V_{\max}$ will


 FIG. 4-8. Determination of $V_{\max}$.

be somewhat less than that determined by this method. However, the theoretical value of $V_{\max}$ does provide a common basis for performance comparison with other flight conditions or missile systems.

2. Rate of Climb. The expression for the rate of climb can be readily derived by inspection of Fig. 4-7 and is as follows:

$$\sin \gamma_C = \frac{T - D}{W} \quad (4-29)$$

where γ_C = flight-path angle

Since $\sin \gamma_C$ is also equal to $(R/C)/V$, Eq. (4-29) becomes

$$\frac{R}{C} = \frac{(T - D)V}{W} \quad (4-30)$$

Equation (4-30) is more commonly written as

$$\frac{R}{C} = \frac{88(T - D)V}{W} \quad (4-31)$$

where R/C is in fpm and V is in mph. The rate of climb at various speeds can be calculated using the information in Fig. 4-8 and

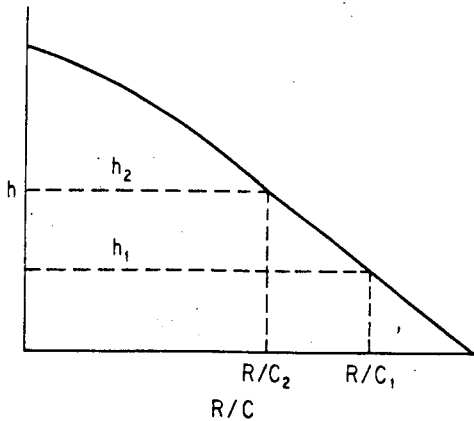


FIG. 4-9. Determination of time to climb.

Eq. (4-31). The maximum rate of climb $(R/C)_{\max}$ is readily determined by plotting R/C vs. V , from which the speed for best rate of climb $V_{(R/C)_{\max}}$ is also determined. The above steps may be repeated at other flight altitudes to determine R/C for the entire range of operating altitudes in order to determine the time to climb to various altitudes.

3. Time to Climb. The time to climb to altitude may be calculated since the variation of R/C with altitude is now known. This may be done by adding the time increments required to climb between any two altitudes h_1 and h_2 , as shown in Fig. 4-9. In general

$$dt = \frac{dh}{dh/dt} = \frac{dh}{R/C} \quad (4-32)$$

or

$$\int_{t_1}^{t_2} dt = \int_{h_1}^{h_2} \frac{1}{R/C} dh \quad (4-33)$$

Since R/C is assumed to be linear between h_1 and h_2 , the time to climb from h_1 to h_2 may be derived from Fig. 4-9:

$$\begin{aligned}\frac{R}{C} &= \frac{R}{C_1} - \frac{R/C_1 - R/C_2}{h_2 - h_1} (h - h_1) \\ &= \frac{(R/C_1)h_2 - (R/C_2)h_1 + h(R/C_2 - R/C_1)}{h_2 - h_1}\end{aligned}\quad (4-34)$$

$$\text{or} \quad t_2 - t_1 = \Delta t = \int_{h_1}^{h_2} \frac{(h_2 - h_1) dh}{(R/C_1)h_2 - (R/C_2)h_1 + h(R/C_2 - R/C_1)} \quad (4-35)$$

$$\text{hence} \quad \Delta t = \frac{h_2 - h_1}{R/C_2 - R/C_1} \ln \frac{R/C_2}{R/C_1} \quad (4-36)$$

If very small increments in altitude are used, the following simplified expression may be used:

$$\Delta t = \frac{2(h_2 - h_1)}{R/C_1 + R/C_2} \quad (4-37)$$

The above assumes a constant speed during climb in that all the excess thrust was utilized to climb. For most modern high-performance aircraft and missiles the excess thrust is quite large so that the flight path approaches vertical. Moreover, the speed for best climb generally increases rapidly with altitude, and a programmed climbing schedule is more practical.

The optimum climbing schedule can be approximated by plotting the rate of climb vs. speed for various altitudes and drawing a straight line which most nearly goes through the maximum.

For this case a considerable percentage of the excess thrust is used to accelerate the aircraft, and a correction to the rate of climb is required. Equation (4-29) then becomes

$$\sin \gamma_C = \frac{T - D}{W} - \frac{a}{g} \quad (4-29a)$$

where a = acceleration. But

$$a = \frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} = \frac{dV}{dh} V \sin \gamma_C$$

and the equation of motion may be written

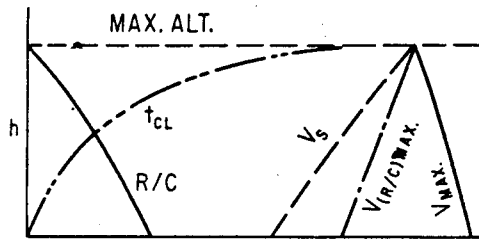
$$(RC)_a = \frac{(RC)_{a=0}}{1 + (V/g)(dV/dh)} \quad (4-31a)$$

where $(RC)_a$ = rate of climb in accelerated flight

$(RC)_{a=0}$ = rate of climb at constant speed

In practice, it is sufficiently accurate to use an average dV/dh from the curve previously described. The time to climb may then be computed as before, utilizing the reduced rates of climb $(RC)_a$.

4. Stall Speed. The stall or minimum speed of the missile may be calculated by the following expression:



$$V_S = \sqrt{\frac{2W}{C_{L_{max}} \rho S}} \quad (4-38)$$

A typical summary plot of the above-mentioned performance parameters for the cruise missile is shown in Fig. 4-10.

5. Maximum Range. The maximum range of a cruise missile is the sum of the distances traversed during climb and level-flight portions of its trajectory. The distance traveled during climb may be calculated by the following equation:

$$R_{CL} = \bar{V}_{(R/C)_{max}} t_{CL} \cos \gamma_{CL} \quad (4-39)$$

where $\bar{V}_{(R/C)_{max}}$ = average velocity for the best rate of climb

t_{CL} = time to climb to cruise altitude

γ_{CL} = flight-path angle in climb

The range for a given level-flight condition may be readily determined by the procedures outlined below:

1. Select three or four different flight speeds and missile-weight conditions.
2. Calculate $C_L = W/qS$.
3. Determine C_D from C_D vs. C_L plot or $C_D = C_{D_0} + KC_L^2 + C_{D_\delta} \delta_{TR}$ at the appropriate Mach-number condition.
4. Calculate $D = C_D q S$ = thrust required.
5. Determine the engine fuel consumption, lb/hr, from the engine specifications.
6. Convert the engine fuel consumption in terms of miles per pound of fuel consumed by

$$\frac{V}{\text{lb/hr}} = \frac{\text{miles}}{\text{hr}} \times \frac{\text{hr}}{\text{lb}} = \text{miles/lb}$$

7. Plot miles/lb vs. V as shown in Fig. 4-11.
8. Read from Fig. 4-11 the maximum values of miles/lb and plot these maximum values vs. missile weight as shown in Fig. 4-12.
9. The range during cruise is the area under the curve defined in Fig. 4-12.

Steps 1 through 9 are then repeated for other flight or cruise altitudes to determine the over-all range of the missile for the various mission profiles.

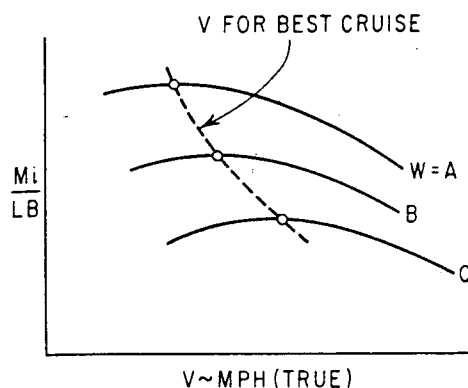


FIG. 4-11. Determination of best miles per pound and V for best cruise.

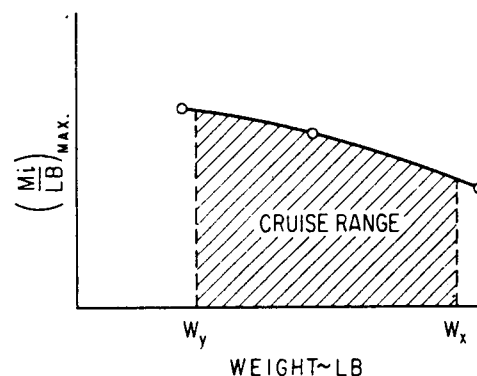


FIG. 4-12. Determination of cruise range. W_x = weight at start of cruise; W_y = weight at end of cruise.

4-9. LONG-RANGE BALLISTIC TRAJECTORY

The general nature of the trajectory flown by long-range ballistic missiles is shown in Fig. 4-13. Because of the enormous weight involved, the missile is launched vertically from the launching pad. Because of the low values of dynamic pressure during lift-off, no large control or guidance commands other than those required to keep the missile in a vertical ascent are required. As previously noted, auxiliary jet controls are used during lift-off until sufficient q is attained, whereupon aerodynamic controls are used. In certain cases, jet controls are sufficiently adequate to eliminate completely the aerodynamic control surfaces. After a few seconds of vertical rise, the missile is commanded into a "gravity" (or zero-lift) turn by simply tilting the velocity vector of the missile for a short period. This "gravity" turn is continued until the missile reaches a sufficiently high altitude, at which point the missile is commanded to fly a constant-attitude trajectory as shown in Fig. 4-13. The altitude at which

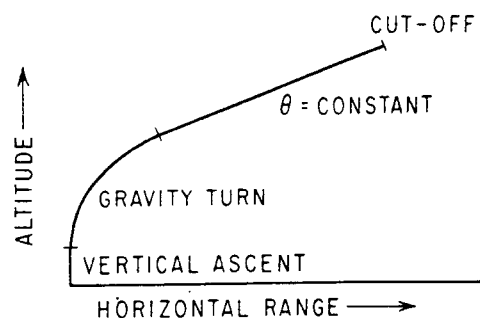


FIG. 4-13. Typical ballistic-missile trajectory during powered flight.

the missile starts its constant-attitude flight is primarily dependent on the severity of the air loads associated during this transition phase. Powered flight is continued in this predetermined optimum attitude or angle until the required velocity is attained, whereupon thrust termination is effected. Immediately upon thrust termination, the re-entry body is separated from the last stage (in the case of multistaged configuration) and commences its free-flight ballistic trajectory to the target. The above-described trajectory may be modified somewhat by certain limitations of the guidance system in the missile.

The methods used to calculate the performance characteristics of ballistic missiles are somewhat different from those previously described in this chapter. While these methods appear to be relatively simple, careful considerations must be given to the various design and trajectory parameters in order to realize the desired optimum performance. The following sections will be concerned with the methods of calculation as well as a brief discussion of the effects on performance of several of the important design and trajectory parameters.

The problem of determining performance characteristics of ballistic missiles may be conveniently divided into two parts, powered flight and unpowered free-flight trajectories, which are discussed below.

1. Powered Flight. The equation of motion for the missile in powered flight may be written as follows:

$$\frac{W_E + W_p}{g} \frac{dV}{dt} = \underbrace{\frac{V_j}{g} \frac{dW_p}{dt}}_{\text{Thrust}} + \underbrace{A_e(p_e - p_o)}_{\text{Drag}} - \underbrace{C_D \frac{1}{2} \rho V^2 S}_{\text{Gravity}} - (W_E + W_p) \sin \gamma \quad (4-40)$$

where W_E = weight of missile at burnout

W_p = weight of propellant

V_j = jet-exhaust velocity (gI_{sp} at SL)

A_e = nozzle exit area

p_e = nozzle exit pressure

p_o = ambient exit pressure

It is apparent that, because of the drag term, no closed solution to Eq. (4-40) is obtainable. Hence it is necessary to solve Eq. (4-40) by either of two methods: (1) the iteration method outlined in Sec. 4-6 or (2) by Eq. (4-12), by making an approximate correction for drag and gravity losses as previously outlined. One further simplification may be made in that the missile is assumed to fly a constant-rate-of-turn course instead of a "gravity" turn in actual flight. This last assumption permits one to apply an average correction readily for gravity losses [Eq. (4-12) or (4-40)] during powered flight.

TABLE 4-2. METHOD OF SOLUTION TO BALLISTIC MISSILE PERFORMANCE DURING POWERED FLIGHT

Flight time t, sec	Missile weight W, lb	M = V/c	$\frac{1}{2} \rho V^2 S C_D = D$	T, lb	T - D lb	$\frac{a_1}{ft/sec^2} = \frac{M}{T - D}$	$a_2 = g \sin \theta$ ft/sec ²	$\Sigma a, ft/sec^2$	$\Delta V, fps$ $\frac{2}{\Delta t} (V - V_{n-1}) = \Delta V$	V, fps $V + \Delta V$	θ $\theta + \Delta \theta$	$\bar{V} \sin \theta$ $(\bar{V} + V_{n-1}) \sin \theta = \bar{V} \sin \theta$	$\bar{V} \cos \theta$ $(\bar{V} + V_{n-1}) \cos \theta = \bar{V} \cos \theta$	$\Delta h = \bar{V} \sin \theta \Delta t$	$\Sigma h = h_{n-1} + \Delta h$	$\gamma = \gamma_{n-1} + \Delta \gamma$
0	20,000	0	0	75,000	75,000	120	32	88	0	0	90	0	0	0	0	0
2	19,400	0.161	38	75,000	74,962	124	32	92	180 est.	180	88	90	≈ 0	180	180	90
4	18,800	0.330	159	75,187	74,869	128	32	96	188	368	86	274	14	548	722	451

Flight condition: $W_L = 20,000$ lb. $W_p = 15,000$ lb. $W_E = 5,000$ lb. $t_b = 50$ sec. $\dot{W}_p = 300$ lb/sec. $I_{0,p} = 250$ sec (SL).
 $S = 10$ ft². $A_e = 5.5$ ft². $p_e = 2,116$ psf. $\theta_{co} = 40^\circ$ (from horizontal). $\dot{\theta} = 1^\circ/\text{sec}$ (assumed constant).

Flight time t, sec	p, fps at Σh	$p_e - p$	c, fps at Σh	$\Delta R, (\bar{V} \cos \theta) \Delta t$	$\Sigma R, R_{n-1} + \Delta R$
0	2,116	0	1,117	0	0
2	2,116	0	1,117	≈ 0	≈ 0
4	2,082	34	1,115	28	28

At the start of a new missile design, it is desirable to calculate the missile-performance characteristics for a few typical flight conditions using the iteration procedure previously outlined. For the type of trajectory involved, it is necessary to modify the procedure somewhat to account for the changing flight path during powered flight. The modified procedure is shown in Table 4-2. Having thus determined the general nature of the trajectory, one can make first-order approximation for other missile designs or flight conditions using Eq. (4-12) to determine the velocity of the missile at burnout. The range and altitude at burnout may be approximated by the following expressions:

$$x = \bar{V}(\cos \bar{\gamma})t_b \quad (4-41)$$

$$y = \bar{V}(\sin \bar{\gamma})t_b \quad (4-42)$$

Since the distance the missile traversed during powered flight is only a small percentage of the total range of the missile, Eq. (4-41) appears to be adequate. Equation (4-42) also appears to be satisfactory since inaccuracy in the determination of altitude at burnout has only a small effect on the range of the missile, as will be shown subsequently. In the above discussion, a flat earth is assumed. This assumption is valid since the traverse during powered flight is relatively small.

2. Unpowered Flight. Once the conditions at burnout are established (i.e., cutoff velocity, altitude, and angle), the range for the

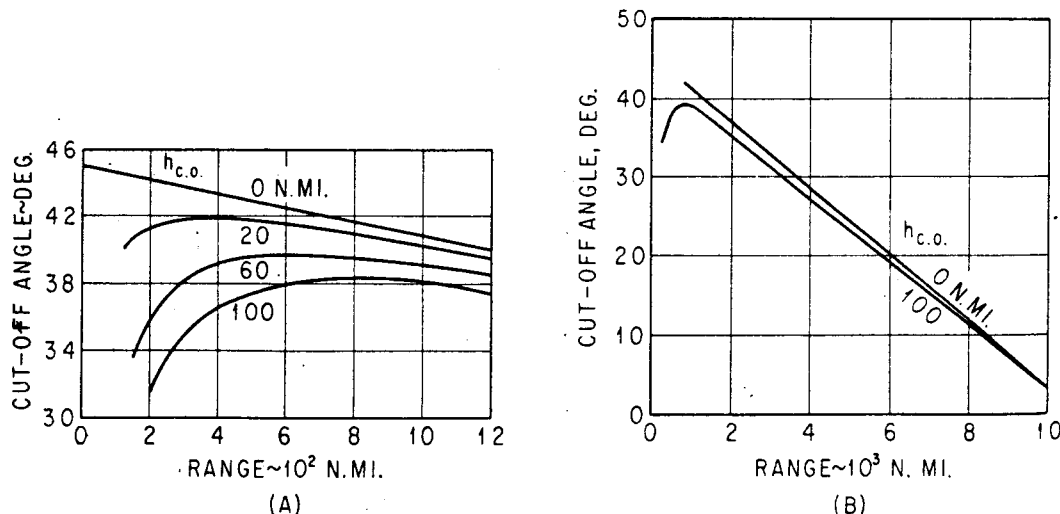


FIG. 4-14. Cutoff angle vs. range.

free-flight portion of the trajectory is readily determined. The free-flight ballistic trajectory is essentially identical to a vacuum trajectory of an object traveling within the gravitational field of a spherical non-rotating earth. The resulting trajectory is one of Kepler's planetary

ellipses with the far focus located at the center of the earth. Expressions which relate the range of the missile as a function of the various trajectory parameters, at the cutoff condition, have been used to calculate the missile-performance characteristics in free flight. The results of these calculations are shown in Fig. 4-14. The maximum range (and corresponding times of flight) shown in Fig. 4-15 were obtained by

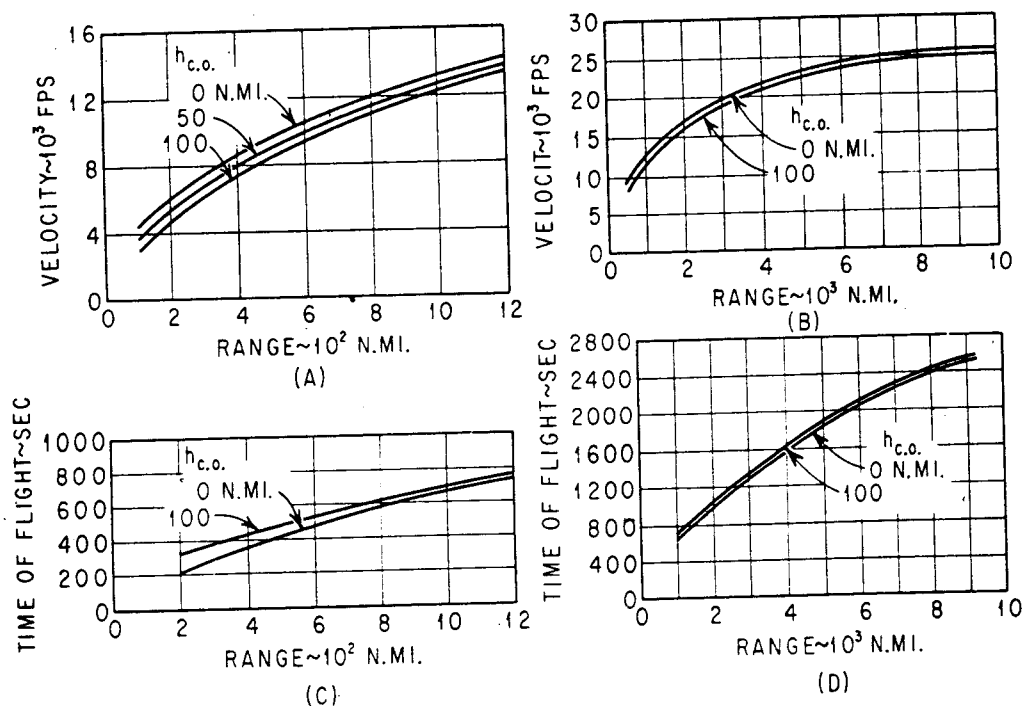


FIG. 4-15. (A and B) Velocity vs. range (optimum launch angle, free flight). (C and D) Flight time vs. range (free flight).

using the optimum-cutoff-angle information from Fig. 4-14. It is apparent from Fig. 4-15 that the cutoff velocity is the primary parameter from the standpoint of range. The cutoff altitude has only a small effect on range. This is particularly true for the long-range conditions.

By neglecting the drag effects on the range traversed during powered flight, the range of a ballistic missile was shown to be¹⁴

$$R = r_0 \varphi = 2r_0 \tan^{-1} \frac{\sin \theta_f \cos \theta_f}{(1/\bar{V}_f^2) - \cos^2 \theta_f} \quad (4-43)$$

where $\bar{V}_f = V_f/V_s$ = ratio of cutoff velocity to satellite velocity

θ_f = angle of incidence as shown in Fig. 4-16

The optimum trajectory (defined as one yielding maximum range for

a given velocity V_f) was determined by differentiating Eq. (4-43) with respect to θ_f and equating to 0. The resulting expressions became

$$\begin{aligned} \bar{V}_f^2 &= 1 - \tan^2 \theta_f \\ \varphi &= \frac{R}{r_0} = \pi - 4\theta_f \end{aligned} \quad (4-44)$$

The results derived from the above equations are shown in Fig. 4-17.

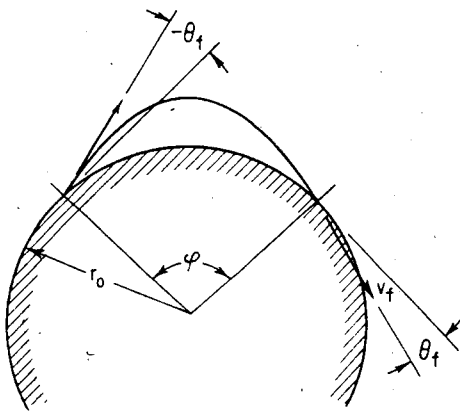


FIG. 4-16. Idealized ballistic-missile trajectory.

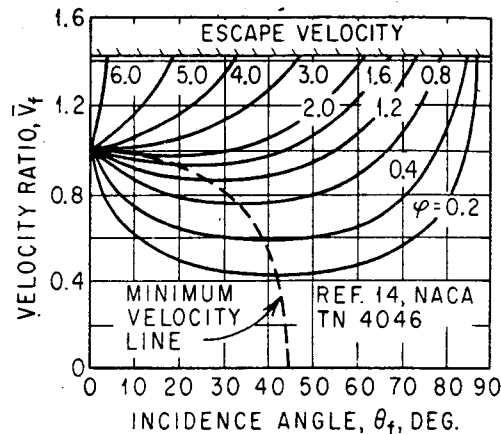


FIG. 4-17. Variation of velocity with incidence angle for various values of range of ballistic vehicles.

3. Design Considerations. Perhaps one of the most important design considerations which demand careful analysis is the requirement for rocket-motor staging. For relatively short range missiles (range less than approximately 200 miles, which varies somewhat depending upon the pay load) a single-stage rocket motor is generally adequate. However, for longer ranges, multistaging is required in order to keep the launch weight of the missile to a minimum. Also, for a given allowable launch weight, increased range is possible by multistaging. By optimizing the distribution of structures and propellant between the various stages of a multistage or "step rocket,"¹⁵⁻²³ maximum range may be realized. The pronounced effects of staging on range may readily be shown with two simplified examples presented below:

Example 1: Launch weight = 20,000 lb, top weight = 5,850 lb.

Case a: Single stage.

Total rocket-motor weight = 14,150 lb

Propellant weight = 12,000 lb

Mass ratio = 0.85

Specific impulse $I_{sp} = 240$ sec

Neglecting drag and gravity,

$$\Delta V_1 = I_{sp} g \ln \frac{W_L}{W_E} = 240 \times 32.2 \ln \frac{20,000}{8,000} = 7,070 \text{ fps}$$

Case b: Assumed two stages of equal motor weight, mass ratio, and specific impulse as above.

Weight of one rocket motor = 7,075 lb

Weight of propellant (one motor) = 6,000 lb

$$V_1 = 240 \times 32.2 \ln \frac{20,000}{20,000 - 6,000} = 2,750 \text{ fps}$$

$$\Delta V_2 = 240 \times 32.2 \ln \frac{20,000 - 7,075}{20,000 - 7,075 - 6,000} = 4,820 \text{ fps}$$

$$\Sigma V = \Delta V_1 + \Delta V_2 = 7,570 \text{ fps}$$

From the above calculation, it is seen that the maximum velocity for the two-staged missile is approximately 7 per cent greater than that using a single-stage design. Since the range varies approximately as the velocity squared, a 14 per cent increase in range is thus realized by staging. By optimizing the mass distribution, a maximum increase in range of approximately 16 per cent may be realized.

Example 2: Launch weight = 20,000 lb, top weight = 1,150 lb.

Case a: Single stage.

Total rocket-motor weight = 18,850 lb

Propellant weight = 16,000 lb

Mass ratio = 0.85

Specific impulse I_{sp} = 240 sec

$$\Delta V = 240 \times 32.2 \ln \frac{20,000}{4,000} = 12,420 \text{ fps}$$

Case b: Assumed two stages of equal motor weight, mass ratio, and specific impulse as above.

Weight of one rocket motor = 9,425 lb

Weight of propellant (one motor) = 8,000 lb

$$\Delta V_1 = 240 \times 32.2 \ln \frac{20,000}{12,000} = 3,930 \text{ fps}$$

$$\Delta V_2 = 240 \times 32.2 \ln \frac{20,000 - 9,425}{20,000 - 9,425 - 8,000} = 10,870 \text{ fps}$$

$$\Sigma V = 14,800 \text{ fps}$$

From the above calculation, it is seen that the maximum velocity

for the two-staged design is approximately 19 per cent greater than for the single-stage design, corresponding to a 30 per cent increase in range. By optimum staging, a maximum increase in range of 67 per cent may be realized over the single-stage design. Hence it is apparent that both multistaging and optimum staging are mandatory from the standpoint of obtaining the maximum possible range for a given launch-weight condition.

Another important design consideration is the thrust level of the rocket motor. For a given total rocket-motor impulse, a high thrust level will result in a higher drag loss, whereas a low thrust level (which entails a longer burning time) may result in large losses due to gravity. Hence it is necessary to conduct a detailed analysis in order to determine the optimum level of thrust to yield the greatest incremental velocity, or range. The operating pressure of the rocket-motor units must also be carefully analyzed since an increase in mass ratio (defined as the ratio of propellant weight to total rocket-motor weight) may be realized with a low operating-chamber pressure. On the other hand, the specific impulse of the propellant (defined as the ratio of total impulse to weight of propellant) is degraded with decreasing operating pressure.

SYMBOLS

A	aspect ratio
A_e	exit area of nozzle
C_D	drag coefficient D/qS
C_{D_i}	induced-drag coefficient
C_{D_N}	drag (coefficient) due to normal force
C_{D_w}	wave-drag coefficient
C_{D_0}	drag coefficient for zero lift
C_{D_δ}	drag (coefficient) due to control-surface deflection
C_f	skin-friction-drag coefficient
C_{f_0}	skin-friction-drag coefficient for incompressible flow
C_L	lift coefficient
$C_{L_{\max}}$	maximum lift coefficient
C_N	normal-force coefficient
C_{N_α}	normal-force-curve slope $\partial C_N / \partial \alpha$
D	drag forces
I	total impulse
I_{sp}	fuel specific impulse
K	drag-rise factor

 a d h h_{co} l m n p_0 p_e q r_0 t t_0 α β γ γ_{CL} γ_L δ

K'	a factor to account for drag effects [see Eq. (4-12)]
M	Mach number
P	pressure coefficient
R	range of missile
$R/C, RC$	rate of climb
Re	Reynolds number
S	reference area, or range of missile
S_l	wetted area over length l shown in Fig. 4-2
S_x	wetted area over length x shown in Fig. 4-2
T	thrust
V	forward speed
$\bar{V}$	average velocity
V_f	cutoff velocity
$\bar{V}_f$	ratio of cutoff velocity to satellite velocity V_f/V_s
V_j	jet-exhaust velocity
V_S	stall speed
V_s	satellite velocity
W	weight
W_E	weight of missile in burnout condition
W_L	weight of missile at launch
W_p	weight of propellant
a	acceleration
d	body diameter
h	altitude
h_{co}	altitude at cutoff
l	length
m	a correlation constant used in Eqs. (3-58a) and (4-10)
n	a slope used in Fig. 4-5
p_o	ambient static pressure
p_e	static pressure at the nozzle exit
q	dynamic pressure ($\frac{1}{2}\rho V^2$)
r_0	radius of the earth
t	(flight) time
t_b	burning time of rocket motor
α	angle of attack
β	$\sqrt{M^2 - 1}$
γ	specific heat of air, or flight-path angle
γ_{CL}, γ_c	flight-path angle (in climbing flight)
γ_L	launch attitude
δ	semivertex angle of leading and trailing edges of air-foil, or control-surface deflection

δ_{TR}	control-surface deflection required for trim
θ	flight path of a ballistic-missile trajectory (see Fig. 4-13)
$\bar{\theta}$	average flight-path angle [see Eqs. (4-41) and (4-42) and Table 4-2]
θ_i	angle of incidence as shown in Fig. 4-16
ρ	density of air
φ	subtended angle shown in Fig. 4-16

Subscripts

CL	in climbing flight
E	empty or burnout condition
L	launch condition, or laminar-flow condition
N	due to normal force
T	turbulent-flow condition
TR	trim condition
a	with acceleration
b	during boost or due to boost
co	cutoff condition
$n - 1, n - 2$	denote time interval before and twice the time interval before the point in question [see Eq. (4-21)]
unc	uncorrected condition
0	incompressible as used in Eq. (4-4), or zero-lift condition as used with C_D
$1, 2$	denote points on trajectory (see Figs. 4-5 and 4-6)

REFERENCES

1. Coles, D.: Measurements in the Boundary Layer on a Smooth Flat Plate in Supersonic Flow I. The Problem of the Turbulent Boundary Layer, *Rept. 20-69*, C.I.T. Jet Propulsion Lab., Pasadena, Calif., June 1, 1953.
2. Smith, D. W., and J. H. Walker: Skin-friction Measurements in Incompressible Flow, *NACA Tech. Note 4231*, March, 1958.
3. Chapman, D., and R. H. Kester: Measurements of Turbulent Skin Friction on Cylinders in Axial Flow at Subsonic and Supersonic Velocities, *J. Aeronaut. Sci.*, vol. 20, no. 7, July, 1953.
4. Frankl, F., and V. Voishel: Turbulent Friction in the Boundary Layer of a Flat Plate in a Two-dimensional Compressible Flow at High Speeds, *NACA Tech. Mem. 1053*, 1943.
5. Rubesin, M. W., R. C. Maydew, and S. A. Varga: An Analytical and Experimental Investigation of the Skin Friction of the Turbulent Boundary Layer on a Flat Plate at Supersonic Speeds, *NACA Tech. Note 2305*, February, 1951.
6. Cope, W. F.: The Turbulent Boundary Layer in Compressible Flow, NPL Engr. Dept. British ARC 7634, November, 1943.

7. Wilson, R. E.: Turbulent Boundary-layer Characteristics at Supersonic Speeds—Theory and Experiment, *J. Aeronaut. Sci.*, vol. 17, no. 9, p. 585, September, 1950.
8. Tucker, M.: Approximate Calculation of Turbulent Boundary-layer Development in Compressible Flow, *NACA Tech. Note* 2337, 1951.
9. Estabrooks, B. B.: An Analysis of the Pressure Distribution Measured on a Body of Revolution at Transonic Speeds in the Slotted Test Section of the Langley 8-ft. Transonic Tunnel, *NACA Research Mem.* L52D21a, June, 1952.
10. Cole, R. I.: Pressure Distributions on Bodies of Revolution at Subsonic and Transonic Speeds, *NACA Research Mem.* L52D30, July, 1952.
11. Young, A. D.: The Calculation of the Total Skin Friction Drags of Bodies of Revolution at Zero Incidence, R & M 1814, April, 1939.
12. Whitcomb, R. T.: A Study of the Zero-lift Drag-rise Characteristics of Wing-body Combinations Near the Speed of Sound, *NACA Research Mem.* L52H08, September, 1952.
13. Jones, R. T.: Theory of Wing-body Drag at Supersonic Speeds, *NACA Research Mem.* A53H18a, 1953.
14. Eggers, A. J., H. J. Allen, and S. E. Neice: A Comparative Analysis of the Performance of Long-range Hypervelocity Vehicles, *NACA Tech. Note* 4046, October, 1957.
15. Malina, F. J., and M. Summerfield: The Problem of Escape from the Earth by Rocket, *J. Aeronaut. Sci.*, vol. 14, pp. 471-480, August, 1947.
16. Vertregt, M.: A Method for Calculating the Mass Ratio of Step Rockets, *J. Brit. Interplanet. Soc.*, vol. 15, no. 2, p. 95, March-April, 1956.
17. Goldsmith, M.: On the Optimization of Two-stage Rockets, *Jet Propulsion*, vol. 27, pp. 415, 416, April, 1957.
18. Schurmann, E. E. H.: Optimum Staging Technique for Multistaged Rocket Vehicles, *Jet Propulsion*, vol. 27, pp. 863-865, August, 1957.
19. Weisbord, L.: A Generalized Optimization Procedure for N-staged Missiles, *Jet Propulsion*, vol. 28, pp. 164-167, March, 1958.
20. Arens, M.: On a Generalized Optimization Procedure for N-staged Missiles, *Jet Propulsion*, vol. 28, p. 766, November, 1958.
21. Subotowicz, M.: The Optimization of the N-step Rocket with Different Construction Parameters and Propellant Specific Impulses in Each Stage, *Jet Propulsion*, vol. 28, pp. 460-463, July, 1958.
22. Hall, H. H., and E. D. Zambelli: On the Optimization of Multistage Rockets, *Jet Propulsion*, vol. 28, pp. 463-465, July, 1958.
23. Weisbord, L.: Optimum Staging Technique, *ARS J.*, vol. 29, no. 6, pp. 445-446, June, 1959.

CHAPTER 5

STATIC LONGITUDINAL STABILITY AND CONTROL

5-1. INTRODUCTION

The effectiveness of a guided missile weapon system, in terms of accuracy and probability of kill, depends greatly on the response characteristics of the complete guidance, control, and airframe loop. One of the most important jobs of the aerodynamicist or configuration designer is to see that sufficient natural static and dynamic airframe stability and damping are provided in the airframe system. Once this is achieved, fewer components and "servo or electronic fixes," which require additional components, are needed to meet a given response requirement. In addition, he must design the airframe with sufficient load factor or maneuverability in order that the missile can execute reasonably low turning radii for interception with the target or flight-path correction due to launch dispersions. Unfortunately, these two design requirements, i.e., stability and maneuverability, are conflicting in that maneuverability decreases as stability is increased. The purpose of this chapter is to analyze some of the various aspects of missile stability and control. Methods of analysis are presented for two general types of design: (1) forward control—where the control surfaces are located ahead of the missile center of gravity, and (2) aft control—where the control surfaces are located behind the center of gravity.

5-2. TWO-DEGREE-OF-FREEDOM ANALYSIS

It is convenient to study the static longitudinal stability and control characteristics of a missile with the aid of a two-degree-of-freedom (one rotational and one translational motion) analysis. Constant forward

velocity is therefore assumed in this case. The equations of motion are as follows:

$$\begin{aligned} M &= I\ddot{\theta} = M_\alpha\alpha + M_\delta\delta + M_{\dot{\theta}}\dot{\theta} + M_{\dot{\alpha}}\dot{\alpha} + \dots \\ &= (C_{m_\alpha}\alpha + C_{m_\delta}\delta)qSd + (C_{m_{\dot{\theta}}}\dot{\theta} + C_{m_{\dot{\alpha}}}\dot{\alpha})\frac{qSd^2}{2V} \end{aligned} \quad (5-1)$$

and

$$\begin{aligned} F &= mV\dot{\gamma} = F_\alpha\alpha + F_\delta\delta \\ &= (C_{L_\alpha}\alpha + C_{L_\delta}\delta)qS \end{aligned} \quad (5-2)$$

Since $C_{L_\alpha} \cong C_{N_\alpha}$, Eq. (5-2) becomes

$$mV\dot{\gamma} = (C_{N_\alpha}\alpha + C_{N_\delta}\delta)qS \quad (5-2a)$$

The geometric relationship between θ , α , and γ (shown in Fig. 5-1) is expressed as

$$\theta = \alpha + \gamma \quad (5-3)$$

In steady-flight condition (i.e., missile in equilibrium), the summation of moments about the center of gravity of the missile is zero. Consequently Eq. (5-1) is equated to zero. Since the damping terms $C_{m_{\dot{\theta}}}$, $C_{m_{\dot{\alpha}}}$, etc., can be neglected, the pitching moment due to control-surface deflection must be balanced by that due to angle of attack. Mathematically, this equilibrium condition may be expressed as follows:

$$C_{m_\alpha}\alpha_{TR} = -C_{m_\delta}\delta \quad (5-4)$$

The condition for static stability is that the sign of C_{m_α} must be negative, i.e., a negative, or nose-down, pitching moment is realized with an increase in angle of attack or lift. Hence, when an upgust strikes a missile flying in equilibrium condition, the missile tends to nose down, or "weathercocks" in the direction of the gust. The controllability or maneuverability of the missile is defined as the magnitude of load factor it can execute in its trimmed or equilibrium condition. The load factor n is defined as the ratio of total trimmed normal force to the weight of the missile. In terms of the rate of change of flight-path angle $\dot{\gamma}$ the aerodynamic maneuverability of the missile may be expressed as

$$mV\dot{\gamma} = nW \quad (5-5)$$

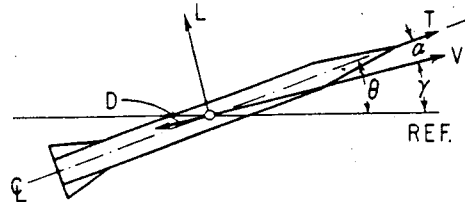


FIG. 5-1. Angle relationship.

In order to evaluate the degree of static stability and the magnitude of the maneuverability of the missile, the aerodynamic derivatives C_{m_α} , C_{m_δ} , etc., must first be determined. The following sections deal with the determination of these derivatives for two general types of design previously mentioned.

5-3. COMPLETE MISSILE AERODYNAMICS—FORWARD CONTROL

The aerodynamic derivatives shown in Eqs. (5-1) and (5-2) are composed of contributions from the various components of the missile.

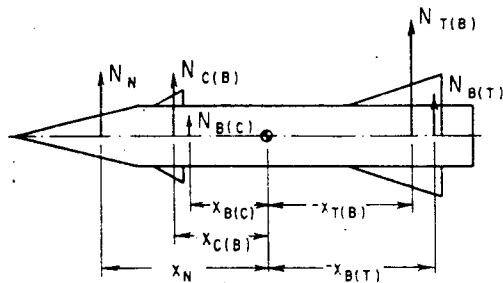


FIG. 5-2. Typical force diagram—front control.

In this section the model build-up will be analyzed for a missile which has its control surfaces placed forward of the center-of-gravity location. This type of configuration may be either the canard- or wing-control design shown in Fig. 2-2. For the purpose of illustration, the former design will be used. Since the condition of equilibrium is of

interest in this section, the damping terms will be temporarily neglected (but will be treated in Chap. 9 in the analysis of the dynamics of the missile). The model build-up equations for the pertinent aerodynamic force and moment derivatives are derived as follows:

$$N_{CM} = N_N + N_{C(B)} + N_{B(C)} + N_{T(B)} + N_{B(T)} \quad (5-6)$$

$$M_{CM} = N_N x_N + N_{C(B)} x_{C(B)} + N_{B(C)} x_{B(C)} + N_{T(B)} x_{T(B)} + N_{B(T)} x_{B(T)} \quad (5-7)$$

where N_{CM} , M_{CM} = normal force and pitching moment of the complete model or missile

N_N = normal force on nose section

$N_{C(B)}$ = normal force on canards in presence of body

$N_{B(C)}$ = normal force on body due to canard

$N_{T(B)}$ = normal force on tail in presence of body

$N_{B(T)}$ = normal force on body due to tail

x_N , $x_{C(B)}$ = moment arms (measured from center of pressure to missile center of gravity) of respective normal forces as shown in Fig. 5-2

Equations (5-6) and (5-7) may be written in coefficient form as follows:

$$(C_{N_\alpha})_{CM} \alpha q S_\pi = (C_{N_\alpha})_N \alpha q S_\pi + (C_{N_\alpha})_C \alpha K_{C(B)} q S_C + (C_{N_\alpha})_C \alpha K_{B(C)} q S_C' + (C_{N_\alpha})_T \alpha_T K_{T(B)} q S_T + (C_{N_\alpha})_T \alpha_T K_{B(T)} q S_T \quad (5-8)$$

$$(C_{N_\delta})_{CM} \delta q S_\pi = (C_{N_\alpha})_C \delta k_{C(B)} q S_C + (C_{N_\alpha})_C \delta k_{B(C)} q S_C + (C_{N_\alpha})_T \alpha_T K_{T(B)} q S_T + (C_{N_\alpha})_T \alpha_T K_{B(T)} q S_T \quad (5-9)$$

$$(C_{m_\alpha})_{CM} \alpha q S_\pi d = (C_{N_\alpha})_N \alpha q S_\pi x_N + (C_{N_\alpha})_C \alpha K_{C(B)} q S_C x_{C(B)} + (C_{N_\alpha})_C \alpha K_{B(C)} q S_C x_{B(C)} + (C_{N_\alpha})_T \alpha_T K_{T(B)} q S_T x_{T(B)} + (C_{N_\alpha})_T \alpha_T K_{B(T)} q S_T x_{B(T)} \quad (5-10)$$

$$(C_{m_\delta})_{CM} \delta q S_\pi d = (C_{N_\alpha})_C \delta k_{C(B)} q S_C x_{C(B)} + (C_{N_\alpha})_C \delta k_{B(C)} q S_C x_{B(C)} + (C_{N_\alpha})_T \alpha_T K_{T(B)} q S_T x_{T(B)} + (C_{N_\alpha})_T \alpha_T K_{B(T)} q S_T x_{B(T)} \quad (5-11)$$

where (1) $K_{C(B)}$ and $K_{T(B)}$ are factors which account for the effect of body upwash at angles of attack ($\delta = 0^\circ$); (2) $K_{B(C)}$ and $K_{B(T)}$ are factors which account for the effect of "body carry-over" lift due to angles of attack ($\delta = 0^\circ$); (3) $k_{C(B)}$ and $k_{T(B)}$ are factors which account for body effect due to control deflection ($\alpha = 0^\circ$); (4) $k_{B(C)}$ and $k_{B(T)}$ are factors which account for body carry-over lift due to control-surface deflection; (5) subscripts CM , N , C , and T denote complete model, nose, canards, and tails, respectively; (6) the subscripts within the parentheses, i.e., $x_{C(B)}$, denote influence by the presence of the body; (7) S_C and S_T denote exposed canard and tail areas, respectively; and (8) $(C_{N_\alpha})_C$ and $(C_{N_\alpha})_T$ are based on S_C and S_T , respectively.

Since $\alpha_T = \alpha - \epsilon$ in Eqs. (5-8) and (5-10) and $\alpha_T = -\epsilon$ in Eqs. (5-9) and (5-11), Eqs. (5-8) through (5-11) become

$$(C_{N_\alpha})_{CM} = (C_{N_\alpha})_N + (C_{N_\alpha})_C [K_{C(B)} + K_{B(C)}] \frac{S_C}{S_\pi} + (C_{N_\alpha})_T [K_{T(B)} + K_{B(T)}] \frac{S_T}{S_\pi} (1 - \epsilon_\alpha) \quad (5-12)$$

$$(C_{N_\delta})_{CM} = (C_{N_\alpha})_C [k_{C(B)} + k_{B(C)}] \frac{S_C}{S_\pi} + (C_{N_\alpha})_T [K_{T(B)} + K_{B(T)}] \frac{S_T}{S_\pi} (-\epsilon_\delta) \quad (5-13)$$

$$\begin{aligned}
 (C_{m_\alpha})_{CM} = & (C_{N_\alpha})_N \left(\frac{x}{d} \right)_N + (C_{N_\alpha})_C K_{C(B)} \frac{S_C}{S_\pi} \left(\frac{x}{d} \right)_{C(B)} \\
 & + (C_{N_\alpha})_C K_{B(C)} \frac{S_C}{S_\pi} \left(\frac{x}{d} \right)_{B(C)} + (C_{N_\alpha})_T K_{T(B)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{T(B)} (1 - \epsilon_\alpha) \\
 & + (C_{N_\alpha})_T K_{B(T)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{B(T)} (1 - \epsilon_\alpha) \quad (5-14)
 \end{aligned}$$

$$\begin{aligned}
 (C_{m_\delta})_{CM} = & (C_{N_\alpha})_C k_{C(B)} \frac{S_C}{S_\pi} \left(\frac{x}{d} \right)_{C(B)} + (C_{N_\alpha})_C k_{B(C)} \frac{S_C}{S_\pi} \left(\frac{x}{d} \right)_{B(C)} \\
 & + (C_{N_\alpha})_T K_{T(B)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{T(B)} (-\epsilon_\delta) \\
 & + (C_{N_\alpha})_T K_{B(T)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{B(T)} (-\epsilon_\delta) \quad (5-15)
 \end{aligned}$$

Since the normal force on the nose section of the body constitutes the majority of the normal force of the complete body configuration, the value of $(C_{N_\alpha})_B$ for the complete body may be substituted in place

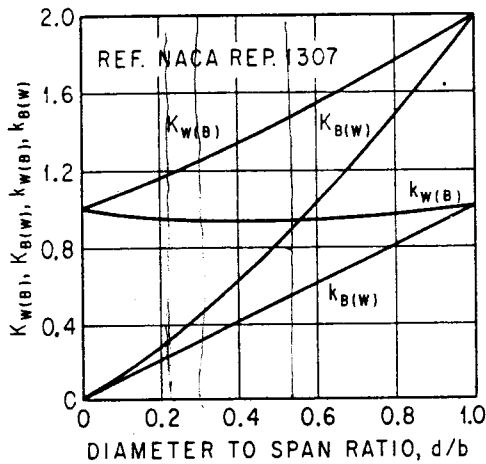


FIG. 5-3. Values of K and k based on slender-body theory.

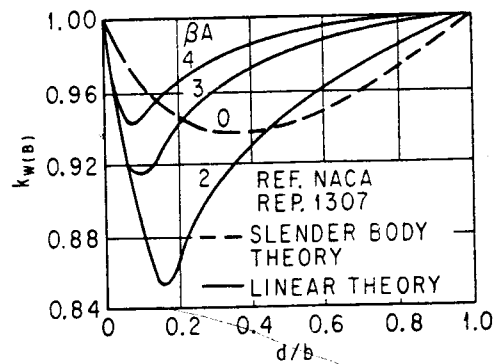


FIG. 5-4. Values of $k_{W(B)}$.

of $(C_{N_\alpha})_N$ in Eqs. (5-12) and (5-14). The normal-force-curve slope for the canard and tail surfaces may be determined as shown in Chap. 3. The values of $K_{C(B)}$ or $K_{T(B)}$ were determined by slender-body theory^{1,2,3,*} and are shown in Fig. 5-3 as a function of diameter-to-span ratio. It is interesting to note that these values do not differ significantly from those derived by linear theory for large-aspect-ratio

* Superscript numbers indicate references listed at the end of the chapter.

wings, or those derived by Beskin's body-upwash theory.⁴ The values of $k_{C(B)}$ or $k_{T(B)}$ were also determined by slender-body theory^{1,2,3} and are shown in Fig. 5-3. These values are applicable for slender triangular wing and body combination. For rectangular wing and body combinations the results derived from exact linear theory⁵ should be used¹ (see Fig. 5-4). The values of $K_{B(C)}$ or $K_{B(T)}$ were determined primarily by the slender-body theory and are shown in Fig. 5-3. For high-aspect-ratio wings at supersonic speeds, Fig. 5-5 should be used.¹

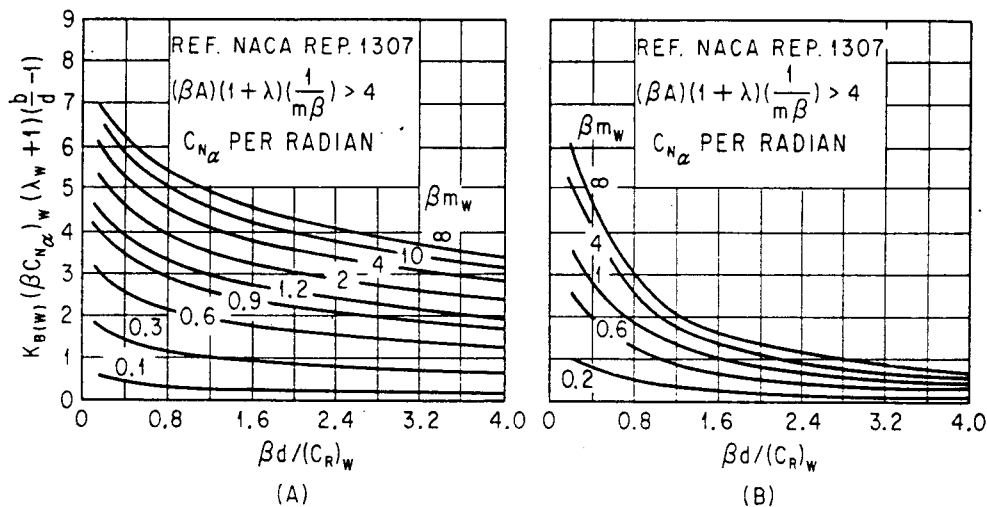


FIG. 5-5. Design charts for determining $K_{B(W)}$ for high-aspect-ratio range at supersonic speeds. (A) With afterbody. (B) No afterbody.

With the exception of the latter, these wing-body interference factors are independent of Mach number. The effect of Mach number is felt only through the values $C_{N\alpha}$ of the various missile components.

The center-of-pressure location for the nose, canard, and tail may be determined by the methods presented in Chap. 3. The effect of the presence of the body on canard or tail center-of-pressure location appears to be negligible, i.e., $x_{C(B)} = x_C$. The location of the center of pressure of the normal force on the body due to canard or tail is a function of many parameters, among which are Mach number, aspect ratio, and taper ratio. Detailed design charts for the center-of-pressure location of these normal-force components may be found in *NACA Report 1307*.¹

Accurate values of ϵ_α and ϵ_δ are very difficult to determine. Theoretical methods are available⁶⁻¹¹ and may be used for preliminary design. However, whenever possible, it is recommended that experimental test results of similar or comparable configurations be used. The need for a comprehensive correlation of both theoretical and

experimental results is obvious, especially for missiles employing relatively low aspect ratio aerodynamic surfaces. Methods for evaluating downwash derivatives ϵ_α and ϵ_δ from wind-tunnel-test results are presented in Appendix F.

Typical complete model normal-force and pitch-moment curves for forward-control (canard- and wing-control) designs are shown in Figs. 5-6 and 5-7. It is seen from Fig. 5-6 that the value of C_{N_δ} is relatively low. As previously noted in Chap. 2, the down load on the tail due

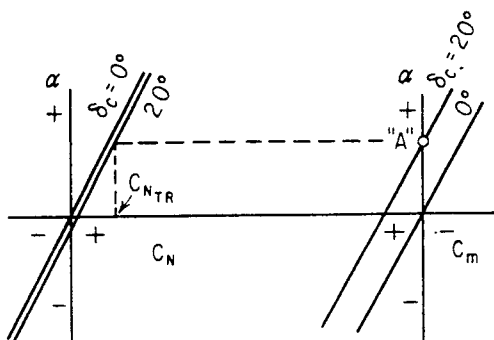


FIG. 5-6. Typical complete-model normal-force and pitching-moment curves—forward (canard) control.

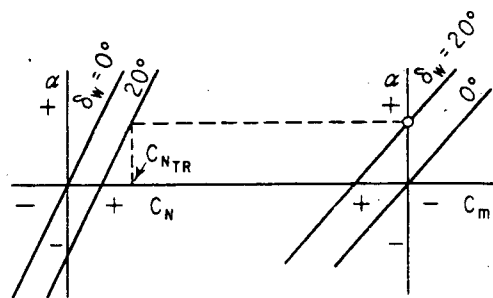


FIG. 5-7. Typical complete-model normal-force and pitching-moment curves—forward (wing) control.

to downwash from the deflected canards may partially or completely nullify the load on the deflected canards. This is readily apparent from Eq. (5-13). On the other hand, the value of C_{N_δ} is high for a wing control because of the large wing area normally associated with this design.

5-4. STATIC STABILITY MARGIN—FORWARD CONTROL

The static stability margin of a missile is defined as the distance between the center of pressure and center of gravity of the missile.

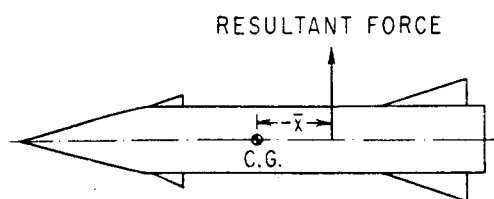


FIG. 5-8. Static stability margin.

Generally, this stability margin is defined in terms of missile body diameters since the nondimensionality factor is d , the body diameter, in lieu of the mean aerodynamic chord $\bar{c}$ used in aircraft terminology. Once the complete missile derivatives $C_{N_\alpha}^*$ and $C_{m_\alpha}^*$ [Eqs. (5-12) and (5-14)] are known, the magnitude of the static stability

* The subscript CM is deleted since the complete model is implied.

margin may be readily calculated with the aid of Fig. 5-8. From Fig. 5-8, the following derivation of the static stability margin $\bar{X}/d$ is made:

$$M = N\bar{X} \quad (5-16)$$

dividing by qSd , Eq. (5-16) becomes

$$C_m = C_N \frac{\bar{X}}{d} \quad (5-17)$$

differentiating with respect to α , Eq. (5-17) becomes

$$C_{m_\alpha} = C_{N_\alpha} \frac{\bar{X}}{d} \quad (5-18)$$

or

$$\frac{\bar{X}}{d} = \frac{C_{m_\alpha}}{C_{N_\alpha}} \equiv C_{m_{c_N}} \quad (5-19)$$

5-5. LOAD-FACTOR CAPABILITY—FORWARD CONTROL

In order to evaluate the aerodynamic load factor or maneuvering capability of the missile, the complete model aerodynamic derivatives C_{N_α} , C_{N_δ} , C_{m_α} , and C_{m_δ} as defined by Eqs. (5-12) through (5-15) must be known. The load factor is defined mathematically as follows:

$$n = \frac{N}{W} = C_{N_{TR}} \frac{qS}{W} \quad (5-20)$$

where $C_{N_{TR}}$ = trimmed normal-force coefficient

W = weight of missile

N = normal force

The value of $C_{N_{TR}}$ may be readily determined with the aid of Fig. 5-6. The trim condition is defined as the point of zero moment, i.e., point A in Fig. 5-6. The value of $C_{N_{TR}}$ is derived as follows:

$$-C_{m_\alpha} \alpha_{TR} = C_{m_\delta} \delta \quad (5-21)$$

or

$$\alpha_{TR} = \frac{-C_{m_\delta} \delta}{C_{m_\alpha}} \quad (5-22)$$

but

$$C_{N_{TR}} = C_{N_\alpha} \alpha_{TR} + C_{N_\delta} \delta \quad (5-23)$$

Substituting Eq. (5-22) into (5-23), we get

$$C_{N_{TR}} = C_{N_\alpha} \left(-\frac{C_{m_\delta} \delta}{C_{m_\alpha}} \right) + C_{N_\delta} \delta \quad (5-24)$$

or

$$n = \left(C_{N\delta} - \frac{C_{m\delta}}{C_{m\alpha}} C_{N\alpha} \right) \delta \frac{qS}{W} \quad (5-25)$$

The load factor per unit control deflection is thus

$$\frac{n}{\delta} = \left(C_{N\delta} - C_{N\alpha} \frac{C_{m\delta}}{C_{m\alpha}} \right) \frac{qS}{W} \quad (5-26)$$

For a typical canard configuration which has a negligibly small value of $C_{N\delta}$, Eq. (5-26) becomes

$$\frac{n}{\delta} = - \left(C_{N\alpha} \frac{C_{m\delta}}{C_{m\alpha}} \right) \frac{qS}{W} \quad (5-27)$$

The above expressions are very useful in the determination of trim load factor provided that linear aerodynamics are realized. If non-linearity exists, the value of n/δ must be obtained directly from the C_N and C_m vs. α curves such as those shown in Figs. 5-6 and 5-7.

5-6. COMPLETE MISSILE AERODYNAMICS—REAR CONTROL

Another common type of design is one in which the control surfaces are located aft of the missile center of gravity. For this type of design

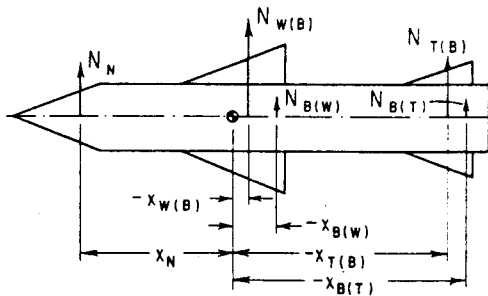


FIG. 5-9. Typical force diagram—rear control.

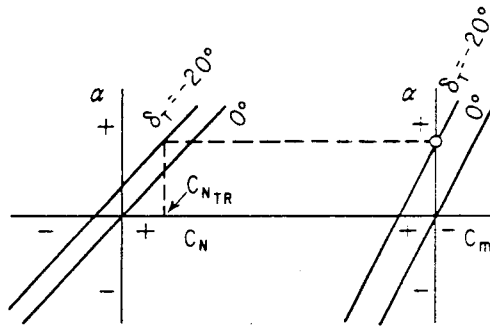


FIG. 5-10. Typical complete-model normal-force and pitching-moment curves—rear control.

the complete model or missile aerodynamics are similar to those previously discussed, as shown below. From Fig. 5-9, the forces and moments for a typical tail-control design are as follows:

$$N_{CM} = N_N + N_{W(B)} + N_{B(W)} + N_{T(B)} + N_{B(T)} \quad (5-28)$$

$$M_{CM} = N_N x_N + N_{W(B)} x_{W(B)} + N_{B(W)} x_{B(W)} + N_{T(B)} x_{T(B)} + N_{B(T)} x_{B(T)} \quad (5-29)$$

Using the procedure previously described, the following relationships are obtained:

$$(C_{N_\alpha})_{CM} = (C_{N_\alpha})_N + (C_{N_\alpha})_W [K_{W(B)} + K_{B(W)}] \frac{S_W}{S_\pi} + (C_{N_\alpha})_T [K_{T(B)} + K_{B(T)}] \frac{S_T}{S_\pi} (1 - \epsilon_\alpha) \quad (5-30)$$

$$(C_{N_\delta})_{CM} = (C_{N_\alpha})_T [k_{T(B)} + k_{B(T)}] \frac{S_T}{S_\pi} \quad (5-31)$$

$$(C_{m_\alpha})_{CM} = (C_{N_\alpha})_N \left(\frac{x}{d} \right)_N + (C_{N_\alpha})_W K_{W(B)} \frac{S_W}{S_\pi} \left(\frac{x}{d} \right)_{W(B)} + (C_{N_\alpha})_W K_{B(W)} \frac{S_W}{S_\pi} \left(\frac{x}{d} \right)_{B(W)} + (C_{N_\alpha})_T K_{T(B)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{T(B)} (1 - \epsilon_\alpha) + (C_{N_\alpha})_T K_{B(T)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{B(T)} (1 - \epsilon_\alpha) \quad (5-32)$$

$$(C_{m_\delta})_{CM} = (C_{N_\alpha})_T k_{T(B)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{T(B)} + (C_{N_\alpha})_T k_{B(T)} \frac{S_T}{S_\pi} \left(\frac{x}{d} \right)_{B(T)} \quad (5-33)$$

The values of the various terms appearing in Eqs. (5-30) through (5-33) may be evaluated as noted previously.

Typical complete missile aerodynamic curves of C_m and C_N vs. angle of attack for a rear-control configuration are shown in Fig. 5-10. It is readily apparent that the sign of $(C_{m_\delta})_{CM}$ is negative (δ_T is considered positive with leading edge deflected upward) for a tail-control design as previously noted in Chap. 2 (Fig. 2-6).

5-7. STATIC STABILITY MARGIN—REAR CONTROL

The static stability margin for a rear-control design may be determined in a manner similar to that previously described. The resultant expression is

$$\frac{\bar{X}}{d} = \frac{C_{m_\alpha}}{C_{N_\alpha}} \equiv C_{m_{c_N}} \quad (5-34)$$

which is identical to Eq. (5-19).

5-8. LOAD-FACTOR CAPABILITY—REAR CONTROL

The expression for n/δ may be derived from Fig. 5-10 using Eqs. (5-21) through (5-25). The resultant expression becomes

$$\frac{n}{\delta} = \left(C_{N_\delta} - C_{N_\alpha} \frac{C_{m_\delta}}{C_{m_\alpha}} \right) \frac{qS}{W} \quad (5-35)$$

which is identical to Eq. (5-26) for the forward-control case; the exception is that the sign of $C_{m\delta}$ for the rear-control case is negative. Hence it is apparent that the normal force due to control-surface deflection is always opposite to the resultant force. Hence it always decreases the load-factor capability of the missile.

SYMBOLS

A	aspect ratio
C_L	lift coefficient
C_m	pitching-moment coefficient
$C_{m_{CN}}$	static stability margin $\bar{X}/d$
C_N	normal-force coefficient
C_R	root chord
I	moment of inertia
$K_{B(C)}, K_{B(W)}, K_{B(T)}$	factors which account for the effect of "body carry-over" lift (or normal force) due to angle of attack
$K_{C(B)}, K_{W(B)}, K_{T(B)}$	factors which account for the effect of body upwash at angle of attack
M	(pitching) moment
$N_{B(C)}, N_{B(W)}, N_{B(T)}$	normal-force "carry-over" on the body due to aerodynamic surfaces
$N_{C(B)}, N_{W(B)}, N_{T(B)}$	normal force on the aerodynamic surfaces in the presence of the body
N_N	normal force on the nose section of the body
S	reference area
S_π	body frontal (reference) area
V	missile forward velocity
W	weight
$\bar{X}$	resultant center of pressure
d	body diameter
$k_{B(C)}, k_{B(W)}, k_{B(T)}$	factors which account for the effect of "body carry-over" lift (or normal force) due to control-surface deflection
$k_{C(B)}, k_{W(B)}, k_{T(B)}$	factors which account for body effect on the aerodynamic surfaces due to deflection
m	mass, or cotangent of leading-edge sweep angle (see Fig. 5-5A and B)
n	load factor
q	dynamic pressure

x	moment arm (distance between missile center of gravity and center of pressure of components)
α	angle of attack
β	$\sqrt{M^2 - 1}$
γ	flight-path angle
δ	control-surface deflection
ϵ	downwash angle
θ	missile attitude
λ	taper ratio (ratio of tip chord to root chord)

Subscripts

C	canard
CM	complete model
N	nose (or body)
W	wing
T	tail

Note: The following form is used throughout to denote aerodynamic derivatives; i.e., C_{N_α} is $\partial C_N / \partial \alpha$, C_{m_α} is $\partial C_m / \partial \alpha$, etc.

REFERENCES

1. Pitts, W. C., J. N. Nielsen, and G. E. Kaattari: Lift and Center of Pressure of Wing-body-tail Combinations at Subsonic, Transonic, and Supersonic Speeds, *NACA Rept.* 1307, 1959.
2. Spreiter, J. R.: The Aerodynamic Forces on Slender Plane- and Cruciform-wing and Body Combinations, *NACA Rept.* 962, 1950 (formerly *Tech. Notes* 1662 and 1897).
3. Morikawa, G.: Supersonic Wing-body Lift, *J. Aeronaut. Sci.*, vol. 18, no. 4, pp. 217-228, April, 1951.
4. Beskin, L.: Determination of Upwash around a Body of Revolution at Supersonic Velocities, Johns Hopkins University, Applied Physics Laboratory, *Rept.* CM-251, May 27, 1946.
5. Nielsen, J. N., and W. C. Pitts: Wing-body Interference at Supersonic Speeds with an Application to Combinations with Rectangular Wings, *NACA Tech. Note* 2677, 1952.
6. Lomax, H., L. Sluder, and M. A. Heaslet: The Calculation of Downwash behind Supersonic Wings with an Application to Triangular Plan Forms, *NACA Rept.* 957, 1950.
7. Lagerstrom, P. A., and M. E. Graham: Downwash and Sidewash Induced by Three-dimensional Lifting Wings in Supersonic Flow, Douglas Aircraft Co., Inc., *Rept.* SM-13007, April, 1947.
8. Spreiter, J. R., and A. H. Sacks: The Rolling-up of the Trailing Vortex Sheet and Its Effect on Downwash behind Wings, *J. Aeronaut. Sci.*, vol. 18, no. 1, pp. 21-32, 72, January, 1951.

9. Rogers, A. W.: Application of Two-dimensional Vortex Theory to the Prediction of Flow Fields behind Wings of Wing-body Combinations at Subsonic and Supersonic Speeds, *NACA Tech. Note 3227*, 1954.

10. Silverstein, A., and S. Katzoff: Design Charts for Predicting Downwash Angles and Wake Characteristics behind Plain and Flapped Wings, *NACA Rept. 648*, 1939.

11. Decker, J. L.: Prediction of Downwash at Various Angles of Attack for Arbitrary Tail Locations, *IAS Preprint 615*, 24th Annual Meeting, Jan. 23-26, 1956.

CHAPTER 6

MANEUVERING FLIGHT

6-1. INTRODUCTION

For certain types of missiles, such as an air-to-air missile attacking a fast-moving and maneuvering target, the maneuvering-flight characteristics are of particular interest. For the cruise-type or long-range ballistic missiles, the demand for maneuvering flight, though greatly reduced, must be satisfied in order to achieve a complete missile weapon system. The following sections deal with the maneuvering-flight characteristics of missiles with both monowing and cruciform designs. The effect of static stability margin on maneuverability or load-factor capability of the missile will be illustrated both analytically and graphically. The effect of aerodynamic damping on load-factor capability is also discussed.

6-2. FLAT TURNS

1. Cruciform. In order to change the flight path of a missile flying in an equilibrium (level or straight) flight condition, an unbalance force must be introduced. For a missile of cruciform design, this unbalance force may be obtained by deflecting the appropriate set or sets of control surfaces. As indicated in Fig. 6-1, the cruciform missile is not required to bank to produce the centripetal force for a flat turn. For the cruciform missile designed to fly with all the aerodynamic control surfaces in the "plus" (+) position during normal flight, only one set of (yaw) controls needs to be deflected to maneuver the missile in a flat turn. However, if these control surfaces are in the X position, both

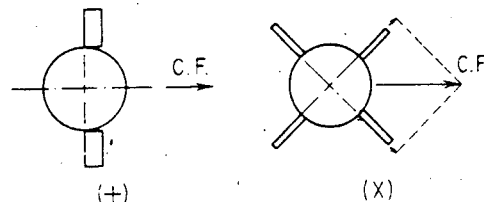


FIG. 6-1. Control deflections for "plus" and "X" designs.

sets of controls must be deflected. The magnitude of each set of control deflections is approximately 0.7 of that required for the case in which the aerodynamic surfaces are in the "plus" position.

The radius of a flat turn may be related to the forward speed and trim load-factor capability of the missile as follows:

$$\text{Centripetal force } CF = \frac{WV^2}{gR} = nW \quad (6-1)$$

or
$$R = \frac{V^2}{ng} \quad (6-2)$$

It is apparent from Eq. (6-2) that the turn radius is directly proportional to the square of the forward velocity and inversely proportional

to the trim load-factor capability of the missile (also see Fig. 6-2). The turn radius can also be expressed in terms of the trim normal force (strictly speaking, it should be the lift force) as follows:

$$nW = N = C_{NTR} \frac{1}{2} \rho V^2 S \quad (6-3)$$

substituting Eq. (6-3) in Eq. (6-1) or (6-2), we get

$$R = \frac{2W}{\rho g S C_{NTR}} \quad (6-4)$$

For certain types of configurations, the value of C_{NTR} is nearly inde-

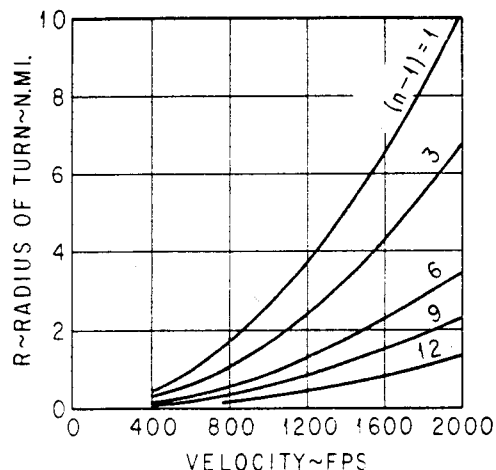


FIG. 6-2. Pull-up performance.

pendent of Mach number. Consequently the turn radius is nearly constant over a relatively wide Mach-number range at a given flight altitude. This feature is desirable from the standpoint of control-system design since a near-constant "aerodynamic-gain" condition is realized.

Quite frequently, it is convenient to express the maneuvering-flight characteristics of a missile in terms of the rate of change in flight-path angle $\dot{\gamma}$. From Eq. (5-5), the rate of change in flight-path angle can be written as

$$\dot{\gamma} = \frac{ng}{V} \quad (6-5)$$

or in terms of C_{NTR} , Eq. (6-11) becomes

$$\dot{\gamma} = \frac{\rho g S V}{2W} C_{NTR} \quad (6-6)$$

2. Monowing. A monowing missile must bank to produce the required centripetal force in a flat turn. Hence, from the force diagram in Fig. 6-3, the following pertinent relationships can be written:

For level flight $W = nW \cos \varphi$ (6-7)

Centripetal force $CF = nW \sin \varphi = \frac{WV^2}{gR}$ (6-8)

Hence, from Eqs. (6-7) and (6-8),

$$R = \frac{V^2}{g \tan \varphi} \quad (6-9)$$

and

$$n = \frac{1}{\cos \varphi} \quad (6-10)$$

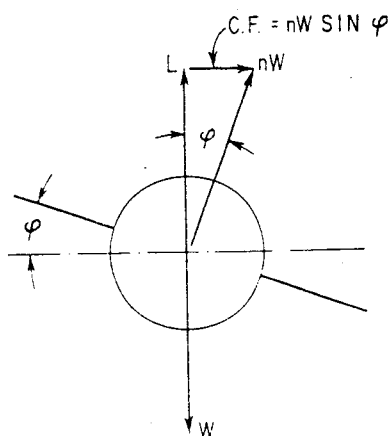


FIG. 6-3. Force diagram for a monowing missile in a flat turn.

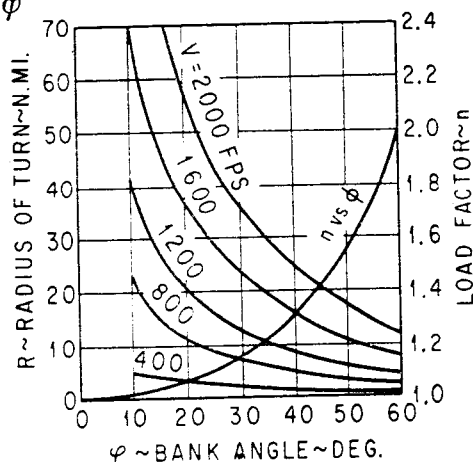


FIG. 6-4. Flat-turn performance.

The results of Eqs. (6-9) and (6-10) are shown graphically in Fig. 6-4. Equations (6-9) and (6-10) can also be expressed in terms of $C_{N_{TR}}$ as

$$R = \frac{2W}{\rho g S C_{N_{TR}} \sin \varphi} \quad (6-11)$$

and

$$\frac{1}{\cos \varphi} = \frac{\rho V^2 S}{2W} C_{N_{TR}} \quad (6-12)$$

It is interesting to note that, if the monowing missile could theoretically bank into a 90° attitude for the flat turn, Eq. (6-11) would be identical to Eq. (6-4), which is applicable for the cruciform missile.

The rate of change of flight path may be written as

$$\dot{\gamma} = \frac{ng \sin \varphi}{V} \quad (6-13)$$

or

$$\dot{\gamma} = \frac{\rho g S V \sin \varphi}{2W} C_{N_{TR}} \quad (6-14)$$

6-3. PULL-UPS

The relationship for the turn radius for pull-ups or pitch-up maneuver from level flight is given as

$$CF = nW - W = \frac{WV^2}{gR} \quad (6-15)$$

or
$$R = \frac{V^2}{g(n-1)} \quad (6-16)$$

The rate of change in flight-path angle $\dot{\gamma}$ may be written as

$$\dot{\gamma} = g \left(\frac{\rho V}{2W} C_{N_{TR}} - \frac{1}{V} \right) \quad (6-17)$$

For pitch-down maneuvers, Eqs. (6-16) and (6-17) become

$$R = \frac{V^2}{g(n+1)} \quad (6-18)$$

and
$$\dot{\gamma} = g \left(\frac{\rho V}{2W} C_{N_{TR}} + \frac{1}{V} \right) \quad (6-19)$$

6-4. RELATIONSHIP OF MANEUVERABILITY AND STATIC STABILITY MARGIN

It was noted in Chap. 5 that the requirements for large static stability margin and great load factor or maneuvering capability are conflicting in that increasing stability margin results in a decrease in load-factor capability. The load factor per unit control deflection n/δ (assuming linear aerodynamic derivatives) given in Eq. (5-27) is expressed as

$$\frac{n}{\delta} = \left(C_{N_\delta} - \frac{C_{m_\delta}}{C_{m_{c_N}}} \right) \frac{qS}{W} \quad (6-20)$$

where

$$C_{m_{c_N}} = \frac{\partial C_m / \partial \alpha}{\partial C_N / \partial \alpha} = \frac{\bar{X}}{d}$$

in accordance with Eq. (5-19).

Since the sign of C_{m_δ} is positive and $C_{m_{c_N}}$ is negative for all practical design conditions, it is apparent from Eq. (6-20) that increasing the static stability margin $C_{m_{c_N}}$ results in a decrease in load-factor capability. This phenomenon may also be illustrated graphically as in Fig. 6-5. The solid line represents the normal-force and pitching-moment curves for a given missile design for a given static stability

margin. The dotted line represents the pitching-moment curves for a condition of greater static stability margin (i.e., a more stable missile) which could be realized by a more forward center-of-gravity location. The normal-force curves are not affected by center-of-gravity location since they are primarily a function of the external configuration. It is apparent from Fig. 6-5 that the trim angle of attack and hence the trim normal-force coefficient $C_{N_{TR}}$ is noticeably lower for the more stable missile than the corresponding values for the less stable missile. Thus, if the requirement for both a large static stability margin and a large load-factor capability exists for a given missile design, relatively large aerodynamic surfaces and/or control surfaces (i.e., larger values of C_{N_α} and C_{m_δ}) must prevail.

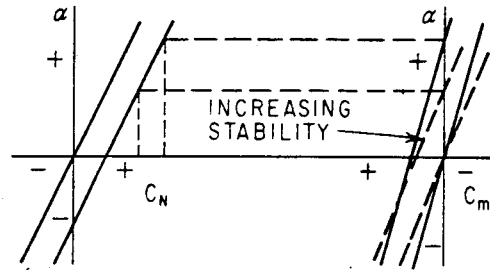


FIG. 6-5. Effect of static stability margin on trim load-factor capability.

Heretofore the effect of aerodynamic damping on the maneuverability or load-factor capability per unit control deflection has been neglected. In actuality, aerodynamic damping, though small, is present when the missile is commanded into a maneuvering-flight trajectory. This is apparent from the equations of motion [Eqs. (5-1) and (5-2a)] and the angle relationship [Eq. (5-3)]:

$$I\ddot{\theta} = (C_{m_\alpha}\alpha + C_{m_\delta}\delta)qSd + (C_{m_\dot{\theta}}\dot{\theta} + C_{m_\ddot{\alpha}}\ddot{\alpha})\frac{qSd^2}{2V} \quad (6-21)$$

$$mV\dot{\gamma} = (C_{N_\alpha}\alpha + C_{N_\delta}\delta)qS \quad (6-22)$$

$$\theta = \alpha + \gamma \quad (6-23)$$

For steady-state trimmed maneuvering flight, $\ddot{\theta}$ and $\ddot{\alpha}$ become zero. Hence the following relationships may be written:

$$C_{m_\alpha}\alpha_{TR} + C_{m_\delta}\delta + C_{m_\dot{\theta}}\dot{\theta}\frac{d}{2V} = \quad (6-24)$$

$$mV\dot{\gamma} = nW = (C_{N_\alpha}\alpha_{TR} + C_{N_\delta}\delta)qS \quad (6-25)$$

$$\text{and} \quad \dot{\theta} = \dot{\gamma} \quad (6-26)$$

substituting Eqs. (6-26) and (6-24), Eq. (6-25) may be expressed as

$$C_{m_\alpha}\alpha_{TR} + C_{m_\delta}\delta + C_{m_\dot{\theta}}\dot{\gamma}\frac{d}{2V}\left[\frac{g}{VW}(C_{N_\alpha}\alpha_{TR} + C_{N_\delta}\delta)qS\right] = 0 \quad (6-27)$$

hence
$$\alpha_{TR} = - \frac{[C_{m_\delta} + C_{m_{\dot{\theta}}}(qSdg/2V^2W)C_{N_\delta}]\delta}{C_{m_\alpha} + C_{m_{\dot{\theta}}}(qSdg/2V^2W)C_{N_\alpha}} \quad (6-28)$$

and
$$\frac{n}{\delta} = \left[C_{N_\delta} - \frac{C_{m_\delta} + C_{m_{\dot{\theta}}}(qSdg/2V^2W)C_{N_\delta}}{C_{m_{c_N}} + C_{m_{\dot{\theta}}}(qSdg/2V^2W)} \right] \frac{qS}{W} \quad (6-29)$$

substituting $q = \frac{1}{2}\rho V^2$ into Eq. (6-29), we get

$$\frac{n}{\delta} = \left[C_{N_\delta} - \frac{C_{m_\delta} + C_{m_{\dot{\theta}}}(\rho Sdg/4W)C_{N_\delta}}{C_{m_{c_N}} + C_{m_{\dot{\theta}}}(\rho Sdg/4W)} \right] \frac{qS}{W} \quad (6-30)$$

The effect of the damping term in Eq. (6-30) is to decrease the value of n/δ . For missiles with relatively high damping characteristics, the decrease in n/δ is in the order of a few per cent over that calculated by Eq. (6-20), which neglects damping. Hence, for most practical applications, the damping may be neglected in the determination of trim load factor.

SYMBOLS

CF	centripetal force
C_m	pitching-moment coefficient
$C_{m_{c_N}}$	static stability margin $\bar{X}/d$
C_N	normal-force coefficient
$C_{N_{TR}}$	trim normal-force coefficient
R	turn radius
S	reference area
V	forward speed
W	weight
$\bar{X}$	resultant center of pressure
d	body diameter
n	load factor
q	dynamic pressure
α	angle of attack
α_{TR}	trim angle of attack
γ	flight-path angle
θ	missile attitude
ρ	air density
φ	bank angle

CHAPTER 7

DIRECTIONAL STABILITY AND CONTROL

7-1. INTRODUCTION

In Chap. 5 the longitudinal static stability and control characteristics were treated independently of those for the other two axes (i.e., yaw and roll). This was made possible by the fact that the motions in pure pitch do not produce aerodynamic forces or moments in either the yaw or roll axes. Similarly, motions in the roll and/or yaw axes do not result in aerodynamic forces or moments in the pitch plane. Strictly speaking, the motions in the yaw axis for a monowing design must be studied simultaneously with those in the roll axis. In addition, for a cruciform design, the combined pitch and yaw motions must be analyzed together with those in roll because of aerodynamic cross-coupling effects. These motions are further complicated by the existence of inertia cross-coupling effects, particularly for missiles whose moment of inertia in roll is small relative to those in pitch and/or yaw. These effects will be discussed in Chap. 9.

For the sake of clarity, the static directional stability and control characteristics in steady flight will be treated separately in this chapter. The lateral stability and control characteristics will be treated separately in Chap. 8. The problem of directional stability and control is twofold: (1) to provide static stability and adequate damping characteristics similar to those for the longitudinal axis and (2) to provide sufficient control to maintain a desired flight path or trim load-factor capability.

In the case of a cruciform design, if satisfactory pitch characteristics are obtained, then by symmetry, the yaw characteristics should also be satisfactory. In the case of the monowing design, sufficiently large stabilizing tail surfaces must be used to obtain the desired static stability and damping characteristics. The following sections deal with the contributions of the different missile components toward these desired characteristics for both monowing and cruciform designs.

Before going into the detailed discussion of the directional stability and control characteristics, it is well first to define some of the nomenclature commonly used in such an analysis (see Fig. 7-1).

1. The sideslip angle β is the angle between the velocity vector and the plane of symmetry of the missile. It is positive by definition when

the nose of the missile is pointed to the left of this velocity vector. Hence it can be seen that β is analogous to the angle of attack α for the longitudinal case.

2. The yaw angle ψ is the angle between a reference line fixed in space and the plane of symmetry. It is positive when the nose of the missile is pointed to the right of the reference direction. Hence ψ is analogous to the pitch angle θ in the longitudinal case. For steady-state conditions such as equilibrium sideslips, $\psi = -\beta$. However, for general application where the missile is not in equilibrium sideslips but is in an

accelerated or turning-flight maneuver, $\psi \neq \beta$. A certain amount of confusion has arisen from the fact that some wind-tunnel data are plotted as a function of ψ instead of the more precise parameter β . Therefore, before these data can be used, the static directional-stability derivative $C_{n\psi}$ must be used as $-C_{n\beta}$, since no such derivative as $C_{n\psi}$ actually exists. The directional-stability derivative $C_{n\beta}$ is analogous to $C_{m\alpha}$ in the pitch case. Defining a positive yawing moment as one which rotates the nose of the missile to the right, the sign of $C_{n\beta}$ must be positive for static stability to exist in yaw.

7-2. CRUCIFORM CONFIGURATION

In the majority of our missile configurations, a cruciform design is used to obtain fast and equal response characteristics in both the pitch (longitudinal) and yaw (directional) axes. For this type of design, the discussions presented in Chap. 5 for the longitudinal case are applicable to the directional case. Therefore, it is considered appropriate to discuss here the pertinent relationships for combined pitch

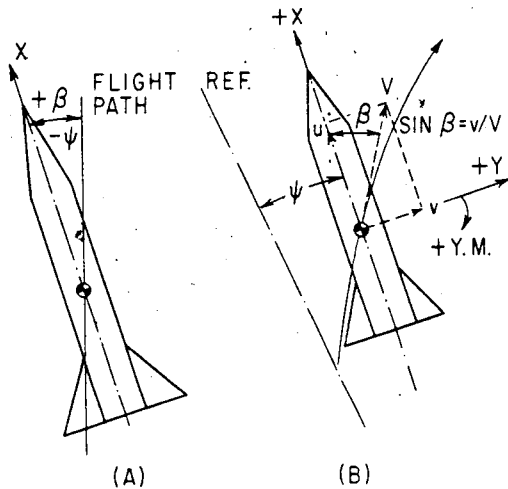


FIG. 7-1. Nomenclature for directional-stability analysis. (A) Equilibrium sideslip. (B) General sideslip.

and yaw maneuvers. From Fig. 7-2, the following angular relationships are defined:

$$\alpha = \tan^{-1} (\tan i \cos \varphi) \quad (7-1)$$

$$\beta = \tan^{-1} (\tan i \sin \varphi) \quad (7-2)$$

where φ = angle between reference plane and plane of resultant wind
 i = angle between longitudinal axis of missile and resultant wind

α and β = angle of attack and sideslip, respectively

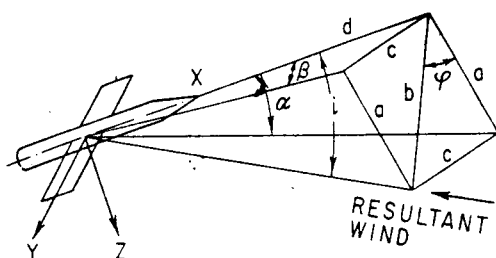


FIG. 7-2. Angle notation.

$$\tan \alpha = \frac{a}{d} \quad \tan \beta = \frac{c}{d}$$

$$a = b \cos \varphi$$

$$c = b \sin \varphi$$

$$d = b / \tan i$$

$$\tan \alpha = \cos \varphi \tan i$$

$$\tan \beta = \sin \varphi \tan i$$

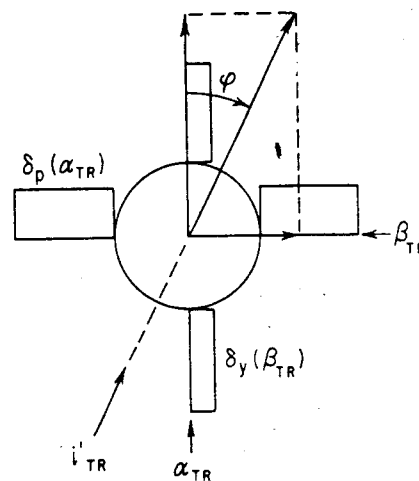


FIG. 7-3. Combined pitch and yaw maneuver.

For small angles, Eqs. (7-1) and (7-2) may be reduced to

$$\alpha \cong i \cos \varphi \quad (7-3)$$

$$\beta \cong i \sin \varphi \quad (7-4)$$

or
$$\tan \varphi \cong \frac{\beta}{\alpha} \quad (7-5)$$

Hence, for a combined maneuver which requires equal pitch and yaw commands, the orientation of the force or velocity vector will be 45° to the reference plane. If i'_{TR} is defined as the trim angle of attack for a unit load factor, then

$$\alpha \cong 0.7i'_{TR} \quad (7-6)$$

$$\beta \cong 0.7i'_{TR} \quad (7-7)$$

For other combinations of two-plane maneuvers, such as that shown in Fig. 7-3, the following relationships may be written (linear aerodynamics assumed):

$$\alpha \cong i'_{TR} \cos \varphi \quad (7-8)$$

$$\beta \cong i'_{TR} \sin \varphi \quad (7-9)$$

where φ is the desired orientation of the velocity or force vector and is specified.

7-3. BODY CONTRIBUTION

The force and moment contributions of the body consist principally of (1) those of the nose or forebody of the missile and (2) those resulting from body upwash (or sidewash) effects as described in Chap. 5. The latter effect is normally treated together with the tail contribution.

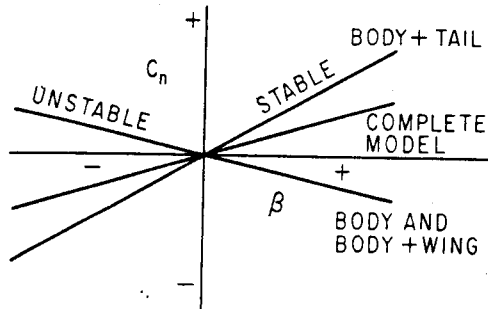


FIG. 7-4. Model build-up in directional-stability curve (monowing design).

Hence the force and moment from the nose or forebody are generally considered as the basic body contribution to the directional-stability characteristics of the missile and are essentially independent of Mach number. For large length-to-diameter ratio l/d , nose or forebody, its large destabilizing effect on directional stability is very undesirable and must be overcome by large tail surfaces. This destabilizing effect may become critical, particularly at the higher supersonic Mach numbers, since the stabilizing contribution of the tail surfaces decreases as the result of decreasing C_{N_α} with increasing Mach number. Hence it may be necessary to reduce the nose l/d to ameliorate this problem even at the sacrifice of missile performance (i.e., increase in wave drag due to foreshortened nose).

The body contribution to yawing moment can be expressed as follows:

$$(\Delta C_{n_\beta})_B = (C_{y_\beta})_B \left(\frac{x}{d} \right)_B \quad (7-10)$$

where $|(C_{y_\beta})_B| = |(C_{N_\alpha})_B|$ for a body of revolution.

Since the sign of $(C_{y_\beta})_B$ is negative (and x is positive measured forward of the center of gravity) $(\Delta C_{n_\beta})_B$ is also negative, indicating an unstable contribution as shown in Fig. 7-4.

7-4. WING CONTRIBUTION

It was noted in Chaps. 2 and 5 that, for either a wing- or tail-control design, the wing has a pronounced effect on the static longitudinal-stability and control characteristics of the missile. However, in the directional-stability and control case, the wing contribution is small,

particularly for the mid-wing design generally used on monowing missile configurations. This is apparent from the fact that, for a straight and unswept wing, the change in angle of attack due to sideslip on one side is the same as that on the opposite side. The effects of sweep, wing planforms, etc., have been extensively treated from the theoretical standpoint.^{1-5*} Except for the dihedral effects, which will be discussed in Chap. 8, the effects of the wing on the directional-stability and control characteristics are generally small and may be neglected in preliminary design studies. This is justified in the light of numerous experimental test results. It should be noted, however, that for monowing designs the effects of wing height on the body may be significant as a result of body-wing mutual interference as well as shock-wave interference, and they should be carefully analyzed in the wind tunnel.

7-5. TAIL CONTRIBUTION

The magnitude of the (vertical) tail contribution to yawing moment required is the sum of the unstable yawing moment due to the nose or forebody and that required to yield the desired static directional-stability margin. For the cruciform design, this tail contribution is identical to that determined for the pitch-stability case. However, for the monowing design the problem of determining the vertical tail contribution is complicated by several factors. Among these factors are (1) body-tail interference, (2) horizontal tail and vertical tail mutual interference, and (3) sidewash introduced by body, wing, and wing-body interference.

The tail contribution to side force and yawing moment (see Fig. 7-4) may be expressed as follows:

$$(\Delta C_{Y\beta})_T = (C_{Y\beta})_T \frac{S_T}{S_\pi} \left(1 + \frac{d\sigma}{d\beta} \right) \quad (7-11)$$

$$\text{and} \quad (\Delta C_{n\beta})_T = (C_{Y\beta})_T \frac{S_T}{S_\pi} \left(1 + \frac{d\sigma}{d\beta} \right) \left(\frac{x}{d} \right)_T \quad (7-12)$$

where $(C_{Y\beta})_T$ = normal-force-curve slope of vertical tail in presence of body and horizontal tail and is based on S_T (tail area)

$d\sigma/d\beta$ = rate of change in sidewash angle (see Fig. 7-5) with angle of sideslip

* Superscript numbers indicate references listed at the end of the chapter.

From the previous discussion, the complete model side-force and yawing-moment derivatives may be expressed as

$$(C_{y\beta})_{CM} = (C_{y\beta})_B + (C_{y\beta})_T \frac{S_T}{S_\pi} \left(1 + \frac{d\sigma}{d\beta}\right) \quad (7-13)$$

and

$$(C_{n\beta})_{CM} = (C_{y\beta})_B \left(\frac{x}{d}\right)_B + (C_{y\beta})_T \frac{S_T}{S_\pi} \left(1 + \frac{d\sigma}{d\beta}\right) \left(\frac{x}{d}\right)_T \quad (7-14)$$

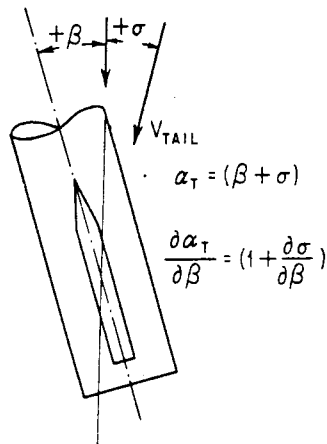


FIG. 7-5. Angle of attack on the (vertical) tail.

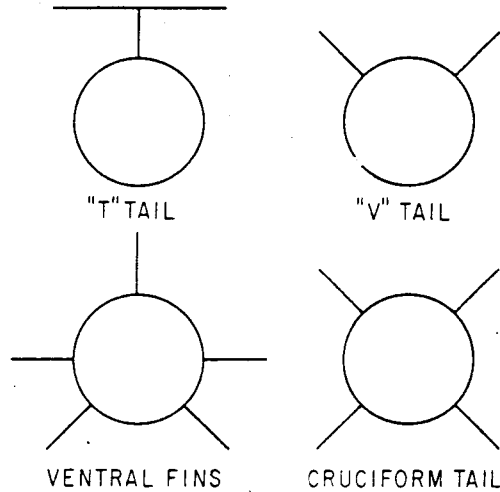


FIG. 7-6. Typical tail designs.

The static stability in yaw may be expressed as

$$\text{Static margin} = \frac{C_{n\beta}}{C_{y\beta}} = \left(\frac{\bar{X}}{d}\right)_{CM} \quad (7-15)$$

and is analogous to $C_{m\alpha}/C_{N\alpha}$ for the longitudinal case. Hence for a given static stability requirement the vertical-tail size may be determined with the aid of Eqs. (7-13) through (7-15).

It will be recalled from Chap. 3 that the normal-force-curve slope of an aerodynamic lifting surface decreases with increasing Mach number in the supersonic region. Therefore, it is apparent that the static stability margin (and control) characteristics deteriorate with increasing Mach number. Consequently, the area of the vertical tail of a monowing design may become extremely large in order to satisfy the stability requirement in the high-Mach-number regions. The tail span for such a large area may be impractical from the standpoint of structural efficiency, physical-span limitation (particularly for air-launched missiles), and aerodynamic cross-coupling effects. In order to circumvent this design inefficiency, several types of tail design may

be used. These include the T tail, V tail, ventral fins, and cruciform X tails as shown in Fig. 7-6. From the standpoint of keeping the span to a minimum, the latter design is optimum. Ventral fins may also be used advantageously, particularly for cases where the pitch angle of attack is relatively large, which results in fuselage blanketing of the top portion of the vertical tail.

7-6. DIRECTIONAL CONTROL

For a cruciform design the primary requirement of directional control is to assure that adequate load-factor capability is realized in the yaw plane. As previously mentioned, if the pitch-control characteristics are satisfactory, the yaw control should also be adequate. If linear aerodynamics are assumed, the control deflections for combined pitch and yaw maneuvers may be readily determined. Since the control-surface deflection is proportional to the trim angle of attack (or sideslip) as expressed in Eq. (5-22), the pitch-and-yaw-control deflections δ_p and δ_y may be related with the aid of Eq. (7-3) as follows:

$$\tan \varphi \sim \frac{\delta_y}{\delta_p} \quad (7-16)$$

$$\text{Hence} \quad \delta_p \cong \delta' \cos \varphi \quad (7-17)$$

$$\delta_y \cong \delta' \sin \varphi \quad (7-18)$$

where δ' is defined as the control-surface deflection required for a unit load factor in a single plane with i'_{TR} as its corresponding trim angle of attack [see Eqs. (7-8) and (7-9)].

For a monowing configuration, the primary requirement of directional control is to maintain zero sideslip. Rudder is generally preferred over the all-movable vertical tail. This results from the fact that the hinge moment and consequently servo-power requirement is minimized and that only a portion of the vertical tail surface is needed to provide the desired control. In general, detailed analysis of the dynamics of the missile about both the yaw and roll axes must be performed before the exact requirement for direction control can be established.

SYMBOLS

C_m	pitching-moment coefficient
C_n	yawing-moment coefficient
C_y	side-force coefficient

S	body frontal (reference) area
V	resultant forward velocity
$\bar{X}$	resultant center of pressure
YM	yawing moment
d	body diameter
i	angle between the longitudinal axis of the missile and the resultant wind
i'_{TR}	trim angle of attack for a unit load-factor length
l	length
u	velocity along the X axis
v	velocity along the Y axis
x	distance between missile center of gravity and center of pressure
α	angle of attack
β	angle of sideslip
δ	control-surface deflection
δ'	control-surface deflection for unit load factor in a single plane
σ	sidewash angle
φ	angle between reference plane and plane of resultant wind
ψ	yaw angle

Subscripts

B	body
CM	complete model
T	tail
p	pitch
y	yaw

REFERENCES

1. Harmon, S. M.: Stability Derivatives of Thin Rectangular Wings at Supersonic Speeds. Wing Diagonals ahead of Tip Mach Lines, *NACA Rept.* 925, 1949.
2. Ribner, H. S.: The Stability Derivatives of Low-aspect-ratio Triangular Wings at Subsonic and Supersonic Speeds, *NACA Tech. Note* 1423, 1947.
3. Ribner, H. S., and F. S. Malvestuto, Jr.: Stability Derivatives of Triangular Wings at Supersonic Speeds, *NACA Rept.* 908, 1948.
4. Moeckel, W. E.: Effect of Yaw at Supersonic Speeds on Theoretical Aerodynamic Coefficients of Thin Pointed Wings with Several Types of Trailing Edge, *NACA Tech. Note* 1549, 1948.
5. Jones, A. L., and A. Alksne: A Summary of Lateral-stability Derivatives Calculated for Wing Plan Forms in Supersonic Flow, *NACA Rept.* 1052, 1949.
6. Love, E. S.: Supersonic Wave Interference Affecting Stability, *NACA Tech. Note* 4358, September, 1958.

CHAPTER 8

LATERAL STABILITY AND CONTROL

8-1. INTRODUCTION

Since there is no natural static stability about the lateral axis such as that associated with either the longitudinal or directional axis, intelligent inputs, such as those from the human pilot in the airplane or the automatic pilot or control-system sensor in the missile, are necessary to control the lateral attitude and/or motion of the airframe. The absence of the human pilot from the missile airframe-control system has to a certain extent simplified the problem of lateral stability and control. Hence the problem is one which deals primarily with the determination of the nature and magnitude of the induced rolling moments and lateral-control effectiveness of the missile. The following sections present some of the more important design aspects with regard to realizing acceptable lateral-stability and control characteristics for both cruciform and monowing configuration.

8-2. INDUCED ROLL—CRUCIFORM

Perhaps the one most difficult parameter to determine accurately is the induced rolling-moment characteristics of a cruciform missile. These rolling moments arise whenever the missile simultaneously executes pitch and yaw maneuvers which are unequal in magnitude. Such maneuvers result in unequal or asymmetric flow patterns over the aerodynamic lifting surfaces, and consequently rolling moments are "induced" on the airframe. An illustration of the general effects causing induced rolling moments is presented in Fig. 8-1 for a wing-control configuration. It can be seen that these effects may be divided into four general classes: (1) tip effect, (2) root effect, (3) body-blanketing effect, and (4) downwash effect. Of these, the most important effect results from the downwash on the tail surfaces. Because of the complex nature of the flow pattern under combined

maneuvers, any attempt to evaluate these rolling moments must, by necessity, be only approximate and quite lengthy. Hence, in most preliminary design analyses, the determination of this parameter is omitted, since it is most conveniently evaluated in the wind tunnel. However, the designer should bear in mind the importance of these induced rolling moments in the determination of a satisfactory and adequate lateral-control system.

8-3. LATERAL CONTROL—CRUCIFORM

The exact magnitude of the lateral-control effectiveness required is not easily determined since it depends chiefly on the magnitude of the induced rolling moments. However, from the standpoint of control-system simplification, it is desirable to provide sufficient lateral control to overcome the aerodynamic induced rolling moments and also those due to surface malalignments encountered throughout the entire missile flight regime of speed and angle of attack and yaw. However, in some cases, it may be necessary to incorporate a more complicated lateral-control system to compensate for a less efficient aerodynamic lateral control in order to simplify the pitch-and-yaw-control systems and consequently result in a more optimum over-all missile system. Such a case may be one in which the main movable wing (or tail) surfaces are used for both pitch or yaw and roll control in lieu of a separate set of lateral-control surfaces such as ailerons or tip controls.

In the selection of a lateral-control system the following important design considerations must be taken into account: (1) control effectiveness, (2) hinge moments, and (3) complexity in terms of structural design and control-system design. For a canard configuration (see Fig. 2-2), the canard surfaces are completely inadequate for lateral control primarily because of (1) the small surface area and its short moment arm and (2) the induced rolling moments from the aft surfaces

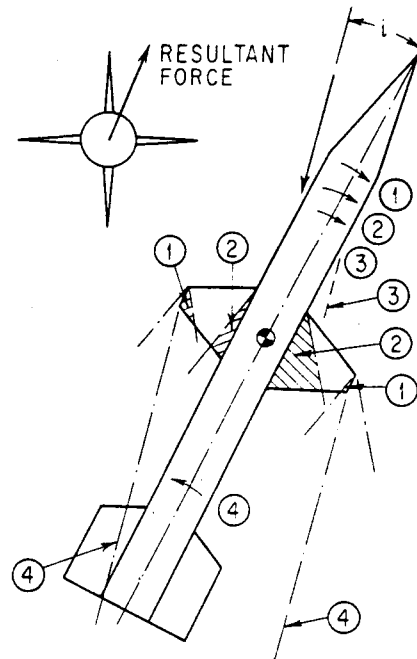


FIG. 8-1. Effects causing induced roll. (1) Tip effect. (2) Root effect. (3) Blanketing effect. (4) Downwash effect.

due to downwash effects. For this type of design, a separate set of lateral controls such as wing-tip ailerons is needed. For a wing-control configuration the movable wing surfaces may be used as lateral-control devices if proper design is realized. As illustrated in Fig. 8-2, the opposing rolling moments from the tail surfaces may be of sufficient magnitude to nullify completely the effectiveness of the wing surfaces even for single-plane maneuvers; hence roll reversal may result. These opposing rolling moments may also be considered as induced rolling moments and are present in addition to those due to combined maneuvers previously described. By proper design, adequate lateral-control effectiveness may be realized from the movable wings. These design considerations will be discussed in a latter section. For a tail-control configuration, the lateral-control effectiveness obtained by differential tail deflections may be marginal because of (1) the downwash from the wing due to angle of attack and (2) the relatively short moment arm for producing rolling moments.

The magnitude of the lateral-control effectiveness obtained by differential wing or tail deflection is also difficult to determine because of the effect of downwash. This is particularly so for combined maneuvering conditions where knowledge of the magnitude of the lateral-control effectiveness (as well as the induced rolling moments) is most desirable. Hence its determination must also be made in the wind tunnel. For the more simplified lateral-control devices, such as wing-tip ailerons, spoilers, or flaps, sufficient experimental data^{1,2,3,*} may be available to permit a preliminary determination of its control effectiveness.

The adequacy of the lateral-control effectiveness may be determined from wind-tunnel-test results by plotting the rolling-moment coefficient C_l vs. i (defined in Chap. 6) for the complete model configuration with and without lateral-control surface deflections superimposed on various combined maneuvers (i.e., different φ 's) as shown in Fig. 8-3. The "trim" point A is the maximum value of i for which lateral control is adequate to trim out the induced rolling moments. If the magnitude of i thus obtained is greater than that anticipated in the required combined pitch and yaw maneuver (including the effect of dynamic overshoot), the lateral-control effectiveness is deemed satisfactory. However, if the magnitude of i is less than that anticipated in flight, lateral control appears inadequate. In such a case, several alternatives may be taken: (1) restrict the magnitude of the combined pitch and yaw maneuver, thereby restricting the effectiveness of the missile

* Superscript numbers indicate references listed at the end of the chapter.

weapon system; (2) modify the pitch-and-yaw-control system to reduce the dynamic overshoot of the airframe or i ; and (3) modify the configuration. The selection of one or a combination of these alternatives depends on the degree of lateral-control deficiency.

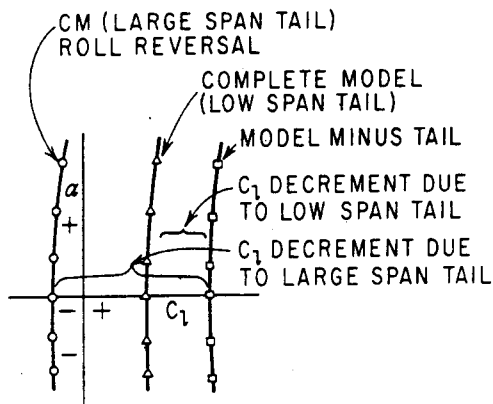


FIG. 8-2. Effect of tail on roll-control effectiveness (single-plane maneuver).

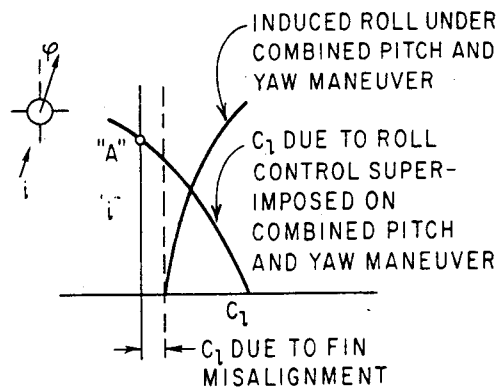


FIG. 8-3. Lateral-control effectiveness when superimposed on combined pitch and yaw maneuver.

8-4. SPECIAL DESIGN CONSIDERATIONS—CRUCIFORM

In view of the complexity of the lateral stability and control problem, certain design practices should be observed during the preliminary or development phase of a missile project in order that satisfactory lateral characteristics may be realized. From the standpoint of keeping the aerodynamic induced rolling moments to a minimum, the span of the tail surface is perhaps the one most important design one must consider. From Fig. 8-1, it is apparent that, for a given wing configuration, the majority of the tail surface is directly affected by the wing downwash. Consequently, a reduction in tail span will materially decrease the roll moment arm and hence the induced rolling moment. The reduction in span has the added advantage that the aspect ratio is reduced, thereby decreasing the lift-curve slope. The latter is, however, a disadvantage from the standpoint of tail effectiveness in that its contribution to the static longitudinal (or directional) stability is also decreased. Hence a relatively large increase in tail area is necessary. The angle of attack should also be kept relatively low in order to avoid the large induced rolling moments generally associated with large values of i , as shown in Fig. 8-3. This may be accomplished by (1) increasing the static longitudinal and directional stability margin and (2) restricting the dynamic overshoot of the angle of attack and/or yaw

by proper control-system design. The former method has the disadvantage in that relatively large aerodynamic surfaces are required to satisfy the maneuverability required (see Sec. 6-4). On the other hand, the second method requires a somewhat more complicated and hence less reliable control system.

From the standpoint of lateral-control effectiveness, separate wing-tip-aileron roll control appears to be the most effective. However, structural and control-system design is somewhat more involved. The hinge moments associated with this type of control appear to be reasonably low. The use of the main wing surfaces for roll control may be acceptable, provided that proper tail design is realized. The hinge line for this type of control must be carefully selected to minimize the hinge moments. The use of the tail surfaces for roll control is generally avoided since it is generally marginal because of its small area and small roll moment arm. For tail-control design (see Fig. 2-2) a separate set of lateral controls on the wing tip is therefore more acceptable.

8-5. DAMPING IN ROLL

In the study of the (dynamic) lateral motion of the missile, the damping-in-roll characteristics must be known. This damping term arises primarily from lift and moment from the wing (or tail) surfaces whenever the missile develops a rolling velocity. The effect of such a rolling velocity is to induce an angle of attack which varies linearly along the span as shown in Fig. 8-4 and can be expressed as

$$\Delta\alpha \cong \frac{v_t}{V} = \frac{py}{V} \quad (8-1)$$

where $\Delta\alpha$ = induced angle of attack at point y along span

V_t = tangential velocity (equal to py)

p = rolling velocity, radians/sec

V = missile forward speed

Consequently the damping-in-roll derivative may be derived as follows:

$$\mathcal{L} = -N\bar{y} = \int_{-b/2}^{b/2} (C_{N_\alpha})_1 \Delta\alpha yqc dy \quad (8-2)$$

where $\mathcal{L}$ = rolling moment

N = normal force

$\bar{y}$ = resultant roll moment arm

$(C_{N_\alpha})_1$ = section normal-force coefficient

c = local chord at span y (see Fig. 8-5)

By using common missile terminology, the rolling-moment coefficient C_l is defined as

$$C_l = \frac{\mathcal{L}}{qS_\pi d} \quad (8-3)$$

Defining C_{l_p} as $\partial C_l / \partial (pd/2V)$ and combining Eqs. (8-2) and (8-3), we get

$$C_{l_p} = \frac{2}{S_\pi d^2} \int_{-b/2}^{b/2} (C_{N_\alpha})_1 cy \, dy \quad (8-4)$$

The magnitude of the damping in roll depends primarily on the aspect ratio, planform, and Mach number and has been determined

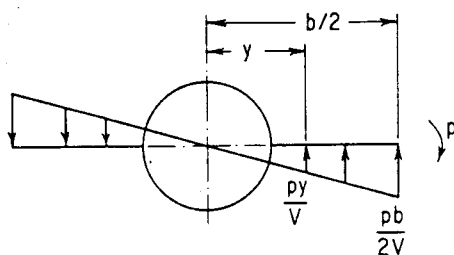


FIG. 8-4. Spanwise variation of induced angle of attack caused by rolling velocity.

both theoretically and experimentally.⁴⁻¹⁶ For preliminary design, the experimental results should be used wherever possible because of the limitations imposed on most of the theoretical treatments. In general, the experimental values of C_{l_p} are somewhat lower than those predicted by theory and may be attributed to such effects as wing thickness, body interference, and possible aeroelasticity.

In the conversion of either the theoretical or experimental results, extreme care must be exercised to assure that proper units or definitions are adhered to. This is particularly important for configurations (i.e., cruciform wing or tail control) which have several aerodynamic surfaces contributing to roll damping. For preliminary design, the damping in roll for four equal wing panels may be assumed to be twice the value of the two panels. This assumes no mutual wing interference. The tail contribution may also be assumed to be independent of the wing interference (i.e., downwash effects). However, experimental results indicate that such a method generally yields values

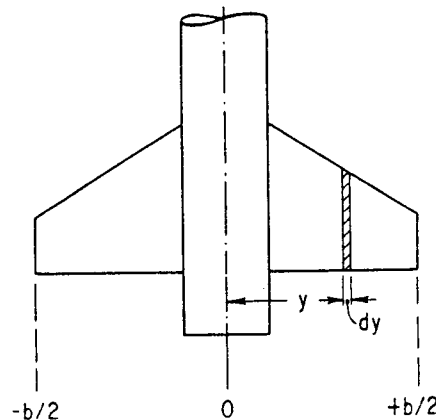


FIG. 8-5. Strip integration for C_{l_p} .

of C_{l_p} which are somewhat larger than the actual value since the downwash decreases the tail contribution to roll damping. For certain applications where a "tight" roll control is required to assure the

desired guidance accuracy, it is desirable to determine the roll-damping term experimentally. This may be accomplished in the wind tunnel with a scaled-down model or by flight-testing a full-scale model.

8-6. INDUCED ROLL—MONOWING

The induced-roll problem for the monowing configuration is substantially less severe than that associated with the cruciform configuration previously described. This results primarily from the following differences in the basic design of the monowing and cruciform configurations: (1) the omission of one set of wings or control surfaces for the monowing design, (2) the bank-to-turn feature, and (3) the separate set of lateral controls generally employed on the monowing configuration. The omission of one set of wings results in a drastic reduction in induced rolling moments since the asymmetric flow due to downwash is essentially eliminated. The bank-to-turn feature, however, does result in a nominal amount of flow asymmetry and hence induced rolling moments, the magnitude of which is generally acceptable. Finally, the use of a separate set of lateral-control surfaces such as wing-tip ailerons generally provides adequate roll-control effectiveness over the entire flight regime.

In the analysis of the (dynamic) lateral (and directional) stability characteristics, the rolling moment due to sideslip C_{l_β} is one of the most influential parameters. This factor is often referred to as the effective dihedral effect and has a finite value even for wings with no geometric dihedral angle. For this condition of no geometric dihedral, the rolling moment due to sideslip in supersonic flow is the result of an increase in lift within the Mach cone from the leading wing tip coupled with a corresponding decrease from the trailing wing tip. The magnitude of C_{l_β} has been determined theoretically for various wing planforms^{4,17,18} and may be used for preliminary design analysis. However, these values should be checked by wind-tunnel tests, particularly when geometric dihedral or cathedral (negative dihedral) is used.

Generally speaking, excessive effective dihedral may result in highly oscillatory motion which must be artificially damped by the lateral-control-system servo loop.

8-7. LATERAL CONTROL—MONOWING

As previously mentioned, a separate set of lateral-control surfaces is generally used in a monowing design. This stems from the fact that

the canards or tail surfaces which are usually employed for pitch control on this type of design are generally inadequate for lateral control. The most common types of lateral control used are (1) flaps, (2) spoilers, and (3) wing-tip ailerons. A detailed analysis in terms of their lateral-control effectiveness,¹⁻³ hinge-moment characteristics, and design complexities must be made before an optimum lateral-control configuration can be realized.

SYMBOLS

C_l	rolling-moment coefficient
C_{l_p}	damping-in-roll coefficient
$(C_{N_x})_1$	section normal-force coefficient
$\mathcal{L}$	rolling moment
N	normal force
S_π	body frontal (reference) area
V	forward speed
c	local chord
d	body diameter
p	rolling velocity
q	dynamic pressure
v_t	tangential velocity py
y	spanwise distance measured from center line of missile
$\bar{y}$	resultant spanwise roll moment arm
α	angle of attack
β	angle of sideslip
q	angle between reference plane and plane of resultant wind

REFERENCES

1. Strass, H. K., E. W. Stephens, E. M. Fields, and E. D. Schult: Collection and Summary of Flap-type-aileron Rolling Effectiveness Data at Zero Lift as Determined by Rocket-powered Model Test at Mach Numbers between 0.6 and 1.6, *NACA Research Mem.* L55F14, Sept. 2, 1955.
2. Brewer, J. D.: Description and Bibliography of NACA Research on Wing Controls—January 1946–February 1955, *NACA Research Mem.* 54K24, Mar. 24, 1955.
3. Lowry, J. G.: Recent Control Studies, *NACA Research Mem.* L55L22a, Feb. 16, 1956.
4. Harmon, S. M.: Stability Derivatives at Supersonic Speeds of Thin Rectangular Wings with Diagonals ahead of Tip Mach Lines, *NACA Rept.* 925, 1949 (supersedes *NACA Tech. Note* 1706).
5. Malvestuto, F. S., Jr., K. Margolis, and H. S. Ribner: Theoretical Lift

and Damping in Roll at Supersonic Speeds of Thin Sweptback Tapered Wings with Streamwise Tips, Subsonic Leading Edges, and Supersonic Trailing Edges, *NACA Rept. 970*, 1950 (supersedes *NACA Tech. Note 1860*).

6. Tosti, L. P.: Low-speed Static Stability and Damping-in-roll Characteristics of Some Swept and Unswept Low-aspect-ratio Wings, *NACA Tech. Note 1468*, October, 1947.

7. Ribner, H. S.: Damping in Roll of Cruciform and Some Related Delta Wings at Supersonic Speeds, *NACA Tech. Note 2285*, 1951.

8. Piland, R. O.: Summary of the Theoretical Lift, Damping-in-roll, and Center-of-pressure Characteristics of Various Wing Plan Forms at Supersonic Speeds, *NACA Tech. Note 1977*, October, 1949.

9. Adams, G. J.: Theoretical Damping in Roll and Rolling Effectiveness of Slender Cruciform Wings, *NACA Tech. Note 2270*, January, 1951.

10. Hedgepeth, J. M., and R. J. Kell: Rolling Effectiveness and Aileron Reversal of Rectangular Wings at Supersonic Speeds, *NACA Tech. Note 2067*.

11. Adams, G. J., and D. W. Dugan: Theoretical Damping Roll and Rolling Moment Due to Differential Wing Incidence for Slender Cruciform Wings and Wing-body Combinations, *NACA Tech. Rept. 1088*, 1952.

12. Bleviss, Z. O.: Some Roll Characteristics of Cruciform Delta Wings at Supersonic Speeds, *J. Aeronaut. Sci.*, vol. 18, no. 5, pp. 289-297, May, 1951.

13. Michael, W. H.: Analysis of the Effects of Wing Interference on the Tail Contributions to the Rolling Derivatives, *NACA Rept. 1086*, 1952.

14. Stone, D. G.: A Collection of Data for Zero-lift Damping-in-roll of Wing-body Combinations as Determined with Rocket-powered Models Equipped with Roll-torque Nozzles, *NACA Tech. Note 3955*, April, 1957.

15. Polhamus, E. C.: A Simple Method of Estimating the Subsonic Lift and Damping in Roll of Sweptback Wings, *NACA Tech. Note 1862*, 1949.

16. Bland, W. M., Jr., and A. E. Dietz: Some Effects of Fuselage Interference, Wing Interference, and Sweepback on the Damping in Roll of Untapered Wings as Determined by Techniques Employing Rocket-propelled Vehicles, *NACA Research Mem. L51D25*, 1951.

17. Jones, A. L., J. R. Spreiter, and A. Alksne: The Rolling Moment Due to Sideslip of Triangular, Trapezoidal, and Related Plan Forms in Supersonic Flow, *NACA Tech. Note 1700*, September, 1948.

18. Ribner, H. S., and F. S. Malvestuto, Jr.: Stability Derivatives of Triangular Wings at Supersonic Speeds, *NACA Tech. Note 1572*, May, 1948.

CHAPTER 9

DYNAMIC STABILITY

9-1. INTRODUCTION

Since the accuracy or effectiveness of a guided missile depends greatly on the dynamics of the missile, particularly during the terminal phase of its flight, it is often desirable to predict its flight dynamics in the early preliminary-design phase to assure that a reasonably satisfactory missile configuration is realized. In this chapter, the problems associated with the dynamics of the airframe as well as the pertinent equations of motion are discussed. In addition, derivations of few of the more commonly used transfer equations are included for automatic-control analysis.

A missile is a free body with six degrees of freedom, translational and rotational motions about the X , Y , and Z axes. In addition, with free controls, each freed control surface permits another degree of freedom for the rotation of the control about its hinge line. In general, the control surfaces on missiles are "locked" in place by the control actuators and are not free to move under the influence of inertia or external aerodynamic applied forces or moments. Hence, in most missile dynamic analyses, six degrees of freedom are adequate. The dynamic-stability characteristics can be evaluated by solution of the appropriate set of equations of motion for each degree of freedom. Each of these equations can be treated independently only if motions along and about one axis under consideration do not result in forces or moments about another axis. Such a case may be motions in pure pitch or roll.

In general, the response characteristics of a missile (or any dynamic system), when disturbed from equilibrium, consist of four modes. These modes may be either oscillatory or nonoscillatory (i.e., aperiodic), damped or undamped, as shown in Fig. 9-1. These oscillations correspond to the natures of the roots of the differential equations of motion. This can be illustrated by considering a simple harmonic equation:

$$A\ddot{\theta} + B\dot{\theta} + C\theta = K \quad (9-1)$$

By substituting the operator p to denote differentiation with respect to time p^2 for the second derivative, etc., Eq. (9-1) becomes

$$(p^2A + pB + C)\theta = K \quad (9-2)$$

The term $(p^2A + pB + C)$ equated to zero is called the characteristic equation. The roots of this characteristic equation may be found by the binomial theorem as follows:

$$P_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (9-3)$$

The solution consists of a "characteristic solution" and a "particular solution." The former solution is determined by the roots of the characteristic equation [Eq. (9-3)] whereas the latter solution is determined by the forcing function. For the case illustrated, the latter is K . The complete solution is the sum of the characteristic and particular solution and may be expressed as

$$\theta = a_1 e^{p_1 t} + a_2 e^{p_2 t} + K \quad (9-4)$$

where a_1 and a_2 are determined by the initial conditions.

From Eq. (9-3), the value of the roots p_1 and p_2 may be one of the following possibilities:

1. Two positive real roots
2. Two negative real roots
3. One positive and one negative real root
4. Conjugate complex root with positive real parts
5. Conjugate complex root with negative real parts

It is apparent that a positive real root indicates a divergent motion since $e^{p t}$ [in Eq. (9-4)] increases with time whereas a negative real root indicates a convergent motion since $e^{p t}$ decreases with time. A complex root indicates oscillatory motion. The latter type of motion is damped if the real part of the root is negative and undamped for a positive real root.

A higher-order differential equation may be conveniently factored into quadratic terms similar to those just discussed. Hence a complicated motion consists of many modes which can be superimposed to

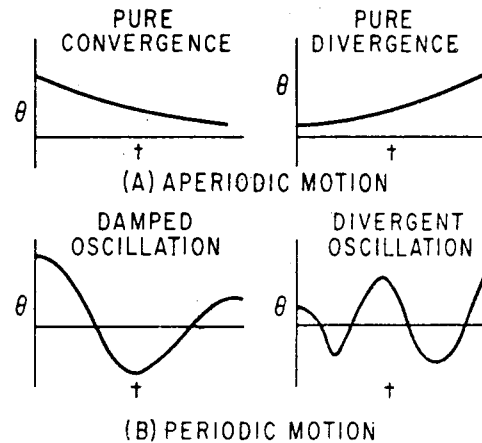


FIG. 9-1. Typical modes of motion.

form the total resultant motion. Such an example is the longitudinal equation of motion which results in a quartic differential equation which can be factored into two quadratics. In this case, two distinct modes are obtained: one short period which is heavily damped and one long period which is very lightly damped and is commonly known as the phugoid motion. These motions will be discussed further in Sec. 9-3.

9-2. EQUATIONS OF MOTION

The equations of motion of a missile with controls fixed may be derived from Newton's second law of motion, which states that the rate of change of momentum of a body is proportional to the summation of forces applied to the body and that the rate of change of the moment of

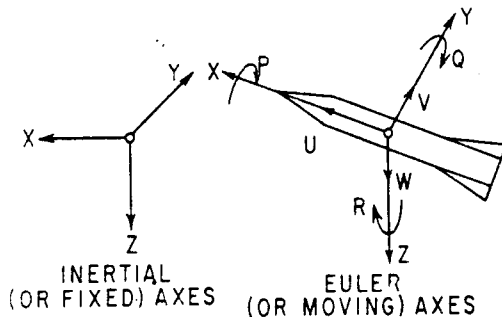


FIG. 9-2. Axes systems.

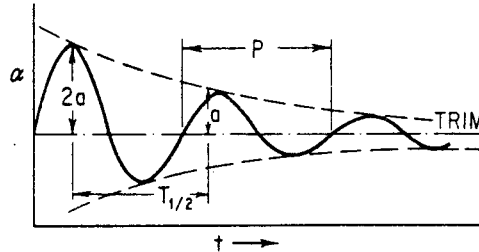


FIG. 9-3. Typical longitudinal short-period oscillation.

momentum is proportional to the summation of moments applied to the body. Mathematically, this law of motion may be written as

$$\begin{aligned}\Sigma F_x &= \frac{d(mU)}{dt} & \Sigma M_x &= \frac{dh_x}{dt} \\ \Sigma F_y &= \frac{d(mV)}{dt} & \Sigma M_y &= \frac{dh_y}{dt} \\ \Sigma F_z &= \frac{d(mW)}{dt} & \Sigma M_z &= \frac{dh_z}{dt}\end{aligned}\quad (9-5)$$

where ΣF_x , ΣF_y , and ΣF_z = summation of forces along x , y , and z axis (see Fig. 9-2)

ΣM_x , ΣM_y , and ΣM_z = summation of moments about the x , y , and z axis

h_x , h_y , and h_z = moments of momentum about x , y , and z axis

The moment of momentum h_x , h_y , and h_z may be written in terms of the

moments of inertia and products of inertia and angular velocities P , Q , and R of the missile as follows:

$$\begin{aligned} h_x &= PI_x - QI_{xy} - RI_{xz} \\ h_y &= QI_y - RI_{yz} - PI_{xy} \\ h_z &= RI_z - PI_{xz} - QI_{yz} \end{aligned} \quad (9-6)$$

Substituting Eq. (9-6) into (9-5) and assuming m is a constant, we get

$$\begin{aligned} \Sigma F_x &= \frac{m dU}{dt} \\ \Sigma F_y &= \frac{m dV}{dt} \\ \Sigma F_z &= \frac{m dW}{dt} \end{aligned} \quad (9-7)$$

$$\begin{aligned} \Sigma M_x &= \dot{P}I_x + P\dot{I}_x - \dot{Q}I_{xy} - Q\dot{I}_{xy} - \dot{R}I_{xz} - R\dot{I}_{xz} \\ \Sigma M_y &= \dot{Q}I_y + Q\dot{I}_y - \dot{R}I_{yz} - R\dot{I}_{yz} - \dot{P}I_{xy} - P\dot{I}_{xy} \\ \Sigma M_z &= \dot{R}I_z + R\dot{I}_z - \dot{P}I_{xz} - P\dot{I}_{xz} - \dot{Q}I_{yz} - Q\dot{I}_{yz} \end{aligned}$$

It is readily apparent that Eqs. (9-7), which are the equations of motion written about an inertial or fixed axis, are extremely cumbersome and must be modified before the motion of the missile can be conveniently analyzed. Hence a moving-axis system called the Eulerian axes is commonly used. This axis system is a right-hand system of orthogonal coordinate axes whose origin is at the center of gravity of the missile and whose orientation is fixed with respect to the missile (see Fig. 9-3). The two main reasons for the use of the Eulerian axes in the dynamic analysis of the airframe are (1) the velocities along these axes are identical to those measured by instruments mounted in the missile and (2) the moments and products of inertia are independent of time. Consequently, Eqs. (9-7) may be rewritten about the Eulerian axes as

$$\begin{aligned} \Sigma F_x &= m(\dot{U} + QW - RV) \\ \Sigma F_y &= m(\dot{V} + RU - PW) \\ \Sigma F_z &= m(\dot{W} + PV - QU) \\ \Sigma M_x &= \dot{P}I_x - \dot{Q}I_{xy} - \dot{R}I_{xz} + QR(I_z - I_y) - PQI_{xz} - Q^2I_{zy} \\ &\quad + R^2I_{yz} + PRI_{xy} \\ \Sigma M_y &= \dot{Q}I_y - \dot{R}I_{yz} - \dot{P}I_{xz} + PR(I_x - I_z) - QRI_{xy} - R^2I_{xz} \\ &\quad + P^2I_{xz} + PQI_{xz} \\ \Sigma M_z &= \dot{R}I_z - \dot{P}I_{xz} + PQ(I_y - I_x) - PRI_{yz} - P^2I_{xy} + Q^2I_{xy} + QRI_{xz} \end{aligned} \quad (9-8)$$

For a monowing configuration, the xz plane is a plane of symmetry. Consequently $I_{yz} = I_{xy} = 0$. Hence Eq. (9-8) may be simplified as follows:

$$\begin{aligned}
 \Sigma F_x &= m(\dot{U} + QW - RV) \\
 \Sigma F_y &= m(\dot{V} + RU - PW) \\
 \Sigma F_z &= m(\dot{W} + PV - QU) \\
 \Sigma M_x &= \dot{P}I_x + QR(I_z - I_y) - I_{xz}(\dot{R} + PQ) \\
 \Sigma M_y &= \dot{Q}I_y + PR(I_x - I_z) + I_{xz}(P^2 - R^2) \\
 \Sigma M_z &= \dot{R}I_z + PQ(I_y - I_x) - I_{xz}(\dot{P} - QR)
 \end{aligned} \tag{9-9}$$

For a cruciform configuration, further simplifications may be made since (1) the xy plane (as well as the xz) is also a plane of symmetry (i.e., $I_{xz} = 0$) and (2) the moment of inertia about the y axis is generally equal to that about the z axis (i.e., $I_y \approx I_z$). Hence the resultant simplified equations are

$$\begin{aligned}
 \Sigma F_x &= m(\dot{U} + QW - RV) \\
 \Sigma F_y &= m(\dot{V} + RV - PW) \\
 \Sigma F_z &= m(\dot{W} + PV - QU) \\
 \Sigma M_x &= \dot{P}I_x \\
 \Sigma M_y &= \dot{Q}I_y + PR(I_x - I_z) \\
 \Sigma M_z &= \dot{R}I_z + PQ(I_y - I_x)
 \end{aligned} \tag{9-10}$$

Detailed derivations of these classical equations of motion may be found in many published works.^{1-6*}

9-3. CLASSICAL SOLUTION—LONGITUDINAL DYNAMICS

Quite frequently it is desirable to evaluate the dynamic characteristics of the airframe alone (with controls locked) in order to determine the type and complexity of the autopilot or control system required to realize a satisfactory complete missile system. The following sections deal with the classical method of analysis for the longitudinal dynamics in two and three degrees of freedom.

1. Two Degrees of Freedom. For this simplified two-degree-of-freedom longitudinal dynamic analysis, Eq. (9-10) reduces to the following expressions:

$$\Sigma F_z = m(\dot{W} - QU) \tag{9-11}$$

$$\Sigma M_y = \dot{Q}I_y \tag{9-12}$$

* Superscript numbers indicate references listed at the end of the chapter.

Since $\dot{W} = U\dot{\alpha}$ and substituting $\dot{\theta}$ for Q , Eqs. (9-11) and (9-12) become

$$\Sigma F_z = mU(\dot{\alpha} - \dot{\theta}) \quad (9-13)$$

$$\Sigma M_y = I_y \ddot{\theta} \quad (9-14)$$

Since $\theta = \alpha + \gamma$ (9-15)

Eq. (9-13) becomes

$$\Sigma F_z = -mU\dot{\gamma} \quad (9-16)$$

For most simplified analyses such as this, it is common to reverse the sign convention by denoting F_z as positive when it is directed upward and also to use V as the missile forward velocity in lieu of U used in the complete six-degree-of-freedom analysis. With these changes made, Eqs. (9-14) and (9-16) become

$$I_y \ddot{\theta} = \Sigma M_y = M_\alpha \alpha + M_\delta \delta + M_{\dot{\theta}} \dot{\theta} + M_{\dot{\alpha}} \dot{\alpha} + M_{\dot{\delta}} \dot{\delta} \quad (9-17)$$

$$mV\dot{\gamma} = \Sigma F_z = N_\alpha \alpha + N_\delta \delta \quad (9-18)$$

Dividing Eq. (9-17) by I_y and Eq. (9-18) by mV , we get

$$\ddot{\theta} = m_\alpha \alpha + m_\delta \delta + m_{\dot{\theta}} \dot{\theta} + m_{\dot{\alpha}} \dot{\alpha} + m_{\dot{\delta}} \dot{\delta} \quad (9-19)$$

and $\dot{\gamma} = f_\alpha \alpha + f_\delta \delta$ (9-20)

where $m_\alpha = M_\alpha/I_y$, $f_\alpha = N_\alpha/mV$, etc.

Equations (9-19) and (9-20) may be solved by using the Heaviside factor, i.e., $\theta = \theta_0 e^{\lambda t}$, $\dot{\theta} = \lambda \theta_0 e^{\lambda t}$, $\ddot{\theta} = \lambda^2 \theta_0 e^{\lambda t}$, etc. Hence, using this factor and factoring out $\theta_0 e^{\lambda t}$ from Eqs. (9-17), (9-19), and (9-20), we get

$$\lambda^2 \theta = m_\alpha \alpha + m_\delta \delta + \lambda m_{\dot{\theta}} \theta + \lambda m_{\dot{\alpha}} \alpha + \lambda m_{\dot{\delta}} \delta \quad (9-21)$$

$$\lambda \gamma = f_\alpha \alpha + f_\delta \delta \quad (9-22)$$

Since the above equations are homogeneous, the determinant of their coefficient is equal to zero. Rearranging the terms, the following determinant is obtained:

$$\begin{vmatrix} 1 & -1 & 1 \\ (m_\alpha + \lambda m_{\dot{\alpha}}) & \lambda(m_{\dot{\theta}} - \lambda) & 0 \\ f_\alpha & 0 & -\lambda \end{vmatrix}$$

By expanding this determinant, the following quadratic is obtained:

$$\lambda[\lambda^2 - (m_{\dot{\theta}} + m_{\dot{\alpha}} - f_\alpha) - (m_\alpha + f_\alpha m_{\dot{\theta}})] = 0 \quad (9-23)$$