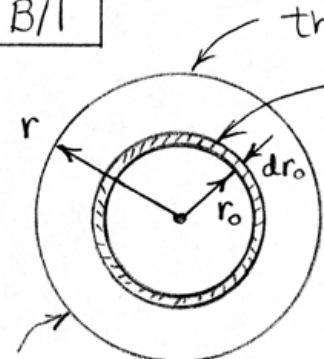


B/1



thickness (depth) = t

$$\begin{cases} dm = \rho dV = \rho 2\pi r_0 dr_0 t \\ dI = dm r_0^2 \\ = 2\pi \rho t r_0^3 dr_0 \end{cases}$$

$$I = \int dI = \int_0^r 2\pi \rho t r_0^3 dr_0^3$$

$$= 2\pi \rho t \frac{r^4}{4} \left(\frac{m}{\pi r^2 t \rho} \right)$$

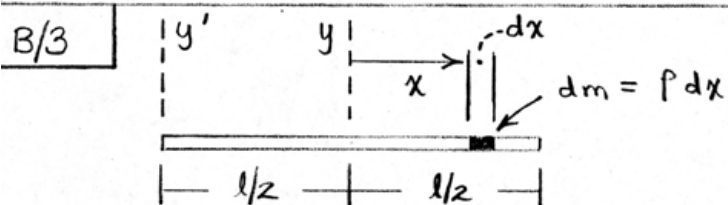
$$= \underline{\underline{\frac{1}{2} m r^2}}$$

$m = \pi r^2 t \rho$

B/2

Complete sphere of mass $2m$, $I_{zz} = \frac{2}{5}(2m)r^2$

For hemisphere $I_{zz} = I_{xx} = \frac{2}{5}mr^2$



$$dI_{yy} = dm x^2 = (\rho dx) x^2 = \rho x^2 dx$$

$$I_{yy} = \int dI_{yy} = \int_{-l/2}^{l/2} \rho x^2 dx = \frac{1}{12} \rho l^3$$

But the total mass $m = \rho l$

$$\text{So } I_{yy} = \frac{1}{12} \rho l^3 \left(\frac{m}{\rho l} \right) = \underline{\underline{\frac{1}{12} m l^2}}$$

$$I_{y'y'} = I_{yy} + m \left(\frac{l}{2} \right)^2 = \underline{\underline{\frac{1}{3} m l^2}}$$

B/4

Conical shell has same radial distribution of mass as does a circular disk of the same mass and radius. Thus

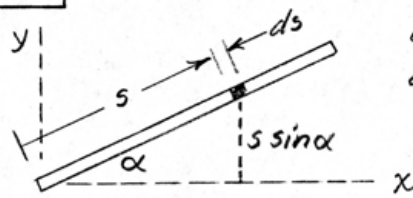
$$\underline{I_{zz} = \frac{1}{2}mr^2}$$

8/5	$\text{error} = \frac{(\frac{1}{2}mr^2 + md^2) - md^2}{\frac{1}{2}mr^2 + md^2} = \frac{1}{1 + 2(d/r)^2}$
-----	--

(a) $d = 10r$, % error $e = \frac{100}{1 + 200} = \underline{0.498\%}$

(b) $d = 2r$, % error $e = \frac{100}{1 + 8} = \underline{11.11\%}$

B/6

Let $p =$ mass per unit length

$$dm = p ds$$

$$dI_{xx} = (s \sin \alpha)^2 dm$$

$$I_{xx} = p \sin^2 \alpha \int_0^b s^2 ds$$

$$= \frac{1}{3} p b^3 \sin^2 \alpha$$

$$= \frac{1}{3} m b^2 \sin^2 \alpha$$

By inspection, $I_{yy} = \frac{1}{3} m b^2 \cos^2 \alpha$

For negligible z -dimensions, $I_{xx} + I_{yy} = I_{zz}$

$$I_{zz} = \frac{1}{3} m b^2 (\sin^2 \alpha + \cos^2 \alpha) = \frac{1}{3} m b^2$$

B/7

Error is
$$\frac{md^2 - (\frac{2}{5}mr^2 + md^2)}{\frac{2}{5}mr^2 + md^2}$$

$$= \frac{-1}{1 + \frac{5}{2}(\frac{d}{r})^2}$$

So percent error $e = \frac{-100}{1 + \frac{5}{2}(\frac{d}{r})^2}$

(a) $\frac{d}{r} = 2$: $|e| = 9.09\%$

(b) $\frac{d}{r} = 10$: $|e| = 0.398\%$

B/8

$$\begin{aligned}\text{Percent error } e &= \frac{\frac{1}{12}ml^2 - (\frac{1}{12}ml^2 + \frac{1}{4}mr^2)}{\frac{1}{12}ml^2 + \frac{1}{4}mr^2}(100) \\ &= \frac{-100}{1 + \frac{1/3}{(r/l)^2}} \quad (\text{in percent})\end{aligned}$$

Values :

r/l	e
0.01	-0.030 %
0.1	-2.91 %
0.5	-42.9 %

B/9

x_0 Table D/3: $I_{xx} = \rho t \left(\frac{bh^3}{12} \right) = \frac{1}{6} m h^2$

$m = \frac{1}{2} \rho h b t$

$h = b \frac{\sqrt{3}}{2}$

G

y_0

$h/3$

y

So $I_{yy} = \frac{1}{6} m \left(b \frac{\sqrt{3}}{2} \right)^2$
 $= \frac{1}{8} m b^2$

$I_{y_0 y_0} = I_{yy} - m \left(\frac{h}{3} \right)^2$
 $= \frac{1}{8} m b^2 - m \frac{b^2}{12}$
 $= \frac{1}{24} m b^2$

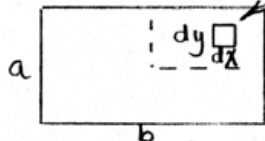
Again from Table D/3:

$$I_{x_0 x_0} = 2 \left(\frac{1}{6} \frac{m}{2} \left(\frac{b}{2} \right)^2 \right) = \frac{1}{24} m b^2$$

$$I_{zz} = I_{x_0 x_0} + I_{y_0 y_0} = \frac{1}{12} m b^2$$

B/10 $m = \rho A = \rho ab$ ($\rho = \text{mass/area}$)

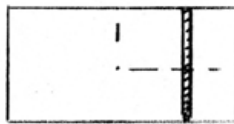
(a) y | $dm = \rho dx dy$



$$I_{xx} = \int (y^2 + z^2) dm = \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} y^2 \rho dy dx$$

$$= \rho \left(\frac{1}{12} a^3 \right) \int_{-\frac{b}{2}}^{\frac{b}{2}} dx = \frac{1}{12} \rho a^3 b \left(\frac{m}{\rho ab} \right) = \frac{1}{12} m a^2$$

(b) y | $dm = \rho a dx$, $dI_{xx} = \frac{1}{12} dm (a^2)$



$$I_{xx} = \int dI_{xx} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{12} \rho a^3 dx$$

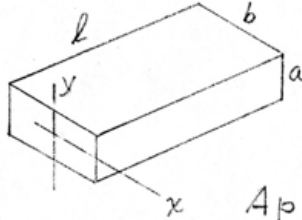
$$= \frac{1}{12} \rho a^3 b \left(\frac{m}{\rho ab} \right) = \frac{1}{12} m a^2$$

Note that method (b) begins in the middle of method (a). By inspection, $I_{yy} = \frac{1}{12} m b^2$

$$I_{zz} = I_{xx} + I_{yy} = \frac{1}{12} m (a^2 + b^2)$$

B/11

$$l = 0.3 \text{ m}, a = 0.03 \text{ m}, b = 0.2 \text{ m}, \bar{m} = 15 \text{ kg}$$



From Sample Problem B/3

$$I_{yy} = \frac{1}{12} m (b^2 + 4l^2)$$

$$I_{xx} = \frac{1}{12} m (a^2 + 4l^2)$$

$$\text{Approx. } I'_{xx} = \frac{1}{3} m l^2$$

$$\text{Fractional error } e = (I_{xx} - I'_{xx}) / I_{xx}$$

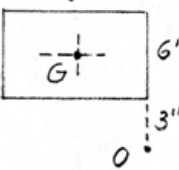
$$= \frac{a^2/12}{a^2/12 + l^2/3} = \frac{1}{1 + (2l/a)^2}$$

$$I_{yy} = \frac{1}{12} (15) (0.2^2 + 4(0.3)^2) = 0.5 \text{ kg} \cdot \text{m}^2$$

$$e = \frac{1}{1 + \left(\frac{2 \times 0.3}{0.03}\right)^2} (100\%) = \frac{100}{401} = 0.25\%$$

B/12

From Sample Problem B/3



$$I_G = \frac{1}{12} m(a^2 + b^2)$$

$$= \frac{1}{12} \frac{6(8)(10)}{1728} \frac{489}{32.2} \left(\left[\frac{6}{12} \right]^2 + \left[\frac{8}{12} \right]^2 \right)$$

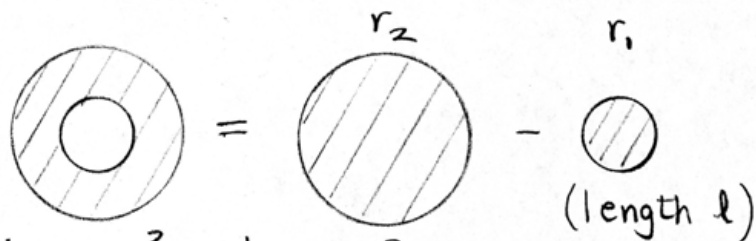
$$= \frac{1}{12} (4.218) \left(\frac{1}{4} + \frac{4}{9} \right) = 4.218 \frac{25}{12(36)}$$

$$\overline{OG} = \sqrt{6^2 + 4^2} = 7.21 \text{ in.}$$

$$I_O = I_G + md^2$$

$$= 4.218 \left[\frac{25}{12(36)} + \left(\frac{7.21}{12} \right)^2 \right]$$

$$I_{OO} = I_O = 4.218 \frac{60.33}{144} = \underline{1.77 \text{ lb-ft-sec}^2}$$



$$\begin{aligned}
 I_{xx} &= \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2 \\
 &= \frac{1}{2} (\rho \pi r_2^2 l) r_2^2 - \frac{1}{2} (\rho \pi r_1^2 l) r_1^2 \\
 &= \frac{1}{2} \rho \pi l (r_2^4 - r_1^4) = \frac{1}{2} \rho \pi l \underbrace{(r_2^2 - r_1^2)}_m (r_2^2 + r_1^2) \\
 &= \underline{\underline{\frac{1}{2} m (r_2^2 + r_1^2)}}
 \end{aligned}$$

B/14

$$I = I_{20} - I_{10} = \frac{1}{2} m_2 r_2^2 - \left[\frac{1}{2} m_1 r_1^2 + m_1 d^2 \right]$$

$$= \rho \pi t \left[\frac{r_2^4}{2} - \frac{r_1^4}{2} - r_1^2 d^2 \right]$$

With $m = \rho \pi t [r_2^2 - r_1^2]$,

$$k_0^2 = \frac{I}{m} = \frac{1}{2} (r_2^2 + r_1^2) - \frac{r_1^2 d^2}{(r_2^2 - r_1^2)}$$

$$= \frac{1}{2} (12^2 + 6^2) - \frac{6^2 4^2}{12^2 - 6^2} = 84.7 \text{ in.}^2$$

$$k_0 = 9.20 \text{ in.}$$

B/15 From Sample Problem B/3, $I_{yy} = \frac{1}{12} m(a^2 + 4l^2)$

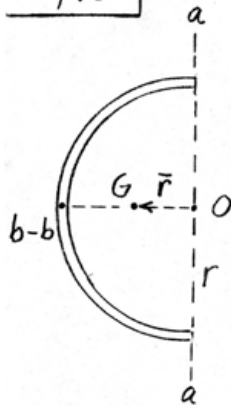
$$m = 1300(0.4)(0.36)(0.10) = 18.72 \text{ kg}$$

$$I_{yy} = \frac{1}{12}(18.72)([0.36]^2 + 4[0.4]^2) = \underline{1.201 \text{ kg}\cdot\text{m}^2}$$

$$\text{For } I_{xx}, \text{ \% error } |e| = \frac{\frac{1}{12} m a^2}{\frac{1}{12} m(a^2 + 4l^2)} 100\% = \frac{100\%}{1 + (2l/a)^2}$$

$$\text{where } a = 0.1 \text{ m}, l = 0.4 \text{ m} \text{ so } |e| = \frac{100\%}{1 + (0.8/0.1)^2} = \underline{1.538\%}$$

B/16



For complete ring of mass $2m$,

$$I_o = (2m)r^2 \quad \& \quad I_{aa} = \frac{1}{2}(2m)r^2$$

So for half ring $I_{aa} = \frac{1}{2}mr^2$

$$I_{bb} = \bar{I} + (r - \bar{r})^2 m$$

$$= I_o - m\bar{r}^2 + (r - \bar{r})^2 m$$

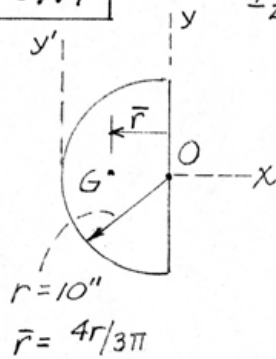
$$= I_o + m(r^2 - 2r\bar{r})$$

$$= mr^2 + m(r^2 - 2r\bar{r})$$

$$= 2mr^2 \left(1 - \frac{\bar{r}}{r}\right) \quad \text{where } \bar{r} = \frac{2r}{\pi}$$

$$\text{So } I_{bb} = 2mr^2 \left(1 - \frac{2}{\pi}\right)$$

8/17



$$I_{zz} = I_O = \frac{1}{2} m r^2 = \frac{1}{2} \frac{5}{32.2} \left(\frac{10}{12} \right)^2 = 0.0539 \text{ lb-ft-sec}^2$$

$$I_{xx} = I_{yy} = \frac{1}{2} I_{zz} = 0.0270 \text{ lb-ft-sec}^2$$

$$I_{yy'} = \bar{I} + m(r - \bar{r})^2$$

$$= I_{yy} - m\bar{r}^2 + m(r - \bar{r})^2$$

$$= I_{yy} + mr(r - 2\bar{r}) = I_{yy} + mr^2 \left(1 - \frac{8}{3\pi} \right)$$

$$= 0.0270 + \frac{5}{32.2} \left(\frac{10}{12} \right)^2 \left(1 - \frac{8}{3\pi} \right)$$

$$= 0.0270 + 0.0163 = 0.0433$$

$$\text{lb-ft-sec}^2$$

B/18

Disk : $I_{zz} = \frac{1}{2}mr^2$, $I_{xx} = \frac{1}{4}mr^2$

Both rods : $I_{zz} = 0$, $I_{xx} = 2\left(\frac{1}{3}\frac{m}{2}L^2\right) = \frac{1}{3}mL^2$

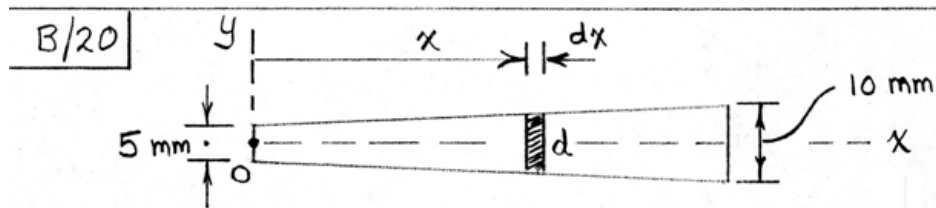
$I_{zz} = I_{xx}$: $\frac{1}{2}mr^2 + 0 = \frac{1}{4}mr^2 + \frac{1}{3}mL^2$

$\frac{3}{4}r^2 = L^2$ or $L = \frac{r\sqrt{3}}{2}$

B/19

Handle mass is ρL ; frame mass is $\rho(\pi \frac{L}{2})$.

$$\begin{aligned} I_{yy} &= \frac{1}{12} \rho L \cdot L^2 + \rho L \left(\frac{3L}{8} \right)^2 + \frac{1}{2} \left(\rho \pi \frac{L}{2} \right) \left(\frac{L}{4} \right)^2 + \rho \pi \frac{L}{2} \left(\frac{9}{8} L \right)^2 \\ &= \left[\frac{43}{192} + \frac{83}{128} \pi \right] \rho L^3 \end{aligned}$$



Diameter $d = 5 + kx$: $10 = 5 + k(100)$, $k = 0.05$

So $d = 5 + 0.05x$

$$dm = \rho dV = \rho \frac{\pi d^2}{4} dx, \quad dI_o = x^2 dm$$

$$= \frac{\rho \pi}{4} (5 + 0.05x)^2 x^2 dx$$

$$I_o = \frac{\rho \pi}{4} \int_0^{100} (25x^2 + 0.5x^3 + 0.0025x^4) dx$$

$$= \frac{\rho \pi}{4} (25.8(10^6)) \text{ kg} \cdot \text{mm}^2$$

With $\rho = 7830 (10^{-9}) \text{ kg/mm}^3$:

$$\underline{I_o = 158.9 \text{ kg} \cdot \text{mm}^2}$$

$$B/21 \quad \text{Rim: } I = \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2 = \frac{1}{2} \rho \pi r_2^4 b - \frac{1}{2} \rho \pi r_1^4 b$$

$$= \frac{1}{2} \rho \pi (r_2^4 - r_1^4) (b)$$

$$= \frac{1}{2} (7830) \pi (0.2^4 - 0.15^4) (0.075)$$

$$= 1.009 \text{ kg} \cdot \text{m}^2$$

$$\text{Hub: } I = \frac{1}{2} \rho \pi (r_2^4 - r_1^4) (b)$$

$$= \frac{1}{2} (7830) \pi (0.05^4 - 0.025^4) (0.12)$$

$$= 0.00865 \text{ kg} \cdot \text{m}^2$$

$$\text{Spokes: } I = 8 \left[\frac{m \ell^2}{12} + m d^2 \right] = 8m \left[\frac{\ell^2}{12} + d^2 \right]$$

$$= 8(7830)(0.1)(200 \times 10^{-6}) \left[\frac{0.1^2}{12} + 0.1^2 \right]$$

$$= 0.01357 \text{ kg} \cdot \text{m}^2$$

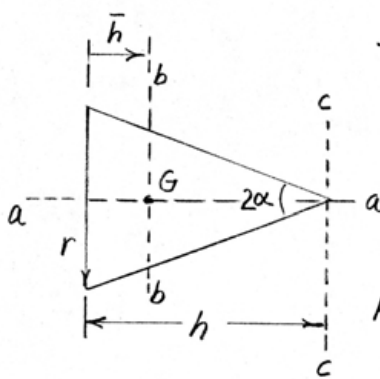
$$\text{Total } I = 1.009 + 0.00865 + 0.01357$$

$$= 1.031 \text{ kg} \cdot \text{m}^2$$

$$n = \frac{1.009}{1.031} (100) = \underline{97.8\%}$$

B/22

$$I_{aa} = \frac{3}{10} mr^2 ; I_{cc} = \frac{3}{20} mr^2 + \frac{3}{5} mh^2 \quad (\text{Table D/4})$$



$$I_{bb} = I_{cc} - m(h - \bar{h})^2, \quad \bar{h} = h/4$$

$$= I_{cc} - m \frac{9}{16} h^2$$

$$= \frac{3}{20} mr^2 + \frac{3}{5} mh^2 - \frac{9}{16} mh^2$$

$$= \frac{3}{20} mr^2 + \frac{3}{80} mh^2$$

$$\text{For } I_{aa} = I_{bb}, \quad \frac{3}{10} mr^2 = \frac{3}{20} mr^2 + \frac{3}{80} mh^2$$

$$\frac{3}{20} r^2 = \frac{3}{80} h^2$$

$$\text{Thus } \left(\frac{r}{h}\right)^2 = \frac{1}{4}, \quad r = \frac{1}{2}h \quad \& \quad \tan \alpha = \frac{r}{h} = \frac{1}{2}, \quad \alpha = 26.6^\circ$$

B/23

For slice of mass dm

$$dI_{zz} = \frac{1}{2} dm (r^2)$$

$$dI_{x'x'} + dI_{y'y'} = dI_{zz}$$

$$\& dI_{x'x'} = \frac{1}{2} dI_{zz}$$

$$= \frac{1}{4} r^2 dm$$

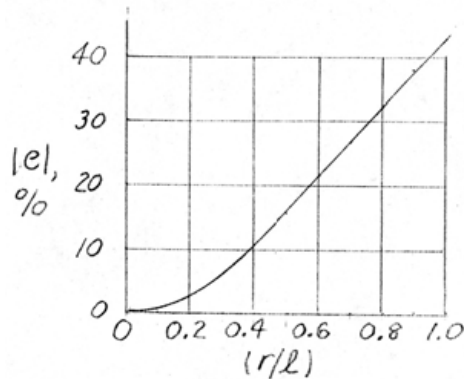
$$dI_{xx} = dI_{x'x'} + z^2 dm$$

So $dI_{xx} = \frac{1}{4} r^2 dm + z^2 dm$ but $dm = \pi r^2 \rho dz$
 where $\rho = \text{density}$

$\& dI_{xx} = \pi r^2 \rho (\frac{r^2}{4} + z^2) dz; I_{xx} = \pi r^2 \rho \int_0^l (\frac{r^2}{4} + z^2) dz$

$$I_{xx} = \pi r^2 \rho (\frac{r^2 l}{4} + \frac{l^3}{3}) = \underline{m (\frac{r^2}{4} + \frac{l^2}{3})}$$

B/24 % error $|e| = \frac{m(\frac{r^2}{4} + \frac{l^2}{3}) - m\frac{l^2}{3}}{m(\frac{r^2}{4} + \frac{l^2}{3})} 100\% = \frac{100\%}{1 + \frac{4/3}{(r/l)^2}}$



For $r/l = 0.2$

error $|e| = \underline{2.91\%}$

B/25

From the results of Prob. B/23 & the transfer theorem

$$I_{xx} = I_{yy} = m\left(\frac{r^2}{4} + \frac{l^2}{3}\right) - m\left(\frac{l}{2}\right)^2 = m\left(\frac{r^2}{4} + \frac{l^2}{12}\right)$$

$$I_{zz} = \frac{1}{2}mr^2$$

$$\text{For } I_{xx} = I_{yy} = I_{zz}, \quad \frac{1}{2}mr^2 = \frac{1}{4}mr^2 + \frac{1}{12}ml^2, \quad \underline{l = r\sqrt{3}}$$

B/26 The mass distribution is essentially the same as that of a cylindrical shell.

From Table D/4: $I_{zz} = mr^2$

$$I_{xx} = I_{yy} = \frac{1}{2}mr^2 + \frac{1}{12}mh^2$$

$$\text{So } I_{zz} = \frac{4}{32.2} \left(\frac{3/2}{12} \right)^2 = \underline{0.001941 \text{ lb-ft-sec}^2}$$

$$\begin{aligned} I_{xx} = I_{yy} &= \frac{0.001941}{2} + \frac{1}{12} \frac{4}{32.2} \left(\frac{10}{12} \right)^2 \\ &= \underline{0.00816 \text{ lb-ft-sec}^2} \end{aligned}$$

B/27

Part 1:

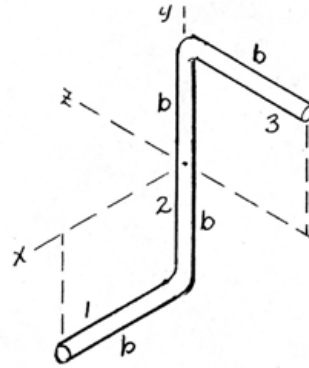
$$m_1 = m/4$$

$$I_{xx} = \frac{m}{4} b^2$$

$$I_{yy} = \frac{1}{3} \frac{m}{4} b^2 = \frac{1}{12} m b^2$$

$$I_{zz} = \frac{1}{12} \frac{m}{4} b^2 + \frac{m}{4} \left[\left(\frac{b}{2} \right)^2 + b^2 \right]$$

$$= \frac{1}{3} m b^2$$



Part 2:

$$m_2 = \frac{1}{2} m$$

$$I_{xx} = \frac{1}{12} \frac{m}{2} (2b)^2 = \frac{1}{6} m b^2$$

$$I_{yy} = 0$$

$$I_{zz} = \frac{1}{6} m b^2$$

Part 3:

$$m_3 = m/4$$

$$I_{xx} = \frac{1}{12} \frac{m}{4} b^2 + \frac{m}{4} \left[\left(\frac{b}{2} \right)^2 + b^2 \right] = \frac{1}{3} m b^2$$

$$I_{yy} = \frac{1}{3} \frac{m}{4} b^2 = \frac{1}{12} m b^2$$

$$I_{zz} = \frac{m}{4} b^2$$

Total:

$$I_{xx} = m b^2 \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{3} \right) = \frac{3}{4} m b^2$$

$$I_{yy} = m b^2 \left(\frac{1}{12} + 0 + \frac{1}{12} \right) = \frac{1}{6} m b^2$$

$$I_{zz} = m b^2 \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4} \right) = \frac{3}{4} m b^2$$

$$\boxed{B/28} \quad \bar{r} = \frac{4r}{3\pi} = \frac{4(100)}{3\pi} = 42.4 \text{ mm}$$

Table D/1: $\rho_{\text{Steel}} = 7830 \text{ kg/m}^3$

$$m = \rho V = 7830 \frac{\pi (0.1)^2}{2} (0.060) = 7.38 \text{ kg}$$

$$I_{oo} = \frac{1}{2} m r^2 = \frac{1}{2} (7.38) (0.1)^2 = 0.0369 \text{ kg} \cdot \text{m}^2$$

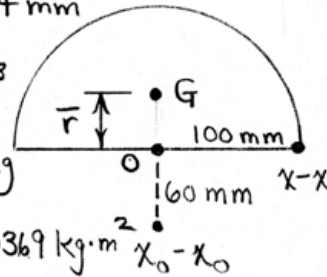
By symmetry, $I_{xx} = I_{oo} + m r^2 = 0.0369 + 7.38 (0.1)^2$
 $= 0.1107 \text{ kg} \cdot \text{m}^2$

$$I_{x_0 x_0} = I_{GG} + m d^2 = (I_{oo} - m \bar{r}^2) + m d^2$$

$$= I_{oo} + m (d^2 - \bar{r}^2)$$

$$= 0.0369 + 7.38 [(0.060 + 0.0424)^2 - 0.0424^2]$$

$$= \underline{0.1010 \text{ kg} \cdot \text{m}^2}$$



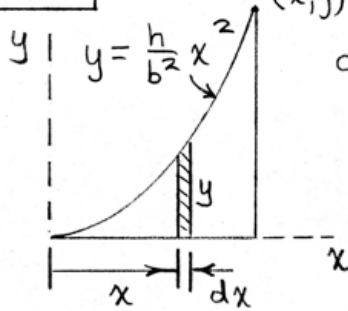
$$\boxed{B/29} \quad I_0 = \frac{1}{3} m l^2 + 7 m x^2 = \underline{m \left(7 x^2 + \frac{1}{3} l^2 \right)}$$

$$\text{For } x = \frac{3}{4} l: I_0 = m \left(7 \cdot \left(\frac{3}{4} l \right)^2 + \frac{1}{3} l^2 \right) = \frac{205}{48} m l^2$$

$$\text{For } x = l: I_0 = m \left(7 l^2 + \frac{1}{3} l^2 \right) = \frac{22}{3} m l^2$$

$$R = \frac{205/48}{22/3} = \underline{0.582}$$

B/30

 $(x, y) = (b, h)$ Plate thickness t 

$$dm = \rho dV = \rho y dx t$$

$$= \rho t \frac{h}{b^2} x^2 dx$$

$$dI_{xx} = \frac{1}{3} dm y^2$$

$$= \frac{1}{3} \rho t \frac{h^3}{b^6} x^6 dx$$

$$dI_{yy} = x^2 dm = \rho t \frac{h}{b^2} x^4 dx$$

$$m = \int dm = \int_0^b \rho t \frac{h}{b^2} x^2 dx = \rho t \frac{h}{b^2} \frac{b^3}{3} = \frac{1}{3} \rho t h b$$

$$I_{xx} = \int dI_{xx} = \int_0^b \frac{1}{3} \rho t \frac{h^3}{b^6} x^6 dx$$

$$= \frac{1}{3} \rho t \frac{h^3}{b^6} \frac{b^7}{7} = \frac{1}{21} \rho t h^3 b \left(\frac{m}{\frac{1}{3} \rho t h b} \right)$$

$$= \underline{\underline{\frac{1}{7} m h^2}}$$

$$I_{yy} = \int dI_{yy} = \int_0^b \rho t \frac{h}{b^2} x^4 dx$$

$$= \rho t \frac{h}{b^2} \frac{b^5}{5} = \frac{1}{5} \rho t h b^3 \left(\frac{m}{\frac{1}{3} \rho t h b} \right) = \underline{\underline{\frac{3}{5} m b^2}}$$

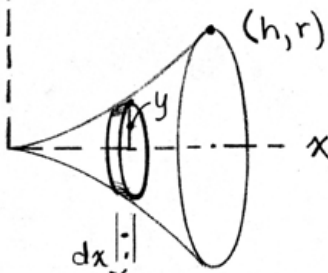
$$I_{zz} = I_{xx} + I_{yy} = \underline{\underline{m \left(\frac{3b^2}{5} + \frac{h^2}{7} \right)}}$$

B/31

y

$$y = kx^2 : r = kh^2 \Rightarrow k = r/h^2$$

$$y = \frac{r}{h^2} x^2$$



$$dm = \rho dV = \rho \pi y^2 dx$$

$$dI_{xx} = \frac{1}{2} dm y^2$$

$$= \frac{1}{2} \rho \pi y^4 dx$$

$$= \frac{1}{2} \rho \pi \frac{r^4}{h^8} x^8 dx$$

$$I_{xx} = \int dI_{xx} = \frac{1}{2} \rho \pi \frac{r^4}{h^8} \int_0^h x^8 dx = \frac{1}{18} \rho \pi r^4 h$$

The mass is $m = \rho V = \int_0^h \rho \pi y^2 dx$

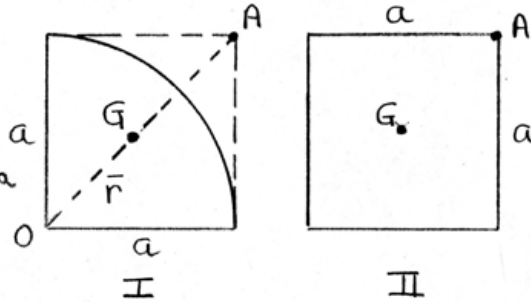
$$= \int_0^h \rho \pi \frac{r^2}{h^4} x^4 dx = \frac{1}{5} \rho \pi r^2 h$$

$$\text{So } I_{xx} = \frac{1}{18} \rho \pi r^4 h \left(\frac{m}{\frac{1}{5} \rho \pi r^2 h} \right) = \frac{5}{18} m r^2$$

B/32

$$I_A = I_{AII} - I_{AI}$$

Let $\rho = \text{mass/unit area}$



For II:

$$I_A = I_G + m \left(\frac{a}{\sqrt{2}} \right)^2 = \frac{1}{6} ma^2 + \frac{1}{2} ma^2 = \frac{2}{3} ma^2$$

$$= \frac{2}{3} (\rho a^2) a^2 = 0.667 \rho a^4$$

For I:

$$I_O = \frac{1}{2} ma^2, \quad I_G = I_O - m \bar{r}^2 = \frac{1}{2} ma^2 - m \left(\frac{4a}{3\pi} \sqrt{2} \right)^2$$

$$= 0.1397 ma^2$$

$$I_A = I_G + m \left(a\sqrt{2} - \frac{4a}{3\pi} \sqrt{2} \right)^2 = 0.802 ma^2$$

$$= 0.802 \left(\frac{1}{4} \pi a^2 \rho \right) a^2 = 0.630 \rho a^4$$

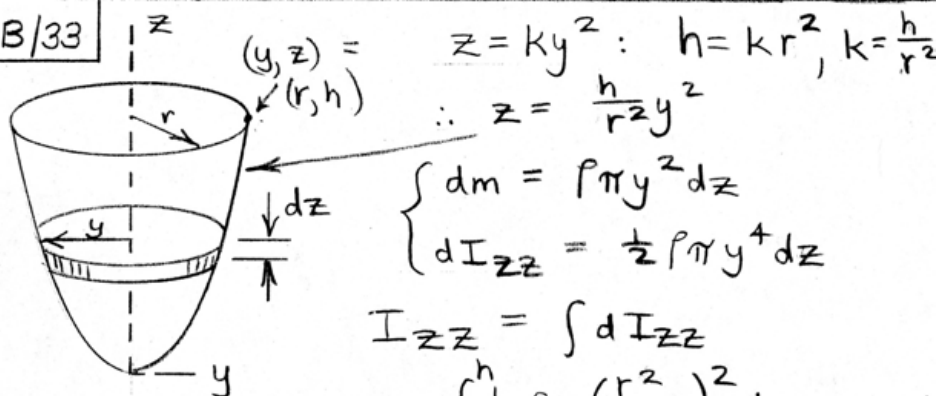
$$\text{So overall, } I_A = 0.667 \rho a^4 - 0.630 \rho a^4$$

$$= 0.0365 \rho a^4$$

$$\text{But } m = \rho \left(a^2 - \frac{1}{4} \pi a^2 \right) = 0.215 \rho a^2$$

$$\text{So } I_A = 0.0365 \rho a^4 \left(\frac{m}{0.215 \rho a^2} \right) = \underline{0.1701 ma^2}$$

B/33



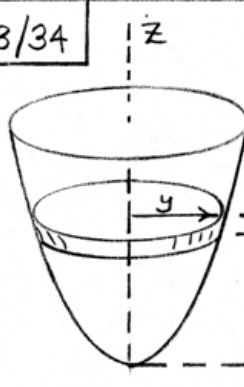
$$I_{zz} = \frac{1}{2} \rho \pi \frac{r^4}{h^2} \cdot \frac{h^3}{3} = \frac{1}{6} \rho \pi r^4 h$$

$$\begin{aligned}
 m &= \rho V = \rho \int_0^h \pi y^2 dz = \rho \pi \int_0^h \frac{r^2}{h} z dz \\
 &= \rho \pi \frac{r^2}{h} \frac{h^2}{2} = \frac{1}{2} \rho \pi r^2 h
 \end{aligned}$$

$$\text{So } I_{zz} = \frac{1}{6} \rho \pi r^4 h \left(\frac{m}{\frac{1}{2} \rho \pi r^2 h} \right) = \frac{1}{3} m r^2$$

$$k_z = \sqrt{\frac{I_{zz}}{m}} = \frac{r}{\sqrt{3}}$$

B/34



$$z = \frac{h}{r^2} y^2$$

$$dm = \rho \pi y^2 dz$$

$$dI_{yy} = dI_{y'y'} + dm z^2$$

$$= \frac{1}{4} dm y^2 + dm z^2$$

$$= dm \left(\frac{1}{4} \frac{r^2}{h} z + z^2 \right)$$

$$= \rho \pi \frac{r^2}{h} z dz \left(\frac{1}{4} \frac{r^2}{h} z + z^2 \right)$$

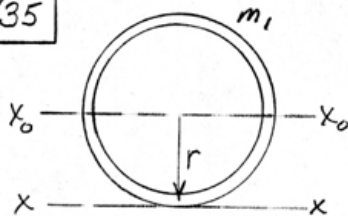
$$= \rho \pi \frac{r^2}{h} \left(\frac{1}{4} \frac{r^2}{h} z^2 + z^3 \right) dz$$

$$\begin{aligned} I_{yy} &= \int dI_{yy} = \rho \pi \frac{r^2}{h} \int_0^h \left(\frac{1}{4} \frac{r^2}{h} z^2 + z^3 \right) dz \\ &= \frac{\rho \pi r^2 h}{4} \left(\frac{r^2}{3} + h^2 \right) \end{aligned}$$

From solution to Prob. B/33, $m = \frac{1}{2} \rho \pi r^2 h$

$$\begin{aligned} \text{So } I_{yy} &= \frac{\rho \pi r^2 h}{4} \left(\frac{r^2}{3} + h^2 \right) \left(\frac{m}{\frac{1}{2} \rho \pi r^2 h} \right) \\ &= \underline{\underline{\frac{1}{2} m \left(h^2 + \frac{r^2}{3} \right)}} \end{aligned}$$

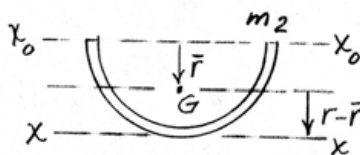
B/35



Full ring:

$$I_{xx} = I_{x_0 x_0} + m_1 r^2$$

$$= \frac{1}{2} m_1 r^2 + m_1 r^2 = \underline{\underline{\frac{3}{2} m_1 r^2}}$$

Half ring: $\bar{r} = 2r/\pi$

$$I_{x_0 x_0} = \frac{1}{2} \left(\frac{1}{2} 2m_2 r^2 \right) = \frac{1}{2} m_2 r^2$$

$$I_G = \frac{1}{2} m_2 r^2 - m_2 \bar{r}^2$$

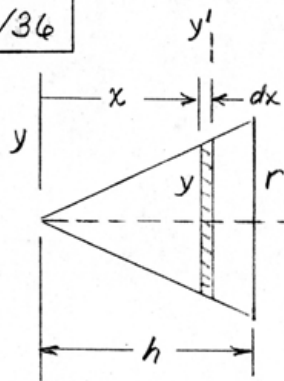
$$= \frac{1}{2} m_2 r^2 - m_2 \frac{4r^2}{\pi^2}$$

$$I_{xx} = I_G + m_2 (r - \bar{r})^2 = \frac{1}{2} m_2 r^2 - m_2 \frac{4r^2}{\pi^2} + m_2 r^2 \left(1 - \frac{2}{\pi} \right)^2$$

$$= \frac{3}{2} m_2 r^2 + m_2 r^2 \left(1 - \frac{4}{\pi} + \frac{4}{\pi^2} \right) - m_2 r^2 \frac{4}{\pi^2}$$

$$= \underline{\underline{m_2 r^2 \left(\frac{3}{2} - \frac{4}{\pi} \right)}}$$

B/36



$$y = \frac{r}{h} x;$$

$$dI_{xx} = \frac{1}{2} dm (y^2) = \frac{1}{2} (\pi y^2 \rho dx) y^2$$

$$= \frac{\pi}{2} \rho \frac{r^4}{h^4} x^4 dx$$

$$I_{xx} = \frac{\pi}{2} \rho \frac{r^4}{h^4} \int_0^h x^4 dx$$

$$= \frac{\pi}{10} \rho \frac{r^4}{h^4} h^5 = \frac{\pi}{10} \rho h r^4$$

$$\text{But } m = \frac{1}{3} \pi r^2 h \text{ so } I_{xx} = \frac{3}{10} m r^2$$

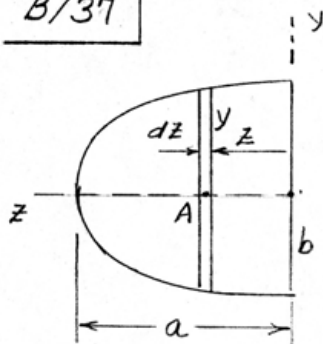
$$dI_{yy} = dI_{y'y'} + x^2 dm = \frac{1}{4} dm y^2 + x^2 dm = \left(\frac{y^2}{4} + x^2 \right) dm$$

$$= \left(\frac{1}{4} \frac{r^2}{h^2} + 1 \right) x^2 \rho \pi y^2 dx = \left(\frac{r^2}{4h^2} + 1 \right) \rho \pi \frac{r^2}{h^2} x^4 dx$$

$$I_{yy} = \frac{\rho \pi r^2}{h^2} \left(\frac{r^2}{4h^2} + 1 \right) \int_0^h x^4 dx = \frac{\rho \pi r^2 h^3}{5} \left(\frac{r^2}{4h^2} + 1 \right)$$

$$I_{yy} = \frac{3}{5} m \left(\frac{r^2}{4} + h^2 \right)$$

B/37



$$dI_{x'x'} = \frac{1}{4} dm y^2$$

$$= \frac{1}{4} \rho \pi y^2 dz y^2$$

$$= \frac{1}{4} \rho \pi y^4 dz$$

$$\frac{y^2}{b^2} + \frac{z^2}{a^2} = 1, \text{ so } y^2 = b^2 \left(1 - \frac{z^2}{a^2}\right)$$

$$\text{Thus } dI_{xx} = dI_{x'x'} + z^2 dm$$

$$dI_{xx} = \frac{1}{4} \rho \pi y^4 dz + (\rho \pi y^2 dz) z^2$$

$$= \rho \pi \left(\frac{1}{4} b^4 \left[1 - \frac{z^2}{a^2}\right]^2 + b^2 z^2 \left[1 - \frac{z^2}{a^2}\right] \right) dz$$

$$I_{xx} = \int_0^a \rho \pi b^2 \left[\frac{b^2}{4} + \left(1 - \frac{b^2}{2a^2}\right) z^2 + \left(\frac{b^2}{4a^2} - 1\right) \frac{z^4}{a^2} \right] dz$$

$$= \rho \pi b^2 \left[\frac{ab^2}{4} + \frac{a^3}{3} \left(1 - \frac{b^2}{2a^2}\right) + \frac{a^5}{5} \left(\frac{b^2}{4a^2} - 1\right) \right] = \frac{2}{15} \rho \pi ab^3 (a^2 + b^2)$$

$$\text{But } m = \int \rho dV = \rho \int \pi y^2 dz = \rho \pi \int_0^a b^2 \left(1 - \frac{z^2}{a^2}\right) dz = \frac{2}{3} \rho \pi b^2 a$$

$$\text{so } I_{xx} = \frac{1}{5} m (a^2 + b^2)$$

B/38

Axis 1-1; Shell; $I_{1-1} = mr^2 = \rho 2\pi r l (r^2) = 2\pi \rho r^3 l$ where $\rho = \text{mass/unit area}$

$$\text{Panels; } I_{1-1} = 2 \left\{ \frac{1}{12} m (2r)^2 + m (2r)^2 \right\}$$

$$= 8\rho (2rl) \left\{ \frac{r^2}{12} + r^2 \right\} = \frac{52}{3} \rho r^3 l$$

$$\text{Total } I_{1-1} = \rho r^3 l (2\pi + 52/3) = 23.62 \rho r^3 l$$

$$\text{Axis 2-2; Shell; } I_{2-2} = \frac{1}{2} m (r^2 + \frac{1}{6} l^2) = \frac{1}{2} \rho (2\pi r l) (r^2 + \frac{l^2}{6})$$

$$\text{Panels; } I_{2-2} = 2 \left\{ \frac{1}{12} m l^2 \right\} = \frac{1}{6} \rho (2rl) l^2 = \frac{1}{3} \rho r l^3$$

$$\text{Total } I_{2-2} = \rho r l \left(\pi \left[r^2 + \frac{l^2}{6} \right] + \frac{1}{3} l^2 \right) = \rho r l \left(l^2 \left[\frac{\pi}{6} + \frac{1}{3} \right] + \pi r^2 \right)$$

$$\text{For critical condition } I_{1-1} = I_{2-2}$$

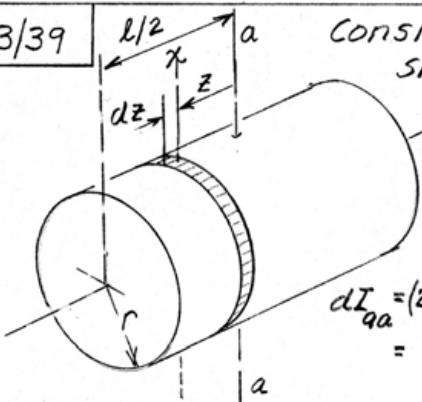
$$\text{so } 23.62 \rho r^3 l = \rho r l (0.857 l^2 + \pi r^2)$$

$$0.857 l^2 = (23.62 - \pi) r^2$$

$$l^2 = 23.89 r^2, \quad l = 4.89 r$$

$$I_{1-1} < I_{2-2} \text{ if } \underline{l > 4.89 r}$$

B/39



Consider complete cylindrical shell with mass ρ per unit area

$$dI_{xx} = \frac{1}{2} dm r^2$$

$$dI_{aa} = dI_{xx} + dm (z^2)$$

$$dI_{aa} = (2\pi r \rho dz) \left(\frac{1}{2} r^2 + z^2 \right)$$

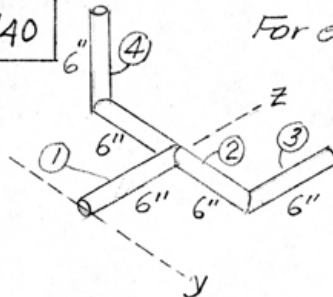
$$= \pi \rho r (r^2 + 2z^2) dz$$

$$I_{aa} = \pi \rho r \int_{-l/2}^{l/2} (r^2 + 2z^2) dz = \pi \rho r \left[r^2 z + \frac{2z^3}{3} \right]_{-l/2}^{l/2}$$

$$= 2\pi \rho r l \left(\frac{r^2}{2} + \frac{l^2}{12} \right)$$

For half shell of mass $m = \pi \rho r l$, $I_{aa} = \frac{m}{2} \left(r^2 + \frac{l^2}{6} \right)$

B/40



For each 6" segment

$$m = \frac{0.30}{32.2} = 0.00932 \text{ lb-ft}^{-1}\text{-sec}^2$$

Let $b = 6\text{-in. length}$

$$\textcircled{1} I_{yy} = \frac{1}{3} m l^2 = \frac{1}{3} m b^2$$

$$\textcircled{2} I_{yy} = (2m) b^2 = 2m b^2$$

$$\textcircled{3} I_{yy} = \frac{1}{12} m b^2 + m \left(b + \frac{b}{2} \right)^2 = \frac{7}{3} m b^2$$

$$\textcircled{4} I_{yy} = \frac{1}{12} m b^2 + m \left(b^2 + \left[\frac{b}{2} \right]^2 \right) = \frac{4}{3} m b^2$$

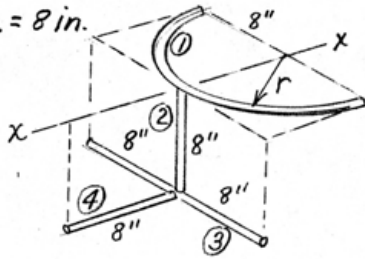
$$\text{Thus } I_{yy} = m b^2 \left(\frac{1}{3} + 2 + \frac{7}{3} + \frac{4}{3} \right) = 6 m b^2 = 6 (0.00932) \left(\frac{6}{12} \right)^2$$

$$= \underline{0.01398 \text{ lb-ft-sec}^2}$$

B/41

$$\mu = \text{weight/in.} = \frac{0.667}{12} = 0.0556 \text{ lb/in.}$$

$$r = a = 8 \text{ in.}$$



mass per unit length is

$$\rho = \frac{0.0556}{32.2(12)} = 1.438(10^{-4}) \frac{\text{lb-sec}^2}{\text{in.}^2}$$

$$m_1 = \pi r \rho = \pi a \rho$$

$$m_2 = a \rho$$

$$m_3 = 2a \rho$$

$$m_4 = a \rho$$

$$\textcircled{1} \quad I_{xx} = \frac{1}{2} m_1 r^2 = \frac{1}{2} \pi a \rho r^2 = \frac{1}{2} \pi \rho a^3$$

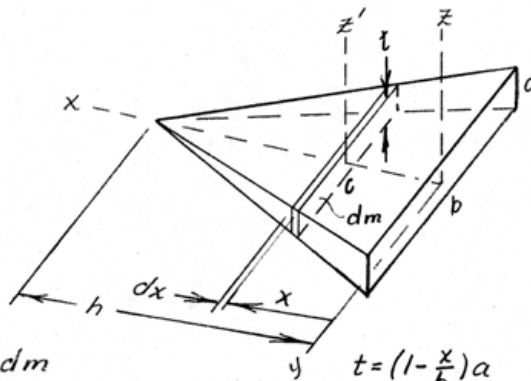
$$\textcircled{2} \quad I_{xx} = \frac{1}{3} m_2 a^2 = \frac{1}{3} \rho a^3$$

$$\textcircled{3} \quad I_{xx} = \frac{1}{12} m_3 (2a)^2 + m_3 a^2 = \frac{4}{3} (2a \rho) a^2 = \frac{8}{3} \rho a^3$$

$$\textcircled{4} \quad I_{xx} = m_4 a^2 = \rho a^3$$

$$\text{Total } I_{xx} = \rho a^3 \left(\frac{\pi}{2} + \frac{1}{3} + \frac{8}{3} + 1 \right) = 5.57 \rho a^3$$

$$= 5.57 (1.438)(10^{-4})(8^3) = \underline{0.410 \text{ lb-in.-sec}^2}$$



For small a , $I_{yy} = \int x^2 dm$

$$t = \left(1 - \frac{x}{h}\right)a$$

$$c = \left(1 - \frac{x}{h}\right)b$$

$$dm = \rho dV = \rho t c dx = \rho \left(1 - \frac{x}{h}\right)a \left(1 - \frac{x}{h}\right)b dx$$

$$= \rho ab \left(1 - \frac{x}{h}\right)^2 dx, \quad \rho = \text{density}$$

$$m = \rho ab \int_0^h \left(1 - \frac{x}{h}\right)^2 dx = \rho ab \left(x - \frac{x^2}{h} + \frac{x^3}{3h^2}\right)_0^h = \frac{1}{3} \rho abh$$

$$I_{yy} = \rho ab \int_0^h \left(1 - \frac{x}{h}\right)^2 x^2 dx = \rho ab \left(\frac{x^3}{3} - \frac{2x^4}{4h} + \frac{x^5}{5h^2}\right)_0^h = \frac{1}{30} \rho abh^3,$$

$$\underline{I_{yy} = \frac{1}{10} m h^2}$$

B/43 From the figure in the solution for Prob. B/42 & the expression for dm , $\rho ab(1-\frac{x}{h})^2 dx$, the moment of inertia of dm about the z -axis is $dI_{zz} = dI_{z'z'} + x^2 dm$ by the transfer-of-axis theorem. Also, from the results of Prob. B/42 or Table D/4

$$dI_{z'z'} = \frac{1}{12} dm c^2 = \frac{1}{12} dm \left(1 - \frac{x}{h}\right)^2 b^2$$

$$\text{so } dI_{zz} = \left[\frac{1}{12} b^2 \left(1 - \frac{x}{h}\right)^2 + x^2 \right] dm = \rho ab \left[\frac{1}{12} b^2 \left(1 - \frac{x}{h}\right)^4 + x^2 \left(1 - \frac{x}{h}\right)^2 \right] dx$$

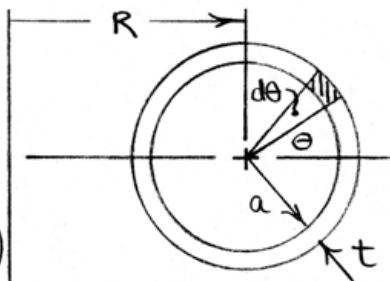
$$I_{zz} = \rho ab \int_0^h \left[\frac{1}{12} b^2 \left(1 - \frac{x}{h}\right)^4 + x^2 \left(1 - \frac{x}{h}\right)^2 \right] dx$$

$$= \rho ab \left[\frac{b^2 h}{60} + \frac{h^3}{30} \right] = \frac{\rho ab h}{30} \left(\frac{b^2}{2} + h^2 \right)$$

$$\& \text{ from solution to Prob. B/42, } m = \frac{1}{3} \rho ab h, \text{ so } \underline{I_{zz} = \frac{1}{10} m \left(\frac{b^2}{2} + h^2 \right)}$$

B/44

Let ρ = mass per unit surface area. For elemental ring of cross section $a d\theta(t)$



and circumference $2\pi(R+a \cos \theta)$:

$$dm = \rho (a d\theta) 2\pi (R+a \cos \theta)$$

$$dI = (R+a \cos \theta)^2 dm = 2\pi \rho (R+a \cos \theta)^3 a d\theta$$

$$\text{So } m = 2\pi \rho a \int_0^{2\pi} (R+a \cos \theta) d\theta = 4\pi^2 \rho a R$$

$$I = 2\pi \rho a \int_0^{2\pi} (R^3 + 3R^2 a \cos \theta + 3R a^2 \cos^2 \theta + a^3 \cos^3 \theta) d\theta$$

$$= 2\pi \rho a [\textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4}]$$

$$\textcircled{1} = \int_0^{2\pi} R^3 d\theta = 2\pi R^3$$

$$\textcircled{2} = 3R^2 a \int_0^{2\pi} \cos \theta d\theta = 0$$

$$\textcircled{3} = 3R a^2 \int_0^{2\pi} \cos^2 \theta d\theta = 3R a^2 \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{2\pi} = 3\pi R a^2$$

$$\textcircled{4} = a^3 \int_0^{2\pi} \cos^3 \theta d\theta = \frac{a^3}{3} [\sin \theta (\cos^2 \theta + 2)]_0^{2\pi} = 0$$

$$\text{So } I = 2\pi^2 \rho a R (2R^2 + 3a^2) \left(\frac{m}{4\pi^2 \rho a R} \right)$$

$$= \frac{1}{2} m (2R^2 + 3a^2)$$

B/45

$$dm = \rho dV = \rho t 2\pi x r d\theta = 2\pi \rho t r^2 \sin \theta d\theta$$

$$m = 2\pi \rho t r^2 \int_0^{\pi/2} \sin \theta d\theta = 2\pi \rho t r^2$$

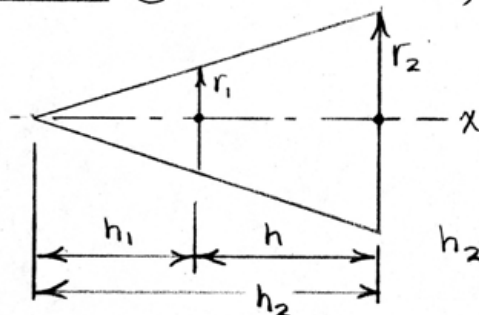
$$\begin{aligned} I_{zz} &= \int x^2 dm = 2\pi \rho t r^2 \int_0^{\pi/2} r^2 \sin^3 \theta d\theta \\ &= 2\pi \rho t r^4 \left[-\frac{\cos \theta}{3} (2 + \sin^2 \theta) \right]_0^{\pi/2} \\ &= \frac{4}{3} \pi \rho t r^4 = \frac{2}{3} m r^2 \end{aligned}$$



Also, $I_{xx} = I_{zz} = \frac{2}{3} m r^2$ since each is half that for whole shell of mass $2m$

B/46

② : Entire cone ; ① : "Missing" top



$$\frac{h_1}{r_1} = \frac{h_1 + h}{r_2}$$

$$h_1 = \frac{hr_1}{r_2 - r_1}$$

$$h_2 = h_1 + h = \frac{hr_1}{r_2 - r_1} + h$$

$$= \frac{hr_2}{r_2 - r_1}$$

$$I_2 = \frac{3}{10} m_2 r_2^2 = \frac{3}{10} \left(\rho \frac{1}{3} \pi r_2^2 \frac{hr_2}{r_2 - r_1} \right) r_2^2 \quad \left(\begin{array}{l} \text{See} \\ \text{Table D/4} \end{array} \right)$$

$$= \frac{1}{10} \rho \pi \frac{hr_2^5}{r_2 - r_1}$$

$$I_1 = \frac{3}{10} m_1 r_1^2 = \frac{3}{10} \left(\rho \frac{1}{3} \pi r_1^2 \frac{hr_1}{r_2 - r_1} \right) r_1^2$$

$$= \frac{1}{10} \rho \pi \frac{hr_1^5}{r_2 - r_1}$$

$$\text{Frustum mass } m = \rho \frac{1}{3} \pi \left[r_2^2 \frac{hr_2}{r_2 - r_1} - r_1^2 \frac{hr_1}{r_2 - r_1} \right]$$

$$= \frac{1}{3} \rho \pi h \frac{r_2^3 - r_1^3}{r_2 - r_1}$$

$$\text{So } I = I_2 - I_1 = \frac{1}{10} \rho \pi h \frac{r_2^5 - r_1^5}{r_2 - r_1} \left(\frac{m}{\frac{1}{3} \rho \pi h \frac{r_2^3 - r_1^3}{r_2 - r_1}} \right)$$

$$= \frac{3}{10} m \frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}$$

*B/47 | Let ρ = mass per unit area

$$\text{Panels: } \begin{cases} I_{xx} = 2 \left\{ \frac{1}{12} m (2r)^2 + m (2r)^2 \right\} = \frac{104}{3} \rho r^4 \\ I_{yy} = 2 \left\{ \frac{1}{12} m (2r)^2 \right\} = \frac{2}{3} m r^2 = \frac{8}{3} \rho r^4 \\ I_{zz} = 2 \left\{ \frac{1}{6} m (2r)^2 + m (2r)^2 \right\} = \frac{28}{3} m r^2 = \frac{112}{3} \rho r^4 \end{cases}$$

$(m = \rho (2r)^2 = \text{mass of each panel})$

Cylindrical shell (see Table D/4):

$$\begin{cases} I_{xx} = I_{yy} = \frac{1}{2} m r^2 + \frac{1}{12} m L^2 = \frac{m}{2} \left(r^2 + \frac{L^2}{6} \right) \\ I_{zz} = m r^2 = 2 \pi r^3 L \rho \end{cases} = \pi r L \rho \left(r^2 + \frac{L^2}{6} \right)$$

Complete model:

$$\begin{cases} I_{xx} = \frac{104}{3} \rho r^4 + \pi r L \rho \left(r^2 + \frac{L^2}{6} \right) \\ I_{yy} = \frac{8}{3} \rho r^4 + \pi r L \rho \left(r^2 + \frac{L^2}{6} \right) \\ I_{zz} = \frac{112}{3} \rho r^4 + 2 \pi r^3 L \rho \end{cases}$$

Since $I_{yy} < I_{xx}$, I_{zz} must be less than I_{yy} :

$$\left(\frac{112}{3} \rho r^4 + 2 \pi r^3 L \rho \right) < \left(\frac{8}{3} \rho r^4 + \pi r L \rho \left(r^2 + \frac{L^2}{6} \right) \right)$$

or $\frac{\pi}{6} \left(\frac{L}{r} \right)^3 - \pi \frac{L}{r} - \frac{104}{3} > 0$

Solve cubic for value of $\frac{L}{r}$ making left side zero to obtain $\frac{L}{r} = 4.54$

Inequality is satisfied if $L > 4.54r$

► B/48

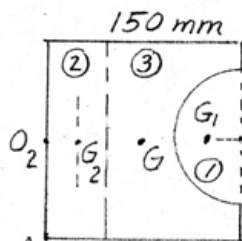
Groove ① $I_{aa} = \bar{I} + m(\bar{G}, \bar{A}^2)$

$$= I_{G_1} - m(\bar{G}, \bar{O}_1^2) + m(\bar{G}, \bar{A}^2)$$

$$= m\left(\frac{1}{2}r^2 - \bar{G}, \bar{O}_1^2 + \bar{G}, \bar{A}^2\right)$$

$$= (11370) \frac{\pi(0.05)^2(0.15)}{2} \left[\frac{0.05^2}{2} - 0.0212^2 + 0.0222 \right]$$

$$= 0.1541 \text{ kg} \cdot \text{m}^2 \text{ (negative)}$$



(a-a)

$$\bar{G}, \bar{O}_1 = \bar{G}, \bar{O}_2 = \frac{4(0.05)}{3\pi}$$

$$= 0.0212 \text{ m}$$

$$\bar{G}, \bar{A}^2 = (0.15 - 0.0212)^2 + (0.075)^2$$

$$= 0.0222 \text{ m}^2$$

Groove ② $I_{aa} = \frac{1}{2}$ that for complete cyl. by symmetry

So from Table D/A, $I_{aa} = \frac{1}{2} \frac{\pi m}{12} (3r^2 + 4l^2)$

$$= \frac{(11370) \pi (0.05)^2 (0.15)}{2} \left[\frac{3(0.05)^2}{12} + 4(0.15)^2 \right]$$

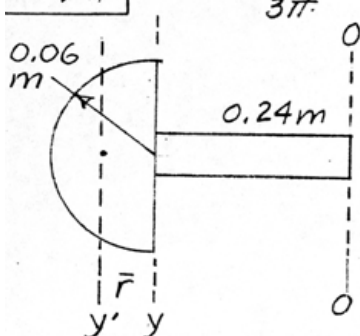
$$= 0.0544 \text{ kg} \cdot \text{m}^2 \text{ (negative)}$$

$$\textcircled{3} I_{aa} = \frac{1}{12} m(a^2 + a^2) + m\left(\frac{a^2}{4} + \frac{a^2}{4}\right) = \frac{2}{3} ma^2$$

$$= \frac{2}{3} (11370) (0.15)^3 (0.15)^2 = 0.576 \text{ kg} \cdot \text{m}^2$$

$$\text{Total } I_{aa} = 0.576 - 0.0544 - 0.1541 = \underline{0.367 \text{ kg} \cdot \text{m}^2}$$

► B/49



$$\bar{r} = \frac{4r}{3\pi} = \frac{4(0.06)}{3\pi} = 0.0255 \text{ m}$$

From Table D/4 for semicylinder

$$\begin{aligned} I_{yy} &= \frac{1}{4}mr^2 + \frac{1}{12}ml^2 \\ &= \frac{1}{4}(0.8)(0.06)^2 + \frac{1}{12}(0.8)(0.120)^2 \\ &= 7.20(10^{-4}) + 9.60(10^{-4}) \\ &= 16.80(10^{-4}) \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\begin{aligned} I_{y'y'} &= I_{yy} - m\bar{r}^2 \\ &= 16.80(10^{-4}) - 0.8(0.0255)^2 \\ &= 11.61(10^{-4}) \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\begin{aligned} I_{oo} &= I_{y'y'} + md^2 = 11.61(10^{-4}) + 0.8(0.24 + 0.0255)^2 \\ &= 0.0575 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

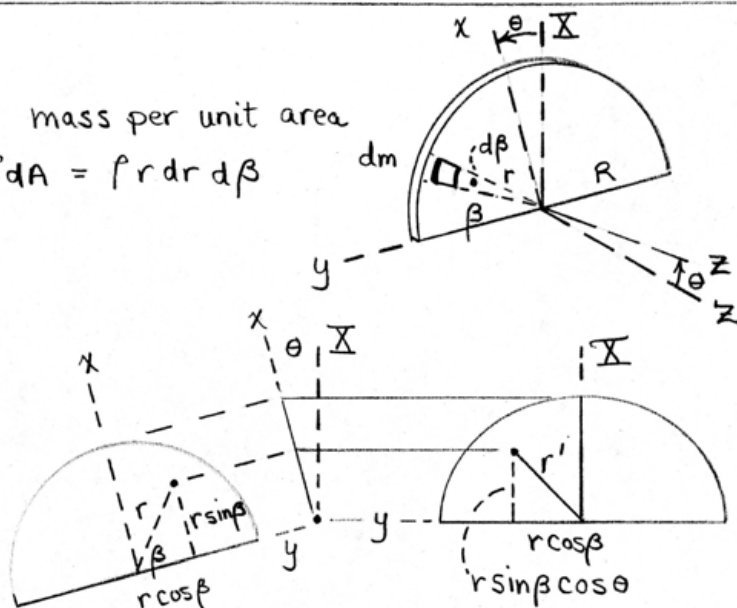
$$\text{Handle: } I_{oo} \approx \frac{1}{3}ml^2 = \frac{1}{3}(0.5)(0.24)^2 = 0.0096 \text{ kg} \cdot \text{m}^2$$

$$\text{Total } I_{oo} = 0.0575 + 0.0096 = \underline{0.0671 \text{ kg} \cdot \text{m}^2}$$

► B/50

Let ρ = mass per unit area

$$dm = \rho dA = \rho r dr d\beta$$



r' = distance from dm to Z -axis.

$$r'^2 = r^2 (\cos^2 \theta \sin^2 \beta + \cos^2 \beta)$$

$$dI_{zz} = r'^2 dm = \rho r^3 (\cos^2 \theta \sin^2 \beta + \cos^2 \beta) dr d\beta$$

$$\begin{aligned} I_{zz} &= \rho \frac{R^4}{4} \int_0^\pi (\cos^2 \theta \sin^2 \beta + \cos^2 \beta) d\beta \\ &= \rho \frac{R^4}{4} \left[\frac{\pi}{2} (1 + \cos^2 \theta) \right] \cdot \frac{m}{\rho \pi R^2 / 2} \\ &= \frac{1}{4} m R^2 (1 + \cos^2 \theta) \end{aligned}$$

B/51

$$I_{xy} = \underline{0}$$

$$I_{xz} = m(-l)(2l) = \underline{-2ml^2}$$

$$I_{yz} = m(l)(-l) + m(-l)(l) = \underline{-2ml^2}$$

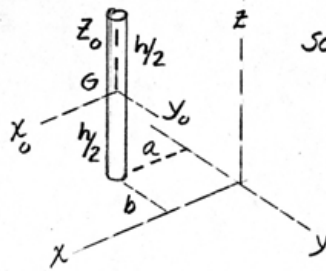
$$\boxed{B/52} \quad I_{xy} = m(\ell)(-\ell) + m(-\ell)(\ell) = \underline{-2m\ell^2}$$

$$I_{xz} = m(2\ell)(-\ell) + m(-2\ell)(\ell) = \underline{-4m\ell^2}$$

$$I_{yz} = \underline{0}$$

B/53

$$\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{xz} = 0$$

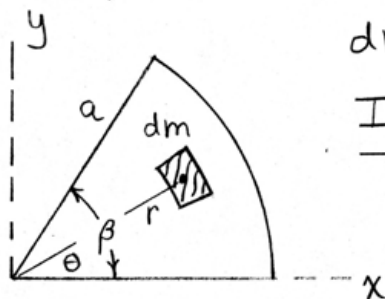


so $I_{xy} = 0 + m a (-b) = -mab$

$$I_{yz} = 0 + m (-b) \frac{h}{2} = -\frac{1}{2} m b h$$

$$I_{xz} = 0 + m a \frac{h}{2} = \frac{1}{2} m a h$$

B/54



$$dm = \rho t r dr d\theta$$

$$\underline{I_{xz} = I_{yz} = 0}$$

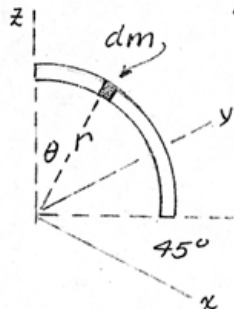
$$\begin{aligned}
 I_{xy} &= \int xy \, dm = \int_0^\beta \int_0^a (r \cos \theta)(r \sin \theta) \rho t r dr d\theta \\
 &= \rho t \int_0^\beta \int_0^a r^3 \cos \theta \sin \theta dr d\theta \\
 &= \rho t \int_0^\beta \frac{a^4}{4} \cos \theta \sin \theta d\theta \\
 &= \frac{\rho t a^4}{4} \frac{\sin^2 \beta}{2} = \frac{\rho t a^4 \sin^2 \beta}{8} \left(\frac{m}{\rho t \beta a^2 / 2} \right) \\
 &= \underline{\underline{\frac{m a^2 \sin^2 \beta}{4 \beta}}}
 \end{aligned}$$

B/55

$$I_{xy} = -\left(\frac{b}{4}\right)\left(\frac{b}{4}\right)\left(\rho\pi\left(\frac{b}{8}\right)^2\right) - \left(-\frac{b}{4}\right)\left(-\frac{b}{4}\right)\left(\rho\pi\left(\frac{b}{8}\right)^2\right) = -\frac{\rho\pi b^4}{512}$$

$$I_{xz} = I_{yz} = 0$$

B/56

Let ρ = mass per unit length of rod

$$dm = \rho r d\theta$$

$$I_{xy} = \int_0^{\pi/2} (r \sin \theta \cos 45^\circ)^2 \rho r d\theta$$

$$= \rho r^3 \frac{1}{2} \int_0^{\pi/2} \sin^2 \theta d\theta = \frac{1}{8} \pi \rho r^3$$

$$= \frac{1}{4} m r^2$$

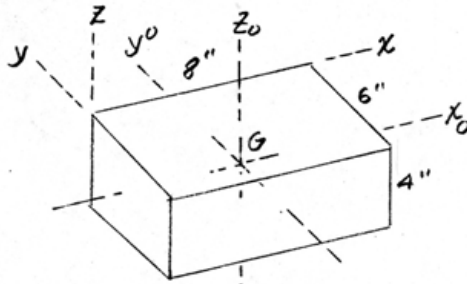
$$I_{xz} = I_{yz} = \int yz dm = \int (r \sin \theta \sin 45^\circ)(r \cos \theta) \rho r d\theta$$

$$= \rho r^3 \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sin \theta \cos \theta d\theta = \frac{\rho r^3}{\sqrt{2}} \left[\frac{-\cos 2\theta}{4} \right]_0^{\pi/2}$$

$$= \frac{1}{2\sqrt{2}} \rho r^3 = \frac{1}{\pi\sqrt{2}} m r^2$$

B/57

$$m = \frac{50}{32.2} \frac{1}{12} = 0.1294 \text{ lb-in}^{-1}\text{-sec}^2$$



$$I_{xy} = I_{x_0y_0} + m d_x d_y = 0 + 0.1294 (4)(-3) = \underline{-1.553 \text{ lb-in.-sec}^2}$$

$$I_{yz} = I_{y_0z_0} + m d_y d_z = 0 + 0.1294 (-3)(-2) = \underline{0.776 \text{ lb-in.-sec}^2}$$

$$I_{xz} = I_{x_0z_0} + m d_x d_z = 0 + 0.1294 (4)(-2) = \underline{-1.035 \text{ lb-in.-sec}^2}$$

B/58

Let $\rho = \text{mass/unit length}$

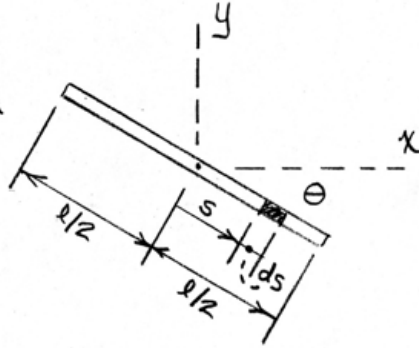
$$dm = \rho ds, \quad m = \rho l$$

$$dI_{xy} = xy dm$$

$$= (s \cos \theta)(-s \sin \theta) \rho ds$$

$$= -\frac{1}{2} \rho \sin 2\theta s^2 ds$$

$$I_{xy} = -\frac{1}{2} \rho \sin 2\theta \int_{-l/2}^{l/2} s^2 ds = -\frac{1}{24} m l^2 \sin 2\theta$$



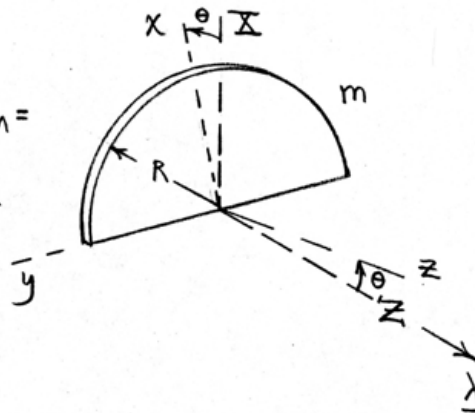
B/59

Use Eq. B/10: $I_{zz} = I_M =$

$$I_{xx} l^2 + I_{yy} m^2 + I_{zz} n^2$$

$$- 2I_{xy} lm - 2I_{xz} ln$$

$$- 2I_{yz} mn$$



Use the unit vector $\underline{\lambda} = l\underline{i} + m\underline{j} + n\underline{k}$

with $l = -\sin\theta$, $m = 0$, $n = \cos\theta$

$$I_{xx} = \frac{1}{4}mR^2, I_{yy} = \frac{1}{4}mR^2, I_{zz} = \frac{1}{2}mR^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

$$\text{So } I_{zz} = \frac{1}{4}mR^2(\sin^2\theta) + \frac{1}{4}mR^2(0) + \frac{1}{2}mR^2(\cos^2\theta)$$

$$= \frac{1}{4}mR^2(\sin^2\theta + 2\cos^2\theta)$$

$$= \underline{\underline{\frac{1}{4}mR^2(1 + \cos^2\theta)}}$$

B/60

Part 1: $m_1 = m/4$

$$I_{xy} = \frac{m}{4} \left(\frac{b}{2} \right) (-b) = -\frac{1}{8} m b^2$$

$$I_{yz} = 0, I_{xz} = 0$$

Part 2: $m_2 = m/2$

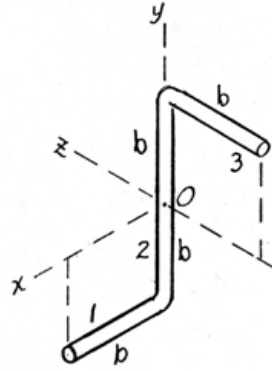
$$I_{xy} = 0, I_{yz} = 0, I_{xz} = 0$$

Part 3: $m_3 = m/4$

$$I_{xy} = 0, I_{xz} = 0$$

$$I_{yz} = \frac{m}{4} (b) \left(-\frac{b}{2} \right) = -\frac{1}{8} m b^2$$

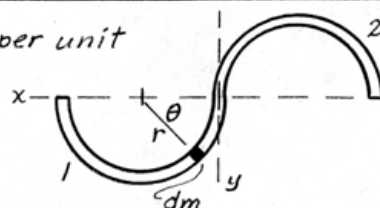
Combined: $\underline{I_{xy} = -\frac{1}{8} m b^2}, \underline{I_{xz} = 0}, \underline{I_{yz} = -\frac{1}{8} m b^2}$



B/61

$I_{xy_1} = I_{xy_2}$; Let $\rho =$ mass per unit length

$$m_1 = \pi r \rho, m = 2\pi r \rho$$

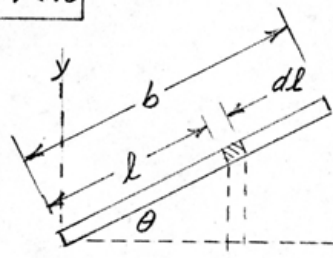


$$I_{xy_1} = \int xy dm = \int_0^\pi r(1 - \cos \theta)(r \sin \theta) \rho r d\theta$$

$$= \rho r^3 \int_0^\pi (\sin \theta - \sin \theta \cos \theta) d\theta = \rho r^3 \left[-\cos \theta - \frac{1}{2} \sin^2 \theta \right]_0^\pi = 2\rho r^3$$

$$I_{xy} = 2I_{xy_1} = 4\rho r^3 = \underline{2mr^2/\pi}, \quad \underline{I_{xz} = I_{yz} = 0}$$

B/62



$$I_{xy} = \int xy \, dm$$

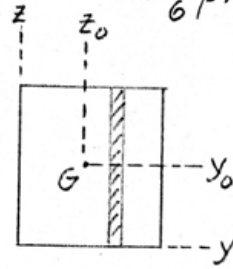
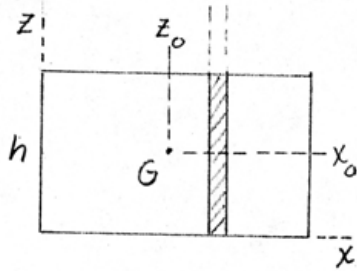
$dm = \rho h \, dl$ where ρ = mass per unit of plate area

$$x = l \cos \theta, \quad y = l \sin \theta$$

$$I_{xy} = \rho h \sin \theta \cos \theta \int_0^b l^2 \, dl$$

$$= \frac{1}{6} \rho h b^3 \sin 2\theta$$

$$= \frac{1}{6} m b^2 \sin 2\theta$$



$$I_{xz} = \bar{I}_{xz} + m d_x d_z = 0 + m \left(\frac{b}{2} \cos \theta \right) \left(\frac{h}{2} \right) = \frac{1}{4} m b h \cos \theta$$

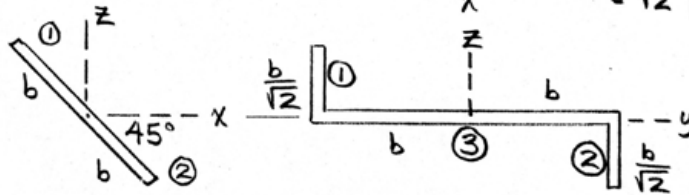
$$I_{yz} = \bar{I}_{yz} + m d_y d_z = 0 + m \left(\frac{h}{2} \right) \left(\frac{b}{2} \sin \theta \right) = \frac{1}{4} m b h \sin \theta$$

B/63

Part ①: $I_{xy} = \frac{m}{4}(-b)\left(-\frac{b}{2\sqrt{2}}\right) = \frac{mb^2}{8\sqrt{2}}$

②: $I_{xy} = \frac{m}{4}(b)\left(\frac{b}{2\sqrt{2}}\right) = \frac{mb^2}{8\sqrt{2}}$

③: $I_{xy} = 0$



For Parts ① & ② of combined mass $m/2$, it can be shown via integration that $I_{xz} = -\frac{1}{12}mb^2$.

Part ③: $I_{xz} = 0$

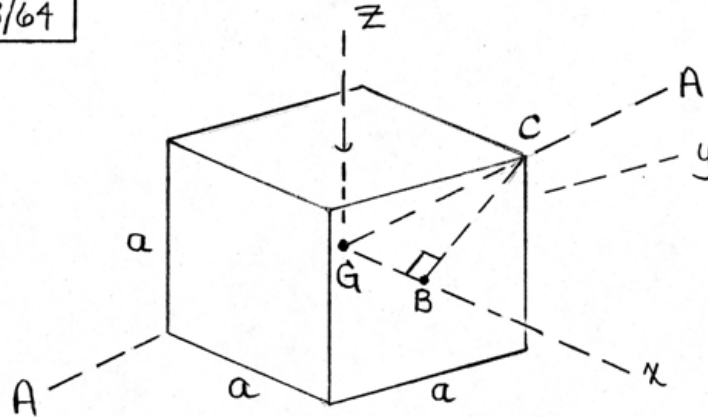
Part ①: $I_{yz} = \frac{m}{4}(-b)\left(\frac{b}{2\sqrt{2}}\right) = -\frac{mb^2}{8\sqrt{2}}$

②: $I_{yz} = \frac{m}{4}(b)\left(-\frac{b}{2\sqrt{2}}\right) = -\frac{mb^2}{8\sqrt{2}}$

③: $I_{yz} = 0$

Totals:
$$\begin{cases} I_{xy} = \frac{mb^2}{4\sqrt{2}} \\ I_{xz} = -\frac{1}{12}mb^2 \\ I_{yz} = -\frac{mb^2}{4\sqrt{2}} \end{cases}$$

B/64



Choose origin of x - y - z axes at center G

By symmetry, the direction cosines of AA

are $l = m = n = \cos \angle BGC$

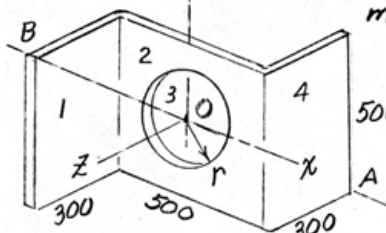
$$\overline{GB} = \frac{a}{2}, \overline{BC} = \frac{a\sqrt{2}}{2}, \overline{GC} = \frac{a}{2}\sqrt{3}, \text{ so } l = m = n = \frac{1}{\sqrt{3}}$$

$$I_{xx} = I_{yy} = I_{zz} = \frac{1}{6}ma^2, I_{xy} = I_{xz} = I_{yz} = 0$$

$$\begin{aligned} \text{From Eq. B/10: } I_{AA} &= 3\left(\frac{1}{6}ma^2\right)\left(\frac{1}{\sqrt{3}}\right)^2 \\ &= \underline{\underline{\frac{ma^2}{6}}} \end{aligned}$$

B/65 $\overline{OB} = \sqrt{(250)^2 + (250)^2 + (300)^2} = 464 \text{ mm}$
 $r = 150$ Direction cosines of OB are $l = -\frac{250}{464} = -0.539$
 $m = \frac{250}{464} = 0.539$, $n = \frac{300}{464} = 0.647$

Masses are
 $m_1 = m_4 = 7830(0.3)(0.5)(0.015) = 17.62 \text{ kg}$
 $m_3 = -7830(17)(0.150)^2(0.015) = -8.30 \text{ kg}$
 $m_2 = 7830(0.5)^2(0.015) = 29.36 \text{ kg}$



Dim. in mm

	1	2	3	4	Totals (kg·m ²)
I_{xx}	0.896	0.612	-0.047	0.896	2.356
I_{yy}	1.630	0.612	-0.047	1.630	3.825
I_{zz}	1.468	1.223	-0.093	1.468	4.067
I_{xz}	-0.661	0	0	-0.661	-1.321

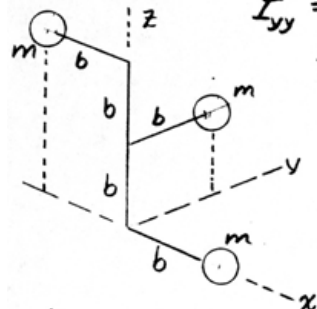
Substitute into Eq. B/10 & get
 $I_{AB} = 2.356(-0.539)^2 + 3.825(0.539)^2 + 4.067(0.647)^2$
 $- 2(-1.321)(-0.539)(0.647)$ where $I_{xy} = I_{yz} = 0$
 $I_{AB} = 2.58 \text{ kg·m}^2$

$$\boxed{B/66} \quad I_{xx} = I_{yy} = I_{zz} = 2\left(\frac{2}{5}mr^2 + mb^2\right) + \frac{2}{5}mr^2 \\ = m\left(\frac{6}{5}r^2 + 2b^2\right) = I$$

$$I_{xy} = I_{yz} = I_{xz} = 0$$

Thus for any axis OM through O , Eq. B/10 gives $I_M = I(l^2 + m^2 + n^2) = I$ independent of l, m, n .

*B/67



$$I_{xx} = m(2b^2) + m(2b)^2 = 6mb^2$$

$$I_{yy} = m(b^2 + b^2 + b^2 + (2b)^2) = 7mb^2$$

$$I_{zz} = m(b^2 + b^2 + b^2) = 3mb^2$$

$$I_{xy} = 0, I_{yz} = mb^2, I_{xz} = -2mb^2$$

Let $I_0 = I/mb^2$ so Eq. B/11 becomes

$$\begin{vmatrix} 6-I_0 & 0 & +2 \\ 0 & 7-I_0 & -1 \\ +2 & -1 & 3-I_0 \end{vmatrix} mb^2 = 0$$

Expansion gives $I_0^3 - 16I_0^2 + 86I_0 - 92 = 0$

Solution by computer or algebraic formula gives

$$I_1 = 7.53 mb^2$$

$$I_2 = 6.63 mb^2$$

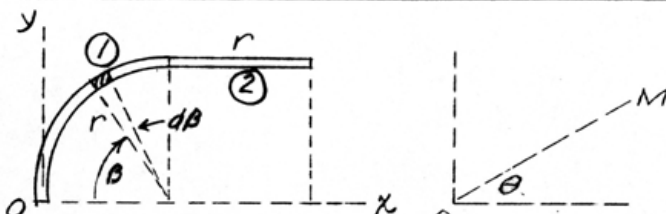
$$I_3 = 1.844 mb^2$$

For I_1 direction cosines substitute in Eq. B/12 & get

$$\begin{cases} (6-7.525)l_1 - 0 + 2n_1 = 0 \\ 0 + (7-7.525)m_1 - n_1 = 0 \\ 2l_1 - m_1 + (3-7.525)n_1 = 0 \end{cases} \quad \text{with } l_1^2 + m_1^2 + n_1^2 = 1$$

solve & get $l_1 = 0.521, m_1 = -0.756, n_1 = 0.397$

* B/68



$$\textcircled{1} I_x = \frac{1}{4} \frac{1}{2} (4mr^2) = \frac{1}{4} \rho \pi r^3, \quad I_y = \int_0^{\pi/2} [r(1-\cos\beta)]^2 \rho r d\beta = \rho r^3 \left(\frac{3\pi}{4} - 2 \right)$$

$$I_{xy} = \int_0^{\pi/2} r(1-\cos\beta) r \sin\beta \rho r d\beta = \rho r^3 \left(-\cos\beta - \frac{1}{2} \sin^2\beta \right)_0^{\pi/2} = \frac{1}{2} \rho r^3$$

$$\textcircled{2} I_x = \rho r^3, \quad I_y = \rho r \left(\frac{r^2}{12} + \left[\frac{3r}{2} \right]^2 \right) = \frac{7}{3} \rho r^3, \quad I_{xy} = \rho r \left(\frac{3r}{2} \right) r = \frac{3}{2} \rho r^3$$

$$\text{Totals, } I_x = \rho r^3 \left(\frac{\pi}{4} + 1 \right) = 1.785 \rho r^3, \quad I_y = \rho r^3 \left(\frac{3\pi}{4} - 2 + \frac{7}{3} \right) = 2.69 \rho r^3$$

$$I_{xy} = \rho r^3 \left(\frac{1}{2} + \frac{3}{2} \right) = 2 \rho r^3$$

$$\text{Eq. A/9, } I_{OM} = I = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

Set up computer program
& solve for $0 < \theta < 90^\circ$

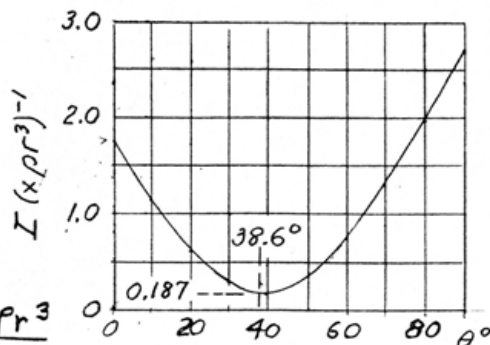
check: Eqs. A/10 & A/11

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4}{2.69 - 1.785}$$

$$2\alpha = 77.3^\circ, \quad \alpha = 38.6^\circ$$

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= (2.24 - 2.05) \rho r^3 = 0.1870 \rho r^3$$



$$\text{*B/69} \quad I_{xx} = m(\sqrt{2}l)^2 + m(\sqrt{2}l)^2 + m(2l)^2 \\ = 8ml^2$$

$$I_{yy} = ml^2 + ml^2 + m(\sqrt{5}l)^2 = 7ml^2$$

$$I_{zz} = ml^2 + ml^2 + ml^2 = 3ml^2$$

$$\text{Eq. B/11, with } I_0 = \frac{I}{ml^2} :$$

$$ml^2 \begin{vmatrix} (8-I_0) & 0 & +2 \\ 0 & (7-I_0) & +2 \\ +2 & +2 & (3-I_0) \end{vmatrix} = 0 \quad \left\{ \begin{array}{l} \text{Notes :} \\ I_{xy} = 0 \\ I_{xz} = I_{yz} = -2ml^2 \\ \text{from Prob. B/51} \end{array} \right.$$

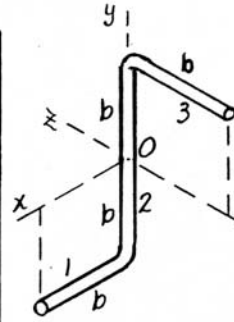
Numerical solution of cubic :

$$\underline{I_1 = 9ml^2}, \quad \underline{I_2 = 7.37ml^2}, \quad \underline{I_3 = 1.628ml^2}$$

For I_1 , solution of Eqs. B/12 along with

$$l_1^2 + m_1^2 + n_1^2 = 1 \quad \text{yields} \quad \begin{cases} l_1 = 0.816 \\ m_1 = 0.408 \\ n_1 = 0.408 \end{cases}$$

*B/70	Part			
	1	2	3	Total
Prob. $\left\{ \begin{array}{l} I_{xx} \\ I_{yy} \\ I_{zz} \end{array} \right.$	$\frac{1}{4}mb^2$	$\frac{1}{6}mb^2$	$\frac{1}{3}mb^2$	$\frac{3}{4}mb^2$
B/27 $\left\{ \begin{array}{l} I_{xy} \\ I_{yz} \end{array} \right.$	$-\frac{1}{8}mb^2$	0	0	$-\frac{1}{8}mb^2$
Prob. $\left\{ \begin{array}{l} I_{xx} \\ I_{yy} \\ I_{zz} \end{array} \right.$	$\frac{1}{4}mb^2$	$\frac{1}{6}mb^2$	$\frac{1}{3}mb^2$	$\frac{3}{4}mb^2$
B/60 $\left\{ \begin{array}{l} I_{xy} \\ I_{yz} \end{array} \right.$	$-\frac{1}{8}mb^2$	0	0	$-\frac{1}{8}mb^2$



Substitute in Eq. B/11 and let $I = I_0 mb^2$

$$\begin{vmatrix} \frac{3}{4} - I_0 & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{6} - I_0 & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{3}{4} - I_0 \end{vmatrix} = 0$$

Expand & get $I_0^3 - \frac{5}{3}I_0^2 + \frac{25}{32}I_0 - \frac{9}{128} = 0$

Solve by computer program or by algebraic formula. (In

this case expansion of the determinant yields a common factor $(\frac{3}{4} - I_0)$ so the cubic becomes

$$(\frac{3}{4} - I_0)(\frac{1}{6} - I_0)(\frac{3}{4} - I_0) - \frac{1}{32} = 0 \text{ or } (\frac{3}{4} - I_0)(I_0^2 - \frac{11}{12}I_0 + \frac{3}{32}) = 0$$

$$\begin{array}{ll} \text{so } I_0 = 0.750 & I_1 = 0.750 mb^2 \\ I_0' = 0.799 & \text{or } I_2 = 0.799 mb^2 \\ I_0'' = 0.1173 & I_3 = 0.1173 mb^2 \end{array}$$

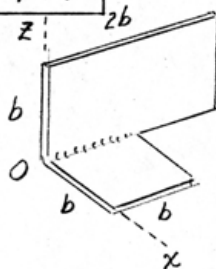
For $I_3 = 0.1173 mb^2$ (minimum moment of inertia) the direction cosines satisfy Eq. B/12 & $l^2 + m^2 + n^2 = 1$ so

$$\left. \begin{array}{l} 0.633l + 0.125m + 0 = 0 \\ 0.125l + 0.0494m + 0.125n = 0 \\ 0 + 0.125m + 0.633n = 0 \end{array} \right\} \text{ Sol. gives } l=m=n=0$$

and also

$$\begin{array}{l} l = 0.1903, \\ m = -0.963, \\ n = 0.1903 \end{array}$$

* B/71



$$I_{xx} = \frac{1}{3} \rho b^4 + \frac{1}{12} \rho (2b^2)(b^2 + 4b^2) + \rho (2b^2) \left(\frac{b^2}{4} + b^2 \right) = \frac{11}{3} \rho b^4$$

$$I_{yy} = \frac{1}{3} \rho b^4 + \frac{1}{3} \rho (2b^2) b^2 = \rho b^4$$

$$I_{zz} = \frac{1}{12} \rho b^2 (b^2 + b^2) + \rho b^2 \left(\frac{b^2}{4} + \frac{b^2}{4} \right) + \frac{1}{3} \rho (2b^2) (2b)^2 = \frac{10}{3} \rho b^4$$

$$I_{xy} = \rho b^2 \left(\frac{b}{2} \cdot \frac{b}{2} \right) = \frac{1}{4} \rho b^4, I_{xz} = 0, I_{yz} = \rho b^4$$

Substitute in Eq. B/11 letting $I = I_0 \rho b^4$ & get

$$\begin{vmatrix} \frac{11}{3} - I_0 & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 - I_0 & -1 \\ 0 & -1 & \frac{10}{3} - I_0 \end{vmatrix} = 0$$

Expand & get

$$I_0^3 - 8I_0^2 + 18.160I_0 - 8.347 = 0$$

Solve by computer program or algebraic formula & get

$$I_1 = 3.78 \rho b^4$$

$$I_2 = 0.612 \rho b^4$$

$$I_3 = 3.61 \rho b^4$$

*B/72

Part ①: $I_{xx} = 0$, $I_{yy} = I_{zz} = \frac{1}{3} \rho b^3$

$I_{xy} = I_{xz} = I_{yz} = 0$

Part ②: I_{xx} , I_{yy} , & I_{zz}

by symmetry are one-half those of semicircular ring.

$I_o = I_G + m(b-\bar{r})^2$

$= I_C - m\bar{r}^2 + m(b-\bar{r})^2$

$= mb^2 + m(b^2 - 2b\bar{r})$

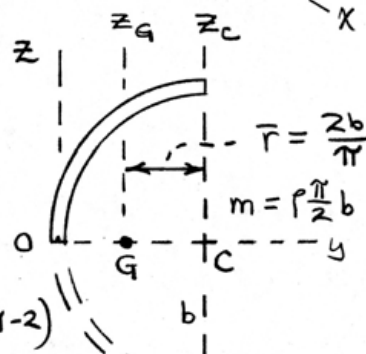
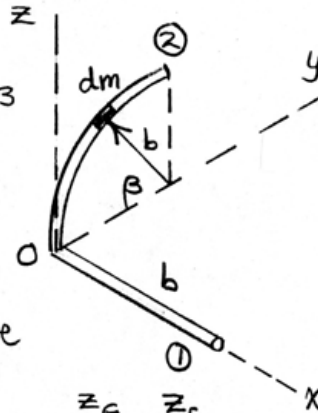
$= \rho \frac{\pi}{2} b^3 \left[1 + 1 - \frac{4}{\pi} \right] = \rho b^3 (\pi - 2)$

$I_{yy} = \frac{1}{2} mb^2 = \frac{1}{4} \rho \pi b^3$, $I_{zz} + I_{yy} = I_o$, so

$I_{zz} = \rho b^3 \left(\frac{3\pi}{4} - 2 \right)$

$I_{xy} = 0$, $I_{xz} = 0$, $I_{yz} = \int yz dm$

$I_{yz} = \int_0^{\pi/2} (b - b \cos \beta)(b \sin \beta) \rho b d\beta = \frac{1}{2} \rho b^3$



For complete rod :

$$I_{xx} = pb^3(\pi - 2) = 1.1416 pb^3, \quad I_{xy} = 0$$

$$I_{yy} = pb^3\left(\frac{\pi}{4} + \frac{1}{3}\right) = 1.1187 pb^3, \quad I_{xz} = 0$$

$$I_{zz} = pb^3\left(\frac{3\pi}{4} - \frac{5}{3}\right) = 0.6895 pb^3, \quad I_{yz} = \frac{1}{2} pb^3$$

Substitute in Eq. B/11 & obtain for

$$\frac{I}{pb^3} = I' :$$

$$\begin{vmatrix} (1.1416 - I') & -0 & -0 \\ -0 & (1.1187 - I') & -0.5 \\ -0 & -0.5 & (0.6895 - I') \end{vmatrix} = 0$$

$$\text{Expand: } I'^3 - 2.950 I'^2 + 2.586 I' - 0.5952 = 0$$

Numerical solution (or by cubic formula)

$$I_1 = \underline{1.448 pb^3}, \quad I_2 = \underline{0.360 pb^3}, \quad I_3 = \underline{1.142 pb^3}$$

From Eq. B/12, the direction cosines for I_2 -axis are

$$(1.1416 - 0.360)l - (0)m - (0)n = 0 \quad (1)$$

$$- (0)l + (1.1187 - 0.360)m - 0.5n = 0 \quad (2)$$

$$- 0(l) - 0.5m + (0.6895 - 0.360)n = 0 \quad (3)$$

$$\text{Solution: } \underline{l = 0}, \quad \underline{m = 0.5503}, \quad \underline{n = 0.8350}$$